UPKAR'S

COMMON ADMISSION TEST

CAT

COMPLETE COURSE

3C

- All You Want to Know About CAT
- This Book Before CAT – Why ?
- Just in Three Steps You Can Crack CAT
- F A Q on CAT
- How to Use PAT Technique to Crack CAT ?
- Examination Tips and 3000 Questions

Dharmendra Mittal

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By

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Dean, International Institution of Management

Delhi

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This Book is Dedicated
To my Lovely &
Supportive Family.

☯ ♦ ☯

J.D.K. Mittal, Raj Bala, Sharvan, Vinay,
Renu, Kavita, Rakhee
Vidushi, Mittali, Sonali, Nikunj, Vansh
Jai Raj Mittal.

Basic Introduction

- Just in three steps you can crack CAT
- F A Q on CAT
- *THIS BOOK BEFORE CAT – Why ?*
- How to use PAT Technique To crack CAT ?
- Examination Tips
- More than 3000 questions
- All you want to know about CAT
- 2 Mock Test on CAT patterns

Dear students,

You are not just appearing for CAT the ultimate destination of tomorrow's leaders in the field of management. You are about to embark on a journey to cross formidable frontier. The unpredictable and seemingly unbreakable Common Admission Test (CAT).

Thousands of students are plagued by the haunting question. "How do I tackle CAT?" Which is notorious for being unpredictable and of a very, very high standard my advice to you is 'first relax' CAT is just another tough exam. To surf through it all you need is to raise the bar of preparation and execute a few well planned intelligent strategies. You will need the help of PAT to crack CAT.

PAT is the first teaching technique of it kind on CAT. It will help the students become adept and adroit at CAT level. Further, plan early with skilled guidance to provide the right approach to meet CAT head on. Equip yourself with the right skills and prowess to excel in your mission of cracking CAT. And I'm sure the results will be outstanding and the butterflies in your stomach will be on vocation.

Here are some tips to help you overcome CAT anxiety and gain confidence.

F A Q ?

Q. How many hours of regular study is necessary in the preparation of CAT ?

Ans. After 2-3 hrs of coaching, 4-5 hrs of self study is necessary. Student should go for the balance study of all portion(subjects). More time should be devoted to a weak subject. Students can take help of their teacher in setting a proper daily schedule.

Q. What sort of Study material and Book should a student use in the preparation ?

Ans. Generally, recommended book of previously selected students and teachers are helpful. Apart from this, coaching institutes provide some sort of study material etc. Generally good coaching institute provide a book list also.

Q. What is the right time to start the preparing for CAT ?

Ans. When a student starts his/her graduation.

THIS BOOK BEFORE CAT – Why ?

If you are planning to take the CAT (Common Admission Test), this book will be indispensable for a higher score.

You are well aware that the CAT is one of the most important examinations that you will ever take. Your entire future may well depend on your performance on the CAT. The results of this test will determine, in great measure, whether you will be admitted to the Institute of your choice. There will be many candidates taking the CAT and not all will score well enough to be accepted by the Institute they choose.

This book is designed to guide you in your study so that you will score high on the CAT. This claim that this book will help you to achieve a higher ranking has both educational and psychological validity, for these reasons—

1. **You will know what to study :** A candidate will do better on a test if he or she knows what to study. The questions in this book will show you what is required and therefore help you get the most benefit from your study time.
2. **You will spot your weaknesses :** Using this book, you will discover where your weaknesses lie. This self-diagnosis will provide you with guidelines for spending your time where it will do the most good.
3. **Exam before Exam :** You will get the 'feel' of the exam. It is important to get the 'feel' of the entire examination. Gestalt (meaning configuration or pattern) psychology stresses that true learning results in a grasp of the entire situation. Gestaltists tell us that we learn by 'insight'. One of the salient principles of this kind of learning is that we succeed in 'seeing through' a problem as a consequence of experiencing previous similar situations. This book contains many 'similar situations' as you will discover when you take the actual examination.
4. **You will gain confidence :** While preparing for the exam you will build up confidence, and you will retain this confidence when you enter the examination hall. This feeling of confidence will be a natural consequence of getting the 'feel' of the exam.
5. **You will add to your knowledge :** In going over the practice questions in this book, you will not if you use this book properly be satisfied merely with the answer to a particular question. You will want to do additional research on the other choices for the same question. In this way, you will broaden your; background to be prepared adequately for the exam to come, since it is quite possible that a question on the exam which you are going to take may require your knowing the meaning of one of these other choices.

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How to STUDY the whole YEAR ?

"Direction is more important than speed"

Hard work is the key to success and there is no alternative to it. You should pay more attention towards study round the year. Do not waste time on gossiping with unreliable sources. Main Aim of your study is to achieve success in life, therefore, one should not leave any stone unturned that comes in your way to gain more knowledge and confidence.

You are required to study round the year with dedication and devotion. You must be serious in studies in the class as well as at home.

Make sure of the following :

(because Revolutions are not made, they come.)

- Make a timetable for daily routine.
- Get up early in the morning and go for a morning walk and read the newspaper.
- Must be attentive in the class.
- Listen to the teachers carefully.
- Make a note of all points discussed in your notebook.
- Put as many questions to the teacher again and again if you cannot follow the method or are not clear about the concept.
- Note down the points discussed by the teacher in the notebook, use pens of many colours, which will indicate unclear concepts, revision required, important concept etc.
- Practice at home and revise all the problems in detail and make short notes for further study.
- Even if there are still some points you could not follow, you must put questions to the teacher the next day.
- Provide time to each subject daily.
- Give extra time to the subject in which you are weak.
- You may request the teacher to give extra time for you. He will surely help if you are truthful and honest.
- Discuss your weak subjects with your friends who are good in that subject.
- Enjoy study in the class and do not feel nervous. ***Rich men have no faults.***
- Must help your family members as and when you find free time. This will give you satisfaction. This will also enhance your confidence because human being is a social animal.
- One who has worked hard round the year keeping all the suggestions given above in mind is very well on the right track.
- Hopefully, at this stage you will find yourself more confident about the problems discussed in the class. As such you will find yourself at a stage to face any challenge boldly posed by the so called examination.
- Read Books beside your syllabus like—Alchemist, Made in China, Think Big, One Minute Manager, Best quotations of each day, You can win, Puzzle A.

Remember—"Books are for nothing but to inspire".

How to Take the CAT ?

TEST-TAKING STRATEGY

- Approach the test confidently. Take it calmly.
- Arrive well before time. No one will be admitted once the test has begun.
- Bring everything you will need including :

  Your admit card or Roll number card.

  At least two sharpened medium-soft pencils and an eraser.

  A watch, if possible, to keep track of the time.
- Listen carefully to all directions. If you have any questions, ask them now, before the test begins.
- Read all directions carefully-twice if necessary.
- Note the time allotted for each section and budget your time intelligently.
- Work quickly but carefully.
- Do all the questions that are easy for you first. Then go back and do the more difficult questions.
- Read each question carefully. Make sure that you understand the question before you answer.
- Stay calm and confident throughout the test. Don't let anything upset you.
- Use all your time for each section. The supervisor will tell you when to start and when to stop.

Time : A time limit is set for each section on the examination. Therefore, follow the time instructions carefully. This book tells you how much time is available for each section of the test. You should calculate in advance about how much time you can afford to spend on each question. Your watch can help you here. Even if you haven't finished a section when the time limit is up, you must pass on to the next section.

Pay Close Attention : Be sure you understand what you're doing at all times. It is natural to feel stress when taking an examination, and it is easy to skip a word or jump to a false conclusion, which may cost you points off your score. Examiners sometimes deliberately insert plausible-appearing false answers, in order to each the candidate who is not alert.

What To Do Before The Test ?

WHEN THE COUNTDOWN BEGINS

The road to **IIMs-CAT** is no easy ride. There are forms to fill out, last minute updates, time schedules, yes the formidable entrance exam itself before you win a ticket and jump onto the bandwagon.

DOING THE TEST

1. **BE A WISE TEST-TAKER –** A CAT exam is standardised. For every question the points come only if they are answered correctly regardless of their being particularly difficult. So, there is no point plodding through a 'hard' question to prove your ability, it will fetch the same marks as a soft one.

2. **DO NOT ANSWER QUESTIONS IN A RIGID ORDER –** Use PAT Technique.
3. **THINK BEFORE ANSWERING –** Paper setters love to confuse they provide answers and alternatives that look right but are not. If you do not think before answering, for sure you will fall into an open trap. Beware!
4. **FAMILIARITY AIDS SCORING –** The best thing a student can do is to get familiar with questions and practise papers. It makes you go through the sections faster knowing what to expect. Do CAT Mock .
5. **MEMORIZE THE DIRECTIONS FOR EACH TYPE OF QUESTIONS –** Directions do not change, once familiar with what you are expected to do, you gain time.

WHAT TO PACK

- Atleast 2 HB pencils, pen, rubber.
- Hall ticket/Admission ticket.
- A watch, which is accurate, has served you well or just plain lucky for you.

A Day Before Exam
"You cannot teach an old dog new tricks"

You might have covered the whole syllabus by now. It is enough to have studied the whole year leaving no stone unturned. Yes, you can go through the notes to recall your thoughts and refresh your memory. You should enjoy with family members and watch T.V. Serials to remove mental stress. This will give you more energy and enhance your confidence. Keep the stationery items like pen, HB pencil, rubber, sharpener, scale and wrist watch ready for the exam. Get your conveyance ready and fit in all respect. Do not keep any thing in mind that can put a unwarranted stress and bad impression on you. You should go to the bed as early as possible and have a sound sleep at night.

On The Exam Day

After a very light, leisurely meal, get to the examination room ahead of time, perhaps ten minutes early. The reason for coming early is to help you get accustomed to the room. It will help you to a better start.

***Bring All Necessary Equipment* :** Three or four sharpened pencils, watch and eraser are needed. No pencils or erasers will be provided at the test centre. You may not take any books, dictionaries, notes, etc., into the examination room. Scores of those individuals taking part in any form of cheating will be cancelled.

***Get Settled* :** Find your seat and stay in it. The test paper will be given by a test supervisor who reads the directions and otherwise tells you what to do. The people who walk about passing out the test papers and assisting with the examination are test proctors. If you're not able to see or hear properly, notify the supervisor or a proctor. If you have any other difficulties during the examination, like, if it's too hot or cold or dark or drafty, let them know. You're entitled to favourable test conditions, and if you don't have them you won't be able to do your best. Don't be a crank, but don't be shy either. An important function of the proctor is to see to it that you have favourable test conditions.

***Relax* :** Don't bring on unnecessary tenseness by worrying about the difficulty of the examination. If necessary, wait a minute before beginning to write. If you're still tense, take a couple of deep breaths, look over your test equipment.

Review

Early Rising	–	Get up early in the morning check the headlines of T.V. channel.
Exercise	–	Go for a short morning walk or do yoga.
Regards	–	Wish everybody you meet in the way and feel more energetic than ever before and create a joyful atmosphere around you.
Prayer	–	Pay your prayer to God.
Breakfast	–	Have a light breakfast.
Wearing	–	Wear comfortable clothes and shoes of your choice.
Conveyance	–	Ensure that your conveyance is ready and fit in all respect.
Admit Card	–	Collect your Admit Card.
Stationery	–	Collect your stationery items and wrist watch.
Well wishes	–	Take the well wishes of elderly people and your parents so that you may feel heavenly and blessed.
Punctual	–	Reach at the exam centre at least 15 minutes before it starts.
Confidence	–	Walk confidently to the information board to know the room and seat number. Go to the exam room as and when you are allowed to go. Take your seat, organise yourself, feel comfortable and get ready to take the exam. By that time you will observe that the Invigilator is distributing the exam paper.
Filling of entries	–	Fill up all the entries as directed by the invigilator at the appropriate space.

D. Mittal

"The Smile of God is Victory
BEST OF LUCK"

How To Crack CAT in the first attempt ?

Tips – To Crack CAT in First Attempt

An important goal of your test preparation is to help you to give the best possible account of yourself by effectively using your knowledge to answer the examination questions.

First, get rid of any negative attitudes toward the test. Your attitude is negative if you view the test as a device to 'trip you up' rather than an opportunity to show how effectively you have learned.

***Approach the Test with Self-Confidence* :** Working through this book is a difficult job, and after you've done it you will probably be better prepared than 90 per cent of the people taking the CAT. Self-confidence is one of the biggest strategic assets you can bring to the testing room.

Nobody likes tests, but some people actually permit themselves to get upset or angry when they see what they think is an unfair test. This can only hurt your score. Keep calm and move right ahead. After all, everyone is taking the same test. Anger, resentment, and fear all slow you down and impair your judgment.

Besides, every test you take, including this one, is a valuable experience that improves your skill. Since, you will undoubtedly be taking other tests in the years to come, it may help you to regard the CAT as training to perfect your skill.

Keep calm; there's no point in panicing. If you've done your work, there's no need for it; if you haven't, a cool head is your very first requirement.

At the very least, this book should remove some of the fear and mystery that surrounds examinations. A certain amount of concern is normal and good, but excessive worry saps your strength and keenness. In other words, be prepared *emotionally*.

Pre Test Review

If you know any others who are taking this test, you'll probably find it helpful to review the book and your notes with them. The group should be small, certainly not more than twelve. Team study at this stage should seek to review the material in a different way from the way you learned it originally; strive for an exchange of ideas. Be selective in sticking to important ideas, and stress the vague and the unfamiliar rather then that which you all know well. End sessions as soon as you get tired.

One of the worst strategies in test-taking is to try to do all your preparation the night before the exam. Cramming is a very good way to guarantee poor test results. Schedule your study properly so as not to suffer from the fatigue and emotional disturbance that come from cramming the night before.

However, you would be wise to review your notes in the 48 hours preceding the exam. You shouldn't have to spend more than two or three hours in this way. Stick to salient points. The other will fall into place quickly.

Don't confuse cramming with a final, calm review that helps you focus on the significant areas of this book and further strengthens your confidence in your ability to handle the test questions. In other words, prepare yourself *factually*.

Keep Fit. Mind and body work together. Poor physical condition will lower your mental efficiency. In preparing for an examination, observe the common sense rules of health. Get sufficient sleep and rest, eat proper foods, plan recreation and exercise. In relation to health and examinations, two cautions are in order. Don't miss your meals prior to an examination in order to get extra time for study. Likewise, don't miss your

regular sleep by sitting up late to 'cram' for the examination. Cramming is an attempt to learn in a very short period of time what should have been learned through regular and consistent study. Not only are these two habits detrimental to health, but seldom do they pay off in terms of effective learning. It is likely that you will be more confused rather than better prepared on the day of the examination if you have broken into your daily routine by missing your meals or sleep.

On the night before the examination, go to bed at your regular time and try to get a good night's sleep. Don't go to the movies. Don't date. In other words, prepare yourself *physically*.

Review

- **AIM**

  It is the foremost requirement of a student to decide and choose his/her Aim before he/she is going to start preparation. You must be sincere and honest enough in assessing your suitability for the Aim.

  "I awoke one morning and found myself famous"

- **GET-SET**

  For success in achieving your aim, it is required to start preparation right from Graduation Ist year with dedication and devotion sincerely.

- **PLANNING**

  For a college going student it may be little difficult to go for academic as well as competitive studies together. If you plan and make a timetable it will be an easy task for you.

- **PUNCTUALITY**

  "Be not slow to visit the sick"

  It is very much important to be punctual while preparing for exam, else you may find that some part of your syllabus is not covered and may cause lack of confidence.

- **IDENTIFY YOUR WEAK TOPICS**

  You must analyse in which AREAS you are weak and needs to put in more efforts to improve.

- **HONEST APPROACH**

  "All or Nothing"

  You must solve the problems honestly at your own. Bring your answer to the question first and then compare with the given Answer Sheet. Do not depend on readymade solutions only.

- **CHOOSING QUALITY MATERIAL AND GUIDANCE**

  "The Tree is known by its fruit"

  You must consult good quality reading material/books. Consulting poor quality books may result in danger or may create confusion in your mind. To get confidence, better go for choosing good quality material, guidance and depend on self study only.

- **SERIOUS APPROACH**

  "We must learn to walk before we can run"

  You must take your study as a entertaining instrument. You must enjoy your study. This will create a conducive atmosphere for study and avoid unwarranted tension.

- **MOCK TESTS**

  You must undergo frequent MOCK TESTS within time frame to know your position about the examination standard you are appearing.

- **CONSULTATION**

  In case you find yourself stuck in solving problems, you must consult your friends and teachers to save time or mail me. My email:- **drdimperfection@gmail.com.**

P A T Teaching for CAT

CAT is not just a competition, it is a battle ……..where only the fittest wins. Yes, CAT is dream destination of future managers of India. Thousand of students just ask one question, "I am studying well but how do I tackle the Common Admission Test (CAT) of IIM's which is so well-known to be an unpredictable exam"?

But now, panacea is in front of you in the form of PAT Technique CAT is just another exam. To do well in it, all one needs is a high level of preparation and a few intelligently planned strategies. PAT Teaching is the first Technique of its kind on CAT which is wholly dedicated to meeting the needs of the CAT (MBA) aspirants. There are no shortcuts to success. Therefore, I believe in beginning early to plan, guide and provide direction to the career path of all our students. So, that they may acquire the necessary skills and vision to excel in the preparation of their destination.

Flow chart of PAT Technique :-

PAT = PASS – AVERAGE – TOP

Aim : How to leave 35% of Question in minimum time that a student (you) can get maximum time for 65% of Questions.

Examination Tips : Today many competition exam consist of objective type questions. All questions of multiple-choice type and there are four or five choices per question. Perhaps you score four marks for every correct answer and deduct one mark for every incorrect answer. Here we emphasis on strategy to enhance your score in these tests. Although, PAT is best.

Sound knowledge and deaf Fundamentals : In these types of exams you must have every strong concepts because every objective question is based on an particular concepts. Last moment cramming may be dangerous for you, so begin with good general textbooks. Concurrently you must take practice tests and use the results to guide your study.

Conscious guessing : In most situations you can resort to guessing where unsure. If you were to simply put a random answer, you would get approximately 25 percentage right and 75 percentage wrong , that is three answer for every that is three wrong answer for every right answer. Since, a right answer is worth four marks as a wrong answer deducts one mark each from your score, complete random guessing would make no difference in your score. Since, the odds are that you know something about some aspect about a question, or about one of the choices, talk a guess on questions of which you are completely sure, and you can eliminate one or two choices as obviously wrong answer.

Time To Jump : Do not waste time on difficult questions. As PAT Technique also suggests. Skip them and mark your examination book to show that you have done so, and proceed to an easier one. Once you have finished return and try to attempt the marked questions. Do not rush. Thoroughly re-check.

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Part A

BASIC TOPICS OF MBA MATHEMATICS

1 Basic Mathematical Operation

1. Polynomial : Algebraic expressions in which the variables involved with only non-negative integral exponents are called polynomials.

2. Terms of a polynomial and their coefficients : If $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n$ is polynomial in variable x, then $a_0x^n, a_1x^{n-1}, \ldots\ldots, a_{n-1}x, a_n$ are known as the terms of polynomial $p(x)$ and $a_0, a_1, a_2, \ldots\ldots a_n$ are known as their coefficients.

3. Degree of a polynomial : Highest power of the variable in a polynomial is called the degree of polynomial.

4. Constant polynomial : A polynomial of degree zero is called a constant polynomial.

5. Zero polynomial : The constant polynomial 0 is called zero polynomial. Degree of zero polynomial is not defined.

6. Monomials : Polynomials having one term are known as monomials.

7. Binomials : polynomials having two terms only are known as binomials.

8. Trinomials : Polynomials having three terms only are known as trinomials.

9. Linear polynomials : A polynomial of degree 1 is called linear polynomial, *e.g.*, $ax + b$.

10. Quadratic polynomial : A polynomial of degree 2 is called quadratic polynomial or in other words, any polynomial of the form $ax + bx^2 + cx + d$, where a, b and c are real numbers and $a \neq 0$, is called quadratic polynomial.

11. Cubic polynomial : A polynomial of degree 3 is called cubic polynomial. In other words, a polynomial of the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are real numbers and $a \neq 0$, is called cubic polynomial.

12.The value of polynomial $p(x)$ at k : If $p(x)$ is a polynomial in x and if k is any real constant, then real value (number) obtained by replacing x by k in $p(x)$, is called the value of $p(x)$ at k and is represented by $p(k)$.

13. Zero of a polynomial : A real number k is called a zero of polynomial $p(x)$ if $p(k) = 0$.

14. Geometrical meaning of zeros of a polynomial : To draw the graph of a polynomial $p(x)$, consider $y = p(x)$ and substitute values of x to find out corresponding y to get (x,u) *i.e.*, points on the graph.

Zero of a polynomial is precisely the x-co-ordinate of the point where the graph intersects x-axis.

(i) A linear polynomial has exactly one zero.

(ii) A quadratic polynomial has at most two zeros.

(iii) A cubic polynomial has at most three zeros.

(iv) A polynomial $p(x)$ of degree n has at most zeros *i.e.*, it can cross x-axis n times.

(v) The graph of a quadratic polynomial $(ax^2 + bx + c)$ is $\cup$- shaped called as parabola.

If $a > 0$ in $ax^2\ bx + c$, the shape of parabola is $\cup$ (opening upward).

If $a < 0$ in $ax^2 + bx + c$, the shape of parabola is $\cap$ (opening downward).

15. Relationship between the zero and the coefficient of a polynomial : If α and β are the zeros of a quadratic polynomial $p(x) = ax^2 + bx + c$; $a \neq 0$, then

$$\alpha + \beta = \frac{-b}{a} \quad \text{i.e., the sum of zeroes}$$

$$= \frac{-\text{(Coefficient of } x)}{\text{Coefficient of } x^2}$$

and $$\alpha\beta = \frac{c}{a} \quad \text{i.e., the product of zeroes}$$

$$= \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

If α, β, γ are the zeroes of cubic polynomial $ax^3 + bx^2 + cx + d, a \neq 0$.

Then, $$\alpha + \beta + \gamma = \frac{-b}{a},$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}, \alpha\beta\gamma = \frac{-d}{a}$$

16. Division algorithm : From Euclid division Algorithm we have

Divided = Divisor × Quotient + Remainder

Quotient is always be an integer.

Remainder is always be either equal or greater than zero or less than the Divisor.

Using this concept in polynomials, we can say that for any given polynomial $f(x)$ and $g(x) \neq 0$; there exist unique polynomials, $q(x)$ and $r(x)$ satisfying

$$f(x) = g(x) \times q(x) + r(x)$$

$$\Rightarrow \quad \frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$$

where $r(x) = 0$ or degree of $r(x)$ will have smaller than $g(x)$ this result is known as "Division Algorithm" for polynomials.

17. Synthetic division : Synthetic division is a method of performing polynomial long division without having to maintain long record of the process of long division. However, it only deals with division by monic linear polynomials, that is, binomials of the form $(x - b)$. Change the sign of b when divided by $(x + b)$ *i.e.*, $- b$.

18. Rational Expressions : If $p(x)$ and $q(x)$ where $q(x) \neq 0$ are two polynomials over integers, then the rational expression is said to be $\frac{p(x)}{q(x)}$.

19. Reduction of rational expression to lowest terms using factorization : A rational expression is said to be in its lowest if the H.C.F. of its numerator and denominator is 1.

In other words, we can say that a rational expression is in its lowest terms if there is no common factor in the numerator and denominator of the rational expression.

If $p(x)$ and $q(x)$ where $q(x) \neq 0$ are two polynomials over integers then the rational expression $\frac{p(x)}{q(x)}$ is said to be in its lowest terms, if the H.C.F of $p(x)$ and $q(x)$ is 1.

For example, the rational expression $\frac{3x+5}{5x+7}$ is in its lowest terms since the H.C.F. of $(3x + 5)$ and $(5x + 7)$ is 1, where as the rational expression $\frac{2(x-1)^2 (2x+5)(x+2)^3}{4(x-1)(3x+7)(x+2)^2}$ is not in its lowest terms because here the H.C.F. of numerator and denominator is $2(x - 1)(x + 2)^2$ which is not 1.

Illustration 1. Examine if the rational expression $\frac{(x-6)(x+1)}{(x+2)(x+1)}$ is in lowest terms. If not express it in lowest terms.

Solution :

Here, $\quad p(x) = (x - 6)(x + 1)$

$\quad q(x) = (x + 2)(x + 1)$

H.C.F. of $p(x)$ and $q(x) = x + 1$

$\therefore$ H.C.F. of $p(x)$ and $q(x)$ is not 1.

$\therefore$ The given rational expression is not in lowest terms.

Now, $\quad \frac{p(x)}{q(x)} = \frac{(x-6)(x+1)}{(x+2)(x+1)} = \frac{x-6}{x+2}$

Illustration 2. Reduce the following rational expression into lowest form :

$$\frac{x^4 - 10x^2 + 9}{x^3 - 4x^2 + 3x}.$$

Solution :

$$\begin{aligned}
\frac{x^4 - 10x^2 + 9}{x^3 - 4x^2 + 3x} &= \frac{x^4 - 9x^2 - x^2 + 9}{x(x^2 - 4x + 3)} \\
&= \frac{x^2(x^2 - 9) - 1(x^2 - 9)}{x(x^2 - x - 3x + 3)} \\
&= \frac{(x^2 - 9)(x^2 - 1)}{x[x(x-1) - 3(x-1)]} \\
&= \frac{(x+3)(x-3)(x+1)(x-1)}{x(x-1)(x-3)} \\
&= \frac{(x+3)(x+1)}{x}
\end{aligned}$$

Illustration 3. Express $\frac{x+y}{x-y}$ to a rational expression whose

(i) numerator is $x^3 + y^3$,

(ii) denominator is $x^2 - y^2$.

Solution :

(i)
$$\begin{aligned}
\frac{x+y}{x-y} &= \frac{(x+y)(x^2 - xy + y^2)}{(x-y)(x^2 - xy + y^2)} \\
&= \frac{x^3 + y^3}{x^3 - 2x^2y + 2xy^2 - y^3}
\end{aligned}$$

(ii)
$$\begin{aligned}
\frac{x+y}{x-y} &= \frac{(x+y)(x+y)}{(x-y)(x+y)} \\
&= \frac{x^2 + 2xy + y^2}{x^2 - y^2}
\end{aligned}$$

20. Addition and subtraction of rational expressions : We perform operations of addition subtraction and multiplication on rational expressions in the same manner as we do on fractions in number system.

Thus, the sum of two rational expression (having same denominators)

$$\frac{p(x)}{q(x)} \text{ and } \frac{r(x)}{q(x)} \text{ is given by}$$

$$\frac{p(x)}{q(x)} + \frac{r(x)}{q(x)} = \frac{p(x) + q(x)}{q(x)}$$

and sum of two rational expression (having different denominators)

$$\frac{p(x)}{q(x)} \text{ and } \frac{r(x)}{s(x)} = \frac{P(x)\, s(x) + r(x)\, q(x)}{q(x)\, s(x)}$$

Note : Like rational numbers, sum of two rational expressions is a rational expression. Now, we solve some example to make the concept clear.

21. Additive inverse of a rational expression : We know that additive inverse of $\frac{3}{4}$ is $\frac{-3}{4}$, since $\frac{3}{4} + \left(\frac{-3}{4}\right)$ $= \frac{3 + (-3)}{4} = 0.$

Similarly, the additive of the rational expression $\frac{p(x)}{q(x)}$ is $\frac{-p(x)}{q(x)}$.

Since, $\frac{p(x)}{q(x)} + \left(\frac{-p(x)}{q(x)}\right) = \frac{p(x) + (-p(x))}{q(x)} = \frac{0}{q(x)} = 0$

$\therefore \frac{-p(x)}{q(x)}$ is the additive inverse of $\frac{p(x)}{q(x)}$.

Thus, the additive inverse of $\frac{2x-3}{x^2-1}$ is $\frac{-(2x-3)}{x^2+1}$ or $\frac{-2x+3}{x^2+1}$.

Note : $\frac{p(x)}{q(x)}$ and $\frac{-p(x)}{q(x)}$ are additive inverses of each other.

Now, we define the subtraction of two rational expressions as the sum of the first rational expression and the additive inverse of the second rational expression, thus

$$\frac{p(x)}{q(x)} - \frac{r(x)}{s(x)} = \frac{p(x)}{q(x)} + \left\{\frac{-r(x)}{s\,(x)}\right\} = \frac{p(x)\,s(x) - r(x)\,q(x)}{q(x)\,s(x)}.$$

Note : The difference of two rational expressions is a rational expression.

Illustration 4. Given that

$a^2(b-c) + b^2(c-a) + c^2(a-b) = -(a-b)(b-c)(c-a)$ simplify the following expression :

$$\frac{a^2}{(a-b)(b-c)} + \frac{b^2}{(b-c)(b-a)} + \frac{c^2}{(c-a)(c-b)}$$

Solution : Given expression :

$$= \frac{-a^2}{(a-b)(c-a)} - \frac{b^2}{(b-c)(a-b)} - \frac{c^2}{(c-a)(b-c)}$$

$$= \left[\frac{a^2(b-c) + b^2(c-a) + c^2(a-b)}{(a-b)(b-c)(c-a)}\right]$$

$$= \left[\frac{-(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}\right]$$

$= [\because\ a^2(b-c) + b^2(c-a) + c^2(a-b) = -(a-b)(b-c)(c-a)\ldots\text{Given}]$

$= 1.$

22. Multiplication of rational expressions : We know that the product of two rational numbers

$$\frac{a}{b} \text{ and } \frac{c}{d} \text{ is } \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

Similarly, the product of two rational expressions

$$\frac{p(x)}{q(x)} \text{ and } \frac{r(x)}{s(x)} \text{ is } \frac{p(x)}{q(x)} \times \frac{r(x)}{s(x)} = \frac{p(x) \times r(x)}{q(x) \times s(x)}$$

In other words, the product of two or more rational expressions is the product of their numerators divided by the product of their denominators. When we cancel all factors common to the two terms of the new expression, the resulting expression will be in its lowest terms.

Note : The product of two rational expressions is rational expression.

23. Reciprocal of a rational expression : Let $\frac{p(x)}{q(x)}$, $q(x) \neq 0$ be a non zero rational expression. Then exists another rational expression $\frac{q(x)}{p(x)}$ such that $\frac{p(x)}{q(x)} \times \frac{q(x)}{p(x)} = 1$

$\frac{q(x)}{p(x)}$ is called the reciprocal (or multiplicative inverse) of $\frac{p(x)}{q(x)}$.

Note : $\frac{P(x)}{q(x)}$ and $\frac{q(x)}{p(x)}$ are reciprocal of each other. For example $\frac{3x+2}{2x^2-1}$, is the reciprocal of $\frac{2x^2-1}{3x+2}$.

24. Division of rational expressions : We know that to divide a rational number by another rational number, we multiply the first number by the reciprocal of the second number (if it exists). Similarly, to divide one rational expression by another rational expression, we multiply the first expression by the reciprocal of the second expression.

Note : $\frac{p(x)}{q(x)} + \frac{r(x)}{s(x)}$ is a rational expression.

25. Raising Rational Numbers to a power with a natural exponent. Taking the root : In addition to the fundamental operations of arithmetic, raising to a power (or involution) and taking the root (or evolution) are also introduced for rational numbers.

Let a be a rational number and let n be a natural number.

Definition : The power of a number a with a natural exponent n ($n \geq 2$) is defined as product of n factors each of which is equal to a :

$$a_n = a.a.\ldots.a \;(n \text{ times})$$

Besides, by definition, $a^1 = a$.

The number a repeated as a factor is called the *base* of the power; the number n which indicates how many times the base is to be used as a factor is called the **exponent** of the power. The result (a_n) is called the power with a natural exponent.

The second power of a number $a_2 = a.a$ s also called the square of the number a (or a squared); the third power $a_3 = a.a.a$ is also called the cube of the number a (or a cubed).

It follows from the above definition that :

(1) An even power of a negative number is a positive number : for instance, $(-5)^{20} > 0$;

(2) An odd power of a negative number is a negative number : for instance, $\left(-\frac{2}{3}\right)^{15} < 0$;

(3) Any power of a positive number is a positive number: $a_n > 0$ if $a > 0$;

(4) The result of raising zero unity to a power with any natural exponent is zero : $0_n = 0$;

(5) The result of raising unity to a power with any natural exponent is unity : $1_n = 1$.

If an expression contains no parentheses, then the operations should be performed in the following order: first involution, then multiplication and division in the sequence in which they are indicated, and, finally, addition and subtraction, again in the sequence in which they are given.

For example,

$$3 \cdot 5^2 - 16 : 2.\left(\frac{1}{2}\right)^3 + 26 = 3 \cdot 25 - 16 : 2.\frac{1}{8} + 26$$

$$= 75 - 8 \cdot \frac{1}{8} + 26 = 100.$$

Evolution is defined as an operation inverse to involution.

Definition : An nth root of a number $a(n \geq 2)$ is a number which, when takes as a factor n times (raised to the nth power), produces the number a.

The nth root of a number a is symbolized as $\sqrt[n]{a}$. the notation $\sqrt[n]{a} = b$ means that $b^n = a$. The exponent (the natural number n ($n \geq 2$) here is the index of the root, and the number a is termed the integrand.

If $n = 2$, the root is usually called the square root; in this case the index 2 is usually omitted. For instance, instead of $\sqrt[2]{7}$ we write $\sqrt{7}$. The third root ($n = 3$) is called the cube root.

On the set of rational numbers the operation of taking the root is not always performable. For instance, there is no rational number equal to the square root of 2. Let us prove this.

Suppose the contrary : $\sqrt{2}$ is a rational number :

$$\sqrt{2} = \frac{p}{q}$$

where p and q are whole numbers and $q \neq 0$. The fraction $\frac{p}{q}$ will be regarded as irreducible (this can always be achieved by applying the basic property of the fraction).

According to the definition of the root, we have $\left(\frac{p}{q}\right)^2 = 2$ or $p^2 = 2q^2$, *i.e.*, p is an even number : $p = 2p_1$, where p_1 is an integer. Then $(2p_1)^2 = 2q^2$ or $q^2 = 2p^2{}_1$, *i.e.*, q is also an even number $q = 2q_1$, where q_1 is an integer. Consequently, the fraction $\frac{p}{q} = \frac{2p_1}{2q_1}$ is reducible, which contradicts the hypothesis.

From the obtained contradiction it follows that $\sqrt{2}$ is not a rational number.

Definition : A rational number $b > 0$ is termed an approximate value of the nth root $\sqrt[n]{a}$. ($a > 0$) with deficit and with an accuracy to α (α is a positive rational number) if

$$b^n < a < (b + \alpha)^n.$$

In this case the number $b + \alpha$ is said to be an approximate value of the root $\sqrt[n]{a}$ with excess and with an accuracy to α.

It has been proved that approximate values of roots of positive numbers always exist for any rational number $\alpha > 0$.

Illustration 5. Find $\sqrt{2}$ to within $\frac{1}{7}$.

Solution : Note that $2 = \frac{2 \cdot 7^2}{7^2} = \frac{98}{49}$; $\sqrt{2} = \frac{\sqrt{98}}{\sqrt{49}} = \frac{\sqrt{98}}{7}$.

Therefore, it suffices to find $\sqrt{98}$ with an accuracy to 1 and divide the obtained number by 7. Since, $\sqrt{98} \approx 9$ (with an accuracy to 1), $\sqrt{2} \approx \frac{9}{7}$ (with an accuracy to $\frac{1}{7}$).

Indeed, $\left(\frac{9}{7}\right)^2 < 2 < \left(\frac{9}{7} + \frac{1}{7}\right)^2$.

In most case the measure of accuracy of α is taken to be equal to $\frac{1}{10^m}$ (m is a natural number), and the approximate value of the root is taken to be equal to a decimal fraction with m digits after the decimal point.

Solving a couple of example, let us consider the rule for extracting the square root of a number.

Illustration 6. Find $\sqrt{72 \cdot 6115}$ correct to 0.01.

Solution : Perform the following operations :

(1) Partition the digits in the radicand by pairs in the following manner : the integral part from right to left and the fractional part from left to right : $\sqrt{72 \cdot 6115}$;

(2) Take (correct to 1) the square root of the first pair of digits, *i.e.*, of number 72 and carry over the second pair of digits (61) :

$$\begin{array}{l} \sqrt{72' \cdot 61' 15} = 8\ldots\ldots; \\ \quad\ \ \mathbf{-64} \\ \quad\ \ \overline{)861} \end{array}$$

(3)Double the found root and write the result on the left :

$$\begin{array}{l} \sqrt{72' \cdot 61' 15} = 8\ldots\ldots; \\ \quad\ \ \mathbf{-64} \\ 16\overline{)861} \end{array}$$

(4) Add the greatest possible digit to the right of the number 16, so that the product of the obtained three- digit number by this digit does not exceed 861. In our example such a digit will be 5:165.5 = 825 < 861, we obtain

$$\begin{array}{r|l} & \sqrt{72' \cdot 61' 15} = 8 \cdot 5\ldots\ldots; \\ & \mathbf{-64} \\ \hline 165 & 861 \\ 5 & -825 \\ \hline & 36 \end{array}$$

(5) Double the found root, carry over the third pair of digits (15) and proceed as in (4) :

$$\sqrt{72'\cdot61'15} = 8\cdot52\ldots\ldots;$$

$$\begin{array}{r|l}
 & \mathbf{-64} \\
\hline
165 & 861 \\
5 & -825 \\
\hline
1702 & 3615 \\
2 & -3404 \\
\hline
 & 211 \quad \text{(remainder)}
\end{array}$$

The square root $\sqrt{72\cdot6115}$ = 8.52 (with deficit to within 0.01), *i.e.,* $8.52^2 < 72.6115 < (8.52 + 0.01)^2$.

Illustration 7. Find the approximate value of $\sqrt{113\cdot5}$ with excess to within 0.001.

Solution : Let us separate the radicand in pairs of digits : $\sqrt{113\cdot5} = \sqrt{1'13'\cdot50'00'00}$. Then proceed as in the preceding example :

$$\sqrt{1'13'\cdot50'00'00} = 10\cdot653\ldots\ldots;$$

$$\begin{array}{r|l}
 & \mathbf{-1} \\
\hline
20 & 13 \\
206 & 1350 \\
6 & -1236 \\
\hline
2135 & 11400 \\
5 & -10625 \\
\hline
21303 & 77500 \\
3 & -63909 \\
\hline
 & 13591
\end{array}$$

The square root $\sqrt{113\cdot5} \approx 10\cdot653$ (with deficit to within 0.01) : $\sqrt{113\cdot5} \approx 10\cdot654$ (with excess to within 0.001).

26. Concept of an Irrational number : Let us extract the square root of 2 with an accuracy to $\frac{1}{10}$, $\frac{1}{10^2}$, $\ldots, \frac{1}{10^n}$ and so on. Continuing this process without bound, we shall obtain a non-terminating decimal : $\sqrt{2}$ = 1.41421… This fraction cannot be periodic, since, as $\sqrt{2}$ is not a rational number, and we know that a periodic fractions represents only a rational number.

Thus, in the process of taking roots there appears non-terminating non-periodic decimals. Fractions of this type define new, irrational numbers.

Definition : Any non-terminating non-periodic decimal of the form

$$A = c\cdot b_1 b_2 \ldots b_n \ldots$$

($c \geq 0$; b_1, b_2,……are digits) is called a positive irrational number.

Every positive irrational number a is associated with an opposite to it negative number

$$-a = -c\cdot b_1 b_2 \ldots\ldots b_n \ldots\ldots$$

Definition : Two irrational numbers

$a = c \cdot b_1 b_2 \ldots\ldots b_n \ldots\ldots$ and $a' = c' \cdot b'_1 b'_2 \ldots b'_n \ldots$

are regarded to *equal* if and only if $c = c'$, $b_1 = b'_1$, $b_2 = b'_2, \ldots, b_n = b'_n, \ldots\ldots$ and so on.

Of two positive numbers the number a is greater than the number a' if $c > c'$ or if $c = c'$ but $b_1 > b'_1$ or if $c = c'$ and $b_1 = b'_1$ but $b_2 > b'_2$, and so on.

If $a > a' > 0$, then we regard that $-a < -a'$ and *vice-versa*.

Let $a = c.\,b_1 b_2 \ldots b_n \ldots\ldots > 0$. The fractions $c.b_1$, $c.b_1 b_2$, etc., are said to be decimal approximations to the irrational number a with deficit. The decimals $c.b_1$+1, $c.b_1(b_2+1)$. etc., obtained by adding a unity to the last retained decimal digit of number a are called decimal approximations to the irrational number a with excess.

In order to compare an irrational number with a rational, the latter can be represented in the form of a periodic fraction and then it is possible to compare the decimal approximations to these numbers using the same rule as in the comparison of two irrational numbers. In this case a terminating decimal is regarded as a periodic fraction with period zero.

For example, $\sqrt{2} > 1\cdot41$, since $\sqrt{2} = 1.4142\ldots$ and $1.41 = 1.4100\ldots$

Irrational numbers can be exemplified by square and cube roots of natural numbers which are not squares and cubes of natural numbers, respectively.

$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\ldots \sqrt[3]{2}, \sqrt[3]{3}, \sqrt[3]{4}, \sqrt[3]{5}, \sqrt[3]{6}, \sqrt[3]{7}, \ldots$$

Irrational numbers are obtained not only in taking roots.

For instance, the number $\pi = 3.14\ldots$ used to denote the ratio of the circumference of a circle to its diameter is an irrational number. The values of logarithms of positive numbers and trigonometric functions, as a rule, are also irrational numbers.

Irrational numbers are subject to the arithmetic operations, subtraction and division being defined as the inverse operations to addition and multiplication. The basic properties of arithmetic operations with rational numbers hold for irrational numbers as well. A rigorous proof of these operations and their properties is given in higher mathematics.

27. Real numbers. Arithmetical roots. Rectangular Co-ordinates in the Plane : The union of the sets of rational and irrational numbers forms the set of *real numbers*. Thus, a real number means either rational or irrational number. Every real number can be approximately replaced by a terminating decimal.

The following arithmetic operations are established to be performed with real numbers: addition, subtraction, multiplication and division. The basic properties of the arithmetic operation for whole numbers also hold for real numbers.

The sum or difference of a rational or an irrational number is always an irrational number. This is also true for a product and a quotient if only the rational number is not equal to zero.

But arithmetic operations on two irrational number can lead to rational numbers.

For instance,

$$(5 + \sqrt{2}) - \sqrt{2} = 5, \sqrt[3]{3} \cdot \sqrt[3]{9} = 3.$$

Any real number can be represented in the form of a sum of two addends; this can be in various ways.

For example, the number 27.2 can be represented as the sum of the numbers 10 and 17.2 or 20 and 7.2 or 27 and 0.2 or – 3 and 30.2 etc., we shall represented a real number in the form of a sum of two such addends, one of which is the integral part of the given number and the other its fractional part.

Definition : The integral part of a number x is the greatest integer not exceeding x. It is denoted by the symbol $[x]$.

For instance,

$$[27{\cdot}2] = 27, [0{\cdot}54] = 0, [-3] = -3, [-4{\cdot}5] = -5$$

If x is a whole number (or integer) then $[x] = x$. If x is a non-integral number (or no integral), then $[x] < x$; in this case the number x is enclosed between two consecutive integers $[x] < x < [x] + 1$. Thus, for any x the inequality $[x] \le x < [x] +1$ holds true.

Definition : The fractional part of a number x is the difference between the number x and its integral part. It is denoted as $\{x\}$. Hence, $\{x\} = x - \{x\}$.

$$\begin{aligned} \text{For instance, } \{27{\cdot}2\} &= 27.2 - [27.2] = 0.2, \{0{\cdot}54\} \\ &= 0{\cdot}54 - [0{\cdot}54] = 0{\cdot}54 \\ \{-3\} &= -3 - [-3] = 0, \\ \{-4{\cdot}5\} &= -4{\cdot}5 - [-4{\cdot}5] \\ &= -4{\cdot}5 - [-5] = 0{\cdot}5. \end{aligned}$$

Since, $[x] \le x < [x] + 1, 0 \le x - [x] < 1$ *i.e.,* for any x the inequality $0 \le \{x\} <1$ holds. The fractional part of a number is a non-negative number less than 1.

According to the definition of the fractional part of a number, $\{x\} = x - [x]$. Hence, $x = [x] + \{x\}$, *i.e.,* any number can be represented as the sum of its integral and fractional parts.

For instance,

$$\begin{aligned} 27{\cdot}2 &= 27 + 0{\cdot}2, \\ 0{\cdot}54 &= 0 + 0.54, \\ -3 &= -3 + 0, -4{\cdot}5 = -5 + 0{\cdot}5. \end{aligned}$$

Raising to a power with a natural exponent and taking the root are defined for real numbers just in the same way as for rational numbers.

Le a be a real number and let n be natural number. By definition,

$$a^n = \begin{cases} aaa\ldots.a \text{ ; } n \text{ times if } n \ge 2, \\ a \quad\quad \text{ ; if } n = 1 \end{cases}$$

and $\sqrt[n]{a} = b$ if $b^n = a$ $(n \ge 2)$.

Alongwith powers with natural exponents, we shall also consider powers with any real exponent.

For real numbers taking of the root $\sqrt[n]{a}$ is always feasible except for the case when n is even and $a < 0$. But this operation is not always single valued.

For instance, $\sqrt{16} = 4$ and $\sqrt{16} = -4$, since $4^2 = (-4)^2 = 16$, we would have to write $\sqrt{16} = \pm 4$. To avoid the two-valued property of the root, the notion of the principal or arithmetic root is introduced.

Definition : The arithmetic nth of a non-negative number a is a non-negative number b for which $b^n = a$.

In what follows we shall consider only the arithmetic value of the root, *i.e.,* $\sqrt[n]{a}$ has sense only for $a \ge 0$ and takes on only non -negative values.

For instance, $\sqrt{16} = 4$ is the arithmetic value of the square root 16.

The arithmetic value of a root exits for every $a \ge 0$. Let us now prove its uniqueness.

Let $\sqrt[n]{a} = b_1$ and $\sqrt[n]{a} = b_2$, where $a \ge 0$ and $b_1 \ge 0$ and $b_2 \ge 0$, then $b_1^n = b_2^n = a$.

If $b_1 \ne b_2$, say $b_1 < b_2$, then, by the property of inequalities, $b_1^n < b_2^n$, which is incorrect. The obtained contradiction implies the uniqueness of the arithmetical root, *i.e.,* $b_1 = b_2$.

If x is a real number, then $|x|$ is the modulus (or absolute value) of the number x. By definition,

$$|x| = \begin{cases} x & \text{if } x \ge 0, \\ -x & \text{if } x < 0. \end{cases}$$

Like rational numbers, real numbers can be represented by points on the number or co-ordinate line.

Let x_1 and x_2 be real numbers and let $M_1(x_1)$ and $M_2(x_2)$ be the corresponding points on the number line.

The following formula is valid for the distance between any two points on the number line :

$$|M_1M_2| = |x_2 - x_1| \text{ , ...(1)}$$

where $|M_1M_2|$ is the length of the line segment $M_1 M_2$. This formula is proved in the same way as in the case of rational numbers x_1 and x_2.

Let us pass from the straight line to the plane.

Two mutually perpendicular number axes with a common origin O from a rectangular co-ordinate system in the plane. The horizontal axis is called the axis of abscissa, or the x-axis the vertical axis is termed the axis of ordinates, or the y-axis (Fig. 1). The plane on which a co-ordinate system is chosen is called the co-ordinate plane.

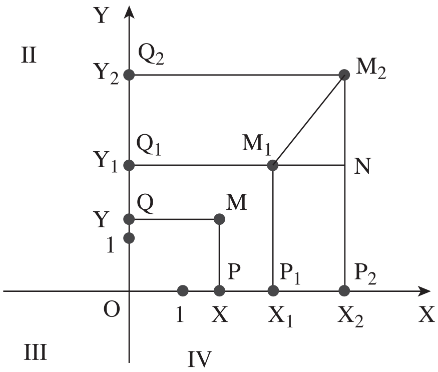


Fig. 1

The co-ordinate plane is divided by the x-axis and y-axis into four parts called the co-ordinate quarters, or quarters, or quadrants. These are known as the first, second, third, and fourth quadrants. Their enumeration is shown in the figure. Right angles formed by the co-ordinate axes are called the co-ordinate angles.

Let M be an arbitrary point of the co-ordinate plane and let us project it on the axis of abscissa and on the axis of ordinate, *i.e.*, drop perpendicular from this point on to the co-ordinate axes (see fig. 1).

Definition : The co-ordinate of the projection of the point M on the x-axis is called the abscissa of the point M, and the co-ordinate of projection of the point M on the y-axis the ordinate of the point M. The abscissa and ordinate of the point M are called the co-ordinates of the point M. We use the following notation: M (x, y) (the abscissas is always written in the first place).

Thus, to every point M of the co-ordinate plane there corresponds an ordered pair of numbers (x, y) –its coordinates.

Conversely, to every pair of numbers x and y there corresponds a unique point M of the co-ordinate plane with the co-ordinates (x, y). Hence, the co-ordinate x and y determine the position of appoint (or locate a point) in a plane.

Indeed, mark a point on the axis of abscissa with co-ordinate x and draw through this point a perpendicular to this axis: then mark a point on the axis of ordinates with co-ordinate y and draw through it a perpendicular to the axis of ordinate. The intersection of these perpendicular just yields the required point M (Fig. 1 shows the case when $x > 0$ and $y > 0$).

If a point lies on the axis of abscissa, then its ordinate is equal to zero. If a point lies on the axis of ordinate, then its abscissa is zero. The converses are also true. The origin has the abscissa and ordinate equal to zero: O (0,0).

Let there be given two points $M_1(x_1,y_1)$ and $M_2\,(x_2, y_2)$ on a co-ordinate plane. For finding the distance between them, the following formula holds :

$$|M_1M_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

To prove this, let us consider the right- angled triangle M_1M_2 N (see fig. 1) in which, by formula (1), the length of the leg M_1N to $|x_2 - x_1|$ and the length of the leg M_2N to $|y_2 - y_1|$. By the Pythagoras theorem,

$$\begin{aligned} |M_1M_2| &= \sqrt{|M_1N|^2 + |M_2N|^2} \\ &= \sqrt{|x_2-x_1|^2 + |y_2-y_1|^2} \\ &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \end{aligned}$$

Formula (2) is also correct for the case when $x_1 = x_2$ or $y_1 = y_2$. Then this formula yields either

$$|M_1M_2| = \sqrt{(y_2-y_1)^2} = |y_2-y_1|$$

$$|M_1M_2| = \sqrt{(x_2-x_1)^2} = |x_2-x_1|.$$

For instance, let us find the distance between the points $M_1(1,3)$ and $M_2(-3, 0)$ by formula (2),

$$\begin{aligned} |M_1M_2| &= \sqrt{(-3-1)^2+(0-3)^2} \\ &= \sqrt{25} = 5. \end{aligned}$$

If a point M has the co-ordinate (x,y), then its distance from the x-axis is equal to $|y|$, the distance from the y-axis to $|x|$, and from the point O – $\sqrt{x^2+y^2}$: $|MP| = |y|$, $|MQ| = |x|$, and $|OM| = \sqrt{x^2+y^2}$.

28. Powers with a Natural Exponent : By the definition of a power with a natural exponent n,

$$a^n = \begin{cases} aa\cdot a\ ;\ n \text{ times if } n \geq 2 \\ a\ ;\ \text{if } n = 1 \end{cases}$$

(the base a is any real number). Let us prove the following properties of raising to a power :

(1) When multiplying powers with equal bases, the exponents are added, *i.e.*, $a^m\ a^n = a^{m+n}$ (m, n-natural numbers).

Proof :

$$a^m\ a^n = \underbrace{(aa\ldots a)}_{m \text{ times}} \times \underbrace{(aa\ldots a)}_{n \text{ times}} = \underbrace{aa\ldots\ldots a}_{(m+n)\text{ times}}$$

(by the associative property of multiplication). Hence, $a^m\ a^n = a^{m+n}$.

(2) When dividing powers with equal bases, the exponents are subtracted, *i.e.*,

$$\frac{a^m}{a^n} = a^{m-n}\ (a \neq 0, m, n - \text{natural number}, m > n).$$

$$\frac{a^m}{a^n} = \frac{\overline{aa\ldots a}\ (m \text{ times})}{aa\ldots a\ (n \text{ times})}$$

Reducing the fraction, we obtain

$$\frac{a^m}{a^n} = \frac{\overline{aa\ldots\ldots a}\ [(m-n)\text{ times}]}{1}\ a^{m-n}.$$

Remark. If $m < n$, then $a^m : a^n = \frac{1}{a^{n-m}}$; if $m = n$, then $a^m : a^n = 1$.

$$\text{Thus, } \frac{a^m}{a^n} = \left\{\begin{array}{l} a^{m-n} \text{ if } m > n, \\ 1 \text{ if } m = n\ (a \neq 0)\cdot \\ \frac{1}{a^{n-m}} \text{ if } m < n, \end{array}\right\}$$

(3) When raising a power, the exponents are multiplied, *i.e.*,

$$(a^m)^n = a^{mn} \ (m.n\text{–natural numbers}).$$

Proof :

$$(a^m)^n = \underbrace{a^m a^m \ldots a^m}_{n \text{ times}} = a^{\overbrace{m+m+m+\ldots+m}^{n \text{ times}}}$$

(by the property of multiplication of powers with equal exponents).

Hence, $(a^m)^n = a^{mn}$

(4) $$(-a)^n = \begin{cases} a^n \text{ if } n \text{ is even} \\ -a^n \text{ if } n \text{ is odd} \end{cases}$$

This property follows directly from the definition of a power with a natural exponent.

(5) When raising a product to a power, each factor is raised to this power, *i.e.*,

$$(ab)^n = a^n b^n \ (n\text{-natural number}).$$

Proof :

$$(ab)^n = \underbrace{(ab)(ab)\ldots(ab)}_{n \text{ times}} = \underbrace{(aa.aa)}_{n \text{ times}} \ \underbrace{(bb\ldots b)}_{n \text{ times}}$$

(by the associative and commutative properties of multiplication). Hence, $(ab)^n = a^n b^n$

(6) When raising a fraction to a power, both the numerator and denominator are raised to this power, *i.e.*,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \ (b \neq 0, n\text{-natural number}).$$

Proof :

$$\left(\frac{a}{b}\right)^n = \frac{a}{b}\cdot\frac{a}{b}\ldots\frac{a}{b} \ (n \text{ times})$$

$$= \frac{aa\ldots a \ (n \text{ times})}{bb\ldots b \ (n \text{ times})}$$

(according to the rule for multiplication of fraction).

Hence, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

Let us show how this property are applied when performing computations and identical transformations.

Illustration 8. Compute $\dfrac{15^3 \cdot 21^2}{35^2 \cdot 3^4}$.

Solution : $$\frac{15^3 \cdot 21^2}{35^2 \cdot 3^4} = \frac{(3\cdot5)^3 \cdot (3\cdot7)^2}{(5\cdot7)^2 \cdot 3^4}$$

$$= \frac{3^3\cdot5^5\cdot3^2\cdot7^2}{5^2\cdot7^2\cdot3^2} = 3\cdot5 = 15.$$

Illustration 9. Evaluate the expression

$$(-1\cdot4)^3 . \left(3\frac{4}{7}\right)^3.$$

Solution :

$$(-1\cdot4)^3\left(3\frac{4}{7}\right)^3 = (-1\cdot4)^3\left(\frac{27}{7}\right)^3$$

$$= -\left(1\cdot4\cdot\frac{25}{7}\right)^3$$

$$= -\left(\frac{14}{10}\cdot\frac{25}{7}\right)^3$$

$$= -(5)^3 = -125.$$

Illustration 10. Perform the indicated operations :

$$\frac{(-2)\cdot(-3)^{17} - (-3)^{16}}{9^7 \cdot 15}$$

Solution :

$$\frac{(-2)\cdot(-3)^{17} - (-3)^{16}}{9^7 \cdot 15} = \frac{(-2)\cdot(-3)^{17} - (-3)^{16}}{9^7\cdot15}$$

$$= \frac{2\cdot3^{17} - 3^{16}}{9^7\cdot15} = \frac{3^{16}(2\cdot3-1)}{(3^2)^7\cdot3\cdot5}$$

$$= \frac{3^{16}(6-1)}{3^{14}\cdot3\cdot5} = \frac{3^{16}\cdot5}{3^{15}\cdot5} = 3.$$

Illustration 11. Arrange the following numbers in increasing order : $\left(-\frac{3}{4}\right)^3, \left(-\frac{2}{5}\right)^2, 0\cdot3^2, (-1\cdot2)^2$.

Solution : We find

$$\left(-\frac{3}{4}\right)^3 = -\left(\frac{3}{4}\right)^3 = -\frac{27}{64}, -\left(\frac{2}{5}\right)^2$$

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25} = 0\cdot16,$$

$$(0\cdot3)^2 = 0\cdot09, (-1\cdot2)^2$$

$$= (1\cdot2)^2 = 1\cdot44.$$

Hence, $$\left(-\frac{3}{4}\right)^3 < 0\cdot3^2 < \left(-\frac{2}{5}\right)^2 < (-1\cdot2)^2.$$

Illustration 12. Which of the given numbers is greater :

(a) 2^{300} or 3^{200} (b) 54^4 or 21^{12};

(c) $(0.4)^4$ or $(0.8)^3$?

Solution : (a) $2^{300} = (2^3)^{100} = 8^{100}$,

$3^{200} = (3^2)^{100} = 9^{100}$;

Hence, $3^{200} > 2^{300}$;

(b) $54^4 = (2\cdot27)^4 = (2\cdot3^3)^4 = 2^4\cdot3^{12}$,

$21^{12} = (3\cdot7)^{12} = 3^{12}\cdot7^{12}$

$(7^3)\ 4\cdot3^{12} = (343)^4\cdot3^{12}$

Hence, $21^{12} > 54^4$.

(c) $$(0\cdot4)^4 = \left(\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)^3\cdot\frac{2}{5},$$

$$(0\cdot8)^3 = \left(\frac{4}{5}\right)^3 = \left(\frac{2}{5}\right)^3\cdot2^3$$

$$= \left(\frac{2}{5}\right)^3\cdot8.$$

Hence, $(08)^3 > (0\cdot4)^4$.

29. Arithmetical square root

Definition : The arithmetical square root of a non-negative number a is a non-negative number b whose square is equal to a : $\sqrt{a} = b$.

For instance, $\sqrt{16} = 4, \sqrt{0} = 0$. The symbol $\sqrt{\ }$ is the sign of the arithmetical square root, and a is called the radicand. The expression $\sqrt{a}$ is read: "the arithmetical square root of the number a" or, simply, "the square root of a"$(a \geq 0)$.

From the definition of the arithmetical square root it follows that :

(1) The expression $\sqrt{a}$ has sense only for $a \geq 0$.

(2) For any number $a \geq 0$ the inequality $\sqrt{a} \geq 0$ is fulfilled.

(3) For any number $a \geq 0$ the equality $(\sqrt{a})^2 = a$ is fulfilled.

In order to prove that b is an arithmetical square root of a number $a \geq 0$, check to see that the following two co-ordinates are fulfilled : (1) $b \geq 0$; (2) $b^2 = a$.

For instance, $\sqrt{25} = 5$, since $5^2 = 25$ and $5 > 0$.

Theorem : *Of any real number a ≥ 0 it is possible to take only one arithmetical square root.*

We are not going to prove that $\sqrt{a}$ $(a \geq 0)$ exists (the proof of this statement is rather difficult). Let us prove the uniqueness of the arithmetical square root.

Let $\sqrt{a} = b_1$ and $\sqrt{a} = b_2$, where $a \geq 0, b_1 ‡ 0$, and $b_2 \geq 0$ then $b_1{}^2 = b_2{}^2$. From the obtained contradiction there follows the uniqueness of the arithmetical square root, *i.e.*, $b_1 = b_2$.

Theorem : *For any real number a*

$$\sqrt{a^2} = |a|.$$

Proof : Consider two cases :

(1) If $a \geq 0$, then by the definition of the arithmetical square root, $\sqrt{a^2} = a$.

(2) If $a < 0$, then $(-a) > 0$. The number $(-a)$ is positive and $(-a)^2 = a^2$. Therefore, $\sqrt{a^2} = -a$.

Thus,

$$\sqrt{a^2} = \begin{cases} a \text{ if } a \geq 0, \\ -a \text{ if } a < 0 \end{cases}$$

Or, by the definition of the modulus of a real number, $\sqrt{a^2} = |a|$ which was just stated.

For instance, $\sqrt{(-5)^2} = |-5| = 5$

$$\sqrt{(a-b)^2} = |a-b| = \begin{cases} a-b \text{ if } a \geq 0, \\ b-a \text{ if } a < 0 \end{cases}$$

The equality $\sqrt{a^2} = |a|$ is fulfilled for any values of a. Hence, this equality is an identity on the set of real numbers.

Substituting the number $a = b_n$ (n is natural number) into the identity $\sqrt{a^2} = |a|$, we obtain the identity

$$\sqrt{b^{2n}} = |b^n|.$$

If $b \geq 0$, we have $\sqrt{b^{2n}} = b^n$

For instance,

$$\sqrt{5^4} = 5^2 = 25, \sqrt{(-3)^6} = |(-3)^2| = 27.$$

Let us prove the following properties of the arithmetical square root :

(1)*The root taken of the product of non-negative factors is equal to the product of the root taken factors, i.e.,*

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \qquad (a \geq 0, b \geq 0).$$

Proof : We have to established that

$$\sqrt{a} \cdot \sqrt{b} \geq 0 \text{ and } \quad (\sqrt{a} \cdot \sqrt{b})^2 = ab$$

Since, $a \geq 0$ and $b \geq 0$, by the definition of the arithmetical root, we have $(\sqrt{a})^2 = a$ and $(\sqrt{b})^2 = b$. Further, the square of a product is equal to the product of the squares of its factor. Therefore,

$$(\sqrt{a} \cdot \sqrt{b})^2 = (\sqrt{a})^2 \,.\, (\sqrt{b})^2 = ab.$$

Since, $\sqrt{a} \geq 0$ and $\sqrt{b} \geq 0$, we have $\sqrt{a}.\sqrt{b} \geq 0$

The property has been proved.

Hence, it follows that

$$\sqrt{a}.\sqrt{b} = \sqrt{ab}\ (a \geq 0, b \geq 0)$$

(2) *The root taken of a fraction with a non-negative numerator and a positive denominator is equal to the root taken of the numerator divided by the root taken of the denominator, i.e.,*

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}\ (a \geq 0, b > 0).$$

Proof : It is necessary to show that

$$\frac{\sqrt{a}}{\sqrt{b}} \geq 0 \text{ and } \left(\frac{(\sqrt{a})}{(\sqrt{b})}\right)^2 = \frac{a}{b}.$$

Since, $a \geq 0$ and $b > 0$, by the definition of the arithmetical root, we have $(\sqrt{a})^2 = a$ and $(\sqrt{b})^2 = b$, squaring the fraction $\frac{\sqrt{a}}{\sqrt{b}}$, we obtain

$$\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2 = \frac{(\sqrt{a})^2}{(\sqrt{b})^2} = \frac{a}{b}.$$

Since, $\sqrt{a} \geq$ and $\sqrt{b} > 0$, we have $\frac{\sqrt{a}}{\sqrt{b}} \geq 0$.

The property has been proved.

Hence, it follows that

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}\ (a \geq 0, b > 0)$$

(3) If $a > b > 0$, then $\sqrt{a} > \sqrt{b}$.

Proof : Let us assume that $\sqrt{a} \leq \sqrt{b}$. Then, squaring both members of the inequality, we obtain $a \leq b$, which contradicts the hypothesis. Hence, $\sqrt{a} > \sqrt{b}$.

Hence, it follows that

If $a > 0, b > 0$ and $\sqrt{a} > \sqrt{b}$, then $a > b$.

Consider the transformation of square roots.

1. Removing a Factor from the Radicand : This is the term for the transformation of the form

$$\sqrt{a^2 b} = a\sqrt{b}\ (a \geq 0, b \geq 0)$$

Indeed, $\sqrt{a^2b} = \sqrt{a^2}\cdot\sqrt{b} = |a|\sqrt{b} = a\sqrt{b}$.

Remark If $a < 0$ and $b \geq 0$, then

$$\sqrt{a^2b} = -a\sqrt{b},$$

Since, $|a| = -a$ for $a < 0$.

2. Bringing a Factor in the Radicand : This is how the transformation of the form

$$a\sqrt{b} = \sqrt{a^2b}\ (a \geq 0, b \geq 0)$$

is called. Let us show the application of the properties and transformation of arithmetical square roots when performing operations with them.

Illustration 13. Remove a factor from the radicand in the expression $\sqrt{4a^2b^3}$, where $a < 0$ and $b > 0$.

Solution : We have

$$\sqrt{4a^2b^3} = \sqrt{4}\cdot\sqrt{a^2}\cdot\sqrt{b^3}$$
$$= 2|a|\sqrt{b^2}\cdot\sqrt{b} = 2|a|\,|b|\cdot\sqrt{b}\ .$$

Since, $a < 0$ and $b > 0$, we get $|a| = -a$ and $|b| = b$.

Therefore, $\sqrt{4a^2b^3}, = -2ab\sqrt{b}$.

Illustration 14. Remove a factor from the radicand in the expression $\sqrt{16a^4b^6c^3}$, where $b < 0$ and $c > 0$.

Solution : We have

$$\sqrt{16a^4b^6c^3} = \sqrt{16}\cdot\sqrt{a^4}\cdot\sqrt{b^6}\cdot\sqrt{c^3}$$
$$= 4\,|a^2|\,|b^3|\,|c|\cdot\sqrt{c}$$

The number a^2 is always non-negative; $|a^2| = a^2$ since $b < 0$ and $c > 0$, $|b^3| = -b^3$ and $|c| = c$, therefore $\sqrt{16a^4b^6c^3}$ $= -4a^2b^3c\sqrt{c}$.

Illustration 15. Bring a factor in the radicand in the expression $\frac{\sqrt{x}}{y}$, where $x \geq 0$ and $y < 0$.

Solution : Since, $y < 0$, $\sqrt{y^2} = |y| = -y$.

Hence, $y = -\sqrt{y^2}$.

Therefore, $\frac{\sqrt{x}}{y} = \frac{\sqrt{x}}{-\sqrt{y^2}} = -\sqrt{\frac{x}{y^2}}$.

Illustration 16. Perform the indicated operations : $\sqrt{343} - \sqrt{252} - \sqrt{7}$.

Solution : Note that $343 = 49.7$, $252 = 36.7$.

Therefore, $\sqrt{343} - \sqrt{252} - \sqrt{7}$

$$= 7\sqrt{7} - 6\sqrt{7} - \sqrt{7} = 0.$$

Illustration 17. Compare the numbers $3\sqrt{5}$ and $4\sqrt{3}$.

Solution : Bringing the factors 3 and 4 in the respective radicands, we obtain

$$3\sqrt{5} = \sqrt{9\cdot 5} = \sqrt{45}, \text{ more } 4\sqrt{3}$$
$$= \sqrt{16\cdot 3} = \sqrt{48}.$$

Since, $45 < 48$, by the property of comparison of roots, we obtain that $\sqrt{45} < \sqrt{48}$ or $3\sqrt{5} < 4\sqrt{3}$.

Illustration 18. Simplify the expression $\sqrt{\frac{x^2}{y}}\cdot\sqrt{\frac{y}{x}}$.

Solution : The root $\sqrt{\frac{x^2}{y}}$ has sense only for $y > 0$ and the root $\sqrt{\frac{y}{x}}$ when x and y have the same sign. Therefore, for the given expression, $x > 0$ and $y > 0$. We have $\sqrt{\frac{x^2}{y}}\cdot\sqrt{\frac{y}{x}} = \sqrt{\frac{x^2}{y}\cdot\frac{y}{x}} = \sqrt{x}$, where $x > 0$ and $y > 0$.

30. Powers with an Integral Exponent : The concept and properties of a power with a natural exponent were considered in previous section. Generalizing the notion of power, we shall introduce here powers with a zero or an integral negative exponent.

Definition : If $a \neq 0$, then $a^0 = 1$.

The expression 0^0 has no sense.

Definition : If $a \neq 0$ and n is a natural number, then

$$a^{-n} = \frac{1}{a^n}.$$

The expression 0^{-n} has no sense.

Using the notions of powers with a zero or an integral negative exponent, the property of a power with a natural exponent

$$\frac{a^m}{a^n} = \begin{cases} a^{m-n} \text{ if } m > n, \\ 1 \text{ if } m = n\ (a \neq 0) \\ \frac{1}{a^{n-m}} \text{ if } m < n, \end{cases}$$

can be written in the form

$$\frac{a^m}{a^n} = a^{m-n} \qquad (a \neq 0)$$

For any natural exponents m and n.

The following properties hold true for any power with any integral exponent :

(1) $a^pa^q = a^{p+q}$;

(2) $(a^p)^q = a^{pq}$;

(3) $\frac{q^p}{a^q} = a^{p-q}$;

(4) $(ab)^p = a^pb^p$;

(5) $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$;

Illustration19. Compute $\frac{18^{-3}\cdot 3^7}{2^{-5}}$.

Solution :
$$\frac{18^{-3}\cdot 3^7}{2^{-5}} = \frac{2^5\cdot 3^7}{18^3}$$
$$= \frac{2^5\cdot 3^7}{(2\cdot 3^2)^3} = \frac{2^5\cdot 3^7}{2^3\cdot 3^6}$$
$$= 2^2\cdot 3 = 12.$$

Illustration 20. Find the value of the expression $1\cdot 7^{-3} : 5\cdot 1^{-3} \times 6^{-3}$.

Solution :

$$1\cdot 7^{-3} : 5\cdot 1^{-3} \times 6^{-3} = \left(\frac{1\cdot 7 \times 6}{5\cdot 1}\right)^{-3}$$
$$= \left(\frac{6}{3}\right)^{-3} = 2^{-3} = \frac{1}{8}.$$

Illustration 21. Write the expression

$$\frac{(ab)^4}{(a^{-2}\cdot b^3)^{-3}} \text{ more } (a \neq 0, b \neq 0)$$

in the form $a^p b^q$ (p, q are integers)

Solution :

$$\frac{(ab)^4}{(a^{-2}\cdot b^3)-3} = \frac{a^4b^4}{(a^{-2})^{-3}\,(b^3)^{-3}}$$

$$= \frac{a^4\cdot b^4}{a^6\cdot b^{-9}} = a^{-2}b^{13}.$$

31. The Arithmetical *n*th Root : Let $a \geq 0$ be a real number and let $n \geq 2$ be a natural number.

Definition : The arithmetical nth root of non-negative number a is a non-negative number b if $b^n = a$, which is written : $\sqrt[n]{a} = b$. For $n = 2$, we have the arithmetical square root. The definition of the arithmetical nth root implies that :

(1) The expression $\sqrt[n]{a}$ has sense only for $a \geq 0$;

(2) The expression $\sqrt[n]{a}$ is always non-negative, *i.e.,* $\sqrt[n]{a} \geq 0$;

(3) The equality $(\sqrt[n]{a})^n = a$ is true for any $a \geq 0$.

Let us prove following basic rules for operations with arithmetical roots :

(1) The Fundamental Property of the Arithmetical Root : The value of an arithmetical root remains unchanged if its index is multiplied by any natural number k and the radicand is simultaneously raised to the power k, *i.e.,*

$$\sqrt[n]{a} = \sqrt[nk]{a^k}\ (a \geq 0)$$

Indeed, let $\sqrt[n]{a} = b(b \geq 0;)$. This means that $b^n = a$. Then, by the property of a power,

$$(b^n)^k = b^{nk} = a^k$$

Hence it follows that $b = \sqrt[nk]{a^k}$. Thus, $\sqrt[n]{a} = b = \sqrt[nk]{a^k}$, which was just stated.

(2) When multiplying arithmetical roots with equal indices, the radicands are multiplied and the index of the root remains unchanged, *i.e.,*

$$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}\ (a \geq 0, b \geq 0).$$

Indeed, by the property of a power, we have

$$(\sqrt[n]{a}\,\sqrt[n]{b})^n = (\sqrt[n]{a})^n\,(\sqrt[n]{b})^n = ab,$$

Since, $(\sqrt[n]{a})^n = a$ and $(\sqrt[n]{b})^n = b$ for $a \geq 0$; and $b \geq 0$; Hence, by the definition of the arithmetical root, it follows that

$$\sqrt[n]{ab} = \sqrt[n]{a^n}\,\sqrt[n]{b} \text{ or } \sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}.$$

The statement has been proved.

In particular,

$$\sqrt[n]{a^n b} = \sqrt[n]{a^n}\,\sqrt[n]{b} = a\sqrt[n]{b} \quad (a \geq 0, b \geq 0)$$

(the rule for removing a factor from the radicand).

(3) When dividing arithmetical roots with equal indices, the radicands are divided and the index of the root remains unchanged, *i.e.,*

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}\ (a \geq 0, b > 0)$$

The property is proved in the same way as the preceding property, in particular,

$$\sqrt[n]{\frac{a}{b}} = \sqrt[n]{\frac{ab^{n-1}}{b^n}}$$

$$= \frac{\sqrt[n]{ab^{\,n-1}}}{b} \qquad (a \geq 0, b > 0)$$

(the rule for riding the radicand of the denominator).

(4) When raising an arithmetical root to a power with a natural exponent, the radicand is raised to this power and the index of the root remain unchanged, *i.e.,* radicand remain unchanged, *i.e.,*

$$(\sqrt[n]{a})\ m = \sqrt[n]{a^m}$$

($a \geq 0$, m – a natural number)

This rule follows from the rule multiplying roots root.

(5) When taking the root of, the indices of the roots are multiplied and he radicand remains unchanged, *i.e.,*

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

($a \geq 0$; m, n – natural numbers, $m \geq 2, n \geq 2$).

Indeed, according to the rule for raising a root to a power, we have

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m]{\sqrt[n]{a}} = \sqrt[m]{((\sqrt[n]{a})\ n)^m}$$

Hence,

$$\left(\sqrt[m]{\sqrt[n]{a}}\right)^{mn} = \sqrt[m]{a^m} = a\ (a \geq 0).$$

Consequently, by the definition of the arithmetical root, $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$, which was required to be proved.

Definition : Roots are said to be like (or similar) if their radicands are equal and their indices are also equal.

In the general case, the sum or the difference of two distinct roots cannot be simplified. Simplifications are possible only in the case of like roots.

For instance,

$(6\sqrt{2} + 5\sqrt{3}) - (6\sqrt{2} + 7\sqrt{3}) = 6\sqrt{2} + 5\sqrt{3} - 6\sqrt{2} - 7\sqrt{3} = -2\sqrt{3}$.

The rule for comparing arithmetical roots is based on the following property: if $a > b > 0$, then $\sqrt[n]{a} > \sqrt[n]{b}$, and conversely, if $\sqrt[n]{a} > \sqrt[n]{b}$ ($a > 0$ and $b > 0$), then $a > b$.

It follows from the properties of inequalities for instance, $\sqrt[3]{3} > \sqrt{2}$. To prove this, let us first, applying the fundamental property of the root, reduce $\sqrt[3]{3}$ and $\sqrt{2}$ to the common index 6 (the least common multiple of the indices of the given roots) : $\sqrt[3]{3} = \sqrt[6]{3^2} = \sqrt[6]{9}$ and $\sqrt{2} = \sqrt[6]{2^3} = \sqrt[6]{8}$. We then use the rule for comparing roots and obtain that $\sqrt[6]{9} > \sqrt[6]{8}$ or $\sqrt[3]{3} > \sqrt{2}$.

We shall always consider only arithmetical roots. Therefore, it is necessary to check attentively that both the radicand and the result of taking the root be non-negative. For instance, we do not consider the product $\sqrt{2}\cdot\sqrt[3]{-3}$, since $\sqrt[3]{-3}$ is not an arithmetical root. Multiplying $\sqrt{2}$ and $\sqrt[3]{3}$, we obtain $\sqrt{2}\cdot\sqrt[3]{3} = \sqrt[6]{2^3}\cdot\sqrt[6]{3^2} = \sqrt[6]{2^3\cdot 3^2} = \sqrt[6]{72}$.

In the case of an arithmetical square root the identity $\sqrt{a^2} = |a|$ holds for any real number. Analogously, we obtain that

$$\sqrt[n]{a^n} = \begin{cases} |a| \text{ if } n \text{ is an even number, } n \ge 2; \\ a \text{ if } a \ge 0 \text{ and } n \text{ is an odd number, } n \ge 3; \end{cases}$$

For instance,

$$\sqrt[3]{x^3} = x\ (x \ge 0),\ \sqrt[4]{x^4} = |x|,\ \sqrt[6]{x^6} = |x|;$$

In the transformations

$$\sqrt[6]{x^3} = \sqrt[3]{\sqrt[3]{x^3}} = \sqrt{x}\ (x \ge 0),\ \sqrt[4]{x^2} = \sqrt{\sqrt{x^2}} = \sqrt{|x|},$$

$$\sqrt[15]{x^3} = \sqrt[5]{\sqrt[3]{x^3}} = \sqrt[5]{x}\ (x \ge 0), \text{ and so on.}$$

Let us work out some examples illustrating how the obtained properties are applied.

Illustration 22. Bring a factor in the radicand in the expression $a\sqrt[3]{1 + \frac{1}{a^3}}$, where $a > 0$.

Solution : Since, $a = \sqrt[3]{a^3}$ $(a > 0)$,

$$a\sqrt[3]{1 + \frac{1}{a^3}} = \sqrt[3]{a^3}\sqrt[3]{1 + \frac{1}{a^3}}$$

$$= \sqrt[3]{a^3\left(1 + \frac{1}{a^3}\right)} = \sqrt[3]{a^3 + 1}$$

Illustration 23. Remove a factor from the radicand in the expression $\sqrt[6]{1 + \frac{1}{a^6}}$, where $a < 0$.

Solution : We know that $\sqrt[6]{a^6} = |a| = -a$, since $a < 0$. Therefore,

$$\sqrt[6]{1 + \frac{1}{a^6}} = \sqrt[6]{\frac{a^6 + 1}{a^6}} = \frac{\sqrt[6]{a^6 + 1}}{\sqrt[6]{a^6}} = -\frac{\sqrt[6]{a^6 + 1}}{a}$$

Illustration 24. Remove a factor from the radicand in the expression $\sqrt[3]{x^8y^{10}z^5}$, where $y > 0$ and $z > 0$.

Solution : We have

$$\sqrt[3]{x^8y^{10}z^5} = \sqrt[3]{x^6x^2y^9yz^3z^2}$$

$$= \sqrt[3]{x^6}\sqrt[3]{y^9}\sqrt[3]{z^3}\sqrt[3]{x^2yz^2}$$

$$= x^2y^3z\sqrt[3]{x^2yz^2}$$

Since, $\sqrt[3]{x^6} = x^2$ for any x, $\sqrt[3]{y^9} = y^3$, $\sqrt[3]{z^3} = z$ for $y > 0$ and $z > 0$.

If we change the conditions and assume that $y < 0$ and $z > 0$, then $\sqrt[3]{x^8y^{10}z^5}$, $= \sqrt[3]{x^8}\sqrt[3]{y^{10}}\sqrt[3]{z^5}$. It is know that $\sqrt[3]{x^8} = x^2\sqrt[3]{x^2}$; for $y < 0$ the root $\sqrt[3]{y^{10}} = \sqrt[3]{(-y)^9(-y)} = (-y)^3\sqrt[3]{-y} = -y^3\sqrt[3]{-y}$; for $z > 0$ the root $\sqrt[3]{z^5} = z\sqrt[3]{z^2}$.

The arithmetical root $\sqrt[3]{y^{10}}$ has sense for any y, while the arithmetical root $\sqrt[3]{z^5}$ only for $z \ge 0$. We obtain that $\sqrt[3]{x^8y^{10}z^5}$, $= -x^2y^3z\sqrt[3]{-x^2yz^2}$ if $y < 0$ and $z > 0$.

Illustration 25. Prove that $\sqrt[4]{ab} = \sqrt[4]{-a}\sqrt[4]{-b}$ if $a \le 0$ and $b \le 0$.

Solution : If $a \le 0$ and $b \le 0$, then $ab \ge 0$. Therefore, the root $\sqrt[4]{ab}$ has sense. Since, $ab = (-a)(-b)$ and both factors are non-negative for $a \le 0$ and $b \le 0$, by the rule for multiplying roots, we have

$$\sqrt[4]{ab} = \sqrt[4]{-a}\sqrt[4]{-b} \qquad (a \le 0, b \le 0)$$

which was required to be proved.

Illustration 26. Perform the indicated operations $\sqrt[3]{2\sqrt{2\sqrt[3]{2}}}$.

Solution : We first transform $2\sqrt[3]{2} = \sqrt[3]{2^4}$.

Therefore, $\sqrt{2\sqrt[3]{2}} = \sqrt{\sqrt[3]{2^4}} = \sqrt[6]{2^4}$.

We then have :

$$\sqrt[3]{2\sqrt{2\sqrt[3]{2}}} = \sqrt[3]{2\sqrt[6]{2^4}}$$

$$= \sqrt[3]{\sqrt[6]{2^{10}}} = \sqrt[18]{2^{10}}$$

$$= \sqrt[9]{2^5} = \sqrt[9]{32}$$

(We have applied the fundamental property of the root and the rule for extracting the root of the root).

32. Powers with a Rational Exponent : The notion and properties of powers with any integral exponent were considered in previous section. Let us now introduce powers with rational fractional exponent.

Definition : If $a > 0$ and x is an arbitrary rational number represented by the fraction $\frac{p}{q}$, where p is an

integer and $q \geq 2$ is a natural number, then $a^x = a^{\frac{p}{q}} = \sqrt[q]{a^p}$;
If $a = 0$ and $x > 0$, then $a^x = 0$.

For instance, $a^{\frac{2}{5}} = \sqrt[5]{a^2}$ for any

$$a \geq 0, b^{-\frac{3}{4}} = b^{-\frac{3}{4}} = \sqrt[4]{b^{-3}} \text{ or } b^{-\frac{3}{4}} = \sqrt[4]{\frac{1}{b^3}} = \frac{1}{\sqrt[4]{b^3}} = \frac{1}{b^{\frac{3}{4}}}$$

for $b > 0$.

Remark 1. If $\frac{p}{q}$ is an irreducible fraction, then for any fraction of the form $\frac{pm}{pm} = \frac{p}{q}$ (n-natural number), we have

$$a^{pn/qn} = \sqrt[qn]{a^{pn}} = \sqrt[q]{a^p} = a^{p/q}$$

(we have made use here of the fundamental property of the arithmetical root).

Remark 2. If $a > 0$ and x is a whole number, represented by a fraction of the from $\frac{p}{q}$, where p is an integer and $q \geq 2$ is a natural number, then the equality $a^{p/q} = \sqrt[q]{a^p}$ is also true, not by the definition of a power with a fractional exponent but by the definition of the arithmetical root. Indeed, if $\frac{p}{q} = x$, then $\sqrt[q]{a^p} = a^x$ $(a > 0)$, since $(a^x)^q = a^{qx} = a^p$ and $a^x > 0$, hence

$$a^x = a^{p/q} = \sqrt[q]{a^p}.$$

The definition of a power with a fractional exponent implies that $a^{-p/q} = \frac{1}{a^{p/q}}$, where $a > 0$ indeed,

$$a^{-\frac{p}{q}} = a^{-\frac{p}{q}} = \sqrt[q]{a^{-p}}$$

$$= \sqrt[q]{\frac{1}{a^p}}$$

or $$a^{-p/q} = \frac{1}{\sqrt[q]{a^p}} = \frac{1}{a^{p/q}}.$$

Since, for $a \neq 0$, $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ (n–natural number), for any rational number x, we have : $a^{-x} = \frac{1}{a^x}$ (if $a > 0$).

The properties of a power with an integral exponent also extend to a power with any rational exponent and a positive base.

Let us give some worked examples illustrating applications of the above considered properties.

Illustration 27. Compute $8^{\frac{2}{3}} - 16^{\frac{1}{4}} + 9^{\frac{1}{2}} - 8^{\frac{2}{3}} - 16^{\frac{1}{4}} + 9^{\frac{1}{2}}$.

Solution :

$$8^{\frac{2}{3}} - 16^{\frac{1}{4}} + 9^{\frac{1}{2}} = \sqrt[3]{8^2} - \sqrt[4]{16} + \sqrt{9}$$
$$= \sqrt[3]{64} - \sqrt[4]{16} + \sqrt{9}$$
$$= 4 - 2 + 3 = 5.$$

Illustration 28. Find the value of the expression

$$(0{\cdot}04)^{-1{\cdot}5} \cdot (0{\cdot}125)^{-\frac{4}{3}} - \left(\frac{1}{121}\right)^{-\frac{1}{2}}.$$

Solution :

$$(0{\cdot}04)^{-1{\cdot}5} \cdot (0{\cdot}125)^{-\frac{4}{3}} - \left(\frac{1}{121}\right)^{-\frac{1}{2}}$$
$$= \left(\frac{1}{25}\right)^{-\frac{3}{2}} \cdot \left(\frac{1}{8}\right)^{-\frac{4}{3}} - \left(\frac{1}{121}\right)^{-\frac{1}{2}}$$
$$= \left(\frac{1}{5^2}\right)^{-\frac{3}{2}} \cdot \left(\frac{1}{2^3}\right)^{-\frac{4}{3}} - \left(\frac{1}{11^2}\right)^{-\frac{1}{2}}$$
$$= (5^{-2})^{-\frac{3}{2}} \cdot (2^{-3})^{-\frac{4}{3}} - (11^{-2})^{-\frac{1}{2}}$$
$$5^3{\cdot}2^4 - 11 = (5{\cdot}2)^3{\cdot}2 - 11 = 2000 - 11 = 1989$$

Illustration 29. Perform the indicated operations :

$$\frac{2{\cdot}4^{-2} + \left(81^{-\frac{1}{2}}\right)^3 \cdot \left(\frac{1}{9}\right)^3}{125^{-\frac{1}{3}} \cdot \left(\frac{1}{5}\right)^{-2} + (\sqrt{3})^0 \cdot \left(\frac{1}{2}\right)^{-2}}$$

Solution : (1) $2{\cdot}4^{-2} + \left(81^{-\frac{1}{2}}\right)^3 \cdot \left(\frac{1}{9}\right)^{-3}$

$$= 2{\cdot}\frac{1}{4^2} + \left(\frac{1}{\sqrt{81}}\right)^3 \cdot 9^3$$
$$= \frac{1}{8} + \frac{1}{9^3} \cdot 9^3$$
$$= \frac{1}{8} + 1 = \frac{9}{8}.$$

(2) $125^{-\frac{1}{3}} \cdot \left(\frac{1}{5}\right)^{-2} + (\sqrt{3})^0 \cdot \left(\frac{1}{2}\right)^{-2}$

$$= \frac{1}{\sqrt[3]{125}} \cdot 5^2 + 1{\cdot}2^2$$
$$= \frac{1}{5} \cdot 5^2 + 1{\cdot}2^2$$
$$= 5 + 4 = 9.$$

(3) $\frac{9}{8} : 9 = \frac{1}{8}$ the fractional is equal to $\frac{1}{8}$.

Illustration 30. Simplify the expression

$$\sqrt[3]{x^{-1/2}\,y^{-2}} \cdot (x^{-5/2}\,y^{-4})^{-\frac{1}{3}} \cdot \sqrt{x^{-2}\,y^{-\frac{1}{3}}}$$

Solution : We have

$$x^{-1/2}\,y^{-2} = \frac{1}{y^2\sqrt{x}} \qquad \text{If } x > 0, y \neq 0;$$

$$x^{-\frac{5}{2}}\,y^{-4} = \frac{1}{y^4\sqrt{x^5}} = \frac{1}{x^2y^4\sqrt{x}} \qquad \text{If } x > 0, y \neq 0;$$

$$\sqrt{x^{-2}\,y^{-\frac{1}{3}}} = \frac{1}{\sqrt{x^2\sqrt[3]{y}}} = \frac{1}{|x|\sqrt[6]{y}} \qquad \text{If } x \neq 0; y > 0.$$

The given expression has sense for $x > 0$ and $y > 0$, we obtain

$$= \sqrt[3]{\frac{1}{y^2\sqrt{x}}}\sqrt[3]{x^2y^4\sqrt{x}} \times \frac{1}{x\sqrt[6]{y}}$$

$$= \sqrt[3]{\frac{x^2 y^4 \sqrt{x}}{y^2 \sqrt{x}}} \cdot \frac{1}{x \sqrt[6]{y}}$$

$$= \sqrt[3]{x^2 y^2} \cdot \frac{1}{x \sqrt[6]{y}}$$

$$= \frac{1}{x} \sqrt[6]{\frac{x^4 y^4}{y}} = \frac{1}{x} \sqrt[6]{x^4 y^3}$$

If $x > 0$ and $y > 0$.

33. Notion of a Power with an Irrational Exponent :

The Properties of a Power with a Real Exponent

Let a be a positive real number and let x be a positive irrational number. As any irrational number, x is a non-terminating non-periodic decimal : $x = c.\, b_1 b_2 \ldots .b_n \ldots\ldots$ deficit $c.b_1, c\ b_1 b_2 \ldots . b_n, \ldots ..$ and with excess $c.b_1 + 1$, $c.b_1(b_2 + 1), \ldots c.b_1 b_2 \ldots .(b_n + 1), \ldots ..$

Let us form two new sequences :

$$a^{c.b_1}, a^{c.b_1 b_2}, \ldots\ldots, a^{c.b_1 b_2 \ldots b_n} \ldots\ldots..$$

$$a^{c.b+1}, a^{c.b_1(b_2+1)}, \ldots\ldots\ldots . a^{c.b_1 b_2 \ldots (b_n+1)} \ldots ..$$

This number is denoted by a^x and is called a power of the number $a>0$ with an irrational exponent x.

For instance, the power $2^{\sqrt{2}}$ is a number for which the sequences $2^{1.4}$, $2^{1.41}$, $2^{1.414}$, $2^{1.4142}$……and $2^{1.5}$, $2^{1.42}$, $2^{1.415}$, $2^{1.4143}$,……..are sequences of decimal approximations with deficit and with excess, respectively ($\sqrt{2}$ = 1·4142…).

By definition, $\quad a^{-x} = \dfrac{1}{a^x} \ (a > 0)$,

where x is a positive irrational number.

Theorem : *For any real x and y and admissible a and b the following equalities are valid* :

(1) $a^x \cdot a^y = a^{x+y}$;

(2) $(a^x)^y = a^{xy}$;

(3) $\dfrac{a^x}{a^y} = a^{x-y}$;

(4) $(ab)^x = a^x \cdot b^x$;

(5) $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$.

34. Worked problems

Illustration 31. Prove that $16^5 + 2^{15}$ is divisible by 33.

Solution : $16^5 + 2^{15} = (2^4)^5 + 2^{15} = 2^{20} + 2^{15} = 2^{15}(2^5 + 1) = 2^{15} \cdot 33$.

Illustration 32. Prove that the numbers $\sqrt{3}$ and log 3 are irrational.

Solution : (1) Suppose that $\sqrt{3}$ is a rational number :

$$\sqrt{3} = \frac{p}{q},$$

where p and q are integers and $q \neq 0$. The fraction $\dfrac{p}{q}$ will be regarded as irreducible. According to the definition of the root, we have

$$\left(\frac{p}{q}\right)^2 = 3 \text{ or } p^2 = 3q^2$$

Hence, it follows that p is divisible by 3 : $p = 3p_1$ where p_1 is a whole number. Then $(3p_1)^2 = 3q^2$ = or $q^2 = 3p^2{}_1$. Hence, q is also divisible by 3 : $q = 3q_1$ where q_1 is a whole number. Consequently the fraction $\dfrac{p}{q} = \dfrac{3p_1}{3q_1}$ is reducible, which contradicts the hypothesis. We conclude that $\sqrt{3}$ is an irrational number.

Remark. We have used the following statements: if the square of a whole number is divisible by 3, then the number itself is also divisible by 3. The reader is invited to prove this statement.

(2) Let $\log_2 3$ be a rational number $\dfrac{p}{q}$, where p and q are whole numbers and $q \neq 0$. Then $\log_2 3 = \dfrac{p}{q}$. By the definition of logarithm, $2^{\frac{p}{q}} = 3$ or $(2^{p/q})^q = 3^q$, we obtain $2^p = 3^q$. But this equality is impossible since 2^p is an even number and 3^q is an odd number. Hence, our supposition is false and $\log_2 3$ is an irrational number.

Illustration 33. Simplify the expression $3\sqrt{\dfrac{2}{3}} - 2\sqrt{\dfrac{3}{2}} + \sqrt{6} + \sqrt{150}$.

Solution :

$$3\sqrt{\frac{2}{3}} - 2\sqrt{\frac{3}{2}} + \sqrt{6} + \sqrt{150}.$$

$$= \sqrt{3^2 \cdot \frac{2}{3}} - \sqrt{2^2 \cdot \frac{3}{2}} + \sqrt{6} + \sqrt{25 \cdot 6}$$

$$= \sqrt{6} - \sqrt{6} + \sqrt{6} + 5\sqrt{6}.$$

$$= 6\sqrt{6}.$$

Illustration 34. For what values of x are the following equalities true :

(a) $x + |x| = 0$; (b) $x + |x| = 2x$;

(c) $\dfrac{x}{|x|} = -1$; (d) $x\sqrt{3} = -\sqrt{3x^2}$;

(e) $3x\sqrt{2} = \sqrt{18x^2}$?

Solution : (a) $x + |x| = 0$; or $|x| = -x$. The equality is true for any $x \leq 0$.

(b) $x + |x| = 2x$ or $|x| = x$. The equality is true for any $x \geq 0$.

(c) $\dfrac{x}{|x|} = -1$ or $|x| = -x$ where $x \neq 0$. The equality is true for $x < 0$.

(d) $x\sqrt{3} = -\sqrt{3x^2}$ or $x\sqrt{3} = -|x|\sqrt{3}$, $x = -|x|$. The equality is true for $x \leq 0$.

(e) $3x\sqrt{2} = \sqrt{18x^2}$ or $3x\sqrt{2} = 3|x|\sqrt{2}$, $x = -|x|$. The equality is true for $x \geq 0$.

Illustration 35. Simplify the expression $\sqrt{x^2-2x+1} - \sqrt{x^2+2x+1}$.

Solution : Note that $x^2 - 2x + 1 = (x-1)^2$, $x^2 + 2x + 1 = (x+1)^2$. Therefore,

$$\begin{aligned}\sqrt{x^2-2x+1}-\sqrt{x^2+2x+1}. &= \sqrt{(x-1)^2}-\sqrt{(x+1)^2}\\ &= |x-1|-|x+1|.\end{aligned}$$

By the definition of modulus,

$$|x-1| = \begin{cases} x-1 \text{ if } (x-1) \geq 0, \\ -(x-1) \text{ if } (x-1) < 0 \end{cases}$$

Or $$|x-1| = \begin{cases} x-1 \text{ if } x \geq 1, \\ -(x-1) \text{ if } x < 1; \end{cases}$$

$$|x+1| = \begin{cases} x+1 \text{ if } (x+1) \geq 0, \\ -(x+1) \text{ if } (x+1) < 0. \end{cases}$$

Or $$|x+1| = \begin{cases} x+1 \text{ if } x \geq -1, \\ -(x+1) \text{ if } x < -1. \end{cases}$$

The points x = – 1 and x =1 divides the number axis into three intervals : $(-\infty, -1)$, $(-1, 1)$, $(1, +\infty)$.

If $x \leq -1$, then $|x-1| - |x+1|$

$$\begin{aligned} &= -(x-1) - (-(x+1)) \\ &= -x + 1 + x + 1 = 2. \end{aligned}$$

If $-1 < x < 1$, then $|x-1| - |x+1|$

$$\begin{aligned} &= -(x-1) - (x+1) \\ &= -2x. \end{aligned}$$

If $x \geq 1$, then $|x-1| - |x+1|$

$$= (x-1) - (x+1) = -2.$$

Thus,

$$\sqrt{x^2-2x+1}-\sqrt{x^2+2x+1} = \begin{cases} 2 & \text{if} \quad x \leq -1, \\ -2x & \text{if} \quad -1 < x < 1, \\ -2 & \text{if} \quad x \geq 1. \end{cases}$$

Illustration 36. For what values of x are the following inequalities true :

(a) $|-x| \leq x$; (b) $x|x| \geq x^2$;

(c) $\sqrt{x^2} \leq -x$; (d) $x\sqrt{2} > \sqrt{2x^2}$?

Solution : (a) $|-x| = |x|$. The inequality $|x| \leq x$ is correct for $x \geq 0$.

(b) The inequality is not fulfilled for $x < 0$. For $x \geq 0$, $|x| = x$, and the slack inequality $x^2 \geq x^2$ is correct.

(c) $\sqrt{x^2} = |x|$; $|x| \leq -x$ for $x \leq 0$.

(d) $\sqrt{2x^2} = |x|\sqrt{2}$; $x\sqrt{2} > |x|\sqrt{2}$ or $x > |x|$ is incorrect for any x.

Illustration 37. Prove that the inequality $3\sqrt{2} + 2\sqrt{7} > 3\sqrt{3} + 4$ is valid.

Solution : Let us bring the factors in the radicands and rewrite the given inequality as follows; $\sqrt{3^2 \cdot 2} + \sqrt{2^2 \cdot 7} > \sqrt{3^2 \cdot 3} + \sqrt{16}$ or $\sqrt{18} + \sqrt{28} > \sqrt{27} + \sqrt{16}$. This equality is reliable, since it is obtained by adding up two correct inequality $\sqrt{18} > \sqrt{16}$ and $\sqrt{28} > \sqrt{27}$.

Illustration 38. Arrange the following numbers in increasing order: $\sqrt[3]{3}$, $\sqrt[4]{4}$ and $\sqrt[5]{5}$.

Solution : Note that $\sqrt[4]{4} = \sqrt[4]{2^2} = \sqrt{2}$. Compare first $\sqrt[3]{3}$ and $\sqrt{2}$. Reduce these two numbers to a common index 6 : $\sqrt[3]{3} = \sqrt[6]{3^2} = \sqrt[6]{9}$ and $\sqrt{2} = \sqrt[6]{2^3} = \sqrt[6]{8}$, hence $\sqrt[3]{3} > \sqrt{2}$.

Compare now $\sqrt{2}$ and $\sqrt[5]{5}$. : $\sqrt{2} = \sqrt[10]{2^5} = \sqrt[10]{32}$ and $\sqrt[5]{5}. = \sqrt[10]{5^2}$

$= \sqrt[10]{25}$. Consequently, $\sqrt{2} > \sqrt[5]{5}$.

Thus, $\sqrt[5]{5}. < \sqrt[4]{4} < \sqrt[3]{3}$.

Illustration 39. Evaluate the expression

$$\left(\frac{a^{5/12}\, a^{-3/8}}{a^{7/24}}\right)^{-\frac{4}{3}} \text{ For } a = 125.$$

Solution : We first simplify the given expression, applying the property of powers with a fractional exponent :

$$\left(\frac{a^{5/12}\, a^{-3/8}}{a^{7/24}}\right)^{-\frac{4}{3}} = (a^{5/12}\, a^{-3/8}\, a^{7/24})^{-4/3}$$

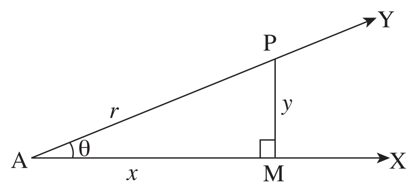
$$(a^{1/4})^{-4/3} = a^{1/3}\sqrt[3]{a}$$

For $a = 125$, we obtain $\sqrt[3]{125} = 5$.

Trigonometry

Important Point and Formula

1. In the right triangle AMP right-angled at M, Base = AM = x, perpendicular = PM = y and Hypotenuse = AP = r.



We define the following six trigonometric ratios :

(i) $\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{y}{r}$, and is written as $\sin\theta$.

(ii) Cosine $\theta = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{x}{r}$, as written as $\cos\theta$.

(iii) Tangent $\theta = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{y}{x}$, and is written as $\tan\theta$.

(iv) Cosecant $\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{r}{y}$, and is written as cosec θ.

(v) Secant $\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{r}{x}$, and is written as sec θ.

(vi) Cotangent $\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{x}{y}$, and is written as cot θ.

Note : It should be noted that sin θ is an abbreviation for "since of angle θ", it is not the product of sin and θ. Similar is the case for other trigonometric ratios.

2. Some useful trigonometric identities

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $\cos^2\theta = 1 - \sin^2\theta$

(iii) $\sin^2\theta = 1 - \cos^2\theta$

(iv) $1 + \tan^2\theta = \sec^2\theta$

(v) $\sec^2\theta - \tan^2\theta = 1$

(vi) $\sec^2\theta - 1 = \tan^2\theta$

(vii) $1 + \cot^2\theta = \text{cosec}^2\theta$

(viii) $\text{cosec}^2\theta - \cot^2\theta = 1$

(ix) $\text{cosec}^2\theta - 1 = \cot^2\theta$

(x) $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$

(xi) $\text{cosec}\,\theta + \cot\theta = \frac{1}{\text{cosec}\,\theta - \cot\theta}$

3. Complementary angle are the angles whose sum is 90^0.

Trigonometric ratios of complementary angles :

(i) $\sin(90^0 - \theta) = \cos\theta$

(ii) $\cos(90^0 - \theta) = \sin\theta$

(iii) $\tan(90^0 - \theta) = \cot\theta$

(iv) $\cot(90^0 - \theta) = \tan\theta$

(v) $\sec(90^0 - \theta) = \text{cosec}\,\theta$

(vi) $\text{cosec}(90^0 - \theta) = \sec\theta$

Trigonometric Ratios of Particular Angles :

T-ratio/θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

2 H.C.F. and L.C.M. of Polynomials

We have studied greatest common divisor and least common multiple of numbers in our lower classes. Here, we shall discuss highest common factor (H.C.F. or Greatest Common Divisor—G.C.D.) and least common multiple (L.C.M.) or polynomials with integral coefficients.

Highest Common Factor

A common factor of two or more polynomials is a polynomial which divides each of them exactly. For example, x is a common factor of x^2y and xz; xy or x^2y or xy^2 or x^2y^2 or x^2y^3 etc., x^2y^3 are common factors of x^3y^3 and x^2y^5. Similarly, $x - 1$ is a common factor of $x^2 - 1$ and $x^3 - 1$.

Illustration 1.

Find the H.C.F of $a^2b^4c^3$ and $a^3b^2c^5$.

Solution :

H.C.F of a^3 and a^2 is a^2,

H.C.F. of b^2 and b^4 is b^2 and H.C.F. of c^5 and c^3 is c^3

$\therefore$ Required H.C.F. is $a^2b^2c^3$.

Illustration 2.

Find the G.C.D. of $24x^2y^3z^3$, $36x^3y^5z^{11}$ and $18xy^2z^7$.

Solution :

We have

$$\begin{aligned} 24x^2y^3z^3 &= 3^1 \times 2^3 \times x^2y^3z^3 \\ 36x^3y^5z^{11} &= 3^2 \times 2^2 \times x^3y^5z^{11} \\ 18xy^2z^7 &= 3^2 \times 2^1 \times x \times y^2 \times z^7 \\ \therefore \text{ Required G.C.D.} &= 3^1 \times 2^1 \times x \times y^2 \times z^3 \\ &= 6xy^2z^3. \end{aligned}$$

Illustration 3.

Find the H.C.F. of $x^2 + 5x + 6$ and $x^2 + x - 6$.

Solution :

$$\begin{aligned} \text{Here,}\quad x^2 + 5x + 6 &= x^2 + 2x + 3x + 6 \\ &= x(x + 2) + 3(x + 2) \\ &= (x + 2)(x + 3) \\ x^2 + x - 6 &= x^2 + 3x - 2x - 6 \\ &= x(x + 3) - 2(x + 3) \\ &= (x + 3)(x - 2) \end{aligned}$$

Only common factor is $x + 3$

$\therefore$ H.C.F. $= x + 3$.

Illustration 4.

Find the G.C.D. of the polynomials $2x^2 - x - 1$ and $4x^2 + 8x + 3$.

Solution :

$$\begin{aligned} \text{Here,}\quad 2x^2 - x - 1 &= 2x^2 - 2x + x - 1 \\ &= 2x(x - 1) + 1(x - 1) \\ &= (2x + 1)(x - 1) \\ 4x^2 + 8x + 3 &= 4x^2 + 2x + 6x + 3 \\ &= 2x(2x + 1) + 3(2x + 1) \\ &= (2x + 1)(2x + 3) \end{aligned}$$

Only common factor is $(2x + 1)$.

G.C.D. $= 2x + 1$.

Illustration 5.

Find the H.C.F. of the polynomials P(x) and Q(x) where P(x) $= 10x^2y(x^3 + y^3)$ and Q(x) $= 15xy^2(x^4 - y^4)$.

Solution :

$$\begin{aligned} P(x) &= 10x^2y(x^3 + y^3) \\ &= 10x^2y(x + y)(x^2 - xy + y^2) \\ Q(x) &= 15xy^2(x^4 - y^4) \\ &= 15xy^2(x^2 + y^2)(x^2 - y^2) \\ &= 15xy^2(x + y)(x - y)(x^2 + y^2) \end{aligned}$$

Now, H.C.F. of 10 and 15 is 5.

Common factors are x, y and $x + y$.

$\therefore$ H.C.F. of P(x) and Q(x) $= 5xy(x + y)$.

Illustration 6.

Find the H.C.F. of $a^3 + b^3 + c^3 - 3abc$ and $(a + b + c)^3$, given that $ab + bc + ca = 0$.

Solution :

$$\begin{aligned} \text{Let}\quad P(a, b, c) &= a^3 + b^3 + c^3 - 3abc \\ \text{and}\quad Q(a, b, c) &= (a + b + c)^3 \end{aligned}$$

We know that

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a + b + c)^2 - 3(ab + bc + ca)]$$

We are given that $ab + bc + ca = 0$

$$\begin{aligned} \therefore\ a^3 + b^3 + c^3 - 3abc &= (a + b + c)(a + b + c)^2 \\ &= (a + b + c)^3 \\ P(a, b, c) &= (a + b + c)^3. \end{aligned}$$

Thus, from (i) and (ii), required H.C.F. is $(a + b + c)^3$.

Illustration 7.

If H.C.F. of the polynomial $x^3 - 2x^2 + px + 6$ and $x^3 - x^2 - 5x + q$ is $x - 3$, find the value of $3p + 2q^2$.

Solution :

As $x - 3$ is the H.C.F. of the given polynomials

$x - 3$ is a factor of

$x^3 - 2x^2 + px + 6$ ...(i)

and $x^3 - x^2 - 5x + q$ ...(ii)

$\therefore$ When $x - 3 = 0$ or $x = 3$, then the value of polynomials (i) and (ii) is zero.

Putting $x = 3$ in (i) , we get

$$(3)^3 - 2(3)^2 + p(3) + 6 = 0$$

Or $\quad 27 - 18 + 3p + 6 = 0$ or $3p = -15$

Or $\quad p = -5$

Similarly, putting $x = 3$ in (ii), we get

$$(3)^3 - (3)^2 - 5(3) + q = 0$$

Or $\quad 27 - 9 - 15 + q = 0$

Or $\quad 3 + q = 0$ or $q = -3$

$\therefore \quad 3p + 2q^2 = 3(-5) + 2(-3)^2$

$= -15 + 18 = 3.$

Illustration 8.

If $x^2 - x - 2$ is the G.C.D. of the expressions $(x - 2)(2x^2 + ax + 1)$ and $(x + 1)(3x^2 + bx + 2)$, find the values of a and b.

Solution :

Here, $\quad$ G.C.D. $= x^2 - x - 2$

$= x^2 - 2x + x - 2$

$= x(x - 2) + 1(x - 2)$

$= (x - 2)(x + 1)$

We are given that $(x - 2)(x + 1)$ is also a factor of $(x - 2)(2x^2 + ax + 1)$.

$\therefore \quad (x + 1)$ is a factor of $2x^2 + ax + 1$

$\therefore \quad$ When $x + 1 = 0$ *i.e.*, $x = -1$, $2x^2 + ax + 1 = 0$

i.e., $2(-1)^2 + a(-1) + 1 = 0$

or $\quad 2 - a + 1 = 0$ or $a = 3$

Similarly, as $(x - 2)(x + 1)$ is also a factor of $(x + 1)(3x^2 + bx + 2)$.

$\therefore \quad (x - 2)$ is a factor of $3x^2 + bx + 2$

When $x - 2 = 0$ or $x = 2$, $3x^2 + bx + 2 = 0$

i.e., $\quad 3(2)^2 + b(2) + 2 = 0$

or $\quad 12 + 2b + 2 = 0$

or $\quad 2b = -14$

or $\quad b = -7$

Thus, $a = 3$, $b = -7$.

Lowest Common Multiple

A common multiple of two or more polynomials is a polynomial which is exactly divisible by each of them. For example, x^3 is a common multiple of x^2 and x^3 and xyz is a common multiple of xy and yz.

Illustration 9.

Find the L.C.M. of $12a^2b^3c^2$ and $18a^4b^2c^3$

Solution :

$12a^2b^3c^2 = 2^2 \times 3 \times a^2 \times b^3 \times c^2$

and $\quad 18a^4b^2c^3 = 2 \times 3^2 \times a^4 \times b^2 \times c^3$

$\therefore$ Required L.C.M. $= 2^2 \times 3^2 \times a^4 \times b^3 \times c^3$

$= 36a^4b^3c^3.$

Illustration 10.

Find the L.C.M. of the following polynomials :

$(x + 3)^2(x + 4)$, $(x - 3)(x + 4)^2$

Solution :

Let $\quad P(x) = (x - 3)^2 (x + 4)$

and $\quad Q(x) = (x - 3)(x + 4)^2$

Now, L.C.M of $(x - 3)^2$ and $(x - 3)$ is $(x - 3)^2$

and L.C.M. of $(x + 4)$ and $(x + 4)^2$ is $(x + 4)^2$

Thus, L.C.M. of P(x) and Q(x) is $(x - 3)^2 (x + 4)^2$.

Illustration 11.

Find the L.C.M. of $x^3 + x^2 + x + 1$ and $x^3 + 2x^2 + x + 2$.

Solution :

$x^3 + x^2 + x + 1 = x^2(x + 1) + 1(x + 1)$

$= (x^2 + 1)(x + 1)$

and $\quad x^3 + 2x^2 + x + 2 = x^3 + x + 2x^2 + 2$

$= x(x^2 + 1) + 2(x^2 + 1)$

$= (x^2 + 1)(x + 2)$

Here, $(x^2 + 1)$, $(x + 1)$ and $(x + 2)$ each has highest degree 1.

L.C.M. $= (x^2 + 1)(x + 1)(x + 2)$

Illustration 12.

Find the L.C.M of the following polynomials :

$P(x) = 12(x^4 - 36)$; $Q(x) = 8(x^4 + 5x^2 - 6)$

Solution :

Here, $\quad P(x) = 12(x^4 - 36)$

$= 12[(x^2)^2 - (6)^2]$

$= 12(x^2 - 6)(x^2 + 6)$

and $\quad Q(x) = 8(x^4 + 5x^2 - 6)$

$= 8(x^4 + 6x^2 - x^2 - 6)$

$= 8[x^2(x^2 + 6) - 1(x^2 + 6)]$

$= 8(x^2 - 1)(x^2 + 6)$

Now, L.C.M of 12 and 8 is 24.

The factors (x^2-6), (x^2+6), (x^2-1) each has highest degree 1.

$\therefore$ L.C.M. of P(x) and Q(x)

$$= 24\,(x^2-6)(x^2+6)(x^2-1)$$

Product of Two Polynomials

Product of two polynomials = The product of their H.C.F. and L.C.M.

$$f(x) \times g(x) = \text{H.C.F.} \times \text{L.C.M.}$$

Let α and ∂ be two polynomials and H be their H.C.F. and L be their L.C.M.

Then, $\quad \text{LH} = \pm\,\alpha\partial$

Or $\quad \text{L} = \pm\dfrac{\alpha\partial}{\text{H}}$

and $\quad \text{H} = \pm\dfrac{\alpha\partial}{\text{L}}$

Notes :

1. If $f(x)$, $g(x)$ and one of L.C.M., H.C.F. are given, the other can be found without ambiguity because L.C.M. and H.C.F., are unique except for a factor of (-1). The sign + or – is so chosen that the coefficient of the highest degree term becomes positive.

2. If L.C.M., H.C.F. and one of the polynomials are given, then the second polynomial can have both + and – signs.

3. If both L.C.M. and H.C.F. are to be found, then we first find H.C.F. and then L.C.M.

Illustration 13.

Find the H.C.F and L.C.M of the following given polynomials and verify that the product of the L.C.M and H.C.F differs from the product of the polynomials if at all by a factor (-1).

(a) $f(x) = (2x+1)^2\,(3x-1)$, $g(x) = (2x+1)(3x-1)^3$

(b) $f(x) = 4(x^2-1)$, $g(x) = 12\,(1-x^3)$

Solution :

(a) We are given

$$f(x) = (2x+1)^2\,(3x-1)$$
$$g(x-1) = (2x+1)(3x-1)^3$$

Here, common factor are $(2x+1)$ and $(3x-1)$ and their least powers are 1 and 1 respectively.

$\therefore \quad \text{H.C.F.} = (2x+1)(3x-1)$

The highest powers of all the factors $(2x+1)$ and $(3x-1)$ are 2 and 3 respectively.

$$\text{L.C.M.} = (2x+1)^2\,(3x-1)^3$$

Product of H.C.F and L.C.M $= (2x+1)\,(3x-1)\,(2x+1)^2\,(3x-1)^3$

$$= (2x+1)^3\,(3x-1)^4 \qquad \ldots\text{(i)}$$

And product of $f(x)$ and $g(x) = (2x+1)^2\,(3x-1)\,(2x+1)\,(3x-1)^3$

$$= (2x+1)^3\,(3x-1)^4 \qquad \ldots\text{(ii)}$$

Comparing (i) and (ii), we get

$$\text{H.C.F.} \times \text{L.C.M.} = f(x) \times g(x)$$

(b) Here, $\quad f(x) = 4(x^2-1) = 2^2(x+1)(x-1)$

and $\quad g(x) = 12(1-x^3)$

$$= 2^2 \times 3(1-x)(1+x+x^2)$$
$$= -\,2^2 \times 3(x-1)(1+x+x^2)$$

The least powers of the common factors 2, $(x-1)$ are 2 and 1 respectively.

$\therefore \quad \text{H.C.F.} = 2^2(x-1) = 4(x-1)$

The highest powers of all the factors 2, 3, $(x-1)$, $(x+1)$ and $(1+x+x^2)$ are 2, 1, 1, 1 and 1.

$\therefore \quad \text{L.C.M.} = 2^2 \times 3(x-1)(x+1)(1+x+x^2)$

$$= 12(x+1)(x^3-1)$$

Product of H.C.F. and L.C.M.

$$= 4(x-1) \times 12\,(x+1)(x^3-1)$$
$$= 48(x^2-1)(x^3-1) \qquad \ldots\text{(iii)}$$

and product of $f(x)$ and $g(x) = 4(x-1) \times 12\,(1-x^3)$ $= 48(x^2-1)(x^3-1)$ $\qquad \ldots$(iv)

Comparing (iii) and (iv), we get

$$\text{H.C.F.} \times \text{L.C.M.} = -f(x) \times g(x)$$

which prove the required result.

Illustration 14.

The H.C.F. of two polynomials $f(x) = (x-1)(x^2-x-6)$ and $g(x) = (x-2)(x^2-1)$ is $x-1$. Find their L.C.M.

Solution :

$$f(x) = (x-1)(x^2-x-6)$$
$$= (x-1)(x-3)(x+2)$$

and $\quad g(x) = (x-2)(x^2-1)$

$$= (x-1)(x+1)(x-2)$$

Given that H.C.F. $= x-1$

Then, L.C.M. $= \pm\dfrac{\alpha\partial}{\text{H}}$

$$= \pm\frac{(x-1)(x-3)(x+2)(x-1)(x+1)(x-2)}{(x-1)}$$
$$= \pm\,(x-3)(x+2)(x-1)(x+1)(x-2)$$
$$= \pm\,(x^2-1)(x^2-4)(x-3)$$

To make the coefficient of x^5 positive, we take + ve sign

$\therefore \quad \text{L.C.M.} = (x^2-1)(x^2-4)(x-3).$

Illustration 15.

The G.C.D. of two polynomials α and ∂ is $(x^2-4x-5)(x^2-5x+6)$. If $\alpha = 5(x+1)(x^2-7x+10)(x^2-x-6)$ and $\partial = 9(x-2)(x^2-4x-5)(x^2-9)$, then find the L.C.M. of α and ∂.

Solution :

Here, $\quad \alpha = 5(x+1)(x^2-7x+10)(x^2-x-6)$

$$= 5(x+1)(x-2)(x-5)(x-3)(x+2)$$

and $\quad \partial = 9(x-2)(x^2-4x-5)(x^2-9)$

$$= 9(x+1)(x-2)(x-5)(x-3)(x+3)$$

and G.C.D. of α and ∂

$$= (x^2-4x-5)(x^2-5x+6)$$
$$= (x+1)(x-2)(x-5)(x-3)$$

We know that the L.C.M. $= \pm \frac{\alpha \partial}{H}$

$= \pm 45(x^2 + 3x + 2)(x^2 - 7x + 10)(x^2 - 9)$

We take + ve sign to make the coefficient of the highest degree term positive.

Thus, the L.C.M. of α and ∂ is $45(x^2 + 3x + 2)(x^2 - 7x + 10)(x^2 - 9)$.

Exercise A

1. Find the G.C.D. of the following polynomials—

   $18(x^3 - x^2 + x - 1)$; $12(x^4 - 1)$
2. Find the G.C.D. of the following polynomials—

   $2(x^4 - y^4)$; $3(x^3 + 2x^2y - xy^2 - 2y^3)$
3. Find the G.C.D. of $(x^3 + 2x^2 - 3x)$ and $(2x^3 + 5x^2 - 3x)$
4. Find the G.C.D. of the following polynomials—

   $6x^3 + 3x^2 - 3x$, $8x^3 + 8x^2 - 6x$
5. If $(x^2 - x - 2)$ is the G.C.D. of the expressions $(x - 2)(2x^2 + ax + 1)$ and $(x + 1)(3x^2 + bx + 2)$. Find the values of a and b.
6. If $(x^2 - x - 6)$ is the G.C.D. of the expressions $(x + 2)(2x^2 + ax + 3)$ and $(x - 3)(3x^2 + bx + 8)$. Find the values of of a and b.
7. $(x^2 + x - 2)$ is the G.C.D. of the expressions $(x - 1)(2x^2 + ax + 2)$ and $(x + 2)(3x^2 + bx + 1)$. Find the values of a and b.
8. The L.C.M. and G.C.D. of the polynomials. P(x) and Q(x) are $56(x^4 + x)$ and $4(x^2 - x + 1)$ respectively. If P(x) $= 28(x^3 + 1)$. Find Q(x).
9. The L.C.M. and G.C.D. of two polynomials P(x) and Q(x) are $2(x^4 - 1)$ and $(x + 1)(x^2 + 1)$ respectively. If P(x) $= x^3 + x^2 + x + 1$ find Q(x).
10. If P(x) $= (x^3 + 1)(x - 1)$ and Q(x) $= (x^2 - x + 1)(x^2 - 3x + 2)$. Find G.C.D. and L.C.M. of P(x) and Q(x).
11. Find the G.C.D. and L.C.M. of two polynomials P(x) and Q(x). where P(x) $= (x^3 - 27)(x^3 - 3x + 2)$ and Q(x) $= (x^2 + 3x + 9)(x^2 - 5x + 6)$.
12. $(x - k)$ is the G.C.D. of $(x^2 + x - 12)$ and $(2x^2 - kx - 9)$. Find the value of k.
13. $(x + 1)(x - 4)$ is the G.C.D. of the polynomials $(x - 4)(2x^2 + x - a)$ and $(x + 1)(2x^2 + bx - 12)$. Find a and b.
14. For what value of k, the G.C.D. of the $x^2 + x - (2k + 2)$ and $(2x^2 + kx - 12)$ is $(x + 4)$?
15. $(x - 3)$ is the G.C.D. of $(x^3 - 2x^2 + px + 6)$ and $(x^2 - 5x + q)$. Find $(6p + 5q)$.

Find the values of a and b so that the polynomials P(x), Q(x) and have H(x) as their H.C.F. :

16. P(x) $= (x^2 + 3x + 2)(x^2 - 7x + a)$

    Q(x) $= (x^2 - 2x - 8)(x^2 - 2x + b)$

    H(x) $= (x + 1)(x - 4)$
17. P(x) $= (x^2 - 3x + 2)(x^2 + 7x + a)$

    Q(x) $= (x^2 + 5x + 4)(x^2 - 5x + b)$

    H(x) $= (x - 1)(x + 4)$
18. P(x) $= (x^2 + 3x + 2)(x^2 + 2x + a)$

    Q(x) $= (x^2 + 7x + 12)(x^2 + 7x + b)$

    H(x) $= (x + 1)(x + 3)$
19. Find the G.C.D. of $4(x^4 - 1)$ and $6(x^3 - x^2 - x + 1)$.
20. If the G.C.D. of $(x - 5)(x^2 - x - a)$ and $(x - 4)(x^2 - 2x - b)$ is $(x - 4), (x - 5)$. Find the value of a, b.
21. Find the value of k for which the G.C.D. of $(x^2 - 2x - 24)$ and $(x^2 - kx - 6)$ is $(x - 6)$.

Find the G.C.D. of the following polynomials :

22. $x^2 - x - 6$; $x^3 - 27$
23. $2x^2 - 9x + 4$; $8x^3 - 1$
24. $18(6x^4 + x^3 - x^2)$; $45(2x^6 + 3x^5 + x^4)$
25. $12(3x^4 - 14x^3 - 5x^2)$; $30(3x^5 + 4x^4 + x^3)$
26. $18x^3 + 45x^2 - 27x$; $15x^4 - 135x^2$

Exercise B

Find the L.C.M. of the following polynomials :

1. $18x^4 - 36x^3 + 18x^2$; $45x^6 - 45x^3$
2. $12x^4 + 324x$; $36x^3 + 90x^2 - 54x$
3. The G.C.D. of polynomials P(x) and Q(x) is $(x - 3)$. If P(x) $= (x - 3)(x^2 + x - 2)$ and Q(x) $= x^2 - 5x + 6$. Find the L.C.M. of P(x) and Q(x).
4. The G.C.D. of polynomials G(x) and H(x) is $10(x + 3)(x - 1)$. If the polynomial G(x) $= 10(x^2 - 9)(x^2 - 3x + 2)$ and H(x) $= 10x(x + 3)(x - 1)^2$. Find the L.C.M. of G(x) and H(x).
5. The G.C.D. and L.C.M. of the polynomials P(x) and Q(x) respectively are $5(x + 3)(x - 1)$ and $20x(x^2 - 9)(x^2 - 3x + 2)$. If P($x$) $= 10(x^2 - 9)(x - 1)$. Find Q(x).
6. The G.C.D. and L.C.M. of two polynomials P(x) and Q(x) are $x(x + a)$ and $12(x + a)(x^2 - a^2)$ respectively. If P(x) $= 4x(x + a)$, find Q(x).
7. The G.C.D. and L.C.M. of two polynomials P(x) and Q(x) are $x(x + a)$ and $12x^2(x - a)(x^2 - a^2)$ respectively. If P(x) $= 4x^2(x + a)^2$, find Q(x).
8. The G.C.D. of two polynomials P(x) $= 4x^2(x^2 - 3x + 2)$ and Q(x) $= 12x(x - 2)(x^2 - 4)$ is $4x(x - 2)$. Find the L.C.M. of the polynomials P(x) and Q(x).
9. Find the L.C.M. of the following polynomial—

   P(x) $= x^4 - 1$; Q(x) $= x^3 - x$.
10. Find the L.C.M. of the following polynomial—

    P(x) $= 12(x^4 - 25)$; Q(x) $= (x^4 + 4x^2 - 5)$.

11. Find the L.C.M. of the following polynomial—

$P(x) = 12(x^4 - 36)$; $Q(x) = 8(x^4 + 5x^2 - 6)$.

12. Find the G.C.D. of $P(x) = 8(x^4 - 16)$; $Q(x) = 12(x^3 - 8)$.

13. Find L.C.M. of the following polynomials—

$4x^2(x^2 - a^2)$; $9x^2(x^3 - a^3)$

14. Find the L.C.M. of the following polynomial—

$3(x^2 - 7x + 12)$; $24(x^2 - 9x + 20)$.

15. The L.C.M. and H.C.F. of two polynomials $P(x)$ and $Q(x)$ are $27x^3(x + a)(x^3 - a^3)$ and $x^2(x - a)$ respectively. If $P(x) = 3x^2(x^2 - a^2)$, find $Q(x)$.

16. Find the G.C.D. of the following polynomials—

$(1 + x + x^3 - x^5)$ and $(1 - x^4 - x^6 + x^7)$

17. If $(x - k)$ be the H.C.F. of $ax^2 + bx + c$ and $cx^2 + ax + b$. Prove that $a^3 + b^3 + c^3 - 3abc = 0$

18. If $H(x)$ and $L(x)$ denote the H.C.F. and L.C.M. of two polynomials $f(x)$ and $g(x)$ such that $H(x) + L(x) = f(x) + g(x)$. Then Prove that $\{H(x)\}^3 + \{L(x)\}^3 = \{f(x)\}^3 + \{g(x)\}^3$.

19. Find all the common roots of the equations—

$x^4 + 5x^3 - x^2 - 5x = 0$ and $x^3 + 4x^2 - x - 4 = 0$

Answer

Exercise A

1. Let

$$\begin{aligned} P(x) &= 18(x^3 - x^2 + x - 1) \\ &= 18[x^2(x - 1) + 1(x - 1)] \\ &= 18(x^2 + 1)(x - 1) \end{aligned}$$

and

$$\begin{aligned} Q(x) &= 12(x^4 - 1) \\ &= 12(x^2 - 1)(x^2 + 1) \\ &= 12(x - 1)(x + 1)(x^2 + 1) \\ \text{G.C.D.} &= 6(x - 1)(x^2 + 1). \end{aligned}$$

2. Let

$$\begin{aligned} P(x) &= 2(x^4 - y^4) \\ &= 2(x^2 - y^2)(x^2 + y^2) \\ &= 2(x - y)(x + y)(x^2 + y^2) \end{aligned}$$

and

$$\begin{aligned} Q(x) &= 3(x^3 + 2x^2y - xy^2 - 2y^3) \\ &= 3(x + 2y)(x^2 - y^2) \\ &= 3(x + 2y)(x + y)(x - y) \\ \text{G.C.D.} &= (x - y)(x + y) = (x^2 - y^2). \end{aligned}$$

3. Let

$$\begin{aligned} P(x) &= x^3 + 2x^2 - 3x \\ &= x(x^2 + 2x - 3) \\ &= x(x^2 + 3x - x - 3) \\ &= x(x + 3)(x - 1) \end{aligned}$$

and

$$\begin{aligned} Q(x) &= 2x^3 + 5x^2 - 3x \\ &= x(2x^2 + 6x - x - 3) \\ &= x(x + 3)(2x - 1) \\ \text{G.C.D.} &= x(x + 3). \end{aligned}$$

4. Let

$$\begin{aligned} P(x) &= 6x^3 + 3x^2 - 3x \\ &= 3x(2x^2 + x - 1) \\ &= 3x(2x^2 + 2x - x - 1) \\ &= 3x(2x - 1)(x + 1) \end{aligned}$$

and

$$\begin{aligned} Q(x) &= 8x^3 + 8x^2 - 6x \\ &= 2x(4x^2 + 4x - 3) \\ &= 2x(4x^2 + 6x - 2x - 3) \\ &= 2x(2x - 1)(2x + 3) \\ \text{G.C.D.} &= x(2x - 1). \end{aligned}$$

5.

$$\begin{aligned} x^2 - x - 2 &= x^2 - 2x + x - 2 \\ &= x(x - 2) + 1(x - 2) \\ &= (x + 1)(x - 2) \\ x &= -1 \text{ or } 2 \end{aligned}$$

Let $P(x) = (x - 2)(2x^2 + ax + 1)$

When $x = -1$

$$\begin{aligned} P(-1) &= (-1 - 2)[2(-1)^2 + a(-1) + 1] \\ &= -3(2 - a + 1) = -3(3 - a) \end{aligned}$$

Since, $(x + 1)$ is a factor of $P(x)$

$$\begin{aligned} P(-1) &= 0 \\ -3(3 - a) &= 0 \\ 3 - a &= 0 \\ a &= 3 \end{aligned}$$

Let $Q(x) = (x + 1)(3x^2 + bx + 2)$

When $x = 2$

$$\begin{aligned} Q(2) &= (2 + 1)(12 + 2b + 2) \\ &= 3(14 + 2b) \end{aligned}$$

Since, $(x - 2)$ is a factor of $Q(x)$

$$\begin{aligned} Q(2) &= 0 \\ 3(14 + 2b) &= 0 \\ 14 + 2b &= 0 \\ 2b &= -14 \\ b &= -7 \\ a &= 3, b = -7. \end{aligned}$$

6.

$$\begin{aligned} x^2 - x - 6 &= x^2 - 3x + 2x - 6 \\ &= x(x - 3) + 2(x - 3) \\ &= (x - 3)(x + 2) \\ x &= 3 \text{ or } -2 \end{aligned}$$

Let $P(x) = (x + 2)(2x^2 + ax + 3)$

When $x = 3$

$$\begin{aligned} P(3) &= (3 + 2) \times [2 \times 9 + 3a + 3] \\ &= (5)(18 + 3a + 3) \\ &= 5(3a + 21) \end{aligned}$$

Since, $(x - 3)$ is a factor of $P(x)$

$$\begin{aligned} P(3) &= 0 \\ 5(3a + 21) &= 0 \\ 3a + 21 &= 0 \\ 3a &= -21 \\ a &= -7 \end{aligned}$$

Let $Q(x) = (x-3)(3x^2+bx+8)$

When $x = -2$

$$\begin{aligned} Q(-2) &= (-2-3)\times(3\times4-2b+8) \\ &= (-5)\times(12-2b+8) \\ &= -5(20-2b) \end{aligned}$$

Since, $(x+2)$ is a factor of $Q(x)$

$$\begin{aligned} Q(-2) &= 0 \\ -5(20-2b) &= 0 \\ 20-2b &= 0 \\ -2b &= -20 \\ b &= 10 \\ a &= -7, b = 10. \end{aligned}$$

7. $$\begin{aligned} x^2+x-2 &= (x+2)(x-1) \\ x &= -2 \text{ or } 1 \end{aligned}$$

Let $P(x) = (x-1)(2x^2+ax+2)$

When $x = -2$

$$\begin{aligned} P(-2) &= (-2-1)\times(8-2a+2) \\ &= -3(10-2a) \end{aligned}$$

Since, $(x+2)$ is a factor of $P(x)$

$$\begin{aligned} P(-2) &= 0 \\ -3(10-2a) &= 0 \qquad \text{[Putting the value } p(-2)] \\ -30+6a &= 0 \\ 6a &= 30 \\ a &= 5 \end{aligned}$$

Let $Q(x) = (x+2)(3x^2+bx+1)$

When $x = 1$

$$Q(1) = (1+2)\times(3+b+1) = 3(4+b)$$

Since, $(x-1)$ is a factor of $Q(x)$

$$\begin{aligned} Q(1) &= 0 \\ 3(4+b) &= 0 \\ 12+3b &= 0 \\ 3b &= -12 \\ b &= -4 \\ a &= 5, b = -4. \end{aligned}$$

8. $$\begin{aligned} P(x) &= 28(x^3+1) \\ Q(x) &= \frac{(\text{L.C.M.})\times(\text{G.C.D.})}{P(x)} \\ &= \frac{56(x^4+x)\times4(x^2-x+1)}{28(x^3+1)} \\ &= \frac{56x(x^3+1)\times4(x^2-x+1)}{28(x^3+1)} \\ &= 8x(x^2-x+1). \end{aligned}$$

9. $$\begin{aligned} P(x) &= x^3+x^2+x+1 \\ &= x^2(x+1)+1(x+1) \\ &= (x^2+1)(x+1) \\ Q(x) &= \frac{(\text{L.C.M.})\times(\text{G.C.D.})}{P(x)} \\ &= \frac{2(x^4-1)\times(x+1)(x^2+1)}{(x^2+1)(x+1)} \\ &= 2(x^4-1). \end{aligned}$$

10. $$\begin{aligned} P(x) &= (x^3+1)(x-1) \\ &= (x+1)(x^2-x+1)(x-1) \\ Q(x) &= (x^2-x+1)(x^2-3x+2) \\ &= (x^2-x+1)[x(x-2)-1(x-2)] \\ &= (x^2-x+1)(x-1)(x-2) \\ \text{G.C.D.} &= (x-1)(x^2-x+1) \\ \text{and L.C.M.} &= (x+1)(x^2-x+1)(x-1)(x-2) \\ &= (x^3+1)(x^2-3x+2). \end{aligned}$$

11. $$\begin{aligned} P(x) &= (x^3-27)(x^2-3x+2) \\ &= (x^3-3^3)(x^2-2x-x+2) \\ &= (x-3)(x^2+3x+9)(x-1)(x-2) \\ Q(x) &= (x^2+3x+9)(x^2-5x+6) \\ &= (x^2+3x+9)(x^2-3x-2x+6) \\ &= (x^2+3x+9)(x-3)(x-2) \\ \text{G.C.D.} &= (x^2+3x+9)(x-2)(x-3) \\ &= (x^3-27)(x-2) \\ \text{and L.C.M.} &= (x-3)(x^2+3x+9)(x-1)(x-2). \end{aligned}$$

12. Let $P(x) = x^2+x-12$

Put $x = k$

$$P(k) = k^2+k-12$$

Since, $(x-k)$ is a factor of $p(x)$.

$$\begin{aligned} P(k) &= 0 \\ \Rightarrow \quad k^2+k-12 &= 0 \\ \Rightarrow \quad (k+4)(k-3) &= 0 \\ \Rightarrow \quad k &= -4 \text{ or } k = 3 \qquad \ldots\text{(i)} \end{aligned}$$

Let $Q(x) = 2x^2-kx-9$

Put $x = k$, $Q(x) = 2k^2-k^2-9$

$$= k^2-9$$

Since, $(x-k)$ is a factor of $Q(x)$

$$\begin{aligned} Q(k) &= 0 \\ k^2-9 &= 0 \\ k^2 &= 9 \\ k &= \pm 3 \qquad \ldots\text{(ii)} \end{aligned}$$

From (i) and (ii)

$k = 3$ is the only common solution.

13. Let $P(x) = (x-4)(2x^2+x-a)$

Put $x = -1$

$$\begin{aligned} P(-1) &= (-1-4)\times(2-1-a) \\ &= -5(1-a) \end{aligned}$$

Since, $(x+1)$ is a factor of $P(x)$

$$P(-1) = 0$$

$$-5(1-a) = 0$$
$$-5+5a = 0$$
$$a = 1$$

Let $Q(x) = (x+1)(2x^2+bx-12)$

Put $x = 4$

$$Q(4) = (4+1)\times(2\times 16+4b-12)$$
$$= 5(4b+20)$$

Since, $(x-4)$ is a factor of $Q(x)$

$$Q(4) = 0$$
$$5(4b+20) = 0$$
$$4b+20 = 0$$
$$b = -5.$$

14. Let $P(x) = x^2+x-(2k+2)$

$$x+4 = 0$$
$$x = -4$$

Then $P(-4) = 16-4-2k-2 = 10-2k$

Since, $(x+4)$ is a factor of $P(x)$

$$P(-4) = 0$$
$$10-2k = 0$$
$$2k = 10$$
$$k = 5.$$

15. $A(x) = x^3 - 2x^2+px+6$

$$A(3) = 27-18+3p+6 = 3p+15$$

Since, $(x-3)$ is a factor of $A(x)$

$$A(3) = 0$$
$$3p+15 = 0$$
$$3p = -15$$
$$p = -5$$
$$B(x) = x^2-5x+q$$
$$B(3) = 9-15+q = q-6$$

Since, $(x-3)$ is a factor of $B(x)$.

$$B(3) = 0$$
$$q-6 = 0$$
$$q = 6$$
$$6p+5q = 6(-5)+5(6)$$
$$= -30+30 = 0.$$

16. $P(x) = (x^2+3x+2)(x^2-7x+a)$

$$= (x+1)(x+2)(x^2-7x+a)$$
$$x-4 = 0$$

Put $x = 4$

Then, $P(4) = (4+1)(4+2)(16-28+a)$

$$= (5)(6)(-12+a) = 30(a-12)$$

Since, $(x+1)(x-4)$ as their H.C.F. ……… (Given)

$$P(4) = 0$$
$$30(a-12) = 0$$
$$a = 12$$
$$Q(x) = (x^2-2x-8)(x^2-2x+b)$$
$$= (x-4)(x+2)(x^2-2x+b)$$

$(x+1) = 0$ Put $x = -1$

Then, $Q(-1) = (-1-4)(-1+2)(1+2+b)$

$$= (-5)(1)(3+b)$$
$$= -5(3+b)$$
$$Q(-1) = 0$$
$$-5(3+b) = 0$$
$$b = -3.$$

17. $P(x) = (x^2-3x+2)(x^2+7x+a)$

$$= (x-2)(x-1)(x^2+7x+a)$$
$$x+4 = 0$$

Put $x = -4$

Then, $P(-4) = (-6)(-5)(16-28+a)$

$$= 30(a-12)$$

Since, $(x-1)(x+4)$ as their HCF ……… (Given)

$$P(-4) = 0$$
$$30(a-12) = 0$$
$$a = 12$$
$$Q(x) = (x^2+5x+4)(x^2-5x+b)$$
$$= (x+1)(x+4)(x^2-5x+b)$$

$(x-1) = 0$; Put $x = 1$

Then, $Q(1) = (2)(5)(1-5+b)$

$$= 10(b-4)$$
$$Q(1) = 0$$
$$10(b-4) = 0$$
$$b = 4.$$

18. $P(x) = (x^2+3x+2)(x^2+2x+a)$

$$= (x+1)(x+2)(x^2+2x+a)$$

Put $x = -3$

Then, $P(-3) = (-3+1)(-3+2)(9-6+a)$

$$= (-2)(-1)(3+a)$$
$$= 2(3+a)$$

Since, $(x+1)(x+3)$ is H.C.F. ……… (Given)

$$P(-3) = 0$$
$$2(3+a) = 0$$
$$3+a = 0$$
$$a = -3$$
$$Q(x) = (x^2+7x+12)(x^2+7x+b)$$
$$= (x+3)(x+4)(x^2+7x+b)$$

Put $x = -1$

Then, $Q(-1) = (-1+3)(-1+4)(1-7+b)$

$$= (2)(3)(b-6)$$
$$= 6(b-6)$$

$$Q(-1) = 0$$
$$6(b-6) = 0$$
$$b-6 = 0$$
$$b = 6$$
$$a = -3, b = 6.$$

19. $4(x^4-1) = 4(x^2-1)(x^2+1)$
$= 4(x+1)(x-1)(x^2+1)$
$6(x^3-x^2-x+1) = 6(x-1)^2(x+1)$
G.C.D. $= 2(x-1)(x+1)$
$= 2(x^2-1).$

20. Let $P(x) = (x-5)(x^2-x-a)$
Put $x = 4$
Then, $P(4) = (4-5)(16-4-a)$
$= -1(12-a)$
Since, $(x-4)$ is a factor of $P(x)$
$P(4) = 0$
$-1(12-a) = 0$
$a = 12$
Let $Q(x) = (x-4)(x^2-2x-b)$
Put $x = 5$
Then, $Q(5) = (5-4)(25-10-b)$
$= (15-b)$
Since, $(x-5)$ is a factor of $Q(x)$
$Q(5) = 0$
$15-b = 0$
$b = 15.$

21. Let $P(x) = x^2-kx-6$
$x = 6$
$P(6) = 36-6k-6 = 30-6k$
Hence, $(x-6)$ is the G.C.D. .... (Given)
$\therefore$ $(x-6)$ is the factor of P(6).
$\therefore$ $P(6) = 0$
$30-6k = 0$
$-6k = -30$
$k = 5.$

22. $x^2-x-6 = (x-3)(x+2)$
$x^3-27 = (x-3)(x^2+3x+9)$
G.C.D. $= x-3.$

23. $2x^2-9x+4 = (2x-1)(x-4)$
$8x^3-1 = (2x-1)(4x^2+2x+1)$
G.C.D. $= (2x-1).$

24. $18(6x^4+x^3-x^2) = 18x^2(6x^2+x-1)$
$= 18x^2(2x+1)(3x-1)$
$45(2x^6+3x^5+x^4) = 45x^4(2x^2+3x+1)$
$= 45x^4(x+1)(2x+1)$
G.C.D. $= 9x^2(2x+1)$

25. $12(3x^4-14x^3-5x^2)$
$= 12x^2(3x^2-14x-5)$
$= 2\times 2\times 3x^2(x-5)(3x+1)$
$30(3x^5+4x^4+x^3) = 30x^3(3x^2+4x+1)$
$= 2\times 3\times 5x^3(3x+1)(x+1)$
G.C.D. $= 2\times 3x^2(3x+1)$
$= 6x^2(3x+1).$

26. $18x^3+45x^2-27x = 9x(2x^2+5x-3)$
$= 3\times 3x(x+3)(2x-1)$
$15x^4-135x^2 = 15x^2(x^2-9)$
$= 3\times 5x^2(x-3)(x+3)$
G.C.D. $= 3x(x+3).$

Exercise B

1. $18x^4-36x^3+18x^2 = 18x^2(x^2-2x+1)$
$= 18x^2(x-1)^2$
$45x^6-45x^3 = 45x^3(x^3-1)$
$= 45x^3(x-1)(x^2+x+1)$
L.C.M. of 18 and 45 = 90
L.C.M. $= 90x^3(x-1)^2(x^2+x+1)$
$= 90x^3(x-1)(x^3-1).$

2 $12x^4+324x = 12x(x^3+27)$
$= 12x(x+3)(x^2-3x+9)$
$36x^3+90x^2-54x = 18x(2x^2+5x-3)$
$= 18x(2x-1)(x+3)$
L.C.M. of 12 and 18 = 36
L.C.M. $= 36x(x+3)(x^2-3x+9)$
$(2x-1)$
$= 36x(x^3+27)(2x-1).$

3. Factors of $x^2+x-2 = x^2+2x-x-2$
$= x(x+2)-1(x+2)$
$= (x+2)(x-1)$
Factors of $x^2-5x+6 = x^2-3x-2x+6$
$= x(x-3)-2(x-3)$
$= (x-3)(x-2)$
L.C.M. $\times$ G.C.D. $= P(x)\times Q(x)$
$$\text{L.C.M.} = \frac{P(x)\times Q(x)}{\text{G.C.D.}}$$
$$= \frac{(x-3)(x+2)(x-1)(x-2)(x-3)}{(x-3)}$$
$= (x-2)(x-3)(x-1)(x+2).$

4. G.C.D. $\times$ L.C.M. of two polynomials = Product of two polynomials
$10(x+3)(x-1)\times$ L.C.M. $= 10(x^2-9)(x^2-3x+2)$
$10x(x+3)(x-1)^2$
Factors of $x^2-3x+2 = x^2-2x-x+2$
$= x(x-2)-1(x-2)$
$= (x-2)(x-1)$

$$\text{Factors of} \quad x^2 - 9 = (x-3)(x+3)$$

$$\text{L.C.M.} = \frac{\dfrac{10\,(x^2-9)(x^2-3x+2)}{10x\,(x+3)(x-1)^2}}{10(x+3)(x-1)}$$

$$= \frac{\dfrac{10\,(x-3)(x+3)(x-2)(x-1)}{10x\,(x+3)(x-1)^2}}{10\,(x+3)(x-1)}$$

$$= (x-3)(x-2)\,10x\,(x+3)(x-1)^2$$

$$= 10x\,(x-2)(x-3)(x+3)\,(x-1)^2$$

$$= 10x\,(x^2-9)(x-2)(x-1)^2 .$$

5. $$\text{G.C.D.} = 5\,(x+3)(x-1)$$

$$\text{L.C.M.} = 20x\,(x^2-9)(x^2-3x+2)$$

$$P(x) = 10\,(x^2-9)(x-1)$$

$$Q(x) = ?$$

$$\text{Now, } P(x) \times Q(x) = \text{G.C.D.} \times \text{L.C.M.}$$

$$\Rightarrow 10\,(x^2-9)(x-1) \times Q(x) = 5(x+3)\,(x-1) \times 20x\,(x^2-9)(x^2-3x+2)$$

$$\Rightarrow Q(x) = \frac{5(x+3)(x-1) \times 20x\,(x^2-9)(x^2-3x+2)}{10\,(x^2-9)(x-1)}$$

$$= 10x\,(x+3)(x^2-3x+2)$$

$$= 10x\,(x+3)(x-1)(x-2).$$

6. We know that :

$$\text{L.C.M.} \times \text{G.C.D.} = P(x) \times Q(x)$$

$$x \times (x+a) \times 12(x+a)(x^2-a^2) = 4x\,(x+a) \times Q(x)$$

$$Q(x) = \frac{x \times (x+a) \times 12\,(x+a)(x^2-a^2)}{4x\,(x+a)}$$

$$= 3(x+a)(x^2-a^2).$$

7. We know that :

$$\text{L.C.M.} \times \text{G.C.D.} = P(x) \times Q(x)$$

$$x \times (x+a) \times 12x^2\,(x-a)(x^2-a^2) = 4x^2\,(x+a)^2 \times Q(x)$$

$$Q(x) = \frac{x\,(x+a) \times 12x^2\,(x-a)(x^2-a^2)}{4x^2\,(x+a)^2}$$

$$= \frac{3x\,(x-a)(x^2-a^2)}{(x+a)}$$

$$= \frac{3x\,(x-a)(x+a)(x-a)}{(x+a)}$$

$$= 3x\,(x-a)^2.$$

8. G.C.D. × L.C.M. = Product of the two polynomials

$$4x\,(x-2) \times \text{L.C.M.}$$

$$= 4x^2\,(x^2-3x+2) \times 12x\,(x-2)\,(x^2-4)$$

$$= 4x^2\,(x^2-2x-x+2) \times 12x\,(x-2)\,(x-2)(x+2)$$

$$= 48x^3\,(x-2)(x-1)(x-2)^2\,(x+2)$$

$$= 48x^3\,(x-2)^3\,(x-1)(x+2)$$

$$\text{L.C.M.} = \frac{48x^3\,(x-2)^3(x-1)(x+2)}{4x\,(x-2)}$$

$$= 12x^2\,(x-2)^2\,(x-1)(x+2)$$

$$= 12x^2\,(x-2)(x-1)(x-2)(x+2)$$

$$= 12x^2\,(x^2-3x+2)(x^2-4).$$

9. $$P(x) = (x^4-1)$$

$$= (x-1)(x+1)(x^2+1)$$

$$Q(x) = x^3 - x$$

$$= x\,(x^2-1)$$

$$= x\,(x+1)(x-1)$$

$$\text{L.C.M.} = x\,(x-1)(x+1)(x^2+1)$$

$$= x\,(x^4-1).$$

10. $$P(x) = 12\,(x^4-25)$$

$$= 12\,(x^2-5)(x^2+5)$$

$$Q(x) = x^4 + 4x^2 - 5$$

$$= (x^2-1)(x^2+5)$$

$$\text{L.C.M.} = 12\,(x^2-5)(x^2+5)(x^2-1)$$

$$= 12\,(x^4-25)(x^2-1).$$

11. $$P(x) = 12\,(x^4-36)$$

$$= 12\,(x^2-6)(x^2+6)$$

$$Q(x) = 8x^4 + 5x^2 - 6$$

$$= 8\,(x^2+6)(x^2-1)$$

$$\text{L.C.M.} = 24\,(x^2-6)(x^2+6)(x^2-1)$$

$$= 24\,(x^4-36)(x^2-1).$$

12. $$P(x) = 8\,(x^4-16)$$

$$= 8\,(x^2+4)(x-2)(x+2)$$

$$Q(x) = 12(x^3-8)$$

$$= 12\,(x-2)\,(x^2+2x+4)$$

$$\text{G.C.D.} = 4\,(x-2).$$

13. $$4x^2\,(x^2-a^2) = 4x^2\,(x+a)(x-a)$$

$$9x^2\,(x^3-a^3) = 9x^2\,(x-a)(x^2+xa+a^2)$$

L.C.M. of 4 and 9 = 36

$$\text{L.C.M.} = 36x^2\,(x-a)(x+a)(x^2+xa+a^2)$$

$$= 36x^2\,(x^3-a^3)(x+a).$$

14. $$3\,(x^2-7x+12) = 3\,(x-4)(x-3)$$

$$24\,(x^2-9x+20) = 24\,(x-4)\,(x-5)$$

L.C.M. of 3 and 24 = 24

$$\text{L.C.M.} = 24\,(x-4)(x-3)(x-5).$$

15. G.C.D. × L.C.M. = Product of two polynomials

$$27x^3\,(x+a)(x^3-a^3) \times x^2\,(x-a) = 3x^2\,(x^2-a^2) \times Q(x)$$

$$Q(x) = \frac{27x^3\,(x+a)(x^3-a^3) \times x^2\,(x-a)}{3x^2\,(x^2-a^2)}$$

$$= 9x^3\,(x^3-a^3).$$

16. $$P(x) = 1 + x + x^3 - x^5$$

$$= (1+x) + x^3\,(1-x^2)$$

$$= (1+x) + x^3\,(1-x)(1+x)$$

$$= (1+x)[1+x^3(1-x)]$$
$$= (1+x)(1+x^3-x^4)$$
$$Q(x) = (1-x^4-x^6+x^7)$$
$$= (1-x^4)-(x^6-x^7)$$
$$= (1+x^2)(1-x^2)-x^6(1-x)$$
$$= (1-x)[(1+x^2)(1+x)-x^6]$$
$$= (1-x)[1+x+x^2+x^3-x^6]$$
$$= (1-x)[(1+x)+x^2+x^3(1-x^3)]$$
$$= (1-x)[(1+x+x^2)+x^3(1-x)(1+x+x^2)]$$
$$= (1-x)(1+x+x^2)(1+x^3-x^4)$$
$$\text{G.C.D.} = 1+x^3-x^4.$$

17. Let $p(x) = ax^2+bx+c$ and $q(x) = cx^2+ax+b$.

Since, $(x-k)$ is the H.C.F. of the given polynomials.

This means k is a zero of both the given polynomials $p(x)$ and $q(x)$.

Now, k is a zero of ax^2+bx+c

$$ak^2+bk+c = 0 \quad \ldots\text{(i)}$$

Also, k is a zero of $cx^2+ax+b = 0$

$$ck^2+ak+b = 0 \quad \ldots\text{(ii)}$$

Solving (i) and (ii), by cross multiplication, we get

$$\frac{k^2}{b^2-ac} = \frac{k}{c^2-ab} = \frac{1}{a^2-bc} \quad \ldots\text{(iii)}$$

From first two members of (iii), we get

$$k = \frac{b^2-ac}{c^2-ab} \quad \ldots\text{(iv)}$$

From last two members of (iii), we get

$$k = \frac{c^2-ab}{a^2-bc} \quad \ldots\text{(v)}$$

From (iv) and (v), we get

$$\frac{b^2-ac}{c^2-ab} = \frac{c^2-ab}{a^2-bc}$$
$$\Rightarrow (b^2-ac)(a^2-bc) = (c^2-ab)^2$$
$$\Rightarrow a^2b^2-b^3c-a^3c+abc^2 = c^4-2c^2ab+a^2b^2$$
$$\Rightarrow a^3c+b^3c+c^4-3abc^2 = 0$$
$$\Rightarrow c(a^3+b^3+c^3-3abc) = 0$$
$$\Rightarrow a^3+b^3+c^3-3abc = 0 \quad [\because c=0]$$

18. We have

$$H(x)+L(x) = f(x)+g(x) \quad \ldots\text{(i)}$$
$$\Rightarrow \{H(x)+L(x)\}^3 = \{f(x)+g(x)\}^3$$
$$\Rightarrow \{H(x)^3\}+\{L(x)\}^3+3H(x).\,L(x).\,\{H(x)+L(x)\}$$
$$= \{f(x^3)\}+\{g(x^3)\}+3\{f(x)\,g(x)\,\{f(x)+g(x)\}$$
$$\Rightarrow \{H(x)^3+\{L(x)\}^3+3\,H(x).\,L(x)\,\{H(x)+L(x)\}$$
$$= \{f(x)\}^3+\{g(x)\}^3+3\,H(x)\,L(x)\,\{H(x)+L(x\}$$
$$[\text{Using (i) and } f(x).\,g(x) = H(x).\,L(x)]$$
$$\Rightarrow \{H(x)\}^3+\{L(x)\}^3 = \{f(x)^3+\{g(x)\}^3$$

19. Given in Ist expression,

$$= x^4+5x^3-x^2-5x$$
$$= x^3(x+5)-x(x+5)$$
$$= (x+5)(x^3-x)$$
$$= x(x^2-1)(x+5)$$

and IInd expression,

$$= x^3+4x^2-x-4$$
$$= x^2(x+4)-1(x+4)$$
$$= (x+4)(x^2-1)$$

So, common of both expression in (x^2-1).

3 Percentage

Introduction

The word per cent means per hundred. The sign of per cent is %.

If a student gets 20% marks, it means that he gets 20 marks out of 100 marks.

If my mother gives me Rs. 40 out of Rs. 100. It means I get 40% of what my mother had.

If a fraction number has denominator 100. It is known as percentage and its numerator is called as rate per cent.

Some Important Points

[A] To find the percentage of a fraction, decimal or a whole number just multiply these numbers by 100 with sign % —

(i) $\frac{2}{5} = \frac{2}{5} \times 100\% = 40\%$

(ii) $\frac{1}{6} = \frac{1}{6} \times 100\% = 16 \cdot 66\%$

(iii) $\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%$

(iv) $2 \cdot 5 = 2 \cdot 5 \times 100\% = 250\%$

(v) $0 \cdot 05 = 0 \cdot 05 \times 100\% = 5\%$

(vi) $1 \cdot 75 = 1 \cdot 75 \times 100\% = 175\%$

(vii) $4 = 4 \times 100\% = 400\%$

(viii) $12 = 12 \times 100\% = 1200\%$

(ix) $3 = 3 \times 100\% = 300\%$

[B] The percentage of fraction, decimal or a whole number can be converted as fractional number as follow—

$$\frac{2}{5}\% = \frac{2}{5} \times \frac{1}{100} = \frac{2}{500} = \frac{1}{250}$$

or $$\frac{3}{4}\% = \frac{3}{4} \times \frac{1}{100} = \frac{3}{400}$$

$$2 \cdot 5\% = 2 \cdot 5 \times \frac{1}{100} = \frac{25}{1000}$$

or $$0 \cdot 35\% = 0 \cdot 35 \times \frac{1}{100} = \frac{35}{100} \times \frac{1}{100} = \frac{35}{10000}$$

$$4\% = \frac{4}{100} \text{ and } 3\% = \frac{3}{100}$$

Illustration 1.

Represent the following percentages as fractional numbers—

(a) $1\frac{1}{3}\% = ?$

(b) $\frac{12}{5}\% = ?$

(c) $\sqrt{2}\% = ?$

(d) $\log_{10} 100\% = ?$

(e) $\frac{1}{2^{-2}}\% = ?$

(f) $5\% = ?$

(g) $\sqrt[5]{32}\% = ?$

(h) $\sqrt[3]{64}\% = ?$

(i) $\frac{1}{\tan 30°}\% = ?$

(j) $\frac{1}{\sqrt{2}\,(\sin 45° + \cos 45°)}\% = ?$

(k) $\left(\frac{1}{2^2 + 3^2}\right)^{1/2}\% = ?$

(l) $\left(\frac{1}{5^2 - 3^2}\right)^{1/2}\% = ?$

Solution :

(a) $\frac{4}{3}\% = \frac{4}{3} \times \frac{1}{100} = \frac{1}{75}$.

(b) $\frac{12}{5} \times \frac{1}{100} = \frac{3}{125}$.

(c) $1 \cdot 41\% = \frac{1 \cdot 41}{100} = \frac{141}{10000}$.

(d) $\log_{10} 10^2\% = 2 \log_{10} 10\%$.

$$= 2 \times 1 \times \frac{1}{100} = \frac{1}{50}$$

(e) $2^2\% = 4\% = \frac{4}{100} = \frac{1}{25}$.

(f) $\frac{5}{100} = \frac{1}{20}$.

(g) $\sqrt[5]{2^5}\% = 2^{\frac{5}{5}}\% = 2\% = \frac{1}{50}$.

(h) $\sqrt[3]{4^3}\% = 4^{\frac{3}{3}}\% = 4\% = \frac{4}{100} = \frac{1}{25}$.

(i) $\frac{1}{1/\sqrt{3}}\% = \sqrt{3}\% = 1 \cdot 732\%$

$$= \frac{1 \cdot 732}{100} = \frac{1732}{100000}$$

(j) $\dfrac{1}{\sqrt{2}\sqrt{2}\left(\sin 45°.\dfrac{1}{\sqrt{2}} + \cos 45°.\dfrac{1}{\sqrt{2}}\right)}\%$

$$= \frac{1}{2} \times \frac{1}{\sin 90°}\% = \frac{1}{2} \times 1\%$$

$$= \frac{1}{2} \times \frac{1}{100} = \frac{1}{200}.$$

(k) $\left(\dfrac{1}{4+9}\right)^{1/2}\% = \dfrac{1}{\sqrt{13}}\%$

$$= \frac{\sqrt{13}}{13} \times \frac{1}{100} = \frac{3{,}501}{13} \times \frac{1}{100}$$

$$= \frac{3501}{1300000}$$

(l) $\left(\dfrac{1}{25-9}\right)^{1/2}\% = \left(\dfrac{1}{16}\right)^{1/2}\% = \dfrac{1}{4}\% = \dfrac{1}{400}.$

Illustration 2.

Represent the following percentages as decimals—

(i) $25\% = ?$

(ii) $3{\cdot}5\% = ?$

(iii) $120\% = ?$

(iv) $\dfrac{12}{5}\% = ?$

(v) $90\% = ?$

Solution :

(i) $\dfrac{25}{100} = 0{\cdot}25$

(ii) $\dfrac{35}{100 \times 10} = 0{\cdot}035$

(iii) $\dfrac{120}{100} = 1{\cdot}2$

(iv) $\dfrac{12}{5} \times \dfrac{1}{100} = \dfrac{3}{125} = 0{\cdot}024$

(v) $\dfrac{90}{100} = 0{\cdot}9$

Illustration 3.

Represent the following decimals as rate percent—

(i) $0{\cdot}25 = ?$

(ii) $0{\cdot}015 = ?$

(iii) $1{\cdot}75 = ?$

(iv) $3{\cdot}33 = ?$

(v) $0{\cdot}005 = ?$

Solution :

(i) $(0{\cdot}25 \times 100)\% = 25\%.$

(ii) $(0{\cdot}015 \times 100)\% = \dfrac{15}{10}\% = \dfrac{3}{2}\%.$

(iii) $(1{\cdot}75 \times 100)\% = 175\%.$

(iv) $(3{\cdot}33 \times 100)\% = 333\%.$

(v) $(0{\cdot}005 \times 100)\% = \dfrac{5}{10}\% = \dfrac{1}{2}\%.$

Illustration 4.

What per cent of—

(i) 20 is 80 (ii) 125 is 25

(iii) 3 is 18 (iv) 150 is 5

Solution :

(i) Let required percentage be $a\%$

Now, $20 \times a\% = 80$

or $20 \times \dfrac{a}{100} = 80$

$\Rightarrow a = \dfrac{80 \times 100}{20}$

$\Rightarrow a = 400$

Hence, 80 is 400% of 20.

(ii) Required percentage of

$$125 = \frac{25 \times 100}{125} = 20$$

Hence, 25 is 20% of 125.

(iii) Required percentage of

$$3 = \frac{18 \times 100}{3} = 600$$

Hence, 18 is 600% of 3.

(iv) Required percentage of

$$150 = \frac{5 \times 100}{150} = \frac{10}{3}$$

Hence, 5 is $\dfrac{10}{3}\%$ of 150.

Illustration 5.

Evaluate—

(i) 30% of $900 = ?$

(ii) $1\frac{1}{2}\%$ of $250 = ?$

(iii) 42% of $700 = ?$

(iv) $2{\cdot}5\%$ of $12500 = ?$

Solution :

(i) $\dfrac{30}{100} \times 900 = 270$

(ii) $\dfrac{3}{2} \times \dfrac{1}{100} \times 250 = \dfrac{15}{4}$

(iii) $\dfrac{42}{100} \times 700 = 294$

(iv) $\dfrac{25}{1000} \times 12500 = 312{\cdot}5$

Illustration 6.

Ram pays 5% Income Tax. If his annual income is Rs. 12500. How much does he pay as Income Tax annually ?

Solution :

Income Tax = 5% of Annual Income = $5\% \times 12500$

$= \dfrac{5}{100} \times 12500 =$ Rs. 625

Illustration 7.

My mother has Rs. 5000 and she gives me 20%. How much I get ?

Solution :

$$\text{I get} = 20\% \text{ of } 5000$$
$$= \frac{20}{100} \times 5000 = \text{Rs. } 1000.$$

Illustration 8.

The monthly salary of Shyam is Rs. 10,000. He spends 25% on the education of his children, 40% on food, 10% on house rent, 3% on travels and 5% on miscellaneous and rest he save for the future. Find his savings.

Solution :

$$\text{Total expenditure} = 25 + 10 + 40 + 3 + 5 = 83\ \%$$

Out of Rs. 100 Shyam expense Rs. 83. So he saves Rs. 17 out of Rs. 100.

$$\text{Now, His savings} = 17\% \text{ of } 10000 = \frac{17}{100} \times 10000$$
$$= \text{Rs. } 1700.$$

Percentage Increase / Percentage Decrease

(i) Percentage Change

$$\% \text{ Change} = \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} \times 100$$
$$= \frac{\Delta \text{ Value}}{\text{Initial value}} \times 100$$
$$= \frac{\text{Difference}}{\text{Initial value}} \times 100$$

(ii) Percentage Increase

$$\text{Percentage Increase} = \frac{\text{Increase value}}{\text{Base value}} \times 100$$

(iii) Percentage Decrease

$$\text{Percentage Decrease} = \frac{\text{Decrease value}}{\text{Base value}} \times 100$$

Percentage increase/decrease is considered with respect to the Base Value (Initial Value) unless mentioned otherwise.

Illustration 9.

The salary of Ram increases from Rs. 5000 to Rs. 6000. Find the percentage increase in salary ?

Solution :

Percentage Increase in salary

$$= \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} \times 100\%$$
$$= \frac{6000 - 5000}{5000} \times 100\% = 20\%.$$

Illustration 10.

The price of potato increases from Rs. 10/kg to Rs. 12/kg. Find percentage increase in the price of potato.

Solution :

Increase in price of potato (F.V. – B.V.) = Rs. 2/kg, and Base Value = Rs. 10/kg

$$\text{Percentage Increase} = \frac{2}{10} \times 100\% = 20\%.$$

Illustration 11.

If Rahim's salary is 40% more than that of Ram's salary, then how much per cent Ram's salary less than that of Rahim ?

Solution :

$$\text{Let Ram's salary} = \text{Rs. } 100$$
$$\text{Salary of Rahim} = \text{Rs. } 140$$

Clearly Ram's salary is Rs. 40 less than that of Rahim.

When Rahim's salary is 140, then Ram's salary = 40 less

Rahim's salary is Re. 1, then Ram's salary $= \frac{40}{140}$ less

Rahim's salary is Rs. 100, then Ram's salary $= \frac{4}{14} \times 100 = \frac{200}{7}\% = 28\frac{2}{7}\%$ less.

Illustration 12.

If initial cost of pen is increased by 25%, then find the final cost.

Solution :

$$\text{Let initial cost of pen} = a$$
$$\text{Increased cost price} = a + a \times 25\%$$
$$= a + \frac{25a}{100}$$
$$\text{Clearly final cost} = \frac{100 + 25}{100} \times a = \frac{125}{100} a$$
$$\text{Final cost} = \text{Initial price} \times \left(\frac{100 + \text{percentage increase}}{100}\right)$$

[C] Two different percentage of a number—

If $a\%$ and $b\%$ of a number N is x_1 and x_2 respectively. Find the number N.

Solution :

$$x_1 = \text{N} \times a\% = \frac{\text{N}a}{100} \quad \ldots(1)$$
$$x_2 = \text{N} \times b\% = \frac{\text{N}b}{100} \quad \ldots(2)$$

From (1) and (2), we get

$$\text{N} = \frac{x_1 + x_2}{a + b} \times 100$$

$$\text{So, Original Number} = \frac{\text{Sum of results}}{\text{Sum of percentage}} \times 100$$

Similarly,

$$\text{Original Number} = \frac{\text{Difference of results}}{\text{Difference of percentages}} \times 100$$

Illustration 13.

If 15% and 25% of a number is 60 and 100 respectively. Find the number N.

Solution :

If N be the required number

$$N = \frac{60 + 100}{15 + 25} \times 100$$

$$= \frac{160}{40} \times 100 = 400.$$

Illustration 14.

If the sum of numbers obtained by taking percentage 10% and 30% of a certain number is 200. Find the original number.

Solution :

Using Formula :

$$\text{Original Number} = \frac{\text{Sum of results}}{\text{Sum of percentages}} \times 100$$

$$= \frac{200}{(10 + 30)} \times 100$$

$$= \frac{200}{40} \times 100 = 500.$$

Illustration 15.

The difference between the Number obtained by increasing a certain number by 3% and that obtained by diminish it by 7% is 81. Find the original number.

Solution :

Using formula :

$$\text{Original number} = \frac{\text{Difference of result values}}{\text{Difference of per cents}} \times 100$$

$$= \frac{81}{[3 - (-7)]} \times 100$$

$$= \frac{81}{10} \times 100 = 810.$$

Illustration 16.

x% of a Number a is equal to y% of a Number b. Find what per cent of a is b ?

Solution :

$$\frac{x}{100} \times a = \frac{y}{100} \times b$$

$$\Rightarrow \quad \frac{a}{b} = \frac{y}{x}$$

$$\text{So,} \quad b = \frac{x}{y} a$$

$$b = \left(\frac{x}{y} a \times 100\right)\%$$

Hence, $b = \left(\frac{x}{y} \times 100\right) a\%$.

Illustration 17.

The Sum of two numbers is C. Five times of the greater number exceeds thrice the smaller one by zC. Find by what per cent the greater number is more than the smaller one ?

Solution :

Let x and y be the required numbers where $x > y$

According to question—

$$x + y = C \quad \ldots(1)$$

$$5x - 3y = zC \quad \ldots(2)$$

Putting value of y from (1) in equation (2), we get

$$5x - 3(C - x) = zC$$

$$\text{or,} \quad 5x - 3C + 3x = zC$$

$$\text{or,} \quad x = \frac{zC + 3C}{8}$$

$$\text{So,} \quad y = C - x = C - \frac{zC + 3C}{8}$$

$$= \frac{5C - zC}{8}$$

$$\text{Difference} = x - y = \left(\frac{z+3}{8} - \frac{5-z}{8}\right)$$

$$C = \frac{2z - 2}{8} \times C$$

$$\frac{\%\text{ Difference}}{x} \times 100\% = \frac{2(z-1)}{8} C \times \frac{8}{C(z+3)} \times 100\%$$

$$= \frac{2(z-1)}{(z+3)} \times 100\%$$

$$= \frac{z-1}{z+3} \times 200\%$$

So, greater number x is $\frac{z-1}{z+3} \times 200\%$ more than the smaller number y.

[D] Problems on expenditure —

(I) When expenditure is not constant

If a be the percentage change in the price of commodity and b be the percentage change in the consumption of the commodity.

Then net % change in expenditure is given by

$$= \left(a + b + \frac{ab}{100}\right)\%$$

(II) When the expenditure does not change

(1) If price increases by a% consumption must decrease by $\left(\frac{a}{100 + a} \times 100\right)\%$.

(2) If consumption increases by a%, price must decrease by $\left(\frac{a}{100 + a} \times 100\right)\%$.

Illustration 18.

The price of petrol is decreased by 5% and its consumption increases by 12%. Find the net percentage change in expenditure and ratio of initial and final expenditure.

Solution :

By using formula :

$$\frac{\text{Initial expenditure}}{\text{Final expenditure}} = \frac{(100)^2}{(100+a)(100+b)}$$

$$\frac{\text{Initial expenditure}}{\text{Final expenditure}} = \frac{100 \times 100}{(100+5)(100+12)}$$

$$= \frac{100 \times 100}{105 \times 112} = \frac{125}{147}$$

Net percentage change in expenditure

$$= \left(a + b + \frac{ab}{100}\right)\%$$

$$= \left(-5 + 12 - \frac{5 \times 12}{100}\right)\%$$

$$= \left(7 - \frac{60}{100}\right)\% = \frac{32}{5}\%$$

Illustration 19.

The price of coal gas is increased by 25%. Find how much per cent its consumption must be decreased if expenditure remains constant ?

Solution :

By using Formula

$$\text{Decrease in consumption} = \left(\frac{a}{100+a} \times 100\right)\%$$

$$= \left(\frac{25}{100+25} \times 100\right)\%$$

$$= \left(\frac{25}{125} \times 100\right)\% = 20\%$$

[E] Problems on revenue—

(A) If price of a commodity is decreased by X% and its consumption is increased by Y%.

(B) Or if the price of a commodity is increased by X% and its consumption is decreased by Y%, then effect on revenue.

= Increased % – Decreased% value

$$- \frac{\text{(Increase \% value)(Decrease \% value)}}{100}$$

$$\text{Clearly } Y - X - \frac{XY}{100} \qquad \ldots(A)$$

$$X - Y - \frac{XY}{100} \qquad \ldots(B)$$

Proof : Let the price of the commodity be Rs. A/unit and consumption be B units.

Total revenue expensed = Rs. A. B

Now, New price = A – X% of A

$$= A - \frac{AX}{100} = A\frac{(100-X)}{100}$$

New consumption = B + Y% of B

$$= B + \frac{BY}{100} = B\frac{(100+Y)}{100}$$

$$\text{New revenue expenses} = \frac{AB}{100 \times 100} \times (100 - X)$$

$$(100 + Y)$$

Change in revenue expenses

$$= \frac{AB\,(100-X)(100+Y)}{100 \times 100} - AB$$

Clearly % effect on revenue

$$= Y - X - \frac{XY}{100}$$

[F] Examination problems—

(I) If passing marks in an examination is X% and a candidate who secures Y marks fails by Z marks, then the maximum marks,

$$M = \frac{100(Y+Z)}{X}$$

(II) A candidate scoring X% in an examination fails by 'a' marks while another candidate who scores Y% marks gets 'b' marks more than the minimum required passing marks. Then maximum marks for that examination is

$$M = \frac{100(a+b)}{Y-X}$$

Let the maximum marks for the examination be M thus, marks obtained by the first candidate = M of x% and marks obtained by second candidate = M of y%

Now, Passing marks for both the candidate is equal so

$$M \text{ of } x\% + a = M \text{ of } y\% - b$$

$$\frac{Mx}{100} + a = \frac{My}{100} - b$$

$$M = \frac{100(a+b)}{y-x}$$

(III) In a class test x% failed in English, y% failed in Hindi. If z% students failed in both English and Hindi, the percentage of students who passed in both the subject is given by $[100 - (x + y - z)]\%$.

Proof : % of student who failed in English only = $(x - z)\%$

% of student who failed in Hindi only = $(y - z)\%$

% of students who failed in either subject = $x\% + (x - z)\% + (y - z)\% = (x + y - z)\%$

So, % of students who passed in both subjects

$$= 100\% - (x + y - z)\%$$

$$= [100 - (x + y - z)]\%$$

[G] Formulae for population—

If the population of a town is p and it increases at the rate of $r\%$ per annum, then

(i) Population after n years

$$= p\left(1 + \frac{r}{100}\right)^n$$

(ii) Population n years ago

$$= \frac{p}{\left(1 + \frac{r}{100}\right)^n}$$

(iii) If r changes every year

Such as r_1 in first year

r_2 in second year and so on.

Population after n years

$$= P\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\left(1 + \frac{r_3}{100}\right)\ldots\ldots\left(1 + \frac{r_n}{100}\right)$$

Illustration 20.

The population of Delhi is 40,00,000 it increases at the rate of 10% per annum. What was its population 2 years ago and also what will be its population after 2 years ?

Solution :

Using formula :

$$\text{Population } n \text{ years ago} = \frac{P}{\left(1 + \frac{r}{100}\right)^n}$$

$$\text{Population 2 years ago} = \frac{40,00,000}{\left(1 + \frac{10}{100}\right)^2}$$

$$= \frac{40,00,000}{11 \times 11} \times 100$$

$$= 3305785$$

$$\text{Population after } n \text{ years} = P\left(1 + \frac{r}{100}\right)^n$$

$$= 40,00,000\left(1 + \frac{10}{100}\right)^2$$

$$= 40,00,000 \times \frac{121}{100}$$

$$= 48,40,000.$$

Illustration 21.

The population of a city is 25,000. If increased by 15% during first year. During second year it decreased by 25% and increased by 40% during the third year. What is the population after 3 years ?

Solution :

By using formula :

Population after 3 years

$$= 25,000\left(1 + \frac{15}{100}\right)\left(1 - \frac{25}{100}\right)\left(1 + \frac{40}{100}\right)$$

$$= 25,000 \times \frac{115}{100} \times \frac{75}{100} \times \frac{140}{100}$$

$$= \frac{250 \times 23 \times 21}{4} = \frac{120750}{4}$$

$$= 30,187{\cdot}5 = 30,187.$$

Illustration 22.

The tax on a commodity is diminished by 2.5% and its consumption increases by 5%. Find the effect on revenue.

Solution :

Let A be tax on a commodity and B be the consumption.

Now, new revenue = tax × consumption

$$= \left(1 - \frac{2{\cdot}5}{100}\right)\text{A}.\left(1 + \frac{5}{100}\right)\text{B}$$

$$= \frac{97{\cdot}5}{100} \times \frac{105}{100}\text{AB}$$

Effect on revenue

$$= x + y + \frac{xy}{100 \times 100}$$

$$= -2{\cdot}5 + 5 - \frac{5 \times 2{\cdot}5}{100 \times 100}$$

$$= 2{\cdot}5 - \frac{5}{4000} = 2{\cdot}5 - 0{\cdot}00125 = 2{\cdot}5$$

Illustration 23.

Gita scores 30% and fails by 20 marks, while Sita who scores 60% marks gets 40 marks more than the minimum required marks for the examination. Find the maximum marks for the examination.

Solution :

By using formula :

$$\text{Maximum marks} = \frac{100(20 + 40)}{(60 - 30)}$$

$$= \frac{100 \times 60}{30} = 200.$$

Illustration 24.

In an examination 75% students failed in Economics, 55% failed in Maths and 35% failed in both the subjects and 500 passed in both the subjects. Find the total number of students.

Solution :

By using formula :

% of students who passed in both subjects

$$= [100 - (x + y - z)]\ \%$$

$$= [100 - (75 + 55 - 35)]\%$$

$$= (100 - 95)\%$$
$$= 5\%$$

Since, 5 % students = 500 students

$\Rightarrow$ 100% (5 × 20) students

$$= 500 \times 20$$
$$= 10{,}000 \text{ students.}$$

Illustration 25.

There are 500 students in an examination 85% students passed in Geography and some of the students passed in Civics while 65% students passed in both the subjects. If 300 students failed in both the subjects. Find the % of students who passed in Civics.

Solution :

Let the required % of students who passed in Civics is x.

Now, by using Set Theory formula

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

No. of students passed in one or both the subjects

$$= x\% + 85\% - 65\%$$
$$= (x + 20)\%$$

No. of students passed one or both the subjects

$$= (x + 20)\%$$

Now, Number of students failed

$$= [100 - (x + 20)]\%$$
$$= (80 - x)\%$$

Since, in 100 students, $(80 - x)$ students is failed

$\therefore$ In 500 students, $\dfrac{80 - x}{100} \times 500$ students is failed

According to questions—

$$\frac{80 - x}{100} \times 500 = 300$$

$\therefore$ $80 - x = 60;\ x = 80 - 60 = 20$

Required % of students who passed in Civics = 20.

Illustration 26.

In an examination a candidates scores 35% but fails by 45 marks. If the passing marks is 65%. What is the maximum marks ?

Solution :

Let the maximum marks = 100

Secured marks = 35

Passing marks = 65

Difference = 65 – 35 = 30

When he fails by 30 marks maximum marks = 100

$\therefore$ He fails by 1 marks, maximum marks $= \dfrac{100}{30}$

$\therefore$ He fails by 45 marks maximum marks

$$= \frac{100}{30} \times 45 = 150.$$

Illustration 27.

The maximum marks in Civics is 120. A candidates scores 60 marks but fails by 12 marks. What is the percentage pass marks ?

Solution :

Since candidates get total (60 + 12) marks

$\therefore$ Candidates get 72 marks out of 120 marks

i.e., In 120 marks, candidates get 72 marks

$\therefore$ In 100 marks, candidates get $\dfrac{72}{120} \times 100\%$

Hence, percentage pass marks = 60%.

Illustration 28.

Gita and Reeta appears for an examination. Gita scores 40% and fails by 24 marks. While Reeta scores 45% which is 16 marks more than the pass marks. What are the maximum marks ?

Solution :

Let the maximum marks = M

Gita gets pass marks $= \dfrac{40}{100}M + 24$

While Reeta gets pass marks $= \dfrac{45}{100}M - 16$

Since, both are equal,

So, $\dfrac{40}{100}M + 24 = \dfrac{45}{100}M - 16$

or $40 = \dfrac{5}{100}M$

$\therefore$ $M = 800.$

Illustration 29.

In a school library, 25% books are in Hindi, 80% of the remaining are in English and the remaining 9,000 are in various languages. What are the total number of books in English ?

Solution :

Let the total books in Library = 100

Now, books in Hindi = 25

Books in English $= 75 \times \dfrac{80}{100} = 60$

Remaining books = 100 – (25 + 60) = 15

When remaining books are 15, then total books = 100

$\therefore$ If remaining books are 9,000, then total books

$$= \frac{100}{15} \times 9{,}000 = 60{,}000$$

Hence, the books in English are

$$= \frac{60}{100} \times 60{,}000 = 36{,}000$$

Illustration 30.

An Army lost 25% of its men in war, 10% of the remaining due to diseases and 5% of the rest declared war disabled. Thus, the strength was reduced to 6,41,250 active men. Find the original strength.

Solution :

Let the original strength = 100

Due to war lost men = 25

Due to diseases lost men = $75 \times \frac{10}{100} = 7{\cdot}5$

Disabled men = $(75 - 7{\cdot}5) \times \frac{5}{100}$

= $3{\cdot}375$

Reduced strength = $100 - (25 + 7{\cdot}5 + 3{\cdot}375)$

= $64{\cdot}125$

When 64·125 is active then original is 100.

6,41,250 is active men, then original strength = 10,00,000.

Exercise A

1. If $12\frac{1}{2}\%$ and 5% of a number is 550 and 220. Find the number.
2. If the sum of numbers obtained by taking percentage of 25% and 10% of a certain number is 350. Find the certain number ?
3. The difference between the numbers obtained by increasing a certain number by 2% and that obtained by diminishing it by 1·5% is 350. Find the original number ?
4. P% of a number C is equal to Q% of a number D. Find the per cent of C relative to D ?
5. The sum of two numbers is A. Seven times of greater number exceeds five times of the smaller one by kA. Find what per cent the greater number is more than the smaller one ?
6. A shopkeeper marks prices at $12\frac{1}{2}\%$ higher than the original price. Due to increase inflation he further increases the price by $6\frac{1}{2}\%$. How much % profit will he get ?
7. The price of rice is increased by 40%. Find how much per cent its consumption must be decreased if expenditure remains constant ?
8. The price of wheat decreases by 24·5 %. Find by how much per cent its consumption must be increased if expenditure remains constant ?
9. The price of kerosene decreases by 25%. Find how much per cent must its consumption be increased if expenditure remains constant ?
10. The population of Bombay was 15,00,000, 3 years ago. If population increased by 10% during first year, decreased by 5% during the second year and again increased by 15%during the third year. What is its population now ?
11. In a factory, the present price of a machine is Rs. 75,000. What was its price 2 years ago and will be its price 2 year hence. If annual rate of depreciation of the machine is 20%.
12. In an election be two candidates Ram and Rahim, Ram got 40% of the votes polled and is defeated by 1600 votes. Find the number of votes polled for Rahim.
13. An ore contains 35% of mass as impurity, whole the metal extracted from this ore contains 5% impurity. How much metal will 240 tonnes of the ore yield ?
14. In April, a cricket team that played 70 games had won 40% of its games. After a phenomenal arising streak this team raised its average to 60%. How many games the team have won in a row to attain this average ?
15. Sheela's income increases by Rs. 2,000 but the rate of tax being reduced from 15% to 12%. She pays the same amount of tax as before. What is her increased income, if 10%of her income is exempted from tax in both the cases ?
16. Two numbers are 20% and 25% less than a third number. How much % is the second number less than the first ?
17. A reduction of Rs. 2/kg enables a man to purchases 8 kg. more sugar for Rs. 32. Find the original price of rice.
18. A reduction of 60% in the price of chocolates enables a person to buy 10 kg. more for Rs. 240. Find the original price of chocolates per kg.
19. A two digital number obtained by after interchanging the initial number increases by 9. What is percentage increase in number, if the sum of numbers is 3 ?
20. 'A' spends 75% of his income and saves the rest. When the cost of living increased, his expenses increased by $25\frac{1}{2}\%$ and his income also increased by $12\frac{1}{2}\%$. What percentage of his income does he save now ?
21. If the length of a rectangle is increased by 60% and the breadth is decreased by 40%, then find the % change in area of the rectangle.
22. Nitin's contributions to charity, religious donation and community welfare are in the ratios of 10 : 20 : 40 respectively. Express the contribution in percentage terms.
23. Which is greatest in $\frac{50}{2}\%$ 0·3, $\frac{2}{15}$?
24. Gita's salary is 20% less than Sita's salary but 30% more than Rita's salary. If Rita's salary is Rs. 75 less than Sita's salary, find the salary of each.

25. Ramesh gets a lumpsum amount of retirement. First he spends 70% to buy a house 33% of the remained, he spends on his daughter's marriage from the balance he invest 60% in a business and finally he is left with Rs. 2,25,500. How much did he get on retirement ?
26. The price of a saree is reduced by successive discounts of 10% and 15%. Find the net percentage decrease in price.
27. The price of a Maruti car is first increased by 12% and later on reduced by 10%. If the original price was Rs. 4,00,000. What is the final price and the % change in the price ?
28. In an election between two candidates 60% of the votes cast their votes, out of which 5% of the votes were declared invalid. A candidate got 17,100 votes which were75% of the total valid votes. Find the total number of votes enrolled in that election.
29. Roma's Mathematics test had 75 problems, *i.e.* 10 Arithmetic, 30 Algebra and 35 Geometry problems. Although she answered 60% of arithmetic, 50% of the algebra and 40% of the Geometry problems correctly. She did not pass the test because she got less than 60% of the problems correct. How many more questions she would have needed to answer correctly to score a 60% passing grade ?
30. If 80% of $(x - y) =$ 50% $(x + y)$, then what per cent of x is y ?
31. A salesman's commission is 5% on all sales upto Rs. 10,000 and 4% on all sales exceeding this. He remits Rs. 31,100 to his parent company after deducting his commission. Find the total sales of salesman.
32. Radha spends 80% of her income. After a increment, her income is increased by 40% and she also increased her expenditure by 35%. Find the percentage increase in savings.
33. The weight of the container alone is 25% of the container filled with a certain fluid when some fluid is removed, the weight of the container and remaining fluid is 50% of the original total weight. What fractional part of the liquid has been removed ?
34. In a competitive examination in stage X, 10% candidates got selection from the total appeared candidates. Stage Y had an equal number of candidates appeared and 12% candidates got selected with 80 more candidates got selected than X. What was the number of candidates appeared from each stages ?
35. The price of maruti is Rs. 10,00,000. It was insured to 90% of its price. The maruti was damaged completely man's accident and the insurance company paid 80% of the insurance. What was the difference between the price of maruti and the amount received ?
36. A bag contains 800 coins of 25 (Paise) denomination and 1500 coins of 50 (Paise) denomination. If 12% of 25 (Paise) coins and 24% of 50 (Paise) coins are removed. The percentage of money removed from the bag is nearly ?
37. A reduction of 20% in the price of rice enables a person to buy 10 kg more for Rs. 100. What is the reduced price / kg.
38. A reduction of 30% in the price of wheat enables a person to buys 3 kg. more for Rs. 90. Find the original price of wheat / kg. ?
39. A class has girls and boys in the ratio 2 : 3. Among the girls, the ratio of Mathematics to Physics students is 4 : 5. If the ratio of Mathematics and Physics students in the entire class is 1 : 2. What percentage of class comprises girls studying Mathematics ?
40. If X^2 % of Y = Y^2% of Z. And Z^4 % of X = Y% of Y. Find the relation between X and Y.
41. One type of liquid contains 30% of water and the second type of liquid contains 40% of water. A glass is filled with 10 parts of first liquid and 6 parts of second liquid.What is the percentage of water in the new mixture in the glass ?
42. Due to an increase of 30% in the price of eggs, 3 eggs less are available for Rs. 7·80. What is the present rate of eggs per dozen ?

Exercise B

1. A motorist decreases his distance covered annually (in km) by 10% when the price of petrol is increased by 3%. Find the percentage change in petrol bill.
2. The rate of increase of the price of rice is observed to be 3 per cent more than the inflation rate expressed in percentage. The price of rice on January 1, 2008 is Rs. 25/kg. The inflation rates of years 2008 and 2009 are expected to be 13% each. The expected price of rice on January 1, 2010 would be.
3. Ganesh can buy 40 oranges or 50 mangoes. He retains 10% of the amount for bus fares and buys 20 oranges and of the balance. How many mangoes can he purchase ?
4. Raju bought 5 erasers, 4 pencils and 7 pens. Sonali bought 9 pens, 10 erasers and 8 pencils for an amount which was half more what Raju had paid. What per cent of the total paid by Raju were paid for the pens ?
5. In a survey of children education, 90% of those asked are in the favour of at least one of the proposal : I, II and III, 48% of those asked favoured proposal I, 36% favoured proposal II and 16% favoured, proposal III. If 5% of those asked favoured all three of the proposals. What percentage of those asked favoured more than one of the three proposals ?
6. Jyoti took five papers in an examination where the full marks were for each paper, her marks in these papers were in proportion of 2 : 3 : 4 : 5 : 6. In all papers together, the candidates obtained 50% of the

total marks then what is the number of papers in which she got more than 40% marks ?

7. One bacteria splits into eight bacteria of the next generation. But due to unfavorable conditions only 25% of one generation can produce the next generation. If the fourth generation number is 64 millions. What is the number in first generation ?

8. Sixty per cent of the employees of a TCS company are men and 40% of them are earning more than Rs. 40,000 per year. If 40% of the company's employees earn more than Rs. 40,000 / year. What fraction of the women employed by the company earn Rs. 40,000 / year ?

9. In a factory there are four types of machines x, y, z and A which produces 10%, 20%, 75% and 25% of the total products respectively. Machines x, y, z and A produces 1%, 4%, 5% and 6% defective products respectively. What is the percentage of non-defective products ?

10. Each edge of a cube is increased by 25%, then the percentage increase in surface area of the cube is ?

11. The boys and girls in a college are in the ratio 4 : 5. If 40% of boys and 35% of the girls are adults, the percentage of students who are not adults is ?

12. Given series is $1 + 4 + 9 + 16 + 25 + 36 + \ldots\ldots\ldots\ldots$. What is the percentage of 10th term relative to 15th term ?

13. A driver reduces his distance covered annually (in km) by 25% when the price of petrol is increased by 5%. Find the per cent increase/ decrease in his annual petrol bill ?

14. In an election 8% of the people in the voter's list did not participate and 48 votes were declared invalid. There are only two candidates Shyam and Krishna. Shyam defeated Krishana by 440 votes. It was found that 60% of the people listed in the voter's list voted for Shyam. Find the total number of votes polled.

15. The rate of increase of the price of coal is observed to be five per cent more than the inflation rate expressed in %. The price of coal, on March 4, 2008 is Rs. 100 / tonnes. The inflation rates of years 2008 and 2009 are expected to be 10% each. Find the expected price of coal on March 4, 2009.

16. The number of votes not cast for the Samata Party increased by 20% in the general election over those not cast for it in the previous assembly polls and the Samata Party lost by a majority thrice as large as that by which it had won the assembly polls. If a total 3,60,000 people voted each time, how many voted for the Samata Party in the previous assembly polls ?

17. Gita buys 25 red pens or 50 blue pens. She retains 20% for autorikshaw as fares and buys 10 red pens and of the balance. She purchases blue pens. What is the number of blue pens she can purchase ?

18. Raj Shekhar bought 3 books, 4 pencils and 5 chocolates while Sonali bought 12 books, 16 pencils and 16 chocolates for an amount which was thrice and half more what Raj Shekhar had paid ? What percentage of the total paid by Raj Shekhar was paid for chocolates ?

19. In a survey of political preferences, 80% of those asked were in favour of at least one of the proposals I, II and III. 50% of those asked favoured proposal I, 30% favoured proposal II and 20% favoured proposal III. If 4% of those asked favoured all three of the proposals, what percentage of those asked favoured more than one of the three proposals ?

20. Riya took five proposals in an examination where the full marks were for each paper her marks in these papers were in proportion of 4 : 3 : 5 : 6 : 7. In all papers together, the candidates obtained 60% of the total marks. Then what is the number of papers in which she got more than 50% marks ?

21. Mr. Ivan Royston is working in the Max New York Life Insurance (MAX). He was hired on the basis of commission and he get the bonus only on the first years commission. He got the policies of Rs. 4 lakh having maturity period of 10 years. His commission in the first, second, third, fourth, and for the rest of the years is 25%, 20%, 10%, 8% and 5% respectively. The bonus is 30% of the commission. If annual premium is Rs. 40,000, then what is his total commission if the completion of the maturity of all the policies in mandatory ?

22. A person gives 20% to his wife and 15% of the remaining to a hospital (as a donation) again 10% of the remaining to Prime Minister's Retired Fund. Then he has only Rs. 8,400 with him. What was the initial sum of money with that person ?

23. Find the percentage of the quantity $1^3 + 2^3 + 3^3 + 4^3 + 5^3$ with respect to $1^3 + 2^3 + 4^3 + 5^3 + 6^3 + 7^3$.

24. If a cone and a cylinder have the same height and radius. What is the percentage difference in volume of cone relative to cylinder ?

25. A resistor of length l and cross sectional area A. If it is cut in four part across the length. Find the % change in resistance of the resistor.

26. A wire of length l and cross-sectional area A. If its length increase by 20% and its cross-sectional area increases by 10%. Find the percentage change in the resistance of the wire.

27. A box of wood of thickness 2 cm. If outer dimensions of the box $24 \times 20 \times 16$ cm^3. How much per cent of air occupied inside the box ?

28. Convert the following fraction into percentage

$$\frac{\log 1 + \log 2 + \log 5}{\log 1 + \log 2 + \log 5 + \log 10}.$$

29. A triangle ABC is made inside a semicircle such that BC is its diameter and point A lies on the circumference. If AB = 3, AC = 4. Find the percentage of area not enclosed by the semi-circle.

30. Given series is 5 + 10 + 20 + 40 + 80 + ………… What per cent difference between 11th and 10th term relative to 10th term ?

31. If the temperature of iron rod increased from 80°C to 90°C. Find what is percentage change the Fahrenheit, if the relation between Fahrenheit and degree centigrade is

$$\frac{C}{100} = \frac{F-32}{180}.$$

32. Each edge of a cube is increased by 20%, then the percentage increase in surface of the cube is ?

Answers

Exercise A

1. By using formula :

$$\begin{aligned}\text{Original number} &= \frac{\text{Sum of results}}{\text{Sum of per cents}} \times 100\\ &= \frac{550+220}{\left(12\frac{1}{2}+5\right)} \times 100\\ &= \frac{770 \times 2}{35} \times 100 = 22 \times 200\\ &= 4{,}400.\end{aligned}$$

2. By using formula :

$$\begin{aligned}\text{Certain number} &= \frac{\text{Sum of results}}{\text{Sum of per cents}} \times 100\\ &= \frac{350}{25+10} \times 100\\ &= \frac{350}{35} \times 100 = 1{,}000.\end{aligned}$$

3. By using formula :

$$\begin{aligned}\text{Original number} &= \frac{\text{Difference of results}}{\text{Difference in per cents}} \times 100\\ &= \frac{350}{2+1{\cdot}5} \times 100 = \frac{350}{3{\cdot}5} \times 100\\ &= 1000\end{aligned}$$

4. According to question,

$$\begin{aligned}C \times \frac{P}{100} &= D \times \frac{Q}{100}\\ \Rightarrow \quad \frac{C}{D} &= \frac{Q}{P}\\ \Rightarrow \quad C &= \frac{Q}{P} \times D\\ \Rightarrow \quad C &= \left(\frac{Q}{P} \times 100\right)\% \text{ of D.}\end{aligned}$$

5. Let x and y be the required numbers where $x > y$

$$x + y = A \quad \ldots(1)$$
$$7x - 5y = kA \quad \ldots(2)$$

From (1) and (2)

$$\begin{aligned}\Rightarrow \quad 7x - 5(A - x) &= kA\\ \text{or,} \quad 12x &= A(k+5)\\ \therefore \quad x &= \frac{k+5}{12} \cdot A\\ \text{and} \quad y &= A - \frac{k+5}{12} \cdot A = \frac{7-k}{12} \cdot A\\ \text{Difference} &= x - y = \left(\frac{k+5}{12} - \frac{7-k}{12}\right) A\\ &= \frac{2k-2}{12} \cdot A\\ \frac{\%\text{ Difference}}{x} &= \left(\frac{2(k-1)A.12}{12(k+5)A} \times 100\right)\%\\ &= \left(\frac{k-1}{k+5} \times 200\right)\%\end{aligned}$$

6.

$$\begin{aligned}\%\text{ profit} &= \left(a + b + \frac{ab}{100}\right)\%\\ &= \left[12\frac{1}{2} + 6\frac{1}{2} + \frac{\left(12\frac{1}{2}\right)\left(6\frac{1}{2}\right)}{100}\right]\%\\ &= \left(19 + \frac{25 \times 13}{400}\right)\% = \frac{317}{16}\%\\ &= 19{\cdot}8\%.\end{aligned}$$

7. Decrease in consumption

$$\begin{aligned}&= \frac{a}{100+a} \times 100\%\\ &= \frac{40}{100+40} \times 100\% = \frac{40}{140} \times 100\%\\ &= \frac{200}{7}\% = 28{\cdot}57\%.\end{aligned}$$

8. Increase in consumption

$$\begin{aligned}&= \left(\frac{a}{100-a} \times 100\right)\%\\ &= \left(\frac{24{\cdot}5}{100-24{\cdot}5} \times 100\right)\%\\ &= \left(\frac{24{\cdot}5}{75{\cdot}5} \times 100\right)\%\\ &= \left(\frac{4900}{151}\right)\% = 32{\cdot}45\%.\end{aligned}$$

9. Increase in consumption

$$= \left(\frac{a}{100-a} \times 100\right)\%$$

$$= \left(\frac{25}{100-25} \times 100\right)\%$$

$$= \left(\frac{25}{75} \times 100\right)\% = 33\frac{1}{3}\%$$

10. Here, rate is different for each year. So, using formula for different rates.

Present population

$$= 15,00,000 \left(1 + \frac{10}{100}\right)\left(1 - \frac{5}{100}\right)\left(1 + \frac{15}{100}\right)$$

$$= 15,00,000 \times \frac{11}{10} \times \frac{19}{20} \times \frac{23}{20}$$

$$= 375 \times 209 \times 23 = 18,02,625.$$

11. Price of machine (2 years ago)

$$= \frac{P}{\left(1 - \frac{r}{100}\right)^2} = \frac{75,000}{\left(1 - \frac{20}{100}\right)^2}$$

$$= \frac{75,000}{4 \times 4} \times 25 = \frac{75,000 \times 25}{16}$$

= Rs.1,17,187·50.

Price of machine (after 2 years)

$$= P\left(1 - \frac{r}{100}\right)^2 = 75,000\left(1 - \frac{20}{100}\right)^2$$

$$= 75,000 \times \frac{16}{25} = 48,000.$$

12. Let Rahim got x votes polled.

Number of votes polled for Ram

$$= (x - 1600)$$

$$\text{Total votes polled} = x + x - 1600 = (2x - 1600)$$

According to Questions,

$$x - 1600 = 40\% (2x - 1600)$$

$$\Rightarrow \quad x - 1600 = \frac{40}{100}(2x - 1600)$$

$$x = 8000 - 3200 = 4800.$$

13. From question it is clear that 100 tonnes of ore gives 65 tonnes of metal which contains 5% impurity.

Since, 100 tonnes of metal contain

= 5 tonnes impurity

∴ 1 tonne of metal contain

$$= \frac{5}{100} \text{ tonnes impurity}$$

∴ 65 tonne of metal contain

$$= \frac{5}{100} \times 65 \text{ tonnes}$$

$$= \frac{13}{4} \text{ tonnes impurity}$$

Now, quantity of pure metal

$$= \left[65 - \frac{13}{4}\right] \text{tonnes}$$

$$= \frac{13 \times 9}{4} \text{tonnes}$$

Clearly, 100 tonnes of ore gives

$$= \frac{13 \times 9}{4} \text{tonnes pure metal}$$

$$\therefore \quad 1 \text{ tonne of ore gives} = \frac{13 \times 9}{4 \times 100} \text{tonnes pure metal}$$

∴ 240 tonnes of ore gives = 70·2 tonnes pure metal.

14. Let x be the number of games won in a row.

Now, according to question,

$$\frac{(70 \text{ of } 40\%) + x}{70 + x} = \frac{60}{100} = \frac{3}{5}$$

$$\text{or,} \quad \frac{28 + x}{70 + x} = \frac{3}{5}$$

$$\text{or,} \quad 2x = 210 - 140 = 70$$

$$x = 35.$$

15. Since, same percentage of his income is exempted from tax in the both cases, this data is not be considered.

$$\text{Initial amount of tax} = \text{Final amount of tax}$$

$$(a - 2000) \times 15\% = a.12\ \%$$

$$\text{or,} \quad (a - 2000)\frac{15}{100} = \frac{12a}{100}$$

$$\text{or,} \quad 3a = 2000 \times 15$$

$$\therefore \quad a = 10,000$$

Hence, Increased income = Rs. 10,000.

16. Let the third number = 100

$$\therefore \quad \text{First number} = 100 - 20\% \text{ of } 100$$

$$= 100 - \frac{20}{100} \times 100 = 80$$

$$\therefore \quad \text{Second number} = 100 - 25\% \times 100$$

$$= 100 - \frac{25}{100} \times 100 = 75$$

Now, second number is lessen than first number second number is 5 less than first number.

$$\% \text{ less} = \frac{5}{80} \times 100 = \frac{25}{4} = 6{\cdot}25\%.$$

17. Here, expenditure is fixed Rs. 32.

Since, rate decrease while amount of rice increases with the same expenditure.

Let the original price = Rs. x / kg.

$$x\,(x + \text{rate change}) = \frac{\text{Expenditure} \times \text{rate change}}{\text{Change in available quantity}}$$

$$\text{or,} \quad x\,(x - 2) = \frac{32 \times 2}{8}$$

$$\text{or,} \quad x^2 - 2x - 8 = 0$$

or, $(x-4)(x+2) = 0$

$\therefore \quad x = 4, x \neq -2$

Original price of rice = Rs. 4/kg.

18. Here, expenditure remains constant.

$$\text{Rate change} = (60\%) \text{ of original price}$$

$$= \frac{60}{100} \times x = \frac{3x}{5} = 0{\cdot}6x$$

where x is the original price

$$x\,(x + \text{rate change}) = \frac{\text{Expenditure} \times \text{rate change}}{\text{Change in available quantity}}$$

or, $$x\,(x - 0{\cdot}6x) = \frac{240 \times 0{\cdot}6x}{10}$$

or, $$0{\cdot}4x = 24 \times 0{\cdot}6;$$

$$x = 36.$$

$\therefore$ Original price = Rs. 36/kg.

19. Let units digit $= x$

10th digit $= y$

According to question,

$$\text{Number} = 10y + x$$

$$x + y = 3 \quad \ldots(1)$$

After interchanging digits

$$\text{new number} = 10x + y$$

$$10x + y - (10y + x) = 9$$

or, $$9x - 9y = 9$$

or, $$x - y = 1 \quad \ldots(2)$$

By equations (1) and (2)

$2x = 4, y = 1; x = 2$

Original number = 12

New number = 21

$$\% \text{ increase in number} = \frac{\text{Change}}{\text{Original number}} \times 100$$

$$= \frac{21-12}{12} \times 100$$

$$= \frac{9}{12} \times 100 = 75\%.$$

20. Let the income of A is Rs. x.

	Income	Expenditure	Saving
Before increase	x	$\frac{75}{100} \cdot x = \frac{3}{4}x$	$\frac{1}{4}x$
After increase	$x\left(1+\frac{25}{200}\right)$	$\frac{3}{4}x\left(1+\frac{50}{200}\right)$	Rs. y
	$x.\frac{9}{8}$	$\frac{3}{4}x \cdot \frac{5}{4}$	

Now, $$\text{New saving} = \frac{9x}{8} - \frac{15x}{16}$$

$$= \frac{18x - 15x}{16} = \frac{3}{16}x$$

$$\% \text{ saving of income} = \text{New saving} \times 100$$

$$= \frac{\frac{3}{16}x}{\frac{9x}{8}} \times 100$$

$$= \frac{100}{6} = 16{\cdot}66\%.$$

21. $$\% \text{ change in result} = x + y + \frac{xy}{100}$$

Here, $x = 60$ and $y = -40$

$$= 60 - 40 - \frac{60 \times 40}{100} = 20 - 24 = -4$$

Hence, area of the rectangle decrease by 4%.

22. Given ratios are 10 : 20 : 40

$$\text{Their sum} = 10 + 20 + 40 = 70$$

$\therefore$ $$\text{Charity} = \frac{10}{70} \times 100 = \frac{100}{7} = 14\frac{2}{7}\%$$

$$\text{Religious donations} = \frac{20}{70} \times 100 = \frac{200}{7} = 28\frac{4}{7}\%$$

Community welfare fund

$$= \frac{40}{70} \times 100 = \frac{400}{7} = 57\frac{1}{7}\%$$

23. I : $$\frac{50}{2}\% = \frac{50}{2 \times 100} = 0{\cdot}25$$

II : $$0{\cdot}3 = 0{\cdot}3$$

III : $$\frac{2}{15} = 0{\cdot}13$$

Clearly 0.3 is the greatest number among above.

24. Let Gita's salary = Rs. G

Since, Gita's salary is 20% less than Sita's salary

$\Rightarrow$ G = 0·8 S ; Here S is Sita's salary

and, Gita's salary 30% more than Rita's salary

$\Rightarrow$ G = 1.3 R; Here R is Rita's salary

When Gita's salary is Rs. x, Sita's salary

$$= \text{Rs. } \frac{8}{10} \cdot x$$

Clearly $0{\cdot}8\,S = 1{\cdot}3\,R$

$$\text{Sita's salary} = \frac{13}{8} \text{ Rita's salary}$$

Difference between Gita's salary and Sita's salary

$$S - R = 75$$

$$\Rightarrow \left(\frac{13}{8} - 1\right)R = \text{Rs. } 75$$

Rita's salary = Rs. 120

$\Rightarrow$ G = 1·3 R = 1·3 × 20 = Rs 156 is Gita's salary

$$\Rightarrow \quad S = \frac{G}{0{\cdot}8} = \frac{156}{0{\cdot}8}$$

= Rs. 195 is Sita's salary.

25. Suppose that Ramesh gets a lumpsum amount

$$= \text{Rs. } 100$$

Now, amount spends in house = 70

Daughter's marriage (33% of remain amount)

$$= 30 \times \frac{33}{100} = 10$$

Amount left After daughter's marriage

$$= 30 - 10 = 20$$

In business, 60% of 20, amount

$$= 20 \times \frac{60}{100} = 12$$

Finally left amount = 20 – 12 = 8

When Rs. 8 left then lumpsum = Rs. 100

$\therefore$ 2,25,500 left when lumpsum

$$= \frac{100}{8} \times 2{,}25{,}500$$

$$= \text{Rs. } 28{,}18{,}750.$$

26. $$\text{Net \% change} = \left(x + y + \frac{xy}{100}\right)\%$$

Here, $x = -10$, $y = -15$

$$\text{Net \% change} = \left(-10 - 15 + \frac{10 \times 15}{100}\right)\%$$

$$= (-25 + 1{\cdot}5)\% = -23{\cdot}5\%$$

27. Using formula :

$$\text{Final Price} = \text{Initial Price} \times \frac{100 + a}{100} \times \frac{100 + b}{100}$$

Here, $a = 12$, $b = -10$

$$\text{Final Price} = 4{,}00{,}000 \times \frac{100 + 12}{100} \times \frac{100 - 10}{100}$$

$$= 40 \times 112 \times 90 = \text{Rs. } 4{,}03{,}200$$

$$\text{Net \% change} = \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(12 - 10 - \frac{12 \times 10}{100}\right)\%$$

$$= \left(5 - \frac{6}{5}\right)\% = \frac{4}{5}\% = 0{\cdot}80\%.$$

28. Let the total number of votes enrolled by x, then number of votes cast = 60 % of x.

$$\text{Valid votes} = \frac{95}{100} \text{ of } (60\% \text{ of } x)$$

$$= 95\% \text{ of } 0{\cdot}6x = 0{\cdot}95 \times 0{\cdot}6x = 0{\cdot}57x$$

Now, according to question :

$$75\% \text{ of } 0{\cdot}57x = 17100$$

$$\Rightarrow \quad 0{\cdot}75 \times 0{\cdot}57x = 17100$$

$$\Rightarrow \quad x = \left(\frac{17100 \times 100 \times 100}{75 \times 57}\right)$$

$$x = 40{,}000.$$

29. Numbered questions attempted correctly

$$= 60\% \text{ of } 10 + 50\% \text{ of } 30 + 40\% \text{ of } 35$$

$$= \frac{60}{100} \times 10 + \frac{50}{100} \times 30 + \frac{40}{100} \times 35$$

$$= 6 + 15 + 14$$

$$= 21 + 14 = 35$$

Questions to be answered correctly for 60% grade

$$= 60\ \% \text{ of } 75$$

$$= \frac{60}{100} \times 75 = 45$$

Required number of questions = 45 – 35 = 10.

30. Given $80\% (x - y) = 50\% (x + y)$

Or, $$\frac{4}{5}(x - y) = \frac{1}{2}(x + y)$$

Or, $$8x - 8y = 5x + 5y$$

Or, $$3x = 13y$$

$$\text{Required percentage} = \left(\frac{y}{x} \times 100\right)\%$$

$$= \left(\frac{3}{13} \times 100\right)\% = 23{\cdot}07\%.$$

31. Let his total sales be Rs. x.

Now, Total sales – commission = Rs. 31,100

Or, $x - [5\% \text{ of } 10{,}000 + 4\% \text{ of } (x - 10{,}000)] = 31{,}100$

Or, $$x - \left[500 + \frac{4}{100}(x - 10{,}000)\right] = 31{,}100$$

Or, $$x - \frac{x}{25} = 31{,}200$$

$$x = \frac{31{,}200 \times 25}{24}$$

$$x = \text{Rs. } 32{,}500.$$

32. Let Radha's income = Rs. 100

Expenditure = Rs. 80

Savings = Rs. 20

New income = Rs. 140

$$\text{New expenditure} = \frac{135}{100} \times 80 = \text{Rs. } 108$$

Now, savings = 140 – 108 = Rs. 32

$$\text{\% increase in savings} = \frac{12}{20} \times 100 = 60\%.$$

33. Let the weight of the container = x

The weight of the fluid = y

According to question,

$$x + y = 100\%$$

Hence, $y = 75\% = 3x$

$$y = 3x \qquad \ldots(1)$$

Given, Weight of container + Remaining fluid

$$= \frac{50}{100}(x + y)$$

$$x + \text{Remaining of Fluid} = \frac{1}{2}(x+3x)$$

$$\text{Remaining fluid} = 2x - x = x$$

$$\text{Removed fluid} = 3x - x = 2x$$

Fractional part of the liquid removed

$$= \frac{2x}{3x} = \frac{2}{3}.$$

34. Since, the same number of candidates appeared from each stages.

So, Let we suppose that 100 candidates appeared from each stage.

Now, For stage X,

$$\text{Selected candidates} = \frac{10}{100} \times 100 = 10$$

For stage Y,

$$\text{Selected candidates} = \frac{12}{100} \times 100 = 12$$

Clearly, $12 - 10 = 2$

When 2 candidates are selected more from stage Y, then there are 100 candidates in each stage

$$\text{When 1 candidates ...} = \frac{100}{2}$$

$$\text{When 80 candidates ...} = \frac{100}{2} \times 80$$

= 4,000 candidates in each stage.

35. Price of maruti = Rs. 10,00,000

$$\text{Amount insured} = 10{,}00{,}000 \times \frac{90}{100}$$

= Rs. 9,00,000

$$\text{Received amount} = 9{,}00{,}000 \times \frac{80}{100}$$

= Rs. 7,20,000

Now, Required difference

= Price of maruti – Received amount

= 10,00,000 – 7,20,000

= Rs. 2,80,000.

36. $$\text{Total money} = \left(800 \times \frac{25}{100} + 1500 \times \frac{50}{100}\right)$$

= Rs. 950

$$\text{25 paise coins removed} = \frac{12}{100} \times 800 = \text{Rs. } 96$$

$$\text{50 paise coins removed} = \frac{24}{100} \times 1500 = \text{Rs. } 360$$

$$\text{Money removed} = \left(96 \times \frac{25}{100} + 360 \times \frac{50}{100}\right)$$

= Rs. 204

$$\text{Required percentage} = \frac{204}{950} \times 100\%$$

$$= \frac{408}{19}\% \cong 21{\cdot}5\%.$$

37. Given amount = Rs. 100

$$\text{Reduced amount} = 100 \times \frac{20}{100} = \text{Rs. } 20$$

Since Rs. 20 the person buys 10 kg. rice

Price of rice = Rs. 2 / kg.

This is the reduced price of rice = Rs. 2/kg.

38. Amount given = Rs. 90

$$\text{Reduction in amount} = 90 \times \frac{30}{100} = \text{Rs. } 27$$

From question :

Then Rs. 27, Here 3 kg. wheat

Price of wheat = Rs. 9 / kg.

When reduced price is 70, then original price = 100

When reduced price is 9, then the original price

$$= \frac{100}{70} \times 9 = 12{\cdot}85.$$

39. Let the girls and boys in the class are $2x$ and $3x$ respectively.

Girls students of Mathematics

$$= \frac{4}{9} \times 2x = \frac{8x}{9}$$

Percentage of girls studying Mathematics

$$= \frac{8x}{9 \times 2x} \times 100$$

$$= \frac{400}{9} = 44{\cdot}44\%.$$

40. Given $X^2\%$ of Y = $Y^2\%$ of Z

$$\frac{X^2}{100} \times Y = \frac{Y^2}{100} \times Z$$

Or, $X^2 = Y.Z$ ...(1)

and Z 4% of X = Y% of Y

$$\frac{Z4}{100} \times X = \frac{Y^2}{100}$$

$X \times Z^4 = Y^2$ ...(2)

Putting the value of Z in equation (2), we get

$$X \times \left(\frac{X^2}{Y}\right)^4 = Y^2$$

Or, $$\frac{XX^8}{Y^4} = Y^2$$

$X^9 = Y^6$.

41. Let the amount of first type liquid = X

The amount of second type of liquid = Y

Now, amount of water in first type of liquid = $\frac{3}{10}$ X

Amount of water in second type of liquid = $\frac{4}{10}$ Y

Since, X parts of liquid contains = $\frac{3}{10}$ X parts of water

$$10 \text{ parts of liquid contains} = \frac{3}{10} \times 10 = 3$$

Similarly,

6 parts of 2nd type liquid contains

$$= \frac{4 \times 6}{10} \text{ part of water}$$

$$= 2{\cdot}4 \text{ parts}$$

$$\text{Total amount of water} = 3 + 2{\cdot}4 = 5{\cdot}4 \text{ parts}$$

$$\text{Total amount of mixture} = 10 + 6 = 16$$

$$\% \text{ Change} = \frac{5{\cdot}4}{16} \times 100 = 33{\cdot}75\%.$$

42. Price of eggs increases by the amount

$$= 7{\cdot}8 \times \frac{30}{100} = 2{\cdot}34$$

Hence, price of 3 eggs is Rs. 2·34

$$\text{Price of 1 egg is} = \text{Rs. } \frac{2{\cdot}34}{3} = 78 \text{ paise.}$$

Exercise B

1. Since, petrol bill is directly proportional to the distance covered and price of petrol.

$$\text{Let petrol bill} = a$$

$$\text{Distance covered} = b$$

$$\text{Price of petrol} = c$$

Now, $a \propto b$ and $a \propto c$

$$a = k.b.c$$

(where k = constant, suppose $k = 1$)

$$a = b.c$$

$$\text{New price of petrol} = \frac{103}{100} \cdot c$$

and $$\text{New distance covered} = \frac{90}{100} \cdot b$$

$$\frac{\text{New bill}}{\text{Original bill}} = \frac{103 \times c \times 90 \times b}{100 \times 100 \times c \times b} = \frac{927}{1000}$$

% change in petrol bill

$$= \frac{\text{New bill} - \text{Original bill}}{\text{Original bill}} \times 100$$

$$= \left(\frac{927}{1000} - 1\right) \times 100$$

$$= -\frac{73}{10} = -7{\cdot}3\%.$$

2. Increase in price of rice = (13 + 3)% = 16%

∴ Price of rice on January 1, 2010

$$= 25\left(1 + \frac{16}{100}\right)^2$$

$$= 25 \times \frac{29 \times 29}{25 \times 25}$$

$$= \frac{841}{25} = \text{Rs. } 33{\cdot}64.$$

3. Let us suppose Ganesh has Rs. 100 with him.

Clearly , price of one orange = Rs. 2·5

Price of one mangoe = Rs. 2

Since, Ganesh retains 10% amount for his bus fares.

So, left amount = Rs. 90

He buys 20 organes, hence, cost of 20 oranges = 20 × 2·5 = Rs. 50

Remaining amount = Rs. (90 – 50) = Rs. 40

Since, price of one mango = Rs. 2

Number of mangoes = 20.

4. Let the Raju's amount = Rs. 100; cost of one pen = Rs. a

cost of one pencil = Rs. b and cost of one eraser = Rs. c

According to question,

$$7a + 4b + 5c = 100 \quad \ldots(1)$$

$$9a + 8b + 10c = 150 \quad \ldots(2)$$

Multiplying equation (1) by 2 and subtracting equation (2), we get

$$5a = 50$$

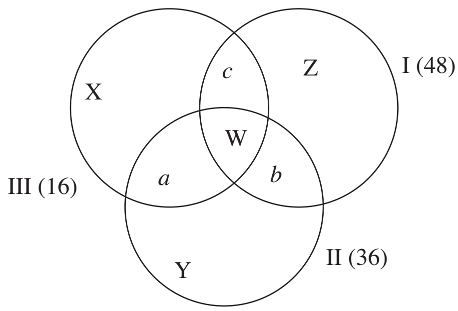
$$\Rightarrow \quad a = 10$$

Total amount paid by Raju

$$= 7a = \text{Rs. } 7 \times 10 = \text{Rs. } 70$$

$$\therefore \quad \text{Required percentage} = \frac{70}{100} \times 100 = 70\%.$$

5.



According to question,

$$X + Y + Z + a + b + c + w = 90$$

$$\Rightarrow 48 - (b + c + 2) + 36 - (a + b + 2) + 16 - (c + a + 2) + (a + b + c + 2) = 90$$

$$\Rightarrow 96 - 2(a + b + c) - 6 + (a + b + c) + 2 = 90$$

$$\Rightarrow 100 - (a + b + c) - 4 = 90$$

$$\Rightarrow 96 - (a + b + c) = 90$$

$$\Rightarrow (a + b + c) = 6$$

∴ Percentage of those asked favoured more than one proposal = 6 + 2 = 8.

6. Let the marks scored in five subjects be $2x$, $3x$, $4x$, $5x$, $6x$.

Total marks in all the five subjects = $20x$

Maximum marks of the five subject given 50% = $20x$

100% = $40x$

Maximum marks in each subject

$$= \frac{1}{5} \times 40x = 8x$$

Hence, percentage in each subject

$$\frac{2.x}{8.x} \times 100 = 25\%$$

$$\frac{3.x}{8.x} \times 100 = 37{\cdot}5\%$$

$$\frac{4.x}{8.x} \times 100 = 50\%$$

$$\frac{5.x}{8.x} \times 100 = 62{\cdot}5\%$$

$$\frac{6.x}{8.x} \times 100 = 75\%$$

She got more than 40% in three papers.

7. Let the number of bacteria in the first generation be x.

$\therefore$ Number of bacteria in the second, third and fourth generation would be

$$8\left(\frac{x}{4}\right), 8\left(\frac{2x}{4}\right), 8\left(\frac{4x}{4}\right), 8\left(\frac{8x}{4}\right) = 2x, 4x, 8x, 16x$$

The above numbers are in G.P. with common ratio 2.

Now, fourth term of these G.P. = $2x.\ 2^3 = x.\ 2^4$

According to question $x.\ 2^4 = 64$

$\Rightarrow$ $x = 4$ millions.

8. Let number of men employees = 60

Number of women employees = 40

Number of men earning more than Rs. 40,000/year

$$= 60 \times \frac{40}{100} = 24$$

Total number of employees earning more than Rs. 40,000 / year = 40

Number of women earning more than Rs. 40,000

$$= 40 - 24 = 16$$

Now, fraction of the women earning Rs. 40,000 or less $= \frac{40-16}{40} = \frac{24}{40} = \frac{3}{5}$

9. Let the total products = a

Now, machine x produces products = $a \times 10\% = 0{\cdot}1a$

and it produces 1% defective products.

It means machine x produces 99% non-defective products.

So, amount of products produced by machines

$$x = (0{\cdot}1a) \times \frac{99}{100}$$

Similarly, machine $y = (0{\cdot}2a) \times \frac{96}{100}$

Machine $z = (0{\cdot}45\ a) \times \frac{95}{100}$

Machine A $= (0{\cdot}25\ a) \times \frac{94}{100}$

Now, Total non-defective products

$$= \frac{a}{100}(9{\cdot}90 + 19{\cdot}20 + 42{\cdot}75) + 23{\cdot}50)$$

$$= \frac{a}{100} \times 95{\cdot}35$$

% non-defective products = 95·35%.

10. Let side of the cube = a m.

Final / new side of the cube

$$= a + a \times \frac{25}{100} = \frac{5}{4}a$$

Initial surface area = $6a^2$

Final/new surface area = $6\left(\frac{5a}{4}\right)^2$

Change in surface area $= 6a^2\left(\frac{25-16}{16}\right) = 6a^2.\frac{9}{16}$

Now, percentage change in surface area

$$= \frac{\text{Change}}{6a^2} \times 100$$

$$= \frac{6a^2\ \frac{9}{16}}{6a^2} \times 100$$

$$= \frac{9}{16} \times 100 = \frac{225}{4} = 56{\cdot}25\%.$$

11. Let the number of boys = $4x$ and the number of girls = $5x$. Total students = $9x$

Adults boys $= 4x.\frac{40}{100} = \frac{8}{5}x$

Adults girls $= 5x.\frac{35}{100} = \frac{7}{4}x$

Total adult students $= \frac{8}{5}x + \frac{7}{4}x$

$$= \frac{32x + 35x}{20} = \frac{67}{20}x$$

Students who are not adults

$$= 9x - \frac{67}{20}x = \frac{113}{20}x$$

Percentage of students who are not adults

$$= \frac{113x}{20 \times 9x} \times 100 = 62{\cdot}78\%.$$

12. Given series is

$1 + 4 + 9 + 16 + 25 + 36 + \ldots\ldots$

$= 1 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + \ldots\ldots$

10th term = 10^2

15th term = 15^2

Now, $\frac{\text{10th term}}{\text{15th term}} \times 100 = \frac{100}{225} \times 100 = 44{\cdot}44\%.$

13. The petrol bill is directly proportional to the distance covered and price of petrol.

Let petrol bill = z and Distance covered = 100 and Rice of petrol = 100

Original petrol bill $z = 100 \times 100$

New petrol price = 105 and New distance covered = 75

New petrol bill $= 105 \times 75$

Percentage change in petrol bill

$$= \frac{\text{New bill} - \text{Original bill}}{\text{Original bill}} \times 100$$

$$= \frac{105 \times 75 - 100 \times 100}{100 \times 100} \times 100$$

Percentage change in petrol bill

$$= \left(\frac{63}{80} - 1\right) \times 100\%$$

$$= -\frac{17}{80} \times 100\%$$

$$= -\frac{85}{4}\% = -21{\cdot}25\%.$$

14. Let the total number of people in the voter's list be x.

The number of people who participated in voting = $0.92x$

Since, 48 votes were declared invalid.

So, valid votes polled = $(0{\cdot}92x - 48)$.

Since, Shyam gets 60% of x.

Number of votes gained by Shyam $= 0{\cdot}6x$.

Votes obtained by Shyam + Votes obtained by Krishna = $(0{\cdot}92x - 48)$.

Or, votes obtained by Krishna = $(0{\cdot}92x - 48 - 0{\cdot}6x)$

$= (0{\cdot}32x - 48)$

Now, According to question; Difference = 440

Or, $0{\cdot}6x - (0{\cdot}32x - 48) = 440$

$\Rightarrow \quad 0{\cdot}28x = 440 - 48$

$\Rightarrow \quad x = \frac{392}{0{\cdot}28} = 1400.$

15. Increase in price of coal

$= (10 + 5)\% = 15\%$

$\therefore$ Price of coal on March 4, 2009

$$= 100\left(1 + \frac{15}{100}\right)$$

$$= 100 \times \frac{115}{100} = \text{Rs. } 115/\text{tonnes}.$$

16. Let 'a' voters voted against the party in the assembly poll.

Then votes in favour $= (3{,}60{,}000 - a)$

Majority of votes by which party won in previous poll $= (3{,}60{,}000 - a) - a = (3{,}60{,}000 - 2a)$

Now, votes polled against the party in general election = $1{\cdot}2a$

And votes polled in favour of the party = $(3{,}60{,}000 - 1{\cdot}2a)$

Majority of votes by which party lost in general election $= 1{\cdot}2a - (3{,}60{,}000 - 1{\cdot}2a)$

$= 2{\cdot}4a - 3{,}60{,}000$

It is given that ,

$2.4a - 3{,}60{,}000 = 3\,(3{,}60{,}000 - 2a)$

$2{\cdot}4a + 6a = 4 \times 3{,}60{,}000$

Or, $\quad a = \frac{4 \times 3{,}60{,}000}{8{\cdot}4}$

$\Rightarrow \quad a = 1{,}71{,}428$

17. Suppose Gita has Rs. 100 with her.

From question :

Price of one red pen = Rs. 4

and Price of one blue pen = Rs. 2

She retain Rs. 20 for autorikshaw fare

$\therefore$ left amount = Rs. 80

The price of 10 red pens = Rs. 40

Remaining amount = Rs. 40

Since, price of one blue pen = Rs. 2

Number of blue pens $= \frac{40}{2} = 20.$

18. Let the Raj Shekhar's amount = Rs. 100

If cost of one book = Rs. x

Cost of one pencil = Rs. y

Cost of one chocolate = Rs. z

According to question,

$3x + 4y + 5z = 100 \quad \ldots(1)$

$12x + 16y + 16z = 300 \quad \ldots(2)$

Multiplying equation (1) by 4 and subtract with (2), we get

$4z = 50$

$\Rightarrow \quad z = 12{\cdot}5$

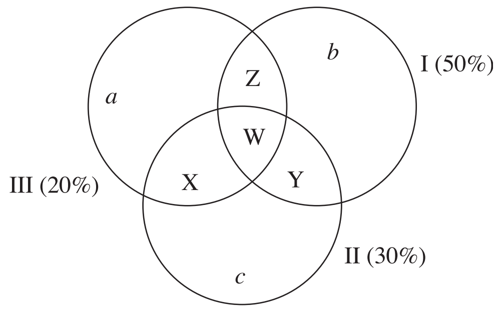
Total amount paid by Raj Shekhar

$= 5 \times 12{\cdot}5 = \text{Rs. } 62{\cdot}5$

Required percentage = 62.5%

19. $[50 - (x + z + w)] + [30 - (x + y + w)] + [20 - (x + z + w)]$

$= 80$



$46 - (x + y) + 26 - (x + z) + 16 - (y + z) + (x + y + z + 4) = 80$

Or, $x + y + z = 92 - 80$

$\Rightarrow$ $x + y + z = 12$

Percentage of those asked favoured more than one proposal = 12 + 4 = 16.

20. Let the marks scored in five subjects be $4x$, $3x$, $5x$, $6x$, $7x$.

Total marks in all the five subjects = $25x$

Maximum marks of the five subjects given 60% = $25x$

$$100\% = \frac{100}{60} \times 25x = \frac{125x}{3}$$

Maximum marks in each subject

$$= \frac{1}{5}\left(\frac{125x}{3}\right) = \frac{25x}{3}$$

Hence, percentage in each subject are

$$= \frac{4x}{25x} \times 3 \times 100 = 48\%$$

$$= \frac{3x}{25x} \times 3 \times 100 = 36\%$$

$$= \frac{5x}{25x} \times 3 \times 100 = 60\%$$

$$= \frac{6x}{25x} \times 3 \times 100 = 72\%$$

$$= \frac{7x}{25x} \times 3 \times 100 = 84\%$$

She got more than 50% in three papers.

21.

Year	Rate of commission	Commission in values
1	25%	$0{\cdot}25 \times 40{,}000 = 10{,}000$
	30% (bonus)	$0{\cdot}3 \times 10{,}000 = 3{,}000$
2	20%	$0{\cdot}2 \times 40{,}000 = 8{,}000$
3	10%	$0{\cdot}1 \times 40{,}000 = 4{,}000$
4	8%	$0{\cdot}08 \times 40{,}000 = 32{,}00$
5-10	5%	$6 \times 0{\cdot}05 \times 40{,}000 = 12{,}000$

Total commission = Rs. 40,200.

22. Let the person has Rs. 100.

His wife gets amount = Rs. 20

Remaining amount = Rs. 80

Donation to hospital $= 80 \times \frac{15}{100} =$ Rs. 12

Again rest money = 80 – 12 = Rs. 68

Prime Minister's relief fund amount

$= 68 \times \frac{10}{100} =$ Rs. 6·8

The rest amount = 68 – 6·8 = Rs. 61·2

When rest amount is 61.2, then initial amount 100

$\therefore$ Rest amount is 8,400, then initial amount should be $= \frac{100 \times 8400}{61{\cdot}2}$

$= \frac{700000}{51} =$ Rs. 13,725·5.

23. Required percentage

$$= \left[\frac{1^3 + 2^3 + 3^3 + 4^3 + 5^3}{1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3} \times 100\right]\%$$

$$= \left[\left(\frac{\left\{5\left(\frac{5+1}{2}\right)\right\}^2}{\left\{7\left(\frac{7+1}{2}\right)\right\}^2}\right) \times 100\right]\%$$

$$= \frac{25}{49} \times \frac{36}{64} \times 100\% = 28{\cdot}70\%.$$

24. Let r be the radius and height h volume of cone is given by

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

Volume of cylinder $V_{cy.} = \pi r^2 h$

Now, $\frac{V_{cone}}{V_{cy}} = \frac{1}{3}$

$$\% \text{ change} = \frac{V_{cy} - V_{cone}}{V_{cy}} \times 100\%$$

$$= \frac{2}{3} \times 100\%$$

$$= 66{\cdot}6\%.$$

25. We know that $R \propto l$; $R \propto \frac{1}{A}$

$$R = \rho \frac{l}{A} \quad \ldots(1)$$

In case of I[st] $R_1 = \rho \frac{l}{A}$

In 2[nd] case $R_2 = \rho \frac{l}{4A} = \frac{R_1}{4}$

Now, Change $= R_2 - R_1 = -\frac{3R_1}{4}$

$$\% \text{ Change} = -\frac{3\frac{R_1}{4}}{R_1} \times 100\% = -75\%$$

Clearly after cutting resistance of the resistor decrease by 75%.

26. Initial length = l; cross-sectional area = A

Resistance is $R = \rho \frac{l}{A}$

New length $= \frac{12}{10} \cdot l$

New cross-sectional area = $\frac{11}{10} \times A$; New resistance is

$$R_1 = \frac{\frac{12}{10} l}{\frac{11}{10} \cdot A} = \rho \times \frac{12}{11} \times \frac{l}{A}$$

$$\text{Change} = R_1 - R = R \times \frac{12}{11} - R = \frac{1}{11} \times R$$

$$\% \text{ Change} = \frac{\frac{1}{11} \cdot R}{R} \times 100\% = 9{\cdot}09\%.$$

27. Original length = 24 cm.;
New length = (24 – 4) = 20 cm.
Original height = 16 cm.;
New height = (16 – 4) = 12 cm.
Original breadth = 20 cm;
New breadth = (20 – 4) = 16 cm.
Original volume = $24 \times 20 \times 16$ cm^3
= 7680 cm^3
New volume = $20 \times 16 \times 12 = 3840$ cm^3
Volume of air inside the box
= 7680 – 3840 = 3840 cm^3

$$\% \text{ Volume of air} = \frac{3840}{7680} \times 100\% = 50\%.$$

28.
$$\text{Percentage} = \frac{\log 1 + \log 2 + \log 5}{\log 1 + \log 2 + \log 5 + \log 10} \times 100\%$$

$$= \frac{\log 1.2.5}{\log 1.2.5.10} \times 100\%$$

$$= \frac{\log 10}{\log 10^2} \times 100\%$$

$$= \frac{1}{2} \times 100\% = 50\%.$$

29. Since, ΔABC is formed inside the circle so it is right angled triangle.

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{3^2 + 4^2} = 5 \text{ cm.}$$

$$\text{Area of semi-circle} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times 3{\cdot}14 \times \left(\frac{5}{2}\right)^2$$

$$= \frac{25 \times 3{\cdot}14}{8} \text{ cm}^2$$

$$\text{Area of triangle ABC} = \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$$

Now, Area not enclosed by semi-circle

$$= \frac{25 \times 3{\cdot}14}{8} - 6 = \frac{78{\cdot}50}{8} - 6$$

$$= \frac{30{\cdot}50}{8}$$

% Area not enclosed by semi-circle

$$= \frac{\frac{30{\cdot}50}{8}}{\frac{25 \times 3{\cdot}14}{8}} \times 100\%$$

$$= \frac{30{\cdot}50}{25 \times 3{\cdot}14} \times 100\%$$

$$= \frac{3050}{78{\cdot}5}\% = 38{\cdot}8\%.$$

30. Given series is
= 5 + 10 + 20 + 40 + 80 + ...
= $5 + 5.2 + 5{\cdot}2^2 + 5{\cdot}2^3 + 5{\cdot}2^4 + \ldots\ldots\ldots$
$a = 5,\ r = 2$
So, 11th term = $a.\,r^{10} = 5{\cdot}2^{10}$
10th term = $a.\,r^9 = 5{\cdot}2^9$
Difference between 11th term and 10th term
= $5{\cdot}2^{10} - 5{\cdot}2^9 = 5{\cdot}2^9$

$$\% \text{ Difference} = \frac{5{\cdot}2^9}{5{\cdot}2^9} \times 100\% = 100\%$$

31. We have the following relationship

$$\frac{C}{100} = \frac{F - 32}{180}$$

Now, $C_1 = 80°C$; $C_2 = 90°$

Now, $\frac{F_1 - 32}{9} = \frac{80}{5}$
$F_1 = 16 \times 9 + 32$;
$F_1 = 144 + 32 = 176°$

When $C_2 = 90°$
$\frac{F_2 - 32}{9} = \frac{90}{5}$
$F_2 = 18 \times 9 + 32$
$F_2 = 162 + 32$
$F_2 = 194°$

Change in Fahrenheit
= $F_2 - F_1$
= 194° – 176° = 18°

$$\text{Now, \% change} = \frac{18°}{176°} \times 100$$

$$= \frac{9}{88} \times 100 = \frac{225}{22} = 10{\cdot}2\%.$$

32. Let Initial edge = a m.

$$\text{Final edge} = \frac{12}{10} \times a \text{ m}$$

$$\text{Initial volume} = a^3\text{m}^3$$

$$\text{Final Volume} = \left(\frac{12}{10} \times a\right)^3 = \frac{1728}{1000} \times a^3\text{m}^3$$

$$\text{Change in Volume} = \left(\frac{1728}{1000} - 1\right) \times a^3$$

$$= \frac{728}{1000} \times a^3$$

$$\% \text{ Change in volume} = \frac{728}{1000} \times 100\% = 72{\cdot}8\%.$$

4 Profit and Loss

Cost Price (C.P.)—The price at which an article is bought is called the cost price (C.P.) of the article.

Example—If a customer buys any object for Rs. X from the shopkeeper / retailer / wholesaler.

Then this price is known as C.P. (Cost Price).

$$\text{C. P.} = \text{Rs. X}$$

Selling Price (S.P.)—The price at which an article is sold is called the selling price (S.P.).

Example—If a person sells an article at any cost to the other person. Then that price is called Selling Price (S.P.).

Profit or Gain—When S.P. > C.P.

$$\text{Profit or Gain} = \text{S.P.} - \text{C.P.}$$

Loss : When C.P. > S.P.

$$\text{Loss} = \text{C.P.} - \text{S.P.}$$

Some fundamental formula

$$\%\text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$\%\text{ loss} = \frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100$$

In case of profit

$$\text{S.P.} = \left(\frac{100 + \%\text{ profit}}{100}\right) \times \text{C.P.}$$

and in case of loss

$$\text{S.P.} = \left(\frac{100 - \%\text{ loss}}{100}\right) \times \text{C.P.}$$

If S.P. and profit are given

$$\text{C.P.} = \left(\frac{100}{100 + \%\text{ profit}}\right) \times \text{S.P.}$$

and

$$\text{C.P.} = \left(\frac{100}{100 - \%\text{ loss}}\right) \times \text{S.P.}$$

Illustration 1.

Sohan buys a pen for Rs. 12 and sells it for Rs. 16. Find his gain per cent.

Solution :

Using formula :

$$\text{Gain }\% = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

Here, C.P. = Rs. 12, S.P. = Rs. 16

$$\text{S.P.} > \text{C.P.}$$

So, $\text{Profit} = \text{S.P.} - \text{C.P.}$

Now, $\text{Gain }\% = \frac{16 - 12}{12} \times 100$

$$= \frac{4}{12} \times 100 = 33 \cdot 3\%.$$

Illustration 2.

Kamala buys a dress for Rs. 72 and after six months sells it for Rs. 90. Find her profit per cent.

Solution :

Given C.P. = Rs. 72

S.P. = Rs. 90

Here, S.P. > C.P.

Profit takes place

$$\text{Profit} = \text{S.P.} - \text{C.P.}$$

$$= \text{Rs. } (90 - 72) = \text{Rs. } 18$$

$$\%\text{ profit} = \frac{\text{Profit}}{\text{C.P.}} \times 100\%$$

$$= \frac{18}{72} \times 100\% = 25\%$$

Illustration 3.

Reena buys a pizza for Rs. 42 and sells it for Rs. 40. Find loss or profit per cent.

Solution :

Given C.P. = Rs. 42

S.P. = Rs. 40

Since, S.P. < C.P.

So, $\text{loss} = \text{C.P.} - \text{S.P.}$

$$= 42 - 40 = \text{Rs. } 2$$

$$\%\text{ loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$\%\text{ loss} = \frac{2}{42} \times 100$$

$$= \frac{100}{21} = 4 \cdot 76\%.$$

Illustration 4.

Rajeev sells an article at 10% gain for Rs. 340. What is its cost price ?

Solution :

By using formula :

S.P. = 340

and % Profit = 10

$$\text{C.P.} = \frac{100}{100 + \%\text{ profit}} \times \text{S.P.}$$

$$\text{C.P.} = \frac{100}{100 + 10} \times 340$$

$$= \frac{100}{110} \times 340 = \text{Rs. } 309.$$

Illustration 5.

Rakesh buys an article for Rs. 400 and sells it at 15% profit. Find its S.P.

Solution :

We have C.P. = Rs. 400

% Profit = 15

$$\text{S.P.} = \frac{100 + \%\text{ profit}}{100} \times \text{C.P.}$$

$$\text{S.P.} = \frac{100 + 15}{100} \times 400$$

$$= \frac{115}{100} \times 400 = \text{Rs. } 460.$$

Illustration 6.

Akash buys a table for Rs. 600 and sells it at a profit of Rs. 120. Find per cent gain.

Solution :

By using formula :

$$\%\text{ Gain} = \frac{\text{Profit/Gain}}{\text{C.P.}} \times 100$$

$$\%\text{ Gain} = \frac{120}{600} \times 100 = \text{Rs. } 20.$$

Illustration 7.

Gita buys 12 pens for Rs. 96 and sells 10 pens for Rs. 90. Find per cent profit or loss.

Solution :

$$\text{Cost price of one pens} = \frac{96}{12} = \text{Rs. } 8$$

$$\text{Selling price of one pens} = \frac{90}{10} = \text{Rs. } 9$$

Since, selling price is greater than cost price.

Profit = 9 – 8 = Re. 1

$$\%\text{ Profit} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$= \frac{1}{8} \times 100 = 12{\cdot}5\%.$$

Illustration 8.

Leela buys 6 dresses for 600 and sells 5 dresses for Rs. 575. Find per cent loss / profit.

Solution :

$$\text{C.P. of one dress} = \frac{600}{6} = \text{Rs. } 100$$

$$\text{S.P. of one dress} = \frac{575}{5} = \text{Rs. } 115$$

Since, S.P. > C.P.

Profit = S.P. – C.P.

= 115 – 100 = Rs. 15

$$\%\text{ profit} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$= \frac{15}{100} \times 100 = 15\%.$$

Illustration 9.

Radha buys A tables for Rs. a and sells B tables for Rs. b. Find net profit / loss per cent.

Solution :

$$\text{C.P. of one table} = \text{Rs. } \frac{a}{A}$$

$$\text{S.P. of one table} = \text{Rs. } \frac{b}{B}$$

In the above C.P. and S.P. we do not know which is greater than other.

So, we have two cases—

Case I : If S.P. > C.P.

Then, profit = S.P. – C.P.

$$= \frac{b}{B} - \frac{a}{A}$$

$$\%\text{ Profit} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$= \left[\left(\frac{b}{B} - \frac{a}{A}\right) \Big/ \frac{a}{A}\right] \times 100$$

$$\%\text{ Profit} = \left(\frac{b}{B} \times \frac{A}{a} - 1\right) \times 100$$

Case II : If C.P. > S.P.

Loss = C.P. – S.P.

$$\text{Loss} = \frac{a}{A} - \frac{b}{B}$$

$$\%\text{ Loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$= \left(\frac{a}{A} - \frac{b}{B}\right) \times \frac{A}{a} \times 100$$

$$\%\text{ Loss} = \left(1 - \frac{b}{B} \times \frac{A}{a}\right) \times 100$$

Problems on Successive Profits and Losses

Illustration 10.

Sita buys a cooker for Rs. 500 and sells it to Soni at 20% profit. Soni sells it to Ritu for 15% profit. Ritu uses it for two years and then sells it to Sonali at a loss of 12%. For how much does Ritu sell the cooker to Sonali ?

Solution :

C.P. of cooker = Rs. 500

$$\text{Sita sells it at} = \text{Rs. } \frac{120}{100} \times 500$$

Soni buys cooker at Rs. $\frac{120}{100} \times 500$

Now, Soni sells it at $= 500 \times \frac{120}{100} \times \frac{115}{100}$

Again, Ritu buys the cooker from Soni at $500 \times \frac{120}{100} \times \frac{115}{100}$

Sonali buys the cooker at

$$= 500 \times \frac{120}{100} \times \frac{115}{100} \times \left(\frac{100 - 12}{100}\right)$$

Ritu sells the cooker to the Ritu at

$$= 500 \times \frac{120}{100} \times \frac{115}{100} \times \frac{88}{100}$$

Clearly :

$$\text{Final S.P.} = \text{C.P.} \times \underbrace{\left(1 + \frac{\%\text{ profit}}{100}\right)\left(1 + \frac{\%\text{ profit}}{100}\right)\left(\text{Loss} - \frac{\%\text{ loss}}{100}\right)}_{\longleftarrow \text{ Product Factors } \longrightarrow}$$

So, In case of successive profits/loses.

Final S.P. = Initial C.P. × Product factors.

Illustration 11.

X sells an article to Y at a loss of 10%, Y to Z at a gain of 15%, Z to U at a loss of 5% and U to V at a profit of 20%. If V had to pay Rs. 450. How much did X pay for it ?

Solution :

From above formula :

$$\text{Product factors} = \left(1 - \frac{10}{100}\right)\left(1 + \frac{15}{100}\right)\left(1 - \frac{5}{100}\right)\left(1 + \frac{20}{100}\right)$$

$$= \frac{90}{100} \times \frac{115}{100} \times \frac{95}{100} \times \frac{120}{100}$$

$$\text{S.P.} = \text{Rs. } 450$$

$$\text{C.P.} = \frac{\text{S.P.}}{\text{Product Factors}}$$

$$= \frac{450}{\frac{90}{100} \times \frac{115}{100} \times \frac{95}{100} \times \frac{120}{100}}$$

$$\text{C.P.} = \frac{450 \times 1000000}{9 \times 115 \times 95 \times 12}$$

$$= \frac{500000}{1311}$$

$$= \text{Rs. } 381 \cdot 4$$

Problems based on Dishonest Seller/Buyer using faulty Measures of Weight/Long

$$\%\text{ Gain} = \frac{\text{Difference between True measure and false measure}}{\text{Amount for which payment made}} \times 100$$

Illustration 12.

Sohan sells cloth at cost price but he uses faulty metre rod. His metre rod measures 80 cms. only. Find his gain per cent.

Solution :

On selling each metre he gets profit as the price of 20 cm. cloth.

C.P. of the cloth sold as one metre = price of 80 cm.

By using formula :

$$\%\text{ Profit} = \frac{20}{80} \times 100 = 25\%,$$

Illustration 13.

A trader buys corn from a farmer using faulty weight at a rate which is same as his selling price in the market. His 1 kg weights 1080 gms. Find his gain per cent.

Solution :

For every 1 kg that the trader buys, he gets

Addition = 1080 – 1000 = 80 gms

Profit = price of 80 gms

C.P. = price of 1 kg

Because trader pays only for 1000 gms and not for 1080 gms.

$$\%\text{ Profit} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$= \frac{80}{1000} \times 100 = 8\%.$$

Illustration 14.

The cost price of 8 pens is equal to the selling price of 10 pens. Find the profit per cent.

Solution :

S.P. of 10 pens = C.P. of 8 pens

S.P. of 1 pen = C.P. of $\frac{4}{5}$ pen

Let C.P. of one pen = Rs. a

Now, S.P. of one pen = Rs. $\frac{4}{5} \cdot a$

C.P. > S.P.

So, Loss = C.P. – S.P.

$$= a - \frac{4}{5} \cdot a$$

$$= \frac{1}{5} \cdot a$$

Now, $\%\text{ Loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100$

$$= \frac{\frac{1}{5} \cdot a}{a} \times 100 = 20\%.$$

Illustration 15.

If the cost price of a articles equals the selling price of b articles. Find the per cent profit/loss.

Solution :

Let C.P. of one article = Rs. X

Since, S.P. of b articles = C.P. of a articles

$$\text{S.P. of one article} = \text{Rs. } \frac{a}{b} \cdot X$$

If $a > b$

Case I : $a > b$

$$\text{S.P.} > \text{C.P.}$$

$$\text{Profit} = \text{S.P.} - \text{C.P.}$$

$$\% \text{ profit} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$\% \text{ profit} = \left(\frac{\frac{a}{b}X - X}{X}\right) \times 100$$

$$\% \text{ profit} = \left(\frac{a}{b} - 1\right) \times 100$$

Case II : $b > a$

$$\text{C.P.} > \text{S.P.}$$

$$\text{Loss} = \text{C.P.} - \text{S.P.}$$

$$\% \text{ loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$\% \text{ loss} = \frac{X - \frac{a}{b}X}{X} \times 100$$

$$\% \text{ loss} = \left(1 - \frac{a}{b}\right) \times 100$$

Illustration 16.

Shyam loses 15% when he sells his camera for Rs. 500. For how much should he sells it in order to gain 10% ?

Solution :

Since, Shyam sells camera at 85 whose C.P. = Rs. 100

∴ Shyam sells camera at 1 whose C.P. = $\frac{100}{85}$

∴ Shyam sells camera at Rs. 500 whose C.P.

$$= \frac{100}{85} \times 500 = \text{Rs. } \frac{10000}{17}$$

If Shyam wants to profit of 10%.

So, when C.P. is Rs. 100, then S.P. = Rs. 110

$$\therefore \text{C.P. is Rs. } \frac{10000}{17} \text{ then S.P.} = \frac{110}{100} \times \frac{10000}{17}$$

$$= \frac{11 \times 1000}{17}$$

$$= \frac{11000}{17}$$

$$\text{Required S.P.} = \text{Rs. } \frac{11000}{17}$$

Illustration 17.

Raju gains $a\%$ when he sells an article for Rs. b. What would be the profit or loss per cent if he sold it for Rs. c ?

Solution :

When Raju gains $a\%$

$$\text{C.P.} = \left(\frac{100}{100 + a}\right) \times b$$

Case I : If C > C.P.

$$\% \text{ profit} = \frac{C - \frac{100}{100 + a} \times b}{\frac{100}{100 + a} \times b} \times 100$$

$$\% \text{ profit} = \left[\frac{C(100 + a)}{100} \times b - 1\right] \times 100$$

Case II : If C.P. > C

$$\% \text{ loss} = \left[1 - \frac{C(100 + a)}{100} \times b\right] \times 100$$

Illustration 18.

Raju loses $a\%$ when he sells an article for Rs. b. What would be the profit or loss per cent if he sold it for Rs. C ?

Solution :

When Raju loses $a\%$

$$\text{C.P.} = \frac{100}{100 - a} \times b$$

Case I : If C.P. > C

$$\text{Loss} = \text{C.P.} - C$$

$$= \frac{100}{100 - a} \times b - C$$

$$\% \text{ Loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$\% \text{ Loss} = \left(1 - \frac{C(100 - a)}{100 \times b}\right) \times 100$$

Case II : If C > C.P.

$$\text{Profit} = C - \text{C.P.}$$

$$= C - \frac{100}{100 - a} \times b$$

$$\% \text{ profit} = \left(\frac{C(100 - a)}{100 \times b} - 1\right) \times 100$$

Illustration 19.

Gita sold an article at a profit of X%. If he had bought if for Y% less and sold it for Rs. W more, he would have made profit of Z%. Find the cost price of the article.

Solution :

Let the original cost price be Rs. K

$$\text{Initial S.P.} = \text{Rs. } \left(\frac{100 + X}{100}\right) K$$

$$\text{New C.P.} = \text{Rs.}\left(\frac{100+Y}{100}\right)K$$

$$\text{New S.P.} = \text{Rs.}\left(\frac{100+Z}{100}\right)\left(\frac{100-Y}{100}\right)K$$

Difference between the two sole price equals Rs. Z.

$$\text{So, }\left(\frac{100+Z}{100}\right)\left(\frac{100-Y}{100}\right)K - \left(\frac{100+X}{100}\right)K = W$$

$$\therefore K = \frac{W \times 100 \times 100}{(100+Z)(100-Y)-(100+X)\times 100}$$

Illustration 20.

Sita buys certain quantity of an article for Rs. z. She sells m^{th} part of the stock at a loss of $x\%$. At what per cent gain should she sell the remaining stock, so as to make an overall profit of $y\%$ on the total transaction ?

Solution :

C.P. of m^{th} part of stock = Rs. $(m \times p)$ or Rs. $m.z$

$$\text{S.P. of } m^{th} \text{ part of stock} = \frac{100-x}{100} \times z$$

For $y\%$ profit on total transaction, its S.P.

$$= \left(\frac{100+y}{100}\right) \times z$$

S.P. of the remaining $(1-m)^{th}$ part of stock

$$= \left(\frac{100+y}{100}\right) \times z - \left(\frac{100-x}{100}\right) \times m \times z$$

C.P. of the remaining $(1-m)^{th}$ part of stock

$$= (1-m).z$$

$$\% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$\% \text{ profit} = \frac{(100+y)-(100-x)m-100(1-m)}{(1-m)z \times 100} \times 1000$$

$$\text{Required \% profit} = \frac{y+xm}{1-m}\%$$

Illustration 21.

Krishna sells a magazine at a profit of 2%. If he had bought it at 5% less and sold it for Rs. 5 more, he would have gained 10%. Find the cost price of the magazine.

Solution :

Here, we have

$$X = 2$$
$$Y = 5$$
$$W = 5$$
$$Z = 10$$

Using formula :

Cost price of magazine

$$= \frac{W \times 100 \times 100}{(100+Z)(100-Y)-(100+X)\times 100}$$

$$= \frac{5 \times 100 \times 100}{110 \times 95 - 102 \times 100}$$

$$= \frac{5 \times 1000}{1045 - 1020}$$

$$= \frac{5 \times 1000}{25}$$

Cost of price of magazine = Rs. 200.

Illustration 22.

Ashu sells an article at a loss of 10%. If he had bought it at 20% less and sold it for Rs. 90 more, he would have made a profit of 20%. Find the cost price.

Solution :

Here, we have

$$X = 10$$
$$Y = 20$$
$$W = 90$$
$$Z = 20$$

Cost price of article

$$= \frac{W \times 100 \times 100}{(100+Z)(100-Y)-(100-X)\times 100}$$

$$= \frac{W \times 100 \times 100}{(100+20)(100-20)-(100-10)\times 100}$$

$$= \frac{W \times 100 \times 100}{120 \times 80 - 90 \times 100} = \frac{90 \times 100}{96-90}$$

$$= \frac{9000}{6} = \text{Rs. } 1{,}500.$$

Illustration 23.

Sohan invests Rs. Z in shares. He sells m^{th} part of it at a profit of X% and the remaining at a loss of Y%. Find his overall % profit or loss.

Solution :

$$\text{C.P. of } m^{th} \text{ part} = m.Z$$

$$\text{S.P. of } m^{th} \text{ part} = \left(\frac{100+X}{100}\right)m.Z$$

C.P. of remaining $(1-m)^{th}$ part = $(1-m)$. Z

S.P. of remaining $(1-m)^{th}$ part

$$= \left(\frac{100-Y}{100}\right)m.Z + \left(\frac{100-Y}{100}\right)$$

$$(1-m).Z$$

$$\text{Total C. P.} = \text{Rs. Z}$$

$$\% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$\% \text{ profit} = (100-Y)(1-m)+(100+X).$$

$$m - 100$$

$$\% \text{ profit} = [m.X-(X-m)Y]\ \%.$$

Illustration 24.

A shopkeeper buys wheat for Rs. 2000. He had to sell one fifth at a loss of 20%. At what per cent gain he should sell the remaining stock, so as to make an overall profit of 10% on the total transaction ?

Solution :

Here,

$$Z = \text{Rs. } 2000$$
$$X = 20$$
$$m = \frac{1}{5}$$
$$Y = 10$$

Using formula :

$$\% \text{ profit} = \frac{Y + X.m}{1 - m}\%$$

$$\% \text{ profit} = \frac{10 + 20 \times \frac{1}{5}}{1 - \frac{1}{5}}\%$$

$$= \frac{14 \times 5}{4}\%$$

$$\% \text{ profit} = 17{\cdot}5\%.$$

Illustration 25.

A trader buys rice for Rs. 4800. He sells one third at a loss of 12%. At what per cent gain should he sell the remaining stock, so as to make an overall profit of 10% ?

Solution :

Here, $Z = 4800$

We have, $m = \frac{1}{3}$

$$X = 12$$
$$Y = 10$$

Using formula :

$$\% \text{ profit} = \frac{Y + X.m}{1 - m}\%$$

$$= \frac{10 + 12 \times \frac{1}{3}}{1 - \frac{1}{3}}\%$$

$$= \frac{14 \times 3}{2}\%$$

$$\% \text{ profit} = 21\%.$$

Illustration 26.

Ramesh buys certain quantity of an article for Rs. a he sells n^{th} part of it at a profit of X%. At what per cent profit or loss should be sell the remaining $(1 - n)^{th}$ part of the stock, so as to make an overall profit of Y% ?

Solution :

$$\text{C.P. of } n^{th} \text{ part} = \text{Rs. } n.a$$

$$\text{S.P. of } n^{th} \text{ part} = \text{Rs.} \left(\frac{100 + X}{100}\right) n.a$$

$$\text{C.P. of total} = \text{Rs. } a$$

$$\text{Total S.P required} = \text{Rs.} \left(\frac{100 + Y}{100}\right) a$$

C.P. of remaining $(1 - n)^{th}$ the part = Rs. $(1 - n).a$

S.P. of remaining $(1 - n)^{th}$ the part

$$= \text{Rs.} \left(\frac{100 + Y}{100}\right).\, a - \left(\frac{100 + X}{100}\right) n.a$$

$$= \text{Rs.} \frac{(100 + Y) - (100 + X).n}{100} \times a$$

$$\% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$= \frac{100 + Y - 100n - Xn - 100 + 100n}{100(1 - n) \times a} \times 100\%$$

$$\% \text{ profit} = \left(\frac{Y - Xn}{1 - n}\right)\%.$$

Illustration 27.

A man buys wheat for Rs. 3000. He sells one-third of it at a profit of 12%. At what per cent gain should he sell remaining two-third, so as to make an overall profit of 15% on the whole transaction ?

Solution :

$$\text{C.P. of } \frac{1}{3}\text{rd} = \frac{3000}{3} = \text{Rs. } 1000$$

$$\text{S.P. of } \frac{1}{3}\text{rd} = \frac{120}{100} \times 1000 = \text{Rs. } 1200$$

$$\text{Total C.P.} = \text{Rs. } 3000$$

$$\text{Required S.P.} = \frac{115}{100} \times 3000 = \text{Rs. } 3450$$

$$\text{C.P. of rest } \frac{2}{3}\text{rd} = \text{Rs. } 2000$$

$$\text{Now, S.P. of } \frac{2}{3}\text{rd} = 3450 - 1200 = \text{Rs. } 2250$$

$$\% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$= \frac{2250 - 2000}{2000} \times 100$$

$$= \frac{250}{2000} \times 100 = 12{\cdot}5\%.$$

Illustration 28.

A hike of X% in the price of an article forces a person to buys Y kg less for Rs. Z. Find the new and the original price per kg of the article.

Solution :

The hike in the price of the article

$$= \frac{X}{100} \times Z$$

This is the cost of Y kg article.

$$\text{C.P. of Y kg of the article} = \frac{X.Z}{100}$$

$$\text{Cost of 1 kg of the article} = \frac{X.Z}{100Y}$$

This is the new price of the article.

$$\text{Now, Original price} = \frac{100}{100+X} \times \text{new price}$$

$$= \frac{100}{100+X} \times \frac{X.Z}{100 \times Y}$$

$$\text{New Price} = \frac{X.Z}{100 \times Y}$$

$$\text{Original Price} = \frac{100}{100+X} \times \frac{X.Z}{100 \times Y}$$

$$\text{Original Price} = \frac{X.Z}{(100+X) \times Y}$$

Illustration 29.

A person sells one-fifth of his stock of good at a profit of 25%. At what per cent profit or loss should he sell his remaining stock, so as to make an overall profit of 12% ?

Solution :

Let C.P. of all the good = Rs. a

Now, C.P. of $\frac{1}{5}$th part of goods

$$= \text{Rs. } \frac{a}{5}$$

$$\text{S.P. of } \frac{1}{5}\text{th part of goods} = \text{Rs. } \frac{125}{100} \times \frac{a}{5} = \frac{25a}{100}$$

$$\text{C.P. of } \frac{4}{5}\text{th part of goods} = \text{Rs. } \frac{4}{5} \times a$$

$$\text{S.P. of the whole goods} = \frac{112}{100} \times a$$

S.P. of the $\frac{4}{5}$th part of goods

$$= \frac{112-25}{100} \times a$$

$$= \frac{87}{100} \times a$$

Clearly, S.P. of $\frac{4}{5}$th part > C.P. of $\frac{4}{5}$th of the goods

$$\text{So, profit} = \text{S.P.} - \text{C.P.}$$

$$= \frac{112}{100} \times a - \frac{87}{100} \times a$$

$$= \frac{25}{100} \times a = \frac{1}{4} \times a$$

$$\% \text{ Profit} = \frac{\frac{1}{4} \cdot a}{\frac{4}{5} \cdot a} \times 100 = \frac{5}{16} \times 100$$

$$= \frac{125}{4} = 31{\cdot}25\%.$$

Illustration 30.

A 20% hike in the price of sugar forces a person to purchase 5 kg less for Rs. 250. Find the new and original price of sugar.

Solution :

By using formula :

We have

$$X = 20$$

$$Y = 5$$

$$Z = 250$$

$$\text{New price} = \frac{X.Z}{100 \times Y}$$

$$= \frac{20 \times 250}{100 \times 5} = \text{Rs. } 10$$

$$\text{Original Price} = \frac{X.Z}{(100+X) \times Y}$$

$$= \frac{20 \times 250}{120 \times 5} = \text{Rs. } 8{\cdot}35.$$

Illustration 31.

A reduction of 5% in the price of rice enables a person to buy $\frac{1}{2}$ kg more for Rs. 190. Find the original price /kg of rice and also its reduced price.

Solution :

Using formula :

$$X = 5$$

$$Y = \frac{1}{2}\text{ kg.}$$

$$Z = 190$$

$$\text{New price} = \frac{X.Z}{100 \times Y}$$

$$= \frac{5 \times 190}{100 \times \frac{1}{2}}$$

$$= \frac{95 \times 2}{10} = \text{Rs. } 19/\text{kg.}$$

$$\text{Original Price} = \frac{X.Z}{(100-X) \times Y}$$

$$= \frac{5 \times 190}{(100-5) \times \frac{1}{2}} \times \text{Rs. } 20/\text{kg.}$$

Illustration 32.

A man purchased m oranges at X a rupee and n oranges at Y a rupee. He mixed them together and sold them at Z a rupee. What is his per cent loss or gain. What happen when $m = n$?

Solution :

$$\text{C.P. of } m \text{ oranges} = \text{Rs. } \frac{m}{X}$$

$$\text{C.P. of } n \text{ oranges} = \text{Rs. } \frac{n}{Y}$$

$$\text{Total C.P.} = \text{Rs. } \left(\frac{m}{X} + \frac{n}{Y}\right)$$

Now, S.P. of $(m+n)$ oranges

$$= \text{Rs. } \frac{m+n}{Z}$$

$$\% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{100} \times 100\%$$

$$\% \text{ profit} = \left[\frac{\frac{m+n}{Z}}{\frac{m}{X} + \frac{n}{Y}} - 1\right] \times 100\%$$

$$\% \text{ profit} = \left[\frac{(m+n)X.Y}{Z(mY + n.X)} - 1\right] \times 100\%$$

When $m = n$

$$\% \text{ profit} = \left(\frac{2m.X.Y}{m(X+Y).Z} - 1\right) \times 100\%$$

$$\% \text{ profit} = \left[\frac{2XY}{(X+Y).Z} - 1\right] \times 100\%.$$

Illustration 33.

A person mixes 30 kg of tea bought at Rs. 120 / kg with 25 kg of tea bought at Rs. 100 /kg. He sells the mixture at Rs. 105 kg. Find his total per cent loss/ gain ?

Solution :

$$\text{C.P. of 30 kg of tea} = 30 \times 120 = 3600$$

$$\text{C.P. of 25 kg of tea} = 25 \times 100 = 2500$$

$$\text{Total C.P.} = \text{Rs. } 6100$$

$$\text{S.P. of 55 kg of tea} = 55 \times 105 = \text{Rs. } 5775$$

Since, $\text{C.P.} > \text{S.P.}$

So, $\text{Loss} = \text{C.P.} - \text{S.P.}$

$$= 6100 - 5775 = 325$$

$$\% \text{ Loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100\%$$

$$= \frac{325}{6100} \times 100\%$$

$$= \frac{325}{61}\% = 5{\cdot}32\%.$$

Illustration 34.

Shyam purchase 100 oranges at 4 a rupee and 200 oranges at 2 a rupee. He mixed them and sells at 3 oranges a rupee. Find his per cent loss or gain.

Solution :

$$\text{C.P. of 100 oranges} = \text{Rs. } 25$$

$$\text{C.P. of 200 oranges} = \text{Rs. } 100$$

$$\text{S.P. of 300 oranges} = \text{Rs. } 100$$

$$\text{C.P. of 300 oranges} = \text{Rs. } 125$$

$$\text{C.P.} > \text{S.P.}$$

$$\text{Loss} = \text{C.P.} - \text{S.P.}$$

$$= 125 - 100 = \text{Rs. } 25$$

$$\% \text{ loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$= \frac{25}{125} \times 100\%$$

$$\% \text{ loss} = 20\%.$$

Illustration 35.

A man buys oranges at 20 a rupee for how many a rupee should he sell it so as to gain 10% ?

Solution :

Let $\text{C.P. of 20 oranges} = \text{Re. } 1$

$$\text{S.P. of 20 oranges} = \text{Rs. } \frac{110}{100} = \text{Rs. } 1{\cdot}1$$

Since, Cost of 20 oranges is Rs. 1·1

$$\text{In Rs. 1 oranges is} = \frac{20}{1{\cdot}1} = \frac{200}{11} = 18{\cdot}1 \cong 18.$$

Illustration 36.

A milkman purchases the milk at Rs. 18/litre and sells it at Rs. 20 liter still he mixes 2 litres of water with every 8 litres of pure milk. What is per cent profit ?

Solution :

For Simplicity :

$$\text{Let pure milk} = 8 \text{ litres}$$

Now, $\text{C.P. of pure milk} = 8 \times 18 = \text{Rs. } 144$

Since, he sell (8 + 2) litres with the cost of 20 /litres

$$\text{S.P.} = 10 \times 20 = \text{Rs. } 200$$

$$\text{Profit} = 200 - 144 = \text{Rs. } 56$$

$$\% \text{ Profit} = \frac{56}{144} \times 100$$

$$= \frac{350}{9} = 38{\cdot}8\%.$$

Illustration 37.

Two articles are sold at the same price. One at a profit of 50% and another one at a loss of 30%. What is the overall profit/loss ?

Solution :

Let $\text{C.P. of Ist article} = X$

$$\text{C.P. of 2nd article} = Y$$

$$\text{S.P. of Ist article} = 1{\cdot}5X$$

$$\text{S.P. of 2nd article} = 0{\cdot}7Y$$

According to question :

$$1{\cdot}5X = 0{\cdot}7Y$$

$$\frac{X}{Y} = \frac{7}{15} \quad \ldots(1)$$

$$\text{Total C.P.} = X + Y$$

$$= Y + \frac{7}{15} \times Y = \frac{22}{15} \times Y$$

$$\text{Total S.P.} = 1{\cdot}5X + 0{\cdot}7Y$$

$$= 1{\cdot}4Y$$

$$\text{Loss} = -\text{S.P.} + \text{C.P.}$$

$$\text{Loss} = -1{\cdot}4Y + \frac{22}{15} \times Y$$

$$= \frac{-21Y + 22Y}{15}$$

$$\text{Loss} = \frac{1}{15} \cdot Y$$

$$\% \text{ loss} = \frac{\frac{1}{15} \cdot Y}{\frac{22}{15} \cdot Y} \times 100$$

$$= \frac{100}{22} = \frac{50}{11} = 4{\cdot}55\%.$$

Illustration 38.

The profit percentage on the three articles X, Y and Z is 5%, 10% and 20% and the ratio of the cost price is 2 : 3 : 5. Also the ratio of number of article sold of X, Y and Z is 1 : 2 : 4. Then what is the overall profit percentage ?

Solution :

$$\text{Let C.P. of one X article} = 2a$$

$$\text{C.P. of one Y article} = 3a$$

$$\text{C.P. of one Z article} = 5a$$

$$\text{Total C.P.} = 2a + 2 \times 3a + 4 \times 5a$$

$$= 2a + 6a + 20a = 28a$$

$$\text{S.P.} = 1{\cdot}05 \times 2a + 6a\,(1{\cdot}1) + 2a \times 1{\cdot}2$$

$$= (2{\cdot}10 + 6{\cdot}6 + 24\,)a$$

$$= 32{\cdot}70a$$

$$\text{Profit} = \text{S.P.} - \text{C.P.}$$

$$= (32{\cdot}70 - 28)a$$

$$= 4{\cdot}70a$$

$$\% \text{ Profit} = \frac{4{\cdot}7a}{28a} \times 100\%$$

$$= \frac{470}{28}\% = 16{\cdot}7\%$$

Exercise A

1. A woman buys 4 tables and 16 chairs for Rs. 3200. She sells the tables at a profit of 20% and chairs at a profit of 10% and makes a profit of Rs. 600. At what price did she buys tables and chairs ?
2. 5 kg of rice costs as much 10 kg of wheat, 20 kg of wheat costs as much as 2 kg of tea, 4 kg of tea costs as much as 24 kg of sugar. Find the cost of 5 kg of sugar if 3 kg of rice costs Rs 30.
3. A fan and a watch are sold of the same price of Rs. 927. The shopkeeper earns a profit of 10% on the fan whereas he incurs a loss of 10% on the watch. Find his overall per cent gain or loss.
4. Rajesh purchased a chair marked at Rs. 800 at two successive discounts of 10% and 15% respectively. He spent Rs. 28 on transportation and sold the chair for Rs. 800. What is his gain per cent ?
5. Kavita buys two tables in Rs. 1,350. She sells one so as to loss 5% and other so as to gain $7\frac{1}{2}$ % on the whole she neither loss or gain. What did each table cost ?
6. Gopal buys some milk contained in 5 vessels of equal size. If he sells his milk at Rs. 6 a liter he losses Rs. 150 while selling it at Rs. 8 a liter he would gain Rs. 250 on the whole. Find the number of liter contain in each case.
7. Ram losses 10% by selling pencils at the rate of 20 a rupee. How many for a rupee must he sell them to gain 10% ?
8. A reduction of 40 per cent in the price of bananas would enable a man to obtain 32 more for Rs. 40. What is the reduced price per dozen ?
9. Gita buys watch for Rs. 1000 and sells it to Sita at 10% loss. Sita sells it to Sonali at 25% profit and Sonali sells it to Pinki at 5% profit. How much did Pinki pay for the watch ?
10. A shopkeeper uses faulty measure of weight of 900 gm. For 1 kg weight and sells wheat at Rs. 20/kg in place of Rs. 24 /kg. Find the per cent change in actual rate.
11. A salesman first marks the price of an article 25% above the cost price but later reduces it by 10% while selling. Find the net per cent profit.
12. Soni sells an article at 10% profit. If she sells it for Rs. 20 more she will make a profit of 12%. Find the cost price of the article.
13. Rahim sells an article at a loss of 15%. If he had bought it for 20% loss and sold it for Rs. 40 more he would have gained 30%. Find the cost price.
14. When a man sold an article for Rs. 900 and made a loss of 10%. At what price should he sell it so as to incur a loss of only 5% ?
15. Mohit bought 80 kg of tomatoes and plans to sell it at 10% profit at Rs. 4·5/kg 10 kg of tomatoes were found rotten. At what price should be now sell rest of the tomatoes in order to make 12% overall profit ?
16. A trader buys 10 shirts for payment of 8 shirts at the marked price and sells them at the marked price. Find his per cent profit.
17. Saurav invested Rs. 25,000 in sugar. He sold $\frac{2}{5}$th at a loss of 15%. At what per cent gain he should sell the remaining stock in order to make overall per cent of 20% ?
18. A reduction of 10% in the price of watch enables a Monu to buy 2 more for Rs. 32000. Find the original and the reduced price per watch.
19. A man buys a certain number of mangoes at 6 a rupee and equal number at 3 a rupee. He mixes them together and sells at a 5 a rupee. Find per cent profit or loss.

20. The marked price of an Apple is Rs. 10 the shop-keeper allows discount of 8% and still make a profit of 10%. What would be his profit if he did not allow discount?
21. Jiya started selling vegetables at Rs. 12 /kg but could not find buyers at this rate. So, she reduced the price to Rs. 10 kg but uses of a faulty weight of 900 gm. For 1 kg. Find the per cent change in actual price or loss.
22. Rohani bought 20 kg of rice at the rate of Rs. 5/kg and 30 kg of rice at the rate of Rs. 6/kg. She mixed the two and sold the mixture at the rate of Rs. 7·5/kg. What was her loss or gain in the total transaction ?
23. Find the difference between a single discount of 60% on Rs. 800 and two successive discounts of 40% and 20% on the same amount.
24. Shyam bought 2 dozen apples at Rs. 20 per dozen and 4 dozen apples at Rs. 15 per dozen. He sold all of them to earn 28%. A what price per dozen did he sell the apples ?
25. Find the cost price of an article, which on being sold at a gain of 15% yields Rs. 5 more than when it is sold at a loss of 12%.
26. 12 kg of potato costs as much as 3 kg of tomato, 9 kg of tomato costs as much as 60 kg of onion, 15 kg of onion costs as much as 21 kg of cabbage. If 12 kg of cabbage costs Rs. 108. Find the cost of 18 kg of potato ?
27. Ravi sells two articles for the same price. On one he incurs 20% loss while on the other he incurs 10% loss. Find his overall per cent loss.
28. Avinash buys a cap for Rs. 160 and sells it to Aman at 8% loss, Aman sells it to Raman at 10% profit and Raman sells it to Rahim at 20% profit. How much did Rahim pay for the cap ?

Answers

1. Let C.P. of one chair

$$= \text{Rs. } X$$

$$\text{C.P. of 16 chairs} = \text{Rs. } 16X$$

$$\text{C.P. of 4 tables} = 3200 - 16X$$

$$\text{S.P. of 4 tables} = \frac{120}{100} \times (3200 - 16.X)$$

$$\text{S.P. of 16 chairs} = \frac{110}{100} \times 4X$$

$$\text{Total S.P.} = \frac{10}{100}[12(3200 - 16X) + 44X]$$

$$= \frac{1}{10}(38400 - 192X + 44X)$$

$$\text{Profit} = \text{Total S.P.} - \text{Total C.P.}$$

$$\text{Profit} = \frac{1}{10}(38400 - 148X) - 3200$$

Or, $$600 = \frac{38400 - 32000 - 148.X}{10}$$

Or, $$148.X = 6400 - 600$$

$$X = \frac{5800}{148} = \text{Rs. } 39{\cdot}18$$

$$\text{C.P. of 16 chairs} = 16 \times 39{\cdot}18 = \text{Rs. } 627$$

$$\text{C.P. of 4 tables} = 3200 - 627 = \text{Rs } 2573.$$

2. $$\text{Cost of 3 kg of rice} = \text{Rs. } 30$$

$$\text{Cost of 5 kg of rice} = \text{Rs. } 50$$

$$\text{Cost of 10 kg of wheat} = \text{Rs. } 50$$

$$\text{Cost of 20 kg of wheat} = \text{Rs. } 100$$

$$\text{Cost of 2 kg of tea} = \text{Rs. } 100$$

$$\text{Cost of 4 kg of tea} = \text{Rs. } 200$$

$$\text{Cost of 24 kg of sugar} = \text{Rs. } 200$$

$$\therefore \text{ Cost of 1 kg of sugar} = \text{Rs. } \frac{200}{24}$$

$$\therefore \text{ Cost of 5 kg of sugar} = \text{Rs. } \frac{100}{12} \times 5$$

$$= \text{Rs. } \frac{500}{12} = \text{Rs. } 41{\cdot}66.$$

3. $$\text{Since, Cost of a fan} = \text{Rs. } 927$$

$$\text{Cost of a watch} = \text{Rs. } 927$$

$$\text{Total S.P.} = \text{Rs. } 1854$$

$$\text{C.P. of a fan} = \frac{100}{110} \times 927 = \text{Rs. } 842{\cdot}72$$

$$\text{C.P. of a watch} = \frac{100}{90} \times 927 = \text{Rs. } 1030$$

$$\text{Total C.P.} = 1030 + 842{\cdot}72$$

$$= \text{Rs. } 1872.72$$

Since, $$\text{C.P.} > \text{S.P.}$$

$$\text{Loss} = \text{C.P.} - \text{S.P.}$$

$$= 1872{\cdot}72 - 1854$$

$$= \text{Rs. } 18.72$$

$$\%\text{ loss} = \frac{\text{loss}}{\text{C.P.}} \times 100$$

$$= \frac{18{\cdot}72}{1872{\cdot}72} \times 100\% = 1\%$$

4. $$1^{st}\text{ discount} = \frac{10}{100} \times 800 = \text{Rs. } 80$$

$$2^{nd}\text{ discount} = \frac{15}{100} \times \text{Rest amount}$$

$$= \frac{15}{100} \times 720 = \text{Rs. } 108$$

$$\text{Rest Amount} = 720 - 108 = \text{Rs. } 612$$

$$\text{Total Cost Price} = 612 + 28 = \text{Rs. } 640$$

$$\text{S.P.} = \text{Rs. } 800$$

$$\text{Profit} = \text{S.P.} - \text{C.P.}$$

$$\text{Profit} = 800 - 640 = \text{Rs. } 160$$

$$\%\text{ Profit} = \frac{160}{640} \times 100 = 25\%$$

5. Let the cost of Ist table = Rs. X

The Cost of IInd table = Rs. (1350 – X)

S.P. of 1st table $= \frac{95}{100} \cdot X = \frac{19}{20} \cdot X$

S.P. of 2nd table $= \frac{107 \cdot 5}{100} \times (1350 - X)$

Since, there is no loss or gain.

So, C.P. = S.P.

$$1350 = \frac{19}{20} \cdot X + \frac{107 \cdot 5}{20}(1350 - X)$$

Or, $135000 = 95.X - 107 \cdot 5X + 107 \cdot 5 \times 1350$

$12 \cdot 5X = 145125 - 135000$

$12 \cdot 5X = 10125$

$$X = \frac{10125}{12 \cdot 5}$$

$$X = \frac{101250}{125} = \text{Rs. } 810$$

Cost of 1st table = Rs. 810

Cost of 2nd table = 1350 – 810 = Rs. 540.

6. Let the vessels contain = X litre of milk

Now, cost price of milk in Ist case

$= 6X + 150$

In 2nd case cost price of milk

$= 8X - 250$

Since, C.P. does not change

$6X + 150 = 8X - 250$

$2X = 400$

$X = 200$

Clearly, 5 vessels contain 200 litres of milk.

7. S.P. of pencils = Re. 1

To get 10% profit

C.P. of pencils = Rs. $\frac{11}{10}$

If Ram sells at the rate of 20 for $\frac{11}{10}$ rupee. He gains 10%.

So, In one rupee $= \frac{20}{11} \times 10$

$= \frac{200}{11} \cong 18$ pencils

8. Reduction in price $= 40 \times \frac{40}{100} =$ Rs. 16

According to question,

Man buys 32 bananas in Rs. 16.

∴ Man buys 12 bananas in Rs. $\frac{16}{32} \times 12 =$ Rs. 6

∴ Rs. 6 / dozens

9. Gita buys watch at = Rs. $\frac{90}{100} \times 1000$

= Rs. 900

Sita buys watch at $= \frac{125}{100} \times 900 =$ Rs. 1125

Sonali buys watch at $= \frac{105}{100} \times 1125$

= Rs. 1181·25

So, Pinki pays Rs. 1181·25.

10. After reduction,

The price of 900 gm wheat = Rs. 20

∴ The price of 1000 gm wheat

$= \frac{20}{900} \times 1000 = \text{Rs. } \frac{200}{9}$

Loss $= \text{Rs. } 25 - \frac{200}{9} = \text{Rs. } \frac{25}{9}$

% Loss $= \frac{\frac{25}{9}}{25} \times 100\% = \frac{100}{9}\%$

= 11·11%.

11. Let the actual price of the article = Rs. X

Marked price in case 1st $= \frac{125}{100} X$

After reduction of 10%

Marked price $= \frac{125}{100} X \times \frac{90}{100}$

$= \frac{5}{4} \times \frac{9}{10} X = \frac{9}{8} X$

Profit = marked price – actual price

$= \frac{9}{8} X - X = \frac{X}{8}$

% profit $= \left(\frac{\text{Profit}}{\text{Actual price}}\right) \times 100$

$= \frac{\frac{X}{8}}{X} \times 100 = 12 \cdot 5\%.$

12. Let the cost price of the article = Rs. X

After 10% profit

S.P. of the article $= \frac{110}{100} \times X$

$= \frac{11}{10} \times X$

When Soni gets 12% profit, then

S.P. $= \frac{11}{10} \times X + 20$

According to question,

When selling price is 112, then C.P. = 100

∴ Selling price is 1, then C.P. $= \frac{100}{112}$

$\therefore$ Selling price is $\left(\frac{11}{10}\times X+20\right)$, then C.P. = $\frac{100}{112}\times\left(\frac{11}{10}\times X+20\right)$

$$\text{This equal to } X = \frac{100}{112}\times\left(\frac{11X}{10}+20\right)$$

$$\text{Or, } X\left(1-\frac{110}{112}\right) = 20\times\frac{110}{112}$$

$$X = \text{Rs. } 1000.$$

13. Let the C.P. of the article = Rs. X

S.P. of the article when Loss is 15% = $\frac{85}{100}\cdot X$

$$\text{He bought the article at} = \text{Rs. } \frac{80}{100}\cdot X$$

$$\text{Now, Rahim sells the article} = \frac{80}{100}\cdot X+40$$

According to question,

$$\frac{80}{100}\cdot X = \left(\frac{80}{100}\cdot X+40\right)\times\frac{100}{113}$$

$$\text{Or, } \frac{8}{10}\cdot X = \left(\frac{8}{10}\cdot X+40\right)\times\frac{100}{113}$$

$$\text{Or, } \frac{4}{5}\cdot X\left(1-\frac{100}{113}\right) = 40\times\frac{100}{113}$$

$$\text{Or, } \frac{4}{5}\cdot X\times\frac{13}{113} = \frac{4000}{113}$$

$$\text{Or, } X = \frac{4000\times 5}{13\times 4}$$

$$X = \text{Rs. } \frac{5000}{13}.$$

14. Given S.P. = Rs. 900

$$\text{C.P.} = \frac{100}{90}\times 900 = \text{Rs. } 1000$$

To get 5% loss the S.P.

$$= \frac{95}{100}\times 1000 = \text{Rs. } 950.$$

15. When Mohit gets 10% profit, then C.P. of tomatoes

$$= \frac{100}{90}\times 4{\cdot}5 = \text{Rs. } 5$$

So, C.P. of 80 kg of tomatoes = 80 × 5 = Rs. 400

Rest tomatoes = 70 kg.

Now, S.P. of 70 kg of tomatoes

$$= \frac{112}{100}\times 400 = \text{Rs. } 448$$

$$\text{S.P. of 1 kg of tomatoes} = \frac{448}{70} = \text{Rs. } 6{\cdot}4.$$

16. Let C.P. of one shirt = Rs. X

Now, C.P. of 8 shirts = Rs. 8X

This is equal to 10 shirts = Rs. 8X

Now, the trader buys 10 shirts at Rs. 8X.

He sells shirts at marked price.

So, S.P. of 10 shirts = Rs. 10X

Now, profit = 10X – 8X = Rs. 2X

$$\%\text{ profit} = \frac{2.X}{8.X}\times 100 = \text{Rs. } 25\%.$$

17. Now, S.P. of $\frac{2}{5}$th stock of sugar

$$= \frac{85}{100}\times 25000\times\frac{2}{5} = \text{Rs. } 8500$$

Let S.P. of $\frac{3}{5}$th stock of sugar = X

Now, Total S.P. = 8500 + X

C.P. of Total sugar = 25000

$$\text{Now, Total S.P.} = \frac{120}{100}\times 25{,}000 = \text{Rs. } 30{,}000$$

According to question,

$$8500 + X = 30{,}000$$

$$X = 30{,}000 - 8{,}500$$

$$X = \text{Rs. } 21{,}500$$

18. The reduction in the price

$$= \frac{10}{100}\times 32{,}000 = 3{,}200$$

From the question,

It is clear that cost of 2 watch = Rs. 3200

Cost of one watch = Rs. 1600

When C.P. is 90 the original price = Rs. 100

C.P. is 1600 the original price

$$= \frac{100}{90}\times 1600$$

$$= \frac{16000}{9} = 1777{\cdot}7.$$

19. Let the total number of mangoes = 2.X

Now, C.P. of 2.X mangoes

$$= \frac{X}{6}+\frac{X}{3} = \frac{3}{6}\cdot X = \frac{X}{2}$$

$$\text{S.P. of 2.X mangoes} = \frac{2}{5}\cdot X$$

$$\text{C.P.} > \text{S.P.}$$

$$\text{So, Loss} = \text{C.P.} - \text{S.P.}$$

$$= \frac{X}{2}-\frac{2X}{5} = \frac{X}{10}$$

$$\%\text{ Loss} = \frac{\text{Loss}}{\text{C.P.}}\times 100\%$$

$$= \frac{\frac{X}{10}}{\frac{3.X}{6}}\times 100\%$$

$$= \frac{60}{3} = 20\%.$$

20. Discount $= 10 \times \frac{8}{100} = 0{\cdot}8$

C.P. of Apple $= 10 - 0{\cdot}8 = 9{\cdot}2$

S.P. $= \frac{110}{100} \times 9{\cdot}2 = $ Rs. 10.12

If discounts is not allowed, then

Profit $= 10{\cdot}12 - 10 = 0{\cdot}12$

% Profit $= \frac{\text{Profit}}{\text{MP}} \times 100\%$

$= \frac{0{\cdot}12}{10} \times 100\% = 1{\cdot}2\%$

21. After reduction

The price of 900 gm of vegetables = Rs. 10

$\therefore$ The price of 1000 gm of vegetables

$= \frac{10}{900} \times 1000 = \frac{100}{9}$

Loss $= 12 - \frac{100}{9} = \frac{8}{9}$

% Loss $= \frac{\frac{8}{9}}{12} \times 100\%$

$= \frac{200}{27}\% = 7{\cdot}4\%$.

22. C.P. of 20 kg of rice $= 20 \times 5 = $ Rs. 100

C.P. of 30 kg of rice $= 30 \times 6 = $ Rs. 180

S.P. of 50 kg of rice $= 50 \times 7{\cdot}5 = $ Rs. 375

Total C.P. $= 280$

Total S.P. $= 375$

Profit = S.P. – C.P.

$= 375 - 280 = $ Rs. 95

% profit $= \frac{\text{Profit}}{\text{C.P.}} \times 100$

$= \frac{95}{280} \times 100$

$= \frac{950}{28} = 33{\cdot}92\%$

23. When discount is 60%

Then discount $= 800 \times \frac{60}{100} = $ Rs. 480

When there are two successive discounts, then

I[st] discount $= \frac{40}{100} \times 800 = $ Rs. 320

Rest Amount $= 800 - 320 = $ Rs. 480

and 2[nd] Amount $= 480 \times \frac{20}{100} = $ Rs. 96

Total amount $= 320 + 96 = $ Rs. 416

Now, Difference $= 480 - 416 = $ Rs. 64.

24. Total C.P. $= 2 \times 20 + 4 \times 15$

$= 40 + 60 = $ Rs. 100

S.P. $= \frac{128}{100} \times 100 = $ Rs. 128

Now, S.P. of 12 apples $= \frac{128}{6} = $ Rs. 21·33.

25. Let C.P. of the article = X

1[st] S.P. $= \frac{115}{100} \cdot X$

2[nd] S.P. $= \frac{88}{100} \cdot X$

Difference $= \left(\frac{115}{100} - \frac{88}{100}\right) X$

$5 = \frac{27}{100} \cdot X$

X = Rs. $\frac{500}{27}$

X = Rs. 18·5.

26. Let C.P. of 1 kg of potato = Rs. X

C.P. of 12 kg of potato = Rs. 12.X

C.P. of 3 kg of tomato = Rs. 12.X

C.P. of 9 kg of tomato = Rs. 36.X

C.P. of 60 kg of onion = Rs. 36.X

C.P. of 15 kg of onion = Rs. $\frac{36}{60} \cdot X \times 15$

= Rs. 9.X

C.P. of 21 kg of cabbage = Rs. 9.X

C.P. of 12 kg of cabbage = Rs. $\frac{9}{21} \cdot X \times 12$

$= \frac{36}{7} \cdot X$

$108 = \frac{36}{7} \cdot X$

$X = \frac{108 \times 7}{36} = $ Rs. 21

Cost of 18 kg of potato = 18.X = 18 (21)

= Rs. 378.

27. Let S.P. of each article = Rs. X

Total S.P. of articles = Rs. 2.X

C.P. of 1[st] article $= \frac{100}{80} \times 2.X = 2{\cdot}5X$

C.P. of 2[nd] article $= \frac{100}{90} \times 2.X = \frac{20}{9} \cdot X$

Total C.P. $= 2{\cdot}5.X + \frac{20}{9} \cdot X$

$= \frac{22{\cdot}5.X + 20.X}{9}$

$= \frac{42{\cdot}5}{9} \cdot X$

C.P. > S.P.

So, $\text{Loss} = \text{C.P.} - \text{S.P.}$

$$= \frac{42 \cdot 5}{9} \cdot X - 2.X$$

$$= \frac{24 \cdot 5}{9} \cdot X$$

$$\% \text{ Loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100\%$$

$$= \frac{\frac{24 \cdot 5X}{9}}{\frac{42 \cdot 5X}{9}} \times 100\%$$

$$= \frac{24 \cdot 5}{42 \cdot 5} \times 100\%$$

$$= 57 \cdot 64\%.$$

28. Aman buys a cap at $= \text{Rs.}\ \frac{92}{100} \times 160$

$= \text{Rs. } 147 \cdot 20$

Raman buys a cap at $= \text{Rs.}\ \frac{110}{100} \times 147 \cdot 20$

$= \text{Rs. } 161 \cdot 92$

Rahim buys a cap at $= \text{Rs.}\ \frac{120}{100} \times 161 \cdot 92$

$= \text{Rs. } 194.304$

Special

1. Ganesh, on his dead bed, keeps half his property for his wife and divides the rest equally among his three sons; Ram, Shyam, Krishna. Some year later, Ram dies leaving half his property to his widow and half to his brothers. Shyam and Krishna, sharing equally. When Shyam makes his will, he keeps half his property for his widow and the rest he bequeaths to his younger brother Krishna when Krishna dies some years later, he keeps half his property for his widow and the remaining for his mother. The mother now has Rs. 18,30,000.
   (i) What was the worth of total property ?
   (ii) What was Shyam's original share ?
   (iii) What was the ratio of the property owned by the window of the three sons, in the end ?
2. Ashu and Tannu both are dealers of classic scooter the price of classic scooter is Rs. 35,000. Ashu gives a discount of 15% on whole, while Tannu gives a discount of 20% on the first Rs. 25,000 and 5% on the rest Rs. 10,000. What is the difference between their selling prices ?
3. A company purchases components X and Y from U.K. and Germany, respectively X and Y from 30% and 40% of the total product cost current gain is 25% due to change in the international scenario, cost of the U.K. mark increased by 30% and that of Germany dollar increased by 20%. Due to market conditions the selling price can not be increased beyond 10%.
   (i) What is the maximum current gain possible.
   (ii) If the Germany dollar becomes cheap by 10% over its original cost and the cost of U.K. mark increased by 15%. What will be the gain ?
4. A on his dead bed, keeps half his property for his wife and divides the rest equally among his three sons; B, C, D. Some year later, B dies leaving half his property to his window and half to his brothers C and D sharing equally. When C makes his will, he keeps half his property for his widow and the rest he begueaths to his younger brother D. When D dies some years later, he keeps half his property for his widow and the remaining for his mother. The mother now has Rs. 63,00,000.
5. A milkman purchases the milk at Rs. Y/litre and sells it at Rs. 1·5Y/litre still he mixes 2·5 litres of water with every 5 litres of pure milk. What is profit percentage ?
6. A Radio dealer incurs on expense of Rs. 150 for producing every radio. He also incurs an additional expenditure of Rs. 10,000 which is independent of the number of radio produced. If he is able to sell a radio during the season, he sells it for Rs. 300. If he fails to do so, he has to sell each watch for Rs. 200.
   (i) If he is able to sell only 120 out of 150 radios he has made in the season, then he has made a profit of ?
   (ii) If he produces 150 radios, what is the number of radios that he must sell during the season in order to breakeven, given that he is able to sell all the radios produced ?
7. What should be the minimum markup percentage such that after giving a discount of 22·5% there will not be a loss ?

Exercise B

1. A retailer bought 25 kg of tea at a discount of 20%. Besides 1 kg tea was freely offered to him by the wholesaler at the purchase of 25 kg tea. Now, he sells all the tea at the marked price to a customer. What is the profit of retailer ?
2. Ram bought 15 oranges for a rupee and sold them at 12 oranges for a rupee. What is the profit percentage ?
3. Tarun purchased the books for Rs. 1,20,000. He sold 50% of it at a profit of 12·5% and rest at a loss. Find the loss percentage on the remaining if the overall loss is 10% ?
4. The ratio of cost price and marked price of an articles is 1·5 : 2 and ratio of percentage profit and percentage discount is 2 : 3. What is the discount percentage ?
5. Shyam sold 10 cameras at a profit of 30% and 6 cameras at a profit of 20%. If he had sold all the 16

cameras at a profit of 25%, then he would have gained Rs. 48 more. What is the cost price of each camera ?

6. A single discount equivalent to three successive discounts of 20%, 40% and 10% ?

7. The market price of an article is increased by 20% and the selling price is increased by 12·5%, then the amount of profit doubles. If the original marked price be Rs. 500 which is greater than the corresponding cost price by 25%. What is the increased selling price ?

8. Sprite and coke, there are two companies, selling the packs of cold-drinks. For the same selling price sprite gives two successive discounts of 5 % and 25%. While coke sells it by giving two successive discounts of 10% and 20%. Which selling price is greatest, if print price of both are the same ?

9. An automobile company launched a scheme for their dealers that if a dealer purchases 5 bikes of Model A, two extra bikes will be free and if he purchases 2 bikes of Model B, he will get one extra bike. If cost of 3 bikes of Model A and cost of 2 bikes of Model B are Rs. 1,20,000 and Rs. 1,00,000. and A Dealer purchases 4 bikes of Model A and 3 bikes of Model B. At what price these bikes should be sold so that the agency can get overall profit of 12·5% ?

10. When a electric bike manufacturer reduced its selling price by 40%. The number of electric bike sold radically increased by 500%. Initially the manufacturer was getting only 120% profit. What is the percentage increase of its revenue ?

11. The ratio of selling price of 2 articles X and Y is 1 : 2 and the ratio of percentage profit is 1 : 2 respectively. If the profit percentage of X is 12·5% and the cost price of Y is Rs. 500. What is overall percentage gain ?

12. X and Y are two partners and they have invested Rs. 1,20,000 and 1,50,000 in a business. After one year X received Rs. 2,000 as his share of profit out of total profit of Rs. 5,600 including his certain commission on total profit since he is a working partner and rest profit is received by Y. What is the commission of X as a percentage of the total profit ?

13. Gita sold her car to Sita at a profit of 20% and Sita sold it to Chandani at a profit of 10%. Chandani sold it to a mechanic at a loss of 8%. Mechanic spend 12% of his purchasing price and then sold it at a profit of 20% to Gita once again. What is the loss of Gita ?

14. In a factory the number of workers reduces in the ratio of 3 : 2 and the salary increases in the ratio 13 : 15. What is the profit percentage of workers over the previous salary ?

15. X, Y and Z invest in the ratio of 1 : 3 : 5 the percentage of return on their investment are in the ratio of 5 : 4 : 3. Find the total earnings if Z earns Rs. 510 more Y.

16. Ram bought a house in Delhi city, whose sale price was Rs. 16 lakh. He availed 20% discount as an early bird offer and then 10% discount due to cash payment. After that he spent 10% of the cost price in interior decoration and lawn of the house. At what price should he sell the house to earn a profit of 25% ?

17. A car mechanic purchased four old cars for Rs. 2 lakh. He spent total 4 lakh in the maintenance and repairing of four cars. What is the average sale price of the rest three cars to get 50% total profit if he already sold one of the four car Rs. 3 lakh ?

18. Cost price of two motorcycles is same. One is sold at a profit of 12% and the other for Rs. 5,000 more than the first. If the net profit is 20%. Find the cost price of each motorcycle.

19. Profit on selling 10 books equal selling price of 3 pens. While loss on selling 10 pens equal to selling price of 4 books. Also profit percentage equals to the loss percentage and cost of books is half of the cost of a pen. What is the ratio of selling price of books to the selling price of a pen ?

20. The cost of setting up a newspapers Rs. 12,000. The cost of paper and ink etc. is Rs. 80 per 100 copies and printing cost is Rs. 120 per 100 copies. In the last month 2500 copies were printed but only 2000 copies could be sold at Rs. 10 each. Total 25% profit on the sale price was realized. There is one more resource of income from the newspaper which is advertising. What sum of money was obtained from the adverting in newspaper ?

21. A milkman mixes 25% water in pure milk but he is not content with it so he again mixes 10% more water in the previous mixture. What is the profit percentage of milkman if he sells it at cost price ?

22. A man sold two flats for 5,00,000 each. On one he gains 25% while on the other he losses 25%. How much does he gain or loss in the whole transaction ?

23. Shyam sold three fourth of his articles at a gain of 40% and the remaining at cost price. Find the gain earned by him in the whole transaction.

24. After getting two successive discounts, a shirt with a list price of Rs. 200 is available at Rs. 160. If the second discount is 4%. Find the first discount.

25. Tarun got 20% concession on the labeled price of an article and sold it for Rs. 9,000 with 25% profit on the price he bought. What is the labeled price ?

26. A retailer marks all his good at 50% above the cost price and thinking that he will still make 20% profit,

offers a discount of 25% on the marked price. What is his actual profit on the sales ?

27. Ramji purchased 20 dozens of toys at the rate of Rs. 375 per dozen. He sold each one of them at the rate of Rs. 33. What was his percentage profit ?

28. A cotton shirt is listed in three different department stores at Rs. 200, Rs. 400 and Rs. 800 respectively. The three stores offers a discount of 10%, 15% and 20% respectively on these prices. Recession force them to offer further discounts of 5%, 10% and 10% respectively on the above reduced price. At this stage the difference between the maximum and minimum and maximum and minimum prices at the initial stage. Find the difference.

29. An alloy is made of three metals X, Y and Z. Metal Y constitutes 40% of the total weight and rest of the weight is distributed in the ratio of 2 : 3 between X and Z. The costprice per unit weight of X, Y and Z is in the ratio of 5 : 2 : 3. The alloy is sold in the market at 40% profit. Due to change in market conditions there is 20% increase in the cost of Y and 25% increases in the cost of Z. The selling price remains the same. Calculate the per cent profit or loss in the transaction.

30. Ram and Shyam have 20 cows with them. Ram sells his cows at a different rate than Shyam. They both receive the same total sum. If Ram had sold his cows at Shyam's price, he would have received Rs. 245. If Shyam sold his cows at Ram's price. He would have received Rs. 320. At what price did Shyam sell each of his cows ?

31. A trader sells widgets at a certain list price if any customer buys a second widget during the week he gives 10% discount on the list price for the second widget and every additional widget that the customer buys during the week is supplied at a price 10% less than that of the second widget. Further, if a customer buys 5 or more widgets during a week the trader gives an addition 2% of the list price as quantity discount. If a certain customer buys 5 widgets during a week what is the average discount the customer gets per widget ?

32. In a garden there are only orange and apple trees fruits available on each tree are equal to the total number of trees of the same kind. Also, orange trees are twice in number as compared to the apple trees. Only 40% of total mangoes and 70% of total apples are in good condition. These have to be stored apples. Rest were wasted.Selling price per orange is Rs. 10 and per apple is Rs. 5. Investment to store (consider only this as the cost price) is Rs. 20 per mango and Rs. 10 per apple. In this transaction, the Garden's owner lost Rs. 15,000. What is the total number of trees in the garden ?

Answer Special

1. Let Total property of the Ganesh = Rs. X

$$\text{Ganesh's wife gets} = \text{Rs. } \frac{X}{2}$$

$$\text{Ram gets} = \text{Rs. } \left(\frac{X}{2}\right) \times \frac{1}{3}$$

$$\text{Shyam gets} = \text{Rs. } \left(\frac{X}{2}\right) \times \frac{1}{3}$$

$$\text{Krishna gets} = \text{Rs. } \left(\frac{X}{2}\right) \times \frac{1}{3}$$

After Ram death,

$$\text{His widow gets} = \text{Rs. } \frac{X}{12}$$

$$\text{Shyam gets} = \text{Rs. } \frac{X}{24}$$

$$\text{Krishna gets} = \text{Rs. } \frac{X}{24}$$

According to Shyam's will,

$$\text{His wife gets} = \text{Rs. } \left(\frac{X}{24} + \frac{X}{6}\right) \times \frac{1}{2}$$

$$\text{Krishna gets} = \text{Rs. } \frac{1}{2} \times \left(\frac{X}{6} + \frac{X}{24}\right) = \text{Rs. } \frac{5X}{48}$$

After Krishna death,

$$\text{His wife gets} = \text{Rs. } \frac{1}{2} \times \left[\left(\frac{5X}{48} \times \frac{1}{2} + \frac{X}{6}\right)\right]$$

$$\text{His mother gets} = \text{Rs. } \frac{1}{2} \times \left[\frac{5X}{48} + \frac{X}{6}\right] = \text{Rs. } \frac{13X}{96}$$

$$\text{Total Mother's money} = 1830000$$

$$\text{Or,} \quad \frac{X}{2} + \frac{13}{96} \cdot X = 1830000$$

$$\text{Or,} \quad \frac{61}{96} \cdot X = 1830000$$

$$\text{Or,} \quad X = \frac{1830000 \times 96}{61}$$

$$X = \text{Rs. } 2880000$$

$$\text{Shyam's share} = \frac{X}{6} + \frac{X}{24}$$

$$= \frac{5}{24} \cdot X$$

$$\text{Shyam's share} = \frac{5}{24} \times 2880000$$

$$= 5 \times 120000$$

$$= \text{Rs. } 6{,}00{,}000$$

$$\text{Ram's wife gets} = \text{Rs. } \frac{X}{12}$$

$$= \text{Rs. } 2{,}40{,}000$$

Shyam's wife gets $= \text{Rs.}\left(\frac{X}{24}+\frac{X}{6}\right)\times\frac{1}{2}$

$= \text{Rs.}\ \frac{1}{2}\times\frac{5}{24}\cdot X$

$= \text{Rs.}\ \frac{5}{2\times 24}\times 2880000$

$= \text{Rs. } 3{,}00{,}000$

Kirshna's wife gets $= \frac{1}{2}\left(\frac{5.X}{24}\times\frac{1}{2}+\frac{X}{6}\right)$

$= \text{Rs.}\ \frac{1}{2}\left(\frac{5.X}{48}+\frac{X}{6}\right)$

$= \text{Rs.}\left(\frac{1}{2}\times\frac{13.X}{48}\right)$

$= \text{Rs.}\ \frac{1}{2}\times\frac{13}{48}\times 2880000$

$= \text{Rs. } 3{,}90{,}000$

and Their ratio $= 8 : 10 : 13$

2. In case of Ashu

Discount on Scooter $= 35000\times\frac{15}{100} = \text{Rs. } 5{,}250$

S.P. $= 35000 - 5250$

$= \text{Rs. } 29{,}750$

In case of Tannu

I^{st} discount $= 25000\times\frac{20}{100} = \text{Rs. } 5{,}000$

2^{nd} discount $= 10000\times\frac{5}{100} = \text{Rs. } 500$

Total discount $= \text{Rs. } 5{,}500$

S.P. $= 35000 - 5500$

$= \text{Rs. } 29{,}500$

Now, Difference $= 29750 - 29500$

$= \text{Rs. } 250$

3. Let the Total production cost = Rs. 100

Cost of component X = Rs. 30

Cost of component Y = Rs. 40

Remaining production cost 30 expenses on other things.

Since, profit is 2.5%

∴ S.P. = 125

New price of component X

$= 30 + 30\times\frac{30}{100} = \text{Rs. } 39$

New price of component Y

$= 40 + 40\times\frac{20}{100} = \text{Rs. } 48$

New production cost $= 39 + 48 + 30 = \text{Rs. } 117$

New S.P. $= 117\times\left(1+\frac{10}{100}\right)$

$= 117\times 1{\cdot}1 = 128{\cdot}7$

Profit $= 128{\cdot}7 - 125 = 3{\cdot}7$

% profit $= \frac{3{\cdot}7}{125}\times 100$

$= \frac{14{\cdot}8}{5} = 2{\cdot}96\%.$

New cost of component A $= 30\times 1{\cdot}1 = \text{Rs. } 33$

New cost of component B $= \text{Rs. } 40\times 0{\cdot}85 = \text{Rs. } 34$

Production cost $= 33 + 30 + 34 = 97$

So, gain $= 125 - 97 = 28$

% gain $= \frac{18}{125}\times 100\% = 22{\cdot}4\%.$

4. Let total property = Rs. X

His wife gets $= \text{Rs.}\ \frac{X}{2}$

B gets $= \text{Rs.}\ \frac{X}{6}$

C gets $= \text{Rs.}\ \frac{X}{6}$

D gets $= \text{Rs.}\ \frac{X}{6}$

Now, after B's death,

B's wife share $= \frac{X}{12}$

C' share $= \text{Rs.}\ \frac{X}{24}$

D' share $= \text{Rs.}\ \frac{X}{24}$

Total C' share $= \frac{X}{24}+\frac{X}{6} = \frac{5}{24}\cdot X$

Total D' share $= \frac{X}{6}+\frac{X}{24} = \frac{5}{24}\cdot X$

After C's death,

His wife's share $= \frac{5}{24\times 2}\cdot X = \frac{5}{48}\cdot X$

D' share $= \frac{5}{48}\cdot X$

Total D'share $= \frac{5}{24}\cdot X + \frac{5}{48}\cdot X = \frac{15}{48}\cdot X$

Now, after D's death,

His wife's share $= \frac{15}{96}\cdot X$

His mother's share $= \frac{15}{96}\cdot X$

Total mother's share $= \frac{15}{96}\cdot X + \frac{X}{2} = \frac{48+15}{96}\cdot X$

$= \frac{63}{96}\cdot X = \frac{21}{32}\cdot X$

(i) What was the worth of total property ?

(ii) What was C'share ?

According to question,

$$\frac{21}{32}\cdot X = 63{,}00{,}000$$

$$X = \frac{6300000 \times 32}{21}$$

$$= \text{Rs. } 96{,}00{,}000$$

$$\text{Total C'share} = \frac{5}{24}\cdot X$$

$$= \frac{5}{24} \times 9600000$$

$$= \text{Rs. } 20{,}00{,}000$$

Since, there is no loss.

So C.P. = S.P.

$$X = \frac{77\cdot5}{100} \times \frac{X\,(100 + Y)}{100}$$

Or, $100 \times 100 = 7750 + 77\cdot5Y$

Or, $10{,}000 - 7750 = 77\cdot5Y$

Or, $2250 = 77\cdot5Y$

$$Y = \frac{22500}{775}$$

$$Y = \frac{900}{31}$$

$$Y = 29\cdot03\%.$$

5. Let pure milk is 5 litres (simplicity).

Now, milk + water

$$5 + 2\cdot5 = 7\cdot5$$

He sells 2.5 litres water at the cost of milk.

So, S.P. $= 7\cdot5 \times 1\cdot5Y = 11\cdot25Y$

C.P. $= 5 \times 1\cdot5\, Y = 7\cdot5Y$

Profit = S.P. – C.P.

$= 11\cdot25Y - 7\cdot50Y = 3\cdot75Y$

$$\% \text{ profit} = \frac{3\cdot75}{7\cdot5} \times 100$$

$$= \frac{4}{3} \times \frac{375}{10}\%$$

$$\% \text{ profit} = \frac{2 \times 75}{3} = 50\%.$$

6. In producing radios

Total cost $= 150 \times 150 + 10000$

= Rs. 32,500

Amount received after the sell of radios

$= 120 \times 300 + 30 \times 250$

$= 36000 + 7500$

= Rs. 43,500

∴ Profit earned $= 43500 - 32500$

= Rs. 11,000

(ii) Production cost of 150 radios = 32,500

Let he sells X radios during the season.

Amount received after selling

$= 300.X + (150 - X) \times 200$

$= 100.X + 30{,}000$

Now, break-even is achieved if production cost is equal to the selling price.

∴ $100.X + 30000 = 32500$

$100.X = 32500 - 30000$

$$X = \frac{2500}{100}$$

$$X = 25$$

7. Let C.P. = X

$$\text{Mark up price} = X + \frac{X.Y}{100} = X.\left(\frac{100 + Y}{100}\right)$$

After giving discount,

$$\text{Discounted price} = \frac{77\cdot5}{100} \times \frac{X(100 + Y)}{100}$$

Answer B

1. Let the marked price of 1 kg tea = Rs. X

Let the marked price of 25 kg tea = Rs. 25X

Now, C.P. of 25 kg tea $= \text{Rs. } 25.X \times \frac{80}{100} = 20X$

S.P. of tea = 26X

Profit = S.P. – C.P.

$= 26X - 20X = 6.X$

$$\% \text{ profit} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$= \frac{6.X}{20.X} \times 100 = 30\%.$$

2. C.P. of one oranges $= \text{Rs. } \frac{1}{15}$

S.P. of one oranges $= \text{Rs. } \frac{1}{12}$

Profit = S.P. – C.P.

$$= \frac{1}{12} - \frac{1}{15} = \frac{5 - 4}{3 \times 5 \times 4} = \frac{1}{60}$$

$$\% \text{ profit} = \frac{\frac{1}{60}}{\frac{1}{15}} \times 100$$

$$= \frac{15}{60} \times 100 = 25\%.$$

3. C.P. of books = Rs. 1,20,000

C.P. of 50% books = Rs. 60,000

$$\text{S.P. of 50\% books} = 60000 \times \frac{112\cdot5}{100}$$

= Rs. 67,500

Since, overall loss is 10%.

$$\text{So, total S.P.} = \frac{90}{100} \times 120000$$

$$= \text{Rs. } 1{,}08{,}000$$

Now, S.P. of the remaining 50% books

$$= 108000 - 67500 = 40{,}500$$

Loss on remaining books

$$= 60000 - 40500 = 19{,}500$$

$$\% \text{ loss} = \frac{19500}{60000} \times 100 = 32{\cdot}5\%.$$

4. Let C.P. of the article $= 1{\cdot}5\ X$

M.P. of the article $= 2X$

Percentage profit $= 2Y\%$

Percentage discount $= 3Y\%$

Since, $$\text{C.P.} = \text{M.P.}\left(1 - \frac{\text{Discount}}{100}\right)$$

$$1{\cdot}5X = 2X\left(1 - \frac{3.Y}{100}\right)$$

Or, $$\frac{3}{4} = 1 - \frac{3.Y}{100}$$

$$Y = \frac{1}{4} \times \frac{100}{3} = \frac{100}{12}$$

$$Y = \frac{25}{3} = 8{\cdot}33\%$$

Percentage profit $= 2 \times 8{\cdot}33 = 16{\cdot}66\%$.

5. Let cost price of each camera = Rs. X

C.P. of 16 cameras $=$ Rs. 16 X

$$\text{S.P. of 10 cameras} = \frac{13}{10} \times 10.X = 13\ X$$

$$\text{S.P. of 6 cameras} = \frac{120}{100} \times 6.X = 7{\cdot}2X$$

$$\text{Total S.P.} = 13X + 7{\cdot}2X$$

$$= 20{\cdot}2X \qquad \ldots(1)$$

When overall profit is 25%.

$$\text{Then, S.P.} = \frac{125}{100} \times 16.X$$

$$= \frac{5}{4} \times 16.X$$

$$= 20X \qquad \ldots(2)$$

Difference (1) and (2) is given by

$$20{\cdot}2X - 20X = 48$$

$$0{\cdot}2X = 48$$

$$X = 240$$

Cost Price of each camera = Rs. 240.

6. Let the marked price $=$ Rs. X

$$\therefore \quad \text{I}^{\text{st}} \text{ discount} = \frac{20.X}{100} = \frac{X}{5}$$

$$\text{Remaining amount} = \frac{80.X}{100} = \frac{8.X}{10}$$

$$2^{\text{nd}} \text{ discount} = \frac{8.X}{10} \times \frac{40}{100} = \frac{32.X}{100}$$

$$= \frac{16.X}{50}$$

$$\text{Remaining amount} = \frac{8.X}{10} - \frac{16.X}{50}$$

$$= \frac{40X - 16X}{50}$$

$$= \frac{24}{50} \cdot X$$

$$3^{\text{rd}} \text{ discount} = \frac{24}{50} \cdot X \times \frac{10}{100}$$

$$= \frac{6}{125} \cdot X$$

If marked price $=$ Rs. 100

Now, equivalent discount

$$= \frac{X}{5} + \frac{16}{50} \cdot X + \frac{6}{125} \cdot X$$

$$= \frac{25 + 40 + 6}{125} \times 100$$

$$= 56{\cdot}8\%.$$

7. Original M.P. $=$ Rs. 500

When M.P. is 125, then C.P. Rs. 100.

$$\therefore \quad 500, \text{ then C.P.} = \frac{100}{125} \times 500 = \text{Rs. } 400$$

After increasing M.P. by 20%

$$\text{Final M.P.} = 500\left(1 + \frac{20}{100}\right)$$

$$= 500 \times \frac{6}{5} = \text{Rs. } 500$$

If profit $= 2.X$

Now, initial profit $= X$

Initial S.P. $= (400 + X)$

After increasing 12.5%

$$\text{S.P.} = \frac{112{\cdot}5}{100} \times (400 + X)$$

According to question,

$$\frac{112{\cdot}5}{100} \times (400 + X) - 400 = 2.X$$

Or, $$\left[\frac{112{\cdot}5}{100} - 1\right] \times 400 = \left(2 - \frac{112{\cdot}5}{100}\right) X$$

Or, $$\left[\frac{12{\cdot}5 \times 400}{100}\right] = \frac{87{\cdot}5}{100} \cdot X$$

$$X = \frac{12{\cdot}5 \times 400}{87{\cdot}5} = \text{Rs. } \frac{400}{7}$$

Increased selling price

$$= \frac{112{\cdot}5}{100}\left[400 + \frac{400}{7}\right]$$

$$= \frac{112{\cdot}5 \times 400 \times 8}{100 \times 7} = \text{Rs. } 514{\cdot}30$$

8. Let the Print Price of both are the same.

$$\text{Let C.P. of sprite} = \text{Rs. P}$$

$$\text{Let C.P. of coke} = \text{Rs. Q}$$

According the question,

$$P \times \frac{95}{100} \times \frac{75}{100} = Q \times \frac{90}{100} \times \frac{80}{100}$$

Or, $\quad P \times 19 \times 15 = 4 \times 9 \times 8 \times Q$

Or, $\quad \frac{P}{Q} = \frac{4 \times 9 \times 8}{15 \times 19} = \frac{3 \times 32}{5 \times 19}$

Or, $\quad \frac{P}{Q} = \frac{96}{95}$

Or, $\quad P = \frac{96}{95} \cdot Q$

Clearly, P is the greatest.

9. Since, 2 bikes of A is free if dealer buys 5 bikes.

So, If Dealer buys 4 bikes of A, then he pays only price of 3 bikes.

Similarly, after buying 2 bikes of B dealer gets one bike extra.

So, he pays only price of 2 bikes B.

∴ Total C.P. of 4 bikes A + 3 bikes B

$$= 120000 + 100000$$

$$= \text{Rs. } 2{,}20{,}000$$

$$\text{S.P. of bikes} = 220000 \times \frac{112{\cdot}5}{100}$$

$$= \text{Rs. } 2{,}47{,}500$$

10. Let C.P. of a bicycles = Rs. 100

Since profit = 120%

S.P. = Rs. 220

Now , sale price of new bicycle

$$= \text{Rs. } 220 \times \frac{60}{100} = \text{Rs. } 132$$

$$\text{Profit} = 132 - 100 = 32$$

$$\%\ \text{profit} = \frac{192}{600} \times 100$$

$$= \frac{192}{6} = 32\%$$

Suppose initially they are selling n bikes after discount scheme the no. of sold bike = $6n$ bike

$$\text{Initial Revenue} = SP \times n = 220 \times n = 220n$$

$$\text{New Revenue} = SP \times n = 132 \times 5n = 660n$$

$$\%\ \text{Increase in Revenue} = \frac{660 - 220}{220} \times 100 = 200\%$$

11. Let S.P. of X = a

Let S.P. of Y = $2a$

Profit of X = b

Profit of Y = $2b$

C.P. of X = $a - b$

C.P. of Y = $2a - 2b$

Since, we get profit of 12·5% in X.

∴ $\quad \frac{12{\cdot}5}{1} = \frac{b}{a-b} \times 100$

Or, $\quad \frac{25}{2} = \frac{b}{a-b} \times 100$

Or, $\quad 8b = a - b$

$$a - 9b = 0 \qquad \ldots(1)$$

C.P. of Y is 500

$$2a - 2b = 500$$

Or, $\quad a - b = 250 \qquad \ldots(2)$

By putting the value of a in equation (2), we get

$$9b - b = 250$$

Or, $\quad 8b = 250$

$$b = \frac{250}{8}$$

$$a = 9 \times \frac{250}{8}$$

$$\text{Total C.P.} = 3a - 3b$$

$$= 3(a - b) = 750$$

$$\text{Total S.P.} = 3a = 3 \times 9 \times \frac{250}{8}$$

$$\text{Profit} = \text{S.P.} - \text{C.P.} = 27 \times \frac{250}{8} - 750$$

$$= 3 \times 250 \times \frac{1}{8}$$

$$\%\ \text{profit} = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{3 \times 250}{8 \times 3 \times 250} \times 100$$

$$= \frac{100}{8} = 12{\cdot}50\%.$$

12. Ratio of profit X : Y = 12 : 15

= 4 : 5

Now, share of profit = 5600 – 2000 = Rs. 3600

Now the share of profit of X

= 3600 – 1600 = 2000

∴ Therefore, required percentage

$$= \frac{2000}{5600} \times 100\% = \frac{10}{28} \times 100$$

$$= \frac{250}{7}\% = 35\frac{5}{7}\%.$$

13. Let cost price of Gita Car = Rs. X

Sita buys the car at

$$= \text{Rs. } \frac{120}{100} \times X$$

Chandani buys the car at

$$= \text{Rs. } \frac{120}{100} \times X \times \frac{110}{100}$$

Mechanic buys the car at

$$= \text{Rs. } \frac{92}{100} \times \frac{110}{100} \times \frac{120}{100} \times X$$

Mechanic spent 12% on the car

Now, price of car

$$= \frac{112}{100} \times \frac{92}{100} \times \frac{110}{100} \times \frac{120}{100} \times \frac{120}{100} \times X$$

∴ Mechanic sold the car at

$$= \frac{112}{100} \times \frac{92}{100} \times \frac{110}{100} \times \frac{120}{100} \times \frac{120}{100} \times X$$

$$= \frac{144 \times 112 \times 92 \times 11}{10000000} \times X$$

Now, at last Gita buys the car

$$\text{So, } \quad \text{Loss} = \left(\frac{16321536}{10000000} - 1\right) \times X$$

$$\text{Loss} = \frac{6321536}{1000000} \times X$$

$$\% \text{ loss} = \frac{6321536}{100000}\% = 63{\cdot}21\%.$$

14. We know that

Total salary = no. of woker × salary per workers

Let initial workers = 3X

Let final workers = 2X

Initial salary = 13Y

Final salary = 15Y

According to question,

Total salary = 3X × 13Y = 39X.Y

Total salary = 2X × 15Y = 30X.Y.

$$\% \text{ profit} = \frac{39X.Y - 30X.Y}{39X.Y} \times 100\%$$

$$= 23.07\ \%.$$

15.

	X	Y	Z
Investment	a	$3a$	$5a$
Rate of return	$\frac{5b}{100}$	$\frac{4b}{100}$	$\frac{3b}{100}$

$$\text{Total} = \frac{5ab}{100} + \frac{12ab}{100} + \frac{15ab}{100}$$

$$= \frac{32ab}{100}$$

Z's earning – Y's earning = Rs. 510

$$\frac{15ab - 12ab}{100} = \frac{3ab}{100} = \text{Rs. } 510$$

$$\text{Total earning} = \frac{32 \times ab}{100}$$

$$= \frac{32 \times 17000}{100} = \text{Rs. } 5{,}440.$$

16. Let marked price of house = Rs. 100

Now, after 20% and 10% discount

$$\text{C.P.} = (100 - 20) - 8 = \text{Rs. } 72$$

Since, Ram spent 10% in interior decoration

$$\therefore \quad \text{C.P.} = 72 + 72 \times \frac{10}{100} = 79{\cdot}2$$

He want to earn 25% profit.

$$\text{So, } \quad \text{S.P.} = 79{\cdot}2 \times \frac{125}{100} = 99$$

When marked price is 100 … sale price = 1600000

$$\therefore \quad \text{Marked price is 99 … sale price} = \frac{1600000}{100} \times 99$$

$$= \text{Rs. } 1{,}58{,}400$$

17. Total cost of 4 cars = 2 + 4 = 6 Lakh

Total S.P. of 4 cars = 6 × 1·5 = Rs. 9 lakh

S.P. of one car = Rs. 3 lakh

Now, S.P. of 3 cars = Rs. 6 lakh

So, average S.P. of all the 3 cars = Rs. 2 lakh.

18. Let C.P. of the 2 motorcycles = Rs. 2.X

$$\text{S.P.} = 1{\cdot}2\,(2.X)$$

$$= 2{\cdot}4X$$

$$\text{S.P. of 1st article} = \frac{112}{100} \cdot X = 1{\cdot}12X$$

$$\text{S.P. of 2nd article} = 1{,}12\,X + 5000$$

According to question,

$$2.4X = 1.12X + 5000 + 1{\cdot}12X$$

$$\text{Or, } \quad 2.4X = 2.24X + 5000$$

$$\text{Or, } \quad 0{\cdot}16X = 5000$$

$$X = \frac{5000}{0{\cdot}16} = \frac{500000}{16}$$

$$X = \text{Rs. } 31{,}250.$$

19. Let

	Book	Pen
C.P.	a	b
S.P.	c	d

According to question,

$$2a = b \quad \ldots(1)$$

$$\text{Profit on selling books} = 10(c - a) = 3d \quad \ldots(2)$$

$$\text{Loss on selling pens} = 10\,(b - d) = 4c$$

$$5(b - d) = 2c \quad \ldots(3)$$

$$\% \text{ profit on books} = \frac{10(c - a)}{a} \times 100$$

$$\% \text{ loss on pens} = \frac{10(b - d)}{b} \times 100$$

These are equal.

$$\therefore \quad \frac{10(c - a)}{10a} \times 100 = \frac{10(b - d)}{10b} \times 100$$

$$\text{Or, } \quad (c - a)b = a(b - d)$$

Or, putting value of b in above equation

$$(c-a).2a = a(b-d)$$
$$2c-2a = b-d$$

Or, $\quad 2c+d = 2b \qquad \ldots(4)$

Now, solving equation (2) and (3), we get

$$\frac{c}{d} = \frac{5}{6}$$

20. Set up cost = Rs. 12,000

$$\text{Paper cost} = \text{Rs. } \frac{80}{100} \times 2500$$
$$= \text{Rs. } 2{,}000$$
$$\text{Printing cost} = \text{Rs. } \frac{120}{100} \times 2500$$
$$= \text{Rs. } 3{,}000$$
$$\text{Total cost} = \text{Rs. } 17{,}000$$
$$\text{Now, Sale price} = \text{Rs. } 2000 \times 10$$
$$= \text{Rs. } 20{,}000$$

Let the amount obtained from adverstising is X, then

$$\therefore \quad (20{,}000 + X) - 17{,}000 = \frac{25}{100} \times 20{,}000$$

Or, $\quad 3000 + X = 5000$

$$X = \text{Rs. } 2{,}000$$

21. Let us suppose that pure milk = 100 of milk

After mixing 25% water

$$\text{Amount of milk} = 100 + 25 = 125$$
$$\text{Again Amount of milk} = 125 \times \frac{10}{100} + 125$$
$$= 125 + 12{\cdot}5 = 137{\cdot}5$$

Since, milk is sold at cost price.

Let cost price of pure milk = Rs. Y

Since, cost price of 100 pure milk = Rs. Y

Now, selling price of 100 pure milk = Rs. Y

$$\therefore \quad \text{Selling price of 137.5 pure milk} = \frac{137{\cdot}5}{100} Y$$
$$\text{Profit} = \frac{37{\cdot}5}{100} Y$$
$$\%\ \text{profit} = \frac{37{\cdot}5}{100} Y \times \frac{100}{Y} \%$$
$$= 37{\cdot}5\ \%$$

22. In such case, there is always a loss.

The selling price is immaterial.

$$\therefore \quad \%\ \text{loss} = \left(\frac{\text{Common loss and gain}}{100}\right)^2$$
$$= \left(\frac{25}{10}\right)^2 \% = \left(\frac{5}{2}\right)^2 \% = \frac{25}{4} \%$$
$$\%\ \text{loss} = 6\frac{1}{4} \%$$

23. Let $\quad$ Cost price = Rs. X

$$\text{Now, C.P. of } \frac{3}{4}\text{th article} = \text{Rs. } \frac{3}{4} X$$
$$\text{C.P. of } \frac{1}{4}\text{th article} = \text{Rs. } \frac{1}{4} X$$
$$\text{Now, S.P. of } \frac{3}{4}\text{th article} = \frac{3}{4} \times \frac{140}{100} X$$
$$= \frac{3}{4} \times \frac{7}{5} X = \frac{21}{20} X$$
$$\text{S.P. of } \frac{1}{4}\text{th article} = \frac{1}{4} X$$
$$\therefore \quad \text{Total S.P. of the article} = \frac{21}{20} X + \frac{1}{4} X$$
$$\text{S.P.} = \frac{21X + 5X}{20}$$
$$= \frac{26}{20} X + \frac{13}{10} X$$
$$\text{Profit} = \text{S.P.} - \text{C.P.}$$
$$= \frac{13}{10} X - X = \frac{3}{10} X$$
$$\%\ \text{gain} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$
$$= \frac{\frac{3X}{10}}{X} \times 100 = 30\%.$$

24. From question it is clear that

$$\text{Market price} = \text{Rs. } 200$$
$$\text{S.P. of shirt} = \text{Rs. } 160$$
$$\text{Let first discount} = \text{Rs. } X\%$$
$$\text{Now, I}^{\text{st}} \text{ discount} = 200 \times \frac{X}{100} = 2X$$
$$2^{\text{nd}} \text{ discount} = (200 - 2X) \times \frac{4}{100}$$
$$= \left(\frac{200 - 2X}{25}\right)$$
$$\text{Now, S.P. of the shirt} = 200 - \left(2X + \frac{200 - 2X}{25}\right)$$
$$160 = 200 - \left[\frac{50X + 200 - 2X}{25}\right]$$

Or, $\quad 40 = \dfrac{48X + 200}{25}$

$$48X = 800$$

Or, $\quad X = \dfrac{50}{3}\%.$

25. Let the labeled price = Rs. X

$$\text{After concession price} = X - X.\frac{20}{100} = \frac{4}{5} X$$

Since, Tarun gets 25% profit

So, S.P. of the price $= \frac{4}{5}X \times \frac{125}{100} = X$

$X = 9000$

So, labeled price = Rs. 9,000

26. Let the marked price = Rs. X

When marked price is 150, then C.P. = 100

X marked price is 150, then C.P. $= \frac{100}{150}X$

$= \frac{2}{3}X$

Now, Discount $= X - \frac{25}{100} = \frac{X}{4}$

Now, S.P. $= X - \frac{X}{4} = \frac{3}{4}X$ …(1)

When C.P. is $\frac{2}{3}X$.

S.P. $= \frac{120}{100} \times \frac{2}{3}X$

$= \frac{6}{5} \times \frac{2}{3}X = \frac{4}{5}X$ …(2)

Now, profit $= \frac{3}{4}X - \frac{2}{3}X = \frac{X}{12}$

Now, % profit $= \frac{\text{Profit}}{\text{C.P.}} \times 100$

$= \frac{\frac{X}{12}}{\frac{2X}{3}} \times 100$

$= \frac{1}{12} \times \frac{3}{2} \times 100 = 12{\cdot}5\%.$

27. Since, cost of one dozen is Rs. 375

cost of 20 dozens is Rs. 375 × 20 = Rs. 7,500

Now, Selling price of one dozen = Rs. 33 × 12

Selling price of 20 dozens = 33 × 12 × 20 = Rs. 7,920

Now, profit = 7920 – 7500 = Rs. 420

% profit $= \frac{420}{7500} \times 100$

$= \frac{420}{75} = \frac{84}{15} = 5{\cdot}6\%$

28. In initial stage :

Difference = Maxmium – Minimum

= 800 – 200 = Rs. 600

After providing successive discounts of 10% and 5% on the price Rs. 200.

Now, Discount $= 200 - \left(200 \times \frac{10}{100}\right)$

= 200 – 20 = Rs. 180

2nd Discount $= 180 \times \frac{5}{100} =$ Rs. 9

Now, Price = 180 – 9 = Rs. 171

When m.p. is 400

Ist discount $= 400 \times \frac{15}{100} =$ Rs. 60

2nd discount = 340 – 34 = Rs. 306

When m.p. = Rs. 800

Ist discount $= 800 \times \frac{20}{100} =$ Rs. 160

2nd discount $= 640 \times \frac{10}{100} =$ Rs. 64

Price = 640 – 64 = Rs. 576

Now Difference = Maximum price – Mainimum price

= 576 – 171 = Rs. 405 …(2)

Now, Difference between equation (1) and equation (2) is

600 – 405 = 195.

29. Let total weight of the alloy = a

	X	Y	Z
Weight =	$\frac{2}{5}a \times \frac{2}{5}$	$\frac{2}{5}a$	$\frac{3}{5} \times \frac{2}{5}a$
Ratio of C.P.	5	2	3

∴ Total C.P.

$= \left(\frac{2}{5}a \times \frac{2}{5} \times 5 + \frac{2}{3}a \times 2 + \frac{6}{5 \times 5}a \times 3\right)$

$= \frac{4}{5}a + \frac{4}{5}a + \frac{18}{25}a = \frac{8}{5}a + \frac{18}{25}a$

$= \frac{58}{25}a$

S.P. $= \frac{140}{100} \times \frac{58}{25}a = \frac{7}{5} \times \frac{58}{25}a$

Due to changed market condition :

C.P. of Y $= \frac{2}{5}a \times 2 \times \frac{120}{100} = \frac{4}{5}a \times \frac{6}{5} = \frac{24}{25}a$

C.P. of Z $= \frac{18}{25}a \times \frac{125}{100} = \frac{9}{11}a$

Now, Total C.P. $= \frac{4}{5}a + \frac{24}{25}a + \frac{9}{10}a$

$= \frac{40a + 48a + 45a}{25 \times 2} = \frac{133}{50}a$

Profit $= \frac{406}{125}a - \frac{133}{50}a = \frac{812 - 665}{25 \times 2 \times 5}a$

$= \frac{147}{10 \times 25}a$

% profit $= \frac{\text{Profit}}{\text{C.P.}} \times 100$

$= \frac{\frac{147a}{25 \times 10}}{\frac{133a}{50}} \times 100$

$$= \frac{147}{5 \times 133} \times 100 = \frac{21}{19} \times 20 = \frac{420}{19}$$

$$= 22\frac{2}{19}\%.$$

30.

	Ram	Shyam
Number of cows	X	$\frac{320}{Y}$
S.P. / cow	Y	$\frac{245}{X}$

Since, both Ram and Shyam have the same amount of money.

$$\therefore \quad \frac{320}{Y} \times \frac{245}{X} = X.Y$$

Or, $(X.Y)^2 = 245 \times 320$

Or, $X.Y = 280$

Given $X + \frac{320}{Y} = 20$

$$X + \frac{320}{280}X = 20$$

Or, $X\left(1 + \frac{8}{7}\right) = 20$

$$X = \frac{7 \times 20}{15} = \frac{7 \times 4}{3} = \frac{28}{3}$$

Now, Shyam sold each of cow at

$$= \frac{245}{\frac{28}{3}} = \frac{105}{4} = 26{\cdot}25$$

31. Price of 1st widget = 100

Price of 2nd widget = 90

Price of 3rd widget $= 90 - 90 \times \frac{10}{100} = 81$

Price of 4th widget = 81

Price of 5th widget = 81

Now, total = 433

Less 2% quantity discount on 5th widget = 10

$\therefore$ Total price of 5 widgets = 423

Average price per widgets $= \frac{423}{5} = 84{\cdot}6$

32. Let there be a oranges on each tree, so there will be a oranges trees in this garden.

Total number of oranges $= a^2$

Total number of apples available $= b^2$

According to question,

$$a = 2b \quad \ldots(1)$$

Oranges available to store $= 0.4a^2$

Apples available to store $= 0{\cdot}7b^2$

Hence, cost price $= 0.4a^2 \times 20 + 0{\cdot}7b^2 \times 10$

$$= 8a^2 + 7b^2 \quad \ldots(2)$$

So, total oranges available for selling

$$= 0{\cdot}75\,(0{\cdot}4a^2)$$

$$= 0{\cdot}3a^2$$

Total apples available for selling

$$= \frac{6}{7} \times 0{\cdot}7b^2 = 0{\cdot}6b^2$$

$\therefore$ Selling price $= 10 \times 0{\cdot}3a^2 + 5 \times 0{\cdot}6b^2$

$$= 3a^2 + 3b^2$$

Loss = C.P. – S.P.

$$= 5a^2 + 4b^2$$

$$15000 = 5a^2 + 4b^2 \quad \ldots(3)$$

Now, $15000 = 5(2b)^2 + 4b^2$ [from equation (3)]

$$15000 = 24b^2$$

$$b^2 = \frac{15000}{25}$$

$$= \frac{5000}{8} = \frac{2500}{4}$$

$$b = \frac{50}{2} = 25$$

$$a = 2b = 50$$

5 Average

Average

The numerical result obtained by dividing the sum of two or more quantities by the number of quantities is called **Average** .

An arithmetic mean of given observations is called **Average** .

Average is defined in so many ways we can say average mean Usual or Normal kind, amount, quality, rate, etc.

Average is a number or value of a set of values carefully defined to typify the set, as a median or mode.

Average refers to the result obtained by dividing a sum by the number of quantities added. For example, the **average** of 15, 12, 27 is $\frac{15+12+27}{3} = \frac{54}{3} = 18$ and in extended use is applied to the usual or ordinary kind, instance, etc.

Average is different from mean and median.

The **Average** of a given set of numbers is a measure of the central tendency of the set. In other words, it is the mean value of a set of numbers or values. Therefore, average of a set of numbers is given by

$$\text{Average} = \frac{(x_1 + x_2 + x_3 + \ldots + x_n)}{n}$$

Or in other words average of some observations

$$= \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Mean commonly designates a figure intermediate between two extremes. For example, the mean temperature for a day with a high of 24°C and a low of 28°C is $\frac{24+28}{2} = 26°C$ and the median is the middle number or point in a series arranged in order of size *i.e.*, the median grade in the group 50, 55, 85, 88, 92 is 85; the **average** is 74.

Norm implies a standard of **average** performance for a given group *i.e.*, a child below the norm for his age in reading comprehension.

Illustration 1.

In a class, the age of four students are 20 years, 22 years, 18 years, and 24 years, then what is the average age of the student of class ?

Solution :

By the above definition average age

$$= \frac{20+22+18+24}{4} = \frac{84}{4} = 21 \text{ years}$$

Therefore, average age of student = 21 years

Weighted Average

The concept of weighted Average is used when we have two or more groups whose individual averages age are known.

Suppose in a class, there are 2 student of 20 years,3 of 21 years ,4 of 22 years and 5 of 23 years, then their average age is given by

$$\frac{(2 \times 20) + (3 \times 21) + (4 \times 22) + (5 \times 23)}{2+3+4+5}$$

$$= \frac{2}{14} \times 20 + \frac{3}{14} \times 21 + \frac{4}{14} \times 22 + \frac{5}{14} \times 23$$

$$= \frac{306}{14} \text{ years}$$

Here, $\frac{2}{14}, \frac{3}{14}, \frac{4}{14}$ and $\frac{5}{14}$ are called the weights of each category of students.

Illustration 2.

What is the average concentration of a mixture if 3L of 36 % sulphuric acid is added to 9L of 24% sulphuric acid solution ?

Solution :

The average concentration of the combined mixture is the weighted average

$$= \left(\frac{3}{12}\right) \times 36 + \left(\frac{9}{12}\right) \times 24$$

$$= 9 + 18 = 27\%$$

In other words, weights are the fraction of the number in that category with respect to the total students in that class. This average is also called the weighted average of that class.

Average Speed

If a (body) certain distance is covered in parts at different speeds , the average speed is given by

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

As, if a body travels $d_1, d_2, d_3, \ldots\ldots d_n$ distance, with speed $s_1, s_2, s_3, \ldots\ldots s_n$ in time $t_1, t_2, t_3, \ldots. t_n$ respectively, then the average speed of the body through the total distance is given by

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Always remember that, Average speed

$$\neq \frac{\text{Sum of speeds}}{\text{Number of different speeds}} \neq \frac{s_1 + s_2 + s_3 + \ldots + s_n}{n}$$

$$\text{Average speed} = \frac{d_1 + d_2 + \ldots + d_n}{t_1 + t_2 + \ldots + t_n}$$

$$= \frac{s_1 t_1 + s_2 t_2 + \ldots + s_n t_n}{t_1 + t_2 + t_3 + \ldots + t_n}$$

$$= \frac{d_1 + d_2 + d_3 + \ldots + d_n}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + \ldots + \frac{d_n}{s_n}}$$

Short cut

If you travel equal distance with speeds u and v, then the average speed over the entire journey is $\frac{2uv}{(u+v)}$.

If a man changes his speed in the ratio $m : n$, then the ratio of times taken becomes $n : m$.

Illustration 3.

Let the distance between two points A and B is d and speed in travelling from point A to B is u km/hr and from point B to A is v km/hr.

Solution :

$$\text{Then, average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{2d}{\frac{d}{u} + \frac{d}{v}}$$

If two speeds are given as u km/hr and v km/hr, then

Average speed (distance being same)

$$= \frac{2uv}{u+v}$$

Illustration 4.

If a person travels two equal distances at 10 km/hr and 30 km/hr .What is the average speed for the entire journey ?

Solution :

$$\text{Average speed} = \frac{2 \times 30 \times 10}{30 + 10}$$

$$= \frac{600}{40} = 15 \text{ km/hr.}$$

Age and Average

If the average age of n persons decreases by x years. Then, the total age of n persons decreases by $(n \times x)$ years. Also, if the average age of n persons increase by x years. Then, the total age of n persons increases by $(n \times x)$ years.

Illustration 5.

The average weight of 6 men decrease by 63 kg when one of them weighing 80 kg is replaced by a new man. Calculate the weight of the new man.

Solutions :

Total weight reduced of 6 men = $6 \times 3 = 18$ kg

This weight of the group is reduced because the man weighing 80 kg is replaced by a man who is 18 kg lighter than him. Therefore, weight of new man = $(80 - 18) = 62$ kg.

Runs and Average

Illustration 6.

A cricketer has a certain average of 9 innings. In the tenth inning he scores 100 runs, thereby increasing his average by 8 runs. Calculate his new average.

Solution :

Let the average of 9 innings be x runs, hence new average will be $(x + 8)$ runs.

Total runs scored for 9 innings = $9x$

Total runs scored for 10 innings = $(9x + 100)$

$$\text{Average for 10 innings} = \frac{\text{Total runs}}{10}$$

$$\Rightarrow \quad (x+8) = \frac{(9x+100)}{10}$$

$$\Rightarrow \quad x = 20$$

$$\text{Therefore, new average} = (20 + 8) = 28 \text{ runs}$$

Average of Some Important Series of Numbers

(a) The average of **odd numbers** from 1 to n is $\frac{(n+1)}{2}$, when n = last odd number

(b) The average of **even numbers** from 2 to n is $\frac{(n+2)}{2}$, when n = last even number

(c) The average of **square of natural numbers** till $n = \frac{n\,(n+1)(n+1)}{6n}$

$$\Rightarrow \frac{(n+1)(2n+1)}{6}$$

(d) The average of **cubes of natural numbers** till n is $= \frac{n^2\,(n+1)^2}{4n}$

$$\Rightarrow \frac{n\,(n+1)^2}{4}$$

(e) The average of **first n consecutive even numbers** is $(n+1)$

(f) The average of **first n consecutive odd numbers** is n.

(g) The average of **squares of first n consecutive even numbers** is $\frac{2(n+1)(2n+1)}{3}$

(h) The average of s**quares of consecutive even numbers till n** is $\frac{(n+1)(n+2)}{3}$

(i) The average of **squares of consecutive odd numbers till n** is $\frac{n\,(n+2)}{3}$

Illustration 7.

What is the average of odd numbers from 1 to 25 ?

Solution :

$$\text{Average} = \frac{25+1}{2} = 13$$

Illustration 8.

What is the average of even numbers from 1 to 40 ?

Solution :

$$\text{Average} = \frac{40+2}{2} = 21$$

Illustration 9.

What is the average of square of natural numbers from 1 to 20 ?

Solution :

$$\text{Average} = \frac{(20+1)(40+1)}{6} = 143{\cdot}5$$

Illustration 10.

What is the average of cubes of natural numbers from 1 to 5 ?

Solution :

$$\text{Average} = \frac{5(5+1)^2}{4} = 45$$

Illustration 11.

What is the average of first 49 consecutive even numbers ?

Solution :

$$\text{Average} = 49 + 1$$

Illustration 12.

What is the average of first 19 consecutive odd numbers ?

Solution :

$$\text{Average} = 19$$

Illustration 13.

What is the average of square of first 10 consecutive even numbers ?

Solution :

$$\text{Average} = \frac{2(10+1)(20+1)}{3}$$

$$= \frac{2 \times 11 \times 21}{3} = 154$$

Illustration 14.

What is the average of square of consecutive even numbers till 10 ?

Solution :

$$\text{Average} = \frac{(10+1)(10+2)}{3} = \frac{11 \times 12}{3} = 44$$

Illustration 15.

What is the average of square of consecutive odd numbers till 12?

Solution :

$$\text{Average} = \frac{12(12+2)}{3}$$

$$= \frac{12 \times 14}{3} = 56$$

Exercise A

1. The average of 5 quantities is 10 and the average of 3 of them is 9. What is the average of the remaining 2 ?
   (A) 11·0 (B) 12·0
   (C) 11·5 (D) 12·5
2. The average of 5 quantities is 6. The average of 3 of them is 8. What is the average of the remaining two numbers ?
   (A) 6·5 (B) 4
   (C) 3 (D) 3·5
3. The average mark in a group of 25 students on a test is reduced by 2 when a new student replaces the topper who scored 95 marks. How many marks did the new student have ?
   (A) 90 marks (B) 50 marks
   (C) 45 marks (D) 95 marks
4. The average of marks obtained by 120 candidates in a certain examination is 35. If the average marks of passed candidates is 39 and that failed candidates is 15. What is the number of candidates who passed the examination ?
   (A) 65 (B) 40
   (C) 100 (D) 35
5. The average of 11 results is 50. If the average of first six results is 49 and that of last six is 52, find the sixth result—
   (A) 55 (B) 45
   (C) 56 (D) 38
6. The average weight of 8 men is increased by 1·5 kg. when one of the men who weights 65 kg is replaced by a new man. The weight of the new man is—
   (A) 76 kg (B) 76·5 kg
   (C) 76·7 kg (D) 77 kg
7. The average of marks in Mathematics for 5 students was found to be 50. Later on it was discovered that in the case of one student the marks 48 were misread as 84. The correct average is—
   (A) 40·2 (B) 40·8
   (C) 42·8 (D) 48·2
8. The average of 7 consecutive numbers is 33. The highest of these numbers is—
   (A) 28 (B) 30
   (C) 33 (D) 36

9. The average monthly salary paid to 75 employees in a company is Rs. 1420. The average salary of 25 of them is Rs. 1350 and that of 30 others is Rs. 1425. The average salary of the remaining employees is—

(A) Rs. 1350 (B) Rs. 1425
(C) Rs. 1500 (D) Rs. 1475

10. Four ropes of increasing length are provided. Their average length is 74 inches, and the difference in length amongst the first three ropes is 2 inches. The difference between the third and the fourth is 6 inches. Thus, the longest rope is—

(A) 72 inch (B) 80 inch
(C) 74 inch (D) 70 inch

11. The average age of a committee of a members is 40 years. A member aged 55 years retired and his place was taken by another member aged 39 years. The average age of the present committee is—

(A) 39 years (B) 38 years
(C) 36 years (D) 35 years

12. The average of 5 quantities is 10 and the average of 3 of them is 9. What is the average of the remaining 2 ?

(A) 11 (B) 12
(C) 11·5 (D) 12·5

Exercise B

1. The average weight of a class of 33 students is 48.5 kg. If the weight of the teacher is included, then the average rises by 500 gm. The weight of the teacher is—

(A) 66·5 kg (B) 65·5 kg
(C) 63·5 kg (D) 69 kg

2. The average age of a couple was 27 years at the time of their marriage. After 20 years of marriage, the average age of the family with 2 children became 30 years. What is the average age of the children ?

(A) 23 years (B) 11 years
(C) 13 years (D) 26 years

3. The average temperature at Delhi for Monday, Tuesday and Wednesday was 27°C while for Tuesday, Wednesday and Thursday, it was 25°C. If the temperature on Thursday was 26°C, then the temperature on Monday was—

(A) 27°C (B) 21°C
(C) 30°C (D) 32°C

4. Sohan scores 80, 40, 90, 50 and 60 per cent marks in the subject Hindi, English, Maths, History and Chemistry respectively. However, the weights attached to each of these subjects are 4, 3, 1, 2, and 5 respectively. Which of the following represents the weighted average of marks scored by Sohan in these five subjects ?

(A) 60 (B) 62
(C) 72 (D) 74

5. The average of eight numbers is 50, that of the first four is 40 and the next three is 25. The eight number is less than the ninth by 7, and less than the tenth by 12. Then, tenth number is—

(A) 81 (B) 93
(C) 91 (D) 90

6. The average amount of sales in a shop per day for 5 days from Monday is Rs1400. The average amount of sales per day for 5 days from Tuesday is Rs1490. By how much does the sale on Saturday exceed that on Monday?

(A) Rs. 400 (B) Rs. 500
(C) Rs. 450 (D) Rs. 600

7. The average age of a class of 20 students is 25 years. The average increases by 1 when the teacher's age is also included. What is the teacher's age ?

(A) 40 years (B) 46 years
(C) 20 years (D) 25 years

8. A batsman has a certain average of runs in 15 innings. In the 16th inning, he makes a score of 90 runs, there by increasing his average by 2. What is the average after the 16th inning ?

(A) 54 (B) 60
(C) 56 (D) None of these

9. In ascending order, the correct sequence of mean, mode and median for 3, 7, 9, 11, 5, 2, 11, 7, 4, 6, 7, 5, 3, 11, 3, 7 is—

(A) mean, median mode
(B) median, mean, mode
(C) mode, median, mean,
(D) median, mode, mean

10. There are five persons A, B, C, D, and E. A weighs twice as much as B. Weight of B is 50% of the weight of C. Weight of D is 40% of that of E. Weight of E is 125% of the weight of A. The person with least weight is—

(A) C (B) E
(C) B (D) A

11. The average of N numbers is X. If one of the numbers A is replaced by another one B, the new average is Y. The new number B equals—

(A) NY – NX – A
(B) Y – X
(C) N(Y – X) + A
(D) (Y – X + A)/N

Directions—(Q. 12 and 13) Read the given data to answer the question that follow. A batsman's average score for a certain number of innings was 22.5 per inning. He played 2 innings more and scored 42 and 46 runs respectively, thus increasing his average by 0.5.

12. How many innings did he play in all ?
    (A) 86 (B) 75
    (C) 42 (D) 27
13. What was his total score?
    (A) 966 (B) 1935
    (C) 1978 (D) None of these
14. What is the average amount of wine in 10 additional casks, which when added to 30 casks with average capacity of 18 litres, increases the average capacity to 90 litres ?
    (A) 300 litres (B) 306 litres
    (C) 210 litres (D) None of these
15. The average of two numbers increases by 25 when one of the numbers is doubled and increases by 30, when the other number is doubled. What must be the original average ?
    (A) 50 (B) 55
    (C) 55·5 (D) None of these

Answers

Exercise A

1. (C) The average of 5 quantities is 10. Therefore, the sum of all '5 quantities is 56.

   The average of 3 of them is 9. Therefore, the sum of the 3 quantities is 27. Therefore, the sum of the remaining two quantities = 50 – 27 = 23.

   Hence, the average of the 2 quantities = 23/2 = 11·5.
2. (C) The average of 5 quantities is 6.

   Therefore, the sum of the 5 quantities is $5 \times 6 = 30$.,

   The average of three of these 5 quantities, is 8.

   Therefore, the sum of these three quantities = $3 \times 8 = 24$.

   The sum of the remaining two quantities = 30 – 24 = 6.

   Average of these two quantities = 6/2 = 3.
3. (C) Let initial average be x, then the initial total is $25x$

   New average will be $x - 2$ and the new total will be $(x-2) \times 25 = 25x - 50$

   The reduction of the 50 is created by the replacement.

   Hence, the new student has 50 marks less than the topper.

   The new student's marks = Topper's marks – the difference in their marks be 95 – 50 = 45
4. (C) Sum of all marks obtained

   $$= \text{Average} \times \text{Number of students} = 35 \times 120 = 4200$$

   Let the number of passed candidates is x.

   Then, Sum of marks obtained by passed candidates $= 39 \times x = 39x$

   Sum of marks obtained by failed candidates

   $$= 15 \times (120 - x) = 1800 - 15x$$

   [Number of failed candidates = $120 - x$]

   Now obviously, Sum of total marks obtained = Sum of marks obtained by passed candidates + Sum of marks obtained by failed candidates

   $$\Rightarrow \quad 4200 = 39x + 1800 - 15x$$
   $$\Rightarrow \quad 4200 - 1800 = 39x - 15x$$
   $$\Rightarrow \quad 2400 = 24x$$
   $$\Rightarrow \quad x = \frac{2400}{24} = 100$$

   100 candidates passed.
5. (C) The total sum of 11 result = $11 \times 50 = 550$

   The total sum of first 6 result = $6 \times 49 = 294$

   The total sum of last 6 result = $6 \times 52 = 312$

   $$\text{Six}^{\text{th}} \text{ result} = 294 + 312 - 550 = 56.$$
6. (D) $65 + 1{\cdot}5 \times 8 = 77$
7. (C) $$\frac{50 \times 5 - 84 + 48}{5} = \frac{250 - 84 + 48}{5} = \frac{214}{5} = 42{\cdot}8.$$

8 (C) $(x-3) + (x-2) + (x-1) + x + (x+1) + (x+2) + (x+3) = 33 \times 7$

   $$\therefore \quad 7x = 33 \times 7$$
   $$x = 33$$

9 (C) $\dfrac{1420 \times 75 - 25 \times 1350 - 30 \times 1425}{20} = 1500.$

10. (B) Let the length of the ropes be x, $x + 2$, $x + 4$ (difference in lengths of the first three ropes being 2 inches) and $x + 10$ (difference in lengths of the third and the fourth ropes being 6 inches).

    $$\text{Thus,} \quad 4x + 16 = 74 \times 4$$
    $$\Rightarrow \quad x = 70 \text{ inches.}$$

    Hence, the length of the longest piece of rope is 80 inches ($x + 10$).
11. (B) $\dfrac{40 \times 8 - 55 + 39}{8} = 38.$
12. (C) The average of 5 quantities is 10.Therefore, the sum of all 5, quantities is 50.

    The average of. 3 of them is 9.

    Therefore, the sum of the 3 quantities is 27.

Therefore, the sum of the remaining two quantities = 50 – 27 = 23.

Hence, the average of the 2 quantities = 23/2 = 11·5.

Exercise B

1. (B) Sum of weight of 33 students

$$= 33 \times 48{\cdot}5 = 1600{\cdot}5 \text{ kg}$$

Let the weight of teacher = x kg

Sum of weight of class (including teacher)

$$= \frac{1600{\cdot}5 + x}{34}$$

$$\Rightarrow \quad x = 65{\cdot}5 \text{ kg.}$$

2. (C) Sum of the age of couple = $27 \times 2 = 54$.

Sum of the age of couple = $47 \times 2 = 94$.

Total ages of all family = $30 \times 4 = 120$

Sum of ages of children = 26 years

$$\text{Average required} = \frac{26}{2} = 13 \text{ years.}$$

3. (D) Sum of temperature at Delhi on Monday, Tuesday and Wednesday

$$M + T + W = 3 \times 27 = 81 \quad \ldots(1)$$

$$T + W + \text{Thus.} = 3 \times 25 = 75 \quad \ldots(2)$$

From equations (1) and (2)

$$M - \text{Thus} = 6$$

$$M = 6 + 26 = 32°$$

So, temperature on Monday was 32°C.

4. (B) Average requires

$$= \frac{(80 \times 4) + (40 \times 3) + (90 \times 1) + (50 \times 2) + (60 \times 5)}{(4 + 3 + 1 + 2 + 5)}$$

$$= \frac{320 + 120 + 90 + 100 + 300}{15}$$

$$= \frac{930}{15} = 62 \text{ marks.}$$

5. (B) Sum of tenth results = $10 \times 50 = 500$

$$\Rightarrow 500 = (40 \times 4 + 3 \times 25 + X + X + 10 + X + 12)$$

$$\Rightarrow 500 = 257 + 3X$$

$$\text{or} \quad X = 81$$

$$\text{Required last not} = 81 + 12 = 93.$$

6. (C) $M + T + Wed + Th + F = 1400 \times 5 \quad \ldots(1)$

$$T + W + Th + F + S = 1490 \times 5 \quad \ldots(2)$$

Solving equations (1) and (2)

$$S - M = 90 \times 5 = \text{Rs. } 450.$$

7. (B) Sum of ages of students = $20 \times 25 = 500$ years

Let teacher's age = x.

$$= \frac{500 + x}{21} = 26$$

$$= 500 + x = 546$$

$$= x = 46 \text{ years.}$$

8. (B) Let the average for 15th inning be X.

$$\Rightarrow \quad \frac{15X + 90}{16} = X + 2 \text{ (Given).}$$

$$\Rightarrow \quad 15X + 90 - 16X = 32$$

$$\Rightarrow \quad -X = -58$$

$$\Rightarrow \quad X = 58$$

$$\text{Requires average} = 58 + 2 = 60.$$

9. (A) Mean $= \frac{3 + 7 + \ldots + 3 + 7}{16} = 6{\cdot}3125$. Mode = 7.

Now in ascending order we can write, the given numbers as

2, 3, 3, 3, 4, 5, 5, 6, 7, 7, 7, 7, 9, 11, 11, 11.

$$\therefore \quad \text{Median} = \frac{\text{8th term + 9th term}}{2} = 6{\cdot}5.$$

10. (C) A's weight is 2 times that of B, so weight of A is not the least.

B's weight is 50% of that of A, so weight of C is not the least.

D's weight is 40% of that of E, so weight of E is not the least.

Hence, the only possible answer is B or D. Let a, b, c, d and e stand for the weight of A, B, C, D and E respectively.

Then, d = 40% of $e = \frac{2}{5}e$. But e = 125% of a

or $e = 1{\cdot}25a$

So, $d = \frac{2}{5} \times 1{\cdot}25a = 0{\cdot}5a$. Since, $a = 2b$, so $b = \frac{1}{2}$

$a = 0{\cdot}5a$.

Hence, $b = d$. So, the least weight is that of B and D.

11. (C) Total of numbers = NX. New Total

$$= NX - A + B$$

$$\text{New Average} = Y = \text{New Total} \div N$$

$$\Rightarrow \quad NY = NX - A + B$$

$$\Rightarrow \quad B = N(Y - X) + A.$$

12. (A) Let he play n innings

$$= \frac{22{\cdot}5x(n-2) + 42 + 46}{n} = 23$$

$$\text{Solving } n = 86.$$

13. (C) Total score $= 86 \times 23 = 1978$.

14. (B) $10X + 30 \times 18 = 40 \times 90$

$$10X = 3600 - 540$$

$$10X = 3060 \text{ litres.}$$

$$X = 306 \text{ litres.}$$

15. (C) When one of the numbers is doubled, the average is increased by 2. So, first number must be 50. ($25 \times 2 = 50$). Similarly, second number must be 60.

So, the original average is $= \frac{50 + 60}{2} = 55{\cdot}5$.

6 Height and Distance

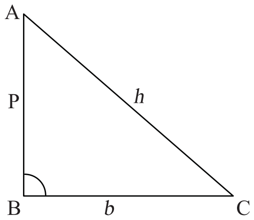
(A) Pythagoras Theorem—This theorem is applicable for a right angled triangle. In right angled triangle there are three basic terms—

(i) **Hypotenuse**—It is the opposite side of the right angle in right-angled triangle. It is the longest side of right-angled triangle.

(ii) Height and

(iii) Base

The other sides of the right angled triangle are called the legs. One of these is height and the other one base of the triangle.



$$\therefore \quad \text{Hypotensue} = AC$$

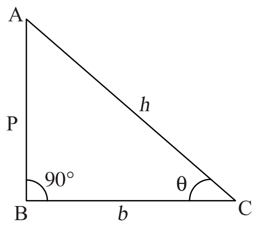
$$\text{Height} = AB$$

$$\text{Base} = BC$$

$$(\text{Hypotensue})^2 = (\text{Height})^2 + (\text{Base})^2$$

$$AC^2 = AB^2 + BC^2$$

(B) Trigonometric Ratios—The most important properties of trigonometric is to find the remaining sides and angles of a triangle, when some of its sides and angles are given.



Consider a right angle triangle ABC such that

AB = P, AC = h, BC = b and $\angle B = 90°$

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{P}{h}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{b}{h}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{P}{b}$$

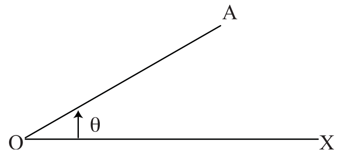
$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{b}{P}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{h}{b}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Height}} = \frac{h}{P}$$

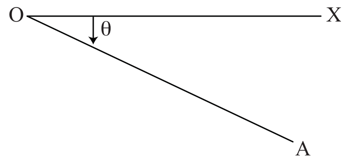
(C) Important Terms Related to Height and Distance—

(i) Angle of Elevation—If 'O' is the observer, 'OX' is the horizontal through 'O' and 'A' is the object of observation at a higher level than the horizontal line OX. Then, the angle XOA is called the angle of elevation.



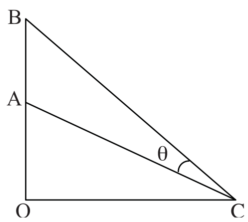
It is called the angle of elevation because the observer has to elevate (raise) his line of sight from the horizontal line OX to see the object A.

(ii) Angle of Depression—If 'O' is the observer 'OX' is the horizontal line through 'O' and 'A' is the object of observation at a lower than the horizontal line, then the angle XOA is called the angle of depression.



Since, the observer has to depress his line of sight from the horizontal line OX to see the object, so it is called the angle of depression.

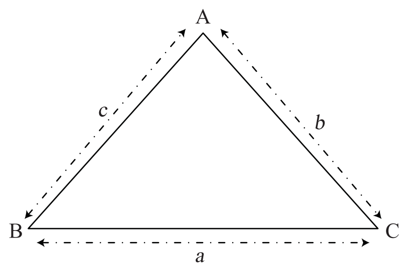
(iii) Angle Subtended by an Object—If the observer is at 'C' and object is 'AB'. Now, the angle subtracted by the object 'AB' at 'C' *i.e.*, at the observer's eye, then it is called angle subtracted by the object. Here, 'θ' is the subtracted angle of an object.



(D) Some Result Useful in Finding Height and Distance—

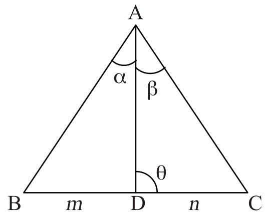
(i) Sine Rule—

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Here, a = length of side BC which is opposite to point A and here A, B, C represent angles.

(ii) In Any Triangle ABC—



If AD divides the angle A into two parts α and β such that

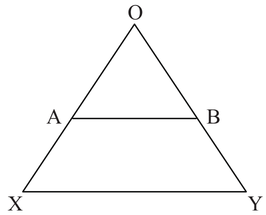
$$\frac{BD}{DC} = \frac{m}{n}$$

$$\angle ADC = \theta$$

Then, $(m + n) \cot \theta = m \cot \alpha - n \cot \beta$

(iii) In a right angled triangle XOY if AB ‖ XY, then

$$\frac{AB}{XY} = \frac{OA}{OX} = \frac{OB}{OY}$$



Exercise A

1. A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m. towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.
2. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance 'a', so that it slides a distance 'b' down the wall making an angle β with the horizontal. Show that $\frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha}$
3. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metres towards the foot of the tower to a point B, the angle of elevation increases to 60°. Find the height of the tower and the distance of the tower from the point A.
4. The angle of elevation of an aeroplane from a point P on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the aeroplane.
5. The angle of elevation θ, of an vertical tower from a point on the ground is such that its tangent is $\frac{5}{12}$. On walking 192 m. towards the tower in the same straight line, the tangent of the angle of elevation is found to be $\frac{3}{4}$. Find the height of the tower.
6. From the top and bottom of a building of height h metres the angles of elevation of the top of a tower are α and β respectively. Prove that the height of the tower is $\frac{h \tan \beta}{\tan \beta - \tan \alpha}$.
7. The angle of elevation of the top of a tower as seen from the point A and B situated in the same line and at a distance p and q respectively from the foot of the tower are complementary. Prove that the height of the tower is $\sqrt{pq}$.
8. From the top of a building 15 m. high, the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower and distance between the tower and the building.
9. From the top of a tower 50 m. high the angles of depression of the top and bottom of pole are observed to be 45° and 60° respectively. Find the height of the pole, and the tower stand in the same plane.
10. The shadow of a vertical tower on level ground increases by 10 metres, when the latitude of the sun changes from angle of elevation 45° to 30°. Find the height of the tower, correct to one place of decimal. (Take $\sqrt{3} = 1{\cdot}73$)
11. A round balloon of radius 'a' subtends an angle θ at the eye of observer while the angle of elevation of its centre is ϕ Prove that the height of the centre of the balloon is a $\sin \phi \operatorname{cosec} \frac{\theta}{2}$.
12. A pole 5 m. high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point 'A' on the ground is 60° and the angle of depression of the point A from the top of the tower is 45°. Find the height of the tower.
13. An aeroplane, when 3000 m. high, passes vertically above another aeroplane at an instant when the angles of elevation of the two aeroplane from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two aeroplanes.
14. From a window (h metres high above the ground) of a house in a street, the angle of elevation and depression of the top and the foot of another house on the opposite side of the street are θ and ϕ respectively. Show that the height of the opposite house is h (1 + tan θ cot ϕ).
15. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the aeroplane in km/hr.
16. A man on the deck of a ship is 12 m above water level. He observes that the angle of elevation of the top of a cliff is 45° and the angle of depression of the

base is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

17. From the top of a hill, the angles of depression of two consecutive stones, 1 kilometre apart, due east, are found to be 30° and 45°. Find the height of the hill.

18. The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is α. On advancing 'p' metres towards the foot of the tower, the angle of elevation becomes β. Show that the height 'h' of the tower is given by $= \dfrac{p \tan \alpha . \tan \beta}{\tan \beta - \tan \alpha}$. Also, determine the height of the tower if $p = 150$ m. $\alpha = 30°$ and $\beta = 60°$.

19. From the top of a building, 60 metres high, the angles of depression of the top and the bottom of a vertical lamp-post are observed to be 30° and 60° respectively. Find (i) the horizontal distance between the building and the lamp post, (ii) The difference between the height of the building and the lamp post.

20. A fire at a building B is reported on telephone to two fire stations F_1 and F_2 10 km apart from each other. F_1 observes that the fire is at angle of 60° from it and F_2 observes that it is at angle of 45° from it. Which station should send its team and how much distance it has to travel ?

21. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 m. away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.

22. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height 5 metres. At a point on the plane, the angles of the elevation of the bottom and the top of the flagstaff are respectively 30° and 60°. Find the height of the tower.

23. A tower is 50 m high. Its shadow is x m shorter when the Sun's altitude is 45°, then when it is 30°, find x correct to the nearest cm.

24. From the top of a tower, the angles of depression of two objects on the same side of the tower are found to be α and β ($\alpha > \beta$). If the distance between the objects is 'p' metres, show that the height 'h' of the tower is given by $h = \dfrac{p \tan \alpha . \tan \beta}{\tan \alpha - \tan \beta}$.

25. A man on the deck of a ship is 16 m above water level. He observes that the angle of elevation of the top of a cliff is 45°, and the angle of depression of the base is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

26. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m above X, the angle of elevation is 45°. Find the height of the tower PQ and the distance XQ.

27. A vertically straight tree, 15 m high, is broken by wind in such a way that its top just touches the ground and makes an angle of 60° with ground. At what length above the ground did the tree break? (Use $\sqrt{3} = 1{\cdot}73$)

Exercise B

1. We have a right angle triangle ABC whose $\angle B = 90°$ and sides are BC = 3, AB = 4 and $\angle ACB = \theta$, then find AC and $\sin \theta, \cos \theta, \tan \theta$—

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$
(C) $\frac{1}{2}$ (D) $\frac{3}{4}$
(E) None of these

2. The string of a kite is 100 metres long and it make angle of 30° with the horizontal. Find the height of the kite—

(A) 15 metre (B) 25 metre
(C) 50 metre (D) 35 metre
(E) 60 metre

3. From a tower 250 high, the angle of depression of a bus is 60°. Find how far the bus is from the tower ?

(A) 125 metre (B) 500 metre
(C) $250\sqrt{3}$ metre (D) $\frac{250}{\sqrt{3}}$ metre
(E) 250 metre

4. If the angle of elevation of the sun changed from 30° to 45°, then find the difference between the lengths of shadows of a pole 30 metre high, made at these positions—

(A) 38 metre (B) 12 metre
(C) 16 metre (D) 22 metre
(E) 30 metre

5. The angle of elevation of the top of a tower 50 metre high, from two points on the level ground on its opposite sides are 60° and 30°. Find the distance between the two points—

(A) 400 metre (B) 100 metre
(C) $\frac{200}{\sqrt{3}}$ metre (D) 200 metre
(E) $200\sqrt{3}$ metre

6. A vertical tower stands on a horizontal plane and is surmounted by a vertical flog staff of height 10 metre. At a point on the plane, the angle of elevation of the bottom and the top of the flag staff are 45° and 60° respectively. Find the height of the tower—

(A) $\frac{10}{\sqrt{3}+1}$ metre (B) $5\left(\sqrt{3}-1\right)$ metre
(C) $5\left(\sqrt{3}+1\right)$ metre (D) 10 metre
(E) $10\left(\sqrt{3}+1\right)$ metre

7. From an observation tower 50 metre above the level of river, the angle of depression of a point on the near shore is 30° and that of a point directly beyond on the far shore is 60°. Find the width of the river—

(A) $\frac{100}{\sqrt{3}}$ metre (B) 100 metre

(C) $100\sqrt{3}$ metre (D) 50 metre

(E) 200 metre

8. From a balloon rising vertically, a person observes two consecutive kilometre stones on the same sides of a straight road and finds their angles of depressions to be 30° and 60°. Find the altitude of the balloon—

(A) $\frac{\sqrt{3}}{2}$ km (B) $\sqrt{3}$ km

(C) $\frac{1}{2}$ km (D) 1 km

(E) $\frac{2}{\sqrt{3}}$ km

9. The angle of elevation of a jet plane from a point on the ground is 60°. After a flight of 18 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $2000\sqrt{3}$ metre. Find the speed of the plane in km/hr—

(A) 800 km/hr (B) 700 km/hr

(C) 600 km/hr (D) 500 km/hr

(E) 400 km/hr

10. A tower sub stands an angle θ_1 at the point A on the same level, as the foot of the tower, and at a second point B. A metre vertically above A the depression of the foot of the tower is θ_2. Find the height of the tower—

(A) $a \tan \theta_1 \cot \theta_2$ (B) $a \tan \theta_2 \cot \theta_1$

(C) $a \frac{\tan \theta_1}{\cot \theta_2}$ (D) $a \frac{\tan \theta_2}{\cot \theta_1}$

(E) a

11. Two pillars of equal height are on either side of a road, which is 100 metre wide. The angles of elevation of the top of the pillars are 60° and 30° at a point on the road between the pillars. Find the position of two pillars and height of each pillar—

(A) $25\sqrt{3}$ metre (B) $50 \frac{1}{\sqrt{3}}$ metre

(C) $25 \frac{1}{\sqrt{3}}$ metre (D) $50\sqrt{3}$ metre

(E) 50 metre

12. A light house is 60 metres high with its base being at the sea level, if the angle of depression of a boat in the sea from the top of the light house is 15°, then what is the distance of the boat from the foot of the light house ?

(A) $60 \times \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ metre

(B) $60 \times \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ metre

(C) $60 \times \left(\frac{1}{\sqrt{3}-1}\right)$ metre

(D) $60 \times \left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ metre

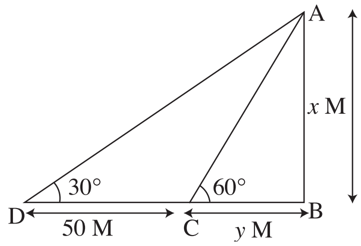
(E) $60\sqrt{3}$ metre

Answers with Hints

Exercise A

1. Suppose height of tower AB = x m.

Distance BC = y m.



In rt. ΔABC,

$$\frac{AB}{BC} = \tan 60°$$

$$\frac{x}{y} = \sqrt{3}$$

$$y = \frac{x}{\sqrt{3}} \quad \ldots(1)$$

In rt. ΔABD,

$$\frac{AB}{DB} = \tan 30°$$

$$\frac{x}{50+y} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}x = 50 + y \quad \ldots(2)$$

Putting the value of y in (2), we get

$$\sqrt{3}x = 50 + \frac{x}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}x = \frac{50\sqrt{3}+x}{\sqrt{3}}$$

$$\Rightarrow \quad 3x = 50\sqrt{3} + x$$

$$\Rightarrow \quad 3x - x = 50\sqrt{3}$$

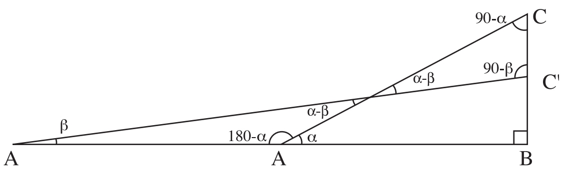
$$\Rightarrow \quad 2x = 50(1{\cdot}73)$$

$$\Rightarrow \quad 2x = 86{\cdot}50 \text{ m}$$

$$\Rightarrow \quad x = 43{\cdot}25 \text{ m.}$$

2. Let the ladder be AC making an angle α with the horizontal. On pulling, the ladder comes in the position $A'C'$ making an angle β with the horizontal.

$A'A = a, C'C = b$



In $\Delta A'AD$ $\quad \dfrac{a}{\sin(\alpha - \beta)} = \dfrac{AD}{\sin\beta} = \dfrac{A'D}{\sin(180° - \alpha)}$

Here, $\quad AD = \dfrac{a \sin\beta}{\sin(\alpha - \beta)}$

$$A'D = \frac{a \sin\alpha}{\sin(\alpha - \beta)}$$

$[\because \sin(180° - \alpha) = \sin\alpha]$

In $\Delta CC'D$ $\quad \dfrac{b}{\sin(\alpha - \beta)} = \dfrac{C'D}{\sin(90° - \alpha)}$

$$= \frac{CD}{\sin(90° + \beta)}$$

$$C'D = \frac{b\cos\alpha}{\sin(\alpha - \beta)}$$

$$CD = \frac{b\cos\beta}{\sin(\alpha - \beta)}$$

$[\because \sin(90° - \alpha) = \cos\alpha$ and $\sin(90° + \beta) = \cos\beta]$

Length of ladder

$$\Rightarrow \quad AD + CD = A'D + C'D$$

$$\Rightarrow \frac{a\sin\beta}{\sin(\alpha - B)} + \frac{b\cos\beta}{\sin(\alpha - \beta)} = \frac{a\sin\alpha}{\sin(\alpha - \beta)} + \frac{b\cos\alpha}{\sin(\alpha - \beta)}$$

$$a\sin\beta + b\cos\beta = a\sin\alpha + b\cos\alpha$$

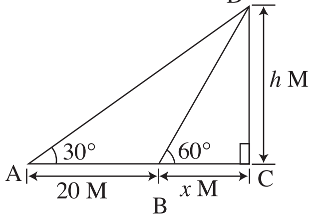
$$a(\sin\beta - \sin\beta) = b(\cos\alpha - \cos\beta)$$

$$\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}.$$

3. Let CD be the tower

$$CD = h \text{ m}$$

$$BC = x \text{ m}$$



In rt., ΔBCD,

$$\tan 60° = \frac{CD}{BC}$$

$$\frac{\sqrt{3}}{1} = \frac{h}{x}$$

$$h = x\sqrt{3} \qquad \ldots(1)$$

In rt., ΔACD,

$$\tan 30° = \frac{h}{x + 20}$$

$$\frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{x + 20} \qquad \text{[From (1)]}$$

$$3x = x + 20$$

$$2x = 20$$

$$x = BC = 10 \text{ cm.}$$

Taking the value of x in (1),

$$h = x\sqrt{3}$$

$$= (10)(1{\cdot}73) \quad [\because \sqrt{3} = 1{\cdot}73]$$

$$= 17{\cdot}3 \text{ m.}$$

$\therefore$ Height of tower $= 17{\cdot}3$ m.

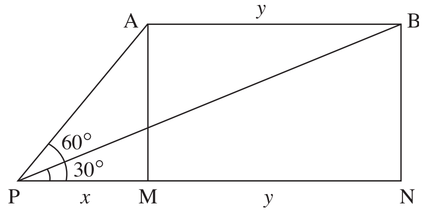
and distance of tower from A

$$= AC = 20 + x$$

$$= 20 + 10 = 30 \text{ m.}$$

4. Let the initial position of the aeroplane be A and after 15 seconds it reaches B.

Let PM = x m and MN = y m



In right ΔAMP,

$$\tan 60° = \frac{AM}{x}$$

$$\sqrt{3} = \frac{1500\sqrt{3}}{x}$$

$$\sqrt{3}x = 1500\sqrt{3}$$

$$\Rightarrow \quad x = 1500 \text{ m}$$

Now, in right ΔBNP,

$$\tan 30° = \frac{BN}{x + y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{1500 + y}$$

$$\Rightarrow \quad 1500 + y = 1500\sqrt{3} \times \sqrt{3}$$

$$\Rightarrow \quad 1500 + y = 4500$$

$$\Rightarrow \quad y = 4500 - 1500$$

$$\Rightarrow \quad y = 3000 \text{ m}$$

$\therefore$ The aeroplane travels 3000 m in 15 sec.

$\therefore$ In 1 sec. it will travel $\frac{3000}{15}$ m = 200 m

$\therefore$ Speed of the aeroplane = 200 m/s

Or, $200 \times \frac{18}{5}$ km/hr = 720 km/hr

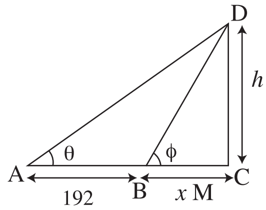
5. Given

$$\tan\theta = \frac{5}{12}$$

$$\tan\phi = \frac{3}{4}$$

$$AB = 192 \text{ m}$$

Let height of the tower DC be h m and BC = x m



In ΔDBC

$$\tan\phi = \frac{h}{x}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{4h}{3} \quad \ldots(1)$$

In ΔDAC,

$$\tan\theta = \frac{h}{x+192}$$

$$\frac{5}{12} = \frac{h}{x+192}$$

$$\Rightarrow \quad 12h = 5x + 960$$

$$\Rightarrow \quad 12h = 5 \times \frac{4h}{3} + 960$$

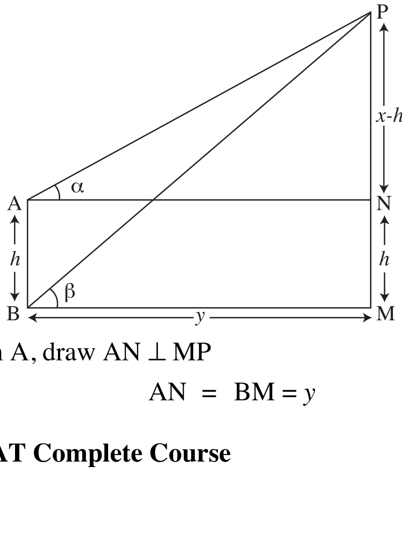
$$\Rightarrow \quad 36h = 20h + 2880$$

$$\Rightarrow \quad 16h = 2880$$

$$\Rightarrow \quad h = \frac{2880}{16}$$

$$\Rightarrow \quad h = 180 \text{ m.}$$

6. AB = h be the building and x be the height of the tower MP S.t. $\angle NAP = \alpha$, $\angle MBP = \beta$



From A, draw AN ⊥ MP

Let AN = BM = y

In right $\angle d$ ΔANP,

$$\frac{y}{x-h} = \cot\alpha$$

$$\therefore \quad y = (x-h)\cot\alpha \quad \ldots(1)$$

In right $\angle d$ ΔBMP,

$$\frac{y}{x} = \cot\beta$$

$$\therefore \quad y = x\cot\beta \quad \ldots(2)$$

From (1) and (2), we get

$$(x-h)\cot\alpha = x\cot\beta$$

$$\therefore \quad \frac{x-h}{\tan\alpha} = \frac{x}{\tan\beta}$$

$$\Rightarrow \quad x\tan\beta - h\tan\beta = x\tan\alpha$$

$$\Rightarrow \quad x(\tan\alpha - \tan\alpha) = h\tan\beta$$

$$x = \frac{h\tan\beta}{\tan\beta - \tan\alpha}$$

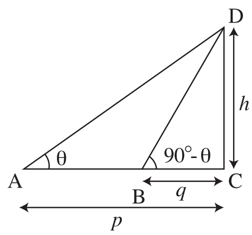
$$\text{Height of the tower} = \frac{h\tan\beta}{\tan\beta - \tan\alpha}.$$

7. Let CD be the tower.

Let CD = h units

and $\angle CAD = \theta$

Then, $\angle CBD = (90° - \theta)$



In rt. ΔACD,

$$\tan\theta = \frac{CD}{AC}$$

$$\tan\theta = \frac{h}{p}$$

$$h = p\tan\theta$$

In rt. ΔBCD,

$$\tan(90° - \theta) = \frac{CD}{BC}$$

$$\cot\theta = \frac{h}{q} \quad [\because \tan(90° - \theta) = \cot\theta]$$

$$h = q\cot\theta$$

Multiplying (1) and (2)

$$h.h = p\tan\theta . q\cot\theta$$

$$h^2 = pq \quad [\because \tan\theta . \cot\theta = 1]$$

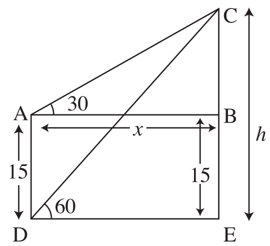
$$h \text{ (height)} = +\sqrt{pq} \text{ unit}$$

[∵ height cannot be – ve]

8. Let AD be the building and CE be the tower.

Let CE = h m

AB = x m



Then, $\quad BC = (h - 15)\,m$

In rt. ΔABC,

$$\tan 30° = \frac{BC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{h-15}{x}$$

$$x = \sqrt{3}\,(h - 15) \qquad \ldots(1)$$

In rt. ΔCED,

$$\tan 60° = \frac{CE}{DE}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{1} = \frac{h}{x}$$

$$\sqrt{3}x = h \quad \text{[Taking value of } x \text{ from (1)]}$$

$$\sqrt{3}[\sqrt{3}(h - 15)] = h$$

$$3h - 45 = h$$

$$3h - h = 45$$

$$\Rightarrow \quad 2h = 45$$

So, $\quad h = \dfrac{45}{2}$ or $22{\cdot}5$ m.

Now, using value of h in (1),

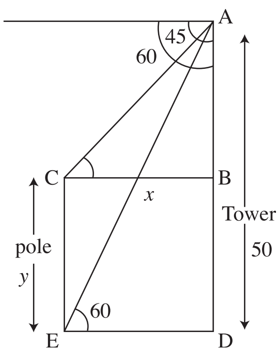
$$x = \sqrt{3}(h - 15)$$

$$= \sqrt{3}\left(\frac{45}{2} - 15\right)$$

$$= 1{\cdot}73 \times \frac{15}{2} \qquad \left[\because \sqrt{3} = 1{\cdot}73\right]$$

$$= \frac{25{\cdot}95}{2} = 12{\cdot}975\,m.$$

9. Let AD and CE be the tower and pole respectively.



Let $\quad BC = DE = x$ m

Let $\quad CE = BD = y$ m

Then, $\quad AB = (50 - y)$ m

In rt. ΔABC

$$\tan 45° = \frac{AB}{BC}$$

$$1 = \frac{50-y}{x}$$

$$x = (50 - y) \qquad \ldots(1)$$

In rt. ΔADE

$$\tan 60° = \frac{AD}{DE}$$

$$\sqrt{3} = \frac{50}{x}$$

$$\Rightarrow \quad x = \frac{50}{\sqrt{3}} \qquad \ldots(2)$$

$$\frac{50}{\sqrt{3}} = 50 - y \quad \text{[From (1) and (2)]}$$

$$y = 50 - \frac{50}{\sqrt{3}}$$

$$y = \frac{50\sqrt{3} - 50}{\sqrt{3}}$$

$$y = \frac{50\left(\sqrt{3}-1\right)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

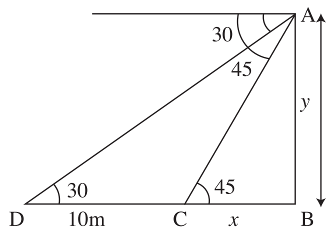
$$y = \frac{50\left(3 - \sqrt{3}\right)}{3}$$

$$y = \frac{50(3 - 1{\cdot}732)}{3} \qquad [\because \sqrt{3} = 1{\cdot}732]$$

$$y = \frac{50 \times 1{\cdot}268}{3} = 21{\cdot}133$$

$\therefore$ Height of the pole = $21{\cdot}13$ m.

10. Let the tower $AB = y$



Angles of altitude are 45° and 30°.

$\therefore \quad m\angle ACB = 45°$

and $\quad m\angle ADB = 30°$

$DC = 10$ m

Let $\quad CB = x$ m

In ΔACB,

$$\frac{AB}{BC} = \tan 45°$$

$$\Rightarrow \quad \frac{y}{x} = 1$$

$$\Rightarrow \quad y = x$$

In ΔADB,

$$\frac{AB}{BD} = \tan 30°$$

$$\Rightarrow \quad \frac{y}{x+10} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{x}{x+10} = \frac{1}{1{\cdot}73} \qquad [\because y = x]$$

$$\Rightarrow \quad 1{\cdot}73x = x + 10$$

$$\Rightarrow \quad 1{\cdot}73x = x + 10$$

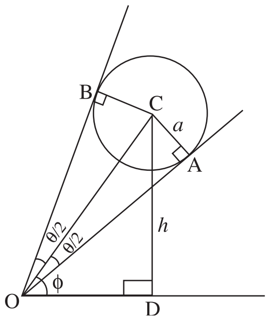
$$\Rightarrow \quad 1{\cdot}73x - x = 10$$

$$\Rightarrow \quad 0{\cdot}73x = 10$$

$$\Rightarrow \quad x = 10 \times \frac{100}{73} = 13{\cdot}7$$

$\therefore$ Height of tower $= y = x = 13{\cdot}7$ m.

11. OA and OB are tangents to the spherical ball.



$\therefore$ AC $\perp$ OA [$\because$ A tangent to a circle is to the radius through the point of contact]

and $\angle$OAC = 90°

Similarly, CB $\perp$ OB

$\therefore$ $\angle$OBC = 90°

In rt. ΔCAO, $\text{cosec}\,\frac{\theta}{2} = \frac{OC}{AC}$

$$\Rightarrow \quad OC = AC.\,\text{cosec}\,\frac{\theta}{2}$$

$$\Rightarrow \quad OC = a.\,\text{cosec}\,\frac{\theta}{2} \qquad \ldots(1)$$

In rt. ΔODC,

$$\sin\phi = \frac{CD}{OC}$$

$$\Rightarrow \quad \sin\phi = \frac{h}{OC}$$

$$\Rightarrow \quad h = OC.\,\sin\phi$$

$$\Rightarrow \quad h = a\,\text{cosec}\frac{\theta}{2}\cdot\sin\phi$$

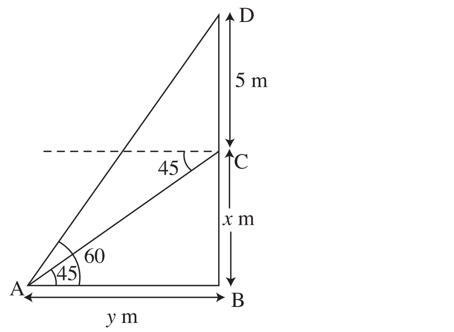
$\therefore$ Height of the centre of balloon

$$= a\sin\phi\ \text{cosec}\,\frac{\theta}{2}$$

12. Let BC be the height of the tower and CD be the height of the pole.

Let BC = x m

and AB = y m



Then in rt. ΔABC,

$$\tan 45° = \frac{BC}{AB}$$

$$\Rightarrow \quad 1 = \frac{x}{y}$$

$$\Rightarrow \quad y = x \qquad \ldots(1)$$

In rt. ΔABD,

$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{x+5}{y}$$

$$\Rightarrow \quad \sqrt{3}y = x + 5$$

$$\Rightarrow \quad \sqrt{3}x = x + 5 \qquad \text{[From (1)]}$$

$$\Rightarrow \quad \sqrt{3}x - x = 5$$

$$\Rightarrow \quad \left(\sqrt{3}-1\right)x = 5$$

$$\Rightarrow \quad x = \frac{5}{\sqrt{3}-1}$$

$$\Rightarrow \quad x = \frac{5}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \quad x = \frac{5\left(\sqrt{3}+1\right)}{3-1}$$

$$\Rightarrow \quad x = \frac{5(1{\cdot}732+1)}{2}$$

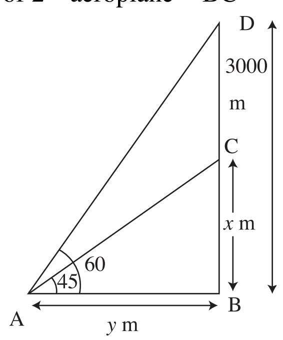
$$x = \frac{5\,(2{\cdot}732)}{2}$$

$$\Rightarrow \quad x = 5 \times 1{\cdot}366 = 6{\cdot}83$$

$\therefore$ Height of the tower $= 6{\cdot}83$ m.

13. Let height of Ist aeroplane = BD

and height of 2nd aeroplane = BC



Let BC = x m and AB = y m.

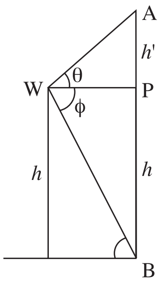
In rt. ΔABC,
$$\tan 45^\circ = \frac{BC}{AB}$$
$$\Rightarrow \quad 1 = \frac{x}{y}$$
$$\Rightarrow \quad y = x \quad \ldots(1)$$

In rt. ΔABD,
$$\tan 60^\circ = \frac{BD}{AB}$$
$$\Rightarrow \quad \frac{\sqrt{3}}{1} = \frac{3000}{y}$$
$$\Rightarrow \quad \sqrt{3}y = 3000$$
$$\Rightarrow \quad \sqrt{3}x = 3000 \quad \text{[From (i)]}$$
$$\Rightarrow \quad x = \frac{3000}{\sqrt{3}}$$
$$\Rightarrow \quad x = \frac{3000}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$\Rightarrow \quad x = \frac{3000\sqrt{3}}{3}$$
$$\Rightarrow \quad x = 1000(1\cdot732)$$
$$\Rightarrow \quad x = 1732\,\text{m}.$$
$$\therefore \quad \text{Difference in height} = 3000 - 1732 = 1268\,\text{m}.$$

14. Let W be the window.

and AB be the house on the opposite side.

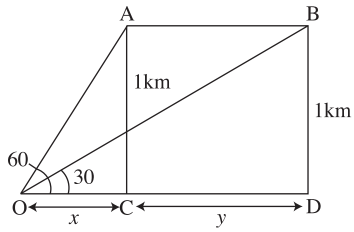
Then, WP is the width of the street



In ΔBPW,
$$\tan \phi = \frac{PB}{WP}$$
$$\frac{h}{WP} = \tan \phi$$
$$WP = h \cot \phi \quad \ldots(1)$$

Now, In ΔAWP,
$$\tan \theta = \frac{AP}{WP}$$
$$\frac{h'}{WP} = \tan \theta$$
$$\Rightarrow \quad h' = WP \tan \theta$$
$$\Rightarrow \quad h' = h \cot \phi \tan \theta$$
$$\therefore \quad \text{Height of house} = h + h' = h + h \tan \theta \cot \phi$$
$$= h(1 + \tan \theta \cot \phi).$$

15. Let O be the point of observation. A be the position of the aeroplane such that $\angle$AOC = 60°, AC = 1 km and OC = x km.



After 10 seconds

Let B be the position of the aeroplane.

Then, $\angle BOD = 30^\circ$,
$$BD = AC = 1\text{ km}$$
and $CD = y$ km

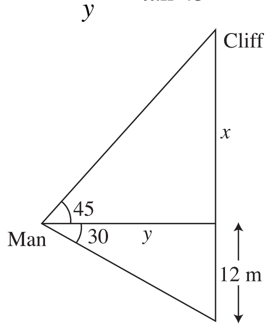
In ΔOAC,
$$\frac{OC}{AC} = \cot 60^\circ$$
$$\Rightarrow \quad x = \frac{1}{\sqrt{3}} \quad \ldots(1)$$

In ΔOBD,
$$\frac{OC}{AC} = \cot 30^\circ$$
$$\Rightarrow \quad x + y = \sqrt{3} \quad \ldots(2)$$

Subtracting (2) from (1), we get
$$y = -\sqrt{3} + \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad y = \sqrt{3} - \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad y = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\text{In 10 seconds distance covered} = \frac{2}{\sqrt{3}}\text{ km}$$
$$\text{In 3600 seconds distance covered} = \frac{2}{\sqrt{3}} \times \frac{3600}{10}$$
$$= 240\sqrt{3}\text{ km}$$
$$\therefore \quad \text{The speed of the aeroplane} = 240\sqrt{3}$$
$$= 240 \times 1\cdot732$$
$$= 415\cdot68\text{ km/hr}.$$

16.
$$\frac{x}{y} = \tan 45^\circ$$



$\Rightarrow \quad x = y \quad [\because \tan 45° = 1]$

$$\frac{12}{y} = \tan 30°$$

$$\Rightarrow \quad y = \frac{12}{\tan 30°}$$

$$\Rightarrow \quad y = 12 \times \sqrt{3}$$

$$\Rightarrow \quad y = 12 \times 1{\cdot}732$$

$$\Rightarrow \quad y = 20{\cdot}784$$

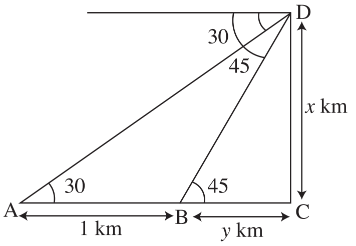
$\therefore$ Distance of ship from cliff = 20·784

$\therefore$ Required Height = 20·784 + 12 = 32·784 m.

17. Suppose height of hill DC = x km.

Distance between two stones AB = 1 km.

Suppose distance of stone B from hill = y km.



In ΔBCD, $\quad \frac{DC}{BC} = \tan 45°$

$$\Rightarrow \quad \frac{x}{y} = 1$$

$$\Rightarrow \quad x = y \quad \ldots(1)$$

In ΔACD, $\quad \frac{CD}{AC} = \tan 30°$

$$\Rightarrow \quad \frac{x}{y+1} = \frac{1}{\sqrt{3}} \quad \ldots(2)$$

Putting the value of $x = y$ in (2), we get

$$\frac{x}{x+1} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}x = x + 1$$

$$\Rightarrow \quad \sqrt{3}x - x = 1$$

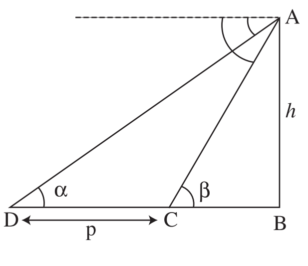
$$\Rightarrow \quad \left(\sqrt{3} - 1\right)x = 1$$

$$\Rightarrow \quad (1{\cdot}73 - 1)x = 1$$

$$\Rightarrow \quad 0{\cdot}73x = 1$$

$$\Rightarrow \quad x = \frac{1}{0{\cdot}73} = \frac{100}{73} = 1{\cdot}37 \text{ km.}$$

18. Let AB be the tower = h



In rt. ΔABC, $\quad \tan\beta = \frac{AB}{BC}$

$$\frac{\tan\beta}{1} = \frac{h}{BC}$$

$$BC.\tan\beta = h$$

$$BC = \frac{h}{\tan\beta} \quad \ldots(1)$$

In rt. ΔABD, $\quad \tan\alpha = \frac{AB}{BD}$

$$\frac{\tan\alpha}{1} = \frac{h}{BC + p}$$

$$h = \tan\alpha.(BC + p)$$

$$= \tan\alpha\left(\frac{h}{\tan\beta} + p\right)$$

$$\left[\because BC = \frac{h}{\tan\beta} \text{ from (1)}\right]$$

$$= \tan\alpha\left(\frac{h + p.\tan\beta}{\tan\beta}\right)$$

$$h\tan\beta = h.\tan\alpha + p.\tan\beta.\tan\alpha$$

$$h\tan\beta - h\tan\alpha = p.\tan\beta.\tan\alpha$$

$$h(\tan\beta - \tan\alpha) = p.\tan\beta.\tan\alpha$$

$$h = \frac{p.\tan\alpha.\tan\beta}{\tan\beta - \tan\alpha}$$

$$p = 150 \text{ m}, \alpha = 30°, \beta = 60°$$

$$h = \frac{p.\tan\alpha.\tan\beta}{\tan\beta - \tan\alpha}$$

$$= \frac{150.\tan 30°.\tan 60°}{\tan 60° - \tan 30°}$$

$$= \frac{150.\left(\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}\right)}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$= \frac{150}{\frac{3-1}{\sqrt{3}}} = 150 \times \frac{\sqrt{3}}{2}$$

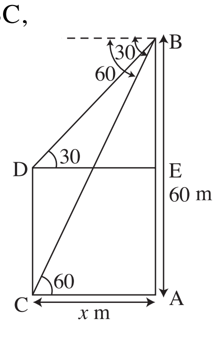
$$= 75 \times (1{\cdot}732) \quad [\because \sqrt{3} = 1{\cdot}732]$$

$$= 129{\cdot}9 \text{ m.}$$

19. Let AB be the building, CD the vertical lamp post.

Let the horizontal distance between the building and the lamp-post be x m.

Now, in ΔABC,



$$\tan 60° = \frac{60}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow \quad x = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

In ΔAEC,

$$\tan 30° = \frac{60-h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{60-h}{20\sqrt{3}}$$

$$60 - h = 20$$

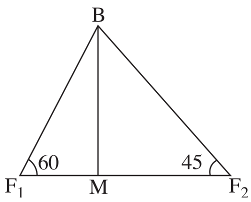
$$h = 40 \text{ m.}$$

(i) Hence, the horizontal distance between the building and lamp-post = $20\sqrt{3} = 34{\cdot}64$ m.

(ii) Difference between the heights of the building and the lamp post

$$60 - h = 60 - 40 = 20 \text{ m.}$$

20. Let M be the foot of the perpendicular from B on $F_1 F_2$.



In rt. ΔBMF_1

$$\frac{BM}{F_1B} = \sin 60°$$

$$= BM = F_1B \sin 60° \quad \ldots(1)$$

In rt. ΔBMF_2

$$\frac{BM}{F_2B} = \sin 45°$$

$$BM = F_2B \sin 45° \quad \ldots(2)$$

From (1) and (2), $F_1B \sin 60° = F_2B \sin 45°$

Since, $\sin \theta$ increases as θ increases from 0 to 90°.

$\sin 60° > \sin 45°$, so that $F_1B < F_2B$

$\therefore$ F_1 is nearer to B than F_2

$\therefore$ The station F_1 should send its team.

Let $\quad F_1B = a$ km.

In rt. ΔBMF_1

$$\frac{F_1M}{F_1B} = \cos 60°$$

$$\Rightarrow \quad F_1M = F_1B \cos 60°$$

$$\Rightarrow \quad F_1M = a \cos 60°$$

In rt. $\Delta BMF_2 \cdot \dfrac{MF_2}{BM} = \cot 45° = 1$

$$MF_2 = BM = F_1B \sin 60°$$

$$= a \sin 60° \quad \text{[Using (1)]}$$

$$F_1F_2 = F_1M + MF_2$$

$$10 = a \cos 60° + a \sin 60°$$

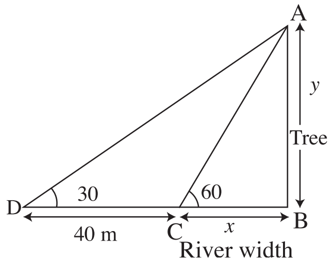
$$10 = a\,(0{\cdot}5 + 0{\cdot}866)$$

$$10 = 1{\cdot}366a$$

$$a = \frac{10}{1{\cdot}366} = 7{\cdot}32 \text{ km.}$$

21. Let height of the tree = y m.

Distance of man standing on bank = x m.



In rt. ΔABC,

$$\frac{AB}{BC} = \tan 60°$$

$$\Rightarrow \quad \frac{y}{x} = \sqrt{3}$$

$$\Rightarrow \quad y = \sqrt{3}x \quad \ldots(1)$$

In rt. ΔABD,

$$\Rightarrow \quad \frac{AB}{BD} = \tan 30°$$

$$\Rightarrow \quad \frac{y}{x+40} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}y = x + 40$$

$$\Rightarrow \quad \sqrt{3}.\sqrt{3}x = x + 40 \quad \text{[From (1)]}$$

$$\Rightarrow \quad 3x = x + 40$$

$$\Rightarrow \quad 3x - x = 40$$

$$\Rightarrow \quad 2x = 40$$

$$\Rightarrow \quad x = 20$$

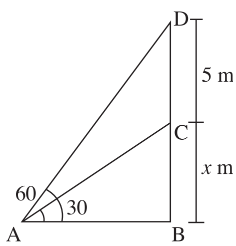
Now, putting the value of $x = 20$ in (1)

$$y = \sqrt{3}x = 1{\cdot}732 \times 20 = 34{\cdot}64$$

$\therefore$ Height of the tree (y) = 34.64 m.

and Width of the river (x) = 20 m.

22. Let BC = x m.



Vertical tower CD = 5 metre in the flagstaff.

Let angle of elevations at A be 30° and 60°.

In ΔABC, $\frac{BC}{AB} = \tan 30°$

$$\frac{x}{AB} = \frac{1}{\sqrt{3}}$$

$$AB = \sqrt{3}x \quad \ldots(1)$$

In ΔABD, $\frac{BD}{AB} = \tan 60°$

$$\frac{x+5}{AB} = \sqrt{3}$$

$$\Rightarrow \quad AB = \frac{x+5}{\sqrt{3}}$$

From (1) and (2), we get

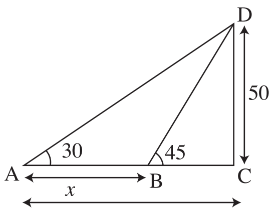
$$\sqrt{3}x = \frac{x+5}{\sqrt{3}}$$

$$3x = x+5$$

$$2x = 5$$

$$x = 2{\cdot}5 \text{ m.}$$

23. Let AB be the tower of height 50 m.



In rt. ΔBCD

$$\frac{BC}{CD} = \cot 45°$$

$$BC = 50 \times 1$$

$$BC = 50 \text{ m.}$$

In rt. ΔACD

$$\frac{AC}{CD} = \cot 30°$$

$$AC = 50 \times \sqrt{3}$$

$$= 50(1{\cdot}732)$$

$$= 86{\cdot}600 \text{ m}$$

Required distance, (1)

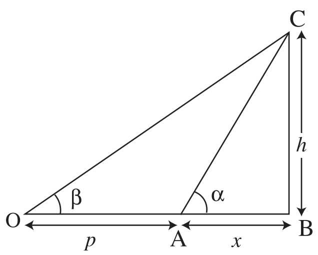
$$x = 86{\cdot}6 - 50$$

$$x = 36{\cdot}6 \text{ m}$$

Also, determine the height of the tower if $p = 50$ metres, $\alpha = 60°, \beta = 30°$.

24. Let BC = h = height of tower

OA = p, AB = x



In ΔABC, $\frac{h}{x} = \tan \alpha$

$$\Rightarrow \quad x = \frac{h}{\tan \alpha} \quad \ldots(1)$$

In ΔOBC, $\frac{h}{p+x} = \tan \beta$

$$\frac{h}{p + \frac{h}{\tan \alpha}} = \tan \beta$$

$$h = p \tan \beta + \frac{h}{\tan \alpha} \cdot \tan \beta$$

$$h\left[1 - \frac{\tan \beta}{\tan \alpha}\right] = p \tan \beta$$

$$h\left[\frac{\tan \alpha - \tan \beta}{\tan \alpha}\right] = p \tan \beta$$

$$h = \frac{p \tan \alpha \tan \beta}{\tan \alpha - \tan \beta}$$

$$p = 50 \text{ m.}, \alpha = 60°, \beta = 30°$$

$$h = \frac{50 \tan 60° \tan 30°}{\tan 60° - \tan 30°}$$

$$= \frac{50 \times \left(\sqrt{3}\right) \times \left(\frac{1}{\sqrt{3}}\right)}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

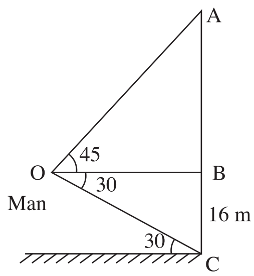
$$= \frac{50}{\left(\frac{3-1}{\sqrt{3}}\right)} = \frac{50}{\frac{2}{\sqrt{3}}} = 50 \times \frac{\sqrt{3}}{2}$$

$$= 25 \times (1{\cdot}73) = 43{\cdot}25$$

$$\left[\because \sqrt{3} = 1{\cdot}73\right]$$

$\therefore$ Height (h) = $43{\cdot}25$ m.

25. Height of deck BC = 16 m.



In rt. ΔOBC, $\frac{BC}{OB} = \tan 30°$

$$\Rightarrow \quad \frac{16}{OB} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad OB = 16\sqrt{3} \text{ m} \quad \ldots(1)$$

In rt. ΔOBA,

$$\frac{BA}{OB} = \tan 45°$$

$$\Rightarrow \quad \frac{BA}{16\sqrt{3}} = 1$$

$$\Rightarrow \quad BA = 16\sqrt{3} \text{ m} \quad \ldots(2)$$

$\therefore$ Height of the cliff $= AB + BC$

$$= 16\sqrt{3} + 16$$

$$= 16\left(\sqrt{3} + 1\right)$$

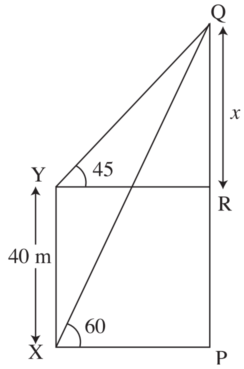
$$= 16(1{\cdot}732 + 1{\cdot}00) \text{ m}$$

$$= 43{\cdot}712 \text{ m}$$

and distance between cliff and ship.

$$= OB = 16\sqrt{3} = 27{\cdot}71 \text{ m.}$$

26. QP (Height) $= 40 + x$



In rt. ΔQPX,

$$\frac{QP}{XP} = \tan 60°$$

$$\frac{40 + x}{XP} = \sqrt{3} \quad \ldots(1)$$

In rt. ΔQRY,

$$\frac{QR}{YR} = \tan 45°$$

$$\Rightarrow \quad \frac{QR}{XP} = \tan 45° \quad [\because YR = XP]$$

$$\Rightarrow \quad \frac{x}{XP} = 1$$

$$\Rightarrow \quad x = XP \quad \ldots(2)$$

Using the value of x in (1), we get

$$\frac{40 + x}{XP} = \sqrt{3}$$

$$\Rightarrow \quad 40 + XP = \sqrt{3}XP$$

$$\Rightarrow \quad 40 = \sqrt{3}XP - XP$$

$$\Rightarrow \quad 40 = \left(\sqrt{3} - 1\right)XP$$

$$\Rightarrow \quad XP = \frac{40}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{40\left(\sqrt{3} + 1\right)}{2}$$

$$= 20(1{\cdot}732 + 1)$$

$$= 20 \times 2{\cdot}732 = 54{\cdot}64$$

$\therefore$ Height (PQ) $= 40 + 54{\cdot}64 = 94{\cdot}64$ metres

In rt. ΔQPX,

$$\frac{PQ}{XQ} = \sin 60°$$

$$\Rightarrow \quad \frac{PQ}{XQ} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad XQ = \frac{2}{\sqrt{3}} \cdot PQ$$

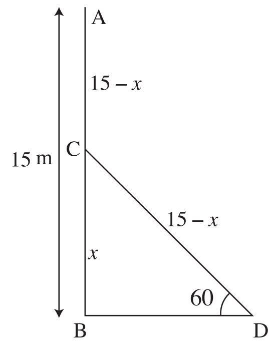
$$\Rightarrow \quad \text{Distance (XQ)} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \cdot PQ$$

$$= \frac{2\sqrt{3}}{3} \times 94.64$$

$$= \frac{2 \times 1{\cdot}732}{3} \times 94{\cdot}64$$

$$= 109{\cdot}3 \text{ m.}$$

27. Height of the tree AB $= 15$ m.



It broke at C. Its top A touches the ground at D.

Now, AC = CD, $\angle$BDC = 60°

$$BC = ?$$

Let $BC = x$

$$AC = 15 - x$$

and $CD = 15 - x \quad [\because AC = CD]$

In rt. ΔCBD,

$$\frac{BC}{CD} = \sin 60°$$

$$\frac{x}{15 - x} = \frac{\sqrt{3}}{2}$$

$$2x = (15 - x)\sqrt{3}$$

$$2x = 15\sqrt{3} - \sqrt{3}x$$

$$2x + \sqrt{3}x = 15\sqrt{3}$$

$$x(2 + \sqrt{3}) = 15\sqrt{3}$$

$$x = \frac{15\sqrt{3}}{2 + \sqrt{3}}$$

$$\Rightarrow \quad x = \frac{15\sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$x = \frac{30\sqrt{3} - 15 \times 3}{4 - 3}$$

$$\Rightarrow \quad x = \frac{30 \times 1{\cdot}73 - 45}{1}$$

$$\therefore \quad x = \frac{51{\cdot}9 - 45}{1}$$

$$\Rightarrow \quad x = 6{\cdot}9 \text{ m}$$

$\therefore$ The tree broke at 6.9 metres from the ground.

Exercise B

1. (B) To find the value of $\sin\theta$ we need hypotenuse. So, using Pythagoras theorem :

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 4^2 + 3^2 = 16 + 9 = 25$$

$$AC = 5$$

Now, $\sin\theta = \frac{\text{Height}}{\text{Hypotenuse}} = \frac{4}{5}$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{3}{5}$$

$$\tan\theta = \frac{\text{Height}}{\text{Base}} = \frac{4}{3}.$$

2. (C) Let C be the position of kite at a height h and AB is horizontal line.

AC represents string. Now, in right angled triangle ABC—

We have to find AC, so using formula :

$$\sin 30° = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{h}{AC}$$

$$h = \frac{AC}{2} = \frac{100}{2} = 50 \text{ metre}$$

$\therefore$ Height of kite above the ground = 50 metre.

3. (D) Here, we have to find AB.

So using formula :

$$\cos 60° = \frac{AB}{BO}$$

and $\tan 60° = \frac{\text{Height}}{\text{Base}}$

$$\sqrt{3} = \frac{250}{\text{Base}}$$

$$\text{Base} = \frac{250}{\sqrt{3}} \text{ metre.}$$

4. (D) First we draw roughly diagram.

Clearly, AB is pole and we have to find CD.

For ΔABC

$$\tan 45° = \frac{30}{BC}$$

$$BC = \frac{30}{\tan 45°}$$

$$BC = \frac{30}{1} = 30 \text{ metre} \qquad \ldots(1)$$

In ΔABC

$$\tan 30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{30}{BD}$$

$$BD = 30\sqrt{3} \qquad \ldots(2)$$

Now, $CD = BD - BC = 30\sqrt{3} - 30$

$$= 30\left(\sqrt{3} - 1\right)$$

$$= 30(1{\cdot}732 - 1) = 30 \times 0{\cdot}732$$

$$CD = 21{\cdot}96 \text{ metre} \cong 22 \text{ metre.}$$

5. (C) OA is the tower.

We have to find AC + AB = CB

In ΔACO

$$\tan 30° = \frac{50}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{50}{AC}$$

$$AC = 50\sqrt{3}$$

In ΔAOB

$$\tan 60° = \frac{50}{AB}$$

$$AB = \frac{50}{\sqrt{3}}$$

$$AB + AC = 50\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) = \frac{200}{\sqrt{3}} \text{ metre.}$$

6. (C) Let AB is flag staff and AO is the tower now we have to find AO.

In ΔAOC

$$\tan 45° = \frac{AO}{CO} = \frac{\text{Height}}{\text{Base}}$$

$$1 = \frac{AO}{CO}$$

$$AO = CO \qquad \ldots(1)$$

In ΔBOC

$$\tan 60° = \frac{OB}{OC}$$

Or, $\sqrt{3} = \frac{OB}{OC}$

$$\sqrt{3} \times OC = OB$$

$$\sqrt{3} \times AO = OB \qquad \text{[By equation (1)]}$$

$$\sqrt{3} \times AO = AB + AO$$

$$AO = \frac{AB}{\sqrt{3} - 1}$$

$$AO = \frac{10}{\sqrt{3} - 1} = \frac{10}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{10(\sqrt{3}+1)}{3-1}$$

$$= 5(\sqrt{3}+1) \text{ metre.}$$

7. (A) Height of tower = AB = 50 metre

In ΔABC

$$\tan 60° = \frac{AB}{BC} = \frac{50}{BC}$$

$$\sqrt{3} = \frac{50}{BC}$$

$$BC = \frac{50}{\sqrt{3}} \quad \ldots(1)$$

In ΔABD

$$\tan 30° = \frac{AB}{BD}$$

Or, $$\frac{1}{\sqrt{3}} = \frac{50}{BD}$$

$$BD = 50\sqrt{3} \quad \ldots(2)$$

Now, we have to find CD

$\therefore$ $$CD = BD - BC$$

$$= 50\sqrt{3} - \frac{50}{\sqrt{3}}$$

$$= \frac{150-50}{\sqrt{3}}$$

$$= \frac{100}{\sqrt{3}} \text{ metre.}$$

8. (A) According to question, C and D are two kilometres stones

$\therefore$ $$CD = 1 \text{ km}$$

Now, In ΔABC,

$$\tan 60° = \frac{AB}{BC} = \frac{h}{BC}$$

$$BC = \frac{h}{\sqrt{3}} \quad \ldots(1)$$

In ΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{BD}$$

$$BD = h\sqrt{3} \quad \ldots(2)$$

Since, $$CD = 1 \text{ km}$$

Or, $$BD - BC = 1 \text{ km}$$

$$h\sqrt{3} - \frac{h}{\sqrt{3}} = 1 \text{ km}$$

$$h = \frac{\sqrt{3}}{2} \text{ km.}$$

9. (A) When the plane is at point A it makes an angle of 60° and when at B it makes an angle of 30° at the same point where the observer is standing. Clearly from Diagram, Distance AB = CD is covered in 18 seconds.

In ΔBCE

$$\tan 30° = \frac{BC}{CE}$$

$$\frac{1}{\sqrt{3}} = \frac{2000\sqrt{3}}{CE}$$

$$CE = 6000 \text{ metre}$$

In ΔADE

$$\tan 60° = \frac{AD}{ED}$$

$$\sqrt{3} = \frac{BC}{ED} \quad [AD = BD]$$

$\therefore$ $$ED = \frac{2000\sqrt{3}}{\sqrt{3}} = 2000 \text{ metre}$$

$\therefore$ $$AB = CD = CE - ED$$

$$= 6000 - 2000 = 4000 \text{ metre}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{AB}{t} = \frac{4000}{18}$$

$$= \frac{4000}{1000 \times 18} \times 3600$$

$$= 800 \text{ km/hr.}$$

10. (A) Here, CD is a tower whose foot is C.

In ΔABC

$$\tan \theta_2 = \frac{AB}{AC}$$

Or, $$AC = \frac{a}{\tan \theta_2} \quad \ldots(1)$$

In ΔACD

$$\tan \theta_1 = \frac{CD}{AC}$$

$$CD = AC \tan \theta_1$$

$$CD = \frac{a}{\tan \theta_2} \times \tan \theta_1$$

$$CD = a \tan \theta_1 \cot \theta_2$$

11. (A) Let h be the height of the pillar. C be the point on the road whose distance from the pole AB is X.

In ΔABC, we have

$$\tan 60° = \frac{AB}{BC} = \frac{h}{X}$$

$$X = \frac{h}{\tan 60°}$$

$$h = \sqrt{3}.X \quad \ldots(1)$$

Now, In ΔCDE

$$\tan 30° = \frac{ED}{CD} = \frac{h}{100 - X}$$

Or, $\dfrac{1}{\sqrt{3}} = \dfrac{h}{100 - X}$ ...(2)

Or, $\dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}.X}{100 - X}$ [From equation (1)]

Or, $100 - X = 3X$

Or, $4.X = 100$

$X = 25$ metre

$h = 25\sqrt{3}$ metre.

12. (A) AB is the light house and C is the boat. We have to find BC.

In ΔABC

$$\tan 15° = \frac{AB}{BC}$$

$$\tan (60° - 45°) = \frac{60}{BC}$$

$$BC = \frac{60}{\tan (60° - 45°)}$$

$$= \frac{60}{\dfrac{\tan 60° - \tan 45°}{1 + \tan 45° \tan 60°}}$$

Or, $$BC = \frac{60}{\dfrac{\sqrt{3} - 1}{1 + \sqrt{3}.1}} = 60 \times \left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1}\right) \text{metre.}$$

7 Ratio and Proportion

Ratio

A ratio is a comparison of two numbers by division—Ratio is the relation which one quantity bears to another of the same kind, the comparison being made by considering what part, multiple, one quantity is of the other. Therefore, every ratio is an abstract quantity.

If a and b are two quantities of the same kind, then $\frac{a}{b}$ is known as the ratio of a and b. Therefore, the ratio of two quantities in the same units is a fraction that one quantity is of the other.

Thus, a to b is a ratio $\left(\frac{a}{b}\right)$, written as $a : b$.

The first term of the ratio is called antecedent, while the second term is called consequent.

In the ratio $a : b$.

Ist term 'a' is known as antecedent.

IInd term 'b' is known as consequent.

Ratio between 60 kg and 100 kg is 3 : 5.

Illustration 1.

Ratio of 50 kg and 60 kg is—

Solution :

$$\frac{50}{60} = \frac{5}{6} = 5 : 6$$

The multiplication or division of each term of ratio by a same non-zero number does not affect the ratio. Hence, 3 : 5 is the same as 6 : 10 or 9 : 15 or 12 : 20 etc.

Ratio can be expressed as percentages. To express the value of a ratio as a Percentage, we multiply the ratio by 100.

Therefore, $\frac{3}{5} = 0{\cdot}6 = 60\%$.

Illustration 2.

Two numbers are in the ratio of 3 : 7. If 4 be added to each, they are in the ratio of 7 : 11. Find the numbers.

Solution :

Let the numbers be denoted by $3x$ and $7x$. Then $\frac{3x+4}{7x+4} = \frac{7}{11}$ · Hence, $x = 1$.

Numbers are 3 and 7.

Proportion

When two ratios are equal, the four quantities composing them are said to be proportional.

The equality of two ratio is called proportion a, b, c, d are said to be in proportion if $a : b = c : d$ or $a : b :: c : d$.

In a proportion, the first and fourth terms are known as extremes, while second and third terms are known as means. Hence, a and d are extremes and b and c are means. Hence a and d are extremes and b and c are means of the proportion $a : b :: c : d$.

In a proportion we always have :

Product of extremes = product of means

$$a \times d = b \times c$$

Illustration 3.

If 0.75 : X : : 5 : 8, then find X.

Solution :

Since, these quantities are in proportion.

So, product of means = product of extreme

Or, $$\frac{0.75}{X} = \frac{5}{8}$$

Or, $$X = 0.15 \times 8$$

Or, $$X = 1.2$$

Illustration 4.

Find a fourth proportional $6mn^2 : 9m^3n :: 4mn^3 : ?$

Solution :

Let fourth proportional is p; then $6mn^2 : 9m^3n :: 4mn^3 : p$

$$\therefore \quad p = 6m^3n^2$$

Continued Proportion

Four Quantities—a, b, c, d, are said to be in a continued proportion , if

$$a : b = b : c = c : d \text{ or } \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Three quantities are said to be in continued proportion, if

$$\text{If } a : b = b : c$$

Or, $$ac = b^2$$

In this relationship, b is said to be the mean proportional between a and c and c is said to be a third proportional to a and b.

Illustration 5.

An object is 20 m long casts a shadow 25 m long. At the same time another object kept merely casts a shadow 30 m. long. Find the length of the second object.

Solution :

Ratio of length of the object to its shadow would be the same.

$$\therefore \quad \frac{20}{25} = \frac{X}{30}$$

$$X = \frac{20 \times 30}{25} = 4 \times 6$$

$$X = 24 \text{ m.}$$

Illustration 6.

An object 1.6 m long casts a shadow 1.4 m long. At the same time another object kept nearby casts a shadow 6.2 m long. Find the length of the second object.

Solution :

Ratio of length of the object to its shadow would be the same.

$$\therefore \quad 1.6 : 1.4 = x : 6.2$$

$$\text{or} \quad x = \frac{1.6 \times 6.2}{1.4} = 7.08 \text{ m}$$

Properties of Proportion

If $\frac{a}{b} = \frac{c}{d}$, then

(i) $\frac{b}{a} = \frac{d}{c}$ (ii) $\frac{a}{c} = \frac{b}{d}$

(iii) $\frac{c}{a} = \frac{b}{d}$ (iv) $\frac{a+b}{a} = \frac{c+d}{d}$

(v) $\frac{a-b}{a} = \frac{c-d}{d}$ (vi) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

(vii) $\frac{a+b}{c+d} = \frac{a-b}{c-d}$ (viii) $\frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$

Relation among More than Two Quantities

(1) The number of quantities are three

If $\frac{A}{B} = \frac{a}{b}$ $\frac{B}{C} = \frac{X}{Y}$

Then, A : B : C = Product of all numerators : First denominator × Second Numerator : Product of all denominator

A : B : C = aX : bX : bY

Pictorial representation is

$$\begin{tikzcd} A : B = & a \arrow[d, "A"'] & b \arrow[dl, "B"'] \arrow[d, "C"] \\ B : C = & X & Y \end{tikzcd}$$

Follow the arrow diagram and multiply to get

A : B : C

(2) Number of quantities are four

If $\frac{A}{B} = \frac{a}{b}$ $\frac{B}{C} = \frac{X}{Y}$ $\frac{C}{D} = \frac{\alpha}{\beta}$

Then, A : B : C : D = aX : bX : bY : bYβ

Some Results on Ratio and Proportion

1. Invertendo – If $a : b :: c : d$, then $b : a :: d : c$
2. Alternendo – If $a : b :: c : d$, then $a : c :: b : d$.
3. Componendo – If $a : b :: c : d$, then $(a + b) : b :: (c + d) : d$
4. Dividendo – If $a : b :: c : d$ then $(a - b) : b :: (c - d) : d$
5. Componendo and Dividendo – If $a : b :: c : d$ then $(a+b) : (a-b) :: (c+d) : (c-d)$.

Equating the components of two and more ratios

If $\frac{a}{b} = \frac{2}{3}, \frac{b}{c} = \frac{4}{5}$. Then find $a : b : c$.

To equate , common component b in the two ratios , take the LCM of 3 and 4 which is 12. Hence, the new ratios are obtained as

$$\frac{a}{b} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}, \frac{b}{c} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

Now, since the common component b in the two ratio has the equal values, $a : b : c = 8 : 12 : 15$.

Therefore, if $\frac{a}{b} = \frac{n_1}{m_1}, \frac{b}{c} = \frac{n_2}{m_2}$

$$\therefore \quad a : b : c = n_1 \times n_2 : m_1 \times n_2 : m_1 \times m_2$$

Illustration 7.

If $\frac{a}{b} = \frac{3}{5}, \frac{b}{c} = \frac{4}{7}$, Find $a : b : c$.

Solution :

$a : b : c = 3 \times 4 : 5 \times 4 : 5 \times 7 = 12 : 20 : 35$

Suppose there are three ratio $\frac{a}{b} = \frac{2}{3} = \frac{b}{c} = \frac{4}{5}$ and $\frac{c}{d} = \frac{7}{15}$

Now, to find the value of $a : b : c : d$. First of all we equate the common term b in the two ratio and then the same process is repeated to equate the common term c.

LCM of 3 and 4 = 12

$\therefore \frac{a}{b} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ and $\frac{b}{c} = \frac{4 \times 3}{5 \times 3}$ two ratio and then the same process is repeated to equate the common term c.

LCM of 3 and 4 = 12

$$\therefore \quad \frac{a}{b} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \text{ and } \frac{b}{c} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}.$$

Now, LCM of 15 and 7 = 105

$$\therefore \frac{b}{c} = \frac{12 \times 7}{15 \times 7} = \frac{84}{105} \text{ and } \frac{c}{d} = \frac{7 \times 15}{15 \times 15} = \frac{105}{225}$$

Again, we have to equate b for first two ratios

$$\therefore \frac{a}{b} = \frac{8 \times 7}{12 \times 7} = \frac{56}{84}, \frac{b}{c} = \frac{84}{105}, \frac{c}{d} = \frac{105}{225}$$

Hence, $a : b : c : d = 56 : 84 : 105 : 225$

Therefore, if $\frac{a}{b} = \frac{n_1}{m_1}$, $\frac{\text{b}}{\text{c}} = \frac{n_2}{m_2}$ and $\frac{c}{d} = \frac{n_3}{m_3}$

Then, $a : b : c : d = n_1 \times n_2 \times n_3 : m_1 \times n_2 \times n_3 :$
$m_1 \times m_2 \times n_3 : m_1 \times m_2 \times m_3$

Illustration 8.

If $\frac{a}{b} = \frac{2}{3}, \frac{b}{c} = \frac{4}{5}$ and $\frac{c}{d} = \frac{7}{11}$.

Solution :

$a = 2 \times 4 \times 7 = 56$, $b = 3 \times 4 \times 7 = 84$, $c = 5 \times 3 \times 7 = 105$ and $d = 11 \times 5 \times 3 = 165$.

$\therefore\ a : b : c : d = 56 : 84 : 105 : 165.$

Illustration 9.

If $\frac{a}{b} = \frac{1}{3}, \frac{b}{c} = \frac{4}{5}, \frac{c}{d} = \frac{7}{9}$. Find $a : b : c : d$.

Solution :

$a = 1 \times 4 \times 7 = 28$, $b = 3 \times 4 \times 7 = 84$, $c = 5 \times 3 \times 7 = 105$, $d = 3 \times 5 \times 9 = 135$

$\therefore\ a : b : c : d = 28 : 84 : 105 : 135$

Illustration 10.

If $2x + 3y : 3x + 5y = 18 : 19$. Find $x : y$.

Solution :

$$\frac{2x+3y}{3x+5y} = \frac{18}{19}$$

Or $$\frac{2\left(\frac{x}{y}\right)+3}{3\left(\frac{x}{y}\right)+5} = \frac{2k+3}{2k+5} = \frac{18}{19}$$

Solving the equation for k, we get $k = \left(\frac{x}{y}\right) = -\frac{33}{16}$

Illustration 11.

Three numbers are in the ratio 3 : 4 : 5, the sum of whose squares is 800. Find the numbers.

Solution :

Let the numbers be $3x$, $4x$ and $5x$.

Then, $9x^2 + 16x^2 + 25x^2 = 800$

Or, $50x^2 = 800 \Rightarrow x^2 = 16$ or $x = 4$.

So, the numbers are 12, 16 and 20.

Illustration 12.

In a mixture of 60 litres, the ratio of milk and water is 2 : 1. What amount of water must be added to make the ratio 1 : 2?

Solution :

Quantity of milk $= \frac{2}{3} \times 60 = 40$ litres and that of water = 20 litres.

Let x litre of water be added to make the ratio 1 : 2.

$$\frac{40}{20+x} = \frac{1}{2} \Rightarrow x = 60 \text{ litres.}$$

Let x litre of water be added to make the ratio 1 : 2.

$$\frac{40}{20+x} = \frac{1}{2} \Rightarrow x = 60 \text{ litres.}$$

Illustration 13.

A bag contains rupee, fifty paisa and five pasia coins whose values are in the proportion of 2 : 3 : 4. If the total number of coins are 480, find the value of each coin and the total amount in rupees.

Solution :

$$\text{Number of coins} = \frac{\text{Amount in rupees}}{\text{Value of coins in rupees}}$$

$\therefore$ Number of one rupee coin $= \frac{2x}{1}$

Number of one 50 paisa $= \frac{3x}{\frac{1}{2}} = 6x$

and number of 25 paisa coin $= \frac{4x}{\frac{1}{2}} = 16x$

Given $2x + 6x + 16x = 480 \Rightarrow x = 20$.

$\therefore$ Value of one rupee coin $= 2x =$ Rs. 40,

Value of 50 paisa coin $= 3x =$ Rs. 60 and

Value of 25 paisa coin $= 4x =$ Rs. 80.

Illustration 14.

Two vessel contain mixture of water and milk in the ratio 1 : 4 and 2 : 5. These mixture of two vessels are mixed in the ratio 1 : 4. Find the ratio of water and milk in the resulting mixture.

Solution.

In vessel 1 quantity of water $= \frac{1}{3}$ and that of milk $= \frac{2}{3}$.

In vessel 2 quantity of water $= \frac{2}{7}$ and that of milk $= \frac{5}{7}$.

From vessel 1, $\frac{1}{5}$ is taken and from vessel 2, $\frac{4}{5}$ is taken. Therefore, the ratio of water to milk in the new vessel.

$$= \left(\frac{1}{3}\times\frac{1}{5}+\frac{2}{7}\times\frac{4}{5}\right) : \left(\frac{2}{3}\times\frac{1}{5}+\frac{5}{7}\times\frac{4}{5}\right)$$

$$= 31 : 74$$

Illustration 15.

A and B are two alloys of gold and copper prepared by mixing metals in proportion 7 : 2 and 7 : 11 respectively. If equal quantities of alloys are melted to form a third alloy c, then find the proportion of gold and copper in C.

Solution :

$$\text{In alloy } c, \quad \text{Gold} = \left(\frac{7}{9}+\frac{7}{18}\right)=\frac{21}{18}$$

$$\text{and} \quad \text{Copper} = \left(\frac{2}{9}+\frac{11}{18}\right)=\frac{15}{18}$$

$$\therefore \text{ Ratio of gold and copper} = \frac{21}{18}:\frac{15}{18}=7:5.$$

Illustration 16.

A and B started a joint firm. A's investment was thrice the investment of B and period of his investment was two times the period of investment of B. If B got Rs. 4000 as profit, find their total profit.

Solution :

$$\begin{aligned}\text{Ratio of investment of A and B} &= 3\times 2 : 1\times 1\\ &= 6:1\\ \text{Share of B} = \frac{1}{7}\times\text{Total profit} &= 4000\\ \therefore \quad \text{Total profit} &= \text{Rs. } 28{,}000\end{aligned}$$

Illustration 17.

If $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$, what is the value of each of the fraction ?

Given that $(a, b, c > 0)$.

Solution :

If each of the ratio is equal to K, then $a = (b + c)$ K, $b = (c + a)$ K and $c = (a + b)$ K.

$$\begin{aligned}\text{Hence,} \quad a+b+c &= (2a+2b+2c)\text{ K}\\ \text{or,} \quad (a+b+c)-2\text{K}(a+b+c) &= 0\\ \text{or,} \quad (a+b+c)(1-2\text{K}) &= 0\\ \text{So,} \quad \text{K} &= \frac{1}{2}\end{aligned}$$

Illustration 18.

If $\quad a : b = c : d$ and $e : f = g : h$,

Find $\quad (ae + bf) : (ae - bf)$.

Solution :

$$\frac{a}{b}=\frac{c}{d} \text{ and } \frac{e}{f}=\frac{g}{h}$$

$$\frac{ae}{bf} = \frac{cg}{dh}$$

$$\frac{ae+bf}{ae-bf} = \frac{cg+dh}{cg-dh}$$

(Applying componendo and Dividendo)

Direct Proportion

If A is direct proportional to B, then as A increases B also increases proportionally. For example the relation between speed, distance and time, speed is directly proportional to distance, when time is kept constant.

If is therefore important to note here that the variation is direct and proportional. If one quantity is doubled the related quantity will also be doubled.

Other examples of direct proportion are :

(a) Simple Interest *Vs* Time (principal and rate being constant)

(b) Density *Vs* Mass (volume being constant)

(c) Force *Vs* Acceleration (mass being constant)

Direct Variation

If A is said to vary directly as B, then as A increases B also increases but not proportionally. This variation is denoted by A $\propto$ B or A = KB, where K is a constant.

For Example, the total cost of production is directly related to the number of items being produced.

Here, the variation is direct but not proportional.

Inverse Proportion

A is inversely proportional to B means if A increases B decreases proportionally. If speed is doubled, time taken to cover the same distance is reduced to half.

Other examples of inverse proportion are

(a) Density *Vs* volume (mass being constant)

(b) Number of person *Vs* time taken to complete the work. (work being same)

Inverse Variation

If A is inversely related to (or) varies inversely as B, then if B increases as A decreases but not proportionally. This relation can be expressed mathematically as A $\propto \frac{1}{B}$

$\Rightarrow$ A = K $\times \frac{1}{\text{B}}$, where K is a constant.

Here, the variation is inverse but not proportional.

Illustration 19.

A can do a piece of work in 12 days, B is 60% more efficient than A. Find the number of days that B takes to do the same piece of work.

Solution :

Ratio of efficiencies of

A and B = 100 : 160 = 5 : 8

Since, efficiency is inversely proportional to the number of days, hence ratio of days taken to complete the job is 8 : 5.

So, number of days taken by B $=\frac{5}{8}\times 12 = 7\frac{1}{2}$ days.

Illustration 20.

The speeds of three cars are in the ratio 2 : 3 : 4. Find the ratio between the times taken by these cars to travel the same distance.

Solution :

Speed is inversely proportional to time taken.

Hence, ratio of time taken by these cars $=\frac{1}{2}:\frac{1}{3}:\frac{1}{4}=$ 6 : 4 : 3.

Exercise A

1. The ratio of the numbers of gents to ladies in a party was 2 : 3. When 20 more gents joined the group, the ratio was reversed. The number of ladies in the party was—
   (A) 16 (B) 24
   (C) 30 (D) 36

2. A man ordered 4 pairs of black socks and some pairs of brown socks. The price of a black pair is double that of a brown pair. While preparing the bill, the clerk interchanged the number of black and brown pairs by mistake which increased the bill by 50%. The ratio of the number of black and brown pair of socks in the original order was—
   (A) 4 : 1 (B) 2 : 1
   (C) 1 : 4 (D) 1 : 2

3. A and B compared their incomes and found that A's income was to that of B as 7 : 9 and that the third of A's income was Rs. 30 greater than the difference of their incomes. Find the difference of the income of two—
   (A) Rs. 180 (B) Rs. 240
   (C) Rs. 320 (D) Rs. 160

4. In a large office, 3/4th of the staff can neither type nor take shorthand. However 1/5th can type and 1/3rd can take shorthand. What proportion of people can do both ?
   (A) 13/40 (B) 17/60
   (C) 1/5 (D) 3/40

5. What must be added to two numbers that are in the ratio 3 : 4, so that they come in ratio 4 : 5—
   (A) 1 (B) 6
   (C) 5 (D) 3

6. A person bought two bikes for Rs. 15,000. He sold one of them for a profit of 10% and another for a loss of 5% and on whole he found that he neither gained nor loss. What is the CP of each bike ?
   (A) Rs. 5,000, Rs. 10,000
   (B) Rs. 7,500, Rs. 7,500
   (C) Rs. 8,000, Rs. 7,000
   (D) Rs. 9,000, Rs. 6,000

7. Divide Rs. 1,350 in three parts such that 12 times the first is equal to 5 times the second and 6 times the third—
   (A) 500,100,750 (B) 250,750, 350
   (C) 300,600,450 (D) 250, 600,500

8. If $x : y :: 5 : 2$, the value of $8x + 9y : 8x + 2y$ is—
   (A) 22 : 29 (B) 29 : 22
   (C) 61 : 26 (D) 26 : 61

9. The monthly incomes of A and B are in the ratio 4 : 5, their expenses are in the ratio 5 : 6. If 'A' saves Rs. 25 per month and 'B' saves Rs. 50 per month, what are their respective incomes ?
   (A) Rs. 400 and Rs. 500
   (B) Rs. 240 and Rs. 300
   (C) Rs. 320 and Rs. 400
   (D) Rs. 440 and Rs. 550

10. Find the ratio of a and b from the equation $12a^2 + 35b^2 - 43ab = 0$—
   (A) 7/3;5/4 (B) 7/4;6/5
   (C) 5/2 only (D) None of the above

11. If x and y are connected by the relation $x^2 + 4y^2 = 4xy$, then ratio of x to y is—
   (A) 2 : 1 (B) 1 : 2
   (C) 1 : 1 (D) 4 : 1

12. If $a, b > 0$, choose the write option—
   (A) $(a^3 + b^3) : (a^2 + b^2)$ is greater than $(a^2 + b^2) : (a + b)$
   (B) $(a^3 + b^3) : (a^2 + b^2)$ is smaller than $(a^2 + b^2) : (a + b)$
   (C) $(a^3 + b^3) : (a^2 + b^2)$ is equal to $(a^2 + b^2) : (a + b)$
   (D) None of the above

13. If $a > b > 0$, choose the write option—
   (A) The ratio $a^2 - b^2 : a^2 + b^2$ is greater than $a - b : a + b$
   (B) The ratio $a^2 - b^2 : a^2 + b^2$ is smaller than a $- b : a + b$
   (C) The ratio $a^2 - b^2 : a^2 + b^2$ is equal to $a - b : a + b$
   (D) None of the above

14. What number must be added to each term of the ratio 2 : 5 so that it may become equal to 5 : 6 ?
   (A) 13 (B) 16
   (C) 15 (D) 30

15. What quantity must be added to the terms of the ratio $p + q : p - q$ to make it equal to $(p + q)^2 : (p - q)^2$?
   (A) p/q (B) $\dfrac{q^2 - p^2}{2p}$
   (C) q/p (D) None of these

16. If two numbers are in the ratio 5 : 7 and if 3 is subtracted from each of them, the ratio becomes 2 : 3. Find the numbers—
   (A) 25; 35 (B) 60; 84
   (C) 15; 21 (D) 30; 42

17. If $\dfrac{3x - 4y}{2x - 3y} = \dfrac{5x - 6y}{4x - 5y}$, find the value of $x : y$—
   (A) 5 : 3 (B) 6 : 5
   (C) 5 : 2 (D) 1 : 1

18. If $(2x + 1) : (3x + 13)$ is the sub-duplicate ratio of 9 : 25, find x—

(A) 35 (B) 54

(C) 52 (D) 34

19. What is the compounded ratio of the following ratio 5 : 14 and 7 : 15 ?

(A) 8 : 3 (B) 9 : 5

(C) 1 : 6 (D) 1 : 7

20. The ages of two persons are in the ratio 5 : 7. Eighteen years ago their ages were in the ratio 8 : 13. Find their present ages—

(A) 53 (B) 10

(C) 52 (D) 11

Exercise B

1. If $a : 5 : : 10 : 25$, then find a ?

(A) 2 (B) 3

(C) 4 (D) 5

(E) 6

2. Find the mean proportion of 0·8 and 0·2 ?

(A) 0·4 (B) 0·6

(C) 0·5 (D) 0·3

(E) 0·7

3· Find the third proportion to 0·016 and 0·024 ?

(A) 0·036 (B) 0·024

(C) 0·016 (D) 0·020

(E) 0·040

4. Find the mean proportional of 9 and 16 ?

(A) 10 (B) 11

(C) 12 (D) 13

(E) 14

5. Find the fourth proportional to 5, 6 and 150 ?

(A) 100 (B) 120

(C) 140 (D) 160

(E) 180

6. Given that A : B = 5 : 2 and B : C = 3 : 2. Find A : C ?

(A) 4 : 7 (B) 5 : 3

(C) 3 : 5 (D) 4 : 5

(E) 5 : 4

7. $\frac{5}{48}$ is what part of $\frac{1}{16}$?

(A) 5/3 (B) 1/3

(C) 2/3 (D) 4/3

(E) 7/3

8. If 2X + 3Y : 3X + 5Y = 7 : 8, then find X : Y ?

(A) 11 : 7 (B) 8 : 11

(C) 7 : 11 (D) 5 : 11

(E) 11 : 5

9. The ratio between two number is 2 : 5, then each number is increased by 2, then the ratio becomes 3 : 2. Find the number?

(A) 11 : 7 (B) 2 : 11

(C) 7 : 11 (D) 5 : 11

(E) 11 : 5

10. The ratio of the number of boys and girls in a school is 2 : 5. If there are 700 students in the school. Find the number of girls in the school?

(A) 200 (B) 350

(C) 500 (D) 650

(E) 50

11. If $\frac{a}{b} = \frac{2}{3}, \frac{b}{c} = \frac{5}{4}, \frac{c}{d} = \frac{6}{7}$ the find $a : b : c : d$?

(A) 20 : 15 : 12 : 14 (B) 10 : 15 : 12 : 3

(C) 10 : 15 : 15 : 4 (D) 10 : 15 : 12 : 14

(E) 20 : 15 : 12 : 3

12. The sum of numbers are in the ratio 5 : 3 : 4. The Sum of whose squares is 800. Find the biggest numbers ?

(A) 20 (B) 16

(C) 12 (D) 8

(E) 4

13. In a mixture of 50 litres, the ratio of milk and water is 2 : 3. What amount of water must be added to make the ratio 1 : 3?

(A) 25 litres (B) 15 litres

(C) 20 litres (D) 30 litres

(E) 40 litres

14. The two vessels contain rupee, fifty paisa, and twenty five paisa coins, whose values are in the proportion of 1 : 2 : 5. If the total number of coins are 1000. Find the value of the total amount in rupees?

(A) Rs. 200 (B) Rs. 400

(C) Rs. 320 (D) Rs. 120

(E) Rs. 40

15. Two vessels contain mixtures of water and milk in the ratio 1 : 2 and 2 : 3. These mixtures of two vessels are mixed in the ratio 1 : 4. Find the ratio of water and milk in the resulting mixture?

(A) 11 : 76 (B) 75 : 76

(C) 11 : 75 (D) 1 : 3

(E) 2 : 3

16. The sum of the present age of A, B and C are 90 years. Six years ago, their ages were in the ratio 3 : 2 : 1 . What is the present age of C?

(A) 12 (B) 14
(C) 16 (D) 20
(E) 18

17. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$ · What is the value of each of the fractions ? (Given $a, b, c > 0$)

(A) 1 (B) 1/2
(C) 1/3 (D) 1/4
(E) 1/5

18. A bag contains rupees, fifty paisa, twenty five paisa and ten paisa coins in the proportion 1 : 3 : 5 : 7. If the total amount is Rs. 22.25. Find the number of coins of fifty paisa kind?

(A) 10 (B) 15
(C) 20 (D) 25
(E) 35

19. When 5 is subtracted from both the numerator and the denominator of a fraction. The fraction reduces to $\frac{1}{2}$. When 2 is add to both the numerator and the denominator the fraction reduces to $\frac{2}{3}$. Find the fraction ?

(A) 4/5 (B) 1/5
(C) 4/3 (D) 2/5
(E) 3/5

20. Sita distributes her pens among four friends A, B, C and D in the ratio $\frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$. What is the minimum number of pens that Sita should have ?

(A) 57 (B) 360
(C) 60 (D) 120
(E) Cannot be determine

21. Ramu divides his property so that his son's share to his wife's and the wife's share to his daughter are 2 : 3 and 4 : 5 respectively. If the daughter gets Rs. 49,000 more than the son. Find the total worth of the property ?

(A) 4,45,000 (B) 2,00,000
(C) 2,45,000 (D) 3,00,000
(E) 3,50,000

22. In what proportion must a number be added, so that $\frac{2}{5}$ of the first part and $\frac{1}{3}$ of the second part are together equal to $\frac{1}{4}$ of the original number ?

(A) 5 : 9 (B) 2 : 5
(C) 1 : 3 (D) 1 : 4
(E) 9 : 5

23. In an express train the passengers travelling in A.C. sleeper class. First class and sleeper class are in the ratio 2 : 3 : 5 and rate for each class in the ratio 1 : 2 : 4. If the total income from the train is Rs. 56,000. Find the income of Indian Railways from A.C. sleeper class?

(A) Rs. 4,000 (B) Rs. 5000
(C) Rs. 40,000 (D) Rs. 50,000
(E) Rs. 16,000

24. The income of Ram and Raju are in the ratio 3 : 2 and expenses are in the ratio 5 : 3. If both save Rs. 2000. What is the income of Ram ?

(A) Rs. 3,000 (B) Rs. 4,000
(C) Rs. 5,000 (D) Rs. 6,000
(E) Rs. 4,500

25. Ram, Shyam and Sohan together earn Rs. 11950 and spend 70%, 60% and 65% of their salaries respectively. If their saving are 14 : 21 : 15. What is the salary of Shyam?

(A) Rs. 4,425 (B) Rs. 4,990
(C) Rs. 6,500 (D) Rs. 5,400
(E) Rs. 3,625

Answer with Hints

Exercise A

1. (B) $G : L = 2 : 3$

$$\Rightarrow \quad \frac{G}{L} = \frac{2}{3} \qquad \text{........(i)}$$

$$\text{or} \quad \frac{G+20}{L} = \frac{3}{2} \qquad \text{.......(ii)}$$

Solving (i) and (ii), we get

$$G = 16$$
$$L = 24$$

2. (C)

3. (A) $A : B = 7 : 9$

$$\Rightarrow \quad B = 9\frac{A}{7}$$

$$\text{Also,} \quad \frac{A}{3} = 30 + B - A$$

Putting the value of B, we get

$$\frac{A}{3} = 30 + \left(\frac{9A}{7}\right) - A$$

$$\Rightarrow \quad \left(\frac{A}{3}\right) = 30 + \frac{2A}{7}$$

$$\Rightarrow \quad A = 630$$

$\therefore$ Difference of income

$$= \left(\frac{630}{3}\right) - 30 = 180$$

4. (B) $-\frac{3}{4} = \frac{1}{5} + \frac{1}{3} - X$

$$X = \frac{7}{60}$$

5. (A) Let x be added to them.

Then, $3 + x : 4 + x : : 4 : 5$

$\Rightarrow (3 + x) \times 5 = (4 \times x) \times 4$

$\Rightarrow 15 + 5x = 16 + 4x$

$\Rightarrow x = 16 - 15$

$= 1$

$\therefore$ 1 must be added.

6. (A) According to the question gain on 1st

= loss on 2nd

= 10% of CP of the 1st

= 5% of CP of the 2nd

$= \frac{10}{100}$ of CP of 1st

$= \frac{5}{100}$ of CP of the 2nd

$= \frac{\text{CP of I}^{st}}{\text{CP of 2}^{nd}}$

$= \frac{5}{100} \times \frac{100}{10} = \frac{5}{10}$

Sum of the ratio $= 5 + 10 = 15$

CP of the 1st bike $= \frac{5}{15} \times 15000 =$ Rs. 5000

and CP of the 2nd bike

$= \frac{10}{15} \times 15000 =$ Rs. 10000

7. (D) We have, $12 \times$ (1st part)

$= 5 \times$ (2nd part)

$= 6 \times$ (3rd part) $= x$(say)

$\Rightarrow$ 1st part $= \frac{x}{12}$

2nd part $= \frac{x}{5}$, 3rd part $= \frac{x}{6}$

$\therefore$ The ratio, 1st part : 2nd part : 3rd part

$= \frac{x}{12} : \frac{x}{5} : \frac{x}{6}$

On multiplying all the terms by 60,

1st part : 2nd part : 3rd part

$= \frac{60x}{12} : \frac{60x}{5} : \frac{60x}{6}$

$= 5x; 12x : 10x$

$= 5 : 12 : 10$

Now, sum of all the ratios

$5 + 12 + 10 = 27$

$\therefore$ 1st part $= \frac{5}{27} \times$ Rs. 1350

= Rs. 250

$\therefore$ 2nd part $= \frac{12}{27} \times$ Rs. 1350 = Rs. 600

$\therefore$ 3rd part $= \frac{10}{27} \times$ Rs. 1350

= Rs. 500.

8. (B) $\frac{x}{y} = \frac{5}{2}$

$$\frac{8x + 9y}{8x + 2y} = \frac{8\frac{x}{y} + 9}{8\frac{x}{y} + 2}$$

$$= \frac{29}{22}$$

9. (A) Let A's income be $= 4x$

A's expenses, therefore $= 4x - 25$

Let B's income be $= 5x$

B's expenses,

Therefore, $= 5x - 50$

We know that the ratio of their expenses

$= 5 : 6$

$\Rightarrow \frac{4x - 25}{5x - 50} = \frac{5}{6}$

$\Rightarrow 24x - 150 = 25x - 250$

$\Rightarrow$ Therefore, $x = 100$

$\Rightarrow$ A's income $4x = 400$ and

B's income $= 5x = 500$.

10. (A) $12a^2 + 35b^2 - 43ab = 0$

$\Rightarrow 12\left(\frac{a}{b}\right)^2 - 43\left(\frac{a}{b}\right) + 35 = 0$

$\Rightarrow 12x^2 + 43x + 35 = 0$

where $x = \frac{a}{b}$

$\Rightarrow (3x - 7)(4x - 5) = 0$

$\Rightarrow x = \frac{7}{3}$

or $x = \frac{5}{4}$

$\Rightarrow \frac{a}{b} = \frac{7}{3}$

or $\frac{a}{b} = \frac{5}{4}$

11. (A) $x^2 + 4y^2 = 4xy$

can be written as

$$x^2 - 4xy + 4y^2 = 0$$

$$\Rightarrow \quad (x - 2y)^2 = 0$$

Therefore, $x = 2y$,

$$x/y = 2,$$

Hence, the ratio of x to y is 2 : 1.

12. (A) Let $A = a^3 + b^3$

$B = a^2 + b^2$

$C = a^2 + b^2$

and $D = a + b$

We know that

$$A : B > C : D$$

iff $AD > BC$

Now, $AD = (a^3 + b^3)(a + b)$

$$= a^4 + b^4 + ab(a^2 + b^2)$$

and $BC = (a^2 + b^2)(a^2 + b^2)$

$$= a^4 + b^4 + 2a^2 b^2$$

$$AD - BC = \{a^4 + b^4 + ab(a^2 + b^2)\} - \{a^4 + b^2 + 2a^2b^2\}$$

$$= ab(a^2 + b^2) - 2a^2b^2 - ab(a^2 + b^2 - 2ab)$$

$$= ab(a - b)^2 > 0$$

$[\because a, b, 0 \therefore ab > 0$ and $(a + b)^2 > 0]$

$\Rightarrow \quad AD > BC$

$\Rightarrow \quad A : B > C : D$

Hence, $(a^3 + b^3) : (a^2 + b^2) > (a^2 + b^2) : (a + b)$

13. (A) Let $A = a^2 - b^2$,

$B = a^2 + b^2$,

$C = a - b$

and $D = a + b$

We know that

$$A : B > C : D$$

iff $AD > BC$

We have $AD = (a^2 - b^2)(a + b)$

$$= (a + b)(a + b)^2$$

and $BC = (a^2 + b^2)(a + b)$

$\therefore \quad AD - BA = (a + b)(a + b)^2 - (a^2 + b^2)(a - b)$

$$= (a + b)\{(a + b)^2 - (a - b)\}$$

$$= (a - b)(2ab)$$

$$= 2ab(a - b) > 0 \qquad [\because a > b > 0]$$

$\Rightarrow \quad AD > BC$

$\Rightarrow \quad A : B > C : D$

$\Rightarrow \quad (a^2 - b^2) : (a^2 + b^2) > (a - b) : (a + b)$

14. (A) Let the number to be added be x.

Then, $\dfrac{2 + x}{5 + x} = \dfrac{5}{6}$

$\Rightarrow \quad 6(2 + x) = 5(5 + x)$

$\Rightarrow \quad x = 13$

15. (B) Let the required quantity be x. Then,

$$\frac{p + q + x}{p - q + x} = \frac{(p + q)^2}{(p - q)^2}$$

$\Rightarrow (p + q + x)(p - q)^2 = (p - q - x)(p + q)^2$

$\Rightarrow (p + q)(p - q)^2 + (p - q)^2 x = (p - q)(p + q)^2 + x(p + q)^2$

$\Rightarrow x[(p + q)^2(p + q)^2] + (p - q)(p - q)^2 - (p + q)(p + q)^2$

$\Rightarrow \quad x[-4pq] = (p + q)\{(p + q) - (p - q)\}$

$\Rightarrow \quad -4xpq = (p + q)(p - q)(2q)$

$\Rightarrow \quad -4xpq = (p^2 - q^2)2q$

$\Rightarrow \quad -4xpq = (p^2 - q^2)2q$

$\Rightarrow \quad x = \dfrac{(p^2 - q^2)2q}{-4pq}$

$$= \frac{p^2 - q^2}{-2p}$$

$$= \frac{q^2 - p^2}{2p}$$

16. (C) Let the two numbers be $5x$ and $7x$.

Then, $\dfrac{5x - 3}{7x - 3} = \dfrac{2}{3}$

$\Rightarrow \quad 15x - 9 = 14x - 6$

$\Rightarrow \quad x = 3$

Hence, the numbers are 15 and 21.

17. (D) $\dfrac{3x - 4y}{2x - 3y} = \dfrac{5x - 6y}{4x - 5y}$

$\Rightarrow \quad \dfrac{3(x/y) - 4}{2(x/y) - 3} = \dfrac{5(x/y) - 6}{4(x/y) - 5}$

[Dividing numerator and denominator by y]

$\Rightarrow \quad \dfrac{3a - 4}{2a - 3} = \dfrac{5a - 6}{4a - 5}$

where $\dfrac{x}{y} = a$

$\Rightarrow \quad (3a - 4)(4a - 5) = (2a - 3)(5a - 6)$

$\Rightarrow \quad 12a^2 - 31a + 20 = 10a^2 - 27a + 18$

$\Rightarrow \quad 2a^2 - 4a + 2 = 0$

$\Rightarrow \quad a^2 - 2a + 1 = 0$

$\Rightarrow \quad (a - 1)^2 = 0$

$\Rightarrow \quad a = 1$

$\Rightarrow \quad \dfrac{x}{y} = \dfrac{1}{1}$

$\Rightarrow \quad x : y = 1 : 1$

18. (D) Since, $(2x + 1) : (3x + 13)$ is the sub-duplicate ratio of 9 : 25, therefore

$$\frac{2x + 1}{3x + 13} = \frac{\sqrt{9}}{\sqrt{25}}$$

$$\Rightarrow \quad \frac{2x+1}{3x+13} = \frac{3}{5}$$

$$\Rightarrow \quad 10x + 5 = 9x + 39$$

$$\Rightarrow \quad x = 34$$

19. (C) The required ratio $= \left(\frac{5}{14} \times \frac{7}{15}\right)$

$$= \frac{1}{6} = 1 : 6$$

20. (B) Let the present ages be $5x$ and $7x$ years.

Then, $\frac{5x-18}{7x-18} = \frac{8}{13}$

$$\Rightarrow \quad x = 10$$

Exercise B

1. (A) $\frac{a}{5} = \frac{10}{25}$

$$a = 2$$

2. (A) If X be the required mean proportion, then

$$\frac{(0{\cdot}8)}{X} = \frac{X}{(0{\cdot}2)}$$

$$X^2 = 0{\cdot}8 \times 0{\cdot}2$$

$$X = 0{\cdot}4$$

3. (A) Let a be the third proportion.

Now, $0{\cdot}016 : 0{\cdot}024 :: 0{\cdot}024 : a$

Or, $a = \frac{0{\cdot}024 \times 0{\cdot}024}{0{\cdot}016}$

$$a = 0{\cdot}036$$

4. (C) Let the mean quantity $=$ X

Now, $9 : X :: X : 16$

$$X^2 = 9 \times 16$$

$$X = 12$$

5. (E) Let the fourth proportional be a.

Now, $5 : 6 :: 150 : a$

Or, $\frac{5}{6} = \frac{150}{a}$

$$a = 180$$

6. (B) Since, $\frac{A}{B} = \frac{5}{2}$

and $\frac{B}{C} = \frac{3}{2}$

$$\frac{A}{B} \times \frac{B}{C} = \frac{5}{2} \times \frac{2}{3}$$

$$= \frac{5}{3}$$

$\therefore$ $A : C = 5 : 3$

7. (A) Required part will be the ratio of $\frac{5}{48}$ and $\frac{1}{16}$.

$\therefore$ Part $= \frac{\frac{5}{48}}{\frac{1}{16}} = \frac{5}{3}$

8. (E) Given $\frac{2X+3Y}{3X+5Y} = \frac{7}{8}$

Or, $\frac{2 \times \frac{X}{Y} + 3}{3 \times \frac{X}{Y} + 3} = \frac{7}{8}$

Or, $16 \times \frac{X}{Y} + 24 = 21 \times \frac{X}{Y} + 35$

Or, $5 \times \frac{X}{Y} = -11$

Or, $\frac{X}{Y} = -\frac{11}{5}$

$$X : Y = -11 : 5$$

9. (B) Let ratio between two numbers is 2 : 5.

$\therefore$ Numerator $= 2X$

Denominator $= 5X$

According to question,

$$\frac{2X+2}{5X+2} = \frac{3}{2}$$

Or, $4X + 4 = 15X + 6$

Or, $11X = -2$

$$X = -\frac{2}{11}$$

10. (C) Let number of boys $= 2X$

Number of girls $= 5X$

According to question,

$$2X + 5X = 700$$

$$X = 100$$

Number of girls $= 5 \times 100$

$$= 500$$

11. (D) **First Method :**

Given : $a : b = 2 : 3$

$$b : c = 5 : 4$$

$$c : d = 6 : 7$$

$$\frac{a}{2} = \frac{b}{3}$$

$$\frac{b}{5} = \frac{c}{4}$$

$$\frac{c}{6} = \frac{d}{7}$$

On multiplying increase by 5, 3 & 2.

We get, $\frac{a}{10} = \frac{b}{15}$

$$\frac{b}{15} = \frac{c}{12}$$

$$\frac{c}{12} = \frac{d}{14}$$

$\therefore$ $a : b : c : d = 10 : 15 : 12 : 14$

Second Method :

$$a = \text{Product of numerators}$$

$$a = 2 \times 5 \times 6 = 60$$

$$b = \text{Denominator of Ist ratio} \times \text{product of numerator of rest ratios}$$

$$= 3 \times 5 \times 6 = 90$$

$$c = 4 \times 3 \times 6 = 4 \times 18$$

$$d = 3 \times 4 \times 7$$

$$a : b : c : d = 10 : 15 : 12 : 14$$

12. (A) Let the numbers be 5X, 3X, 4X.

Then, $(5X)^2 + (3X)^2 + (4X)^2 = 800$

$$50X^2 = 800$$

$$X = 4$$

So, the numbers are 12, 16 and 20.

13. (D) In the first mixture

$$\text{Amount of milk} = \frac{2}{5} \times 50 = 20 \text{ liters}$$

$$\text{Amount of water} = 30 \text{ litres}$$

If X litres of water is added to the mixture, then

$$\frac{20}{30 + X} = \frac{1}{3}$$

$$60 = 30 + X$$

$$X = 30 \text{ litres}$$

14. (C) Number of coins $= \dfrac{\text{Amount in rupees}}{\text{Value of coins in rupees}}$

$$\text{Number of one rupee coin} = X$$

$$\text{Number of 50 paisa coin} = \frac{2X}{\frac{1}{2}} = 4X$$

Number of twenty five paisa coin

$$= \frac{5X}{\frac{1}{4}} = 20X$$

Given : $X + 4X + 20X = 1000$

$$25X = 1000$$

$$X = 40$$

$$\text{Number of rupee coin} = 40$$

$$\text{Number of fifty paisa coin} = 160$$

Number of twenty five paisa coin

$$= 800$$

Total value of coins

$$= 1 \times 40 + \frac{1}{2} \times 160 + \frac{1}{4} \times 800$$

$$= 40 + 80 + 200$$

$$= \text{Rs. } 320$$

15. (A) In First vessel :

$$\text{Quantity of water} = \frac{1}{3} \text{ and milk} = \frac{2}{3}$$

In Second vessel :

$$\text{Quantity of water} = \frac{2}{5}$$

and $\text{milk} = \dfrac{3}{5}$

In resultant vessel :

$\frac{1}{5}$ part of mixture of First vessel is taken and

$\frac{4}{5}$ part of mixture of Second vessel is taken

So,the ratio of water to milk in the new vessel

$$\therefore \left(\frac{1}{3} \times \frac{1}{5} + \frac{2}{5} \times \frac{1}{5}\right) : \left(\frac{2}{3} \times \frac{4}{5} + \frac{3}{5} \times \frac{4}{5}\right)$$

$$= \left(\frac{1}{15} + \frac{2}{25}\right) : \left(\frac{8}{15} + \frac{12}{25}\right)$$

$$= \frac{5 + 6}{75} : \frac{40 + 36}{75}$$

$$= \frac{11}{75} : \frac{76}{75}$$

$$= 11 : 76$$

16. (E) Let six years ago their ages were 3X, 2X and X respectively.

Now, according to question,

$$\text{Sum of present ages} = 90$$

Or, $(3X + 6) + (2X + 6) + (X + 6) = 90$

Or, $6X + 18 = 90$

Or, $6X = 72$

$$X = 12$$

$\therefore$ C's present age $= 12 + 6$

$$= 18 \text{ years}$$

17. (B) Let $\dfrac{a}{b + c} = \dfrac{b}{c + a}$

$$= \frac{c}{a + b} = k$$

$\therefore$

$$a = (b + c) \cdot k \qquad \ldots\ldots(1)$$

$$b = (c + a) \cdot k \qquad \ldots\ldots(2)$$

$$c = (a + b) \cdot k \qquad \ldots\ldots(3)$$

Now, Adding equation (1), (2) and (3), we get

$$a + b + c = (2a + 2b + 2c) \cdot k$$

$$k = \frac{1}{2}$$

18. (B) Let the number of coins be X, 3X, 5X and 7X.

Now, X coin of one rupees = X rupees

$$\text{3X coins of fifty paise} = \frac{3X}{2} \text{ rupees}$$

$$\text{5X coins of twenty paise} = \frac{5X}{4} \text{ rupees}$$

$$\text{7X coins of ten paise} = \frac{7X}{10} \text{ rupees}$$

According to question,

$$X + \frac{3X}{2} + \frac{5X}{4} + \frac{7X}{10} = 22{\cdot}25$$

Or, $$\frac{89X}{20} = 22{\cdot}25$$

Or, $$X = 5$$

$\therefore$ Number of rupee = 5

$\therefore$ Number of 50 paise coin = 15

$\therefore$ Number of 25 paise coin = 25

$\therefore$ Number of 10 paise coin = 35

19. (A) Let numerator = X

Denominator = Y

$$\frac{X-5}{Y-5} = \frac{1}{2} \qquad \ldots\ldots(1)$$

and $$\frac{X+2}{Y+2} = \frac{2}{3} \qquad \ldots\ldots(2)$$

From equation (1)

$$2X - Y = 5 \qquad \ldots\ldots(3)$$

From equation (3)—equation (2), we get

$$X = 10 + 2 = 12$$

$$Y = 20 - 5 = 15$$

Fractional number $= \frac{12}{15} = \frac{4}{5}$

20. (A) L.C.M. of 3, 4, 5 and 6 is 60

So, the pens are distributed among A, B, C and D in the ratio

$$\frac{1}{3} \times 60 : \frac{1}{4} \times 60 : \frac{1}{5} \times 60 : \frac{1}{6} \times 60$$

That is 20 : 15 : 12 : 10

Total number of pens

$$= 20{\cdot}X + 15{\cdot}X + 12{\cdot}X + 10{\cdot}X$$

$$= 57{\cdot}X$$

For minimum number of pens,

$$X = 1$$

So, Sita should have at least 57 pens.

21. (C) According to question,

$$\frac{\text{Son's Share}}{\text{Wife's Share}} = \frac{2}{3}$$

$$\frac{\text{Wife's Share}}{\text{Daughter Share}} = \frac{4}{5}$$

Now, $$\frac{\text{Son's Share}}{2} = \frac{\text{Wife's Share}}{3}$$

$$\frac{\text{Wife's Share}}{4} = \frac{\text{Daughter Share}}{5}$$

$$\frac{\text{Son's Share}}{8} = \frac{\text{Wife's Share}}{12}$$

$$= \frac{\text{Daughter Share}}{15}$$

Now, Son's share = $8{\cdot}X$

Wife's share = $12{\cdot}X$

Daughter's share = $15{\cdot}X$

Now according to question,

$$15{\cdot}X - 8{\cdot}X = 49{,}000$$

Or, $$7{\cdot}X = 49{,}000$$

$$X = 7{,}000$$

Total worth of property

$$= 8X + 12X + 15X$$

$$= 35X$$

$$= 35 \times 7000 = 2{,}45{,}000$$

22. (A) Always remember, when a number is to be divided in the proportion assume the number as First divided in X : 1.

First part $= \frac{X}{X+1}$

Second part $= \frac{1}{X+1}$

Given : $$\frac{2}{5}\left(\frac{X}{X+1}\right) + \frac{1}{3}\left(\frac{1}{X+1}\right) = \frac{1}{4}$$

Or, $$\frac{6{\cdot}X + 5}{15(X+1)} = \frac{1}{4}$$

Or, $$24{\cdot}X + 20 = 15{\cdot}X + 15$$

Or, $$24{\cdot}X - 15{\cdot}X = 15 - 20$$

Or, $$9{\cdot}X = -5$$

$$X = -\frac{5}{9}$$

Required proportion = X : 1

= 5 : 9

23. (A) Number of passengers care $2{\cdot}X$, $3{\cdot}X$ and $5{\cdot}X$

Rate = X, $2X{\cdot}4X$

Since, income = Number of passenger × rate

= income in the ratio

= 2 : 6 : 20

Income from A. C. sleeper class

$$= \frac{2}{28} \times 56000$$

= Rs. 4,000

24. (D) Income of Ram = $3{\cdot}X$

Income of Raju = $2{\cdot}X$

Expense of Ram = $5{\cdot}Y$

Expense of Raju = $3{\cdot}Y$

Now, saving amount of Ram = $3X - 5Y$

Saving amount of Raju = $2X - 3Y$

$$\Rightarrow \quad (3X - 5Y) + (2X - 3Y) = 2000$$

$$\Rightarrow \quad 5X - 8Y = 2000 \quad \ldots\ldots(1)$$

Since saving of Ram = Saving of Raju

$$3X - 5Y = 2X - 3Y$$

$$X = 2Y \quad \ldots\ldots(2)$$

Putting the value of X in equation (1), we get

$$5 \times 2Y - 8Y = 2000$$

$$2Y = 2000$$

$$Y = 1000$$

So, Ram's income $= 3 \cdot X$

$= \text{Rs. } 6{,}000$

25. (A) Let the saving amount of Ram $= 14 \cdot X$

Let the saving amount of Shyam $= 21 \cdot X = 1260$

Let the saving amount of Sohan $= 15 \cdot X = 900$

Since, Rs. 30 is saved when income

$= 100$

$\therefore$ Rs. $14 \cdot X$ is saved when income $= \dfrac{100}{30} \times 14 \cdot X$

Shyam's income $= 100 \times \dfrac{21 \cdot X}{40}$

Raju's income $= 100 \times \dfrac{15 \cdot X}{35}$

Now, $100 \times \left(\dfrac{14 \cdot X}{30} + \dfrac{21 \cdot X}{40} + \dfrac{15 \cdot X}{35} \right)$

$= 11{,}950$

or $\left(\dfrac{14}{5 \times 6} + \dfrac{21}{5 \times 8} + \dfrac{15}{5 \times 7} \right) 100X$

$$= \left(\frac{14}{6} + \frac{21}{8} + \frac{15}{7} \right) \frac{1}{5} \times 100X$$

$$= \left(\frac{7}{3} + \frac{21}{8} + \frac{15}{7} \right) \times 20X$$

$$= \left(\frac{392 + 441 + 360}{168} \right)$$

$$= \frac{1193 \times 20}{168} X$$

$$= 11950$$

Or, $X = \dfrac{11950 \times 168}{1193 \times 20}$

$X \approx 84$

Salary or income of Shyam

$$\approx \frac{100}{40} \times 21 \times 84$$

$\approx$ Rs. 4,410

$=$ Rs. 4,425

8 Mixture or Alligations

(A) Mixture—A mixture of two or more different ingredients is formed when they mix in any ratio.

Alligation rule—Alligation means "linking".

When different quantities of some or different ingredients, of different cost are mixed together to produce a mixture of a mean cost, the ratio of their quantities are inversely proportional to the difference in their cost from the mean cost.

$$\frac{\text{Quantity of smaller cost ingredient}}{\text{Quantity of larger cost ingredient}} = \frac{\text{Larger cost} - \text{Mean cost}}{\text{Mean cost} - \text{Smaller cost}}$$

Now, $\qquad \dfrac{m - C}{d - m} = \dfrac{x_2}{x_1}$

C.P. of a unit Quantity of cheaper (C)

C.P. of unit Quantity of dearer (d)

Mean price $= m$

(B) Price/Value of the mixture—

Case I : When two ingredients X and Y are mixed.

A_1 and A_2 be the amounts of ingredients X and Y respectively.

C_1 and C_2 be their cost price.

Now,Cost price mixture is given by

$$C_m = \frac{C_1A_1 + C_2A_2}{A_1 + A_2}$$

Case II : When more than two ingredients are mixed.

Then , $\quad C_m = \dfrac{C_1A_1 + C_2A_2 + C_3A_3 + \ldots\ldots + C_nA_n}{A_1 + A_2 + A_3 + \ldots\ldots + A_n}$

Illustration 1.

In what proportion should one variety of oil at Rs. 8/kg be mixed with another of Rs. 10 /kg to get a mixture worth Rs. 9 /kg ?

Solution :

Using formula :

$$\text{Ratio} = \frac{10 - 9}{9 - 8} = \frac{1}{1} = 1 : 1$$

$$\text{Ratio} = \frac{C_2 - C_m}{C_m - C_1} = \frac{10 - 9}{9 - 8} = \frac{1}{1} = 1 : 1$$

Illustration 2.

Alcohol cost Rs. 3·5 /litre and kerosene oil cost Rs. 2·5 / litre. In what proportion should these be mixed, so that the resulting mixture may be 2·75/litre ?

Solution :

Using formula :

$$\text{Ratio} = \frac{C_2 - C_m}{C_m - C_1}$$
$$= \frac{0{\cdot}25}{0{\cdot}75} = \frac{1}{3}$$

(C) When two mixtures of same ingredients mixed in different ratios—

Mixture has ingredients (A, B) in X : Y

Mixture has ingredients (A, B) in $a : b$

The First mixture ratio of quantities $= \dfrac{X}{Y}$

Second mixture has ratio of quantities $= \dfrac{a}{b}$

Now, M unit of mixture first and N unit of second mixture are mixed to form a resultant mixture with ingredients (A and B) in $A_A : A_B$.

Case I : When A_A and A_B are to be found out—

$$\frac{\text{Quantity of ingredient A}}{\text{Quantity of ingredient B}} = \frac{A_A}{A_B}$$

$$= \frac{M \times \left(\dfrac{x}{X + Y}\right) + N \times \left(\dfrac{a}{a + b}\right)}{M \times \left(\dfrac{Y}{X + Y}\right) + N \times \left(\dfrac{a}{a + b}\right)}$$

Amount of ingredient A in the resultant mixture

$$= \frac{A_A}{A_A + A_B} \times (M + N)$$

Amount of ingredient B in the resultant mixture

$$= \frac{A_B}{A_A + A_B} \times (M + N)$$

Illustration 3.

Ramu lent out Rs. 12,00 in two parts one at 8% and the other at 10% interest. The yearly comes out to be 9·5%. Find the amount lent in two parts ?

Solution :

So, $\quad$ Ratio of quantity $= \dfrac{0{\cdot}5}{1{\cdot}5} = \dfrac{1}{3}$

$\therefore$ Quantity of money lent at 8%

$$= \frac{1}{4} \times 1200 = \text{Rs. } 300$$

$\therefore$ Quantity of money lent at 10%

$$= \frac{3}{4} \times 1200 = \text{Rs. } 900$$

Case II : When M and N are to be found

Quantity of ingredient A in mixture-I = $\frac{X}{X+y}$

Quantity of ingredient A in mixture-II = $\frac{a}{a+b}$

$$\text{In the resultant mixture} = \frac{A_A}{A_A + A_B}$$

$$\text{Now,} \quad \frac{\text{Quantity of mixture-I}}{\text{Quantity of mixture-II}} = \frac{C}{D}$$

$$= \frac{\frac{a}{a+b} - \frac{A_A}{A_A + A_B}}{\frac{A_A}{A_A + A_B} - \frac{X}{X+Y}}$$

∴ Amount of mixture-I in resultant mixture

$$= \left(\frac{C}{C+D}\right) \times (M+N)$$

Amount of mixture-II in resultant mixture

$$= \left(\frac{D}{C+D}\right) \times (M+N)$$

Illustration 4.

Three litres of water is added to a certain quantity of pure milk costing Rs. 6 /litre. If by selling the mixture at the same price as before, a profit of 20% is made. What is the amount of pure milk in the mixture ?

Solution :

Since, selling price of mixture = cost price of mixture

$$\text{Or,} \quad \frac{120}{100} \times \text{C.P. of mixture} = 6$$

$$\text{Or,} \quad \text{C.P. of mixture} = \frac{6 \times 10}{12} = 5$$

Ratio milk and water is given by

$$\frac{\text{Milk}}{\text{Water}} = \frac{5-0}{6-5} = 5$$

$$\text{Since,} \quad \text{C.P. of water} = 0$$

∴ For 3 litres of water.

$$\text{Quantity of milk} = 5 \times 3 = 15 \text{ litres.}$$

Illustration 5.

How many kg of sugar at 34 paise/kg must Raju mix with 25 kg of salt at 20 paise/kg. So, that he may, on selling the mixture at 40 paise/kg. gain 20% on the outlay ?

Solution :

Using formula :

$$\text{Cost price of mixture} = 40 \times \frac{100}{120} = 32 \text{ paise}$$

Using formula :

$$\text{Ratio of quantity} = \frac{A_{34}}{A_{20}} = \frac{32-20}{34-32} = \frac{12}{2} = 6$$

$$\text{So,} \quad A_{34} = 6 \times A_{20}$$

$$= 6 \times 25 = 150 \text{ kg.}$$

Illustration 6.

Two equal glasses filled with mixtures of alcohol and water in the proportion of 2 : 3 and 2 : 1 respectively were emptied into a third glass. What is the proportion of alcohol and water in the third glass ?

Solution :

Using formula :

$$\frac{A_A}{A_B} = \frac{M \times \left(\frac{X}{X+Y}\right) + N \times \left(\frac{a}{a+b}\right)}{M \times \left(\frac{Y}{X+Y}\right) + N \times \left(\frac{b}{a+b}\right)}$$

When since glasses are equal :

$$\therefore \quad M = N$$

$$\text{Given} \quad \frac{X}{Y} = \frac{2}{3}$$

$$\text{and} \quad \frac{a}{b} = \frac{2}{1}$$

$$\therefore \quad \frac{A_A}{A_B} = \frac{\frac{2}{5} + \frac{2}{3}}{\frac{3}{5} + \frac{1}{3}} = \frac{16}{10} = \frac{8}{5}$$

So, in third glass, alcohol and water are in proportion of 8 : 5.

Removal and Replacement by Equal Amount

(i) Suppose a container contains M unit of mixture of ‘A’ and ‘B’. From this R unit of mixture is taken out and replaced by an equal amount of ingredient B only this process is repeated ‘n’ times, then after n operations.

$$\therefore \quad \frac{\text{Amount of ‘A’ Left}}{\text{Amount of ‘A’ originally present}} = \left(1 - \frac{R}{M}\right)^n$$

$$\text{Amount of B left} = M - \text{Amount of A left}$$

(ii) Consider a container containing only ingredient ‘A’ of X_0 unit. From this X_r unit is taken out and replaced by and equal amount of ingredient B. This process is repeated n times, then after n operations.

$$\frac{\text{Amount of ‘A’ Left}}{\text{Amount of ‘A’ originally present}} = \left(1 - \frac{X_r}{X_0}\right)^n$$

$$\text{So,} \quad \text{Amount of ‘A’ left} = X_0 \times \left(1 - \frac{X_r}{X_0}\right)^n$$

$$\text{So,} \quad \frac{\text{Amount of A left}}{\text{Amount of B left}} = \frac{\left(1 - \frac{X_r}{X_0}\right)^n}{1 - \left(1 - \frac{X_r}{X_0}\right)^n}$$

Illustration 7.

In two alloys, copper and zinc are related in the ratios 2 : 1 and 2 : 3 respectively. After alloying together 12 kg of the first alloy, 10 kg of the second and a certain amount of zinc and alloy is obtained in which copper and

zinc are in equal proportions. Find the weight of pure zinc added ?

Solution :

Using formula :

$$\frac{\text{Amount of Copper}}{\text{Amount of Zinc}} = \frac{12 \times \frac{2}{3} + 10 \times \frac{2}{5}}{12 \times \frac{1}{3} + 10 \times \frac{3}{5} + x}$$

$$= \frac{8+4}{4+6+x}$$

Where x is the amount of pure zinc.

So, Given $\frac{\text{Amount of Copper}}{\text{Amount of Zinc}} = \frac{1}{1}$

Or $\frac{10+x}{12} = 1$

Or, $x = 2$ kg.

Illustration 8.

In two alloys, the ratio of copper and zinc are 5 : 4 and 1 : 2. How many kg of the first alloy and of the second alloy should be melted together to obtain 24 kg of a new alloy with equal contents of copper and zinc ?

Solution :

Here, two alloys having same ingredients are mixed to obtain a new alloy.

Amount of each alloy is to be found out individually.

So, In First mixture copper $= \frac{X}{X+Y} = \frac{5}{9}$

Copper in Second mixture $= \frac{a}{a+b} = \frac{1}{3}$

In the resultant mixture $= \frac{A_A}{A_A + A_B} = \text{copper}$

$$= \frac{1}{2}$$

So, now $\frac{C}{D} = \frac{\frac{X}{X+Y} - \frac{A_A}{A_A+A_B}}{\frac{A_A}{A_A+A_B} - \frac{a}{a+b}}$

$$= \frac{\frac{5}{9} - \frac{1}{2}}{\frac{1}{2} - \frac{1}{3}}$$

$$\frac{C}{D} = \frac{3}{9} = \frac{1}{3}$$

So, Amount of First alloy $= \frac{C}{C+D} \times 24$

$$= \frac{1}{4} \times 24 = 6 \text{ kg}$$

Amount of Second alloy $= 24 - 6 = 18$ kg.

Illustration 9.

Six kilograms of sugar at Rs. 15 /kg and 5 kgs of sugar at Rs. 20 /kg are mixed together and the mixture is sold at a 11% profit. What is the selling price per kg of the mixture ?

Solution :

Cost price per kg of mixture is given by

$$= \frac{6 \times 15 + 5 \times 20}{6+5}$$

$$= \frac{90+100}{11} = \frac{190}{11}$$

Now, Selling price / kg of mixture

$$= \frac{110}{100} \times \frac{190}{11} = \text{Rs. } 19/\text{kg.}$$

Illustration 10.

Two liquids are mixed in the proportion of 2 : 1 and the mixture is sold at Rs. 12 per litre at a 20% profit. If the First liquid costs Rs. 2 more per litre than the second. What does it cost/litre ?

Solution :

Since, mixture is sold at 20% profit.

So, cost price of the mixture $= \frac{100}{120} \times 12 = \text{Rs. } 10$

Let the price of First liquid $=$ Rs. x /litre

The cost price of Second liquid $=$ Rs. $(x+2)$/litre

Now, Cost price of mixture $= \frac{2x + 1\,(x+2)}{3}$

$$10 = \frac{3x+2}{3}$$

$$28 = 3x$$

$$x = \text{Rs. } \frac{28}{3}/\text{litre.}$$

Illustration 11.

Sea water contains 10% salt by water. How many litre fresh water must be added to 80 litre of sea water for the content of salt in solution to be made 4% ?

Solution :

Amount of salt present in 80 litre of sea water

$$= 80 \times \frac{10}{100} = 8 \text{ kg}$$

After adding x litre of fresh water mixture $= 80 + x$

Now, $\frac{8}{80+x} = \frac{4}{100}$

Or, $200 = 80 + x$

Or, $x = 120$ kg.

Illustration 12.

A sum of Rs. 6.25 is made up of so coins which are either 10 paise or 5 paise coins. How many of it are of 5 paise coins ?

Solution :

Let, In the mixture :

$$\text{Number of coins} = m$$

$$\text{Price of mixture} = \frac{6{\cdot}25}{80} = \frac{625}{80} \text{ paise}$$

$$\text{Now, } \frac{5-\text{paise coins}}{10-\text{paise coins}} = \frac{10-\frac{6{\cdot}25}{80}}{\frac{6{\cdot}25}{80}-5}$$

$$= \frac{800-625}{625-400} = \frac{175}{225} = \frac{7}{9}.$$

Illustration 13.

A mixture contain milk and water in ratio of 4 : X. When 30 litres of the mixture and 6 litre of water are mixed the ratio of milk and water becomes 2 : 1. Find the values of x ?

Solution :

$$\text{Amount of mixture} = 36 \text{ litres}$$

$$\text{Amount of milk} = \frac{2}{3} \times 36 = 24 \text{ litre}$$

$$\text{Amount of water} = \frac{1}{3} \times 36 = 12 \text{ litre}$$

On mixing,

$$\frac{\text{Amount of milk}}{\text{Amount of water}} = \frac{4}{X}$$

Or, $$\frac{24}{6} = \frac{4}{X}$$

Or, $$X \times 6 = 6$$

Or, $$X = 1 \text{ litre.}$$

Illustration 14.

A dishonest milkman fills up his bucket which is $\frac{2}{3}$th full of milk, with water. He again removes 2 litres of his mixture from the bucket and adds an equal quantity of water. If milk is now 60% of the mixture. What is the capacity of the bucket in litres ?

Solution :

Let Y be the capacity of bucket.

$$Y = \text{original amount of mixture}$$

Amount of mixture removed = 2 litre

So, $$\text{Milk} = (Y-2) \times \frac{2}{3}$$

$$\text{Water} = \frac{1}{3} \times (Y-2) + 2$$

Now, $$\frac{\frac{2}{3} \times (Y-2)}{\frac{1}{3} \times (Y-2) + 2} = \frac{60}{100} = \frac{3}{5}$$

Or, $$\frac{2 \times (Y-2)}{Y-2+6} = \frac{3}{5}$$

Or, $$(2Y-4) \times 5 = 3 \times (Y+4)$$

Or, $$10Y - 20 = 3Y + 12$$

$$7Y = 32$$

$$Y = \frac{32}{7}$$

Illustration 15.

Three equal glasses are filled with a mixture of spirit and water. The proportion of spirit to water in each glass is 1 : 2, 2 : 3 and 3 : 1 respectively. The contents of three glasses are emptied into a single vessel. What is the proportion of spirit and water in the vessel ?

Solution :

Now, $$\frac{\text{Amount of Spirit}}{\text{Amount of Water}} = \frac{\frac{1}{3}+\frac{2}{5}+\frac{3}{4}}{\frac{2}{3}+\frac{3}{5}+\frac{1}{4}}$$

$$= \frac{20+24+45}{40+36+15} = \frac{89}{91}$$

Exercise A

1. In what proportion must tea at Rs. 72/kg be mixed with tea at Rs. 90/kg in order to obtain the mixture worth Rs. 85/kg ?

   (A) $\frac{5}{18}$ (B) $\frac{7}{3}$

   (C) $\frac{3}{7}$ (D) $\frac{13}{5}$

   (E) $\frac{5}{13}$

2. In what proportion water is mixed with pure milk in order to make a profit of 25% by selling it at cost price?

   (A) 3 : 4 (B) 4 : 5

   (C) 1 : 6 (d) 1 : 5

   (E) 1 : 4

3. Two vessels A and B contain milk and water in the ratio 7: 5 and 17 : 4 respectively. In what ratio mixtures from two vessels should be mixed to get a new mixture containing milk and water in the ratio 3 : 2 ?

   (A) $\frac{22}{105}$ (B) $\frac{7}{12}$

   (C) $\frac{88}{7}$ (D) $\frac{17}{21}$

   (E) $\frac{3}{5}$

4. There are n vessels of sizes, C_1, C_2, C_3, ……… C_n containing mixtures of milk and water in the ratio $a_1 : b_1$, $a_2 : b_2$ ………… $a_n : b_n$ respectively. The contents are emptied into a single large vessel. Find the ratio of milk to water in the resulting mixture ?

5. Three glasses of sizes 3 litres, 4 litres and 5 litres contain mixture of milk and water in the ratio of 2 : 3, 3 : 7 and 4 : 11 respectively. The contents of all the

three glasses are poured into single vessel. Find the ratio of milk to water in the resultant mixture ?

(A) 1 : 1 (B) 14 : 31
(C) 56 : 15 (D) 124 : 15
(E) 9 : 21

6. Four vessels of equal sizes contain mixture of sprit and water. The concentration of spirit in four vessel are 50%, 60%, 75% and 90% respectively. If all the four mixtures are mixed. Find in the resultant mixture the ratio of spirit to water ?

(A) 11 : 5 (B) 13 : 16
(C) 15 : 14 (D) 5 : 4
(E) 25 : 12

7. Three glasses of capacity 2 litres, 3 litres and 5 litres contain mixture of milk and water with concentration 60%, 80% and 90% respectively. The content of three glasses isemptied into a large vessel. Find the milk concentration and ratio of milk to water in the resultant mixture ?

(A) 81 : 19 (B) 1 : 1
(C) 45 : 14 (D) 17 : 19
(E) 7 : 9

8. 80 litres of milk has 75% milk concentration. How much water should be added to make its concentration 60% ?

(A) 14 litres (B) 21 litres
(C) 24 litres (D) 18 litres
(E) 16 litres

9. 42 litre of a mixture has wine and water in the ratio 4 : 5. How much water must be added to get wine to water of 5 : 7 in the resultant mixture ?

(A) 84 litres (B) 62 litres
(C) 57 litres (D) 46 litres
(E) 74 litres

10. In what proportion may three kinds to tea prices @ Rs. 80, Rs. 70 and Rs. 50/kg be mixed to produce a mixture worth Rs. 65 /kg ?

(a) 3 : 2 : 5 (B) 2 : 3 : 5
(C) 1 : 2 : 3 (D) 3 : 3 : 4
(E) 1 : 1 : 1

11. In what proportion may three kinds of wheat bought @ Rs. 8, Rs. 10 and Rs. 14 be mixed to produce a mixture. Which would earn 25% on selling it at Rs. 15/kg. ?

(A) 1 : 1 : 3 (B) 2 : 2 : 3
(C) 3 : 1 : 3 (D) 1 : 2 : 3
(E) 3 : 3 : 4

12. Find the proportion in which 4 types of tea priced @ Rs. 20, Rs. 30, Rs. 50. Rs. 80 be mixed so as to obtain a mixture worth Rs. 70/kg ?

(A) 1 : 4 : 4 : 5 (B) 1 : 2 : 4 : 4
(C) 3 : 2 : 4 : 5 (D) 5 : 2 : 4 : 5
(E) 1 : 2 : 4 : 5

13. Ramu has 60 kgs of rice. He sells a part of it at 20% profit and the rest at 30% profit. If he gain 25% on the whole, find the quantity of each part ?

(A) Quantity sold at 30% profit = 20 kg; Quantity sold at 20% profit = 40 kg
(B) Quantity sold at 30% profit = 10 kg; Quantity sold at 20% profit = 50 kg
(C) Quantity sold at 30% profit = 25 kg; Quantity sold at 20% profit = 35 kg
(D) Quantity sold at 30% profit = 30 kg; Quantity sold at 20% profit = 30 kg
(E) Quantity sold at 30% profit = 35 kg; Quantity sold at 20% profit = 25 kg

14. Rajesh bought two tables for Rs. 2000. He sells one at 10% loss and the other at 10% profit and thus on the whole. He had no gain no lose in whole transaction. Find the cost price of each ?

(A) Rs. 1000 for Loss Rs. 1000 for gain
(B) Rs. 950 for Loss Rs. 1050 for gain
(C) Rs. 800 for Loss Rs. 1200 for gain
(D) Rs. 1200 for Loss Rs. 800 for gain
(E) Cannot be determine

15. Sita bought a certain quantity of sugar for Rs. 100. She sells one fourth of it at 5% loss. At what per cent profit should he sell the remainder stock so as to make an overall profit of 20% ?

(A) $\frac{85}{3}\%$ profit (B) 85% profit
(C) 65% profit (D) $\frac{65}{3}\%$ profit
(E) $\frac{35}{3}\%$ profit

16. In a courtyard there are many chickens and goats. If heads are counted it comes to 80 but when legs are counted it comes 220. Find the number of chickens and goats in the courtyard ?

(A) 45 : 35 (B) 55 : 25
(C) 50 : 30 (D) 60 : 20
(E) 40 : 40

Exercise B

1. Shyam buys spirit at Rs. 75 /litre adds water to it and then sells it at 91 /litre. What is the ratio of spirit to water if his profit in deal is 30%?

(A) 75 : 91 (B) 91 : 130
(C) 71 : 20 (D) 75 : 16
(E) 70 : 30

2. There are two containers; the first contains 600 ml. of alcohol, while the second contains 600 ml. of water. Three cups of alcohol from the first container is taken out and is mixed well in second container. Then three cups of this mixture is taken out and is mixed first container. Let X denote the proportion of water in the first container and Y denote the proportion of alcohol in the second container. Find the relation between X and Y ?

   [Capacity of each cup 100 mL.]

3. Sita has Rs. 2000. She invests a part of it at 2% per annum and the remainder at 5% per annum simple interest. Her total income in 2 years is Rs. 500. Find the sum invested at higher rate of interest ?

   (A) Rs. 1200 (B) Rs. 1166·67
   (C) Rs. 833·33 (D) Rs. 900
   (E) Rs. 1400

4. Raju covers a distance of 200 km. In 20 hours partly by walking at 10 km/hr and rest by running at 22 km/hr. Find the distance covered by higher speed ?

   (A) 8·67 km (B) 115 km
   (C) 19·67 km (D) 86·67
   (E) 191·67 km

5. The expenditure and saving of an employee are in the ratio 3 : 2. His income increases by 15% but at the same time his expenditure also increases by 20%. Find increase or decrease in his savings ?

   (A) Decrease 7·5% (B) Decrease 5%
   (C) Remain same (D) Increase 7·5%
   (E) Increase 5%

6. A sum of Rs. 250 is divided among 10 students. Each boy gets Rs. 20 where as a girl gets Rs. 30. Calculate the number of boys in class ?

   (A) 5 (B) 6
   (C) 4 (D) 3
   (E) 7

7. A sum of Rs. 21 is made up of 60 coins which consist of either 50 paise or 25 paise. How many are there of 25 paise coins ?

   (A) 24 (B) 36
   (C) 40 (D) 20
   (E) 30

8. A vessel contains 'a' litres of liquid A and 'b' litres be with drawn and replaced by liquid B, then 'b' litres of mixture be with drawn and replaced by liquid B and the operation is repeated 'n' times in all. Find the ratio of liquid A left after nth operation to the whole quantity of liquid A initially present in the vessel ?

   (A) $\left[\frac{b}{a}\right]^n$ (B) $\left[\frac{b^n}{a}\right]$
   (C) $\frac{a}{nb}$ (D) $\left(1-\frac{b}{a}\right)^n$
   (E) Cannot be determine

9. From a cask of wine containing 20 litres, 5 litres are with drawn and the cask is refilled with water. The process is repeated a second and then a third time. Find the ratio of wine to water in the resulting mixture?

   (A) 1 : 5 (B) 13 : 15
   (C) 64 : 61 (D) 61 : 64
   (E) 4 : 5

10. A vessel contains mixture of liquids X and Y in the ratio 3 : 2. When 20 litres of the mixture is taken out and replaced by 20 litres of liquid Y, the ratio changes to 1 : 4. How many litres of liquid X was there initially present in the vessel ?

   (A) 8 litres (B) 28 litres
   (C) 18 litres (D) 22 litres
   (E) 12 litres

11. A piece of an alloy of two metals (X and Y) weighs 12 gm and costs Rs. 180. If the weights of two metals be interchanged the new alloy would be worth Rs. 120. If the price of metal X is Rs. 15 /gm. Find the weight of the other metal in the original piece of alloy ?

   (A) 4·8 gm (B) 1·2 gm
   (C) 4 gm (D) 5 gm
   (E) 2 gm

12. A heard of 2 legged and 4 legged animals give a head count as H. When legs are counted, it comes to L numbers. Find the number of 4 legged animals in terms of H and L ?

   (A) $\frac{L}{2}+H$ (B) $\frac{L+H}{2}$
   (C) $L-H$ (D) $\frac{L-H}{2}$
   (E) $\frac{L}{2}-H$

13. There were two different iron alloys of total weight 50 kg. The first contains 40% less iron than the second. Determine the percentage of iron in the first alloys, if it is known that there were 6 kg of iron in the first alloy and 12 kg in the second ?

   (A) 15% (B) 35%
   (C) 20% (D) 55%
   (E) 60%

14. Four glasses of sizes 3 litres, 4 litres, 6 litres and 7 litres contain mixture of milk and water in the ratio of 2 : 1, 5 : 3, 6 : 3 and 9 : 5 respectively. Find the ratio of milk to water if the contents of all the four glasses are poured into one large vessel ?

(A) 2 : 5 (B) 8 : 7
(C) 13 : 7 (d) 5 : 7
(E) 1 : 7

15. An insert has 6 legs while another insect has 4 legs. In a group with both types present the total number of heads is 80 and total number of legs is 420. Find the number of 4 legged insects ?

(A) 10 (B) 30
(C) 20 (D) 40
(E) 23

Answers with Hints

Exercise A

1. (E) Using allegation rule :

$$\frac{\text{Quantity of cheaper tea}}{\text{Quantity of dearer tea}} = \frac{90-85}{85-72} = \frac{5}{13}$$

2. (D) Let cost price of pure milk be Re. 1 /litre

Now, S.P. of mixture = Rs. 1 /litre

Profit = 25%

So, C.P. of 1 /litre of mixture

$$= \frac{100}{125} \times 1 = \frac{4}{5}$$

We assume the C.P. of 1 litre of water is zero.

$$\frac{\text{Quantity of water}}{\text{Quantity of milk}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}$$

Ratio of water to pure milk in mixture 1 : 5.

3. (C) The amount of milk in First mixture $= \frac{7}{12}$

The amount of milk in Second mixture $= \frac{17}{21}$

In combination, the amount of milk $= \frac{3}{5}$

Now, Using allegation rule :

$$\text{Required ratio} = \frac{22}{105} : \frac{1}{60}$$

$$= \frac{22 \times 60}{105} = \frac{88}{7}$$

4. Amount of milk in different vessels

$$= \frac{a_1c_1}{a_1+b_1}, \frac{a_2c_2}{a_2+b_2}, \ldots\ldots, \frac{a_nc_n}{a_n+b_n}$$

Amount of water in different vessels

$$= \frac{b_1c_1}{a_1+b_1}, \frac{b_2c_2}{a_2+b_2}, \ldots\ldots, \frac{b_nc_n}{a_n+b_n}$$

In the resulting mixture :

Milk : Water

$$= \left[\frac{a_1c_1}{a_1+b_1} + \frac{a_2c_2}{a_2+b_2} + \ldots\ldots + \frac{a_nc_n}{a_n+b_n}\right] : \left[\frac{b_1c_1}{a_1+b_1} + \frac{b_2c_2}{a_2+b_2} + \ldots\ldots + \frac{b_nc_n}{a_n+b_n}\right]$$

5. (B) Using previous formula :

Milk : Water

$$= \left[\frac{2\times3}{2+5} + \frac{4\times3}{3+7} + \frac{5\times4}{4+11}\right] : \left[\frac{3\times3}{3+2} + \frac{7\times4}{3+7} + \frac{11\times5}{4+11}\right]$$

$$= \left(\frac{6}{5} + \frac{12}{10} + \frac{20}{15}\right) : \left(\frac{9}{5} + \frac{28}{10} + \frac{55}{15}\right)$$

$$= \left(\frac{56}{15}\right) : \left(\frac{124}{15}\right)$$

$= 56 : 124 = 14 : 31.$

6. (A) Here, the given per cent figures indicate the fraction of spirit in the mixture.

Concentration of water in four vessel are 50%, 40%, 25% and 10% respectively.

In the resultant mixture :

$$\text{Spirit : Water} = (0{\cdot}5 + 0{\cdot}6 + 0{\cdot}75 + 0{\cdot}9) : (0{\cdot}5 + 0{\cdot}4 + 0{\cdot}25 + 0{\cdot}1)$$

$= 2{\cdot}75 : 1{\cdot}25 = 1{\cdot}1 : 0{\cdot}05$

$= 11 : 5$

7. (A) The total amount of milk

$= 2 \times 0{\cdot}6 + 3 \times 0{\cdot}8 + 5 \times 0{\cdot}9$

$= 1{\cdot}2 + 2{\cdot}4 + 4{\cdot}5 = 8{\cdot}1$ litres

Milk concentration in the resultant mixture

$$= \frac{8{\cdot}1}{2+3+5} \times 100$$

$$= \frac{8{\cdot}1}{10} \times 100 = 81\%$$

Water concentration in the resultant mixture

$= 100 - 81 = 19$

$\therefore$ Milk : Water = 81 : 19

8. (E) Given milk has 75% concentration water which is to be added has 0% milk concentration.

$\therefore$ Find concentration of solution is 60%

By allegation rule :

Water should be added to the given milk in the ratio

15 : 60 = 1 : 4

$\therefore$ Quantity of water to be added

$$= \frac{1}{5} \times 80 = 16 \text{ litres.}$$

9. (B) Fraction of water in the mixture = $\frac{4}{9}$

For water to be added fraction = 1

Fraction of water in the resultant mixture = $\frac{5}{12}$

So, water must be added in the ratio

$$= \frac{31}{36} : \frac{7}{12} = \frac{31}{21}$$

So, Quantity of water to be added

$$= \frac{31}{21} \times 42 = 62 \text{ litres.}$$

10. (D) We write the prices according order as

Make pairs by choosing one from each side of the mean price and apply allegation rule then add the quantity obtained under each price.

This will give the ratio in which the ingredient should be mixed.

Required ratio = 15 : 15 : 20

= 3 : 3 : 4.

11. (C) S.P. of mixture = Rs. 15 / kg

Profit = 25%

C.P. of mixture = $15 \times \frac{100}{125}$ = Rs. 12/kg

Now, Required ratio = 6 : 2 : 6

= 3 : 1 : 3.

12. (E) Using allegation rule :

Required ratio = 10 : 20 : 40 : 50

= 1 : 2 : 4 : 5.

13. (D) **First Method :**

Let X kg of rice is sold at a profit of 20%.

Now, (60 – X) kg rice is sold at 30% profit.

Let the cost price of rice be Rs. 100 / kg.

Total C.P. = Rs. 60×100 = Rs. 6000

Gains 25% on the whole, therefore,

$$\text{Total S.P.} = \frac{125}{100} \times 6000 = 7500$$

Also, Total S.P. = $120 \times X + 130 \times (60 - X)$

$7500 = 7800 - 10X$

$X = \frac{300}{10} = 30$

X = 30 kg.

So, he sells 30 kg at 20% profit and rest 30 kg at 30% profit.

Second Method :

Ratio = 5 : 5 = 1 : 1

Quantity sold at 20% profit = $\frac{1}{2} \times 60 = 30$ kg

Quantity sold at 30% profit = 30 kg

14. (A) C.P. of table sold at 10% loss

$$= \frac{1}{2} \times 2000 = \text{Rs. } 1000$$

C.P. of the table sold at 10% gain

$$= \frac{1}{2} \times 2000 = \text{Rs. } 1000$$

15. (A) Let the remainder stock be sold at X% profit.

Now, $\frac{X - 20}{25} = \frac{\frac{1}{4}}{\frac{3}{4}}$

Or, $X - 20 = 25 \times \frac{1}{3}$

Or, $3X = 25 + 60 = 85$

Or, $X = \frac{85}{3}$

$X = \frac{85}{3}$% profit.

16. (C) Let the number of goats be X.

∴ Number of chickens = 80 – X

Total legs = $2 \times (80 - X) + 4X$

Now, 2X + 160 = 220

2X = 60

X = 30

So, Number of goats = 30

Number of chickens = 50

Exercise B

1. (C) S.P. of the mixture = $\frac{91}{130} \times 100$ = Rs. 70

Assuming cost of water is Rs. 0.

Now, using allegation rule, we get Required ratio of spirit is 71 : 20.

2. Given Capacity of each cup be 100 mL.

After first operation, first container will have 300 mL of alcohol and second container will have 300 mL alcohol and 600 mL water.

Ratio water to alcohol in the second container

= 9 : 3 = 3 : 1

After second operation, the quantity of water and alcohol left would be $300 \times \frac{3}{4} = 225$ mL

and $300 \times \frac{1}{4} = 75$ mL

Now, Quantity of water in first container = 225 mL

Alcohol = 75 + 300 = 375 mL

So, ratio of water and alcohol

$$= \frac{225}{375} = \frac{3}{5}$$

So, clearly there are different ratios

Second ratio > First ratio

Y > X

3. (B) Average rate of interest

$$= \frac{500 \times 100}{2000 \times 2} = \frac{50}{4}\%$$

$$= \frac{25}{2}\% = 12{\cdot}5\%.$$

Investment at 2% per annum

$$= \frac{7{\cdot}5}{18} \times 2000 = \frac{2{\cdot}5}{6} \times 2000$$

$$= \frac{25}{60} \times 2000 = \text{Rs. } \frac{2500}{3}$$

Investment at 5% per annum $= \frac{10{\cdot}5}{18} \times 2000$

$$= \frac{3{\cdot}5}{6} \times 2000$$

$$= \frac{35}{3} \times 100$$

$$= \text{Rs. } \frac{3500}{3}$$

$$= \text{Rs. } 1166{\cdot}67$$

4. (E) Average speed $= \frac{200}{5} = 40$ km/hr

Ratio of time taken at 10 km/hr to 10 km/hr

$$= 2 : 10 = 1 : 5$$

Time taken at 10 km/hr

$$= \frac{1}{6} \times 5 = \frac{5}{6} \text{ hr}$$

Distance covered at 10 km/hr

$$= \frac{10}{6} \times 5 = \frac{25}{3}$$

Distance covered at 22 km/hr

$$= 200 - \frac{25}{3} = \frac{575}{3} = 191{\cdot}67 \text{ km.}$$

5. (D) Here, expenditure and saving are two ingredients of income. Therefore we can write as under, assuming x% increases in savings.

$$\frac{15 - x}{5} = \frac{3}{2}$$

Or, $30 - 2x = 15$

Or, $2x = 15$

$$x = \frac{15}{2} = 7{\cdot}5\%.$$

6. (A) Average money per student

$$= \frac{250}{10} = \text{Rs. } 25$$

Now, $\frac{\text{Number of Girls}}{\text{Number of boys}} = \frac{30 - 25}{25 - 20} = \frac{1}{1}$

So, number of boys $= \frac{1}{2} \times 10 = 5.$

7. (B) Average value of 60 coins

$$= \frac{21 \times 100}{60}$$

$$= \frac{7 \times 100}{20} = 35 \text{ paise}$$

$$10 : 15 = 2 : 3$$

Number of 50 paise $= \frac{2}{5} \times 60 = 24$

Number of 25 paise coins

$$= 60 - 24 = 36$$

8. (D) b litres $= \frac{b}{a}$ of litres $= \frac{b}{a}$ of the whole quantity of liquid A after first operation

$$= \left(1 - \frac{b}{a}\right) \text{of whole}$$

Quantity of liquid after second operation

$$= \left(1 - \frac{b}{a}\right)^2 \text{ of whole}$$

$$\frac{\text{Quantity of liquid A left after second operation}}{\text{Quantity of liquid A initially present}} = \left(1 - \frac{b}{a}\right)^2$$

From n times :

$$\frac{\text{Quantity of liquid A left after nth operation}}{\text{Whole quantity of liquid A initially present}} = \left(1 - \frac{b}{a}\right)^n$$

9. (C) $\frac{5}{20} = \frac{1}{4}$th part of cask

Using the formula :

Quantity of wine left third with drawn

$$= \left(1 - \frac{1}{5}\right)^3 = \left(\frac{4}{5}\right)^3 \text{ of the whole}$$

$$= \frac{64}{125} \times 20 \text{ litres}$$

So, Quantity of water left after third withdraw

$$= 20 - \frac{64}{125} \times 20$$

$$= \frac{125 - 64}{125} \times 20 = \frac{61}{125} \times 20 \text{ litre}$$

Final ratio of wine to water

$$= \frac{\frac{64}{125} \times 20}{\frac{61}{125} \times 20} = \frac{64}{61}$$

Wine : Water = 64 : 61.

10. (C) % of liquid Y initially present in the vessel

$$= \frac{2}{3 + 2} \times 100 = 40\%$$

% of liquid Y finally present in the vessel

$$= \frac{4}{1 + 4} \times 100 = 80\%$$

First solution in which the percentage of liquid Y is 40%.

The second solution is liquid Y which is being mixed and it has 100% liquid Y.

So, 80% of liquid Y present in the resultant mixture may be taken as average percentage so using rule of allegation on liquid Y present we can write—

The ratio of liquid left in the vessel to liquid Y being mixed = 1 : 2.

Since, the quantity of liquid Y being mixed is 20 litres, the quantity of liquid left in the vessel in 10 litres.

Therefore, the total quantity of liquid initially present in the vessel

$$= 10 + 20 = 30 \text{ litres}$$

$$\text{Quantity of liquid X} = \frac{3}{2+3} \times 30 = 18 \text{ litres.}$$

11. (A) If the two alloys are mixed, the mixture would contain 12 gm of each metal and it would cost Rs. (180 + 120) = Rs. 300

Cost of (12 gm of metal X + 12 gm of metal Y) = Rs. 300

Cost of (1 gm of metal X + 1 gm of metal Y) = Rs. $\frac{300}{12}$ = Rs. 25

Cost of 1 gm of metal Y = Rs. (25 – 15) = Rs. 10

Average cost of original piece of alloy

$$= \frac{180}{15} = \text{Rs. } 12/\text{gm}$$

$$\text{So, } \frac{\text{Quantity of metal X}}{\text{Quantity of metal Y}} = \frac{12-10}{15-12} = \frac{2}{3}$$

Hence, the Quantity of metal X = $\frac{2}{5} \times 12 = 4{\cdot}8$ gm.

12. (E) Let the number of 2 legged animals be Y, then the Number of 4 legged animals be H – Y.

$$\begin{aligned} \text{Total number of legs} &= 4(\text{H} - \text{Y}) + 2\text{Y} \\ \text{Given } 4(\text{H}-\text{Y}) + 2\text{Y} &= \text{L} \\ 4\text{H} - 2\text{Y} &= \text{L} \\ 2\text{H} - \text{Y} &= \frac{\text{L}}{2} \\ \text{Y} &= 2\text{H} - \frac{\text{L}}{2} \end{aligned}$$

and number of 2 legged animals

$$= 2\text{H} - \frac{\text{L}}{2}$$

Number of 4 legged animals

$$= \text{H} - \left(2\text{H} - \frac{\text{L}}{2}\right)$$

$$= -\text{H} + \frac{\text{L}}{2} = \frac{\text{L}}{2} - \text{H}$$

13. (C) Let the weight of two alloys be a and b respectively.

$$a + b = 50$$

Let the percentage iron content in the second alloy be x%.

Then, the percentage iron content in the first alloy

$$= (x - 40)\%$$

Quantity of iron in the second alloy

$$= \frac{x}{100} \times a$$

Quantity of copper in the first alloy

$$= \frac{x-40}{100} \times a$$

$$\begin{aligned} \frac{xb}{100} &= 12 \\ b &= \frac{1200}{x} \end{aligned}$$

$$\text{and} \quad \frac{x-40}{100} \times a = 6$$

$$\begin{aligned} a &= \frac{600}{x-40} \\ a + b &= \frac{600}{x-40} + \frac{1200}{x} \\ &= \frac{600x + 1200x - 48000}{x(x-40)} \\ &= \frac{1800x - 48000}{x^2 - 40x} \\ a - b &= 50 \end{aligned}$$

$$\text{So,} \quad \frac{1800x - 48000}{x^2 - 40x} = 50$$

$$\begin{aligned} \text{Or,} \quad 1800x - 48000 &= 50x^2 - 2000x \\ \text{Or,} \quad 50x^2 - 3800x + 48000 &= 0 \\ \text{Or,} \quad x^2 - 76x + 960 &= 0 \\ \text{Or,} \quad x^2 - 60x - 16x + 960 &= 0 \\ \text{Or,} \quad x(x-60) - 16(x-60) &= 0 \\ (x-60)(x-16) &= 0 \\ x &= 16, 60 \end{aligned}$$

Since, percentage iron content in an alloy cannot be negative, we discard $x = 16$.

Percentage iron content in the first alloy = 20%

% iron content in the second alloy = 60%

14. (C) Ratio of milk to water is

$$= \left(\frac{2 \times 3}{3} + \frac{5 \times 4}{8} + \frac{6 \times 6}{9} + \frac{9 \times 7}{14}\right) : \left(\frac{1 \times 3}{3} + \frac{3 \times 4}{8} + \frac{3 \times 6}{9} + \frac{5 \times 7}{14}\right)$$

$$= \left(2 + \frac{5}{2} + 4 + \frac{9}{2}\right) : \left(1 + \frac{3}{2} + 2 + \frac{5}{2}\right)$$

$$= \left(\frac{12 + 14}{2}\right) : \left(\frac{6 + 8}{2}\right)$$

$$= \frac{26}{2} : \frac{14}{2} = 13 : 7.$$

15. (B) Let number of 6 legged insects = X

Number of 4 legged insects = 80 – X

According to question ,

$$\begin{aligned} 420 &= 6\text{X} + 4 \times (80 - \text{X}) \\ 420 &= 2\text{X} + 320 \\ \text{X} &= \frac{100}{2} = 50 \end{aligned}$$

$$\begin{aligned} \text{So, } \quad \text{6 legged insects} &= 50 \\ \text{4 legged insects} &= 80 - 50 = 30. \end{aligned}$$

9 Time, Distance and Speed

The speed of a body is given by

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Distance} = \text{Speed} \times \text{Time}$$

The distance covered by the body is equal to the multiplication of speed and time.

Some Fundamental Rules

1. For **Constant Speed**—

$$\text{Distance} \propto \text{Time}$$

It means more distance is covered in more time for a constant speed.

2. For the **Same Distance**—

$$\text{Speed} \propto \frac{1}{\text{Time}}$$

More speed is acquired in less time for same distance.

3. For the **Same Time**—

$$\text{Speed} \propto \text{Distance}$$

More speed is covered more distance in same time.

Conversion of One Unit System to Other

(A) If speed is given in km/hr, then to change it in m/sec or m/minutes.

Solution :

$$\text{Now, speed} = \frac{\text{km}}{\text{hr}}$$

$$= \frac{1000 \text{ metre}}{3600 \text{ sec}}$$

$$\text{Speed 1 km/hr} = \frac{5}{18} \text{ m/sec}$$

If we have v km/hr

Then in m/sec it is given by

$$\text{V km/hr} = \frac{5}{18} \text{V m/sec}$$

Similarly, if speed is V km/hr

$$\text{Speed} = \text{V}.\frac{\text{km}}{\text{hr}}$$

$$= \frac{\text{V} \times 1000 \text{ metre}}{60 \text{ minutes}}$$

$$\text{Speed (km/hr)} = \text{V} \times \frac{50}{3} \text{ metre/minutes}$$

(B) To convert V m/sec in km/hr or km/minutes.

Solution :

If speed is given in V m/sec, then

$$\text{Speed} = \text{V m/sec}$$

$$= \text{V} \times \frac{\frac{1}{1000}\text{km}}{\frac{1}{3600}\text{hr}}$$

$$\text{Speed (m/sec)} = \text{V} \times \frac{18}{5} \text{ km/hr}$$

Again,

$$\text{Speed (m/sec)} = \text{V} \times \frac{60}{1000}$$

$$\text{Speed (m/sec)} = \text{V} \times \frac{3}{50} \text{ km/minutes}$$

Illustration 1.

What is the speed of a maruti car if it travels 132 km in 132 minutes ?

Solution :

$$\text{Distance covered} = 132 \text{ km}$$

$$\text{Time taken} = 132 \text{ min.}$$

$$= 132/60 \text{ hrs.}$$

$$\text{Now, Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{132 \times 60}{132} = 60 \text{ km/hr.}$$

Illustration 2.

If Ram's car speed is 60 km/hr, then find the time taken by the car to travel a distance of 1200 km.

Solution :

$$\text{Since,} \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{1200}{60} = 20 \text{ hours.}$$

Illustration 3.

Find the distance if Ramesh's bike's speed is 80 m/sec and it takes time 60 minutes.

Solution :

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= 80 \text{ m/sec} \times 60 \text{ min}$$

$$= 288000 \text{ m} = 288 \text{ km}$$

Average Speed

$$\text{Average Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$\text{Average Speed} = \frac{d_1 + d_2 + d_3 + \ldots\ldots + d_n}{t_1 + t_2 + t_3 + \ldots\ldots + t_n}$$

$$= \frac{s_1 t_1 + s_2 t_2 + s_3 t_3 + \ldots\ldots + s_n t_n}{t_1 + t_2 + t_3 + \ldots\ldots + t_n}$$

$$= \frac{d_1 + d_2 + d_3 + \ldots\ldots + d_n}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + \ldots\ldots + \frac{d_n}{t_n}}$$

$$\neq \frac{s_1 + s_2 + s_3 + \ldots\ldots + s_n}{n}$$

Illustration 4.

If Gita's car covers 73 km in 68 sec., 135 km in 82 sec., 89 km in 76 sec. and 63 km in 44 sec., respectively. Then, find the average speed of the Gita's car.

Solution :

From formula :

$$\text{Average Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$\text{Average Speed} = \frac{d_1 + d_2 + d_3 + d_4}{t_1 + t_2 + t_3 + t_4}$$

$$\text{Now, Average speed} = \frac{73 + 135 + 89 + 63}{68 + 82 + 76 + 44} \times 60$$

$$= \frac{360 \times 60}{270} = 80 \text{ km/hr.}$$

Illustration 5.

If Sonali's car speed is 36 km/hr, then find car's is speed in metre/second or metre/minutes.

Solution :

$$\text{Sonali's car speed} = 36 \text{ km/hr}$$

$$= 36 \times \frac{5}{18} = 10 \text{ m/sec}$$

$$\text{Again Sonali's car speed} = 10 \text{ m/sec}$$

$$= \frac{10 \text{ m}}{\frac{1}{60} \text{ minutes}}$$

$$= 600 \text{ metre/minutes.}$$

Illustration 6.

Ram's bike speed is 5 m/sec, then find bike speed in km/hr or km/sec or km/minutes.

Solution :

$$\text{Now, Bike speed} = 5 \text{ m/sec}$$

$$= \frac{5 \times \frac{1}{1000} \text{ km}}{\frac{1}{3600} \text{ hr}} = 5 \times \frac{36}{10} = 5 \times \frac{18}{5}$$

$$\text{Bike speed} = 18 \text{ km/hr}$$

$$= 18 \times \frac{\text{km}}{3600 \text{ sec}}$$

$$= \frac{1}{200} \text{ km/sec} = 0{\cdot}005 \text{ km/sec}$$

$$= \frac{1}{200} \times \frac{\text{km}}{\frac{1}{60} \text{ minutes}}$$

$$= \frac{60}{200} = 0{\cdot}3 \text{ km/minutes}$$

Illustration 7.

Sita covers a distance d_1 km at V_1 km/hr and, then d_2 km at V_2 km/hr. Find his average speed during the whole journey.

Solution :

Time taken to travel d_1 km at V_1 km/hr is

$$t_1 = \frac{d_1}{V_1} \quad \ldots(1)$$

Time taken to travel d_2 km at V_2 km/hr

$$\therefore \quad t_2 = \frac{d_2}{V_2} \quad \ldots(2)$$

$$\text{Average Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$\text{Average Speed} = \frac{d_1 + d_2}{t_1 + t_2}$$

$$= \frac{d_1 + d_2}{\frac{d_1}{V_1} + \frac{d_2}{V_2}}$$

$$\therefore \quad \text{Average speed} = V_1 \times V_2 \times \left(\frac{d_1 + d_2}{d_1 . V_2 + d_2 . V_1}\right)$$

Average Speed for Same Distance

If distances are same, then $d_1 = d_2$

$$\text{Then, Average speed} = 2 \times \left(\frac{V_1 . V_2}{V_1 + V_2}\right)$$

$$\text{Average speed} = 2 \times \frac{\text{Product of speeds}}{\text{Sum of speeds}}$$

Illustration 8.

A train covers a distance between A and B. If it travels from A to B with speed 40 km/hr and from B to A with speed 60 km/hr, then find the average speed of the train.

Solution :

Since, distance between A and B is fix.

$$\text{So, Average speed} = 2 \times \left(\frac{V_1 . V_2}{V_1 + V_2}\right)$$

$$\text{Average speed} = 2 \times \frac{40 \times 60}{40 + 60}$$

$$= 48 \text{ km/hr.}$$

Illustration 9.

Sonu covers 50 km of his journey at 20 km/hr and the remaining distance at 30 km/hr. If the total journey is of 110 km. What is his average speed for the whole journey ?

Solution :

Since, distances are different.

So time taken to cover 50 km t_1

$$t_1 = \frac{50}{20} = 2{\cdot}5 \text{ hr}$$

Time taken to cover 60 km t_2

$$t_2 = \frac{60}{30} = 2 \text{ hr}$$

Now, Average speed $= \dfrac{\text{Total Distance Covered}}{\text{Total Time Taken}}$

$$= \frac{110}{2{\cdot}5 + 2} = \frac{110}{4{\cdot}5}$$

$$= \frac{1100}{45} = \frac{220}{9} \text{ km/hr.}$$

Illustration 10.

A train goes from A to B at speed V_1 km/hr and is late by t_1 hour. If it goes at V_2 km/hr it reaches t_2 hours early. Now, find the distance taken between A to B.

Solution :

Let the distance between A to B = d kms

Time taken to reach with speed $V_1 = \dfrac{d}{V_1}$

and time taken to reach with speed $V_2 = \dfrac{d}{V_2}$

According to question,

$$\frac{d}{V_1} - t_1 = \frac{d}{V_2} + t_2$$

Or, $$d\left(\frac{1}{V_1} - \frac{1}{V_2}\right) = t_1 + t_2$$

Now, $$d = \frac{V_1.V_2}{V_2 - V_1} \times (t_1 + t_2)$$

Illustration 11.

Monu goes to school from his house at speed 20 km/hr and is late by one hour. If he goes at 30 km/hr he reaches 2 hours early. Find the distance between his house and school.

Solution :

Given Speeds are V_1 = 20 km/hr

V_2 = 30 km/hr

t_1 = 1 hour

t_2 = 2 hours

From the formula :

$$d = \frac{V_1.V_2}{V_2 - V_1} \times (t_1 + t_2)$$

$$= \frac{20 \times 30}{30 - 20} \times (1 + 2) = \frac{600}{10} \times 3$$

$$d = 180 \text{ km.}$$

Illustration 12.

Ram covers his onward journey at speed V_1 km/hr and covers the return journey of equal distance at speed V_2 km/hr. If the total time taken by Ram is T hours for whole journey. What is the one way journey distance ?

Solution :

Let the one way journey distance = d

Time taken during onward journey $= t_1 = \dfrac{d}{V_1}$

Time taken during return journey $= t_2 = \dfrac{d}{V_2}$

Now, according to question,

$$T = t_1 + t_2$$

$$T = \frac{d}{V_1} + \frac{d}{V_2}$$

$$d = \frac{V_1.V_2}{V_1 + V_2} \times T$$

Illustration 13.

An old man walks to his office at 4 km/hr and returns to his house at 2 km/hr. If he spends total 5 hours on his to and for walking. What is the distance between his house and office ?

Solution :

According to question,

V_1 = 4 km/hr

V_2 = 2 km/hr

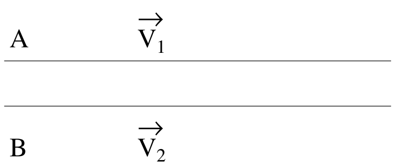
T = 5 hours = $t_1 + t_2$

Now, $$d = \frac{V_1.V_2}{V_1 + V_2} \times T$$

$$= \frac{4 \times 2}{4 + 2} \times 5 = \frac{8}{6} \times 5 = \frac{20}{3} \text{ km.}$$

Relative Speed

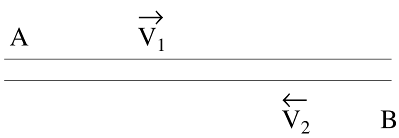
Case I : When two bodies move with speed V_1 km/hr and V_2 km/hr respectively in the same direction.



A's speed in relative with B or B's Speed in relative with A is called Relative Speed.

Relative Speed = $V_1 - V_2$ (If $V_1 > V_2$)

Case II : When two bodies move with speed V_1 km/hr and V_2 km/hr respectively in the opposite direction.



A's speed in relative with B or B's Speed in relative with A is called Relative Speed.

$$\text{Relative Speed} = V_1 + V_2$$

Illustration 14.

Two cars A and B start from the same point at speeds 60 km/hr and 40 km/hr in the same direction. Find the distance between them after 5 hours.

Solution :

Since, both the cars move in the same direction.

$$\therefore \quad \text{Relative Speed} = V_1 - V_2$$

$$= 60 - 40 = 20 \text{ km/hr}$$

Distance between A and B after 5 hours

$$= 20 \times 5 = 100 \text{ km.}$$

Illustration 15.

Two cars A and B start from the same point but move in opposite directions at speeds of 55 km/hr and 45 km/hr respectively. Find (i) their relative speed and (ii) their separation after 3 hours.

Solution :

$$\text{A's speed} = 55 \text{ km/hr}$$

$$\text{B's speed} = 45 \text{ km/hr}$$

(i) Relative speed $= 55 + 45 = 100$ km/hr

(ii) Now, separation after 3 hours

$$= \text{Relative speed} \times \text{time}$$

$$= 100 \times 3 = 300 \text{ km.}$$

Illustration 16.

Car A starts from X at 5 a.m. and reaches Y at 10 a.m. Another car B starts from Y at 7 a.m. and reaches X at 2 p.m. At what time do the two cars meet on their way ?

Solution :

$$\text{A's speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{d}{10-5} = \frac{d}{5}$$

and $\quad \text{B's speed} = \dfrac{d}{7}$

In such case we use the formula,

They will meet at

= Second's starting time

$$+ \frac{\dfrac{\text{(First's arrival time – Second's starting time)}}{\text{(Time taken by second)}}}{\text{Sum of time taken by both}}$$

$T_1 = 5$ am; $T_2 = 10$ am; $T_3 = 7$ am; $T_4 = 2$ pm (14)

$$\text{Required time} = T_3 + \frac{(T_2 - T_3)(T_4 - T_3)}{(T_2 - T_1) + (T_4 - T_3)}$$

$$\therefore \quad \text{Required time} = 7 + \frac{(10-7)(14-7)}{(10-5)+(14-7)}$$

$$= 7 + \frac{3 \times 7}{12}$$

$$= 7 + \frac{7}{4} = 7 + 1\frac{3}{4}$$

$$= 8 \text{ hrs 45 minutes.}$$

Illustration 17.

When a person covers the distance between his house and office at 60 km/hr. He is late by 15 minutes. But when he travels at 80 km/hr he reaches 5 minutes early. What is the distance between his house and office ?

Solution :

We assign,

Late time as positive and early time as –ve.

So, $\quad T_1 = 15$ minute

$T_2 = -5$ minute

Given speeds are $\quad V_1 = 60$ km/hr

$V_2 = 80$ km/hr

According to formula :

Distance between his house and office

$$= \frac{V_1 V_2}{V_2 - V_1}(T_1 - T_2)$$

$$= \frac{80 \times 60}{80 - 60}[(15 + 5) \text{ minute}]$$

$$= \frac{80 \times 60}{20} \times \left(\frac{20}{60} \text{ hrs.}\right) = 80 \text{ km.}$$

IInd Method

$$T + \frac{15}{60} = \frac{d}{60} \quad \ldots(1)$$

$$T - \frac{5}{60} = \frac{d}{80} \quad \ldots(2)$$

By equations (1) and (2)

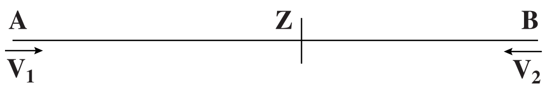
$$\frac{15}{60} + \frac{5}{60} = \frac{d}{60} - \frac{d}{80}$$

$$d = 80 \text{ km.}$$

Illustration 18.

Two persons X and Y start their journey at the same time in opposite directions from two points and after passing each other they complete their remaining journey in T_1 and T_2 hours respectively. Then, find the ratio of speed of X and Y.

Solution :



Let the total distance between the points is d km.

$$\text{X's speed} = V_1$$

$$\text{Y's speed} = V_2$$

Since, X and Y move in opposite direction.

So, $\quad \text{Relative speed} = V_1 + V_2$

$$\text{Time taken when they meet} = \frac{d}{V_1 + V_2}$$

$$\text{Time taken to move from A to Z} = \frac{d}{V_1 + V_2}$$

$$\text{Distance traveled by X} = \overline{AZ} = V_1 \times \frac{d}{V_1 + V_2}$$

Similarly,

$$\text{Distance travelled by Y} = \overline{BZ} = V_2 \times \frac{d}{V_1 + V_2}$$

Remaining distance travelled by X

$$= V_2 \times \frac{d}{V_1 + V_2}$$

Remaining distance travelled by Y

$$= V_1 \times \frac{d}{V_1 + V_2}$$

Now, Time taken by X to travel

$$\overline{BZ}\,(T_1) = \frac{\left(V_2 \times \dfrac{d}{V_1 + V_2}\right)}{V_1} \quad \ldots(1)$$

Time taken by Y to travel

$$\overline{AZ}\,(T_2) = \frac{\left(V_1 \times \dfrac{d}{V_1 + V_2}\right)}{V_1} \quad \ldots(2)$$

According to question,

$$\text{Ratios of X and Y} = T_1 : T_2$$

$$\frac{\left(\dfrac{V_2}{V_1}\right)}{\left(\dfrac{V_1}{V_2}\right)} = \frac{T_1}{T_2}$$

$$\left(\frac{V_2}{V_1}\right)^2 = \frac{T_1}{T_2}$$

$$\therefore \quad \left(\frac{V_2}{V_1}\right) = \sqrt{\frac{T_1}{T_2}}$$

$$\therefore \quad V_1 : V_2 = \sqrt{T_2} : \sqrt{T_1}.$$

Illustration 19.

Ram starts his journey from Patna to Delhi and Simultaneously Shyam starts from Delhi to Patna. After crossing each other they finish their remaining distance in $2\frac{1}{3}$ and 7 hours respectively. What is Shyam's speed if Ram's speed is $21\sqrt{3}$ km/hr ?

Solution :

$$\text{Ram's speed : Shyam's speed} = \sqrt{7} : \sqrt{2\tfrac{1}{3}}$$

$$= \sqrt{7} : \sqrt{\frac{7}{3}}$$

$$= \sqrt{3} : 1$$

$$\therefore \quad \frac{\text{Ram's speed}}{\text{Shyam's speed}} = \sqrt{3}$$

$$\text{Or,} \quad \text{Shyam's speed} = \frac{21\sqrt{3}}{\sqrt{3}}$$

$$= 21 \text{ km/hr.}$$

Illustration 20.

A train leaves Delhi at 6 a.m. and reaches Kanpur at 11 a.m. Another train leaves Kanpur at 7 a.m. and reaches Delhi at 1 p.m. At what time do the two trains meet ?

Solution :

Let the distance between Delhi and Kanpur by d km.

Let the speed of train leaving from Delhi is X

and Let the speed of train leaving from Kanpur is Y.

$$\text{Now,} \quad \text{X's speed} = \frac{d}{11-6} = \frac{d}{5} \text{ km/hr}$$

$$\text{Y's speed} = \frac{d}{6} \text{ km/hr}$$

Let the both the trains meet at t hour after 6 a.m.

Now, Distance travelled by A in t hour

$$= \frac{d}{5} \times t$$

Distance travelled by B in $(t-1)$ hour

$$= \frac{d}{6} \times (t-1)$$

$$\text{Now, Total distance} = d$$

$$\frac{d}{5} \times t + \frac{d}{6} \times (t-1) = d$$

$$\text{Or,} \quad \frac{t}{5} + \frac{t-1}{6} = 1$$

$$\text{Or,} \quad \frac{6t + 5(t-1)}{30} = 1$$

$$\text{Or,} \quad 11t - 5 = 30$$

$$\text{Or,} \quad 11t = 35$$

$$t = \frac{35}{11} \text{ hours}$$

$$t = 3\frac{2}{11} \text{ hours} \quad \text{or at 9·11 am they will meet.}$$

Some Basic Formula For Trains

(A) If a trains crosses a stationary person or pole or signal post, then time taken by the train is given below :

$$\text{Time taken} = \frac{\text{Length of the train}}{\text{Train's speed}}$$

(B) If train cross a platform, bridge, tunnel or standing train of length l km, then time taken is given by

If length of the Train = X km

Its speed = V km/hr

$$\text{Time taken} = \frac{\text{Length train + Length of tunnel/bridge}}{\text{Speed of Train}}$$

$$\text{Time taken} = \frac{X + l}{V}$$

(C) If a train A of length l_1 moving at speed V passes on object B (or train B) of length l_2 whose speed is U in the same direction. Then,

Relative speed between train A and object B or train B

$$= V - U$$

Now, If $\quad V > U$

Time taken by train A to cross the object B

$$T = \frac{l_1 + l_2}{V - U}$$

Illustration 21.

The distance between two stations P and Q is 480 km. Two trains that starts simultaneously from P and Q and meet after 5 hours. If the difference in their speed is 8 km/hr. Find their speeds.

Solution :

According to above formula :

$$a = 8 \text{ km/hr}$$
$$T = 5 \text{ hours}$$
$$S = 480 \text{ km}$$

Now, Speed of the faster train

$$= \frac{1}{2}\left(\frac{S}{T} + a\right)$$
$$= \frac{1}{2}\left(\frac{480}{5} + 8\right) = \frac{1}{2}(96 + 8)$$
$$= \frac{1}{2} \times 104 = 52 \text{ km/hr}$$

Speed of the slower train

$$= \frac{1}{2}\left(\frac{S}{T} - a\right) = \frac{1}{2}\left(\frac{480}{5} - 8\right)$$
$$= \frac{1}{2}(96 - 8) = 44 \text{ km/hr.}$$

Illustration 22.

Ram traveling at 60 km/hr reach his school from his house 40 minutes earlier. If he had traveled at 50 km/hr he would have reached 10 minute late. How far is the school ?

Solution :

According to question,

$$\frac{d}{60} + \frac{40}{60} = \frac{d}{50} - \frac{10}{60}$$

Or, $$d\left(\frac{1}{50} - \frac{1}{60}\right) = \frac{2}{3} + \frac{1}{6} = \frac{4+1}{6}$$

Or, $$d\left(\frac{6-5}{5 \times 6 \times 10}\right) = \frac{5}{6}$$

$$d = 250 \text{ km.}$$

Illustration 23.

A train is moving with speed 36 km/hr. How much time it will take to cross a standing pole, the length of the train is 90 m ?

Solution :

$$\text{Speed of train, } V = 36 \text{ km/hr}$$
$$= 36 \times \frac{5}{18}$$
$$= 10 \text{ m/sec}$$

Now, $$\text{Length of the train} = \frac{\text{Distance}}{\text{Time}} = \frac{l}{V}$$
$$= \frac{90}{10} = 9 \text{ metres.}$$

Illustration 24.

A train of length 120 metres is moving with speed 54 km/hr. How much it will take to cross a plateform of length 120 meters ?

Solution :

$$\text{Given Speed of train, } V = 54 \text{ km/hr} = 54 \times \frac{5}{18}$$
$$= 15 \text{ m/sec}$$
$$\text{Length of the train, } l_1 = 120 \text{ m}$$
$$\text{Length of the platforms } l_2 = 120 \text{ m}$$
$$\text{From formula, Time taken} = \frac{l_1 + l_2}{V} = \frac{120 + 120}{15}$$
$$= \frac{240}{15} = \frac{80}{5}$$
$$= 16 \text{ seconds.}$$

Illustration 25.

Two trains X and Y start simultaneously from stations A and B and move towards each other at speed V_1 and V_2 km/hr respectively. At the point where they meet, one train has covered a km more than the other. Now, find the distance between A and B.

Solution :

Let $\quad V_1 > V_2$

Let us suppose that they meet at time t hr.

Now, Distance covered by the train X = $V_1 \times t$

Distance covered by the train Y = $V_2 \times t$

According to question,

$$V_1 \times t - V_2 \times t = a$$

Or, $$t = \frac{a}{V_1 - V_2}$$

$$\text{Total Distance} = V_1 \times t + V_2 \times t$$
$$= t(V_1 + V_2)$$
$$\text{Total Distance} = a \times \left(\frac{V_1 + V_2}{V_1 - V_2}\right) \text{km.}$$

Illustration 26.

Let two trains start from Delhi and Bombay and move towards with speed 60 km/hr and 50 km/hr respectively. At the meeting point it is found that train from Delhi has traveled 20 km more than the train coming from Bombay. Find the distance between Delhi and Bombay ?

Solution :

In our question,

$$V_1 = 60 \text{ km/hr}$$

$$V_2 = 50 \text{ km/hr}$$

$$a = 20 \text{ km}$$

Then, $$\text{Distance} = a \times \left(\frac{V_1 + V_2}{V_1 - V_2}\right)$$

$$= 20 \times \left(\frac{60 + 50}{60 - 50}\right)$$

$$= 20 \times \frac{110}{10} = 220 \text{ km.}$$

Illustration 27.

Two trains of lengths l_1 and l_2 metres run on parallel tracks. When moving in the same direction, the faster train passes the slower one in T_1 seconds. But when they are moving in opposite directions at same speeds as earlier, they cross each other completely in T_2 seconds. Find the speeds of the trains.

Solution :

Let the speed of faster train = V_1 km/hr

Let the speed of slower train = V_2 km/hr

When they are running in the same direction.

Relative Speed = $(V_1 - V_2)$ km/hr

Then, T_1 = time taken to cross each other in the same direction

$$= \frac{l_1 + l_2}{V_1 - V_2}$$

Or, $$V_1 - V_2 = \frac{l_1 + l_2}{T_2} \quad \ldots(1)$$

When they are running in opposite direction

Relative Speed = $V_1 + V_2$

T_2 = Time taken to cross each other in opposite direction

$$= \frac{l_1 + l_2}{V_1 - V_2}$$

$$V_1 + V_2 = \frac{l_1 + l_2}{T_2} \quad \ldots(2)$$

Solving equation (1) and equation (2), we get

$$V_1 = \left(\frac{l_1 + l_2}{2}\right)\left(\frac{T_1 + T_2}{T_1 . T_2}\right) \text{m/sec}$$

$$V_2 = \left(\frac{l_1 + l_2}{2}\right)\left(\frac{T_1 - T_2}{T_1 . T_2}\right) \text{m/sec}$$

If the both the trains are equal.

$$l_1 = l_2 = l$$

$$V_1 = l \times \left(\frac{T_1 + T_2}{T_1 . T_2}\right) \text{m/sec}$$

$$V_2 = l \times \left(\frac{T_1 - T_2}{T_1 . T_2}\right) \text{m/sec.}$$

Illustration 28.

Two trains of lengths 1 km and 2 km run on parallel tracks. When running in the same direction the faster train crosses the slower one in 20 seconds. When running in opposite directions at speed same as their earlier speeds, they pass each other completely in 10 seconds. Find the speed of each train.

Solution :

$$l_1 = 1 \text{ km}$$

$$l_2 = 2 \text{ km}$$

$$T_1 = 20 \text{ sec}$$

$$T_2 = 10 \text{ sec}$$

Now, according to above formula,

$$V_1 = \left(\frac{l_1 + l_2}{2}\right)\left(\frac{T_1 + T_2}{T_1 . T_2}\right) \text{m/sec}$$

$$= \frac{(1 + 2)1000}{2} \times \left(\frac{20 + 10}{20 \times 10}\right)$$

$$= \frac{3000}{2} \times \frac{30}{20 \times 10}$$

$$= \frac{900}{4} = 225 \text{ m/sec}$$

$$V_2 = \left(\frac{l_1 + l_2}{2}\right)\left(\frac{T_1 - T_2}{T_1 . T_2}\right) \text{m/sec}$$

$$= \frac{3000}{2} \times \left(\frac{20 - 10}{20 \times 10}\right)$$

$$= 1500 \times \frac{10}{200}$$

$$= 75 \text{ m/sec.}$$

Illustration 29.

A train running at 54 km/hr passes a tunnel completely in 5 minutes. While inside the tunnel it meets another train of three-fourth of its length traveling at 72 km/hr and passes it completely in 8 seconds. Find the length of the trains and of the tunnel.

Solution :

Speed of the first train = 54 km/hr

$$= 54 \times \frac{5}{18} = 15 \text{ m/sec}$$

Since, it passes tunnel completely in 5 minutes.

If l be the length of train and d be the length of tunnel.

$$\text{Then, Time required} = \frac{\text{Distance}}{\text{Speed}}$$

$$5 \times 60 \text{ sec} = \frac{l+d}{15}$$

Or, $l + d = 75 \times 60 \text{ m} = 4500 \text{ m}$

Now, relative speed of the trains

$$= 15 + 20 = 35 \text{ m/sec.}$$

Total distance covered in crossing the trains

$$= l + \frac{3}{4}l = \frac{7}{4}l$$

$$= \text{Relative speed} \times \text{Time}$$

$$= 35 \times 8 = 280 \text{ m}$$

Or, $\frac{7}{4}l = 280$

$$l = 160 \text{ m}$$

$$\text{Length of tunnel} = 4500 - 160 = 4340 \text{ m.}$$

Illustration 30.

Two trains C and D starts simultaneously from stations A and B, S km a part and move toward each other. They meet after time T hours. If the difference in speeds of the two trains is a km/hr. Find the speeds of the two trains.

Solution :

Let the speed of trains C and D be V_1 and V_2 km/hr respectively and $V_1 > V_2$.

Now, Given $V_1 - V_2 = a$

Since, both the trains move in opposite direction.

So, Total distance = Distance covered by C + Distance covered by D

$$S = V_1 \times T + V_2 \times T$$

$$V_1 + V_2 = \frac{S}{T} \quad \ldots(1)$$

$$V_1 - V_2 = a \quad \ldots(2)$$

Solving equation (1) and equation (2), we get

$$\therefore \text{ Speed of faster train} = V_1 = \frac{1}{2}\left(\frac{S}{T} + a\right)$$

$$\text{Speed of slower train} = V_2 = \frac{1}{2}\left(\frac{S}{T} - a\right)$$

Boats and Streams

Up-Stream—The motion of boats or ships against the stream is called up-stream.

Let the speed of boat is V m/sec.

While stream flows with U m/sec.

Now, Speed of the boat against the stream

$$= (V - U) \text{ m/sec.} \quad \ldots(1)$$

Down-Stream—The motion of boat and ship along the direction of stream is called down-stream motion.

In this case—

$$\text{Down-stream Speed} = (V + U) \text{ m/sec.} \quad \ldots(2)$$

From equation (1) and equation (2)

Speed of boat in still water

$$= \frac{1}{2}(\text{Down-stream speed} + \text{Up-stream speed})$$

Speed of the stream

$$= \frac{1}{2}(\text{Down-stream speed} - \text{Up-stream speed})$$

Illustration 31.

A swimmer covers a certain distance down-stream in 90 minutes but takes 100 minutes to return up-stream the same distance. If he can swims in still water with the speed of 60 metre per second. Find the speed of the current.

Solution :

Using the formula,

$$V = W \times \left(\frac{T_1 + T_2}{T_2 - T_1}\right)$$

Here, $T_1 = 90$ min

$$T_2 = 100 \text{ min}$$

$$W = 60 \text{ m/sec}$$

Now, $V = W \times \left(\frac{T_1 + T_2}{T_2 - T_1}\right)$

$$= 60 \times \left(\frac{90 + 100}{100 - 90}\right)$$

$$= 60 \times \frac{190}{10}$$

$$V = 1140 \text{ m/sec.}$$

Illustration 32.

A man can run a ship in still water at V km/hr. In a stream flowing at W km/hr. If it takes T hours to run to a point and come back. Find the distance between the two points.

Solution :

$$\text{Down-stream speed} = (V + W) \text{ km/hr}$$

$$\text{Up-stream speed} = (V - W) \text{ km/hr}$$

Let the distance between the two points = d km

Sum of time taken down-stream and up-stream equals total time.

$$\therefore \frac{d}{V+W} + \frac{d}{V-W} = T$$

Or, $d = \frac{T(W+V)(V-W)}{2.V}$

$$d = T \times \left(\frac{V^2 - W^2}{2.V}\right) \text{km.}$$

Illustration 33.

Ramu can swim down-stream 30 km in 3 hours and up-stream 24 km in 3 hours. Find his speed in still water and also the speed of the current ?

Solution :

$$\text{Down-stream} = \frac{30 \text{ km}}{3 \text{ hr}} = 10 \text{ km/hr}$$

$$\text{Up-stream} = \frac{24 \text{ km}}{3 \text{ hr}} = 8 \text{ km/hr}$$

$$\text{Speed in still water} = \frac{1}{2}(10 + 8) = 9 \text{ km/hr}$$

$$\text{Speed of the stream} = \frac{1}{2}(10 - 8) = 1 \text{ km/hr.}$$

Illustration 34.

Gita can Swim down-stream d_1 km in T_1 hours and up-stream d_2 in T_2 hours. Find her speed in still water and speed of current.

Solution :

$$\text{Now, down-stream speed} = \frac{d_1}{T_1} \text{ km/hr}$$

$$\text{Up-stream speed} = \frac{d_2}{T_2} \text{ km/hr}$$

$$\text{Speed in still water} = \frac{1}{2}\left(\frac{d_1}{T_1} + \frac{d_2}{T_2}\right) \text{km/hr}$$

$$\text{Speed of the current} = \frac{1}{2}\left(\frac{d_1}{T_1} - \frac{d_2}{T_2}\right) \text{km/hr.}$$

Illustration 35.

Reena swims a certain distance down-stream in T_1 hours and returns the same distance up-stream in T_2 hours. If the speed of the stream be W km/hr. Find the Reena's speed in still water.

Solution :

Let the speed of Reena in still water = V km/hr

Now, down-stream speed = (V + W) km/hr

Up-stream speed = (V – W) km/hr

According to question,

The distance covered down-stream and up-stream are equal.

$$\text{So,} \quad (V + W) \times T_1 = (V - W) \times T_2$$

$$V = W \times \left(\frac{T_1 + T_2}{-T_1 + T_2}\right)$$

$$V = W \times \left(\frac{T_1 + T_2}{T_2 - T_1}\right) \text{km/hr.}$$

Exercise A

1. In a stream running at 2 km/hr a boat goes 6 km. up-stream and back again to be the starting point is 33 minutes. Find the speed of the boat in still water ?

   (A) 12 km/hr (B) 15 km/hr
   (C) 22 km/hr (D) 25 km/hr
   (E) 30 km/hr

2. A man can row 20 km up-stream and 25 km down-stream in 8 hours. Also he can row 30 km up-stream and 45 km down-stream in 10 hours. Find the speed of the man in still water ?

   (A) $\frac{75}{16}$ km/hr (B) $\frac{75}{8}$ km/hr
   (C) $\frac{75}{4}$ km/hr (D) $\frac{75}{2}$ km/hr
   (E) 75 km/hr

3. A boat covers a certain distance down-stream in 2 hours, while it comes back in $\frac{5}{2}$ hours. If the speed of stream be 4 km/hr. What is the speed of the boat in still water ?

   (A) 45 km/hr (B) 224 km/hr
   (C) 36 km/hr (D) 18 km/hr
   (E) 25 km/hr

4. A train running at 54 km/hr takes 15 seconds to pass a platform. Next it takes 12 seconds to pass a man walking at 6 km/hr in the same direction in which the train is going. Find the ratio between the length of the train and the length of the platform ?

   (A) 43 : 34 (B) 28 : 23
   (C) 32 : 13 (D) 14 : 11
   (E) 7 : 5

5. A train A starts from Patna at 6 p.m. and reaches Banaras at 7 p.m. while another train B starts from Banaras at 6 p.m. and reaches Patna at 7:30 p.m. The two trains will cross each other. Find the time when they will crossing each other ?

   (A) 6 : 24 pm (B) 6 : 36 pm
   (C) 6 : 48 pm (D) 6 : 54 pm
   (E) 6 : 18 pm

6. Two trains, one from Delhi to Patna and the other from Patna to Delhi, start simultaneously. After they meet, the trains reach their destination after 16 hours and 64 hours respectively. What is the ratio of their speeds ?

   (A) 2 : 1 (B) 3 : 2
   (C) 4 : 3 (D) 5 : 3
   (E) 4 : 1

7. Two train 4001 and 1432 are 120 km apart. The train 4001 starts from A at 6 a.m. and travels towards B at 45 km/hr another train starts from B at 7 a.m. and travels towards A at a speed of 25 km/hr. At what time they meet ?

   (A) 8 : 04 am (B) 7 : 36 am
   (C) 7 : 12 am (D) 8 : 44 am
   (E) 9 : 12 am

8. Two train start from A and B respectively and travel towards each other at a speed of 54 km/hr and 36 km/hr respectively. By the time they meet, the first train has traveled 150 m more than the second. Find the distance between A and B ?

(A) 350 m (B) 450 m
(C) 540 m (D) 300 m
(E) 500 m

9. X is twice as fast as fast Y and Y thrice as fast as Z. The journey covered by Z in 60 minutes then. Find time required by X ?

(A) 8 minutes (B) 6 minutes
(C) 10 minutes (D) 12 minutes
(E) 15 minutes

10. Walking at $\frac{4}{5}$ of its usual speed, a train is 10 minutes late. Find the usual time to cover the journey ?

(A) 20 minutes (B) 25 minutes
(C) 30 minutes (D) 35 minutes
(E) 40 minutes

11. A thief is spotted by a policeman from a distance of 1200 m. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 12 km/hr and that of the policeman 16 km/hr, how far the thief will have run before he is catches ?

(A) 1212 metres (B) 1220 metres
(C) 1236 metres (D) 1260 metres
(E) 1272 metres

12. A car driver travels from the plains to the hill stations, which are 108 km apart at an average speed of 18 km/hr. In the return trip, he covers the same distance at average speed of 36 km/hr. Find the average speed of the car cover the entire distance of 500 km. ?

(A) 12 km/hr (B) 18 km/hr
(C) 24 km/hr (D) 45 km/hr
(E) 36 km/hr

13. The average speed of a train in the onward journey is 50% more than that in the return journey. The train halts for one hour on reaching the destination. The total distance taken for the complete to and for journey is 16 hours, covering a distance of 1000 km. What is the speed of the train in the onward journey ?

(A) 55·55 km/hr (B) 33·33 km/hr
(C) 66·66 km/hr (D) Cannot be determine
(E) Data insufficient

14. The distance between Mumbai and Chennai is 1020 km Mumbai–Channai expresses leaves Mumbai CST at 6·00 p.m. towards Chennai on the same day, Chennai–Mumabi Express leaves Chennai for Mumbai at 8.00 p.m. and its speed higher by 10 km/hr than the speed of Mumbai–Chennai Express. Both trains meet at Wadi junction, which is exactly at the centre between Mumbai and Chennai. At what time do both the trains meet ?

(A) 11 : 30 pm same day
(B) 5 : 20 am next day
(C) 5 : 20 pm next day
(D) Sharp 12 same day (mid night)
(E) Sharp 12 next day (mid day)

15. The distance between Patna and Delhi is 300 km two trains simultaneously leave from Patna and Delhi. After they meet, the train traveling towards Delhi reaches there after 9 hours, while the train traveling towards Delhi reaches there after 4 hours. Find the speed of each train in km/hr ?

(A) 30; 20 (B) 35; 25
(C) 40; 30 (D) 25; 12
(E) 60; 35

16. Two trains 100 km a part towards each other on the same track. One train travels at 55 km/hr, other travels at 45 km/hr. A bird starts flying at a speed of 80 km/hr at the location of the faster train. When it reaches the slower train, it turns around and flies in the opposite direction at the same speed. When it reaches the faster train again it turns around and so on. When the trains collide, how far has the bird flown ?

(A) 60 km (B) 70 km
(C) 80 km (D) 90 km
(E) 100 km

17. Two trains 100 m and 120 m long are running in the same direction with speeds of 72 km/hr and 54 km/hr. In how much time will the first train cross the second ?

(A) 35 sec (B) 44 sec
(C) 58 sec (D) 75 sec
(E) 90 sec

18. A Train 4321 running at 81 km/hr takes 180 seconds to pass a platform. Next it takes 75 seconds to pass a man walking at 9 km/hr in the same direction in which the train is going. Find the ratio length of the train and the length of the platform ?

(A) 1 : 2 (B) 4 : 5
(C) 3 : 2 (D) 2 : 3
(E) 1 : 3

19. Two trains running in opposite directions cross a man standing on the platform in 30 seconds and 20 seconds respectively and they cross each other in 25 seconds. Find the ratio of their speeds ?

(A) 2 : 3 (B) 1 : 1
(C) 2 : 5 (D) 3 : 5
(E) 3 : 2

20. There are two different roads between two cities. The first is 20 km longer than the second A car travels along the second road and covers the distance between the cities in $3\frac{1}{2}$ hours. Another car travels along the second road and covers the distance in $2\frac{1}{2}$ hours. If the speed of the first car is 40 km/hr less than that of the second car, then what is the speed of faster car ?

(A) 60 (B) 80

(C) 120 (D) 160

(E) 200

21. Taruna can row a boat d_1 km up-stream and d_2 km down-stream in T_1 hours. He can row d_1' km up-stream and d_2' km down-stream in T_2 hours. Then find the down-stream speeds ?

(A) $\frac{d_1 d_2' - d_1' d_2}{d_1 T_2 - T_1 d_2'}$ (B) $\frac{d_1 d_2' - d_1' d_2}{d_2 T_2 - T_1 d_1'}$

(C) $\frac{d_1 d_2' - d_1' d_2}{d_1 T_1 - T_1 d_1'}$ (D) $\frac{d_1 d_2' - d_1' d_2}{d_1 T_2 - T_2 d_1'}$

(E) $\frac{d_1 d_1' - d_2' d_2}{d_1 T_2 - T_1 d_1'}$

22. A pedestrian and a cyclist start simultaneously towards each other from towns X and Y. Which are 80 km apart and meet two hours from start, then they continue their journey and the cyclist arrive at X 20 hours earlier than the pedestrian arrive at Y. Find the ratio of their speeds ?

(A) 1 : 2 (B) 1 : 3

(C) 1 : 4 (D) 1 : 9

(E) 1 : 8

23. Sita and Soni start simultaneously from point P towards Q, 60 kms. apart. Sita's spee is 4 km/hr less than Soni's speed. Sita after reaching Soni turns back and meets Sita at 12 km from Soni. Find Sita's speed ?

(A) 20 km/hr (B) 16 km/hr

(C) 24 km/hr (D) 12 km/hr

(E) 9 km/hr

Exercise B

1. Sita sees his friends standing at a distance of 240 metres from his position. She increases her speed by 50% and hence takes 20 seconds now to reach her.

(i) If she travels at the original speed, how much time will she take?

(ii) What was her original speed ?

(A) 30 sec; 8 m/sec (B) 40 sec; 6 m/sec

(C) 24 sec; 10 m/sec (D) 36sec.

(E) None of these

2. Ramesh arrives at his office 30 minutes late everyday. On a particular day, he reduces his speed by 25% and hence arrived 50 min. late instead. Find how much speed he should increase so that he will be on time on a particular day ?

(A) 50% (B) 75%

(C) 100% (D) 125%

(E) 150%

3. A car traveled 30% of time at a speed of 20 km/hr, 40% of time at a speed of 30 km/hr and rest of the journey at a speed at 40 km/hr. What is the average speed of the car for the entire journey ?

(A) 25 km/hr (B) 45 km/hr

(C) 30 km/hr (D) 60 km/hr

(E) 15 km/hr

4. A thief steals a car at 10.30 a.m. and drives it at 60 km/hr. The theft is discovered at 11·00 a.m. and the owner sets off in another car at 75 km/hr when will he overtake the thief ?

(A) 12·30 (B) 1·00

(C) 1·15 (D) 1·45

(E) 1·30

5. A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But if he travels 200 km by train and rest by car, he takes half an hour longer. Find the speed of the car ?

(A) 50 km/hr (B) 60 km/hr

(C) 65 km/hr (D) 75 km/hr

(E) 80 km/hr

6. A cart race track has a circumference of 3000 m the length of the race is 9000 m. The fastest and the slowest cart meet for the first time after the start of the race at the end of the length in 10 minute. All the carts start at the same point and fastest cart moves at twice the speed of the slowest cart, what is the time taken by the fastest cart to finish the race ?

(A) 30 minutes (B) 25 minutes

(C) 20 minute (D) 40 minutes

(E) 45 minutes

7. The local shuttle service trains which travel at a uniform speed run at regular intervals, Sunita, walking down along the railway track at uniform speed, found that every 8 minutes there is a local train coming in opposite direction and every 24 minutes, there is a local train overtaking her from behind. What is the time gap between one local train passing a stationary point on the railway route and the immediately next local train in the same direction passing the same point ?

(A) 12 minutes (B) 15 minutes
(C) 18 minutes (D) 20 minutes
(E) 24 minutes

8. Two trains are moving towards each other at speed of 60 m/sec and 90 m/sec. At the time, the two trains are 150 m apart, a fly starts moving alongside one of the trains at 5 m/sec the fly changes its course when the two trains collide and flies a distance of 10 m to the East, then 25 m to the North and then 55 m to the South. At this point it spots two cars moving head long towards each other at speed 20 m/sec and 30 m/sec the fly immediately changes its course again and flies at thrice its original speed. The distance bet-ween the two cars is equal to the distance covered by the fly until it spots the cars. How many metres will the fly travel before the cars collide ?
(A) 128 m (B) 126 m
(C) 122 m (D) 134 m
(E) 114 m

9. Ramesh goes from X to Y via U, V and W. The distance between in the ratio of 2 : 3 : 2 : 3. The time taken by the man to cover these distances in the ratio of 5 : 3 : 2 : 5. If the man takes 5 hours to go from X to Y, then what is the differences between the time taken by the man to go from U to V and that from W to Y ?
(A) 30 minutes (B) 40 minutes
(C) 60 minutes (D) 80 minutes
(E) None of these

Question based on data sufficient (Q. 10 to 16)—

10. A train running at a certain speed crosses another train running in the opposite direction in 5 seconds. To find out the speed of the first train, which of the following information X and Y is sufficient ?
X = the length of the first train
Y = the length of the second train
(A) only X is sufficient
(B) only Y is sufficient
(C) Either X or Y is sufficient
(D) Both X and Y are sufficient
(E) Both X and Y are not sufficient.

11. What is the speed of the train ?
(i) The train crosses a tree in 13 seconds.
(ii) The train crosses a platform of length 250 metres in 27 seconds.
(iii) The train crosses another train running in the same direction in 32 seconds.
(A) (i) and (ii) only (B) (ii) and (iii) only
(C) (i) and (iii) only (D) Any two of the three
(E) None of these

12. What is the speed of train ?
(i) The train crosses 300 metres long platform in 20 seconds.
(ii) The train crosses another stationary train of equal length in 15 seconds.
(iii) The train crosses a signal pole in 10 seconds.
(A) (i) and (ii) only
(B) (i) and either (ii) or (iii) only
(C) (ii) and either (i) or (ii) only
(D) (iii) and either (i) or (ii) only
(E) None of these

13. What is the speed of stream ?
(i) The boat covers 20 km in 5 hours moving up stream.
(ii) The boat covers 20 km in 4 hours moving down stream.
(iii) The ratio between the speed of boat and stream is 2 : 1 respectively.
(A) Any two of the three
(B) (i) and (ii) only
(C) (ii) and (iii) only
(D) (i) and (iii) only
(E) All (i), (ii) and (iii)

14. A tank is fitted with two inlet pipes A and B both the pipes are kept open for 10 minutes. So, that the tank is two-thirds full and then pipe A is closed. How much time will B take to fill the remaining part of the tank ?
(i) Pipe A is thrice as fast as pipe B.
(ii) Pipe B alone can fill the tank in 60 minutes.
(A) Only (i)
(B) Only (ii)
(C) Any one is sufficient
(D) All (i) and (ii)
(E) None (i) & (ii)

15. A tank is fitted with two taps A and B. In how much time will the tank be full if both the taps are opened together ?
(i) A is 50% more efficient than B
(ii) A alone takes 16 hours to fill the tank
(iii) B alone takes 24 hours to fill the tank.
(A) (ii) and (iii) only
(B) All (i), (ii) and (iii)
(C) (i) and (ii) only
(D) (i) and (iii) only
(E) Any two of the three

16. A train running at a certain speed crosses a stationary engine in 20 seconds. To find out the speed of the train, which of the following information is necessary ?
(A) Only the length of the train
(B) Only the length of engine
(C) Either the length of the train or the length of the engine
(D) Both the length of the train and the length of the engine
(E) None of these

Answers with Hints

Exercise A

1. (C) Let speed of boat in still water = V km/hr

$$\text{Now,} \quad \text{down-stream speed} = V + 2$$
$$\text{Up-stream speed} = V - 2$$

According to question,

$$\text{Total time taken} = \frac{6}{V+2} + \frac{6}{V-2} = \frac{33}{60}$$

Or, $11.V^2 - 240.V - 44 = 0$

Or, $(V - 22)(11.V + 2) = 0$

$$V = 22,$$
$$V \neq -\frac{2}{11}$$

$\therefore$ Speed of boat in still water = 22 km/hr.

2. (B) Let the man can row in still water in V km/hr and speed of stream U km/hr.

According to question,

$$\frac{20}{V-U} + \frac{25}{V+U} = 8 \quad \ldots(1)$$
$$\frac{30}{V-U} + \frac{45}{V+U} = 10 \quad \ldots(2)$$

Now, multiplying equation (1) with 3 and equation (2) with 2, then subtracting, we get

$$\therefore \quad \frac{75}{V+U} - \frac{90}{V+U} = 24 - 20$$
$$-\frac{15}{V+U} = 4$$
$$V + U = -\frac{15}{4} \quad \ldots(3)$$

Now, putting the value of (V + U) equation (1), we get

$$\frac{20}{V-U} + \frac{25}{\frac{15}{4}} = 8$$
$$\frac{20}{V-U} = 8 - \frac{20}{3} = \frac{4}{3}$$
$$V - U = 15 \quad \ldots(4)$$

Now, subtract equation (4) – equation (3), we get

$$2.V = \frac{75}{4}$$
$$V = \frac{75}{8} \text{ km/hr.}$$

3. (C) Now, Let the speed of the boat in still water = W km/hr

Now, according to question,

Since, Distance is fixed/constant.

Distance in down-stream motion

$$= 2(4 + W)$$

Distance in up-stream motion

$$= \frac{5}{2}(W - 4)$$

Now, $2(4 + W) = \frac{5}{2}(W - 4)$

Or, $16 + 4W = 5W - 20$

Or, $W = 36$ km/hr.

4. (C) Let length of the platform and train are l_1 and l_2 respectively.

Now, $54 \text{ km/hr} = 54 \times \frac{5}{18} = 15$ m/sec

Or, $\frac{l_1 + l_2}{15} = 15$

$$l_1 + l_2 = 225 \text{ m} \quad \ldots(1)$$

In second case man and train both move in the same direction. So,

$$\text{Relative Speed} = (54 - 6) \times \frac{5}{18}$$
$$= \frac{48 \times 5}{18} = \frac{8 \times 5}{3} = \frac{40}{3} \text{ m/sec}$$

Now, $\frac{l_2}{\frac{40}{3}} = 12$

$$l_2 = 160 \text{ m} \quad \ldots(2)$$

Putting the value of l_3 in equation (1), we get

$$l_1 = 225 - 160 = 65 \text{ m}$$

$\therefore$ Length of the train = 160 m

$\therefore$ Length of the platform = 65 m

Hence, required ration = 160 : 65 = 32:13

5. (B) Since, the distance between Patna and Banaras is constant which is X m.

Now, Speed of train A $= \frac{X}{60}$ m/min.

Speed of train B $= \frac{X}{90}$ m/min.

Let they meet at time Y min.

Now, Total Distance = Distance Covered by the both the trains

Or, $X = \frac{X}{60} \cdot Y + \frac{X}{90} \cdot Y$

Or, $1 = \frac{Y}{60} + \frac{Y}{90} = \frac{(3+2)Y}{180}$

$\therefore$ $5.Y = 180$

Or, $Y = \frac{180}{5} = 36$ minutes

Clearly, they cross each other at 6:36 pm.

6. (A) According to the formula,

The ratio of their speed $= \sqrt{T_2} : \sqrt{T_1}$

$= \sqrt{64} : \sqrt{16}$

$= 8 : 4 = 2 : 1$

7. (A) Now, Let they meet after t hours of 6 am.

Now, Total distance remains fix.

$\therefore$ $120 = 45.t + 25(t-1)$ $(t > 1 \text{ hour})$

Or, $120 = 70.t - 25$

Or, $t = \frac{145}{70}$ hour

Or, $t = \frac{145}{70} \times 60$

$= 124.3$ minutes $= 2$hrs 4 minutes

Hence, they will meet at 8:04 am.

8. (D) Speed of the first train $= 54 \times \frac{5}{18} = 15$m/sec

Speed of the second train $= 36 \times \frac{5}{18} = 10$m/sec

If the distance between A and B = X m

Now, when they meet then time is fix.

So, $\frac{X}{10} = \frac{X+150}{15} = 300$ m.

9. (C) Let speed of Z $= a$ m/min

Now, Speed of Y $= 3.a$ m/min

Speed of X $= 6.a$ m/min

Ratio of speeds X : Y : Z $= 6a : 3a : a$

$= 6 : 3 : 1$

Since, Distance remains constant.

So, Ratio of time $= \frac{1}{6} : \frac{1}{3} : 1$

$= 1 : 2 : 6$

Clearly, when Z takes 6 min. then X takes 1 min. if Z takes 60 min.

X takes $= \frac{1}{6} \times 60 = 10$ minutes.

10. (E) Since, Distance is constant.

$\therefore$ Speed $\propto \frac{1}{\text{Time}}$

New Speed $= \frac{4}{5}$ of the usual speed

New time taken $\frac{5}{4}$ of the usual time.

According to question,

$\frac{5}{4}$ of the usual time – usual time $= 10$

Or, $\frac{1}{4}$ of the usual time $= 10$

Usual time $= 10 \times 4 = 40$ minutes.

11. (D) Since, policeman and thief move in the same direction.

So, Relative speed $= 16 - 12 = 4$ km/hr

Since, Speed of the policeman is greater than the thief.

So, Time taken by the policeman to cover 1200 m

$= \frac{1200 \text{ metre}}{4 \text{ km/hr}} = \frac{1200}{4 \times \frac{5}{18}}$

$= 300 \times \frac{18}{5}$

$= 60 \times 18 = 1080$ seconds

$= \frac{1080}{60}$ minutes $= 18$ minutes

In 18 minutes the distance covered by the thief

$= 12 \times \frac{5}{18} \times 18 = 60$ metres

The total distance covered by the thief before he is catched = 60 metres.

12. (C) According to question,

Distance is fixed.

So, Average speed is given by

$= \frac{2V_1V_2}{V_1 + V_2} = 2 \times \frac{18 \times 36}{18 + 36}$

$= \frac{2 \times 18 \times 36}{18 + 36} = 24$ km/hr.

13. (A) Let the speed of the train in the onward = V km/hr.

Now, the speed of the train in return

$= \frac{150}{100} \cdot V = \frac{3}{2} \cdot V$ km/hr

According to question,

Total time taken $= 16 - 1 = 15$ hours

Average speed $= \frac{\text{Total Distance}}{\text{Total Time}}$

$= \frac{2V_1V_2}{V_1 + V_2}$ (As distance equal)

$\Rightarrow$ $\frac{2 \times V \times \frac{3}{2} V}{V + \frac{3}{2} V} = \frac{1000}{15}$

$$\Rightarrow \quad \frac{6V}{5} = \frac{1000}{15}$$

$$\Rightarrow \quad V = \frac{1000 \times 2{\cdot}5}{18} = 55{\cdot}55 \text{ km/hr.}$$

14. (B) Each train has to cover a distance of $\frac{1020}{2}$ km = 510 km

Let the speed of Mumbai–Chennai Express = x km/hr

So, time taken by it to reach Wadi-junction is = $\frac{510}{x}$ hr

Now, speed of Chennai–Mumbai Express = $(x + 10)$ km/hr

Now, Time taken to reach Wadi-junction = $\frac{510}{x+10}$

According to question ,

$$\frac{510}{x+10} + 2 = \frac{510}{x}$$

Or, $$510 \times \frac{10}{x(x+10)} = 2$$

Or, $$510 \times 5 = x^2 + 10x$$

Or, $$x^2 + 10x - 2550 = 0$$

Or, $$x = \frac{-10 \pm \sqrt{10^2 + 4 \times 2550}}{2}$$

$$= \frac{-10 \pm 10\sqrt{1 + 102}}{2}$$

Or, $$x = -5 \pm 5\sqrt{103}$$

Or, $$x = -5 + 5\sqrt{103} \text{ km/hr}$$

$$x \cong 45 \text{ km/hr}$$

Now, Both the trains meet after

$$= \frac{510}{45} = \frac{102}{9} = \frac{34}{3}$$

$$= 11\frac{1}{3} = 11 \text{ hr } 20 \text{ min.}$$

Hence, meeting time 6 pm + 11 hr 20 min = 5·20 am next day.

15. (A) Suppose both the trains meet at a point which is a km from Patna.

The train traveling towards Delhi travels for $(300 - a)$ km after the meeting point in 9 hours.

Its speed $= \frac{300-a}{9}$ km/hr

But it has travelled a km from Patna to reach the meeting point.

$\therefore$ Time required to travel a km $= \frac{9a}{300-a}$ hr

The train traveling towards Patna travels a km in 4 hours.

Its Speed $= \frac{a}{4}$ km/hr

But it has traveled $(300 - a)$ km to reach the meeting point.

$\therefore$ Time taken to reach the meeting points is

$$= \frac{300-a}{\frac{a}{4}} = 4 \times \left(\frac{300-a}{a}\right)$$

$$\therefore \quad \frac{9a}{300-a} = 4 \times \left(\frac{300-a}{a}\right)$$

Or, $$9a^2 = 4 \times (300-a)^2$$

$$= \{2 \times (300-a)\}^2$$

$$\therefore \quad 3a = 2 \times (300-a)$$

Or, $$5a = 2 \times 300$$

$$\therefore \quad a = 120 \text{ km.}$$

Speed of train moving towards Patna is $\frac{a}{4} = \frac{120}{4} = 30$ km/hr and Speed of train moving towards Delhi is $\frac{300-120}{9} = 20$ km/hr.

16. (C) Since, the trains are 100 km apart.

Since, trains are moving in opposite direction.

So, Relative speed = 55 km/hr + 45 km/hr

= 100 km/hr

So, time taken to collide the train

$$= \frac{\text{Distance}}{\text{Relative speed}}$$

$$= \frac{100}{100} = 1 \text{ hour}$$

So, the distance covered by bird in 1 hour

$$= 80 \times 1 = 80 \text{ km.}$$

17. (B) Since, the trains are running in the same direction.

So, Relative speed = 72 – 54 = 18 km/hr

$$= 18 \times \frac{5}{18} = 5 \text{ m/sec}$$

Time taken by the trains to cover each other

= Time taken to cover

$(100 + 120)$ m at 5 m/sec

$$= \frac{220}{5} = 44 \text{ sec.}$$

18. (B) Let the length of the train 4321 is a m. and length of the platform = b m.

Since, the train and the man is in the same direction.

$\therefore$ Relative speed = 81 – 9 = 72 km/hr

$$= 72 \times \frac{5}{18} = 20 \text{ m/sec}$$

Now, the train crosses the man in 75 seconds.

So, Length of the train $= 20 \times 75 = 1500$ m

$$\text{Now, speed of the train} = 81 \text{ km/hr}$$
$$= 81 \times \frac{5}{18} = \frac{45}{2} \text{ m/sec}$$

Now, $\dfrac{a+b}{\frac{45}{2}} = 20$

$$a + b = 450 \text{ m}$$

Since, $a = 200 \text{ m}$

$$b = 450 - 200 = 250 \text{ m}.$$

Hence, required ratio is 200 : 250 : 4 : 5.

19. (B) Let the speed of first train = x m/sec
and the speed of the second train = y m/sec.

So, length of the first train = $30x$

Length of the second train = $20y$

$\therefore$ They cross each other in 25 seconds

So, $\dfrac{30x + 20y}{x + y} = 25$

$$5y = 5x$$

$\therefore$ $\dfrac{x}{y} = \dfrac{1}{1}$

The ratio of speeds are 1 : 1.

20 (D) Let the length of first road be a km and so the length of the second road will be $(a - 20)$ km.

Speed of car I = x km/hr

Speed of car II = $(x + 40)$ km/hr

According to question ,

$$\frac{a}{x} = \frac{7}{2} \qquad \ldots(1)$$

and $$\frac{a-20}{x+40} = \frac{5}{2} \qquad \ldots(2)$$

Now, putting the value of a in equation (2), we get

$$\frac{\frac{7}{2}x - 20}{x + 40} = \frac{5}{2}$$

Or, $\dfrac{7x - 40}{2(x + 40)} = \dfrac{5}{2}$

Or, $7x - 40 = 5x + 200$

Or, $2x = 240$

Or, $x = 120$ km/hr

$\therefore$ $x + 40 = 120 + 40 = 160$ km/hr

Hence, the speed of fastest car is 160 km/hr.

21. (B) Let the up-stream = V km/hr

Let the down-stream = U km/hr

According to question,

$$\frac{d_1}{\mathrm{V}} + \frac{d_2}{\mathrm{U}} = \mathrm{T}_1 \qquad \ldots(1)$$

$$\frac{d_1'}{\mathrm{V}} + \frac{d_2'}{\mathrm{U}} = \mathrm{T}_2 \qquad \ldots(2)$$

Now, $\dfrac{1}{\mathrm{V}}(d_1 d_2' - d_1' d_2) = \mathrm{T}_1 d_2' - \mathrm{T}_2 d_2$

$$\mathrm{V} = \frac{d_1 d_2' - d_1' d_2}{\mathrm{T}_1 d_2' - \mathrm{T}_2 d_2} \text{km/hr}$$

$$\frac{d_2}{\mathrm{U}} = \mathrm{T}_1 - d_1 \times \frac{\mathrm{T}_1 d_2' - \mathrm{T}_2 d_2}{d_1 d_2' - d_1' d_2}$$

$$= \frac{\mathrm{T}_1 d_1' d_2 - d_1 \mathrm{T}_1 d_2' - d_1 \mathrm{T}_1 d_2' + d_1 \mathrm{T}_2 d_2}{d_1 d_2' - d_1' d_2}$$

$$= \frac{d_2 (d_1 \mathrm{T}_2 - \mathrm{T}_1 d_1')}{d_1 d_2' - d_1' d_2}$$

$\therefore$ $\mathrm{U} = \dfrac{d_1 d_2' - d_1' d_2}{d_1 \mathrm{T}_2 - \mathrm{T}_1 d_1'}$ km/hr.

22. (D) The pedestrian and the cyclist both are moving in opposite direction.

Let the speed of pedestrian = a km/hr

The speed of cyclist = b km/hr

Now, relative speed = $(a + b)$ km/hr

Now, Distance = Speed $\times$ Time

Or, $80 = 2(a + b)$

$$a + b = 40$$

$$-\frac{80}{b} + \frac{80}{a} = 20$$

Or, $\dfrac{4}{a} - \dfrac{4}{40 - a} = 1$

Or, $\dfrac{4(40 - 2a)}{(40 - a)a} = 1$

Or, $160 - 8a = 40a - a^2$

Or, $a^2 - 48a + 160 = 0$

Or, $a = \dfrac{48 \pm \sqrt{(48)^2 - 4 \times 160 \times 1}}{2 \times 1}$

Or, $a = \dfrac{48 \pm \sqrt{1664}}{2}$

Or, $a = \dfrac{48 \pm 40}{2}$

Or, $a = 44; 4$

$\because$ $a + b = 40$ and b cannot be in negative. So, $a = 44$ not possible and by $a = 4$; $b = 36$.

So, the required ratio is 4: 36 = 1 : 9

23. (B) Let Speed of Soni = V km/hr

Let Speed of Sita = (V – 4) km/hr

Now, Time taken by Sita and Soni as equal.

So, Time taken by Sita = $\dfrac{60 - 12}{\mathrm{V} - 4} = \dfrac{48}{\mathrm{V} - 4}$

Time taken by Soni = $\dfrac{60 + 12}{\mathrm{V} + 4} = \dfrac{72}{\mathrm{V} + 4}$

Now, $\dfrac{48}{\mathrm{V} - 4} = \dfrac{72}{\mathrm{V} + 4}$

Or, $4(\mathrm{V} + 4) = 6(\mathrm{V} - 4)$

$$2(V+4) = 3(V-4)$$
$$V = 8+12 = 20$$
$$V = 20 \text{ km/hr}$$
$$\text{Soni's speed} = 20 \text{ km/hr}$$
$$\text{Sita's speed} = 16 \text{ km/hr.}$$

Exercise B

1. (A) (i) Let the Sita's original speed = V

Now, after increasing speed the final speed = $\frac{3}{2}$V

Now, she will take $\frac{2}{3}$ T time to travel the same distance.

∴ According to question,

$$\frac{2}{3}T = 20$$
$$T = 30 \text{ sec.}$$

Now, her original time to reach her friend = 30 sec.

(ii) Original speed $= \frac{\text{Distance}}{\text{Original Time}}$

$$= \frac{240}{30} = 8 \text{ m/sec.}$$

2. (C) If the initial speed = V

$$\text{New speed} = \frac{3}{4}V$$
$$\text{New time} = \frac{4}{3}T \text{ (Where T = original time)}$$

Now, According to question,

$$\frac{4}{3}T - T = 50 - 30 = 20$$

Hence, $T = 60$ minutes

Mean if he will covers his journey in 30 minutes (Half of initial time) he will be on time.

So, Desired Speed = double of initial speed (as we know that Distance is constant).

3. (C) Let time taken = T hr

Now, the distance traveled in 30% of time of

$$20 \text{ km/hr} = 0{\cdot}3\, T \times 20 = 6\, T \text{ km}$$

The Distance traveled in 40% of time of

$$30 \text{ km/hr} = 0{\cdot}4\, T \times 30 = 12\, T \text{ km}$$

The distance traveled in 30% of time in 40 km/hr

$$= 0{\cdot}3\, T \times 40 = 12T$$

$$\text{Now, Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$
$$= \frac{6T + 12T + 12T}{T}$$
$$= 30 \text{ km/hr.}$$

4. (B) In an half hour the distance traveled by the thief

$$= 60 \times \frac{1}{2} = 30 \text{ km}$$

Now, the distance between owner and thief is 30 km.

Since, the owner and the thief move in the same direction.

So, relative speed = 75 – 60 = 15 km/hr

From Time taken by the owner in overtaking the thief

$$= \frac{30}{15} = 2 \text{ hr}$$

At 1.00 pm he will catch the thief.

5. (E) Let the speed of train be V km/hr

and the speed of car = U km/hr

In Ist Case

$$\frac{400}{V} + \frac{200}{U} = 6\frac{1}{2} = \frac{13}{2} \quad \ldots(1)$$

In 2nd Case

$$\frac{200}{V} + \frac{400}{U} = 7 \quad \ldots(2)$$

Now, equation (1) – equation (2), we get

$$\frac{800}{V} - \frac{200}{V} = 13 - 7$$
$$\frac{600}{V} = 6$$
$$V = 100 \text{ km/hr}$$

From putting the value of V in equation (2), we get

$$\frac{200}{100} + \frac{400}{U} = 7$$

Or, $$\frac{400}{U} = 7 - 2 = 5$$
$$U = 80 \text{ km/hr.}$$

6. (A) Let the speed of the slowest cart = x m/min

The speed of the fastest cart = $2x$ m/min

Thus, when they meet for the first time, the fastest cart takes one around more than the slowest cart. That is the fastest cart move 3000 more with relative speed of $(2x - x)$ m/min in 10 minutes.

$$\frac{3000}{2x - x} = 10$$
$$x = 300 \text{ m/min}$$

So, time taken by the fastest cart to complete the race is $= \frac{9000}{300} = 30$ minutes

7. (A) The time internal between the train = t

The distance between any two consecutive trains coming in the same direction as Sunita at

where a = speed of train

b = speed of Sunita

Now, $$\frac{at}{a+b} = 8$$

and $$\frac{at}{a-b} = 24 \text{ min}$$

$$\frac{a+b}{at}+\frac{a-b}{at} = \frac{1}{8}+\frac{1}{24}=\frac{4}{24}$$

$$= \frac{2a}{at}=\frac{1}{6}$$

$$t = 12 \text{ minutes.}$$

8. (E) **In the Ist case,**

When the fly is moving alongside the train.

Relative speed of the trains

$$= 60 + 90 = 150 \text{ m/sec}$$

Time taken for the two trains to collide

$$= \frac{150 \text{ m}}{150} = 1 \text{ second}$$

The speed of the fly $= 5$ m/sec

In the IInd case,

Now, Distance covered by the fly = 5 m

After this it covers a distance of 10 + 25 + 55 = 90 m

Total distance covered by the fly = 90 + 5 = 95 m

Now, this is the distance between two cars.

Now, Relative speed of the cars

$$= 20 \text{ m/sec} + 30 \text{ m/sec}$$

$$= 50 \text{ m/sec}$$

Time taken by the cars to collide

$$= \frac{95}{50}=\frac{19}{10} \text{ sec}$$

Now, speed of the fly $= 2 \times 5 = 10$ m/sec

Distance traveled by the fly

$$= 10 \times \frac{19}{10} = 19 \text{ m}$$

Total Travel by the fly $= 95 + 19 = 114$ m.

9. (B) Time taken by the man is in the ratio of 5 : 3 : 2 : 5 from X to U, U to V, V to W and W to Y respectively.

Let this time will be $5a, 3a, 2a, 5a$.

So, total time taken $= 5a + 3a + 2a + 5a = 5$ hours

Or, $a = \frac{1}{3}$ hour

Time From U to V $= 3a = 1$ hour $= 60$ minutes

Time From W to Y $= 5a = 5 \times \frac{1}{3} = 100$ minutes

Now, Required difference

$$= 100 - 60 = 40 \text{ minutes.}$$

10. (E) Let length of the trains are a metre and b metre respectively and V and U are their speed respectively.

When they move in opposite direction.

Time taken to cross each other

$$= \frac{a+b}{\text{V}+\text{U}} = 5$$

To find V we need a, b and U.

So, length of trains are not sufficient.

Correct answer is (E).

11. (A) Let the speed of the train be a metres/sec.

Time taken to cross a tree $= \frac{\text{Length of the train}}{\text{Speed of the train}}$

and time taken to cross a platform

$$= \frac{\text{Length of train + Length of platform}}{\text{Speed of the train}}$$

So, First gives $13 = \frac{l}{a}$

$$l = 13a$$

Second gives $27 = \frac{l+250}{a}$

$$a = \frac{125}{7} \text{ m/sec.}$$

Thus, First and Second give the speed of the train.

12. (B) Let a be the speed of the train and l be length of the train.

Now, From (i)

$$20 = \frac{l+300}{a} \quad \ldots(1)$$

From (ii) $15 = \frac{2l}{a}$ …(2)

From (iii) $10 = \frac{l}{a}$ …(3)

Clearly, (i) and (ii) gives us answer

and (i) and (iii) also provides answer.

13. (A) From (i)

Speed of up-stream $= \frac{20}{5} = 4$ km/hr

From (ii)

Down-stream speed $= \frac{20}{4} = 5$ km/hr

From (iii)

Speed of boat $= 2a$ km/hr

Speed of stream $= a$ km/hr

Now, From (i) and (ii)

Speed of stream $= \frac{5-4}{2} = \frac{1}{2}$ km/hr

From (ii) and (iii), we get

$$a = 5 \text{ km/hr}$$

So, speed of boat $= 10$ km/hr

Clearly, any two of the three will give the answer.

14. (C) **In First Case,**

In one minute let the pipe B can fill $\frac{1}{x}$ part of the tank

$\therefore$ In one minute pipe A can fill $\frac{3}{x}$ part of the tank

In one minute (A + B) can fill

$$= \frac{1}{x}+\frac{3}{x}=\frac{4}{x} \text{ part of the tank}$$

$\therefore$ In 10 minutes can fill $= \frac{40}{x}$ tank

According to question,

$$\frac{40}{x} = \frac{2}{3}$$

$$\Rightarrow \qquad x = 60$$

So, B can fill the tank in 60 minutes.

A can fill the tank in 20 minutes.

In Second Case,

B can fill the tank in 60 minutes.

So A can fill the tank in 20 minutes.

15. (E) From (ii)

In one hour A can fill $\frac{1}{16}$ part of the tank.

Let us suppose in one hour B can fill $\frac{1}{x}$ part of the tank.

Now, From (i) $\quad \frac{1}{16} = \frac{150}{100} \times \frac{1}{x} = \frac{3}{2x}$

$$x = 24$$

So, B can fill the tank in 24 hours.

In one hour (A + B) can fill $\left(\frac{1}{16} + \frac{1}{24}\right)$ part of the tank.

In one hour (A + B) can fill $= \frac{5}{48}$ part of the tank

Thus, (1) and (2) give the answer.

From (ii)

In one hour B can fill $\frac{1}{24}$ part of the tank

Now, From (ii) and (iii), we get the same answer

and From (i) and (iii), we also get the same answer.

16. (D) We know that the time taken by the train to cross a stationary engine

$$= \frac{\text{Length of train + Length of engine}}{\text{Speed of the train}} = 20$$

Clearly, to find the speed of the train, the length of the train and the length of the engine both must be known in the above statements.

10 Time and Work

Generally, in our daily life we do everything's are known as work. For doing these work we require time.

So, time and work are related to each other.

Since, a person performs / complete work and it takes time to do any work.

So, number of person also affects the work and the time.

Here, we have some important relevant relationship between person, work and time.

1. More work requires more persons.

If time remains constant it means for doing any work time is fixed. Then, Person is directly proportional to the work.

If we have to do a lot of work it need a lot of person.

So, $$\text{Person} \propto \text{Work} \qquad \ldots(1)$$

2. If the work to be done kept constant, more persons will take less time to complete it and *vice-versa*.

Clearly,

$$\text{Person} \propto \frac{1}{\text{Time}}$$

$$\text{Time} \propto \frac{1}{\text{Person}} \qquad \ldots(2)$$

3. If to do any work, the number of person is fixed then, time depends on work and *vice-versa*.

Clearly, more work requires more time.

$$\text{Time} \propto \text{Work}$$

$$\text{Work} \propto \text{Time} \qquad \ldots(3)$$

Now, combining (1), (2) and (3)

We have elegant equation,

$$\frac{\text{Person} \times \text{Time}}{\text{Work}} = \text{Constant}$$

Or $$\frac{P_1 \times T_1}{W_1} = \frac{P_2 \times T_2}{W_2}$$

Illustration 1.

A farmer can dig a trench in 100 days

(i) How much he can dig in 1 day?

(ii) How much he can dig in 4 days?

(iii) How much days he will take to dig $\frac{3}{4}$th of the trench ?

(iv) In how many days he can dig $\frac{1}{20}$th of the trench ?

(v) In how many days he can dig 5 such trench?

Solution :

(i) A farmer in 100 days can dig a trench.

∴ A farmer in one day can dig $\frac{1}{100}$th trench

(ii) A farmer in one day can dig $\frac{1}{100}$th trench

∴ A farmer in 4 days can dig $\frac{4}{100}$th trench

$= \frac{1}{25}$ trench

(iii) Since, the farmer can dig a trench in 100 days

∴ The farmer can dig $\frac{3}{4}$th trench in $\frac{3}{4} \times 100$

$= 75$ days

(iv) The farmer can dig $\frac{1}{20}$th trench in $\frac{100}{20}$ days

$= 5$ days

(v) Since, the farmer can dig one trench in 100 days

∴ The farmer can dig 5 trench in 500 days.

Illustration 2.

7 persons can do a certain piece of work in 21 days.

(i) How many persons required to do the same work in 14 days ?

(ii) In how many days can 14 persons complete the same work ?

Solution :

Since, in 21 days, 7 persons can do a certain work.

So, in 1 day 7×21 persons can do a certain work.

in 14 days $\frac{7 \times 21}{14}$ persons can do a certain work

$= \frac{21}{2}$ persons

(ii) Since, 7 persons can do a certain work in 21 days

∴ 1 persons can do a certain work in 21×7 days

∴ 14 persons can do a certain work in $\frac{21 \times 7}{14}$ days

$= \frac{21}{2}$ days.

Illustration 3.

80 persons can do a certain job in 5 days. How many persons are required to do the same job in 20 days ?

Solution :

Here, we have $W_1 = W_2$

Given $$\frac{P_1 \times T_1}{W_1} = \frac{P_2 \times T_2}{W_2}$$

$$P_1 \times T_1 = P_2 \times T_2$$

Or, $$P_2 = \frac{T_1 \times P_1}{T_2}$$

Here, $P_1 = 80, T_1 = 5$ days, $T_2 = 20$ days

$$80 \times 5 = P_2 \times 20$$

$$P_2 = 20 \text{ persons.}$$

Illustration 4.

10 persons can make 40 toys in 5 hours. How many toys can 8 persons make in 10 hours ?

Solution :

Given that $P_1 = 10, P_2 = 8, T_1 = 5$ hours, $T_2 = 10$ hours, $W_1 = 40$ days, $W_2 = ?$

$$\frac{P_1 \times T_1}{W_1} = \frac{P_2 \times T_2}{W_2}$$

Or, $$\frac{10 \times 5}{40} = \frac{8 \times 10}{W_2}$$

$$W_2 = \frac{40 \times 8}{5}$$

$$W_2 = 64 \text{ toys.}$$

Illustration 5.

25 men can cut 10 trees in 5 hours. In how many hours 20 men can cut 15 trees ?

Solution :

Given that $P_1 = 25, P_2 = 20, W_1 = 10$ trees, $W_2 = 15$ trees, $T_1 = 5$ hours, $T_2 = ?$

$$\frac{P_1 \times T_1}{W_1} = \frac{P_2 \times T_2}{W_2}$$

Or, $$\frac{25 \times 5}{10} = \frac{20 \times T_2}{15}$$

$$T_2 = \frac{25 \times 75}{200} = \frac{75}{8} \text{ hours.}$$

Illustration 6.

A man C can do a work in X days and another man D can do the same work in Y days, then show that C and D together can do the some work in $\frac{XY}{X+Y}$ days ?

Solution :

Man C can do a work in X days

$\therefore$ man C can do in one day $\frac{1}{X}$ work

Similarly, in one day man D can do $\frac{1}{Y}$ work

Now, when C and D work together they can do in one day $\left(\frac{X+Y}{XY}\right)$ work

$$= \left(\frac{X+Y}{XY}\right)$$

Since, $\left(\frac{X+Y}{XY}\right)$ work is done by A and B together in one day

$\therefore$ 1 work is done by A and B together in

$$\frac{1}{\frac{X+Y}{XY}} = \frac{XY}{X+Y}$$

Clearly, C and D work together the same work in $\frac{XY}{X+Y}$ days.

Illustration 7.

C can do a work in 5 days and D in 25 days. How many Hours will they take to do the same work, if both work together and in a day their work hour is 6 ?

Solution :

C and D work together the same work in $\frac{XY}{X+Y}$ days

$$\therefore \quad = \frac{25 \times 5}{5 + 25} = \frac{125}{30} = \frac{25}{6}$$

$$= \frac{25}{6} \times 6 \text{ hours} = 25 \text{ hours.}$$

Illustration 8.

A and B can together does a piece of work in 20 days, B alone can do it in 5 days. In how many days can A alone do it ?

Solution :

A and B can together do a piece of work in $\frac{XY}{X+Y}$ days.

According to question,

$$20 = \frac{5X}{X+5}$$

Or, $$20X + 100 = 5X$$

$$X = \frac{100}{3} \text{ days.}$$

Illustration 9.

Ram, Shyam, Rahim and Abdul can do a piece of work in X, Y, Z and W days respectively. In how many days they can do this work if they work together ?

Solution :

In one day Ram can do $\frac{1}{X}$ work

In one day Shyam can do $\frac{1}{Y}$ work

In one day Rahim can do $\frac{1}{Z}$ work

In one day Abdul can do $\frac{1}{W}$ work

Now, in one day all can do together can do

$$\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}+\frac{1}{W} \text{ work}$$

Then, they can together do the same work in

$$= \frac{1}{\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}+\frac{1}{W}} \text{ days}$$

$$= \frac{X.Y.Z.W}{Y.Z.W + X.Z.W + X.Y.W + X.Y.Z} \text{ days}$$

Illustration 10.

A, B, C and D can do a piece of work in 3, 6, 9 and 12 days respectively. In how many days they can do this work if they work together ?

Solution :

According to formula,

A, B, C and D can do a piece of work in

$$= \frac{3 \times 6 \times 9 \times 12}{6 \times 9 \times 12 + 3 \times 9 \times 12 + 3 \times 6 \times 9 + 6 \times 9 \times 12}$$

$$= \frac{36}{25} \text{ days.}$$

Illustration 11.

X and Y can do a piece of work in a days Y and Z in b days, Z and X in c days. How long would each take to do the same work separately ?

Solution :

In one day X and Y can do together $\frac{1}{a}$ work ...(1)

In one day Y and Z can do together $\frac{1}{b}$ work ...(2)

In one day Z and X can do together $\frac{1}{c}$ work ...(3)

$\therefore$ In one day X, Y and Z can do together $\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$work

$$\text{Now, Z's one day work} = \frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\frac{1}{a}$$

$$= \frac{1}{2}\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)$$

Z's one day work is $\frac{1}{2}\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)$

Clearly, Z alone can do the work in $\dfrac{1}{\frac{1}{2}\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)}$

$$= \frac{2abc}{c.a + a.b - b.c} \text{ days}$$

Similarly,

$$\text{X alone can do the work} = \frac{2abc}{a.b + b.c - c.a}$$

$$\text{Y alone can do the work} = \frac{2abc}{a.c + c.b - b.a}.$$

Illustration 12.

X and Y can do work in 10 days, Y and Z in 20 days, Z and X in 30 days. In how many days can they complete it, if they work together ?

Solution :

$$\text{(X + Y)'s one day work} = \frac{1}{10}$$

$$\text{(Y + Z)'s one day work} = \frac{1}{20}$$

$$\text{(X + Z)'s one day work} = \frac{1}{30}$$

2(X + Y + Z)'s one day work

$$= \frac{1}{10}+\frac{1}{20}+\frac{1}{30}$$

$$= \frac{1}{10}\left(1+\frac{1}{2}+\frac{1}{3}\right)$$

$$= \frac{1}{10} \times \left(\frac{6+3+2}{6}\right)$$

$$= \frac{1}{10} \times \frac{11}{6}$$

So (X + Y + Z)'s one day work $= \frac{11}{120}$

So (X + Y + Z)'s can work $= \frac{120}{11}$ days

$$\cong 11 \text{ days.}$$

Illustration 13.

X and Y together can do a piece of work in 24 days, Y and Z together can do it in 12 days. X starts the work and works on it for 8 days, then Y takes it up and works for 10 days. Finally Z finishes the work in 16 days. In how many days can each do the work when doing it separately ?

Solution :

$$\text{(X + Y)'s one day work} = \frac{1}{24} \quad ...(1)$$

$$\text{(Y + Z)'s one day work} = \frac{1}{12} \quad ...(2)$$

Now, according to question,

$$1 = \frac{8}{24}+\frac{2}{12}+\frac{14}{d}$$

$$1 = \frac{8+4}{24}+\frac{14}{d}$$

$$1 = \frac{1}{2}+\frac{14}{d}$$

$$d = 28$$

Hence, Z can do it in 28 days individually .

$$\text{Y individual} = \frac{1}{12} - \frac{1}{28} = \frac{28-12}{28 \times 12}$$

$$= \frac{16}{28 \times 12} = \frac{1}{21}$$

Hence, Y can do it in 21 days individually .

$$\text{X individual} = \frac{1}{21} - \frac{1}{24}$$

$$= \frac{8-7}{3 \times 7 \times 8} = \frac{1}{168}$$

Hence, X can do it in 168 days individually.

Illustration 14.

X and Y can do a work in 12 days while Y and Z can do it in $6\frac{2}{3}$ days. After X had worked on it for 3 days and Y for 4 days, Z finished the work in 7 days. In how many days could each do the work separately ?

Solution :

Let the daily work of X, Y and Z be a, b, c respectively, then we can write

$$a + b = \frac{1}{12}$$

$$a = \frac{1}{12} - b \qquad \ldots(1)$$

$$b + c = \frac{3}{20}$$

$$c = \frac{3}{20} - b \qquad \ldots(2)$$

$$3.a + 4.b + 7.c = 1 \qquad \ldots(3)$$

Now, putting the value of a and b in equation (3)

$$\therefore \quad 3\left(\frac{1}{12} - b\right) + 4.b + 7\left(\frac{3}{20} - b\right) = 1$$

Or, $$-6b + \frac{1}{4} + \frac{21}{20} = 1$$

Or, $$6.b = \frac{5+21}{20} - 1$$

$$= \frac{26-20}{20}$$

$$6.b = \frac{6}{20}$$

$$b = \frac{1}{20}$$

$$a = \frac{1}{12} - \frac{1}{20} = \frac{5-3}{4 \times 3 \times 5}$$

$$= \frac{2}{4 \times 3 \times 5} = \frac{1}{30}$$

$$c = \frac{3}{20} - \frac{1}{20} = \frac{1}{10}$$

$\therefore$ a, b and c are $\frac{1}{30}, \frac{1}{20}, \frac{1}{10}$ respectively.

Therefore, a, b and c can do the work separately in 30, 20 and 10 days respectively.

Illustration 15.

A tank can be filled in 40 minutes but there is a leakage in it which can empty the full tank in 80 minutes. In how many minutes it can be filled ?

Solution :

In 40 minutes one tank is filled.

In one minute $\frac{1}{40}$ part of tank is filled.

In 80 minutes one tank is made empty.

In one minute $\frac{1}{80}$ part of tank is made empty

Now, In one minute $\left(\frac{1}{40} - \frac{1}{80}\right)$ part of tank is filled

$\therefore$ In one minute $\frac{2-1}{80} = \frac{1}{80}$ part of tank is filled

$\therefore$ One tank is filled in 80 minutes.

Concept of Efficiency

A man has efficiency 50 per day it means he can do $\frac{1}{2}$ half work in a day.

Clearly, efficiency indicates or measure time in which the whole work is done.

Efficiency is related to time, days, year, month, hours, seconds and minutes.

Efficiency is inversely proportional to time

$$\in \; \propto \; \frac{1}{t}$$

Illustration 16.

Ram can do a job in 9 days and Shyam can do the same job in 6 days. Find efficiency and in how many days working together they can complete the job ?

Solution :

$$\text{Ram's one day work} = \frac{1}{9}$$

$$\text{Shyam's one day work} = \frac{1}{6}$$

(Ram + Shyam)'s one day work

$$= \frac{1}{9} + \frac{1}{6} = \frac{2+3}{3 \times 3 \times 2}$$

$$= \frac{5}{3 \times 3 \times 2} = \frac{5}{18}$$

$$\text{Efficiency of Ram} = \frac{1}{9} \times 100\% = 11\frac{1}{9}\%$$

$$\text{Efficiency of Shyam} = \frac{1}{6} \times 100\% = 16\frac{4}{6}\%$$

(Ram + Shyam) can do work in

$$= \frac{18}{5} \text{ days} = 3\frac{3}{5} \text{ days}$$

Illustration 17.

A is twice as efficient as B and is therefore able to finish a piece of work in 40 days less than Q. Find the time in which A and B can complete the work individually ?

Solution :

Efficiency of $A : B = 2 : 1$

A requires 2.X days and B requires X days.

According to question,

$$2.X - X = 40$$

Or, $X = 40$

$\therefore$ $2.X = 2 \times 40 = 80$ days

$X = 40$ days

Thus, A can finish the work in 80 days and B can finish the work in 40 days.

Illustration 18.

If 25 persons can do a piece of work in 5 days then calculate the number of persons required to complete the work in 10 days ?

Solution :

We know that

Number of persons / man / workers X days = work

We represent the person / man / worker = P

Day = D

Work = W

Now, For the same work

$$P_1 . D_1 = W_1 \quad \ldots(1)$$

$$P_2 . D_2 = W_2 \quad \ldots(2)$$

$$P_1 . D_1 = P_2 . D_2$$

$\therefore$ $25 \times 5 = P_2 \times 10$

Or, $P_2 = \dfrac{25 \times 5}{10}$

$P_2 = \dfrac{25}{2}$ persons

Illustration 19.

6 boys and 8 girls finish a job in 6 days and 16 boys and 10 girls finish the same job in 4 days. In how many days working together 1 boy and 1 girl can finish the work ?

Solution :

We have

Job = work = Person $\times$ Day

In Ist case,

$$6(6.B + 8.G) = \text{Work} \quad \ldots(1)$$

$$4(16.B + 10.G) = \text{Work} \quad \ldots(2)$$

Now, from equation (1) and (2), we get

$$(-64 + 36).B + (48 - 40).G = 0$$

$$-28.B + 8.G = 0$$

$$28.B = 8.G$$

Or, $14.B = 4.G$

$$7.B = 2.G \quad \ldots(3)$$

Clearly, the work of 2 girls is equal to 7 boys.

In Ist cast,

$$6 \text{ boys} = 6 \times \frac{2}{7} \text{girls} = \frac{12}{7} \text{ girls}$$

$$6 \text{ boys} + 8 \text{ girls} = \left(\frac{12}{7} + 8\right) = \frac{68}{7} \text{ girls}$$

Now, $\dfrac{68}{7}$ girls finish the a job in 6 days

$\therefore$ 1 girl finish the a job in $\dfrac{68}{7} \times 6$ days

$= \dfrac{408}{7}$ days

1 girl finishes the a job $= \dfrac{408}{7}$ days

Again, $8 \text{ girls} + 6 \text{ boys} = 8 \times \dfrac{7}{2} \text{boys} + 6 \text{ boys}$

$= 34$ boys

$\therefore$ 34 boys finish the job in 6 days

$\therefore$ 1 boys finish the job in $= 34 \times 6$ days

1 boy finish the job in $= 204$ days

One girl's one day job $= \dfrac{7}{408}$

One boy's one day job $= \dfrac{1}{204}$

One boy and girl one day job

$$= \frac{7}{408} + \frac{1}{204} = \frac{9}{408}$$

Hence, they will finish the work together in $\dfrac{408}{9} \cong 46$ days.

Illustration 20.

Pipe X can fill a tank in 12 minutes and pipe Y can fill it in 36 minutes. If both the pipes are opened to fill an empty tank. In how many minutes will it be full ?

Solution :

First method :

In 12 minutes pipe X can fill one tank

$\therefore$ In one minute pipe X can fill $\dfrac{1}{12}$ part of tank

Similarly, in one minute pipe Y can fill $\dfrac{1}{36}$ part of tank

Now, in one minute both pipes can fill $\left(\dfrac{1}{12} + \dfrac{1}{36}\right)$ part of tank

$$= \frac{3 + 1}{36} = \frac{1}{9} \text{ part of tank}$$

$\therefore$ Both the pipes can fill one tank in 9 minutes.

Second Method :

$$\text{Efficiency of pipe X} = \frac{100}{12} = \frac{25}{3}\%$$

$$\text{Efficiency of pipe Y} = \frac{100}{36} = \frac{25}{9}\%$$

Now, Efficiency of both the pipe to fill tank

$$= \left(\frac{25}{3} + \frac{25}{9}\right)\% = \frac{100}{9}\%$$

Now, time required to fill the tank

$$= \frac{100}{\frac{100}{9}} = 9 \text{ minutes.}$$

Illustration 21.

X can do a piece of work in 12 days, Y can do it in 16 days and Z can do it in 20 days. In how many days they can complete the work together ?

Solution :

$$\text{Efficiency of X} = \frac{100}{12}\% = \frac{25}{3}\%$$

$$\text{Efficiency of Y} = \frac{100}{16}\% = \frac{25}{4}\%$$

$$\text{Efficiency of Z} = \frac{100}{20}\% = 5\%$$

Now, if X, Y and Z work together, then

$$\text{Efficiency} = 5 + \frac{25}{4} + \frac{25}{3} = \frac{60 + 75 + 100}{12} = \frac{235}{12}\%$$

$$\text{Now, time taken to do work} = \frac{100}{\frac{235}{12}} = \frac{100}{235} \times 12 = \frac{240}{47} \text{ days.}$$

Illustration 22.

A group of boys can do a certain piece of work in 25 days. If the group had 10 more boys, the work could be done in 10 days less. How many boys are there in the group ?

Solution :

Let the number of boys = X

According to question,

X boys can do the work in 25 days

While (X + 10) can do the work in 15 days

In both the cases total work done remains the same.

$$X \times 25 = (X + 10) \times 15$$

$$10 \times X = 150 = 15 \times 10$$

$$X = \frac{15 \times 10}{10} = 15 \text{ boys}$$

Clearly, Formula :

Original no. of boys

$$= \frac{\text{No. of additional boys} \times \text{No. of days taken by in 2nd case}}{\text{No. of less days}}$$

Illustration 23.

A certain number of men can complete a job in 30 days. If there were 5 men more, it could be completed in 10 days less. How many men were in the beginning ?

Solution :

From formula :

Original no. of men

$$= \frac{\text{No. of additional men} \times \text{No. of days taken by in 2nd case}}{\text{No. of less days}}$$

$$= \frac{5 \times 20}{10}$$

$\therefore$ Original no. of men = 10.

Work and Wages

Illustration 24.

Ram, Shyam and Rajesh can do a piece of work in d_1, d_2, d_3 days respectively. If they work together, in what proportion should their earnings be divided amongst them ?

Solution :

$$\text{Ram's one day work} = \frac{1}{d_1}$$

$$\text{Shyam's one day work} = \frac{1}{d_2}$$

$$\text{Rajesh's one day work} = \frac{1}{d_3}$$

Their earnings are also directly proportional to their ones day work.

Now, Ram's share : Shyam's share : Rajesh's share

$$= \frac{1}{d_1} : \frac{1}{d_2} : \frac{1}{d_3}$$

$$= d_2 \times d_3 : d_1 \times d_3 : d_2 \times d_3$$

The wages earned by a worker is directly proportional to the amount of work he does.

Wages are distributed in direct proportion to the amount of work done by individual workers.

Total wages = One person's one day work × Number of person × Number of days

Illustration 25.

Sita can do a work in 3 days while Gita can do it in 6 days. Both of them work together to do the work. If the total amount paid for the work is Rs. 120. How much is Sita share in it ?

Solution :

$$\text{Sita's one day work} = \frac{1}{3}$$

$$\text{Gita's one day work} = \frac{1}{6}$$

Now, shares ratio of Sita and Gita is given by

$$\text{Sita's share : Gita's share} = \frac{1}{3} : \frac{1}{6} = 2 : 1$$

$$\text{Since, total wage} = \text{Rs. } 120$$

$$\therefore \quad \text{Sita's share} = \frac{2}{3} \times 120 = \text{Rs. } 80$$

$$\therefore \quad \text{Gita's share} = \frac{1}{3} \times 120 = \text{Rs. } 40$$

Illustration 26.

Sanjay and Sheela contract to do a work together for Rs. 360. Sanjay can do it in 8 days and Sheela alone in 12 days. But with the help of Gita they finish it in 4 days. How is the money to be divided among them ?

Solution :

$$\text{Sanjay's one day work} = \frac{1}{8}$$

$$\text{Sanjay's 4 days work} = \frac{4}{8} = \frac{1}{2}$$

Similarly,

$$\text{Sheela's 4 days work} = \frac{4}{12} = \frac{1}{3}$$

Now, Sanjay, Sheela and Gita are working only 4 days.

$$\text{So,} \quad \text{Gita's 4 days work} = 1 - \left(\frac{1}{2} + \frac{1}{3}\right) = \frac{1}{6}$$

$$\therefore \text{ The ratio of their shares} = \frac{1}{2} : \frac{1}{3} : \frac{1}{6} = 3 : 2 : 1$$

$$\text{Sanjay's share} = \frac{3}{6} \times 360 = \text{Rs. } 180$$

$$\text{Sheela's share} = \frac{2}{6} \times 360 = \text{Rs. } 120$$

$$\text{Gita's share} = \frac{1}{6} \times 360 = \text{Rs. } 60$$

Exercise A

1. 10 men can make 20 shirts in 5 days working 5 hours/day. How many shirts can 20 men make in 10 days working 8 hours/day?
2. Ram can type 800 pages in 25 days working 5 hours/ day. In how many days he can type 1000 pages working 10 hours per day?
3. 6 workers can make 12 toys in 5 days working 4 hours per day. How many toys can 12 workers make in 8 days working 10 hours per day?
4. X can do a piece of work in 20 days while Y can do it in 5 days. In how many days can X and Y working together it?
5. Ram and Shyam can do a work in 16 days. If Ram can do the same work in 21 days. In how many days Shyam alone can finish the work?
6. Sita, Gita, Radha, Shalani can do a certain work in 3, 6, 12, 9 days respectively. In how many days can they working together, finish the same job?
7. Ram and Rahim can do a work in 8 days. Rahim and Krishna in 12 days, Krishna and Ram in 16 days. In how many days can they complete it, If they work together?
8. Ram and Raju can do a work in 15 days. Raju and Rakesh in 20 days. Rakesh and Ram in 12 days. In how many days can they complete it. If (i) they work together. (ii) They work separately.
9. 4 men or 5 women can do a certain piece of work in 40 days. How many days will 8 men and 15 women take to do the same work, when working together?
10. 16 men and 8 women can do a certain piece of work in 6 days. 8 men and 6 women can do this work in 10 days. In how many days 32 men and 32 women can finish the same work?
11. 5 men and 7 women can make 20 shirts in 6 days. 3 men and 4 women can make 8 shirts in 2 days. How many women work with 8 men. So that they can make 40 shirts in 8 days?
12. Ram, Shyam and Gita together earn Rs. 900 in 12 days. Ram and Shyam together earn Rs. 300 in 5 days. Shyam and Gita together earn Rs. 320 in 8 days. Find the earning of each?
13. 5 girls and 4 boys earn Rs. 660 in 3 days. 10 boys and 20 grils earn Rs. 3500 in 5 days. In how many days can 6 boys and 4 girls earn Rs. 1300?

14 8 taps are fitted to a water tank. Some of them are water taps to fill the tank and the remaining are outlet taps used to empty the tank. Each water tap can fill the tank in 36 hours and each outlet tap can empty it in 48 hours. On opening all the taps, the tank is filled in 6 hours. Find the number of inlet water taps?

15 The quantity of water flowing through a pipe is proportional to square of its diameter. A tank has three inlets of diameters 2 cm., 4 cm. and 6 cm. If the smallest inlet can fill the tank in 4 hours. In how much time can all the three inlets fill the tank?

16 Men, Women and children are employed in a factory, the amount of work done by a man, a woman and a child in a given being 3 : 2 : 1. They are paid wages according to the amount of work done by each. In the factory there are 20 men, 12 women and 6 children and their total wages amount to Rs. 270. How much will the daily wages be if there are 30 men, 25 women and 20 children in all?

17 An overhead cubical cistern with an edge equal to 2 meter has water flowing into it from four pipes. Pipe A can pour in 80 litres per minute. While the second

pipe B can fill 30 litres per minute. There is another pipe C that can fill 20 litres of water per minute and the last pipe D can fill 10 litres of water per minute. Of the given pipes B, C and D each of them shall act as inlet pipes till the water level in the tank is below their respective heights. The amount of water level rises above the pipe, it starts acting as an outlet pipe with a drain capacity same as the pouring capacity assume that pipes to be of negligible diameter.

If pipes B, C, D are at heights of 0.28 m., 0.5 m., 0.7 m. respectively. Then find the time taken to fill the cistern completely?

18 A, B, C, D and E are five taps capacity of B is twice that of A, capacity of C is 3 times that of A, capacities of D and E are 4 and 5 times that of A respectively. In First Case A, D and E act as input pipes and B and C are out put pipes. In second Case C, D and E act as input pipes and A and B act as out put pipes. If A and D working together as input pipes can fill the tank in 4 hours, then what is the difference in time required to fill the tank in the first and second cases stated above?

19 A task is assigned to a group of 'n' men not all of whom have the same capacity to work. Every day exactly two men out of the group work on the tasks with no same pair of men working together twice. Even after all the possible pairs have worked once, all the men together had to work for exactly one day more to finish the task. Find the number of days that will be required for all the men working together to finish the work?

Exercise B

1. Ram can do a piece of work in 8 days, Shyam can do it in 10 days, with the help of Rahim, they finish it in 4 days. In how many days Rahim complete the whole work alone ?
2. Gita is twice as good a workman as Lila and therefore Gita takes 5 days less than Lila to finish the work individually. If Gita and Lila working together complete the work in 5 days, then how many days are required by Lila to complete the work alone ?
3. Ganga, Jamuna and Janki can do a piece of work, working together in one day. Ganga is five time efficient as Janki and Jamuna takes the trice the number of days as Janki takes to do it alone. What is the difference between the number of days taken by Ganga and Jamuna ?
4. Rajesh is renowned packager of fruits in Patna. He packs 24 apples or 36 guavas everyday working 6 hours per day. His wife Reena also helps him. She packs 20 apples or 25 guavas working 5 hours per day. Rajesh has to pack 4000 apples and 4400 guavas with the help of his wife. They work alternatively, each day 8 hours. His wife started packaging on the first day and works on every alternate days. Similarly Rajesh started his work one second day and worked alternatively till the completion of the work. In how many days the work will be finished ?
5. A tank is connected with 10 pipes. Some of them are inlet pipes and other is working as outlet pipes. Each of the inlet pipes can fill the tank in 12 hours, individually, which each of those that empty the tank can empty it in 8 hours individually. If all the pipes are kept open when the tank is full. It will take exactly 8 hours for the tank to empty. How many of these are inlet pipes ?
6. Two pipes X and Y can fill a tank in 25 hours and 20 hours respectively. Hari opened the pipes X and Y to fill an empty tank and some times later he closed the pipes X and Y when the tank was supposed to be full. After that it was found that the tank was emptied in 5 hours because an outlet pipe Z connected to the tank was open from the beginning. If Hari closed the pipe Z instead of closing pipes X and Y the remaining tank would have been filled in what time ?
7. Pipe P takes $\frac{1}{2}$ of the times required by pipe Q to fill the empty tank individually. When an outlet pipe R is also opened simultaneously with pipe P and pipe Q. It takes $\frac{3}{4}$ more time to fill the empty tank than it takes, when only pipe P and pipe Q are opened together. If it takes to fill 21 hours when all the three pipes are opened simultaneously, then in what time pipe R empty the full tank operating alone ?
8. Sona, Mona and Soni are three friends. Sona and Mona are twins. Soni takes 2 days more than Sona to complete the work. If Mona started a work and 3 days later Sona joins her, then the work gets completed in 3 more days working together Sona, Mona and Soni can complete thrice the original work in 6 days. In how many days Sona can complete twice the original work with double the efficiency working alone ?
9. Rohan can do a work in 15 days while Ramu can do the same work in 10 days. They started work together. After 5 days Rohan left the work and Ramu completed it. For how many days Ramu worked more than the number of days required, when both worked together ?
10. The ratio of efficiency of A is to C is 3 : 2 the ratio of number of days taken by B & A is 2 : 1 A takes 12 days less than C when A and C complete is the work individually. B and C started the work and left after 2 days. Find the number of days taken by A to finish the remaining work ?
11. The number of days required by X, Y and Z to work individually is 4, 6 and 8 respectively. They started a work doing it alternatively. If X has started then followed by Y and so on. How many days are needed to complete the whole work ?

12. Kamal, Krishna and Raju started a work together for Rs. 900. Kamal and Krishna did $\frac{2}{5}$ of the total work while. Krishna and Raju together did $\frac{4}{5}$ of the total work. What is the amount of greatest efficient person ?

13. If 2 men or 3 women or 4 boys can do a piece of work in 52 days. Then the same of piece of work will be done by 1 man,1 women and 1 boy. What is the required number of days to complete the work if they all work together ?

14. If 2 men or 3 women or 4 boys can do a piece of work in 104 days, then the same piece of work will be done by 1 man, 1 woman and 1 boy. What are the required days ?

15. 3 children and 1 man complete a certain piece of work in 6 days. Each child takes twice the time taken by a man to finish the work. In how many days will 5 men finish the same work ?

16. Six men and five women can do a piece of work in 4 days, while four men and two women can do the same work in 7 days. If Rs. 48 is given to a women for her contribution towards work, per day, then what is the amount received by a man per day ?

17. 4 boys and 6 girls can do a piece of work in 12 days. 7 boys and 8 girls can do the same work in 7 days. In how many days 2 boys and 3 girls complete the same work working together ?

18. (Y – 2) men do a piece of work in Y days and (Y + 7) men can do 75% of the same work in (Y – 10) days. Then in how many days can (Y + 10) men finish the work?

19. 33 girls can do a job in 30 days. If 44 girls started the job together and after every day of the work, one girl leaves. What is the minimum number of days required to complete the whole work ?

20. Z is twice efficient as X, Y takes thrice as many days as Z. X takes 8 days to finish the work alone. If they work in pairs (*i.e.* XY, YZ, ZX) starting with XY on the first day then YZ on the second day and ZX on the third day and so on, then how many days are required to finish the work ?

21. Gita is twice efficient as Sita and Sita can do a piece of work in 15 days. Sita started the work and after a few days Gita joined her. They completed the work in 11 days from the starting. For how many days they worked together ?

22. Rani, Sheela and Soni can complete a piece of work in 9, 18, 24 days respectively. They started the work together and Rani left after 2 days before the completion of the work and Sheela left 4 days before the completion of the work. In how many days was the work completed ?

23. A piece of work can be completed by 10 boys and 6 girls in 18 days. Boys works 9 hours per day while Girls works 7.5 hours per day. Per hour efficiency of a girl is $\frac{2}{3}$rd of a boys's efficiency. In how many more days the work will be completed by 5 boys and 9 girls ?

24. Pipes X and Y can fill a tank in 15 hours and 10 hours respectively. Today morning as soon as the pipes were opened, air bubbles were generated in both the pipes due to which pipe X could work with only $\frac{1}{2}$ of its capacity and pipe B could work with $\frac{1}{3}$ of its capacity. After some time, the bubbles in both the pipes burst and then the tank could be filled in 4 hours. For how much time were the air bubbles present in the pipes ?

25. Large, medium and small ships are used to bring water 2 large ships carry as much water as 5 small ships, 2 medium ship carry the same amount of water as one large ships and 2 small ships. 5 large, 8 medium and 12 small ships, each made $42\frac{1}{5}$ journeys and brought a certain quantity of water. In how many journey would 20 large, 14 medium and 24 small ships, bring the same quantity of water ?

26. Two pipes X and Y can separately fill a cistern in 15 and 20 min. respectively and waste pipe C can carry off 10 litres per minutes. If all the pipes are opened when the cistern is full, it is emptied in 2 hours. How many litres does the cistern hold?

27. The tunnel-boring machines, working at the two ends of a tunnel, have to complete a work in 60 days. If the first machine does 30% of the work assigned to it and the second $26\frac{2}{3}$%, then both will drive 60 metres of tunnel. If the first machine had done $\frac{2}{3}$ of the work assigned to the second one, and the second $\frac{3}{10}$ of the work assigned to the first one, then the first machine would have needed 6 days more than would have the second, how many metres of the tunnel are driven by each machine per day ?

28. A tank is filled with three pipes with uniform flow. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. First and Second Pipe open simultaneously will take as much time as third pipe can take individually. Find the time required by the third pipe to fill the tank ?

29. Ram has the job of laying flower beds. He takes 2 hours to lay a bed which is 12 feet × 10 feet. He asks

his friends to work with him sometimes. His friend takes 3 hours to do the same job. Ram has to pay his friend on an hourly basis, so he prefers to work alone. Ram has recently got a contract to lay 6 flower beds, 4 of which have dimensions 10 feet × 15 feet each and two 12 feet × 10 feet each. He has promised to do the job within 9 hours. For how many hours does he need to employ his friend ?

30. If a cistern generally takes 30 min. to fill by a pipe, but due to a leak, it takes 20 extra min. to be filled, then find the time in which the lack can empty the cistern fully ?

31. Men, Women and children are employed in a factory, the amount of work done by a man a woman and a child in a given time being 2 : 5 : 7. They are paid wages according, to the amount of work done by each. In a factor there are 5 men 8 women and 10 children and their total daily wages amount to Rs. 512. How much will the daily wages be if there are 12 men, 15 women and 8 children in all ?

32. Two pipes P and Q can fill a tank 12 hours and 18 hours respectively. If both pipe are open simultaneously, How much time they will take to fill the tank ?

33. Pipes A and B can fill a tank alone in 9 hours and 12 hours respectively. Pipe can empty the full tank in 15 hours. If all the three pipe are open simultaneously, how much time it will take them to fill the tank completely ?

34. The Quantity of water flowing through a pipe is proportional to square of its diameter. A tank has three inlets of diameters 1 cm., 2 cm. and 3 cm. If the smallest inlet alone can fill the tank in 10 hours. In how much time can all the three inlets together fill the tank ?

35. Two pipes X and Y can fill a cistern in 14 hours and 16 hours respectively. The pipers are opened simultaneously and it is found that due to leakage in the bottom, it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it ?

36. Three pipes X, Y and Z can fill a tank from empty to full in 30 minutes, 20 minutes and 10 minutes respectively. When the tank is empty all the three pipes are opened. X, Y and Z discharge chemical solution A, B and C respectively. What is the proportion of solution C in the liquid in the tank after 3 minutes ?

37. In a group of four boys, the second boy is twice as efficient as the first one. Third one is twice as efficient as the second one, and so on. All of them working together will take 5 days to complete a job. How much extra time will the second and third boy take, working together as compared to the fourth boy, working alone to complete the same job ?

Answers with Hints

Exercise A

1. Given that

$$P_1 = 10, \quad P_2 = 20$$
$$W_1 = 20 \text{ shrits}, \quad W_2 = ?$$
$$T_1 = 5 \times 5 = 25 \text{ hours}, \quad T_2 = 8 \times 10 = 80 \text{ hours}$$

From formula,

$$W_2 = P_2 \times T_2 \times \frac{W_1}{P_1 \times T_1}$$

$$W_2 = 20 \times 80 \times \frac{20}{10 \times 25} = 128 \text{ shirts.}$$

2. $W_1 = 800, \quad W_2 = 1000$

$T_1 = 25 \times 5 = 125$ hours, $\quad T_2 = ?$

$P_1 = 1, \quad P_2 = 1$

$$\frac{P_2 \times T_2}{W_2} = \frac{P_1 \times T_1}{W_1}$$

Or, $$\frac{T_2}{1000} = \frac{25 \times 5}{800}$$

$$T_2 = \frac{5}{4} \times 5 \times 25$$

$$\text{Now } 10 \times \text{days} = \frac{5}{4} \times 5 \times 25$$

$$\text{Days} = \frac{5 \times 5 \times 25}{4 \times 10} = \frac{125}{8}$$

$$\text{Days} = 15\frac{5}{8}$$

3. Given

6 workers in 5 days working 4 hours per day can make 12 toys

So, 1 worker in 5 days working 4 hours per day can make $= \frac{12}{6}$ toys

Or, 12 workers in 5 days working 4 hours/day

$$= \frac{12 \times 12}{6} \text{ toys}$$

Or, 12 workers in one day working 4 hours/day

$$= \frac{12 \times 12}{6 \times 5} \text{ toys}$$

Or, 12 workers in 8 days working 4 hours/day

$$= \frac{12 \times 12}{6 \times 5} \times 8$$

Or, 12 workers in 8 days working 10 hours/day

$$= \frac{12 \times 12 \times 8}{6 \times 5 \times 4} \times 10 \text{ toys}$$

$$= 96 \text{ toys}$$

4. In 20 days X can do a piece of work.

∴ 1 day X can do a piece of $\frac{1}{20}$th work.

Similarly,

In 1 day Y can do $= \frac{1}{5}$th work

Now, In 1 day (X + Y) can do together

$$= \left(\frac{1}{20}+\frac{1}{5}\right)\text{th work}$$

$$= \frac{4+1}{20} = \frac{5}{20}$$

$$= \frac{1}{4}\text{th work}$$

$\because$ X and Y together can do $\frac{1}{4}$th work in one day

$\therefore$ X and Y together can do 1 work in 4 days.

5. Ram's one day work $= \frac{1}{24}$th work

(Ram + Shyam)'s one day work $= \frac{1}{16}$th work

Now, Shyam's one day work

$$= \left(\frac{1}{16}-\frac{1}{24}\right)\text{th work}$$

$$= \frac{3-2}{8\times 3\times 2} = \frac{1}{48}\text{th work}$$

Clearly, Shyam can do the work in 48 days.

6. Sita's one day work $= \frac{1}{3}$th work

Gita's one day work $= \frac{1}{6}$th work

Radha's one day work $= \frac{1}{12}$th work

Shalani's one day work $= \frac{1}{9}$th work

So, (Sita + Gita + Radha + Salani)'s one day work

$$= \left(\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{9}\right)\text{th work}$$

$$= \frac{12+6+3+4}{36}\text{th work} = \frac{25}{36}$$

$\therefore$ They can together do the work in $\frac{36}{25}$ days.

7. (Ram + Rahim)'s one day work

$$= \frac{1}{8}\text{th work} \qquad \ldots(1)$$

(Rahim + Krishna)'s one day work

$$= \frac{1}{12}\text{th work} \qquad \ldots(2)$$

(Krishna + Ram)'s one day work

$$= \frac{1}{16}\text{th work} \qquad \ldots(3)$$

Adding equation (1) + equation (2) + equation (3), we get

2(Ram + Rahim + Krishna)'s one day work

$$= \left(\frac{1}{8}+\frac{1}{12}+\frac{1}{16}\right)\text{th work}$$

$$= \frac{6+4+3}{48} = \frac{13}{48}\text{th work}$$

$\therefore$ (Ram + Rahim + Krishna)'s one day work

$$= \frac{13}{96}\text{th work} \qquad \ldots(4)$$

Subtracting equation (1) from equation (4) , we get

Krishna's one day work $= \frac{13}{96}-\frac{1}{8} = \frac{13-12}{96}$

$$= \frac{1}{96}\text{th work}$$

Ram's one day work $= \frac{13}{96}-\frac{1}{12} = \frac{13-8}{96}$

$$= \frac{5}{96}\text{th work}$$

Rahim's one day work $= \frac{13}{96}-\frac{1}{16} = \frac{13-6}{96}$

$$= \frac{7}{96}\text{th work}$$

$\therefore$ Ram can do the work in $\frac{96}{5}$ days

$\therefore$ Rahim can do the work in $\frac{96}{7}$ days

$\therefore$ Krishna can do the work in 96 days.

8. Now, (Ram + Raju)'s one day work $= \frac{1}{15}$ …(1)

(Raju + Rakesh)'s one day work $= \frac{1}{20}$ …(2)

(Ram + Rakesh)'s one day work $= \frac{1}{12}$ …(3)

Now, 2 (Ram + Raju + Rakesh)'s one day work

$$= \frac{1}{15}+\frac{1}{20}+\frac{1}{12}$$

$$= \frac{4+3+5}{60} = \frac{12}{60}$$

(Ram + Raju + Rakesh)'s one day work

$$= \frac{12}{120} = \frac{1}{10} \qquad \ldots(1)$$

Now, Rakesh's one day work

$$= \frac{1}{10}-\frac{1}{15} = \frac{3-2}{30} = \frac{1}{30}$$

Hence, Rakesh can do the work in 30 days.

Ram's one day work $= \frac{1}{10}-\frac{1}{20} = \frac{1}{20}$

Hence, Ram can do the work = 20 days

Raju's one day work $= \frac{1}{10}-\frac{1}{12} = \frac{6-5}{60} = \frac{1}{60}$

Hence, Raju can do the work in 60 days.

9. According to question,

4 men is equivalent to 5 women

$\therefore$ 4 men = 5 women

1 men $= \frac{5}{4}$ women

8 men = 10 women

Now, 8 men + 15 women = 10 women + 15 women

$= 25$ women

5 women can do a piece of work in 40 days

$\therefore$ 25 women can do a piece of work in $\frac{40}{25} \times 5$

$= 8$ days.

10. According to question,

$$\begin{aligned} 16m + 8w &= 6 \text{ days} \\ 6 \times 16m + 48w &= 1 \text{ day} \\ 96m + 48w &= 1 \text{ day} \quad \ldots(1) \end{aligned}$$

and

$$\begin{aligned} 8m + 6w &= 10 \text{ days} \\ 80m + 60w &= 1 \text{ day} \quad \ldots(2) \end{aligned}$$

Equation (1) and equation (2) are equal, so

$$\begin{aligned} 96m + 48w &= 80m + 60w \\ 16m &= 12w \\ 4m &= 3w \quad \ldots(3) \end{aligned}$$

Now 32 men $= (3w) \times 8 = 24w$

Clearly, 32 men + 32 women $= 24w + 32w = 56$ w

and Since 16 men $= 4 \times 4$ m $= 12$ w

16 men + 8 women $= 12w + 8w = 20w$

Now, 20w can do a certain piece of work in 6 days

$\therefore$ 56 w can do a certain piece of work in

$$\frac{6 \times 20}{56} = \frac{15}{7} \text{ days}$$

11. 20 shirts in 6 days formed by 5 men + 7 women

$\therefore$ 1 shirt in 6 days formed by $\frac{5m + 7w}{20}$

$\therefore$ 1 shirt in 1 day formed by $\frac{6}{20}(5m + 6w)$ ...(1)

Now, Similarly in 2[nd] case,

One shirt in 1 day by $\frac{2}{8}(3m + 4w)$

$$= \frac{1}{4}(3m + 4w)$$

Now,

$$\begin{aligned} \frac{1}{4}(3m + 7w) &= \frac{3}{10}(5m + 4w) \\ 5(3m + 7w) &= 6(5m + 4w) \end{aligned}$$

Or,

$$\begin{aligned} 15m + 35w &= 30m + 24w \\ 15m &= 11w \quad \ldots(2) \end{aligned}$$

Now,

$$\begin{aligned} 3m &= 3 \times \frac{11}{15}w = \frac{11}{5}w \\ 3m + 4w &= \frac{11}{5}w + 4w = \frac{31}{5}w \end{aligned}$$

According to question,

One shirt in 1 day is made by $\frac{31}{5}$ w

$\therefore$ One shirt in 8 days is made by $\frac{31}{5 \times 8}$w

$\therefore$ 40 shirts in 8 days is made by $\frac{31}{5 \times 8} \times 40$w

$= 31$w

Now, since $15m = 11w$

$$8m = \frac{11}{15} \times 8w = \frac{88}{15}w$$

$\therefore$ No. of required women

$$= 31 - \frac{88}{15} = 31 - 5$$

$=$ about 26 women

12. (Gita + Ram + Shyam)'s daily earning

$$= \frac{900}{12} = \frac{300}{4} = \text{Rs. } 75 \quad \ldots(1)$$

(Ram + Shyam)'s daily earning

$$= \frac{300}{5} = \text{Rs. } 60 \quad \ldots(2)$$

(Shyam + Gita)'s daily earning

$$= \frac{320}{8} = \text{Rs. } 40 \quad \ldots(3)$$

Now, Gita's daily earning = Rs. 15

Ram's daily earning = Rs. 35

Shyam's daily earning = Rs. 25

13. (5G + 4B)'s daily earning

$$= \text{Rs. } \frac{660}{3} = \text{Rs. } 220 \quad \ldots(1)$$

(10B + 20G)'s daily earning

$$= \text{Rs. } \frac{3500}{5} = \text{Rs. } 700 \quad \ldots(2)$$

According to question,

In 1[st] case Rs. 1 earned by $\frac{5G + 4B}{220}$

In 2[nd] case Rs. 1 earned by $\frac{10B + 20G}{700}$

$$\therefore \quad \frac{5G + 4B}{220} = \frac{10B + 20G}{700}$$

Or,

$$\begin{aligned} 35G + 28B &= 22B + 44G \\ 6B &= 9G \end{aligned}$$

$$\Rightarrow \quad 2B = 3G \quad \ldots(3)$$

In I[st] case,

$$\begin{aligned} 5G &= 5 \times \frac{2}{3} \times B = \frac{10}{3}B \\ 5G + 4B &= \frac{10}{3}B + 4B \\ &= \frac{22}{3}B \end{aligned}$$

Since, $\frac{22}{3}$ B can earn Rs. 220 in one day

$$\therefore \quad 6B + 4G = 6B + \frac{2}{3} \times 4B = \frac{26}{3}B$$

Now, $\frac{26}{3}$ B can earn Rs. 700 in one day

$$= 220 \times \frac{3}{22} \times \frac{26}{3} \times 26$$

$$= \text{Rs. } 260 \text{ in one day}$$

$\therefore$ $\frac{26}{3}$ B earn Rs. 260 in one day

$\therefore$ $\frac{26}{3}$ B earn Rs. 1 in $= \frac{1}{260} \times 1300$

$\frac{26}{3}$ B earn Rs. 1300 in $= \frac{1 \times 1300}{260} = 5$ days

So, In 5 days 6 boys and 4 girls can earn Rs 1300.

14. Let the number of inlet water taps $= a$

$\therefore$ Number of out let taps are $= 8 - a$

Since, the tank can be filled by each tap in 36 hours.

Therefore, part filled by a taps in 1 hours $= \frac{a}{36}$

Now, Similarly

Part emptied by $(8 - a)$ out let taps in 1 hour $= \frac{8-a}{48}$

$\therefore$ Net part filled in 1 hour $= \frac{a}{36} - \frac{8-a}{48}$

$$= \frac{4a - 3(8-a)}{12 \times 3 \times 4}$$

$$= \frac{7a - 24}{12 \times 3 \times 4}$$

The tank will be full in $\frac{36 \times 4}{7a - 24}$ hours on opening all the taps together.

According to question,

$$\frac{36 \times 4}{7a - 24} = 6$$

Or, $24 = 7a - 24$

Or, $48 = 7a$

$\therefore$ $a = \frac{48}{7}$

$a \cong 7$

15. In one hour the inlet of 2 cm diameter can fill $\frac{1}{4}$ of the tank.

$\therefore$ In 1 hour the inlet of 4 cm diameter can fill $\frac{1}{4} \times \frac{4^2}{2^2} = 1$ tank

Clearly, inlet of 4 cm diameter can fill tank in 1 hour.

Similarly,

In 1 hour the inlet of 6 cm diameter can fill

$$= \frac{1}{4} \times \frac{6^2}{2^2} = \frac{36}{16} = \frac{9}{4} = 2\frac{1}{4} \text{ tank}$$

Clearly, inlet of 6 cm diameter can fill tank in $\frac{4}{9}$ hour

Now, the inlet of diameter 2 cm can fill $\frac{1}{4 \times 60}$ part of tank in 1 minute

and the inlet of diameter 4 cm can fill $\frac{1}{60}$ part of tank in 1 minute.

The inlet of diameter 6 cm can fill $\frac{9}{4 \times 60}$ part in 1 minute.

Now, In one minute three inlets can fill $\left(\frac{1}{240} + \frac{1}{60} + \frac{9}{4 \times 60}\right)$ part of tank

$$= \frac{1 + 4 + 9}{240}$$

$$= \frac{14}{240} = \frac{7}{120}$$

Clearly, three inlets can fill $\frac{120}{7}$ minutes = 17·15 min

16. The ratio of amount of work done by man, woman and child is 3 : 2 : 1.

$\therefore$ The amount of work done by a man $= 3X$

The amount of work done by a woman $= 2X$

The amount of work done by a child $= X$

Total work done by men $= 3X \times 20 = 60X$

Total work done by women

$$= 2X \times 12 = 24X$$

Total work done by 6 children

$$= 6 \times X = 6X$$

According to question,

Total daily wages amount = Total work done by men + Total work done by women + Total work done by children

$$270X = 60X + 24X + 6X$$

$$270 = 90X$$

$$X = 3$$

Required daily wages $= 30 \times 3X + 25 \times 2X + X \times 20$

$$= 90X + 50X + 20X$$

$$= 160X$$

$$= 160 \times 3 = \text{Rs. } 480.$$

17. Now, since $1\,m^3 = 1000$ litres

$2\,m^3 = 2000$ litres

Volume up to B $= 0{\cdot}28 \times 2000 = 560$ litres

Volume up to C $= 0{\cdot}5 \times 2000 = 1000$ litres

Volume up to D $= 0{\cdot}7 \times 2000 = 1400$ litres

Volume up to A $= 2000$ litres

Time to fill upto B $= \frac{560}{80 + 30 + 20 + 10}$

$= 4$ minutes

$$\text{Time to fill from B upto C} = \frac{1000-560}{80-30+20+10}$$

$$= \frac{440}{80} = 5{\cdot}5 \text{ minutes}$$

Similarly,

$$\text{Time to fill from C upto D} = \frac{1400-1000}{80-30-20+10}$$

$$= \frac{400}{40} = 10 \text{ minutes}$$

$$\text{Time to fill from D upto Top} = \frac{2000-1400}{80-30-20-10}$$

$$= \frac{600}{20} = 30 \text{ minutes}$$

$$\text{Total time} = 4 + 5{\cdot}5 + 10 + 30$$

$$= 49{\cdot}5 \text{ minutes.}$$

18. Let A can fill or empty X% of the tank in an hour.

So, B, C, D and E can fill / empty 2X, 3X, 4X and 5X per cent of tank in one hour respectively.

Now, A and D can fill (X + 4X) = 5X per cent of tank in one hour.

In 4 hours they can fill 20 X per cent of tank. But they take 4 hours to fill the tank.

$$20X = 100$$

$$X = 5$$

Case II : C, D and E act as input and A and B act as output.

So, $3X + 4X + 5X - (X + 2X) = 9X$

$= 9 \times 5$ per cent of tank gets filled in 1 hour

So, Time taken to fill tank $= \frac{100}{45}$ hours

Case I : A, D and E act as input and B and C act as out put.

$X + 4X + 5X - (2X + 3X) = 5X$

$= 5 \times 5$ per cent of tank gets filled in 1 hour

So, $\text{Time taken} = \frac{100}{25}$

$$\text{Difference} = \frac{100}{25} - \frac{100}{45}$$

$$= \frac{100}{15 \times 2 \times 3} = \frac{10}{9} \text{ hours}$$

19. Let the ith person be able to do the job in X_i days.

Since, 'n' people are working all possible pairs, each person works exactly $(n-1)$ days.

$$\text{Total work done} = (n-1)\left(\frac{1}{X_1} + \frac{1}{X_2} + \frac{1}{X_3} + \ldots\ldots + \frac{1}{X_n}\right)$$

All persons working together in a single day would be able to do

$$W = \left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots\ldots + \frac{1}{x_n}\right) \text{of the work.}$$

Exercise B

1. I[st] method

$$\text{Efficiency of Ram} = \frac{100}{8} = \frac{25}{2} = 12{\cdot}5\%$$

$$\text{Efficiency of Shyam} = \frac{100}{10} = 10\%$$

$$\text{Efficiency of (Rahim + Ram + Shyam)} = \frac{100}{4} = 25\%$$

Now, Rahim's efficiency $= 25 - (10 + 12{\cdot}5)$

$= 2{\cdot}5\%$

Now, Rahim can do the same work alone in $\frac{100}{2{\cdot}5} = 40$ days

Second Method :

Rahim's 1 day work = Ram, Shyam and Rahim's 1 day work – (Shyam 1 day work + Ram 1 day work)

$$= \left[\frac{1}{4} - \frac{1}{10} - \frac{1}{8}\right] = \frac{1}{40}$$

Hence, Rahim can do the same work alone in 40 days.

2. Since, Days $\propto \frac{1}{\text{Efficiency}}$

If, Gita's efficiency = 2

Then, Lila's efficiency = 1

Let Gita requires X days then

Lila requires 2.X days to complete the same job

Now, According to question,

$$2.X - X = 5$$

$$X = 5$$

Lila requires = 2.X = 10 days

Second Method :

Let Gita takes X days, then Lila takes (X + 5) days to complete the same job.

Gita and Lila five days work = 1

$$\frac{5}{x} + \frac{5}{5+x} = 1$$

$$5x + 25 + 5x = x^2 + 5x$$

$$x = 5 \text{ days}$$

Lila takes = 10 days to work completely.

3. Since, Time $\propto \frac{1}{\text{Efficiency}}$

So,	Janki (N)	Ganga (G)	Jamuna (J)
Efficiency	1	5	3

Now, $\frac{1}{G} + \frac{1}{J} + \frac{1}{N} = 1$

$$\frac{1}{5N} + \frac{1}{3N} + \frac{1}{N} = 1$$

$$N = \frac{23}{15}$$

and J = 3N and G = 5N ; Difference = 2N

$$\text{Difference} = \frac{46}{15} \cong 3 \text{ days}$$

So, Number of days taken by Ganga

$$= \frac{100}{\frac{100}{3}} = 3 \text{ days}$$

Number of days taken by Janki = 15 days

Number of days taken by Jamuna = 5 days

4. Rajesh packs 24 apples working 6 hours/day

Rajesh packs $\frac{24}{6}$ apples working 1 hrs./day

Rajesh packs $\frac{24}{6} \times 8$ apples working 8 hrs./day

Rajesh packs 32 apples working 8 hrs./day …(1)

Similarly,

Rajesh's wife packs $\frac{20}{5} \times 8$ apples working 8 hrs./day

∴ Rajesh's wife packs 32 apples working 8 hrs./day

In (Rajesh + his wife) pack 64 apples working 8 hrs./2 days …(2)

Now,64 apples are packed in 2 days

∴ 1 apple is packed in $\frac{2}{64}$ days

∴ 4000 apples are packed in

$$\frac{1}{32} \times 4000 \text{ days} = \frac{1000}{8}$$

$$= 125 \text{ days}$$

Again

Rajesh packs 36 guavas working 6 hours/day

Rajesh packs $\frac{36}{8} \times 8$ guavas working 8 hours/day

Rajesh pack 48 guavas working 8 hours/day …(3)

Similarly,

His wife packs $\frac{25}{5} \times 8$ guavas working 8 hours/day

40 guavas is packed working 8 hours/day …(4)

Now, (Rajesh + his wife) pack 88 guavas working 8 hours/2 day

Since, 88 guavas are packed in 2 days

∴ one guava is packed in $\frac{2}{88}$ days

∴ 4400 guavas are packed in $\frac{1}{44} \times 4400 = 100$ days

Clearly, in 125 + 100 = 225 days they finish the work of packing.

5. Let number of inlet pipes = X

Number of outlet pipes = 10 – X

Part of tank emptied in one hour

$$= \frac{10 - X}{8}$$

$$\text{Part of tank filled} = \frac{X}{12}$$

According to question,

$$\frac{X}{12} - \frac{10 - X}{8} = \frac{1}{8}$$

Or, $5.X = 33$

$$X = \frac{33}{5} \cong 7 \text{ pipes.}$$

6. Efficiency of inlet X $= \frac{100}{25} = 4\%$

Efficiency of inlet Y $= \frac{100}{20} = 5\%$

∴ Efficiency of X and Y together

$$= \frac{100}{25 \times 4} \times 9 = 9\%$$

Pipes X and Y together can fill tank in $\frac{25 \times 4}{9} = \frac{100}{9}$ hours

Now, If the efficiency of Z be $a\%$, then $\frac{100}{9}$ hours the capacity of tank which will be filled

$$= \frac{100}{9}\left(\frac{100}{9} - a\right)$$

According to question,

Amount of water is being emplied by Z at $a\%$ per hour, then

$$\frac{100}{9}\left(\frac{100}{9} - a\right) \times \frac{1}{a} = 5$$

Or, $$\frac{20}{9a}\left(\frac{100 - 9a}{9}\right) = 1$$

Or, $2000 - 180a = 81a$

Or, $261a = 2000$

$$a = \frac{2000}{261}$$

Hence, in $\frac{100}{9}$ hours $\frac{2000}{261}$ % tank is filled only, hence the remaining $\left(100 - \frac{2000}{261}\right)$% of the capacity will be filled by pipes X and Y in

$$= \frac{100 \times 241}{261 \times 100} \times 9 \cong 8 \text{ hours.}$$

7. Let pipe P fill the tank in X hours and then pipe Q fill it in 2X hours.

Therefore, In one hour they will fill

$$= \frac{1}{2X} + \frac{1}{X} = \frac{3}{2X}$$

Time required $= \frac{2X}{3}$ hours

When the pipe R is also opened then it takes

$$= \frac{2X}{3} + \frac{2X}{3} \times \frac{3}{4}$$

$$= \frac{2X}{3} + \frac{X}{2} = \frac{7X}{6}$$

Now, in one hour pipe P, Q and R working together fill

$$= \frac{6}{7X} + \frac{1}{2X} - \frac{1}{R} = \frac{6}{7X}$$

$\therefore$ R = 2.X

Hence, in 2.X hours pipe R can empty the whole tank

$\therefore$ $\frac{7X}{6} = 21$

X = 18

Now, 2.X = 36 hours.

8. From the last statement,

Efficiency of Sona, Mona and Soni = 50%

From the first statement ,

Soni takes 2 days more than Sona.

From the second statement,

Mona had worked for 6 days and Sona had worked for 3 days only.

Number of days taken by Sona = 6

Efficiency = 16·66%

It means Sona has completed 16·66 × 3 = 50% work in 3 days

Therefore, Mona had completed 50% work in 6 days.

Efficiency of Mona $= \frac{50}{6} = 8.33$ %

Efficiency of Soni = 50 – (16·66 + 8·33)

= 25%

$\therefore$ Sona takes 6 days.

9. Rohan can do in 15 days one work

$\therefore$ Rohan can do in 5 days $\frac{1}{3}$ work

Similarly, Ramu can do in 5 days $\frac{1}{2}$ work

Both together can do in one day $\left(\frac{1}{15} + \frac{1}{10}\right)$work

$$= \frac{2+3}{5 \times 3 \times 2} = \frac{1}{6} \text{ work}$$

Both together can do in 5 days $= \frac{5}{6}$ work

Rest work $= \frac{1}{6}$ work

Given Ramu can complete 1 work in 10 days

Ramu can complete $\frac{1}{6}$ work in $\frac{10}{6}$ days

$$= \frac{10}{3} \text{ days} \cong 3 \text{ days}$$

Required days taken by Ramu = 5 + 3 ≅ 8 days

Now, Both together can do the same work in 6 days

$\therefore$ Required difference in the number

= 8 – 6 = 2 days.

10.

	A	B	C
Efficiency	3	6	2
Number of days	3.X	6.X	2.X

According to question,

3.X – 2.X = 12

Or, X = 12

Number of days taken by A = 36

Number of days taken by B = 72

Number of days taken by C = 24

In 72 days B can completes the work

So in 2 days B can completes $\frac{1}{36}$ work

Similarly, in 2 days C can work $\frac{1}{12}$ work

Now, Total work done in 2 days

$$= \frac{1}{36} + \frac{1}{12} = \frac{4}{36} = \frac{1}{9} \text{ work}$$

Rest work $= \frac{8}{9}$

Since, in 36 days A can finish the work.

So, $\frac{8}{9}$ work is finished by A in $36 \times \frac{8}{9}$ days = 32 days.

11.

Days	1	2	3
	X	Y	Z
	4	6	8
Work in one day	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$

Now, in 3 days X, Y, Z can do

$$\frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \frac{6+4+3}{6 \times 4} = \frac{13}{24} \text{ work}$$

Rest work $= \frac{11}{24}$ work

Day	4^{th}	5^{th}	6^{th}
	X	Y	Z
Work in	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$

Now, X and Y can do

$$\frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12} \text{ work}$$

In 5 days Total work done $= \frac{13}{24} + \frac{5}{12} = \frac{23}{24}$ work

Rest work $= \frac{1}{24}$

The work is done by only Z on 6th day.

$\therefore$ Total number of days $= 6$.

12. Kamal (Ka) + Krishan (Kr) $= \frac{2}{5}$

Krishna (Kr) + Raju (R) $= \frac{4}{5}$

Since (Ka + Kr + Kr + R) – (Ka + Kr + R) = Kr

$$\frac{6}{5} - 1 = \frac{1}{5} = \text{kr.}$$

$$\text{Krishna} = \frac{1}{5}$$

Now, $\quad \text{Kamal} = \frac{2}{5} - \frac{1}{5} = \frac{1}{5}$

$$\text{Raju} = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

So, the whole amount will be distributed in the ratio of 1 : 1 : 3.

Now, Raju is greatest efficient.

So, he get his own share $= \frac{3}{5} \times 900 = \text{Rs. } 540$.

13. 2 men can complete the job in 52 days hence 1 man can complete same job in 108 days. So, man's one day job is $\frac{1}{104}$

3 women can complete the job in 52 days hence 1 women can complete same job in 156 days.

So, women's one day job is $\frac{1}{156}$

4 boys can complete the job in 52 days hence 1 boy can complete same job in 208 days. So, boy's one day job is $\frac{1}{208}$

If all work together then their one day work

$$= \frac{1}{104} + \frac{1}{156} + \frac{1}{208}$$

$$= \frac{1}{52}\left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right] = \frac{1}{52}\left[\frac{6+4+3}{12}\right]$$

$$= \frac{1}{52} \times \frac{13}{12} = \frac{1}{48}$$

Hence, work will complete in = 48 days.

14. According to question,

Work done by 2 men = 3 women = 4 boys

$$1 \text{ man} = \frac{3}{2} \text{ women}$$

$$1 \text{ man} = 2 \text{ boys}$$

Or, Again 1 man + 1 woman + 1 boy

$$= 1 \text{ man} + \frac{2}{3} \text{ man} + \frac{1}{2} \text{ man}$$

$$= \frac{6+4+3}{6} = \frac{13}{6} \text{ man}$$

Since, 2 man can finish the work in 104 days

$\therefore$ 1 man can finish the work in 2×104 days

$\therefore$ $\frac{13}{6}$ man can finish the work in

$$= \frac{2 \times 104 \times 6}{13} = 96 \text{ days.}$$

15. According to question,

$$\text{One man} = 2 \text{ children}$$

$$1M = 2C$$

Since, $(3C + 1M)$ complete work in 6 days

$\left(\frac{1}{2} \cdot 3M + 1M\right)$ complete work in 6 days

$\frac{5}{2} \cdot M$ complete the work in 6 days

$\therefore$ $5M$ complete the work in $6 \times \frac{5}{2} \times \frac{1}{5} = 3$ days.

16. We represent men as M and women as W –

$$\therefore \quad 6M + 5W = 4 \qquad \ldots(1)$$

$$4M + 2W = 7 \qquad \ldots(2)$$

In Ist case—Work will be complete in One day by $(24M + 20W)$

and, in IInd case—Work will be complete in One day by $(28M + 14W)$

Hence, $\quad 24M + 20W = 28M + 14W$

$$4M = 6W$$

$$2M = 3W$$

$$\therefore \quad \frac{M}{W} = \frac{3}{2}$$

So, Amount received by men

$$= \frac{3}{2} \times 48 = \text{Rs. } 72$$

17. In 12 days 4 boys and 6 girls can complete one work

$\therefore$ In one day $(4 \times 12B + 6 \times 12G)$ can complete one work ...(1)

Similarly,

In one day $(7 \times 7B + 8 \times 7G)$ can complete one work ...(2)

Now, From (1) and (2), we get

$$48B + 72G = 49B + 56G$$

$$16G = B \qquad \ldots(3)$$

Now, In Ist case—

$$4 \text{ Boys} + 6 \text{ Girls} = 4 \times 16G + 6G$$

$$= (64 + 6)G$$

$$= 70G$$

Again, $2 \text{ Boys} + 3 \text{ Girls} = 2 \times 16G + 3G$

$$= 32G + 3G$$

$$= 35G$$

Since, 70 G can complete the work in 12 days.

So, 35 G can complete the work in 24 days.

18. Since,

In Y days (Y – 2) men do a piece of work

In one day (Y – 2). Y men do a piece of work ...(1)

Again,

In (Y – 10) days (Y + 7) men can do $\frac{3}{4}$ work

$\therefore$ In (Y – 10) days $\frac{4}{3}$ (Y + 7) men can do one work

In one day $\frac{4}{3}$ (Y – 10). (Y + 7) men one work ...(2)

Similarly,

Equation (1) and equation (2) are equal.

So, $$Y.(Y-2) = \frac{4}{3}\cdot(Y-10).(Y+7)$$

$$Y^2 - 6Y - 280 = 0$$

$$Y = 28 \text{ or } Y = -14$$

So, Acceptable value of Y = 28

Now Total men = Y + 10 = 38

Since, 26 men can do the work in 28 days

$\therefore$ 1 men can do the work in 28×26 days

$\therefore$ 38 men can do the work in $\frac{28 \times 26}{38} \cong 14$ days

19. $$\text{Total work} = 33 \times 30$$

$$= 990 \text{ girls} - 1 \text{ day}$$

$$\text{First day's work} = \frac{1}{990} \times 44$$

$$\text{Second day's work} = \frac{1}{990} \times 43$$

$$\text{Third day's work} = \frac{1}{990} \times 42 \text{ and so on}$$

So, the total work in 44 days

$$= \frac{1}{990}(44 + 43 + 42 + \ldots\ldots)$$

We know the Sum of natural Number

$1 + 2 + 3 + 4 + \ldots\ldots + n$ is $\frac{n(n+1)}{2}$

So, Total work done in 44 days

$$= \frac{1}{990} \times \frac{44 \times 45}{2} = 1$$

Hence, in 44 days Total work will be completed.

20. According to question, we have

	X	Y	Z
Efficiency	1	–	2
Days	–	3	1

We know Efficiency $\propto \frac{1}{\text{Days}}$

So,	X	Y	Z
Efficiency	3	2	6
Number of days	2	3	1

$$\text{Number of days taken by X} = 8$$

$$\text{Number of days taken by Y} = 8 \times \frac{3}{6} = 4$$

$$\text{Number of days taken by Z} = 8 \times \frac{1}{6} = \frac{4}{3}$$

$$\text{One day work of (X + Y)} = \frac{1}{8} + \frac{1}{4} = \frac{1+2}{8} = \frac{3}{8}$$

$$\text{One day work of (Y + Z)} = \frac{1}{4} + \frac{3}{4} = 1$$

$$\text{One day work of (X + Z)} = \frac{3}{4} + \frac{1}{8} = \frac{6+1}{8} = \frac{7}{8}$$

Hence, the work will be finish on 2nd day.

21. Since, $$\text{Efficiency} \propto \frac{1}{\text{Number of days}}$$

So, Sita can do a piece of work in 15 days

Gita can do a piece of work in $\frac{15}{2}$ days

$$\text{Sita's one day work} = \frac{1}{15}$$

$$\text{Gita's one day work} = \frac{2}{15}$$

Let Gita joined Sita after X days, then

In X days Sita can do work $= \frac{X}{15}$

Now, Rest day = 11 – X

(Gita + Sita)'s one day work

$$= \frac{1}{15} + \frac{2}{15} = \frac{1}{5}$$

Now, Both can do in (11 – X) day work $= \frac{11-X}{5}$

$$\frac{11-X}{5} + \frac{X}{15} = 1$$

Or, $$\frac{3(11-X)+X}{15} = 1$$

Or, $$33 - 3.X + X = 15$$

Or, $$18 = 2.X$$

Or, $$X = 9 \text{ days}$$

So, they work together for 2 days.

22. Let the work will be finished in d days

$$\text{Rani's one day work} = \frac{1}{9}$$

$$\text{Sheela's one day work} = \frac{1}{18}$$

$$\text{Soni's one day work} = \frac{1}{24}$$

$$\text{Now, in } d \text{ days Soni's work} = \frac{d}{9}$$

$$\text{In } (d-2) \text{ days, Rani's work } = \frac{d-2}{18}$$

$$\text{and in } (d-4) \text{ days, Sheela's work } = \frac{d-4}{24}$$

$$\text{Now,} \quad \frac{d}{9}+\frac{d-2}{18}+\frac{d-4}{24} = 1$$

$$\Rightarrow \quad \frac{8d+4(d-2)+3(d-4)}{9\times2\times4} = 1$$

Hence, $d = \frac{92}{15} \cong 6$ days.

23. Work of a boy for one hour $= \frac{3}{2}$ girl's work for 1 hour

Again, Work of a boy for 1 day

$$= \left(\frac{3}{2}\times\frac{9}{7\cdot5}\right) \text{girl's for 1 day}$$

$$= \frac{9}{5} \text{ girls work for 1 day}$$

$$\text{So,} \quad 1 \text{ boy} = \frac{9}{5} \text{ girl}$$

$$\therefore \quad 10 \text{ boy} + 6 \text{ girls} = 10\times\frac{9}{5}+6 = 24 \text{ girls}$$

$$\text{Again, } 5 \text{ boys} + 9 \text{ girls} = 5\times\frac{9}{5}+9 = 18 \text{ girls}$$

Since, 24 girls can do the work in 18 days

So, 1 girl can do the work in 18×24

18 girls can do the work in $\frac{18\times24}{18} = 24$ days

Hence, 6 more days will be required to complete the job with the help of 5 boys and 9 girls.

24. $$\text{Pipe X can fill } = \frac{1}{15} \text{ part of tank in one hour}$$

$$\text{Pipe Y can fill } = \frac{1}{10} \text{ part of tank in one hour}$$

Now, in one hour both fill

$$= \frac{1}{15}+\frac{1}{10} = \frac{2+3}{5\times3\times2} = \frac{1}{6}$$

In 4 hours both the pipes (X + Y) can fill

$$\frac{4}{6} = \frac{2}{3} \text{ part of the tank}$$

$$\text{Now, Rest part } = 1-\frac{2}{3} = \frac{1}{3}$$

Now, According to question,

If X be required in which both the pipes work in case of air bubbles.

$$\text{Now,} \quad X\left(\frac{1}{2}\times\frac{1}{15}+\frac{1}{3}\times\frac{1}{10}\right) = \frac{1}{3}$$

$$\text{Or,} \quad \frac{2X}{30} = \frac{1}{3}$$

$$\text{Or,} \quad \frac{2X}{30} = \frac{1}{3}$$

$$X = 5 \text{ hours}$$

25. Let the capacities large, medium and small ships are represented by X, Y and Z respectively.

Now, According to question,

$$2.X = 5.Z \quad \ldots(1)$$

$$2.Y = X + 2.Z \quad \ldots(2)$$

From equation (1) and equation (2), we get

$$2.Y = \frac{5.Z}{2} + 2.Z = \frac{9}{2}\cdot Z$$

$$Y = \frac{9}{4}\cdot Z \quad \ldots(3)$$

Let the required number of journey is 'a', then

$$a(20\times X + 14\times Y + 24\times Z)$$

$$= (5\times X + 8\times Y + 12\times Z)\left(42\tfrac{1}{5}\right)$$

$$\text{Or,} \quad a\left(20\times\frac{5.Z}{2}+14\times\frac{9}{4}\cdot Z + 24.Z\right)$$

$$= \left(42\tfrac{1}{5}\right)\left(5\times\frac{5.Z}{2}+8\times\frac{9}{4}\cdot Z+12.Z\right)$$

$$\text{Or,} \quad a\left(50+\frac{63}{2}+24\right) = \left(\frac{25}{2}+18+12\right)\times\left(42\tfrac{1}{5}\right)$$

$$\text{Or,} \quad a\left(\frac{100+63+48}{2}\right) = \left(\frac{25}{2}+18+12\right)\times\left(42\tfrac{1}{5}\right)$$

$$\text{Or,} \quad a\times\frac{211}{2} = \frac{85}{2}\times\left(42\tfrac{1}{5}\right)$$

$$\text{Or,} \quad a = \frac{85}{211}\times\left(42\tfrac{1}{5}\right)$$

$$= 17 \text{ journeys.}$$

26. Let a be the capacity of cistern.

Since, Pipe X in one hour can fill $= \frac{1}{15}$ part of cistern

Pipe X in 2 hours can fill $= \frac{2}{15}$ part of cistern

Similarly, pipe Y in 2 hours can fill $= \frac{2}{20}$ part of cistern

According to question : Cistern if full

$$\text{So,} \quad a + 2\,\text{hr.}\left(\frac{a}{15}+\frac{a}{20}-10\right) = 0$$

$$\text{Or,} \quad a + 2\times60\left(\frac{4.a+3.a-600}{5\times3\times4}\right) = 0$$

$$\text{Or,} \quad a + 2\times60\times\left(\frac{7a-600}{60}\right) = 0$$

$$\text{Or,} \quad a + 14.a - 1200 = 0$$

$$a = 80 \text{ litres}$$

Hence, the cistern capacity is 80 litres.

27. Let the first machine drive a metres/day and second machine b metres/day. Then

$$60a\left(\frac{30}{100}\right)+60b\left(\frac{80}{300}\right) = 60$$

Or, $$18a+48b = 60 \quad \ldots(1)$$

In Second case,

First machine finishes the work in

$$\frac{2}{3}\times\frac{60b}{a} = 40\frac{b}{a} \text{ days}$$

and Second machine finishes the work in

$$\frac{3}{10}\times\frac{60.a}{b} = 18\frac{a}{b} \text{ days}$$

Then $$40\times\frac{b}{a}-18\times\frac{a}{b} = 6$$

We put $$\frac{b}{a} = C$$

$$40.C-\frac{18}{C} = 6$$

Or, $$20.C^2-9 = 3.C$$

Or, $$20.C^2-3.C-9 = 0$$

Or, $$20.C^2-15.C+12.C-9 = 0$$

Or, $$5.C\,(4.C-3)+3\,(4.C-3) = 0$$

Or, $$(4.C-3)(5.C+3) = 0$$

Or, $$C = \frac{3}{4}=\frac{b}{a}$$

28. Let V be the volume of the tank. a be the time taken by the second pipe to fill the tank.

Or, $a+5$ = time taken by the first pipe to fill the pool

$a-4$ = time taken by the third pipe to fill the pool

Or, $$\frac{V}{a+5}+\frac{V}{a} = \frac{V}{a-4}$$

Or, $$\frac{2.a+5}{(a+5)a} = \frac{1}{a-4}$$

Or, $$a^2-8a-20 = 0$$

Or, $$(a-10)(a+2) = 0$$

Or, $$a = 10$$

Hence, the time taken by the third pipe to fill the tank is $10-4 = 6$ hours.

29. Total area to be laid $= 4\times(10\times15)$
$+ 2\times(12\times10)$
$= 600+240 = 840$ sq. feet

In one hour, Ram can lay

$$= \frac{12\times10}{2} = 60 \text{ sq. feet}$$

In one hour, the friend can lay

$$= \frac{12\times10}{3} = 40 \text{ sq. feet}$$

In 9 hours, Ram can lay $= 9\times60 = 540$ sq. feet

Remaining job $= 840-540 = 300$

Now, the number of hours taken by friend

$$= \frac{300}{40} = 7{\cdot}5 \text{ Hrs.}$$

30. The pipe fills the cistern in one minute $=\frac{1}{30}$

Due to a leak cistern in one minute $=\frac{1}{50}$

Now, leak takes required time to empty

$$= \frac{1}{30}-\frac{1}{50}=\frac{20}{1500}$$

So, Required time taken = 75 minutes

31. The ratio of amount of work done by a man, women and a child is 2 : 5 : 7.

The amount of work done by a man = $2.a$

The amount of work done by a women = $5.a$

The amount of work done by a child = $7.a$

Total work done by man = $2.a\times5 = 10.a$

Total work done by women = $5.a\times8 = 40.a$

Total work done by children = $10\times7.a = 70.a$

So, Total daily wages amount = Total work done by men + Total work done by women + Total work done by children

So, $$512 = 120.a$$

$$a = \frac{512}{120}=\frac{128}{30}$$

Total wages of twelve men + Fifteen women + Eight children

$$= 12\times2.a+15\times5.a+8\times7.a$$
$$= 24.a+75.a+56.a$$
$$= 155.a$$
$$= 155\times\frac{128}{30}$$
$$= 31\times\frac{128}{6}=31\times\frac{64}{3}$$
$$= \frac{1984}{3}$$

32. Pipe P can fill in 12 hours one tank

$\therefore$ Pipe P can fill in 1 hours $\frac{1}{12}$th tank

Similarly,

In 1 hour pipe Q can fill $\frac{1}{18}$th part of the tank

Both the pipes (P + Q) can fill $=\left(\frac{1}{12}+\frac{1}{18}\right)$th in one hour

In one hour (P + Q) can fill

$$= \frac{3+2}{4\times9}=\frac{5}{36}\text{th tank}$$

(P + Q) can fill the tank in $\frac{36}{5}$ hours

33. If all the three pipes are open then

In one hour they can fill

$$= \left(\frac{1}{9} + \frac{1}{12} - \frac{1}{15}\right) \text{part of tank}$$

$$= \frac{20 + 15 - 12}{3 \times 4 \times 5 \times 3} = \frac{23}{9 \times 20} \text{part of tank}$$

Clearly, the tank will be full in $\frac{9 \times 20}{23}$ hours $= \frac{180}{23}$ hours.

34. Since, the Quantity of water is proportional to square of its diameter.

$\therefore$ In one hour in inlet of 1 cm. diameter can fill $\frac{1}{10}$ of the tank

In one hour in inlet of 2 cm. diameter can fill $\frac{1}{10} \times \frac{2^2}{1^2} = \frac{4}{10}$ of the tank

Similarly,

In one hour the inlet of 3 cm. diameter can fill $\frac{1}{10} \times \frac{3^2}{1^2} = \frac{9}{10}$ of the tank

In one hour three inlets of diameter can fill $\frac{1}{10} + \frac{4}{10} + \frac{9}{10} = \frac{14}{10}$ of the tank

$$= \frac{7}{5} \text{ of the tank}$$

Hence, the whole tank will get filled in $\frac{5}{7} \times 60$ minutes $= \frac{300}{7}$ minutes

35. Since, pipe X can fill one cistern in 14 hours.

So, In one hour pipe X can fill $= \frac{1}{14}$th part of cistern

In one hour pipe Y can fill $= \frac{1}{16}$th part of cistern

Now, In one hour pipes X and Y can fill

$$= \frac{1}{14} + \frac{1}{16} = \frac{15}{112}$$

Time taken by these pipes to fill the tank $= \frac{112}{15}$

= 7 hours 28 minutes

Since, Due to leakage, time taken is given by

T = 7 hrs. 28 min. + 32 min. = 8 Hrs.

$\therefore$ In one hour (two pipes + leak) can fill $= \frac{1}{8}$ part of cistern

Now, In one hour leak can empty water

$$= \frac{15}{112} - \frac{1}{8} = \frac{1}{112}$$

$\therefore$ Leak will empty the full cistern in 112 hours.

36. Since, In one minute pipe X can fill $\frac{1}{30}$ part of tank

In one minute pipe Y can fill $\frac{1}{20}$ part of tank

In one minute pipe Z can fill $\frac{1}{10}$ part of tank

In one minute (X + Y + Z) can fill

$$= \frac{1}{30} + \frac{1}{20} + \frac{1}{10} = \frac{11}{60}$$

In 3 minutes (X + Y + Z) can fill

$$= 3 \times \frac{11}{60} = \frac{11}{20}$$

Individually Pipe Z can fill in 3 minutes $= \frac{3}{10}$

Now, Required ratio $= \frac{3}{10} \times \frac{20}{11} = \frac{6}{11}$

37. Let the first boy do $a\%$ of the job in a day so 2nd will do $2a\%$ 3rd will do $4a\%$ and 4th will do $8a\%$ in a day.

All of them working together will take 5 days to complete a job *i.e.* in one day they are finishing 20% of the job.

So, we get $a + 2.a + 4.a + 8.a = 20$

Or, $a = \frac{20}{15} = \frac{4}{3}$

Since, 2nd and 3rd working together will finish

$$6.a\% = \frac{4}{3} \times 6$$

= 8% of the job in a day

So, Time taken by 2nd and 3rd

$$= \frac{100}{8} = \frac{25}{2} \cong 12\frac{1}{2} \text{ days}$$

and fourth will finish $8a\% = \frac{32}{3}\%$ of the job in a day

and fourth will take $= \frac{100}{32} \times 3 = \frac{300}{32} \cong 9$ days

Hence, extra days $= 12\frac{1}{2} - 9 = 3\frac{1}{2}$ days.

11 Simple and Compound Interest

There are two types of Interest—

1. Simple Interest
2. Compound Interest

If the interest on a certain sum borrowed for a certain period is calculated uniformly. It is called simple interest.

In Compound Interest—For every unit of time (yearly, half-yearly, quarterly or monthly) principal varies. The simple interest due at the end of the first unit of time is added to the principal and the amount so obtained becomes the principal for the second unit of time. Similarly the amount after the second unit of time becomes the principal for third unit of time and so on.

SIMPLE INTEREST

Transaction of money after takes places among banks, individuals, business and other concerns.

If a person X borrows some money from another person Y for a certain period, then after that specified period, the borrower has to return the money borrowed as well as some additional money. This additional money that borrower has to pay is called Interest.

Here, person Y is called lender, who gives money and person X is known as borrower, who receives money.

Principal (P)—The money given by the lender is called principal / Sum.

$\therefore$ Amount = Principal + Interest

Rate Percent (R)—The interest that the borrower has to pay for every 100 rupees borrowed for every year is known as rate per cent per annum. It is denoted by $r\%$ or R%.

If P = Principal

r = Rate per cent per annum

t = Time Period

Now, Simple Interest is given by

$$\text{S.I.} = \frac{P \times r \times t}{100}$$

Illustration 1.

Find Simple Interest in the following cases—

(i) P = Rs. 1200, $r = 5\ \%$ per annum, $t = 2$ years

(ii) P = Rs. 1800, $r = 10\%$ per annum, $t = 2$ months

(iii) P = Rs. 3650, $r = 12\%$ per annum, $t = 730$ days

Solution :

(i) $$\text{S.I.} = \frac{P \times r \times t}{100} = \frac{1200 \times 5 \times 2}{100} = \text{Rs. } 120$$

(ii) $$\text{S.I.} = \frac{P \times r \times t}{100}$$

Here, time = 2 months = $\frac{2}{12}$ year = $\frac{1}{6}$ year

Now, $$\text{S.I.} = \frac{1800 \times 10}{100} \times \frac{1}{6} = \text{Rs. } 30$$

(iii) Time = t = 730 days = $\frac{730}{365}$ year

Now, $$\text{S.I.} = \frac{3650}{100} \times \frac{730}{365} = 12 \times 73 = \text{Rs. } 876$$

Effect of Change of P, r, t on Simple Interest

(A) If P_1 changes to P_2 and r, t are fixed, then

$$\text{Change in S.I.} = \frac{r \times t}{100}(P_2 - P_1)\ (P_2 > P_1)$$

(B) If P remains constant and r and t change—

$$\text{Then Change in S.I.} = \frac{P}{100}(r_1 t_1 - r_2 t_2)$$

Formula of Amount :

$$\text{Amount} = \text{Principal} + \text{Interest}$$

$$A = P + \frac{P \times r \times t}{100}$$

$$\Rightarrow \quad A = P\left(1 + \frac{r \times t}{100}\right)$$

$$\text{Or,} \quad A = \text{S.I.}\left(1 + \frac{100}{r \times t}\right)$$

(C) Effect of change of r or t on Amount—

If P is fixed and either r or t is variable. So, principal is given by

$$P = \frac{A_1 \times X_2 - A_2 \times X_1}{X_1 - X_2}$$

where X = r or t

where A = Amount

$$\text{Again,} \quad r \text{ or } t = \frac{A_1 - A_2}{A_1 \times X_2 - A_2 \times X_1} \times 100$$

Illustration 2.

If Given P = Rs. 120, r = 2% per annum, and t = 5 years. Then Find A = ?

Solution :

From given formula,

$$A = P\left(1+\frac{r \times t}{100}\right)$$

$$A = 120 \times \left(1+\frac{5 \times 2}{100}\right)$$

$$= 120 \times \frac{11}{10} = \text{Rs. } 132.$$

Illustration 3.

Find the following :

(i) A = Rs. 7200, r = 10% per annum, t = 5 years, P = ?

(ii) P = Rs. 600, r = 3% per annum, t = 12 months, A = ?

(iii) S.I. = 150, P = Rs. 2000, r = 12% per annum, t = ?

(iv) S.I. = 120, P = Rs. 5000, t = 3 months, r = ?

Solution :

(i) $$A = P\left(1+\frac{r \times t}{100}\right)$$

$$7200 = P\left(1+\frac{10 \times 5}{100}\right)$$

Or, $$7200 = P \times \frac{3}{2}$$

$$P = 2400 \times 2 = \text{Rs. } 4800$$

(ii) From given formula

$$A = P\left(1+\frac{r \times t}{100}\right)$$

$$A = 600 \times \left(1+\frac{3}{100} \times \frac{12}{12}\right)$$

$$= 600 \times \frac{103}{100} = \text{Rs. } 618$$

(iii) $$\text{S.I.} = \frac{P \times r \times t}{100}$$

Or, $$150 = \frac{2000 \times 12 \times t}{100}$$

Or, $$t = \frac{15}{2 \times 12} = \frac{5}{4 \times 2} = \frac{5}{8} \text{ years}$$

$$t = \frac{5}{8} \times 12 \text{ months} = \frac{15}{2} \text{ months}$$

(iv) $$\text{S.I.} = \frac{P \times t \times t}{100}$$

$$120 = 5000 \times \frac{3 \times r}{12 \times 100}$$

Or, $$\frac{12 \times 4}{5} = r$$

Or, $$\frac{48}{5} = r$$

$$r = 9{\cdot}6\% \text{ per annum.}$$

Illustration 4.

At what rate of interest per annum will a sum double itself in 5 years ?

Solution :

Let required principal = X

According to question,

Amount after 5 years = 2.X

Interest = 2.X – X = X

$$\text{S.I.} = \frac{P \times r \times t}{100}$$

Or, $$X = \frac{X \times r \times 5}{100}$$

Or, $$r = 20\%$$

(D) Repayment of debt in equal Installments—Let Ram borrows an amount Rs. M and he returns it in equal installments at a rate r% annum if 'i' is annual installment.

Now, Borrowed amount = M

$$= n.i + \frac{r \times i}{100 \times Y} \times \frac{n(n-1)}{2}$$

where Y = no. of installments per annum

Y = 1, when each installment is paid yearly.

Y = 2, when each installment is paid half-yearly.

Illustration 5.

What annual installment will discharge a debt of Rs. 4800 due in 5 years at 10% simple interest

Solution :

Using formula,

$$M = n.i + \frac{r \times i}{100 \times Y} \times \frac{n(n-1)}{2}$$

M = Rs. 4800, n = 5, Y = 1, r = 10%, i = annual installment

$$4800 = 5.i + \frac{10 \times i}{100 \times 1} \times \frac{5 \times 4}{2}$$

$$= 5.i + \frac{i}{10} \times 10$$

$$6.i = 4800$$

$$i = \text{Rs. } 800.$$

COMPOUND INTEREST

The difference between the final amount (A) obtained at the end of the last unit of time and the original principal (P).

Compound Interest C.I. = A – P

Some important formula,

$$\begin{aligned} \text{We have Principal} &= \text{Rs. P} \\ \text{Rate} &= r\% \text{ per annum} \\ \text{Time period} &= t \text{ years} \\ \text{Amount} &= \text{Rs. A} \end{aligned}$$

1. When Interest is compounded annually

$$A = P\left(1 + \frac{r}{100}\right)^t$$

2. When Interest is compound half-yearly

$$A = P\left(1 + \frac{\frac{r}{2}}{100}\right)^{2t}$$

$$A = P\left(1 + \frac{r}{200}\right)^{2t}$$

$$C.I. = A - P$$

Illustration 6.

Find compound interest on Rs. 2000 at 5% per annum, compounded yearly, for 2 years ?

Solution :

P = Rs. 2000, $r = 5\%$, $t = 2$ years

Now,
$$\begin{aligned} A &= P\left(1 + \frac{r}{100}\right)^t \\ &= 2000\left(1 + \frac{5}{100}\right)^2 \\ &= 2000 \times \frac{21 \times 21}{400} = 5 \times 441 \\ A &= \text{Rs. } 2205. \end{aligned}$$

Illustration 7.

Find the compound interest on Rs. 1200 at 25% per annum, compounded quarterly for 1 year ?

Solution :

Since, interest is compounded quarterly.

So,
$$\begin{aligned} \text{Time} &= 4.\text{X}, r = \frac{25}{4}\% \\ A &= P\left(1 + \frac{r}{4 \times 100}\right)^{4n} \\ &= 1200 \times \left(1 + \frac{25}{4 \times 100}\right)^4 \\ &= 1200 \times \left(\frac{17}{16}\right)^4 \\ &= 1200 \times \frac{17 \times 17 \times 289}{16 \times 16 \times 256} \\ &= 1200 \times \frac{289 \times 289}{256 \times 256} \\ C.I. &= A - P \\ &= 1200 \times \frac{289 - 289}{256 \times 256} - 1200 \\ &= 1200\left[\frac{289^2 - 256^2}{256^2}\right] \\ &= \frac{1200 \times 545 \times 33}{256 \times 256} = 329{\cdot}32. \end{aligned}$$

Illustration 8.

Shyam invests Rs. 25,000 in a bond which gives interest at 10% per annum during the first year, 20% during the second year and 25% during the third year. How much does he get at the end of third year ?

Solution :

From question,

P = Rs. 25,000, $r_1 = 10\%$, $r_2 = 20\%$, $r_3 = 25\%$

Amount is given by

$$\begin{aligned} A &= P\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\left(1 + \frac{r_3}{100}\right) \\ &= 25000 \times \left(1 + \frac{10}{100}\right) \times \left(1 + \frac{20}{100}\right) \\ &\qquad \times \left(1 + \frac{25}{100}\right) \\ &= 25000 \times \frac{11}{10} \times \frac{6}{5} \times \frac{5}{4} = \text{Rs. } 41{,}250. \end{aligned}$$

Illustration 9.

Find the compound interest on Rs. 20000 for $3\frac{1}{2}$ years at 20% per annum compounded yearly ?

Solution :

Now, P = Rs. 20000, $r = 20\%$, $t = 3\frac{1}{2}$ years

Now, Amount is given by

$$\begin{aligned} A &= P\left(1 + \frac{r}{100}\right)^3\left(1 + \frac{\frac{r}{2}}{100}\right) \\ &= 20000 \times \left(1 + \frac{20}{100}\right)^3\left(1 + \frac{10}{100}\right) \\ A &= 20000 \times \frac{216}{125} \times \frac{11}{10} = \text{Rs. } 38016 \\ C.I. &= A - P = 38016 - 20000 = \text{Rs. } 18016 \end{aligned}$$

Illustration 10.

A certain sum of money becomes n times in t years when compounded annually. In how many years will it become n^x times at the same rate of compound interest ?

Solution :

Using formula,

$$A = P\left(1 + \frac{r}{100}\right)^t$$

Or,
$$n.P = P\left(1 + \frac{r}{100}\right)^t$$

$$n = \left(1+\frac{r}{100}\right)^t \quad \ldots(1)$$

Let the sum become n^x times in T years.

$$n^x = \left(1+\frac{r}{100}\right)^T$$

$$n = \left(1+\frac{r}{100}\right)^{\frac{T}{x}} \quad \ldots(2)$$

Comparing equation (1) and equation (2), we get

$$\frac{T}{x} = t$$

$$T = x.t \text{ years}$$

Illustration 11.

If the difference between C.I. and S.I. on a certain sum at $r\%$ per annum for 2 years is Rs. X. Find the sum ?

Solution :

Let the sum be Rs. P.

$$\text{S.I.} = \frac{P \times r \times 2}{100} = \frac{2Pr}{100}$$

$$\text{C.I.} = P\left[\left(1+\frac{r}{100}\right)^2 - 1\right]$$

$$= P\left[\left(1+\frac{2r}{100}+\left(\frac{r}{100}\right)^2 - 1\right)\right]$$

$$= P \times \frac{r}{100} \times \left(2+\frac{r}{100}\right)$$

$$\therefore \quad x = CI - SI = \frac{Pr}{100} \times \left(2+\frac{r}{100}\right) - \frac{2pr}{100}$$

$$= \frac{Pr}{100}\left(2+\frac{r}{100}-2\right) = \frac{pr^2}{100^2}$$

$$P = \frac{x(100)^2}{(r)^2}.$$

Illustration 12.

A certain sum of money invested is compounded yearly, becomes Rs. A_1 in n years and Rs. A_2 in $(n+1)$ years. Find the sum and the rate of interest ?

Solution :

Rs. $(A_2 - A_1)$ is the interest on Rs. A_1 in 1 year.

Using Formula :

$$r = \frac{100 \times I}{P.t}$$

$$I = \text{Rs. } (A_2 - A_1)$$

$$P = \text{Rs. } A_1$$

$$t = 1 \text{ year}$$

$$r = \frac{100(A_2 - A_1)}{A_1}\% \text{ per annum}$$

$$A = P\left(1+\frac{r}{100}\right)^t$$

$$P = \frac{A}{\left(1+\frac{r}{100}\right)^t}$$

For n years, we have

$$A = \text{Rs. } A_1$$

$$r = \frac{100(A_2 - A_1)}{A_1}\% \text{ per annum}$$

$$t = n \text{ years}$$

$$P = \frac{A_1}{\left[1+\frac{100(A_2-A_1)}{100 \times A_1}\right]^n}$$

$$P = \frac{A_1}{\left(\frac{A_2}{A_1}\right)^n}$$

$$P = A_1.\left(\frac{A_1}{A_2}\right)^n$$

Illustration 13.

A certain sum of money invested at compound interest, compounded annually becomes Rs. 8820 in 2 years and Rs. 9261 in 3 years. Find the rate of interest and the sum ?

Solution :

Rs. (9261 – 8820) = Rs. 441 is the interest on Rs. 8820 for 1 year.

$$r = \frac{441}{8820} \times 100$$

$$r = 5\% \text{ per annum}$$

$$P = \frac{A}{\left(1+\frac{r}{100}\right)^t}$$

$$= \frac{8820}{\left(1+\frac{5}{100}\right)^2} = \frac{8820}{\frac{441}{400}}$$

$$P = \text{Rs. } 8000.$$

Illustration 14.

Divide Rs. 10387 into two parts, such that the first part after 5 years is equal to the second part after 7 years, compound interest being 10% per annum compounded yearly ?

Solution :

Let the first part = Rs. X

Second part = 10387 – X

Now, Amount after 5 years = $X.\left(1+\frac{10}{100}\right)^5$

$$\text{Amount after 7 years} = (10387 - X) \times \left(1 + \frac{10}{100}\right)^7$$

According to question,

$$X.\left(\frac{11}{10}\right)^5 = (10387 - X) \times \left(\frac{11}{10}\right)^7$$

Or, $$X = (10387 - X) \times \frac{121}{100}$$

Or, $$\frac{221}{100} \times X = 10387 \times \frac{121}{100}$$

Or, $$X = 10387 \times \frac{121}{221}$$

$$= 47 \times 121 = 5687$$

Second Part = Rs. 4700.

Exercise A

1. Find the simple interest on Rs. 15,000 at $2\frac{2}{3}\%$ per annum for 3 months ?
   (A) Rs. 200 (B) Rs. 100
   (C) Rs. 350 (D) Rs. 50
   (E) Rs. 150

2. Find the simple interest on Rs. 20,000 at $7\frac{1}{4}\%$ per annum for the period from 4th Feb., 2005 to 18th April, 2005 ?
   (A) Rs. 290 (B) Rs. 450
   (C) Rs. 190 (D) Rs. 390
   (E) Rs. 150

3. A sum at simple interest of $6\frac{1}{4}\%$ per annum amounts to Rs. 25,000 after 4 years. Find the sum ?
   (A) Rs. 12000 (B) Rs. 15000
   (C) Rs. 8000 (D) Rs. 10000
   (E) Rs. 20000

4. A sum of Rs. 1000 amounts to Rs. 1200 in 5 years at simple interest. If the interest rate is increased by $3\frac{1}{2}\%$, it would amount to how much ?
   (A) Rs. 1000 (B) Rs. 1200
   (C) Rs. 850 (D) Rs. 1500
   (E) Rs. 1375

5. Raju borrows Rs. 25000 for 2 years at 5% p.a. simple interest. He immediately lends it to Rani at $7\frac{1}{2}\%$ p.a. for 2 years. Find his gain in the transaction per year ?
   (A) Rs. 125 (B) Rs. 225
   (C) Rs. 625 (D) Rs. 500
   (E) Rs. 800

6. In how many years Rs. 1000 will produce the same interest at the rate of 6% as Rs. 1200 produce in 5 years @ $5\frac{1}{2}\%$?
   (A) 55% (B) 12·5%
   (C) 5·5% (D) 16·33%
   (E) 35%

7. Two equal sum of money were lent in at simple interest at 12% p.a. for $2\frac{1}{2}$ years and $3\frac{1}{2}$ years respectively. If the difference in interests for two periods was Rs. 720. Then Find the each sum ?
   (A) Rs. 650 (B) Rs. 950
   (C) Rs. 5760 (D) Rs. 1250
   (E) Rs. 4000

8. Gitanjali invested a certain amount in three different schemes X, Y and Z with the rate of interest 5% p.a., 10% p.a. and 15% p.a. respectively. If the total interest accrued in one year was Rs. 4500 and the amount invested in scheme Z was 180% of the amount of the amount invested in scheme X and 120% of the amount invested in scheme Y. What was the amount invested in scheme Y ?
   (A) Rs. 45117 (B) Rs. 34468
   (C) Rs. 90234 (D) Rs. 81000
   (E) Rs. 17234

9. Divide Rs. 25300 into 3 parts so that their amounts 4, 6 and 8 years respectively may be equal, the rate of interest being 5% per annum at simple interest. What is the difference between first and third part ?
   (A) Rs. 1110 (B) Rs. 17500
   (C) Rs. 8400 (D) Rs. 1700
   (E) Rs. 9100

10. The Simple interest on a certain sum of money for $3\frac{1}{2}$ years at 8% per annum is Rs. 57 less than the simple interest on the same sum for $5\frac{1}{2}$ years at 12% per annum. Find the sum ?
    (A) Rs. 600 (B) Rs. 750
    (C) Rs. 150 (D) Rs. 1200
    (E) Rs. 1800

Exercise B

1. Find compound interest on Rs. 225,00 at 4% per annum for 2 years, compounded annually ?
   (A) Rs. 5625 (B) Rs. 1125
   (C) Rs. 1416 (D) Rs. 1500
   (E) Rs. 1836

2. Ramu borrowed Rs. 20,000 from two persons he paid 6% interest to one and 8% per annum to the other. In one year he paid total interest Rs. 1280. How much did he borrow at 8% rate ?

(A) Rs. 6000 (B) Rs. 8000
(C) Rs. 12000 (D) Rs. 4000
(E) Rs. 16000

3. A borrowed Rs. 1200 at 5% per annum and Rs. 1600 at 6% per annum for the same period. He paid Rs. 312 as total interest. Find the time for which he borrowed the sum ?

(A) 0·5 years (B) 1 year
(C) 1·5 years (D) 2 years
(E) 2·5 years

4. Find the annual installment that will discharge a debt of Rs. 1,60,000 due in 5 years at 10% per annum simple interest ?

(A) Rs. 16000 (B) Rs. 20000
(C) Rs. 24000 (D) Rs. 32000
(E) Rs. 28000

5. When the bank reduces the rate interest from 5% to 4% per annum. Rohit with draws Rs. 1000 from his account. If he now Rs. 75 less interest during one year. Find how much total money was there is Rohit's account initially ?

(A) Rs. 3500 (B) Rs. 3200
(C) Rs. 2800 (D) Rs. 2200
(E) Rs. 2500

6. Find the compound interest on Rs. 2000 for 9 months at 8% per annum being given when the interest is reckoned.

(i) Quarterly
(ii) Half-yearly
(iii) Yearly

(A) Rs. 122; 121; 120 (B) Rs. 120; 121; 122
(C) Rs. 122; 122; 122; (D) Rs. 122; 121; 118
(E) Rs. 120; 122; 124

7. A certain sum is interested at compound. The interest accrued in the first two years is Rs. 272 and that in the first three years is Rs. 434. Find the rate per cent ?

(A) 8% (B) 7%
(C) 6% (D) 9%
(E) 10%

8. Rajesh set up a factory by investing Rs. 50,000. During the first three successive years, his profit were 5, 10 and 15% respectively. If each year profit calculated on previous year's capital. Find his total profit ?

(A) Rs. 12000 (B) Rs. 14218
(C) Rs. 26000 (D) Rs. 24000
(E) Rs. 16412

9. A certain sum of money amounts Rs. 4800 in 3 years at 5% per annum simple interest in how many years will it amount to Rs. 5600 at the same rate of interest ?

(A) 5 year 10 months
(B) 6 year 10 months
(C) 7 year 10 months
(D) 3 year 10 months
(E) 4 year 10 months

10. An amount of Rs. 50440 borrowed at 5% per annum compounded yearly, is to be repaid in 3 equal installments. Find the amount of each installment ?

(A) Rs. 18522 (B) Rs. 20202
(C) Rs. 22110 (D) Rs. 19202
(E) Rs. 23110

11. Rs. 24000 is lent out in three parts. The first part is lent at 4% per annum for 5 years, the second at 5% for 3 years and the third at 8% for 4 years. The total interest earned on each part is equal. Find the value of biggest part ?

(A) Rs. 18000 (B) Rs. 16000
(C) Rs. 9000 (D) Rs. 10000
(E) Rs. 12000

12. Rakesh invests $\frac{1}{5}$th of his capital at 5% per annum $\frac{1}{3}$rd at 2% and the remainder at 3%. If his annual income from these is 920. Find this capital invested in thousand ?

(A) 36 (B) 24
(C) 12 (D) 50
(E) 30

13. A sum of money lent out at compound interest increases in value by 50% in 3 years. A person wants to lend three different sums of money X, Y and Z for 6, 9 and 12 years respectively at the above rate, in such a way that he gets back equal respective periods. Find the ratio X : Y : Z ?

(A) 3 : 2 : 1 (B) 11 : 9 : 7
(C) 8 : 4 : 3 (D) 4 : 3 : 2
(E) 9 : 6 : 4

14. The simple interest on a certain sum for 2 years is Rs. 60 and the compound interest is Rs. 72. Find the Sum ?

(A) Rs. 600 (B) Rs. 100
(C) Rs. 75 (D) Rs. 60
(E) Rs. 30

15. Divide Rs. 5115 into two parts such that the first part after 10 years is equal to the second part after 7 years, compound interest being 20% per annum compounded yearly and find the difference between two parts ?

(A) Rs. 3340 (B) Rs. 4440
(C) Rs. 5115 (D) Rs. 1875
(E) Rs. 1465

16. If the difference between C.I. and S.I. on a certain sum at $p\%$ per annum for 3 years is Rs. q. Find the sum ?

17. Krishna lends Rs. a in n parts in such a way that interest on the first part is $r_1\%$ per annum for t_1 years, on the second part r_2 % for t_2 years and so on. If the interest earned from each part for the corresponding periods are equal. Find the ratio of each part ?

Answers with Hints

Exercise A

1. (B) Using formula,

$$\text{S.I.} = \frac{P \times r \times t}{100}$$

Here, P = Rs. 15000, $r = 2\frac{2}{3}\% = \frac{8}{3}\%, t = \frac{3}{12}$ year

Now, $$\text{S.I.} = \frac{15000 \times 8 \times 3}{100 \times 3 \times 12} = \text{Rs. } 100$$

2. (A) $$\text{Principal} = P = \text{Rs. } 20000,$$

$$\text{Rate} = r = 7\frac{1}{4}\% = \frac{29}{4}\%$$

$$\text{Time} = t = \frac{24 + 31 + 18}{365} = \frac{73}{365} = \frac{1}{5} \text{ year}$$

$$\text{S.I.} = \frac{P \times r \times t}{100} = \frac{20000 \times 29}{100 \times 4} \times \frac{1}{5}$$

$$= \frac{50 \times 29}{5} = \text{Rs. } 290.$$

3. (E) Now, $$\text{S.I.} = \frac{P \times r \times t}{100}$$

$$= \frac{X \times 25 \times 4}{100 \times 4} = \frac{X}{4}$$

Now, $$\text{Amount} = \text{Sum} + \text{S.I.}$$

$$= X + \frac{X}{4}$$

$$25000 = \frac{5X}{4}$$

$$X = \frac{25000 \times 4}{5}$$

$$= 5000 \times 4 = \text{Rs. } 20000$$

4. (E) Let initial rate of interest = $r\%$

Given Principal = Rs. 1000; Amount = Rs. 1200; Time = 5 years

$$\text{S.I.} = \frac{P \times r \times t}{100}$$

$$\Rightarrow \quad 200 = \frac{1000 \times r \times 5}{100}$$

$$r = 4\%$$

Now, New rate of interest

$$= 4 \times 3\frac{1}{2} = \left(4 + \frac{7}{2}\right)\% = \frac{15}{2}\%$$

Now, $$\text{New S.I.} = \frac{1000 \times 15 \times 5}{100 \times 2}$$

$$= 25 \times 15 = \text{Rs. } 375$$

Now, $$\text{Amount} = \text{Sum} + \text{S.I.}$$

$$= 1000 + 375 = \text{Rs. } 1375.$$

5. (C) Interest on Rs. 25000

$$= \frac{25000 \times 5 \times 2}{100} = \text{Rs. } 2500$$

Now, Interest on Rs. 25000

$$= \frac{25000 \times 2 \times 15}{100 \times 2} = \text{Rs. } 3750$$

Now, Amount gained by Raju = 3750 – 2500

= Rs. 1250 in two years

Now, Amount gained by Raju = Rs. 625 in one year.

6. (C) Let required time = x years

According to question,

$$\frac{1000 \times 6 \times x}{100} = \frac{1200 \times 11 \times 5}{100 \times 2}$$

Or, $$x = \frac{55}{10} = \frac{11}{2}$$

$$\therefore \quad x = \frac{11}{2}\%$$

7. (C) Now, Let sum of money = Rs. X

Now, $$\text{S.I.} = \frac{X \times 12 \times 5}{100 \times 2} = \frac{3.X}{10}$$

In 2nd case,

$$\text{S.I.} = \frac{X \times 7 \times 5}{100 \times 2} = \frac{7.X}{40}$$

Now difference in simple interest

$$= \frac{3.X}{10} - \frac{7.X}{40} = \frac{5.X}{40}$$

Now, $$720 = \frac{5.X}{40}$$

Or, $$X = \frac{720 \times 40}{5} = \text{Rs. } 5760.$$

8. (E) Let the amount be Rs. a, Rs. b and Rs. c respectively.

Now, $$\frac{a \times 5 \times 1}{100} + \frac{b \times 10 \times 1}{100} + \frac{c \times 15 \times 1}{100} = 4500$$

$$a + 2b + 3c = 90{,}000 \quad \ldots(1)$$

According to question,

$$c = \frac{180}{100} \cdot a = \frac{9}{5} \cdot a \quad \ldots(2)$$

$$c = \frac{120}{100} \cdot b = \frac{6}{5} \cdot b \quad \text{...(3)}$$

Now, putting the values of a and b in equation (1), we get

$$\therefore \quad a + \frac{5}{9} \cdot c + 2 \times \frac{5}{6} \cdot c + 3.c = 90000$$

$$\therefore \quad c \times \frac{10 + 30 + 54}{18} = 90000$$

$$\therefore \quad c \times \frac{94}{18} = 90000$$

$$\therefore \quad c = \frac{90000 \times 18}{94} = \text{Rs. } \frac{810000}{47} = \text{Rs. } 17234$$

9. (E) Let the first part = Rs. X

Let the second part = Rs. Y

Now, Third part = 25300 – (X + Y)

Since, Amount = Sum + S.I.

So, for 4 years

$$\text{Amount} = X + \frac{4 \times X \times 5}{100} = \frac{6X}{5} \quad \text{...(1)}$$

For 6 years,

$$\text{Amount} = Y + \frac{Y \times 5 \times 6}{100} = \frac{13.Y}{10} \quad \text{...(2)}$$

For 8 years,

$$\text{Amount} = [25300 - (X + Y)] + [25300 - (X + Y)] \times \frac{5 \times 8}{100}$$

$$= [25300 - (X + Y)] \left[1 + \frac{2}{5}\right]$$

$$= [25300 - (X + Y)] \left[\frac{7}{5}\right] \quad \text{...(3)}$$

According to question,

$$\frac{6.X}{5} = \frac{13.Y}{10}$$

$$12.X = 13.Y \quad \text{...(4)}$$

$$\text{Now,} \quad \frac{6.X}{5} = \left[25300 - \left(Y + \frac{13}{12} \cdot Y\right)\right]\left(1 + \frac{2}{5}\right)$$

$$\text{Or,} \quad \frac{6.X}{5} = \left(25300 - \frac{25}{12} \cdot Y\right) \times \frac{7}{5}$$

$$\text{Or,} \quad \frac{6}{5} \times \frac{13}{12} \cdot Y = \left(25300 - \frac{25}{12} \cdot Y\right) \times \frac{7}{5}$$

$$\text{Or,} \quad Y\left(\frac{6 \times 13}{5 \times 12} + \frac{25 \times 7}{12 \times 5}\right) = 25300 \times \frac{7}{5}$$

$$\text{Or,} \quad Y \times \frac{253}{5 \times 12} = 25300 \times \frac{7}{5}$$

$$\text{Or,} \quad Y = 100 \times 7 \times 12$$

$$= \text{Rs. } 8400$$

$$\text{Now,} \quad X = \frac{13}{12} \cdot Y = \frac{13}{12} \times 8400 = \text{Rs. } 9100$$

$$Z = 25300 - (9100 + 8400)$$

$$= 25300 - 17500 = \text{Rs. } 7800$$

Now, Sums are Rs. 8400, Rs. 9100, Rs. 17500

Difference between third and first is

$$= 17500 - 8400 = \text{Rs. } 9100.$$

10. (C) Let the sum be Rs. X.

In I[st] case,

$$\text{S.I.} = X \times \frac{7}{2 \times 100} \times 8 = \frac{28}{100}X \quad \text{...(1)}$$

In 2[nd] case,

$$\text{S.I.} = X \times \frac{11}{2} \times \frac{12}{100} = \frac{66}{100}X \quad \text{...(2)}$$

According to question,

$$\frac{66}{100}X - \frac{28}{100}X = 57$$

$$\text{Or,} \quad \frac{38}{100}X = 57$$

$$\text{Or,} \quad X = \frac{57 \times 100}{38}$$

$$= \frac{3}{2} \times 100 = \text{Rs. } 150.$$

Exercise B

1. (E)

$$\text{C.I.} = A - P$$

$$A = P\left(1 + \frac{4}{100}\right)^2 = 22500\left(\frac{26}{25}\right)^2$$

$$\text{C.I.} = 22500\left[\frac{26^2}{25^2} - 1\right]$$

$$= 22500\left(\frac{26^2 - 25^2}{25^2}\right) = 1836$$

2. (D) Let the sum borrowed at 6% interest be Rs. a.

Now, the sum borrowed at 8% = (20000 – a)

Now, time is one year

$$r_1 = 6\%, \qquad r_2 = 8\%$$

$$\text{S.I.}_1 = \frac{P_1 \times r_1 \times t}{100} \qquad \text{S.I.}_2 = \frac{P_2 \times r_2 \times t}{100}$$

$$\text{S.I.}_1 = \frac{a \times 6 \times 1}{100} \qquad \text{S.I.}_2 = \frac{(20000 - a) \times 8 \times 1}{100}$$

$$\text{Since,} \quad \text{S.I.}_1 + \text{S.I.}_2 = 1280$$

$$\text{Or,} \quad \frac{6a}{100} + \frac{(20000 - a) \times 8}{100} = 1280$$

$$\text{Or,} \quad -2a + 20000 \times 8 = 128000$$

$$\text{Or,} \quad 2a = 160000 - 128000$$

$$2a = 32000$$

$$a = 16000$$

$$20000 - 16000 = 4000$$

3. (D) Let required time $= t$ years

Now, when rate is 5%

So, $1_1 = \frac{P \times r \times t}{100} = \frac{1200 \times 5 \times t}{100}$

$1_1 = 60t$ ...(1)

When rate is 6%

$1_2 = \frac{P \times r \times t}{100} = \frac{1600 \times 6 \times t}{100}$

$1_2 = 96t$ ...(2)

According to question,

$$60t + 96t = 312$$

$$t = 2 \text{ years}$$

4. (D) Let each installment per annum = Rs. x

First installment is paid after 1 year and hence will remain with the lender for the remaining (5 – 1) = 4 years.

Similarly, second installment will remain will the lender for 3 years.

Third installment for 2 years and fourth installment for one year.

Now, fifth installment remain Rs. x as such.

Now, Amount $= A_1 + A_2 + A_3 + A_4 + A_5$

Now, $1 = \frac{P \times r \times t}{100}$

$\therefore$ $A = P + 1 = P \times \left(\frac{100 + rt}{100}\right)$

Or, $A = x\left[\frac{100 + 10 \times 4}{100} + \frac{100 + 10 \times 3}{100} + \frac{100 + 10 \times 2}{100} + \frac{100 + 10 \times 1}{100} + 0\right]$

$16000 = x \times 5$

$x = \text{Rs. } 32000$

5. (A) Interest on Rs. 1000 at 5% for 1 year

$= \frac{1000 \times 5 \times 1}{100} = \text{Rs. } 50$

This Rs. 50 Rohit would have lost even if the rate of interest had not been reduced.

So, the loss of interest due to reduction in the rate of interest.

$= 75 - 50 = \text{Rs. } 25$

Reduction in the rate of interest = (5 – 4)% = 1%

Rs. 1 is lost on every Rs. 100

$\therefore$ Rs. 25 is lost when amount = Rs. 2500

So, Rohit's total amount in the account initially

$= 2500 + 1000 = \text{Rs. } 3500$

6. (D) P = Rs. 2000

(i) Interest is compounded quarterly

So, Time $= \frac{9}{12} = \frac{3}{4}$ year

$$\text{C.I.} = P\left[\left(1 + \frac{r}{100 \times n}\right)^{nt} - 1\right]$$

$$= P\left[\left(1 + \frac{8}{100 \times 4}\right)^{4 \times \frac{3}{4}} - 1\right]$$

$$= 2000\left[\left(\frac{51}{50}\right)^{3} - 1\right] = \text{Rs. } 122$$

(ii) $n = 2$ interests compounded Half-yearly.

$$\text{C.I.} = 2000 \times \left[\left(1 + \frac{8}{100 \times 2}\right)^{2 \times \frac{3}{4}} - 1\right]$$

$$= 2000 \times \left[\left(1 + \frac{1}{25}\right)^{\frac{3}{2}} - 1\right] = \text{Rs. } 121$$

(iii) $n = 1$

$$\text{C.I.} = 2000 \times \left[\left(1 + \frac{8}{100 \times 1}\right)^{\frac{3}{4}} - 1\right]$$

$$= 2000 \times \left[\left(\frac{27}{25}\right)^{\frac{3}{4}} - 1\right] = \text{Rs. } 118.$$

7. (A) Let the sum = P

Using formula, $\text{C.I.} = P\left[\left(1 + \frac{r}{100}\right)^{n} - 1\right]$

$$\text{C.I.}_1 = P\left[\left(1 + \frac{r}{100}\right)^{2} - 1\right]$$

Let us suppose $\left(1 + \frac{r}{100}\right) = a$

$\text{C.I.}_1 = P(a^2 - 1) = 272$ ...(1)

In 2nd Case,

$$\text{C.I.}_2 = P\left[\left(1 + \frac{r}{100}\right)^{3} - 1\right]$$

$434 = P(a^3 - 1)$ ...(2)

From equations (1) and (2), we get

$$\frac{a^3 - 1}{a^2 - 1} = \frac{434}{272}$$

$$\frac{a^2 + a + 1}{a + 1} = \frac{434}{272}$$

$$\frac{a^2}{a + 1} + 1 = \frac{434}{272}$$

$$\frac{a^2}{a + 1} = \frac{81}{136}$$

Or, $136a^2 - 81a - 81 = 0$

Or, $a = \frac{9}{8} = 1 + \frac{r}{100}$

$$\frac{r}{100} = \frac{1}{8}$$

$\therefore$ $r = 8\%$.

8. (E) Since, profit for each year is on the previous year's capital.

So, its similar to compound profit

Now, Using formula,

$$\text{C.I.} = 50000\left[\left(1+\frac{5}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{15}{100}\right)-1\right]$$

$$= 50000\left[\frac{21}{20}\times\frac{11}{10}\times\frac{23}{20}-1\right]$$

$$= 50000\times\frac{1313}{20\times 10\times 20}$$

$$= \frac{50}{4}\times 1313 = 16412{\cdot}5.$$

9. (B) Let Sum = P

For First case,

$$I_1 = \frac{P\times r\times t}{100}$$

$$A_1 = I_1 + P = \frac{P\times r\times t}{100} + P$$

Or, $4800 = P\left(1+\frac{3\times 5}{100}\right)$

Or, $4800 = P\left(1+\frac{3}{20}\right) = P\times\frac{23}{20}$

$$P = \frac{4800\times 20}{23} \quad \ldots(1)$$

From Second case,

$$I_2 = \frac{P\times r\times t}{100}$$

$$A_2 = I_2 + P = P + \frac{P\times 5\times t}{100}$$

Or, $5600 = P\left(1+\frac{t}{20}\right) \quad \ldots(2)$

By eqns. (1) and (2)

$$5600 = 4800\times\frac{20}{23}\left(1+\frac{t}{20}\right)$$

Or, $\frac{56}{48}\times\frac{23}{20} = \left(1+\frac{t}{20}\right)$

Or, $\frac{7}{6}\times\frac{23}{20} = \left(1+\frac{t}{20}\right)$

Or, $\frac{41}{120} = \frac{t}{20}$

$$t = \frac{41}{6} \text{ years} = 6 \text{ year } 10 \text{ month.}$$

10. (A) Let each equal installment be of Rs. a.

Since, installment is paid after one year and hence it includes interest for one year and the principal sum.

So, the present worth of the first installment

$$= \frac{a}{1+\frac{5}{100}} = \frac{20}{21}\times a$$

The second installments includes interest for two years along with its principal sum.

$\therefore$ Present worth of the second installment

$$= \frac{a}{\left(1+\frac{5}{100}\right)^2} = \frac{400}{441}\times a$$

Similarly, the present worth of the third installment

$$= \frac{a}{\left(1+\frac{5}{100}\right)^3} = \frac{8000}{9261}\times a$$

Now, according to question –

$$50440 = \frac{20}{21}\times a + \frac{400}{441}\times a + \frac{8000}{9261}\times a$$

$\therefore$ $a = $ Rs. 18522.

11. (D) Let interest of each part of sum = I

In First case,

Let sum = P_1; t = 5 years; r = 4%

So, $I. = \frac{P\times r\times t}{100} = \frac{P_1\times 5\times t}{100} = \frac{P_1}{5}$

In Second case,

Sum = P_2; t = 3 years; r = 5%

So, $I = \frac{P_2\times 3\times 5}{100} = \frac{3\times P_2}{20}$

In Third case,

Sum = P_3; t = 4 years ; r = 3%

So, $I = \frac{P_3\times 4\times 3}{100} = \frac{P_3\times 3}{25}$

Since, $P_1 + P_2 + P_3 = 24000$

$\therefore$ $I\left(\frac{25}{3}+5+\frac{20}{3}\right) = 24000$

Or, $I\times\frac{60}{3} = 24000$

$$I = 1200$$

P_1 = Rs. 6000; P_2 = Rs. 8000; P_3 = Rs. 10000

Hence, the value of biggest part is Rs. 10000.

12. (E) Let capital = Rs. a, time = 1 year

Now, in First case,

$$I_1 = \frac{a}{5} \times \frac{1}{100} \times 5 + \frac{\frac{a}{3} \times 2 \times 1}{100} + \frac{\left[a - \left(\frac{a}{5} + \frac{a}{3}\right)\right] \times 3 \times 1}{100}$$

$$920 = a\left[\frac{1}{100} + \frac{1}{150} + \frac{7}{5 \times 100}\right]$$

$$920 = a \times \left[\frac{30 + 20 + 42}{3000}\right]$$

$$a = \text{Rs. } 30000.$$

13. (E) Since, a certain sum of money increased by 50% in 3 years.

$$\text{In 3 years, } x \text{ becomes } = \frac{3}{2} \cdot X$$

In next 6 years, X will become

$$= \frac{3}{2} \times \frac{3}{2} \cdot X = \left(\frac{3}{2}\right)^2 .X$$

Similarly,

In 9 years Y will become $\left(\frac{3}{2}\right)^3 .Y$

In 12 years Z will become $\left(\frac{3}{2}\right)^4 .Y$

Now, $\left(\frac{3}{2}\right)^3 .X = \left(\frac{3}{2}\right)^3 .Y = \left(\frac{3}{2}\right)^4 .Z$

Or, $X = \frac{3}{2} \cdot Y = \frac{9}{4} \cdot Z$

$\therefore$ $X : Y : Z = 1 : \frac{2}{3} : \frac{4}{9}$

Or, $X : Y : Z = 9 : 6 : 4$

14. (C) Let Sum = P; Rate = r %; Time = 2 years

$$\text{Simple Interest} = \text{S.I.} = \frac{P \times r \times t}{100}$$

Or, $\text{S.I.} = \frac{P \times r \times 2}{100} = 60$

Or, $\frac{P \times r}{100} = 30$ …(1)

Since, $\text{C.I.} = 72$

$\text{S.I.} = 60$

$\text{C.I.} - \text{S.I.} = 12$

$\text{S.I. for First year} = \frac{60}{2} = 30$

Since, C.I. is the sum of simple interest of First year and Second year.

Now, Solving for C.I.

For First year

$$\text{S.I.}_1 = \frac{P \times r \times 1}{100}$$

For Second year

$$\text{Sum} = P + \text{S.I.} = P + \frac{P \times r}{100}$$

So, $\text{S.I.}_2 = P\left(1 + \frac{r}{100}\right) \times \frac{r}{100}$

Now, $\text{C.I.} = \text{S.I.}_1 + \text{S.I.}_2$

$$72 = \frac{P \times r}{100} + P\left(1 + \frac{r}{100}\right) \times \frac{r}{100}$$

$$72 = \frac{P \times r}{100}\left(2 + \frac{r}{100}\right) \quad \text{…(2)}$$

Putting the value of $\frac{pr}{100} = 30$

$$72 = 30\left(2 + \frac{r}{100}\right)$$

$$\frac{72}{30} = \left(2 + \frac{r}{100}\right)$$

$$\frac{r}{100} = \frac{12}{30}$$

Or, $\frac{P \times r}{100} = 30$

Or, $P \times \frac{12}{30} = 30$

Again using eq. (1)

$$P = \frac{900}{12} = \frac{300}{4} = \text{Rs. } 75.$$

15. (E) Let the first part be Rs. a and second part is Rs b.

Now, First part for 10 years

$$= a\left(1 + \frac{20}{100}\right)^{10} = a \times \left(\frac{6}{5}\right)^{10}$$

The second part for 7 years

$$= b\left(1 + \frac{20}{100}\right)^7 = b \times \left(\frac{6}{5}\right)^7$$

According to question ,

$$a \times \left(\frac{6}{5}\right)^{10} = b \times \left(\frac{6}{5}\right)^7$$

Or, $\frac{b}{a} = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$

Since, $a + b = 5115$

$\therefore$ $a + \frac{216}{125} \times a = 5115$

Or, $$a = \frac{125}{341} \times 5115$$

$$a = 125 \times 15$$

$$a = \text{Rs. } 1875$$

$$b = 5115 - 1875 = \text{Rs. } 3340$$

Hence, desired result is $b - a = 3340 - 1875$

$$= \text{Rs. } 1465.$$

16. Let Sum be Rs. a.

$$\text{S.I.} = \frac{\text{Principal} \times \text{rate} \times \text{time}}{100}$$

$$\text{S.I.} = \frac{a \times p \times 3}{100} = \frac{3ap}{100}$$

$$\text{C.I.} = a\left[\left(1 + \frac{p}{100}\right)^3 - 1\right]$$

$$\text{C.I.} = a\left[1 + \frac{p^3}{(100)^3} + \frac{3p^2}{(100)^2} + \frac{3p}{100} - 1\right]$$

$$\text{C.I.} = a \times \frac{p}{100}\left[\frac{p^2}{(100)^2} + \frac{3p}{100} + 3\right]$$

$$\text{C.I.} - \text{S.I.} = \frac{ap}{100}\left[\frac{p^2}{(100)^2} + \frac{3p}{100} + 3\right] - \frac{3ap}{100}$$

$$q = \frac{ap}{100}\left[\frac{p^2}{(100)^2} + \frac{3p}{100} + 3 - 3\right]$$

$$q = \frac{ap^2}{100^2}\left[\frac{p + 300}{100}\right]$$

$$a = \frac{q\,(100)^3}{p^2\,(p + 300)}$$

17. Let Sum a is divided into n parts $P_1, P_2, P_3, \ldots\ldots P_n$ and I be the equal interest earned on each part.

Then, $$P_1 = \frac{100 \times I}{r_1 \times t_1}$$

$$P_2 = \frac{100 \times I}{r_2 \times t_2}$$

....

....

$$P_n = \frac{100 \times I}{r_n \times t_n}$$

$$\therefore\ P_1 : P_2 : P_3 \ldots P_n = \frac{100 \times I}{r_1 \times t_1} : \frac{100 \times I}{r_2 \times t_2} : \frac{100 \times I}{r_3 \times t_3} : \frac{100 \times I}{r_n \times t_n}$$

$$P_1 : P_2 : P_3 \ldots P_n = \frac{1}{r_1 \times t_1} : \frac{1}{r_2 \times t_2} : \frac{1}{r_3 \times t_3} \ldots\ldots : \frac{1}{r_n \times t_n}$$

12 Plane Geometry

Point

Point is a basic concept in Geometry and is hypothetical too.

- Point is represented by a fine dot made by a sharp pencil on a sheet of paper.
- It has no width and no length.
- A circle with zero radii. So, area is also zero.

Straight Line

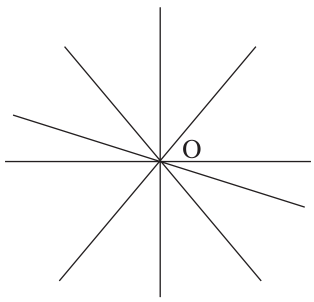
1. At least two distinct points can define a line.

Here A and B are two distinct points.

2. A line contains infinitely many points.

A, B, C, D, E … are infinite points on the line.

3. Through a given point, there pass infinitely many lines.



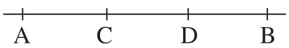
Through point O, infinite lines pass.

4. A line has only length no width.

5. Given two distinct points, there is one and only one line that can contain both the given points.

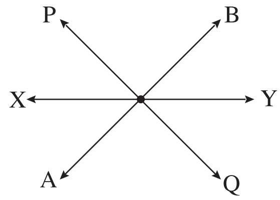
If A and B are two distinct points, then there is only one line which contain both the points.

6. Collinear—If two or more points are said to be collinear if these point lie on the same line.



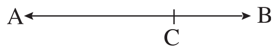
A, B, C and D are four distinct points contained by one straight line, So these four points are collinear.

7. Concurrent—If more than three lines are said to be concurrent if these lines pass through a point.

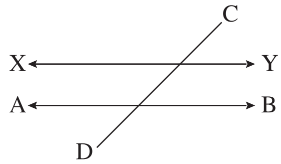


8. A point C is said to be on the line AB.

If $\quad AC + CB = AB$



9. Two distinct lines in the same plane are either parallel or intersecting at only one point.



Lines XY and AB are parallel.

Here, AB and CD are intersecting each other.

Parallel

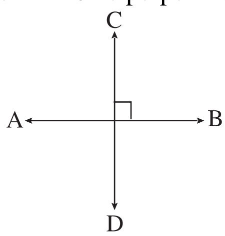
Two lines are said to be in parallel if

(i) They lie in the same plane.

(ii) They do not intersect, though how far they are extended.

Perpendicular Lines

If two lines make an angle of 90° with each other, then these lines are said to be perpendicular.



Clearly, AB is perpendicular to CD.

It is represented by $AB \perp CD$ or $CD \perp AB$.

Line Segment

A line segment is a part of a line.

1. It has definite length and two end points.
2. It can not be extend on both sides.
3. Every line segment has one or only one mid point.

AB is the part of PQ. So, AB is line segment.

Ray

A ray has one end point and extends in the other direction up to infinity.

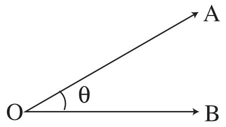
It is represented by the symbol $\rightarrow$

Angles

An angle is the bending between two rays which have same end points but different direction.

Here, are two rays having the same end point O.

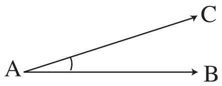
- The end point O is known as vertex.
- OA and OB are called the "arms".
- θ is known as angle. "$\angle$" it is represented by the symbol and $\angle AOB = \angle BOA = \theta$ Every angle divides the whole space around itself in two parts :



Types of Angles

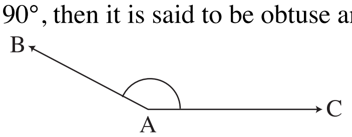
1. Acute Angles—If measurement of an angle is less than 90°, then such angle is said to be acute angle.

$\angle A = \angle CAB = \angle BAC$ is acute angle.

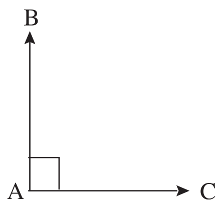


No two acute angles can be supplementary.

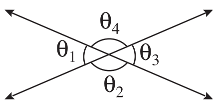
2. Obtuse Angle—If measurement of an angle is greater than 90°, then it is said to be obtuse angle.



3. Right Angle—An angle whose measure is 90° is called a right angle. The angle between two perpendicular lines is 90°.



4. Vertically Opposite Angles—When two lines interest, four angles are formed. The angles opposite to each other are called vertically opposite angles and are equal to each other.



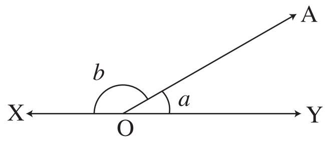
Here, $\theta_1 = \theta_3$

and $\theta_2 = \theta_4$

where, θ_1 and θ_3 are vertically opposite angles.

θ_2 and θ_4 are vertically opposite angles.

5. Supplementary Angles—If sum of two angles is equal to 180°. then these angles are said to be supplementary angles.

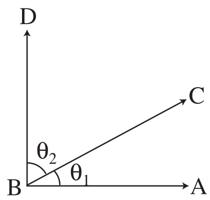


Here, angles a and b are supplementary angles.

$$a + b = 180°$$

Since, XOY is a straight line.

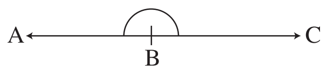
6. Complementary Angles—If sum of two angles is equal to 90°, then these angles are said to be complementary angles.



Here, $\angle ABC + \angle CBD = 90°$

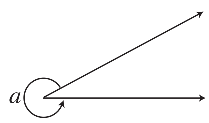
$$\theta_1 + \theta_2 = 90°$$

7. Straight Angle—A straight angles has its sides lying along a straight line. An angle whose measure is 180° is called a straight angle.



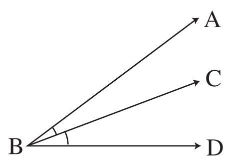
$$\angle ABC = 180°$$

8. Reflex Angle—An angle with measure more than 180° and less than 360° is called a reflex angle.



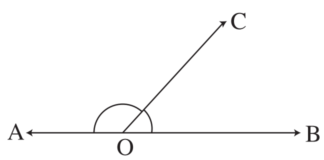
$$180° < a < 360°$$

9. Adjacent Angles—Two angles are adjacent if they share the same vertex and a common side but no angle is inside another angle.



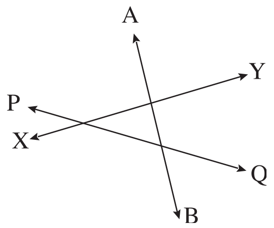
Clearly, from diagram $\angle ABC$ and $\angle CBD$ are adjacent angles.

10. Linear Pair of Angles—Two angles are said to form a linear pair if they have a common side and their other two sides are opposite rays.



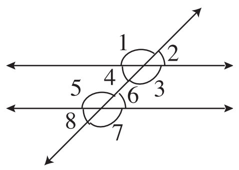
The Sum of the measures of the angle is 180°. The angles that form a linear pair are always adjacent.

11. Angles Made by a Transversal—A line which intersects two or more given coplanar lines in distinct point.



Here, AB is a transversal line.

12. Corresponding Angles—Angles that appear in the same relative position in each group are called corresponding angles.



That is $\angle 1$ and $\angle 5$ are corresponding angles.

$\angle 2$ and $\angle 6$ are corresponding angles.

$\angle 7$ and $\angle 3$ are corresponding angles.

13. Alternate Interior Angles—Interior angles on opposite sides of the transversal are called alternate interior angles.

Here, $\angle 4$ and $\angle 6$ are alternate interior angles.

$\angle 3$ and $\angle 5$ are alternate interior angles.

14. Consecutive Interior Angles—The pairs of the interior angles on the same side of the transversal are called the pairs of consecutive interior angles.

Here, $\angle 4$ and $\angle 5$ are consecutive interior angles.

$\angle 3$ and $\angle 6$ are consecutive interior angles.

Transversal Across the Two Parallel Lines

When a transversal line *mn* cuts the parallel lines XY and AB, then we have the following relationship between angles.

(1) Corresponding angles will be equal—

Here,
$$\angle 1 = \angle 5$$
$$\angle 2 = \angle 6$$
$$\angle 8 = \angle 4$$
$$\angle 7 = \angle 3$$

(2) Alternate interior angles will be equal—
$$\angle 4 = \angle 6$$
$$\angle 8 = \angle 5$$

The sum of consecutive interior angles will be equal to 180°
$$\angle 5 + \angle 4 = 180°$$
$$\angle 3 + \angle 6 = 180°$$

Some Important Points Related to the Line

1. Two adjacent angles are linear pair if they are supplementary.

2. The sum of all the angles rounded at a point is equal to 360°.

3. If two parallel lines are intersected by a transversal, then the bisector of any pairs of alternate interior angles are parallel.

Clearly, OA and O' A' are parallel lines.

If these are bisectors of angles $\angle 1$ and $\angle 2$ which are alternate interior angles of two parallel lines l and m.

Triangles

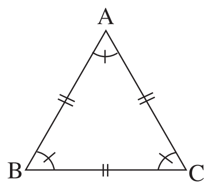
The plane figure bounded by the union of three lines, which join three non-collinear points, is called a triangle. A triangle is denoted by the symbol Δ.

Non-collinear points are called the vertices of the triangle. In ΔABC. A, B and C are the vertices of the triangle.

Line Segments—AB, BC and AC are called sides of the triangle.

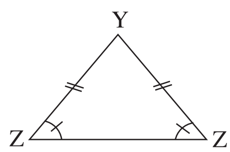
(1) Based on Length of the Sides—There are three types of triangles—

(a) Equilateral Triangle—Equilateral triangles have equal sides and equal angles.



Each angle measures 60°.
$$AB = BC = AC$$
$$A = B = C = 60°$$

(b) Isosceles Triangle—It has two equal sides and two equal angles two equal angles are some times called the base angles and the third angle is called the vertex angle.



Here,
$$XY = YZ$$
$$\angle YXZ = \angle XZY$$

(c) Scalene—Scalene triangles have all three sides of different length and all angles of different measure.
$$PR \neq RQ \neq PQ$$

(2) Based on Measure of the Angles—There are three types of triangles—

(a) Acute Triangle—A triangle in which all the angles are acute (< 90°) is called as an acute angle triangle.

A special case of an acute angle triangle is when all the three acute angles are equal.

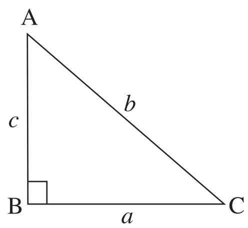
(b) Obtuse Triangle—A triangle in which one of the angles is obtuse is called as an obtuse triangle.

Here, $\angle B > 90°$

(c) Right Angle Triangle—Right angled triangle contains one right angle and other two angles are complementary.

In right angle, the opposite side of the right angle is called hypotenuse.

The other two sides are called base and perpendicular.



$$AC = \text{Hypotenuse}$$
$$BC = \text{Base}$$
$$AB = \text{Perpendicular}$$

Pythagorean Theorem is based on right angled triangle.

Pythagorean Theorem—It states that the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

$$(AC)^2 = (BC)^2 + (AB)^2$$

Some Important Points

1. Sum of angles of a triangle is equal to 180°.

2. Sum of the lengths of any two sides is greater than the length of third side.

3. Difference of any two sides is less than the third side.

4. Side opposite to the greatest angle is greatest and *vice-versa*.

5. A triangle must have at least two acute angles.

6. If a, b, c denote the sides of a triangle, then

(i) If $c^2 < a^2 + b^2$, triangle is acute angled.

(ii) If $c^2 = a^2 + b^2$, triangle is right angled.

(iii) If $c^2 > a^2 + b^2$, triangle is obtuse angled.

7. Triangles on equal bases and between the same parallel have equal area.

8. When two sides are extended in any direction, an angle is formed with another side. This is called the exterior angle.

There are six exterior angles of a triangle.

Similarity—Two triangles are similar if all three pairs of corresponding angles are equal. The sum of the three angles of a triangle is 180°.

Therefore, if two angles of triangle – 1 are equal to two angles of triangle – 2. Then, these triangles are said to be similar.

The length the sides of similar triangles are in proportion to each other.

Here, $ST \parallel QR$

$\therefore$ $\Delta PST \sim \Delta PQR$

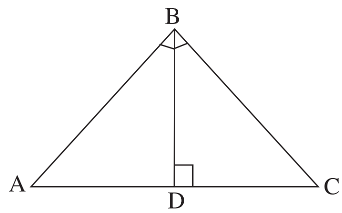
$\therefore$ $\dfrac{PR}{PS} = \dfrac{PQ}{PT}$

$= \dfrac{ST}{RQ}$

Properties of Similar Triangles

(A) Right Triangle—ABC is a right triangle with B as right angle.

Now, BD is perpendicular to AC.



(a) $\Delta ABC \sim \Delta ABD$

$$\frac{AC}{AB} = \frac{AB}{AD}$$

$$(AB)^2 = AC \times AD$$

(b) $\Delta ABC \sim \Delta BCD$

$$\frac{AC}{BC} = \frac{BC}{CD}$$

$$(BC)^2 = AC \times CD$$

(c) $\Delta ABD \sim \Delta BCD$

$$\frac{AD}{BD} = \frac{BD}{CD}$$

$$(BD)^2 = AD \times CD$$

(B) If two triangles are equiangular, their corresponding sides are proportional.

In $\Delta ABC \sim \Delta XYZ$

If $\angle A = \angle X$

$\angle B = \angle Y$

$\angle C = \angle Z$

Then, $\dfrac{AB}{XY} = \dfrac{AC}{XZ} = \dfrac{BC}{YZ}$

Congruence of Two Triangles—Two triangles are congruent if and only if one of them can be made to superpose on the others, so as to cover if exactly.

(A) Side-Angle-Side (SAS) Congruence Postulate—If the two sides and the angle included in one triangle are congruent to the corresponding two sides and the angle included in another triangle then the two triangles are congruent.

$\therefore$ $\Delta ABC \cong \Delta A'B'C'$

$\therefore \quad \angle B = \angle B'$

$AB = A'B'$

$BC = B'C'$

(B) Angle-Side-Angle (ASA) Congruence Postulate—If two angles of one triangle and the side they include are congruent to the corresponding angles and side of another triangle the two triangle are congruent.

$\angle B = \angle E$

$\angle C = \angle F$

$BC = EF$

$\therefore \quad \Delta ABC \cong \Delta DEF$

(C) Angle-Angle-Side (AAS) Congruence Postulate—If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another, then the two triangles are congruent.

$\angle A = \angle P$

$\angle B = \angle Q$

$\therefore \quad BC = QR$

$\therefore \quad \Delta ABC \cong \Delta PQR$

(D) Side-Side-Side (SSS) Congruence Postulate—If all the sides of one triangle are congruent to the corresponding sides of another triangles then the triangles are congruent.

$XY = X'Y'$

$XZ = X'Z'$

$YZ = Y'Z'$

$\therefore \quad \Delta XYZ \cong \Delta X'Y'Z'$

(E) Right-Angle-Hypotenuse-Side (RHS) Congruence Postulate—This postulate is applicable only to right triangles. If the hypotenuse and any one side of a right triangle are congruent to the hypotenuse and the corresponding side of another right another then the two triangles are congruent.

Here, $\quad PR = P'R'$

$\angle Q = Q' = 90°$

$PQ = P'Q'$

$\therefore \quad \Delta PQR \quad P'Q'R'$

1. Altitude—An altitude is the perpendicular dropped from one vertex to the side opposite the vertex. If measures the distance between the vertex and the line which is the opposite side.

Since, every triangle has three vertices, it has three altitudes.

Here, $\quad AD \perp BC$

$\therefore \quad AB = \text{Altitude}$

2. Orthocentre—The perpendiculars drawn from the vertices to opposite sides meet at a point called orthocenter of the triangle.

Clearly, altitudes AB, CF and BE cut at a point O.

So, O is the orthocenter of ΔABC.

3. Median—A line segment from the vertex of a triangle to the mid-point of the side opposite to it is called a median.

Thus, every triangle has three medians.

4. Centroid—In a triangle, the point of intersection of three medians is called centroid. The centroid divides any median in the ratio 2 : 1.

5. Circumcentre—The point at which the perpendicular bisectors of the sides of a triangle meet is the circumcentre of the triangle. The circumcentre O of a triangle is equidistant from the three vertices.

We have $\quad OA = OB = OC$

The circle with centre O and passing through A, B and C is called the circumcircle of ΔABC.

6. Angle Bisector—A line segment from the vertex to the opposite side such that it bisects the angle at the vertex is called as angle bisector. Thus every triangle has three angle bisectors.

$$\angle 1 = \angle 2 = \frac{\angle A}{2}$$

7. Incentre—The point of intersection of the angle bisectors of a triangle is called the incentre I.

The perpendicular distance of I to any one side is in radius and the circle with centre I and radius equal to in radius is called the in circle of the triangle.

Clearly, $\quad IA = \text{inradius}$

Exercise A

1. In a right angle triangle ABC, right angled at A, if $AD \perp BC$, such that $AD = a'$, $BC = b$, $CA = c$, $AB = a$, then—

   (A) $\frac{1}{a'^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (B) $\frac{1}{a'^2} = \frac{b^2}{c^2(b^2 - c^2)}$

   (C) $\frac{1}{a'^2} = \frac{1}{a^2} + \frac{1}{c^2}$ (D) $\frac{1}{a'^2} = \frac{1}{b^2} + \frac{1}{c^2}$

2. If ABC is an equilateral triangle and $AD \perp BC$, then AD^2 is equal to—

   (A) $\frac{3}{2} CD^2$ (B) $2DC^2$

   (C) $3DC^2$ (D) $4DC^2$

3. The circum-radius of an equilateral triangle of side 12 cm will be—

   (A) $2\sqrt{3}$ cm (B) $3\sqrt{3}$ cm

   (C) $4\sqrt{3}$ cm (D) $\sqrt{22}$ cm

4. PQR is a triangle, $PS \perp QR$. If $RS^2 - SQ^2 = \frac{1}{4} QR^2$, then $\frac{RS}{RQ} = ?$

   (A) $\frac{5}{4}$ (B) $\frac{5}{8}$

   (C) $\frac{7}{3}$ (D) $\frac{3}{7}$

5. P is a point on the base BC of the equilateral triangle ABC, such that $BP = \frac{BC}{4}$. Then $\frac{AP^2}{AB^2}$ = —

(A) $\frac{13}{16}$ (B) $\frac{13}{6}$

(C) $\frac{11}{7}$ (D) $\frac{3}{16}$

6. In ΔPQR, $\angle A = 60°$. The internal bisectors of angles B & C meet at D. Then $\angle BDC$ will be—

(A) 90° (B) 115°

(C) 60° (D) 120°

7. The value of X in the adjoining figure will be (given that PQ || RS)—

(A) 3 (B) 3·5

(C) 1·5 (D) 2·0

8. In the figure, AB is parallel to CD. OC = OD = 18 cm, AB = OB = 12 cm and CD = 6 cm. Then the length of AC is—

(A) 14 cm (B) 15 cm

(C) 16 cm (D) 17.1 cm

9. In the triangle PQR, PR = 8 cm, PR = 20 cm and BR = 4 cm. Then the length of AQ is—

(A) 16 cm (B) 18 cm

(C) 32 cm (D) 40 cm

10. In the figure, AB, CD, XY are parallel. If AB = 4 cm, CD = 6 cm, then XY will be equal to—

(A) 3 cm (B) 5 cm

(C) 2·4 cm (D) 2·405 cm

11. AB is a chord of a circle which is equal to its radius. Then the angle subtended by this chord at the minor arc is—

(A) 60° (B) 75°

(C) 1000° (D) 150°

12. In the figure AC = CE = EG = GI. If the area of strip DEGF = 20 units and also BC II DE II FG II HI, then area of strip BCED will be—

(A) 7 units (B) 5 units

(C) 12 units (D) 8 units

13. In the figure, if ST is parallel to QR, then the measure of $\angle TSP$ will be—

(A) 13° (B) 43°

(C) 33° (D) 77°

14. In the diagram, L_1 and L_2 are parallel. The sum of the angles α, β, χ marked in the diagram is—

(A) 180° (B) 270°

(C) 360° (D) less than 270°

15. ABC is an equilateral triangle and PQRS is a square inscribed in it. Which of the following is true ?

(A) $AR^2 = RC^2$ (B) $2AR^2 = RC^2$

(C) $3AR^2 = 4RC^2$ (D) $4AR^2 = 3RC^2$

16. The three sides of a triangle are 12, 20, and 24 cm respectively. What is the area of the triangle ?

(A) 32 cm^2 (B) $32\sqrt{14}$ cm^2

(C) 50 cm^2 (D) None of these

17. The base of a right angled triangle is 5 units and its hypotenuse is 13 units. What is its area ?

(A) 30 sq. units (B) 40 sq. units

(C) 45 sq. units (D) 85 sq.units

18. The length of three sides of a triangle are given. In which of the following cases, a triangle cannot be formed ?

(A) 3 cm, 4 cm, 5 cm

(B) 10 cm, 8 cm, 6 cm

(C) 7.8 cm, 1.8 cm, 9.5 cm

(D) 1.3 cm, 1.2 cm, 0.5 cm

19. The perimeter of a right-angled triangle is 24 cm. If the hypotenuse of the triangle is 10 cm and one of the other side is 8 cm, what would be the area of the triangle ?

(A) 48 sq.cm (B) 24 sq. cm

(C) 72 sq.cm (D) 80 sq. cm

20. From an equilateral triangle, a new triangle was formed by increasing one side x times and reducing the other x times, *i.e,* if the side was 'a' initially, then making one side ax and another side $\frac{a}{x}$. Then which of the following is/are true ?

I. $x < \frac{1+\sqrt{5}}{2}$ II. $x > \frac{2}{1+\sqrt{5}}$

III. x has no upper limit IV. x has no lower limit

(A) I only (B) I and II only

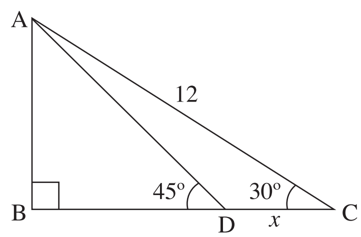
(C) I and IV only (D) II and III only

21. PS, UT are drawn perpendicular to QR in ΔPQR. If PS = 8 cms and UT = 1 cm, the ratio of area of ΔPUR that of ΔURQ is—

(A) 8 : 3 (B) 5 : 3

(C) 5 : 8 (D) 8 : 5

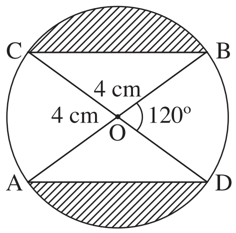
22. The value of x in the given figure is—



(A) $12(\sqrt{3}-1)$ cm (B) $6(\sqrt{3}-1)$ cm

(C) $12\sqrt{3}$ cm (D) $6\sqrt{3}$ cm

23. In the figure, the area of shaded portion is—



(A) $2\left(\frac{\pi}{3}-\sqrt{3}\right)$ cm² (B) $4\left(\frac{\pi}{3}-\sqrt{3}\right)$ cm²

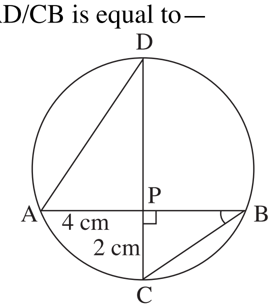
(C) $4\left(\frac{2\pi}{3}-3\sqrt{3}\right)$ cm² (D) $4\left(\frac{2\pi}{3}-\sqrt{3}\right)$ cm²

24. The sum of two angles of a triangle is 100° and their difference is 30°. Then the smallest angle is—

(A) 40° (B) 50°

(C) 65° (D) 35°

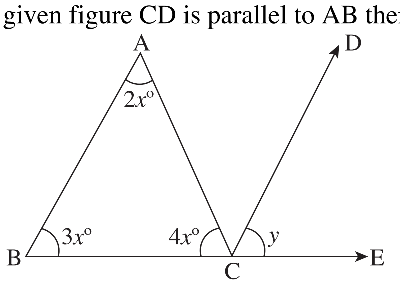
25. In given figure, $\angle B = \angle D$ and CP = 2 cm, AP = 4 cm, then AD/CB is equal to—



(A) 5/2 (B) 3

(C) 2/1 (D) 3/2

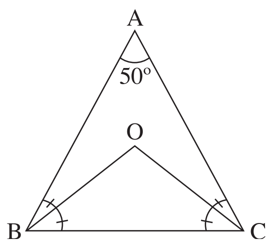
26. In the given figure CD is parallel to AB then $\angle y$ is—



(A) 60° (B) 100°

(C) 80° (D) 40°

27. In a ΔABC, $\angle A = 50°$. If the internal bisectors of angles B and C meet in O, then the measure of $\angle BOC$ is—



(A) 115° (B) 120°

(C) 125° (D) 130°

28. If the angles $(3x - 40)°$ and $(2x - 20)°$ are complementary, then the value of x is—

(A) 45° (B) 35°

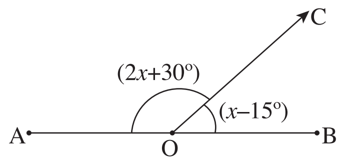
(C) 25° (D) None of these

29. If two supplementary angles differ by 50°, then the smaller angle is—

(A) 115° (B) 75°

(C) 65° (D) None of these

30. In the adjoining figure, if AOB is a line, then the value of x is—



(A) 75° (B) 55°

(C) 45° (D) 35°

Exercise B

1. The sum of interior angles of a polygon is twice the sum of its exterior angles. The polygon is—

(A) Octagon (B) Nonagon

(C) Hexagon (D) Decagon

2. A regular polygon has 54 diagonals. The number of sides of the polygon is—

(A) 9 (B) 10

(C) 12 (D) 15

3. The sides of a Pentagon are produced to meet so as to form a star shaped figure, as shown below. The sum of the angles at vertices of the star is—

(A) 2 right angles (B) 3 right angles

(C) 4 right angles (D) 5 right angles

4. If each interior angle of a regular polygon is 10 times its exterior angle, the number of sides in the polygon is—

(A) 10 (B) 12

(C) 22 (D) 24

5. If a regular hexagon is inscribed in a circle of radius r, then the perimeter and area hexagon will be—

(A) $6r\ \frac{3\sqrt{2}r^2}{4}$ (B) $5r, \frac{3\sqrt{3}r^2}{2}$

(C) $6r, \frac{3\sqrt{3}r^2}{2}$ (D) $5r, \frac{3\sqrt{3}r^2}{4}$

6. The radius of a circle is 20 cm. The radii (in cm) of three concentric circles drawn in such a manner that the whole area is divided into four equal parts, are—

(A) $20\sqrt{2}, 20\sqrt{3}, 20$ (B) $10\sqrt{3}/3, 10\sqrt{2}/3, 10/3$

(C) $10\sqrt{3}, 10\sqrt{2}, 10$ (D) 17, 14, 10

7. Two circles of radii 10 cm and 6 cm are drawn with centers C and C′ respectively. Their transverse common tangents meet CC′ in A. The point divides CC′ in the ratio—

(A) 4 : 5 internally (B) 10 : 16 internally

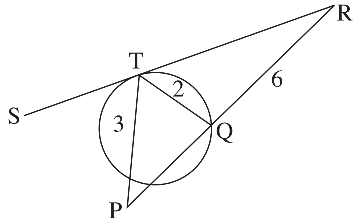
(C) 10 : 6 externally (D) 16 : 10 externally

8. C_1 and C_2 are the centres of the two circles whose radii are 7 cm and 5 cm respectively. The direct common tangents to the circles meet C_1C_2 in P. Then P divides C_1C_2 in the ratio—

(A) 5 : 6 internally (B) 7 : 5 internally

(C) 7 : 5 externally (D) 5 : 7 externally

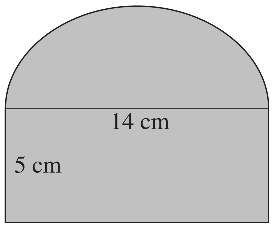
9. In the figure, the length of TR will be—



(A) 6 cm (B) 9 cm

(C) 5 cm (D) 8 cm

10. In the adjoining figure, AD, AE and BC are tangents to the circle at D, E, F respectively. Then—



(A) AD = AB + BC + CA

(B) 2 AD = AB + BC + CA

(C) 3 AD = AB + BC + CA

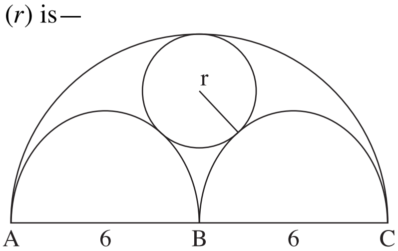
(D) 4 AD = AB + BC + CA

11. The sum of the interior angles of a polygon is 7 times the sum of its exterior angles. The number of sides in the polygon is—

(A) 14 (B) 16

(C) 18 (D) 20

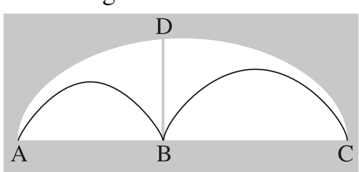
12. In the adjoining figure the radius of the smallest circle (r) is—



(A) $3\sqrt{3}$ cm (B) 3 cm

(C) 2 cm (D) None of these

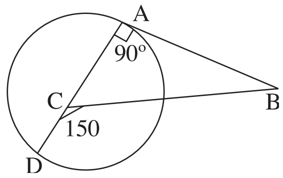
13. In the figure, if AB = 4 cm, BD = $6\sqrt{3}$ cm, then the area of shaded region will be—



(A) 25π (B) 27π

(C) 30π (D) 45π

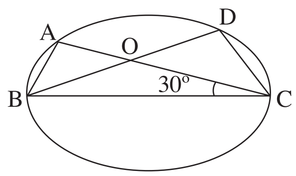
14. In the figure AB is tangent and AD is the diameter of the circle. If $\angle BCD = 150°$, $\angle ABC$ will be—



(A) 40° (B) 50°

(C) 60° (D) 70°

15. In the figure, O is centre of circle, $\angle ACB = 30°$. Then $\angle BDC$ will be—



(A) 30° (B) 45°

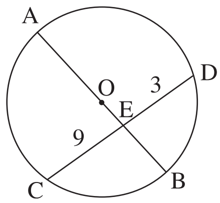
(C) 60° (D) 75°

16. A circle has two parallel chords of length 6 cm and 8 cm. If the chords are 1 cm apart and are on the same side of the diameter- parallel to them, then the diameter of the circle is—

(A) 5 cm (B) 6 cm

(C) 10 cm (D) 12 cm

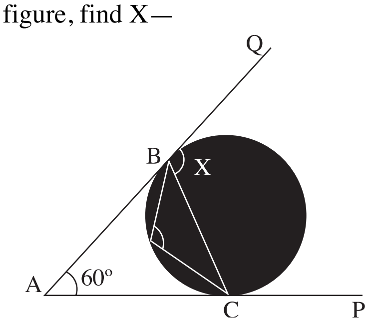
17. In the figure AB is the diameter of the circle with centre O. CD cuts AB at E such that OE = EB. If CE = 9 cm and ED = 3 cm, then the diameter of the circle will be—



(A) 8 cm (B) 10 cm

(C) 11 cm (D) 12 cm

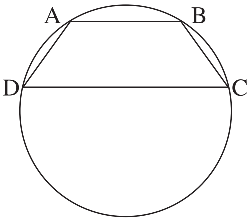
18. In the figure, find X—



(A) 120° (B) 135°

(C) 140° (D) 145°

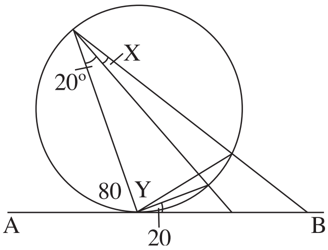
19. In the figure, DC is the diameter of circle and AB is a chord parallel to DC. If AB = AD, then the value of $\angle BAD$ will be—



(A) 100° (B) 120°

(C) 90° (D) 130°

20. In the figure, the value of x is—



(A) 15° (B) 65°

(C) 70° (D) 85°

21. In the above question the value of y is—

(A) 15° (B) 65°

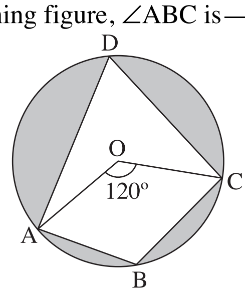
(C) 70° (D) 85°

22. The angle of a hexagon are x, $2x + 4$, $2x + 4$, $x - 10$, $x - 10$, and $2x + 21$, the value of x is—

(A) 45° (B) 60°

(C) 79° (D) 85°

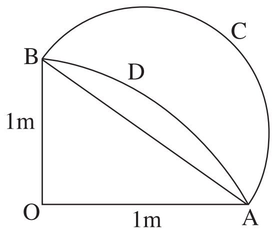
23. In the adjoining figure, ∠ABC is—



(A) 40° (B) 60°

(C) 70° (D) 120°

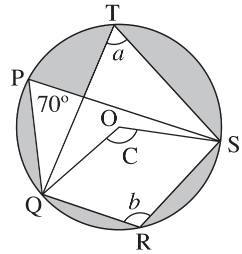
24. In the figure O is the center of the sector OAB, ∠BOA = 90°. A semicircle ABC is constructed with AB as diameter. If OA = 1 m, then the area of the shaded part will be—



(A) πm^2 (B) $\frac{1}{2} m^2$

(C) 300 m^2 (D) 5000 cm^2

25. In the adjoining figure, the respective values of a, b and c are—



(A) 90°, 105°,110° (B) 70°, 95°, 105°

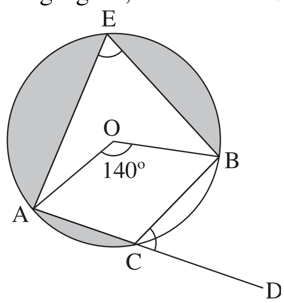
(C) 70°, 120°, 140° (D) 75°, 90°, 85°

26. Two parallel chords of a circle are 6 cm and 8 cm in length. If the diameter of the circle is 10 cm, then the distance between the chords is—

(A) 1 cm (B) 2 cm

(C) 3 cm (D) 3.5 cm

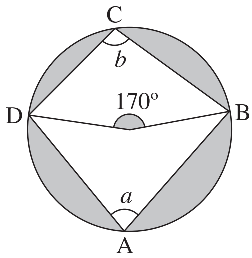
27. In the adjoining figure, the measure of ∠BCD is—



(A) 45° (B) 70°

(C) 90° (D) 120°

28. In the adjoining figure. Find the value of angle a and b—



(A) 110° and 70° (B) 120° and 60°

(C) 85° and 95° (D) 80° and 100°

Answers with Hints

Exercise A

1. (B) Area of triangle $= \frac{1}{2} ac = \frac{1}{2} a'b$ or $a = \frac{a'b}{c}$

Now, $a^2 + c^2 = b^2$

$$\Rightarrow \left(\frac{a'b}{c}\right)^2 + c^2 = b^2$$

$$\Rightarrow a'^2 = \frac{b^2}{c^2}(b^2 - c^2)$$

$$\Rightarrow \frac{1}{a'^2} = \frac{b^2}{c^2(b^2 - c^2)}$$

2. (C) We have $AD = \frac{\sqrt{(3)}}{2} \times BC$

$$= \frac{\sqrt{3}}{2} \times DC \times 2 = \sqrt{3}DC$$

or $AD^2 = 3\ DC^2$

3. (C) Circum – radius $= \frac{a}{\sqrt{3}}$

where a is the side of equilateral triangle.

$$\Rightarrow \quad \frac{12}{\sqrt{3}} = \frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 4\sqrt{3} \text{ cm}$$

4. (B) $RS^2 - SQ^2 = \frac{1}{4} QR^2$

$\Rightarrow (PR^2 - PS^2) - (PQ^2 - PS^2) = \frac{1}{4} QR^2$

$\Rightarrow \quad PR^2 - PQ^2 = \frac{1}{4} QR^2$ ……(i)

$PR^2 - PQ^2 = RS \times RQ - QS \times QR$

$= QR\,[RS - QS]$ …….(ii)

From equations (i) and (ii)

$\Rightarrow \quad QR[RS - QS] = \frac{1}{4} QR^2$

$\Rightarrow RS - (RQ - RS) - \frac{1}{4} QR$

$\Rightarrow \quad RS + RS - RQ = \frac{1}{4} QR$

$\Rightarrow \quad 2RS = \frac{5}{4} RQ$

$\Rightarrow \quad \frac{RS}{RQ} = \frac{5}{8}$

5. (A) $AB = BC = AC$ (given)

$BP = \frac{1}{4} BC$

AD is perpendicular to BC.

$\Rightarrow \quad AD = DC = \frac{1}{2} BC$ or $\frac{1}{2} AB$

$\Rightarrow \quad PD = BD - BP$

$= \frac{1}{2} BC - \frac{1}{4} BC$

$= \frac{1}{4} BC$ or $\frac{1}{4} AB$

$AP^2 = AD^2 + PD^2$

(Pythagorean theorem)

$= (AB^2 - BD^2) + PD^2$

$= AB^2 - \frac{1}{4} AB^2 + \frac{1}{16} AB^2$

$= \frac{AB^2}{16} [16 - 4 + 1]$

$= \frac{13AB^2}{16}$

$\Rightarrow \quad \frac{AP^2}{AB^2} = \frac{13}{16}$

6. (D) Angle $\angle BDC = 90° + \frac{1}{2} \angle A$

$= 90° + \frac{1}{2} (60°)$

$= 120°.$

7. (C) The two triangles are similar.

Therefore, $\frac{3}{4} = \frac{X}{2}$

$4X = 6$

or $X = \frac{6}{4} = 1{\cdot}5.$

8. (A) $p = p_1$ (since OC = OD)

$q = q_1$ (OB = AB)

$\Rightarrow$ Since, AB ∥ CD)

$\Rightarrow \quad q_1 = p$

$\Rightarrow \quad q_1 = p_1$

$= q = q$

$\Rightarrow$ OAB ≅ ≈ CDO (A.A.A.)

$\Rightarrow \quad \frac{OA}{AB} = \frac{CD}{DO}$

$\Rightarrow \quad \frac{OA}{12} = \frac{6}{18}$

$\Rightarrow \quad OA = \frac{6 \times 12}{18} = 4$ cm.

$\Rightarrow \quad AC = OC - OA$

$= (18 - 4) = 14$ cm.

9. (C) Triangle PAB and PRQ

$\angle P = \angle P$ [Common]

$\angle PAB = \angle PRQ$ [Given]

$\angle PBA = \angle PQR$ [Property]

So, $\Delta PAB \cong \Delta PRQ$ [by A.A.A. property]

$\Rightarrow \quad \frac{PQ}{PR} = \frac{PB}{PA}$

$\Rightarrow \quad \frac{PQ}{20} = \frac{PR - BR}{8}$

$\Rightarrow \quad \frac{PQ}{20} = \frac{20 - 4}{8}$

$\Rightarrow \quad PQ = \frac{20 \times 16}{8} = 40$ cm.

$\Rightarrow \quad AQ = PQ - PA$

$\Rightarrow \quad = (40 - 8) = 32$ cm.

10. (C) ΔABC and ADC are similar triangles

$\Rightarrow \quad \frac{AB}{XY} = \frac{CA}{CY}$

$\Rightarrow \quad XY = \frac{AB \times CY}{CA} = \frac{4CY}{CA}$ ……(i)

$\Rightarrow \quad \frac{DC}{XY} = \frac{AC}{AY}$

$\Rightarrow \quad XY = \frac{6AY}{AC}$ ……(ii)

From equations (i) and (ii)

$\frac{4CY}{CA} = \frac{6AY}{AC}$

$\Rightarrow \quad \frac{CY}{AY} = \frac{6}{4} = \frac{3}{2}$

$\Rightarrow \quad \frac{CY + AY}{AY} = \frac{3}{2} + 1$

$= \frac{5}{2}$

or $XY = \frac{6AY}{AC} = \frac{6 \times 2}{5}$

$\Rightarrow \quad XY = \frac{12}{5}$

$= 2{\cdot}4$ cm

11. (D) Given that $OA = OB = AB$

$\therefore \quad \angle AOB = 60°$

Now, extend AO so that it meets the circle at the point C. Join BC,

$\therefore \quad \angle ABC = \frac{1}{2}\angle AOB$

$= \frac{1}{2} \times 60° = 30°$

Again, since APBC is a cyclic quadrilateral,

So, $\angle APB = (180° - 30°) = 150°$

12. (C) If $BC = X$

Then, $DE = 2X$,

$FG = 3X$,

$HI = 4X$, [By similar triangles]

Area of DEGF = 20 units

$= \frac{1}{2}(2X + 3X) \times h = 20$

[$\therefore$DEFG is a trapezium and h = Distance between two parallel lines]

$\Rightarrow \quad Xh = \frac{20 \times 2}{5} = 8$

$\Rightarrow$ Area of BCDE $= \frac{1}{2}(X + 2X) \times h$

$= \frac{1}{2} \times 3X \times h = \frac{3}{2} \times (Xh)$

$= \frac{3}{2} \times 8 = 12$ units.

13. (D) $ST \parallel QR$

$\Rightarrow \angle Q = \angle S$,

$\angle R = \angle T$

$\angle TPY = \angle TSP = 77°$

14. (C) L_3 is drawn parallel to L_1 and L_2.

Let $\beta = \alpha_1 + \chi_1$

Now, $\alpha + \alpha_1 = 18°$

and $\chi + \chi_1 = 18°$

$\therefore \quad \alpha + \alpha_1 + \chi + \chi_1 = 36°$

and $(\alpha_1 + \chi_1) = \beta$

Hence, $\alpha + \beta + \chi = 360°$

Required answer = 360°

15. (D) Since, $SR \parallel BC$

$\therefore \quad AS = AR$

Since, $\angle A = 60°$,

$\therefore$ ΔASR is equilateral.

Let $AR = X$,

$\therefore \quad PQ = SR = X$

In ΔRQC $\angle RQC$,

$\angle C = 60°$

$\sin 60° = \frac{RQ}{RC} = \frac{X}{Y}$

or $\frac{\sqrt{3}}{2} = \frac{X}{Y}$

or $\sqrt{3}Y = 2X$

Squaring both side,

$3Y^2 = 4X^2$;

$3RC^2 = 4AR^2$.

16. (B) $s = \frac{12 + 20 + 24}{2} = \frac{56}{2}$

$= 28$

Here, $a = 10$ cm,

$b = 14$ cm

and $c = 16$ cm

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{28 \times 16 \times 8 \times 4}$

$= \sqrt{4 \times 7 \times 4 \times 4 \times 8 \times 4}$

$= 32\sqrt{14}$ cm^2

17. (A) $BC^2 = AC^2 - AB^2$

$= (13)^2 - (5)^2$

$BC^2 = 144$

$\Rightarrow \quad BC = 12$ units.

Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 5 \times 12$

$= 30$ sq.units.

18. (C) The sum of any two sides should be greater than the third side.

Here, $(1{\cdot}8 + 9{\cdot}5)$ cm = $7{\cdot}8$ cm

19. (B) Perimeter = 24 cm

Then third side is $= 24 - (10 + 8)$

Area of the triangle $= \frac{1}{2} \times 8 \times 6$

$= 24$ sq. cm.

20. (A) Here, the property of the triangle *i.e.,* sum of the two sides of a triangle must be greater than third side is used.

(i) $ax < a + \frac{a}{x}$

$\Rightarrow \quad x < 1 + \frac{1}{x}$

$\Rightarrow \quad x^2 - x - 1 < 0$

Therefore, $x < \frac{1 + \sqrt{5}}{2}$ ….. (i)

(ii) $ax + a > \frac{a}{x}$

$\Rightarrow \quad x + 1 > \frac{1}{x}$

$\Rightarrow \quad x^2 + x - 1 > 0$

Therefore, $x > \frac{-1 + \sqrt{5}}{2}$ ……(ii)

Combining equation (i) and equation (ii)

$$\frac{1 + \sqrt{5}}{2} > x > \frac{-1 + \sqrt{5}}{2}$$

Therefore, only I is the true.

21. (B) Area of ΔPUR = Area of ΔPQR – Area of ΔUQR

$= \frac{1}{2} QR \times PS - \frac{1}{2} QR \times UT$

$= \frac{1}{2} QR[8 - 3]$

$= \frac{5}{2} QR$

$$\text{and Area of } \Delta URQ = 3 \times \frac{1}{2} \times QR$$

Therefore,

$$\Delta PUR : \Delta UQR = \frac{5}{2} QR : \frac{3}{2}. QR$$
$$= 5 : 3$$

22. (B)

$$AB = 12 \sin 30°$$
$$\Rightarrow AB = 12 \times \frac{1}{2} = 6 \text{ cm}$$
$$\tan 45° = \frac{AB}{BD}$$
$$\Rightarrow AB = BD = 6 \text{ cm.}$$
$$\Rightarrow (12)^2 = 6^2 + (6 + x)^2$$
$$\Rightarrow 12^2 - 6^2 = (6 + x)^2$$
$$\Rightarrow 144 - 36 = (6 + x)^2$$
$$\Rightarrow 108 = 36 + 12x + x^2$$
$$\Rightarrow x^2 + 12x - 72 = 0$$
$$x^2 = \frac{-12 \pm \sqrt{144 + 4 \times 72}}{2}$$
$$x = \frac{-12 \pm \sqrt{144 + 288}}{2}$$
$$= \frac{-12 \pm \sqrt{432}}{2} = \frac{-12 \pm 12\sqrt{3}}{2}$$

Taking +ve sign

$$\Rightarrow \frac{-12 + 12\sqrt{3}}{2}$$
$$\Rightarrow 6\sqrt{3} - 6$$
$$\Rightarrow 6\left(\sqrt{3} - 1\right)$$

– ve is not taken because distance can be negative.

23. (D) $\because$

$$\angle AOD = \frac{(360° - 2 \times 120°)}{2}$$
$$= 60°$$

So, area of shaded portion

$$= \frac{\theta}{360°} \pi r^2 - \frac{r^2 \sin \theta}{2}$$
$$= \frac{60°}{360°} \times \pi \times 16 - \frac{16 \times \frac{\sqrt{3}}{2}}{2}$$
$$= \frac{8\pi}{3} - 4\sqrt{3}$$
$$= 4\left(\frac{2\pi}{3} - \sqrt{3}\right) \text{cm}^2$$

24. (D) In ΔABC, given that

$$\angle A + \angle B = 100° \quad \ldots(i)$$
$$\angle A - \angle B = 30° \quad \ldots(ii)$$
$$2\angle A = 130°$$
$$\angle A = \frac{130°}{2}$$
$$= 65°$$
$$\angle B = 35°$$

25. (C) $\because$

$$AP \times PB = DP \times PC$$
$$\frac{AP}{PD} = \frac{PC}{PB}$$

$$\text{and } \angle B = \angle D$$
$$\text{and } \angle APD = \angle CPB$$

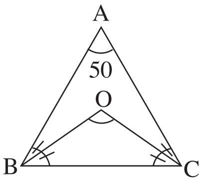
Thus, Δ PCB and Δ APD are similar

$$\text{Hence, } \frac{CP}{PA} = \frac{CB}{AD}$$
$$= \frac{2}{4} = \frac{1}{2}$$
$$\therefore \frac{AD}{CB} = \frac{2}{1}$$

26. (A) $\because$

$$CD \parallel AD$$
$$\therefore \angle ABC = \angle DCE$$
$$\text{i.e., } 3x° = y$$
$$\text{Now, } 3x + 2x + 4x = 180°$$
$$9x = 180°$$
$$x = 20°$$
$$\therefore y = 3x$$
$$y = 3 \times 20° = 60°$$

27. (A) Δ ABC



$$\angle A + \angle B + \angle C = 180$$
$$\angle B + \angle C = 180 - 50$$
$$= 130$$

In Δ BOC

$$\frac{\angle B}{2} + \angle O + \frac{\angle C}{2} = 180$$
$$\Rightarrow \angle O = 180 - \frac{130}{2}$$
$$\angle O = 115$$

28. (D) Given angles are complementary *i.e.*, sum of two angles is 90°.

$$\Rightarrow 3x - 40° + 2x - 20° - 90°$$
$$\Rightarrow 5x = 90° + 60°$$
$$= 150°$$
$$\Rightarrow x = 30°$$

29. (C) **Note**—Two angles are said to be supplementary if their sum is 180°.

$$\Rightarrow x° - (180 - x)° = 50°$$
$$\Rightarrow 2x° = 50° + 180°$$
$$\Rightarrow 2x = 230°$$
$$\Rightarrow x° = \frac{230°}{2} = 115°$$
$$\therefore \text{Smaller angle} = 180° - 115° = 65°.$$

30. (B)

$$2x + 30° + x - 15° = 180°$$
$$\Rightarrow 3x + 15° = 180°$$
$$\Rightarrow 3x = 180° - 15°$$
$$\Rightarrow 3x = 165°$$
$$\Rightarrow x = 55°$$

Exercise B

1. (C) We have, Sum of exterior of a polygon = 360°. Hence, sum of interior angles = 720° which is true for a hexagon.
2. (C) From the formula :

$$= {}^nC_2 - n = 54$$

$$= \frac{\angle n}{\angle 2 \angle n-2} - n = 54$$

$$= \frac{n(n-1)}{2} - n = 54$$

$$\Rightarrow \quad n^2 - 3n - 108 = 0$$

$$\Rightarrow \quad n^2 - 12n + 9n - 108 = 0$$

$$\Rightarrow \quad n(n-12) + 9(n-12) = 0$$

$$\Rightarrow \quad n = 12$$

or $n = -9$

– ve is not possible

⇒ there are 12 sides.

3. (A) Formula for the sum of angles of the vertices of a star of n sided polygon = $(n-4) \times 180°$. For pentagon we have $(5-4) \times 180° = 180° = 2$ right angles.
4. (C) Each exterior angle of n sided polygon = $\frac{360}{n}$, and interior angle = $\left(180 - \frac{360}{n}\right)$.

Hence, as per the condition given $\left(180 - \frac{360}{n}\right)$

$$= 10 \times \frac{360}{n}$$

$$\Rightarrow \quad 180 - \frac{360}{n} = 10 \times \frac{360}{n}$$

$$\Rightarrow \quad \frac{11 \times 360}{n} = 180$$

$$\Rightarrow \quad n = \frac{11 \times 360}{180} = 22$$

5. (C) We have, Radius of circle = side of hexagon = r. Hence, Perimeter = $6r$

$$\text{Area} = \frac{3\sqrt{3}}{2} \times \text{Side}^2 = \frac{3\sqrt{3}}{2} r^2$$

6. (C) Total Area = $\pi (20)^2 = 400\pi$

Each area = 100π

Let R_1 be the radius of the smallest circle.

Then, $\pi R_1^2 = 100\pi$

$\Rightarrow R_1 = 10$ cm

$\pi(R_2^2 - R_1^2) = 100\pi$

or $R_2 = 10\sqrt{2}$ cm

$\pi(R_3^2 - R_2^2) = 100\pi$

or $R_3 = 10\sqrt{3}$ cm

Short cut : From options check the ratio of areas.

7. (C) Transverse common tangent divides the line joining the centres of two circles in the ratio of their radii internally *i.e.*, 10 : 6.
8. (B) The direct common tangent divides the line C_1C_2 in the ratio of their radii *i.e.*, 7 : 5 externally.
9. (B) We have, Triangle TQR similar to triangle PTR or $\frac{TQ}{QR} = \frac{PT}{TR}$ or $TR = PT \times \frac{QR}{TQ} = 3 \times \frac{6}{2} = 9$ cm
10. (B) Knowing that the tangents drawn to a circle from a point outside it are equal, one gets

$$AD = AE$$

$$BD = BF$$

$$CF = CE$$

∴ $AD = AB + BD = AB + BF$

∴ Also, $AD = AE = AC + CE = AC + CF$

∴ $2AD = AB + AC + BF + CF$

$= AB + AC + BC$

11. (B) Let the side of a polygon is n.

According to question,

$$\Rightarrow \quad 2(n-2) \times 90° = 7 \times 360°$$

$$n - 2 = \frac{7 \times 360}{180} = 14$$

$$\Rightarrow \quad n - 2 = 14$$

$$\Rightarrow \quad n = 16$$

∴ The number of sides in the polygon = 16

12. (C) We have in triangle OBD

$$(r+3)^2 = 3^2 + (6-r)^2$$

or $r = 2$ cm

13. (B) Angle ADC is a right angle (angle in a semi-circle).

So, $BD^2 = AB \times BC$

or $36 \times 3 = 4 \times BC$

or $BC = \frac{36 \times 3}{4} = 27$ cm

Now, Shaded Area = Area of bigger semi-circle – Area of smaller semi-circle

$$\Rightarrow \quad \frac{1}{2}\pi\left(\frac{31}{2}\right)^2 - \frac{1}{2}\pi(2)^2 - \frac{1}{2}\pi\left(\frac{27}{2}\right)^2$$

$$\Rightarrow \quad \frac{1}{2}\pi\left[\frac{961}{4} - 4 - \frac{729}{4}\right]$$

$$\Rightarrow \quad \frac{1}{8}\pi[961 - 745]$$

$$\Rightarrow \quad \frac{\pi}{8}[216] = \frac{216\pi}{8}$$

$$= 27\pi$$

14. (C) $\angle ACD = 180° - 150° = 30°$

$\angle ABC = 180° - (90° + 30°)$

So, $\angle ABC = 180° - 120° = 60°$

15. (C) $\angle ACB = 30°$,

$\angle ABC = 90°$ (Angle in a semi-circle)

$\Rightarrow \angle BAC = 60°$

and $\angle BDC = 60° = \angle BAC$

(Angle in a same segment)

16. (C) We have

$$\sqrt{(r^2 - 3^2)} - \sqrt{(r^2 - 4^2)} = 1$$

or $r = 5$ cm

(on trying different options)

or Diameter = $2 \times 5 = 10$ cm.

17. (D) Let a be the radius of the circle.

Then, $\text{OE} = \text{EB}$

and $\text{OE} + \text{EB} = a$

$\Rightarrow \text{OE} = \text{EB} = \frac{a}{2}$

$\Rightarrow \text{AE} \times \text{EB} = \text{CE} \times \text{ED}$

$\Rightarrow \left[a + \frac{a}{2}\right] \times \frac{a}{2} = 9 \times 3$

$\Rightarrow \frac{3a}{2} \times \frac{a}{2} = 9 \times 3$

$\Rightarrow 3a^2 = 4 \times 9 \times 3$

$\Rightarrow a^2 = 4 \times 9 = 36$

$\Rightarrow a = 6 \text{ cm}$

So, $\text{Diameter} = 2 \times 6 = 12 \text{ cm}$

18. (A) $\angle$ACB and $\angle$ABC are equal. Since, two tangents from an external point are equal and so the triangle is equilateral because

$\angle\text{A} = 60°,$

$\angle\text{ACB} = \angle\text{ABC}$

$= 60° \text{ (each)}$

So, $\angle x = \angle\text{A} + \angle\text{ACB}$

$= 60° + 60° = 120°$

19. (B) $a = 90°$ (angle in semi-circle)

$\text{AB} = \text{AD}$ (given)

$\Rightarrow b = d$ (base angles, loss triangle)

$\text{X} = \text{X}_1$ (angle in same segment)

$\text{B} = \text{X}_1$ (alt. angles, AB || CD)

$\Rightarrow b = d = \text{X} = \text{X}_1$

$a + \text{X} + b + d = 180°$

$90° + 3\text{X} = 180°, \text{X} = 30°$

$\angle\text{BAD} = a + \text{X}$

$= 90° + 30° = 120°$

20. (A) $c = 80°$ (angle in alt. seg.)

$z + 20° + 45° = c$ (ext. angle of triangle)

$\Rightarrow z + 20° + 45° = 80°$

$z = 15°,$

$x = z = 15°$(angle in same seg.)

$80° + y + z = 180°$ (adj. angle on st. line)

21. (B) $80° + y + 15° + 20° = 180°$

$y = 65°$

22. (C) The sum of angles of a regular hexagon is

$= 720°$

$\Rightarrow x + 2x + 4 + 2x + 4 + x - 19 + x - 10 + 2x + 21$

$= 720°$

$\Rightarrow 9x + 9 = 720$

$\Rightarrow 9x = 720 - 9$

$9x = 711$

$x = 79°$

23. (D) The measure of arc

$\text{CA} = 360° - 120° = 240°$

Now, measure of arc CA

$= 2\angle\text{ABC} = 240°$

$\therefore \angle\text{ABC} = 120°$

24. (B) Area of segment

$\text{ABD} = \frac{1}{4}\pi r^2 - \frac{1}{2}(\text{OA})(\text{OB})$

$= \frac{\pi}{4} - \frac{1}{2}$

Area of the shaded part

$= \frac{1}{2}\pi\,(\text{AB}/2)^2 - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{1}{2}\pi\left[\sqrt{\frac{(1^2 + 1^2)}{2}}\right]^2 - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{4} - \frac{\pi}{4} + \frac{1}{2} = \frac{1}{2}\text{ m}^2$

25. (C) $a = 70°$

Also, $a + b = 180°$

$\Rightarrow b = 180° - 70° = 110°$

$\angle\text{C} = 2\angle\text{OTS}$

$\angle\text{C} = 2 \times 70° = 140°$

26. (A) $\text{OA} = \text{OB} = 5 \text{ cm}$

$\text{AN} = 4 \text{ cm}$

and $\text{BM} = 3 \text{ cm}$

$\text{ON}^2 = \text{OA}^2 - \text{AN}^2$

$\text{ON}^2 = 5^2 - 4^2 = 25 - 16$

$\text{ON}^2 = 9$

or $\text{ON} = 3 \text{ cm}$

In ΔOBM, by Pythagorean theorem

$\text{OM}^2 = \text{OB}^2 - \text{BM}^2$

$\text{OM}^2 = 5^2 - 3^2 = 25 - 9$

$\text{OM}^2 = 16,$

$\text{OM} = 4$

$\therefore$ Distance between chords

$= \text{OM} - \text{ON}$

$= 4 - 3 = 1 \text{ cm}$

27. (B) $\angle\text{ADB} = 2\angle\text{AEB}$

$\Rightarrow \angle\text{AEB} = 70°$

So, $\angle\text{AEB} + \angle\text{ACB} = 180°$

$\angle\text{ACB} = 180° - 70° = 110°$

and $\angle\text{ACB} + \angle\text{BCD} = 180°$

$\angle\text{BCD} = 180° - 110° = 70°$

28. (C) $\angle\text{BOD} = 2a$

$\Rightarrow a = \frac{170}{2} = 85°$

and $b = 180° - 85°$

$= 95°$

13 Mensuration

Mensuration deals with the measurements of lengths, area, volumes, surfaces etc.

(A) TRIANGLE

Perimeter—Perimeter of a plane figure is the measure of the length of its boundary.

(i) Find the area of triangle when sides are given.

Let a, b, c be the sides of a triangle whose opposite vertices are A, B, C respectively.

If S be the semi perimeter of the ΔABC

Now, $2S = a + b + c$

$$S = \frac{a+b+c}{2} \quad \ldots(1)$$

Area is given by

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

It is known as Hero's Formula.

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

(ii) **Right Angled Triangle—**Here, ABC is right angled triangle of height h and base b.

Now, $\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$\text{Perimeter} = b + h + p$$

$$\text{Area} = \frac{1}{2} \times p \times b$$

(iii) **Isosceles right angled triangle—**Let ABC is a isosceles right angled triangle where

$$AB = BC$$

Now, if $BC = AB = X$

Then, AC be the hypotenuse $AC = \sqrt{2}.X$

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$\text{Area} = \frac{1}{2} \times X^2$$

(iv) **Equilateral Triangle—**Let ABC be an equilateral triangle of each side a.

We draw a perpendicular AD to the side BC.

Now, $CD = BD = \frac{a}{2}$

Now, Applying Phythagoras

In right angled triangle ACD

$$AD = \sqrt{AC^2 - CD^2}$$

$$= \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}}{2} \cdot a$$

$$\text{Perimeter} = 3a$$

$$\text{Height} = \frac{\sqrt{3}}{2} \cdot a$$

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$A = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} \cdot a$$

$$A = \frac{\sqrt{3}}{4} \cdot a^2$$

Illustration 1.

If a, b, c be the sides of a triangle and S the semi-perimeter of the triangle. Find the area of the triangle in terms of a, b, c and S.

Solution :

We draw $AD \perp BC$.

Now, $2S = a + b + c$

$$S = \frac{a+b+c}{2} \quad \ldots(1)$$

If $CD = X$

$\therefore$ $BD = a - X$

If $AD = h$

Now, in right angled ΔACD

$$b^2 = h^2 + X^2 \quad \ldots(2)$$

Again, in right angled ΔABD

Now, $h^2 = c^2 - (a - X)^2 \quad \ldots(3)$

From equation (2) and equation (3), we get

$$b^2 - X^2 = c^2 - (a - X)^2$$

Or, $b^2 - X^2 = c^2 - a^2 - X^2 + 2aX$

Or, $2aX = b^2 - c^2 + a^2$

$$X = \frac{a^2 + b^2 - c^2}{2a}$$

$\therefore$ $h^2 = b^2 - X^2$

$$= b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2$$

Now, Area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times a \times h$$

$$= \frac{1}{2} \times a \times \sqrt{b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2}$$

$$= \frac{1}{2} \times a \times \frac{1}{2a} \times \sqrt{4a^2b^2 - (a^2 + b^2 - c^2)^2}$$

$$= \frac{1}{4} \times \sqrt{(2ab)^2 - (a^2 + b^2 - c^2)^2}$$

$$= \frac{1}{4} \times \sqrt{(2ab + a^2 + b^2 - c^2)(2ab - a^2 - b^2 + c^2)}$$

$$= \frac{1}{4} \times \sqrt{\{(a + b)^2 - c^2\}\{c^2 - (a - b)^2\}}$$

Area of ABC

$$= \frac{1}{4} \times \sqrt{\begin{array}{c}(a + b + c)(a + b - c) \\ (c + a - b)(c - a + b)\end{array}}$$

$$= \frac{1}{4} \times \sqrt{2S.(2S - 2c)(2S - 2b)(2S - 2a)}$$

$$= \sqrt{S\,(S - a)(S - b)(S - c)}\ .$$

Illustration 2.

Find the area of triangle whose sides are 5, 12 and 13 metres.

Solution :

We see that $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

So, given triangle is right angled triangle whose hypotenuse = 13 metre

Now, Area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times 5 \times 12 = 30 \text{ metre}^2.$$

Illustration 3.

The perimeter of an equilateral triangle is 15 metre. Find its area and the length of any of its altitudes.

Solution :

Let a be the side of equilateral triangle.

Since, in equilateral triangle all sides are equal.

So, perimeter $= 3a$

$15 = 3a$

$a = 5$ metre

Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{Side})^2$

$$= \frac{\sqrt{3}}{4} \times (5)^2$$

$$= \frac{\sqrt{3}}{4} \times 25 \text{ metre}^2$$

The length of any of the altitude of the triangle

$$= \frac{\text{Area}}{\text{Base}}$$

$$= \frac{\frac{\sqrt{3}}{4} \times 25}{5} = \frac{\sqrt{3}}{4} \times 5 \text{ metre}^2$$

Illustration 4.

Find the lengths of the three sides of a triangle 20 metre, 51 metre, 37 metre. Find the area of the triangle and hence find the length of the perpendicular on the greatest side from the opposite vertex.

Solution :

Let S be the semi-perimeter.

Then, $a = 20$ metre, $b = 51$ metre, $c = 37$ metre

$$S = \frac{a + b + c}{2} = \frac{20 + 51 + 37}{2}$$

$$= \frac{108}{2} = 54 \text{ metre}$$

Now, Area of the triangle is given by

$$= \sqrt{S\,(S - a)(S - b)(S - c)}$$

$$= \sqrt{54\,(54 - 20)(54 - 51)(54 - 37)}$$

$$= \sqrt{54 \times 34 \times 3 \times 17}$$

$$= \sqrt{6 \times 9 \times 6 \times 4 \times 3 \times 17}$$

$$= 6 \times 3 \times 2 \times \sqrt{51}$$

Greatest side = 51 metre

$\therefore$ Height $= \frac{\text{Area}}{\text{Side}}$

$$= \frac{6 \times 3 \times 2 \times \sqrt{51}}{51} = \frac{36}{\sqrt{51}} \text{metre.}$$

Illustration 5.

The sides of a triangle are in the proportion 3 : 4 : 5 and its perimeter if 48 metres. Find the area of the triangle. Is it a right angle triangle ?

Solution :

Let sides are 3X, 4X and 5X.

Now, perimeter = 3X + 4X + 5X

48 = 12 X

Or, $X = \frac{48}{12} = 4$

$\therefore$ Sides are 12 metre, 16 metre, 20 metre.

Area of triangle $= \sqrt{S\,(S - a)(S - b)(S - c)}$

Since S = Semi perimeter so S = 24

Area of triangle

$$= \sqrt{24\,(24 - 12)(24 - 16)(24 - 20)}$$

$$= 12 \times 8 = 96 \text{ metre}^2$$

Now, $12^2 + 16^2 = 144 + 196 = 340$

$20^2 = 400$

Clearly, Given triangle is not right angle triangle.

Illustration 6.

One side of a triangular lawn is 50 metre and the other two sides are equal. If the cost of paying the lawn is Rs. 3500 at 2·5/square metre. Find the length of each side.

Solution :

Now, we draw a perpendicular AD to the base BC.

Now, In right angled triangle ACD

$$h = \sqrt{a^2 - 25^2}$$

$$\text{Now,} \quad \text{Area of triangle} = \frac{1}{2} \times 50 \times h$$

$$= \frac{1}{2} \times 50 \times \sqrt{a^2 - 25^2} \quad \ldots(1)$$

$$\text{Since,} \quad \text{Rate of cost} = 2{\cdot}5/\text{metre}^2$$

$$\text{Total cost} = 3500$$

$$\text{So, Area of the triangle} = \frac{3500}{2{\cdot}5} \quad \ldots(2)$$

Now, From equation (i) and equation (ii), we get

$$\frac{3500}{2{\cdot}5} = \frac{1}{2} \times 50 \times \sqrt{a^2 - 25^2}$$

$$\text{Or,} \quad \frac{35000}{25} \times \frac{2}{50} = \sqrt{a^2 - 25^2}$$

$$\text{Or,} \quad 56 = \sqrt{a^2 - 25^2}$$

$$\text{Or,} \quad 56^2 + 25^2 = a^2$$

$$\text{Or,} \quad 3136 + 625 = a^2$$

$$a = \sqrt{3761} \text{ metre.}$$

Illustration 7.

In the adjoining, equilateral triangle ABC, three perpendiculars OE, OF, OG are drawn from point 'O' to the three sides. If the perpendicular measure 6 metre, 8 metre and 10 metre respectively. Find the area of the triangle.

Solution :

Now, joining the points B and O and O and C.

We get BOC as a triangle whose heights OE and base is BC.

$$\text{Now, Area of } \Delta BOC = \frac{1}{2} \times BC \times OE$$

$$= \frac{1}{2} \times a \times 6$$

Similarly, Area of ΔACO

$$= \frac{1}{2} \times a \times 8$$

$$\text{Area of } \Delta AOB = \frac{1}{2} \times a \times 10$$

$$\text{Now, Area of } \Delta ABC = \text{Area of } \Delta AOB + \text{Area of } \Delta BOC + \text{Area of } \Delta AOC$$

$$\frac{\sqrt{3}}{4} \times a^2 = \frac{1}{2} \times 6 \times a + \frac{1}{2} \times 8 \times a + \frac{1}{2} \times 10 \times a$$

$$\text{Or,} \quad \frac{\sqrt{3}}{4} \times a^2 = \frac{24}{2} \times a$$

$$\text{Or,} \quad a = \frac{48}{\sqrt{3}}$$

$$\text{Or,} \quad a = 16\sqrt{3} \text{ metre}$$

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4} \times a^2 \times \frac{\sqrt{3}}{4} \times \left(16\sqrt{3}\right)^2$$

$$= \frac{\sqrt{3}}{4} \times 16 \times 16 \times 3$$

$$= 168\sqrt{3} \text{ metre}^2.$$

Illustration 8.

A ladder 25 metre long is placed against a wall with its foot 7 metre from the wall. How far should the foot be drawn out so that the top of the ladder may come down by half the distance that the foot is drawn out ?

Solution :

Let X_1Y_1 be the ladder of length l.

$a =$ distance of point X_1 from foot.

X_2Y_2 be the new position of the ladder.

Now, In X_1OY_1

$$(X_1Y_1)^2 = (OX_1)^2 + (OY_1)^2$$

$$\text{Or,} \quad l^2 = h^2 + 7^2 \quad \ldots(i)$$

In X_2OY_2

$$(X_2Y_2)^2 = (OX_2)^2 + (OY_2)^2$$

$$\text{Or,} \quad l^2 = \left(h - \frac{a}{2}\right)^2 + (7 + a)^2 \quad \ldots(ii)$$

$$\text{Given} \quad l = 25$$

$$h^2 = 25^2 - 7^2$$

$$= (25 + 7)(25 - 7)$$

$$= 32 \times 18$$

$$h = 24 \text{ metre}$$

$$\text{Now,} \quad (25)^2 = \left(24 - \frac{a}{2}\right)^2 + (7 + a)^2$$

$$\text{Or,} \quad 25^2 - 24^2 = -24a + \frac{a^2}{4} + 49 + 14a + a^2$$

$$\text{Or,} \quad 49 = -10a + \frac{5a^2}{4} + 49$$

$$\text{Or,} \quad 5a\left(\frac{a}{4} - 2\right) = 0$$

$$\text{Or,} \quad a = 0 \text{ not possible}$$

$$\text{Or,} \quad a = 8 \text{ metre}$$

Hence, the foot of the ladder should be drawn out by 8 metre.

Illustration 9.

If the perimeter of a right angled isosceles triangle is $(2\sqrt{2}+2)$ metre. Find the hypotenuse.

Solution :

$$a^2 + a^2 = b^2$$

$$2a^2 = b^2$$

$$b = \sqrt{2}a \quad \ldots(i)$$

Now, $\quad$ Perimeter $=$ OA $+$ OB $+$ AB

$$2\sqrt{2}+2 = a + a + \sqrt{2}a$$

Or, $\quad 2(\sqrt{2}+1) = 2a + \sqrt{2}a$

Or, $\quad 2(\sqrt{2}+1) = \sqrt{2}(\sqrt{2}+1)a$

Or, $\quad a = \dfrac{2}{\sqrt{2}} = \sqrt{2}$ metre

$$b = \sqrt{2}\times\sqrt{2} = 2 \text{ metre}$$

$$b = 2 \text{ metre}$$

So, $\quad$ Hypotenuse $=$ 2 metre

(B) AREA AND PERIMETER OF RECTANGLE AND A SQUARE

(a) Rectangle—Let l be the length of the rectangle and b is its breadth.

(i) $\quad$ Perimeter $= l + b + l + b$

$$= 2\,(l + b)$$

(ii) Area of rectangle ABCD = Area of ΔACD + Area of ΔABC

Area of rectangle ABCD

$$= \frac{1}{2}\times l\times b + \frac{1}{2}\times l\times b$$

$$= l\times b$$

Area of rectangle $=$ Length $\times$ Breadth

(b) Square—Since, square has equal sides.

Let each side is a.

(i) Perimeter $= 4a$

(ii) **Area—**Area of square ABCD = Area of ΔACD + Area of ΔABC

$$= \frac{1}{2}\times \text{AD}\times\text{CD} + \frac{1}{2}\times\text{AB}\times\text{BC}$$

$$= \frac{1}{2}\times a\times a + \frac{1}{2}\times a\times a$$

Area of square $= a^2$

Illustration 10.

If the diagonal of a rectangle is 5 cm long and the perimeter of the rectangle is 14 cm. Find the area of the rectangle.

Solution :

Let a and b be the length and breadth of the rectangle.

Then, $\quad a^2 + b^2 = 5^2 \quad \ldots(i)$

Since, $\quad$ Perimeter $= 2(a + b)$

$$14 = 2(a + b)$$

$$a + b = 7 \quad \ldots(ii)$$

Now, putting the value of b in equation (i), we get

$$a^2 + (7 - a)^2 = 5^2$$

Or, $\quad a^2 + 49 - 14a + a^2 = 25$

Or, $\quad 2a^2 - 14a + 49 - 25 = 0$

Or, $\quad 2a^2 - 14a + 24 = 0$

Or, $\quad a^2 - 7a + 12 = 0$

$$(a - 3)(a - 4) = 0$$

$$a = 3, 4$$

$\therefore \quad b = 4, 3$

Now, $\quad$ Area of the rectangle $= a\times b$

$$= 12 \text{ cm}^2.$$

Illustration 11.

The length of a rectangle exceeds its width by 20 meter and the area of the rectangle is 300 metre^2. Find the dimensions of the rectangle.

Solution :

Let the breadth of the given rectangle be a metre.

Now, $\quad$ its length $= (a + 20)$ metre

Area $=$ 300 metre^2

$$a(a + 20) = 300$$

Or, $\quad a^2 + 20a - 300 = 0$

Or, $\quad a^2 + 30a - 10a - 300 = 0$

Or, $\quad a(a + 30) - 10(a + 30) = 0$

Or, $\quad (a - 10)(a + 30) = 0$

$$a = 10, -30$$

If, $\quad a = 10$ metre

Then, $\quad$ length $=$ 30 metre

Illustration 12.

A table cover 4 metre $\times$ 3 metre, is spread on a meeting table. If 35 cm of the table cover is hanging all around the table. Find the cost of polishing the table top at Rs. 2·05 per square metre.

Solution :

ABCD is our required area of table.

Now, $\quad$ length of table $= 4 - 2\times 0{\cdot}35$

$= 4 - 0{\cdot}7 = 3{\cdot}3$ metre

Breadth of the table $= 3 - 2\times 0{\cdot}35$

$= 3 - 7 = 2{\cdot}3$ metre

Now, $\quad$ Area of the table $= 3{\cdot}3\times 2{\cdot}3$

$= 7{\cdot}59$ metre^2

The cost of polishing the table

$= 7{\cdot}59\times 2{\cdot}05$

$=$ Rs. 15·5595

Illustration 13.

There is a square field whose side is 44 metre a square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at Rs. $\frac{11}{4}$ and Rs. $\frac{3}{2}$ /metre^2 respectively is Rs. 4904. Find the width of the gravel path.

Solution :

Let the width of the gravel path = a metre

Now, Side of square flower bed = $(44 - 2a)$ metre

Area of the flower bed = $(44 - 2a)^2$ metre^2

Area of square field = $44 \times 44 = 1936$ metre^2

Now, Area of the gravel path

$$= \text{Area of the field} - \text{Area of the flower bed}$$
$$= 1936 - (44 - 2X)^2$$
$$= 1936 - (1936 - 176X + 4X^2)$$
$$= (176X - 4X^2) \text{ metre}^2$$

Cost of lying the flower bed

$$= (44 - 2X)^2 \times \frac{11}{4}$$
$$= 11(22 - X)^2$$

Cost of gravelling the path

$$= (176X - 4X^2) \times \frac{3}{2}$$
$$= 6(44X - X^2)$$

According to question,

$$11(22 - X)^2 + 6(44 - X^2) = 4904$$

Or, $\quad 5X^2 - 220X + 5324 = 4908$

Or, $\quad X^2 - 44X + 84 = 0$

Or, $\quad X = 2, 42$

But $X \neq 42$ since the side of the square is 44 metre.

$\therefore \quad X = 2$

Hence, the width of the travel path is 2 metre.

Illustration 14.

A rectangular grass plot 8 metre × 6 metre has roads, each 1 metre wide, running in the middle of its one parallel to length and the other parallel to breadth. Find the cost of gravelling the road at Rs. 2 /metre^2.

Solution :

From figure it is clear that Area of rectangular road ABCD = $1 \times 8 = 8$ metre^2

Similar, Area of rectangular road = $6 \times 1 = 6$ metre^2

But A′B′C′D′ is consider twice in the measuring of area of roads.

So, $\quad$ Area of road $= 6 + 8 - 1 \times 1$

$$= 14 - 1 = 13 \text{ metre}^2$$

Now, Cost of gravelling the roads

$$= \text{Rate} \times \text{Area}$$
$$= 2 \times 13 = \text{Rs. } 26$$

Illustration 15.

A rectangular courtyard, 16 metre and 18 metre broad, is to be paved exactly with square tiles, all of the same size. Find the largest size of such a tile and the number of tiles required to pave it.

Solution :

Side of the largest possible square tile

= H.C.F. of length and width of the hall

= 2

Area of square tile $= 2^2 = 4$

$$\text{Number of square tile} = \frac{\text{Area of Courtyard}}{\text{Area of square tile}} = \frac{16 \times 18}{4} = 72.$$

Illustration 16.

A square room is surrounded by a verandah of width 'w'. If the arc of the verandah is A_0. Find the area of the room.

Solution :

Let the side of the room = a metre

Area of the room = a^2 metre^2

Area of room + Vernadah = $(a + 2w)^2$ metre^2

Area of verandah = $A_0 = (a + 2w)^2 - a^2$

$$A_0 = (2a + 2w) \times 2w$$
$$A_0 = 4aw + 4w^2$$
$$a = \frac{A_0 - Aw^2}{4w}$$

$$\therefore \quad \text{Area of the room} = a^2 = \left(\frac{A_0 - Aw^2}{4w}\right)^2 \text{ metre}^2$$

Illustration 17.

A ground of length 120 metre and breadth 100 metre has pavements of uniform width 2·5 metre all around it, both on its outside and inside. Find the total are of the pavements.

Solution :

For rectangle A′B′C′D′ which is interior part of rectangle ABCD.

$\therefore$ Its length = 120 metre − 2 × 2·5 = 115 metre

Breadth = 100 − 5 = 95 metre

Its Area = 115 × 95 metre^2

For rectangle ABCD

Area = 120 × 100 metre^2

For rectangle EFGH

Length = 120 + 5 = 125 metre

Breadth = 100 + 5 = 105 metre

Its Area = Length × Breadth

$= 125 \times 105$ meter^2

Now, Required area is our shaded area

$$= 125 \times 105 - 115 \times 95$$
$$= 25(5 \times 105 - 23 \times 19)$$
$$= 25(525 - 437)$$
$$= 25 \times 88 = 2200 \text{ metre}^2$$

(C) AREA OF PARALLELOGRAM, A RHOMBUS, A TRAPEZIUM, A QUADRILATERAL

(a) Parallelogram—In parallelogram, opposite sides are parallel and equal to each other.

$$\text{Area} = \text{Height} \times \text{Base}$$

(b) Rhombus—In Rhombus, all sides are equal and its diagonals perpendicular bisect to each other.

If d_1 and d_2 are the diagonals of the Rhombus ABCD.

Now, AC = d_1, BD = d_2

Now, In ΔACD

$$\text{Base} = \text{AC} = d_1$$
$$\text{Height} = \text{DO} = \frac{d_2}{2}$$
$$\text{Area of } \Delta\text{ACD} = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$= \frac{1}{2} \times d_1 \times \frac{d_2}{2} = \frac{d_1 d_2}{4}$$

In ΔABC,

$$\text{Base} = \text{AC} = d_1$$
$$\text{Height} = \text{BO} = \frac{d_2}{2}$$
$$\text{Area of } \Delta\text{ABC} = \frac{1}{2} \times \text{Height} \times \text{Base}$$
$$= \frac{1}{2} \times d_1 \times \frac{d_2}{2} = \frac{d_1 d_2}{4}$$

Now, Area of Rhombus = Area of ΔACD + Area of ΔABC

$$\therefore \quad \text{Area of Rhombus} = \frac{d_1 d_2}{4} + \frac{d_1 d_2}{4}$$
$$\text{Area of Rhomubs} = \frac{1}{2} \times d_1 \times d_2$$
$$\text{Area of Rhombus} = \frac{1}{2} \times \text{Product of Diagonals}$$

(c) Trapezium—If an quadrilateral contains two parallel and two non-parallel sides, it is called trapezium.

If non-parallel sides are equal then it is called isosceles trapezium.

Let AB is parallel to CD

AB = a CD = b

and we draw a perpendicular AN to the side CD.

Now, Area of trapezium ABCD = Area of ΔACD + Area of ΔABC

$$\text{Now, Area of } \Delta\text{ACD} = \frac{1}{2} \times \text{Height} \times \text{Base}$$
$$= \frac{1}{2} \times h \times b$$
$$\text{Area of } \Delta\text{ABC} = \frac{1}{2} \times \text{Height} \times \text{Base}$$
$$= \frac{1}{2} \times h \times a$$

Area of trapezium ABCD

$$= \frac{1}{2} \times h \times b + \frac{1}{2} \times h \times a$$
$$= \frac{1}{2} \times h \times (a + b)$$
$$\text{Area of trapezium} = \frac{1}{2} \times \text{Height} \times \text{Sum of sides}$$

(d) Quadrilateral—We draw perpendiculars from A and C to the diagonal BD whose lengths are h_1 and h_2 respectively.

Now, Area of Quadrilateral

$$= \text{Area of } \Delta\text{ABD} + \text{Area of } \Delta\text{BCD}$$
$$= \frac{1}{2} \times h_1 \times \text{BD} + \frac{1}{2} \times h_2 \times \text{BD}$$
$$\text{Area of Quadrilateral} = \frac{1}{2} \times \text{BD} \times (h_1 + h_2)$$

Illustration 18.

Find the area of a quadrilateral inscribed in a circle whose side measures 36 metre, 77 metre, 75 metre and 40 metre respectively.

Solution :

We know that if a Quadrilateral of sides a, b, c and d inscribed inside the circle.

Then, Area of the quadrilateral is given by

$$\text{S} = \sqrt{\text{S}(\text{S} - a)(\text{S} - b)(\text{S} - c)(\text{S} - d)}$$

where S = Semi-perimeter of the quadrilateral

$$\text{Now,} \quad \text{S} = \frac{a + b + c + d}{2} = \frac{36 + 77 + 75 + 40}{2}$$
$$\text{S} = 114$$

Hence, Area of quadrilateral

$$= \sqrt{114(114 - 36)(114 - 77)(114 - 75)(114 - 40)}$$
$$= 2886 \text{ metre}^2.$$

Illustration 19.

The sides of a quadrilateral, taken in order, are 10, 10, 7, 5 metre respectively and angle contained by the first two sides is 60°. Find its area.

Solution :

Let ABCD be the Quadrilateral.

Since, In ΔACD

$$AD = DC \quad \text{and} \quad \angle ADC = 60°$$

$$\therefore \quad \angle DAC = \angle DCA$$

We know that In ΔACD

$$\text{Sum of angles} = 180°$$

Or, $$2\angle DAC + 60° = 180°$$

$$2\angle DAC = 120°$$

$$\angle DAC = 60°$$

$$\therefore \quad \angle DCA = 60°$$

Clearly, ΔACD is an equilateral triangle.

So, $$\text{Area of } \Delta ACD = \frac{\sqrt{3}}{4} \times (\text{Side})^2$$

$$= \frac{\sqrt{3}}{4} \times (10)^2 = 25\sqrt{3} \text{ metre}^2$$

Now, Area of ΔABC :

$$AC = 10 \text{ metre}$$

If S be the semi-perimeter then

$$S = \frac{10+7+5}{2} = 11 \text{ metre}$$

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{11(11-10)(11-7)(11-5)}$$

$$= \sqrt{11 \times 1 \times 4 \times 6} = 2\sqrt{66} \text{ metre}^2.$$

Illustration 20.

The cross-section of a canal is a trapezium in shape if the canal is 12 metre aside at the top, 18 metre wide at the bottom and the area of the cross-section is 2400 metre^2. Find the depth of the canal.

Solution :

We know that Area of trapezium is given by

$$= \frac{1}{2} \times \text{Height} \times (\text{Sum of parallel sides})$$

Here, 12 metre, 18 metre are if depth of the canal is h metre.

Now, $$2400 = \frac{1}{2} \times h \times (12 + 18)$$

$$2400 \times 2 = h \times 30$$

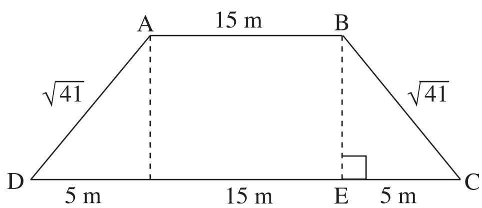
$$h = \frac{2400 \times 2}{30} = 80 \times 2$$

$$h = 160 \text{ metre.}$$

Illustration 21.

Parallel sides of a trapezium are 15 metre and 25 metre, while non-parallel sides of equal length each is $\sqrt{41}$ metre. Now, find the area and height of the trapezium.

Solution :



From the diagram clearly BE is a perpendicular drawn from B to the side CD.

Now, ΔBCE is right angled triangle.

So, $$(BE)^2 = (BC)^2 - (CE)^2$$

$$= 41 - 25$$

$$= 16$$

$$BE = 4 \text{ metre} \quad \text{or} \quad \text{Height} = 4 \text{ metre}$$

Now, $$\text{Area of trapezium} = \frac{1}{2} \times (15 + 25) \times 4$$

$$= \frac{1}{2} \times 40 \times 4 = 80 \text{ metre}^2.$$

Illustration 22.

The perimeter of a rhombus is 20 metre and sum of the lengths of its diagonals is 14 metre. Find the area of the rhombus.

Solution :

Let d_1, d_2 are the diagonals of the perimeter and a be the side.

So, $$\text{Perimeter} = 4a$$

$$20 = 4a$$

$$a = 5 \text{ metre} \qquad \ldots(1)$$

$$\text{Sum of diagonals} = 14$$

$$d_1 + d_2 = 14 \qquad \ldots(2)$$

Since, $$(d_1)^2 + (d_2)^2 = 5^2 \times 4$$

$$(d_1)^2 + (14 - d_1)^2 = 5^2 \times 4$$

$$2(d_1)^2 + 196 - 28d_1 = 25 \times 4$$

$$2(d_1)^2 - 28d_1 + 96 = 0$$

$$d_1 = \frac{28 \pm \sqrt{(28)^2 - 4 \times 96 \times 2}}{2 \times 2}$$

$$d_1 = \frac{28 \pm 2 \times 2\sqrt{49 - 48}}{4}$$

$$d_1 = \frac{28 \pm 4}{4}$$

$$d_1 = \frac{32}{4}, \frac{24}{4}$$

$$d_1 = 8, 6$$

$$d_2 = 6, 8$$

We take $d_1 = 8$ metre and $d_2 = 6$ metre

Now, $$\text{Area} = \frac{1}{2} \times \text{Product of diagonals}$$

$$= \frac{1}{2} \times 8 \text{ metre} \times 6 \text{ metre}$$

$$= 24 \text{ metre}^2.$$

Illustration 23.

In a four sided field, the longer diagonal is 24 metre and the perpendicular from the opposite vertices upon the longer diagonals are 16 metre and 18 metre. Find the area of the field.

Solution :

Let ABCD be the such field whose diagonal AC is 24 metre.

Now, Area of this field $= \frac{1}{2} \times (h_1 + h_2) \times AC$

$$= \frac{1}{2} \times 24 \times (16 + 18)$$

$$= 12 \times 34 = 408 \text{ metre}^2.$$

(D) AREA OF REGULAR POLYGONS

(i) Polygon—A polygon is a plane figure enclosed by the line segments which are known as the sides of the polygon.

(ii) Regular Polygon—A polygon is a regular polygon if its all sides and all angles are equal.

(iii) Internal Angle of Regular Polygon—Each internal angle of a regular polygon of n sides is equal to

$$\left(\frac{n-2}{n} \times 180\right)^\circ$$

(iv) Circum-Circle of a Regular Polygon : A regular polygon can be inscribed in a circle which is known as the circum-circle.

We draw a line $ON \perp AB$.

Since we have a polygon of n sides whose length is a.

Now, Angle subtended by each sides on the centre O $= \frac{360^\circ}{n}$.

Now, In right angled triangle OBN

$$\angle NOB = \frac{360^\circ}{2 \times n} = \frac{180^\circ}{n}$$

Or, $\sin \frac{180^\circ}{n} = \frac{NB}{OB} = \frac{a}{2 \times R}$

$$R = \frac{a}{2} \times \text{cosec}\left(\frac{180^\circ}{n}\right)$$

$$\text{Area of polygon} = n \times \left[\frac{1}{2} \times \text{Height} \times \text{Base}\right]$$

$$= n \times \left[\frac{1}{2} \times ON \times AB\right]$$

$$= n \times \left[\frac{1}{2} \times R \times \cos\frac{180^\circ}{n} \times AB\right]$$

$$= n \times \left[\frac{1}{2} \times R \times \cos\frac{180^\circ}{n} \times a\right]$$

$$= \frac{n}{2} \times \frac{a^2}{2} \times \text{cosec}\frac{180^\circ}{n} \times \cos\frac{180^\circ}{n}$$

$$\text{Area of polygon} = \frac{1}{4} \times n \times a^2 \times \cot\left(\frac{180^\circ}{n}\right)$$

(v) In-Circle of a Regular Polygon—If a circle having centre at the centre of a regular polygon and touching all sides of it is called the in-circle.

Let a is the length of the polygon and R is the radius of the in-circle.

Now, In ΔONB

$$\sin \Delta BON = \frac{NB}{OB} = \frac{\frac{a}{2}}{R}$$

$$\sin\left(\frac{180^\circ}{n}\right) = \frac{a}{2 \times R}$$

$$R = \frac{a}{2} \times \text{cosec}\left(\frac{180^\circ}{n}\right)$$

$$\text{Area of AOB} = \frac{1}{2} \times \text{Height} \times \text{Base}$$

$$= \frac{1}{2} \times ON \times AB$$

$$= \frac{1}{2} \times R \times \cos\left(\frac{180^\circ}{n}\right) \times a$$

$$= \frac{1}{2} \times a \times \frac{a}{2} \times \text{cosec}\left(\frac{180^\circ}{n}\right) \times \left(\frac{180^\circ}{n}\right)$$

$$= \frac{1}{4} \times a^2 \times \cot\left(\frac{180^\circ}{n}\right)$$

If polygon has n sides, then

$$\text{Area of polygon} = n \times \frac{1}{4} \times a^2 \times \cot\left(\frac{180^\circ}{n}\right)$$

If r be the radius of in-circle.

Now, $r = ON = \frac{a}{2} \times \cot\left(\frac{180^\circ}{n}\right)$

Now, Area of in-circle $= \pi r^2$

$$= \pi \times \frac{a^2}{4} \times \cot^2\left(\frac{180^\circ}{n}\right)$$

$$= \frac{\pi \times a^2}{4} \times \cot^2\left(\frac{180^\circ}{n}\right).$$

Illustration 24.

Find the area of a regular hexagon whose side is 5 metre.

Solution :

Using the formula—

$$\text{Area of regular hexagon} = 3a^2 \frac{\sqrt{3}}{2}$$

$$= 3 \times 5^2 \times \frac{\sqrt{3}}{2}$$

$$= 25 \times \frac{3\sqrt{3}}{2}$$

$$= \frac{75\sqrt{3}}{2} \text{ metre}^2.$$

Illustration 25.

Find the difference between the area of a regular hexagon each of whose side is 24 cm and the area of the circle inscribed in it.

Solution :

We know that the area of an sided regular polygon is $n \times \frac{1}{4} \times a^2 \times \cot\left(\frac{180°}{n}\right)$

and area of the in-circle is $\frac{\pi \times a^2}{4} \times \cot^2\left(\frac{180°}{n}\right)$

where a is the side of the polygon.

Now, Number of sides = $n = 6$

$$\therefore \text{ Required area } = n \times \frac{1}{4} \times a^2 \times \cot\left(\frac{180°}{n}\right) - \frac{\pi \times a^2}{4} \times \cot^2\left(\frac{180°}{n}\right)$$

$$= \frac{1}{4} \times 6 \times (24)^2 \times \cot 30° - \frac{1}{4} \times \frac{22}{7} \times (24)^2 \times \cot^2 30°$$

$$= 144 \times 6\sqrt{3} - 144 \times \frac{22}{7} \times 3$$

$$= \left(864\sqrt{3} - \frac{144 \times 66}{7}\right) \text{cm}^2.$$

Illustration 26.

Compare the area of an equilateral triangle, a square and a regular hexagon of equal perimeter.

Solution :

Let the perimeter of each polygon be S.

Now, each side of equilateral triangle = $\frac{S}{3}$

$$\text{Its area} = \frac{\sqrt{3}}{4} \times \left(\frac{S}{3}\right)^2$$

$$= \frac{\sqrt{3}}{36} \times 3^2 \quad \ldots(1)$$

$$\text{Each sides of square} = \frac{S}{4}$$

$$\text{Now, Its area} = \frac{S^2}{16} \quad (2)$$

$$\text{Each side of regular hexagon} = \frac{S}{6}$$

$$\text{Area of this regular hexagon} = \frac{3\sqrt{3}}{2} \times \left(\frac{S}{6}\right)^2$$

Now, Area of equilateral triangle : Square : Regular

$$= \frac{S^2}{12\sqrt{3}} : \frac{S^2}{16} : \frac{S^2}{8\sqrt{3}}$$

$$= 4 : 3\sqrt{3} : 6.$$

Illustration 27.

Each side of a regular hexagon measures X cm. By joining the mid points of each side, another hexagon is formed inside it. Find the ratio of areas of outer and inner hexagons.

Solution :

Let ABCDEF is a regular hexagon of side X cm.

Now, OD bisects angle EDC.

$$\therefore \quad \angle ODK = 60°$$

$$\text{Now,} \quad OK = \frac{x}{2} \times \tan 60° = \frac{x\sqrt{3}}{2} \text{ cm.}$$

In regular hexagon $OD = OE = ED$

Each side of inner hexagon

$$= OK = \angle K = \frac{x\sqrt{3}}{2} \text{ cm.}$$

$$\text{Area of inner hexagon} = 6 \times \frac{\sqrt{3}}{4} \times (\angle K)^2$$

$$= \frac{3\sqrt{3}}{2} \times \frac{x^2 \times 3}{4}$$

$$= \frac{9}{8} \times \sqrt{3} \times x^2 \text{ cm}^2.$$

$$\text{Area of outer hexagon} = 6 \times \frac{\sqrt{3}}{4} \times (AB)^2$$

$$= \frac{3}{2} \times \sqrt{3} \times x^2$$

$$\text{Now,} \quad \text{Ratio} = \frac{\frac{3}{2} \times \sqrt{3} \times x^2}{\frac{9}{8} \times \sqrt{3} \times x^2}$$

$$= \frac{4}{3} = 4 : 3$$

(E) AREA OF A CIRCLE, SECTOR AND SEGMENT OF A CIRCLE

(a) Circle—A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains the same.

The fixed point of a circle is called the **centre**.

The perimeter of a circle is generally known as its **circumference**.

(i) If r be the radius of the circle. Then,

circumference = $2\pi r$

$$\text{Diameter} = 2 \times \text{radius} = d$$

$$\therefore \quad \text{Circumference} = \pi d$$

(ii) $$\text{Area} = \pi r^2 = \pi \times \left(\frac{d}{2}\right)^2$$

$$= \frac{\pi}{4} \times d^2$$

(iii) Area of a quadrant of a circle = $\frac{\pi}{4} \times r^2$

(iv) Area enclosed by two concentric circles—If R and r are radii of two concentric circles, then Area enclosed by the two circles = $\pi R^2 - \pi r^2 = (R^2 - r^2)\pi$.

Some Important Points

(i) If two circles touch internally, then the distance between their centers if equal to the difference of their radii.

(ii) If the circles touch externally, then the distance between their centers is equal to the sum of their radii.

(iii) Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

(iv) The number of revolutions completed by a rotating wheel is one minute

$$= \frac{\text{Distance moved in one minute}}{\text{Circumference}}$$

(b) Sector of a Circle and Its Area—

Minor Sector—If minor arc ACB form the sector AOBCA then this sector is called the minor sector.

Major Sector—The major arc ADB form the major sector.

Since, 360° angle is made at the centre Area = πr^2

$\therefore$ θ° angle is made at the centre area = $\frac{\theta°}{360°} \times \pi r^2$

$$\text{So, Area of sector } \Delta AOB = \frac{\theta°}{360°} \times \pi r^2$$

$$\text{Area of } \Delta AOB = \frac{1}{2} \times \sin \theta . r^2$$

$$= \frac{1}{2} \times r^2 . \sin \theta$$

Now, Area of shaded region ACBD

$$= \text{Area of the sector OACB} - \text{Area of the } \Delta AOB$$

$$= \frac{\theta°}{360°} \times \pi r^2 - \frac{1}{2} \times r^2 . \sin \theta$$

$$= \left[\frac{\pi\theta}{360°} - \frac{\sin\theta}{2}\right] \times r^2$$

(c) Circular Pathway—

(i) Let ABC be the circle whose radius = r there is a path way DEF outside this circle ABC width = w

Now, Area of circular part ABC = πr^2

$$\text{Area of circular part DEF} = \pi . (r + w)^2$$

$$\text{Area of circular pathway} = \pi (r + w)^2 - \pi r^2$$

$$= \pi w (2r + w)$$

(ii) **In Second Case**—If circular path is made inside the circle ABC.

$$\text{Area of the circular path way} = \pi w (2r - w)$$

(iii) **Semi-circle—**

$$\text{Area of Semi-circle} = \frac{\pi r^2}{2}$$

$$\text{Perimeter} = \pi r + 2r = r(\pi + 2)$$

(iv) Let us consider three identical circle of radius r. Now, we have shaded area which is made after touching to each other.

Now, Triangle formed by such geometry is equilateral triangle.

Now, Side of $\Delta ABC = 2r$

$$\text{Its Area} = \frac{\sqrt{3}}{4} \times (\text{Side})^2$$

$$= \frac{\sqrt{3}}{4} \times 4r^2 = \sqrt{3} \times r^2$$

$$\text{Now, Area of sector} = 3 \times \frac{60°}{360°} \times \pi r^2$$

$$= \frac{\pi r^2}{2}$$

$$\text{Now, Shaded area} = \sqrt{3} \times r^2 - \frac{\pi r^2}{2}$$

$$= \left(\sqrt{3} - \frac{\pi}{2}\right) \times r^2$$

(v) **Chords and arcs**—Let us consider that r be the radius of the circle and arc ABC has length l.

Length of the arc which makes an angle θ is given by

$$l = \left(\frac{\theta}{360°}\right) \times \text{circumference of circle}$$

$$l = \left(\frac{\theta}{360°}\right) \times 2r$$

Illustration 28.

A piece of wire is bent in the shape of an equilateral triangle of each side 1·32 metre if it is bent again to form a circular ring. Find the diameter of the ring.

Solution :

Each side of the equilateral triangle = 1·32 metre

Now, perimeter of the equilateral triangle

$$= 3 \times 1{\cdot}32 \text{ metre} = 3{\cdot}96 \text{ metre}$$

Since, the some triangle piece of wire is bent again to form a circular ring.

So, Perimeter of triangle = Circumference of the circular ring

$$\text{Or,} \quad 3{\cdot}96 = 2\pi r$$

$$r = \frac{3{\cdot}96 \times 7}{2 \times 22}$$

$$r = 0{\cdot}63 \text{ metre.}$$

Illustration 29.

The hands of a clock are 14 cm and 7 cm respectively. Find the difference between the distances traversed by their extremities in 2 days 4 hours.

Solution :

$$\text{Total time} = 2 \text{ days} + 4 \text{ hours}$$

$$= 2 \times 24 + 4$$

$$= 52 \text{ hours}$$

Radius of hour hand $= r = 7$ cm.

Radius of minute hand $= R = 14$ cm.

Now, Total distance traversed by the extremity of hour hand

$$= 2\pi r \times n = 2 \times \frac{22}{7} \times 7 \times \text{No. of revolution}$$

$$= 44 \text{ cm} \times 52$$

$$= 2288 \text{ cm}$$

Total distance traversed by the extremity of minute hand

$$= 2\pi R \times \text{no. of revolution}$$

$$= 2 \times \frac{22}{7} \times 14 \times 52$$

$$= 88 \times 52 = 4576 \text{ cm}$$

Required distance $= 4576 - 2288 = 2288$ cm.

Illustration 30.

A circular grass plot 42 metre in radius is surrounded by a ring of gravel. Find the width of the gravel, so that the area of the grass and gravel may be equal.

Solution :

Let r be the radius of grass plot and w = width of the gravel

Using formula :

Area of gravel $= \pi w(2r + w)$

Area of grass $= \pi r^2$

According to question,

Area of grass = Area of gravel

Or, $\pi r^2 = \pi w(2r + w)$

Or, $r^2 = w.2.r + w^2$

Or, $w^2 + 2w \times 42 - (42)^2 = 0$

$$w = \frac{-42 \times 2 \pm \sqrt{(42 \times 2)^2 + 4 \times 1\,(42)^2}}{2}$$

$$w = \frac{-42 \times 2 \pm 42 \times 2\sqrt{2}}{2}$$

$$w = -42 \pm 42\sqrt{2} = 42\,(-1 \pm \sqrt{2})$$

Considering the + ve value—

$$w = -42\,(-1 \pm \sqrt{2}) \text{ cm.}$$

Illustration 31.

Ram by walking diametrically across a circular grass plot, finds that it has taken him 30 sec. less than if he has kept to the path round the outside. If he walks 45 meters per minute. Find the circumference of the glass plot.

Solution :

Let ACBD is the circumference of grass plot whose value is a.

Now, When Ram walks across AOB

$$\text{Distance} = X_1 = \frac{a}{\pi}$$

While, Path ACB $= X_2 = \frac{a}{2}$

Walking speed of Ram $= \frac{45}{60}$ metre/sec.

$= \frac{3}{4}$ metre/sec.

Using the formula,

Distance = Speed × Time interval

$$X_2 - X_1 = \frac{3}{4} \times 45$$

Or, $$\left(\frac{a}{2} - \frac{a}{\pi}\right) = \frac{135}{4}$$

Or, $$a = \frac{135}{4} \times \frac{2 \times \pi}{(\pi - 2)}$$

$$a = 67{\cdot}5 \times \frac{157}{57} \text{ metre}$$

(F) CUBES AND CUBOIDS

Parallelepiped—A solid bounded by three parallel plane surfaces is called a parallelepiped. The plane surfaces are known as the faces of the parallelepiped. A parallelepiped contains 6 sides and 12 edges.

Each face of a parallelepiped is a parallelogram and opposite faces are congruent *i.e.*, equal in all respects.

Let l, b, h be the length, breadth and height of the cuboids.

Total surface area of the cuboids

= Area of all the faces

= Area of ABCD + Area of BCGF + Area of EFGH + Area of CDHG + Area of AEDH + Area of ABFE

$= l \times h + h \times b + l \times h + l \times b + b \times h + l \times b$

$= 2(l \times b + b \times h + l \times h)$

Volume of the cuboids = Area of base × Height

Volume of the cuboids $= l \times b \times h$

(iii) Diagonal of the cuboids $= \sqrt{l^2 + b^2 + h^2}$

(iv) Area of the four walls $= 2h\,(l + b)$

(v) **Cube—**If all the edges of a cuboids are equal in length, it is called a cube.

Now, Total area of the cube $= 6a^2$

Volume of the cube $= a^3$

Diagonal of the circle $= \sqrt{3}a$

Illustration 32.

Find the volume, the surface area and the diagonal of a cuboids 12 cm long, 5 cm wide and 3 cm high.

Solution :

Here, we have

$l = 12$ cm.

$b = 4$ cm.

$h = 3$ cm.

$$\text{Volume of the cuboids} = l \times b \times h$$

$$V = 12 \times 4 \times 3 = 144 \text{ cm}^3$$

Total Surface area of the cuboids

$$= 2(l \times b + b \times h + l \times h)$$

$$A = 2(48 + 12 + 36)$$

$$= 2 \times 96 = 192 \text{ cm}^2$$

$$\text{Diagonal} = \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{(12)^2 + (4)^2 + (3)^2}$$

$$= \sqrt{144 + 25} = 13 \text{ cm}.$$

Illustration 33.

A plot of land in the form of a rectangle has a dimension 120 m × 112 m. A drainlet 20 m wide is dug all around it on the outside and the earth dug out is evenly spread over the plot, increasing its surface level by 34 m. Find the depth of the drainlet.

Solution :

Let a m be the depth of the drainlet.

Now, Volume of drainlet

$$= (160 \times 10 \times a + 160 \times 10 \times a + 112 \times 10 \times a + 112 \times 10 \times a)$$

$$= (5440) \text{ m}^3$$

Now, Volume of earth spread out over the plot = 112 × 120 × 34 m^3

Clearly, The Volume of earth spread over the plot

= Volume of the drainlet

Or, $$5440 \times a = 112 \times 120 \times 34$$

$$a = \frac{112 \times 120 \times 34}{5440}$$

$$a = 84 \text{ m}.$$

Illustration 34.

A class room is 8 m long 6.4 m wide and 5·4 m high. It has one door 2 × 1·6 m and three windows, each measuring 1 m × 1 m. The interior walls are to be colour washed. The contractor charges Rs. 5 per sq. m. Find the cost of colour washing.

Solution :

$$\text{Total area of the four walls} = 2(l + b) \times h$$

Here, we have $l = 8$ m

$b = 6·4$ m

$h = 5·4$ m

$$\therefore \text{ Total area of the four walls} = 2(8 + 6·4) \times 5·4$$

$$= 2 \times 14·4 \times 5·4$$

$$= 155·52 \text{ m}^2$$

$$\text{Area of the door} = 2 \times 1·6 = 3·2 \text{ m}^2$$

$$\text{Area of windows} = 3 \times 1 \text{ m} \times 1 \text{ m} = 3 \text{ m}^2$$

The required area to be washed = Total area of the four walls – (Area of the door + Area of windows)

$$= 155·52 - (3·2 + 3)$$

$$= 155·52 - 6·2 = 149·32 \text{ m}^2$$

Now, Cost of the colour washing

= Required area × Cost

= 149·32 × 5

= Rs. 746·6.

Illustration 35.

The cost of white washing the walls of a room at Rs. 2·4 per m^2 is Rs. 640 and the cost of carpeting the floor at Rs. 6·4 per m^2 is Rs. 320. There are two almirahs each of size 2·4 m × 1 m and four doors each of size 2·4 m × 2 m. If the length and breadth are in the ratio 1 : 2. Find the dimensions of the room.

Solution :

Since, the ratio of length and breadth are 1 : 2.

So, Let length $= a$

Breadth $= 2a$

Area of the floor $= a \times 2a$

$= 2a^2$

Now, Cost of carpeting the floor

$$= 2a^2 \times 6·4$$

$$320 = 2a^2 \times 6·4$$

Or, $$a^2 = \frac{3200}{2 \times 6·4}$$

$$a = 5 \text{ m}$$

Breadth = 10 m

Let height of the wall = h m

Area of two almirahs $= 2 \times 2·4 \times 1 = 4·8 \text{ m}^2$

Area of four doors $= 4 \times 2·4 \times 2 = 19·2 \text{ m}^2$

Now, Area of the four walls

$$= (l + b) \times h$$

$$= (5 + 10) \times h$$

$$= 15\, h \text{ m}^2$$

According to question,

The cost of white washing the walls = Area of the walls excluding almirahs and door × rate

$$640 = \{15 \times h - (19·2 + 4·8)\} \times 2·4$$

Or, $$\frac{640 \times 10}{24} = 15 \times h - 24$$

Or, $$\frac{800}{3} + 24 = 15 \times h$$

Or, $$\frac{872}{3 \times 15} = h$$

$$h = \frac{872}{3 \times 15} \text{ m}.$$

Illustration 36.

One iron-solid is a cuboids of dimensions 12 m × 24 m × 36 m. It is melted and cubes each of side 4 m are molded from it. Find the number of cubes formed.

Solution :

$$\text{Volume of iron-solid} = 12 \text{ m} \times 24 \text{ m} \times 36 \text{ m}$$
$$= 12 \times 24 \times 36 \text{ m}^3$$
$$\text{Volume of one cube} = 4 \times 4 \times 4 \text{ m}^3$$

Now, Number of each cubes

$$= \frac{\text{Volume of iron solid}}{\text{Volume of one cube}}$$
$$= \frac{12 \times 24 \times 36}{4 \times 4 \times 4}$$
$$= 3 \times 6 \times 9 = 162.$$

Illustration 37.

A cuboids has dimensions a m $\times$ b m $\times$ c m. If its length and breadth increases by 25%, then find the percentage increase in the lateral surface area of the cuboids if breadth is the half of length.

Solution :

Let a be the length and b be the breadth of the cuboids respectively.

Now, after 25% increment.

$$\text{Length} = l' = l + l \times \frac{25}{100} = \frac{5}{4} l$$

and $$\text{Breadth} = b' = b + b \times \frac{25}{100} = \frac{5}{4} b$$

Since, lateral surface area = Area of four walls

$$= 2(l + b) \times h$$

$$\text{Final Lateral surface area} = 2\left(\frac{5}{4} l + \frac{5}{4} b\right) \times h$$

Since, it is given $b = \frac{l}{2}$

So, Final lateral surface area

$$= A' = 2\left(\frac{5}{4} l + \frac{5}{4} b\right) \times h$$
$$= 5 \times \frac{3}{4} \times l \times h$$

Initial lateral surface area

$$= A = 2\,(l + b) \times h$$
$$= 2 \times \frac{3}{2} \times l \times h$$

Now, $$\frac{A'}{A} = \frac{\frac{15}{4}}{3}$$
$$= \frac{15}{4} \times \frac{1}{3} = \frac{5}{4} = 3lh$$
$$1 - \frac{A'}{A} = \frac{1}{4}$$

% increase in lateral surface area

$$= \left(1 - \frac{A'}{A}\right) \times 100 = 25\%.$$

Illustration 38.

A water tank of thickness 10 m measures externally 220 m, in length, 180 m in breadth and 120 m in height. Find its external and internal surface area and the maximum weight of water that it can store. 1 m^3 of water weighs 1 gm. What is the weight of the tank if 100 m^3 of its material weight 250 kg ?

Solution :

Let l, b and h be the external length, breadth and height of the tank.

Similarly, l', b' and h' be the internal length, breadth and height of the tank.

Now, $l = 220$ m, $b = 180$ m, $h = 120$ m

$$l' = 220 - 20 = 200 \text{ m}$$
$$b' = 180 - 20 = 160 \text{ m}$$
$$h' = 120 - 20 = 100 \text{ m}$$

$$\text{Now, External volume} = V = 220 \times 180 \times 120 \text{ m}^3$$
$$\text{Internal Volume} = V' = 200 \times 160 \times 100 \text{ m}^3$$

Volume of water in the tank $= V'$

$$\text{Now, Weight of the water} = V' \times 1 \text{ gm}$$
$$= \frac{200 \times 160 \times 100}{1000} \text{kg}$$
$$= 3200 \text{ kg}$$

Volume of material of the tank $= V - V'$

$$= 22 \times 18 \times 12 \times 10^{-3} - 20 \times 16 \times 10^3 \text{ m}^3$$
$$= 1552 \times 103 \text{ m}^3$$

$$\text{Now, Weight of the material} = \frac{1552 \times 10^3 \times 250}{100}$$
$$= 3880000 \text{ kg.}$$

(G) RIGHT CIRCULAR CYLINDER

(a) Let r be the radius of the base and h be the height of the cylinder.

(i) Area of the base $= \pi r^2$

(ii) Curved surface area $= 2\,\pi rh$

(iii) Total surface Area = Curved surface area + Area of circular parts

$$\text{Total surface Area} = 2\,\pi rh + 2\,\pi r^2$$
$$A = 2\,\pi r\,(h + r)$$

(iv) $$\text{Volume} = \text{Area of base} \times \text{height}$$
$$V = \pi r^2 h$$

(b) Surface Area and Volume of a hollow Cylinder

A solid bounded by two co-axial cylinders of the same height and different radii is called a hollow cylinder.

Let R and r be external and internal radii of the cylinder respectively.

(i) Area of circular end $= \pi(R^2 - r^2)$

(ii) Curved surface area of the hollow cylinder

$= \text{External surface} + \text{Internal surface}$

$= 2\,\pi Rh + 2\,\pi rh$

$= 2\,\pi h\,(R + r)$

(iii) Total surface area of the hollow cylinder

$= \text{Curved surface} + 2\,(\text{Area of the base ring})$

$= 2\,\pi Rh + 2\,(\pi R^2 - \pi r^2) + 2\,\pi rh$

$= 2\,\pi h\,(R + r)(h + R - r)$

(iv) Volume of the material = External Volume – Interior Volume

$= \pi R^2 h - \pi r^2 h$

$V = \pi h\,(R^2 - r^2)$

Illustration 39.

Find the curved surface area, volume and total surface area of a cylinder whose height 5 m and circumference of its base is 44 m.

Solution :

Given $h = 5$ m

Circumference of the base is given by $= 2\,\pi r$

Or, $44 = 2 \times \frac{22}{7} \times r$

$r = 7$ m

Now, Curved surface area $= 2\,\pi rh$

$= 44 \times 5$

$= 220\ \text{m}^2$

Total Surface Area $= 2\,\pi r\,(r + h)$

$= 44 \times (7 + 5)$

$= 44 \times 12\ \text{m}^2$

$= 528\ \text{m}^2$

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 5$

$= 154 \times 5\ \text{m}^3$

$= 770\ \text{m}^3$

Illustration 40.

Two cylinders cans have bases of the same size, the diameter of each is 14 cm. One of the can is 10 cm high and the other is 20 cm. If the a can of radius $r = 70$ cm and it has volume equal to the sum of both the cylinders then. Find the height of this can of radius 70 cm.

Solution :

For First cylinder can—

Diameter = 14 cm

Radius = 7 cm

Its Volume $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 10$

$= 1540\ \text{m.}^3$

For, Second cylinder can—

Radius = 7 cm

Height h = 20 cm

Its Volume $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 20$

$= 154 \times 20\ \text{m}^3$

$= 3080\ \text{m}^3$

Since, Volume of Third Can = Volume of First Can + Volume of Second Can

Or, $\pi \times (70)^2 \times h = 1540 + 1540 \times 2$

$\frac{22}{7} \times 70 \times 70 \times h = 1540 \times 3$

Or, $h = \frac{70 \times 3}{700} = \frac{3}{10}$ m.

Illustration 41.

A solid iron rectangular block of dimensions 2·2 m, 2·1 m and 1·5 m is cast into a hollow cylindrical pipe of internal radius 14 cm and thickness 7 cm. Find the length of the pipe.

Solution :

Let l be the length of the pipe.

Internal dimensions of the pipe are

$r = 14$ cm

$h = l$

Internal volume $= \pi r^2 h$

$= \frac{22}{7} \times (14)^2 \times l$

$= \frac{22}{7} \times 14 \times 14 \times l$

$= 22 \times 28 \times l\ \text{cm}^3$

External dimensions of the pipe

$r = 14 + 7 = 21$ cm

$h = l$

External volume $= \frac{22}{7} \times 21 \times 21 \times l\ \text{cm}^3$

$= 22 \times 3 \times 21 \times l\ \text{cm}^3$

Volume of the material of the pipe

$= (22 \times 21 \times 3 \times l - 22 \times 28 \times l)$

$= 22\,l\,(63 - 28)\text{cm}^3$

Volume of solid rectangular block

$= 2{\cdot}2 \times 2{\cdot}1 \times 1{\cdot}5\ \text{m}^3$

Now, we have

$22 \times l \times 35\ \text{cm}^3 = 2{\cdot}2 \times 2{\cdot}1 \times 1{\cdot}5 \times 10^6\ \text{cm}^3$

$l = \frac{21 \times 15}{35} \times 10^3\ \text{cm}$

$= \frac{45}{5} \times 10^3\ \text{cm}$

$= 9 \times 10^3$

Illustration 42.

A goodown building is in the form such that its upper part is semi-cylindrical of radius 7 m while the inner portion of the godown is cuboidal form of dimension 14 m × 5 m × 21 m. Find the volume of the godown and the total interior surface excluding the floor.

Solution :

Here, we have

$$r + h = 21 \text{ m}$$

Since, $r = 7$ m

$h = 14$ m

$l = 14$ m breadth $= b = 5$ m

Since, Diameter of semi-cylindrical part is 14 m, so diameter is along the length of the godown.

Now, Height of cylindrical part $= b = 5$ m

$$\text{Volume of semi-cylindrical part} = \frac{\pi r^2 h}{2}$$

$$= \frac{22}{7} \times \frac{(7)^2 \times 5}{2}$$

$$= 11 \times 7 \times 5 \text{ m}^3$$

$$= 385 \text{ m.}^3$$

Volume of cuboidal form of the godown

$$= l \times b \times h$$

$$= 14 \times 5 \times 14 \text{ m}^3$$

$$= 980 \text{ m}^3$$

$$\text{Total Volume of godown} = 385 + 980$$

$$= 1365 \text{ m}^3$$

Total surface area excluding the floor

= Interior surface of semi-cylindrical part

+ Interior surface of cuboidal form

$$= \pi r (r + h) + 2 (l + b) \times h$$

$$= \frac{22}{7} \times 7 \times (7 + 5) + 2 (74 + 5) \times 14$$

$$= 22 \times 12 + 28 \times 19$$

$$= 264 + 532 = 796 \text{ m}^2$$

Illustration 43.

A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled into it. The diameter of the pencil is 14 mm, the diameter of the graphite is 7 mm and the length of the pencil is 20 cm. Calculate the weight of the whole pencil, if the specific gravity of the wood 0·7 gm/cm^3 and that of the graphite is 3·5 gm/cm^3.

Solution :

Given for pencil,

Radius = 7 mm

Height = 20 cm

Now,

$$\text{Volume of the pencil} = \pi r^2 h$$

$$= \frac{22}{7} \times 7 \times 7 \times 20 \times 10^{-2} \text{ cm}^3$$

$$= 440 \times 7 \times 10 - 2 \text{ cm}^3$$

For Graphite,

$$\text{Radius} = r = \frac{7}{2} = 3{\cdot}5 \text{ cm}$$

Height = 20 cm

$$\text{Volume of the graphite} = \pi r^2 h$$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 10^{-2} \times 20$$

$$= 11 \times 10 \times 7 \times 10^{-2} \text{ cm}^3$$

Volume of wood = Volume of pencil – Volume of graphite

$$= (44 \times 7 \times 10^{-1} - 77 \times 10^{-1}) \text{ cm}^3$$

$$= 7 \times 10^{-1} \times (33) \text{ cm}^3$$

Mass of wood = Volume × Density

$$= 7 \times 33 \times 10^{-1} \times 0{\cdot}7 \text{ gm}$$

$$= 16{\cdot}17 \text{ gm}$$

$$\text{Mass of graphite} = 7{\cdot}7 \times 3{\cdot}5$$

$$= 2{\cdot}695 \text{ gm}$$

So, Total weight $= 2{\cdot}695 + 16{\cdot}170$ gm

$$= 18{\cdot}765 \text{ gm.}$$

(H) CONE

A right circular cone is a solid generated by revolving a line segment which passes through a fixed point and which makes a constant angle with a fixed line.

Height of the Cone—Perpendicular drawn from vertex C to the base on the circular base.

Slant height of the Cone—The length of the segment BC is called the slant height of the cone.

Now, Curved surface area of the cone

$= \frac{1}{2} \times$ length of the arc of the sector × radius of the base

$$\text{Curved surface area of cone} = \frac{1}{2} \times 2\,\pi r \times l$$

$$= \pi r l$$

Total surface area of the cone = Curved surface area + Area of the base

$$= \pi r l + \pi r^2$$

$$= \pi r (l + r).$$

Illustration 44.

The radius of the base and the height of a right circular cone arc respectively 14 cm and 35 cm. Find the curved surface area, total surface area and the volume of the cone.

Solution :

Here, $r = 14$ cm

$h = 35$ cm

Let l be the lateral height or slant height, then

$$l = \sqrt{r^2 + h^2} = \sqrt{14^2 + 35^2}$$
$$= 7 \times \sqrt{4 + 25} = 7\sqrt{29}$$

Curved surface area or lateral surface area $= \pi rl$

$$= \frac{22}{7} \times 14 \times 7\sqrt{29}$$
$$= 22 \times 2 \times 7\sqrt{29}$$
$$= 308\sqrt{29}$$

Total surface area of the cone

$$= (\pi rl + r^2)$$
$$= \pi r(l + r)$$
$$= \frac{22}{7} \times 14 \times \left(7\sqrt{29} + 14\right)$$
$$= 22 \times 2 \times 7\left(2 + \sqrt{29}\right)$$
$$= 308\left(2 + \sqrt{29}\right) \text{ cm}^2$$

Value of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 35$

$$= \frac{22 \times 2 \times 14 \times 35}{3} \text{cm}^3$$

Illustration 45.

An iron pillar has part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 6 cm. The cylindrical part is 120 cm high and the conical part is 8 cm high. Find the weight of the pillar if one cubic cm of iron weighs 10 gm.

Solution :

For conical part $r = 6$ cm

$h = 8$ cm

$$l = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2}$$
$$l = 10 \text{ cm}$$

Volume $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 6^2 \times 8$$
$$= 12 \times 8 \times \pi = 96\pi \text{ cm}^3$$

For cylindrical part $h = 120$ cm, $r = 6$ cm

Volume $= \pi r^2 h = \pi \times 6^2 \times 120$

$$= 36 \times 120 \times \pi$$
$$= 4320\pi \text{ cm}^3$$

Volume of iron = Volume of conical part + Volume of cylindrical part

$$= 96\pi + 4320\pi$$
$$= 5416\pi \text{cm}^3$$

Mass of iron = Volume of iron × density

Mass of iron $= 5416\pi \times \frac{10}{1000}$

$$= 5416 \times 3{\cdot}14 \times \frac{1}{100}$$
$$= 170{\cdot}07 \text{ kg.}$$

Illustration 46.

The interior of a building is in the form of cylinder of diameter 14 m and height 10 m, surmounted by a cone whose vertical angle is a right angle. Find the area of the surface and the volume of the building.

Solution :

According to question,

Radius of the base of cylinder $= 7 \text{ m} = r_1$

$\therefore$ Radius of the cone $= r_2 = 7$ m

Now, In ΔAOB

$$\angle BAO = 45°$$
$$\sin 45° = \frac{BO}{AB}$$
$$AB = \frac{BO}{\sin 45°} = \frac{7}{\frac{1}{\sqrt{2}}} = 7\sqrt{2} \text{ m}$$

AB is the lateral height of the cone.

Now, $AO = \sqrt{(AB)^2 - (BO)^2}$

$$= \sqrt{(7\sqrt{2})^2 - (7)^2} = 7 \text{ m}$$

Now, Surface area of the building = Surface area of the cylinder + Surface area of cone

Surface area of the building

$$= 2\pi r_1 h + \pi r_2 l$$
$$= 2 \times 3{\cdot}14 \times 7 \times 10 + 3{\cdot}14 \times 7 \times 7\sqrt{2}$$
$$= 440 + 154 \times 1{\cdot}41$$
$$= 440 + 217{\cdot}14$$
$$= 657{\cdot}14 \text{ m.}^2$$

Volume of the building

= Volume of the cylinder + Volume of the cone

$$= \pi(r_1)^2 h + \frac{1}{3}(r_2)^2 h'$$
$$= \frac{22}{7} \times 7 \times 7 \times 10 + \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$
$$= 1540 + \frac{154 \times 7}{3}$$
$$= 1540 + 359{\cdot}33 = 1899{\cdot}33 \text{ m.}^3$$

Illustration 47.

A semi-circular sheet of metal of diameter 28 cm is bent into an open conical cup. Find the depth and capacity of cup.

Solution :

When the semi-circular sheet is bent into an open conical cup, the radius of the sheet becomes the slant height of the cup and the circumference of the sheet becomes the circumference of the base of the cone.

$\therefore$ Slant height of the conical cup = 14 cm.

If r be the radius of the conical cup.

$\therefore$ Circumference of the base of the conical cup = Circumference of the sheet

$$\therefore \quad 2\pi r = \pi \times 14$$

$$\Rightarrow \quad r = 7$$

Now, $l^2 = r^2 + h^2$

$$\Rightarrow \quad h = \sqrt{l^2 - r^2}$$

$$h = \sqrt{(14)^2 - (7)^2} = 7\sqrt{3} \text{ cm}$$

$$h = 7 \times 1{\cdot}732 = 12{\cdot}12 \text{ cm}$$

$\therefore$ Depth of the cup = 12·12 cm

Also, capacity of the cup = Volume of the cup

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12{\cdot}12$$

$$= 602{\cdot}26 \text{ cm.}^3$$

Illustration 48.

The radii of the ends of a bucket of height 24 cm are 21 cm and 14 cm. Find its capacity.

Solution :

Using the properties of similar triangles between AOB and AO′E.

Now, $\frac{OB}{O'E} = \frac{AO}{AO'}$ or $\frac{7}{\frac{21}{2}} = \frac{h'}{h' + 24}$

Or, $\frac{2}{3} = \frac{h'}{h' + 24}$

$h' = 48$ cm.

Now, Volume of cone ADE

$$= \frac{1}{3}\pi\left(\frac{21}{2}\right)^2 \times (24 + 48)$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 72$$

$$= 11 \times 21 \times 36$$

$$= 396 \times 21$$

$$= 8316 \text{ cm}^3$$

Volume of cone ACB $= \frac{1}{3} \times \pi \times (7)^2 \times 48$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 48$$

$$= 154 \times 16 = 2464 \text{ cm}^3$$

Capacity of bucket $= 8316 + 2464$

$$= 10780 \text{ cm.}^3$$

(I) PYRAMIDS

Right Pyramid :

Volume $= \frac{1}{3} \times \text{Base area} \times \text{Height}$

$$= \frac{1}{3} \times A \times h$$

Slant Surface area $= \frac{1}{2} \times \text{Slant height} \times \text{base side}$

$$= \frac{1}{2} \times l \times b \times n$$

n = number of sides

Whole Surface Area = Base Area + Total Slant Arc

For A right pyramid on a triangular base, each edge of the pyramid is 'a' metre.

OR A regular tetrahedron of 'a' metre edge.

Volume $= \frac{a^3}{6\sqrt{2}}$

For Square Base $= \frac{a^3}{3\sqrt{2}}$

Illustration 49.

A right pyramid 6 m high has a square base of which the diagonal is 10 m. Find its volume.

Solution :

Let ABCDE be our required pyramid

OA = h = 6 m.

If side of the base be a.

Now, diagonal $= a\sqrt{2}$

Or, $10 = a\sqrt{2}$

$$a = \frac{10}{\sqrt{2}}$$

Now, Area of base $= A = a^2 = \left(\frac{10}{\sqrt{2}}\right)^2$

$$= 50 \text{ m}^2$$

Now, Volume of the pyramid $= \frac{1}{3} \times A \times h$

$$= \frac{1}{3} \times 50 \times 6$$

$$= 100 \text{ m}^3$$

Illustration 50.

Find the volume of a pyramid formed by cutting off a corner of a cube whose edge is 8 m by a plane bisects three conterminous edges.

Solution :

Let the edge of the given cube = 8 m

ABCO is our required pyramid.

Now, AO = BO = OC = 4 m

Here, AO, BO, OC are perpendicular to each other.

If ΔBOC is base.

$$\text{Now its Area} = \frac{1}{2}\times 4\times 4 = 8\ m^2$$

$$\text{Volume of pyramid} = \frac{1}{3}\times A\times h$$

$$= \frac{1}{3}\times 8\times 4$$

$$= \frac{32}{3}\ m^3$$

Illustration 51.

A right angled triangle of which the sides are 10 m and 24 m in length, is made to turn round its hypotenuse. Find the volume of the double cone thus formed.

Solution :

Let ABC be the right angled triangle.

where $\angle ABC = 90°$

$$AB = 10\ m$$

$$BC = 24\ m$$

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{(10)^2 + (24)^2} = 26\ m$$

Since, cone is made by rotating ΔABC along AC.

So, radius of base of the required cone = BO = OD

Now, ΔABC and ΔBOC are similar.

So, $$\frac{AB}{BO} = \frac{AC}{BC}$$

Or, $$\frac{10}{X} = \frac{26}{24} = \frac{13}{12}$$

$$X = \frac{120}{13}\ m$$

So, In right angled triangle AOB

$$AB = 10$$

$$BO = \frac{120}{13}$$

$$AO = \sqrt{(10)^2 - \left(\frac{120}{13}\right)^2}$$

$$= \left(\frac{\sqrt{100\times 169 - 14400}}{13}\right)$$

$$= \frac{10}{13}\times 5 = \frac{50}{13}\ m$$

So, $$CO = 26 - AO = 26 - \frac{50}{13}$$

$$= \frac{288}{13}\ m$$

$$\text{Volume of cone ABC} = \frac{1}{3}\times (BO)^2\times AO$$

$$= \frac{1}{3}\times\left(\frac{120}{13}\right)^2\times\frac{50}{13}$$

$$\text{Volume of cone BCD} = \frac{1}{3}\times (OB)^2\times OC$$

$$= \frac{1}{3}\times\left(\frac{120}{13}\right)^2\times\frac{288}{13}$$

$$\text{Now, Total volume} = \frac{1}{3}\times\left(\frac{120}{13}\right)^2\times\left(\frac{50}{13}+\frac{288}{13}\right)$$

$$= \frac{1}{3}\times\left(\frac{120}{13}\right)^2\times\frac{388}{13}\ m^3$$

Illustration 52.

A cone is 100 m high and its slant height is inclined 30° to horizon. Find the area of its curved surface.

Solution :

According to question,

$$\angle ABO = 30°$$

Since, in right angle ΔAOB

$$\sin 30° = \frac{h}{l} = \frac{AO}{AB}$$

$$\frac{1}{2} = \frac{100}{AB}$$

$$l = 200\ m$$

Now, $$\cos 30° = \frac{BO}{AB} = \frac{r}{l}$$

$$\frac{\sqrt{3}}{2} = \frac{r}{200}$$

$$r = 100\sqrt{3}\ m$$

$$\text{Area of lateral surface} = \pi r l$$

$$= \frac{22}{7}\times 100\sqrt{3}\times 200$$

$$= 3{\cdot}14\times 2\times 104\times\sqrt{3}\ m.^2$$

Illustration 53.

The base of a prism is quadrilateral ABCD if its height is 12 m and AB = 9 m, BC = 40 m, CD = 28 m, AB = 9 m, BC = 40 m, CD = 28 m, DA = 15 m, $\angle B = 90°$. Find the volume.

Solution :

According to question,

$$AB = 9, BC = 40$$

Now, In right angled triangle ABC

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= (9)^2 + (40)^2 = 1681$$

$$AC = 41\ m.$$

$$\text{Now, Area of the base} = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \frac{1}{2}\times 9\times 40 + \text{Area of } \Delta ACD$$

In ΔACD

AD = 15, DC = 28, AC = 41

If $\quad S = \text{Semi-metre} = \dfrac{15+28+41}{2} = 42 \text{ m}$

$$\therefore \quad \text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{42(42-41)(42-15)(42-28)}$$

$$= \sqrt{42 \times 1 \times 27 \times 14}$$

$$= \sqrt{6 \times 7 \times 9 \times 3 \times 2 \times 7}$$

$$= 6 \times 7 \times 3 = 42 \times 3 = 126 \text{ m}^2$$

$$\text{Area of the base} = 180 + 126$$

$$= 306 \text{ m}^2$$

$$\text{Volume of the prim} = \text{Area of base} \times \text{Height}$$

$$= 306 \times 12 = 3672 \text{ m.}^3$$

(J) SPHERE

A sphere is a solid figure generated by a complete revolution of a Semi-circle around of its diameter which is kept fixed.

The centre and the radius of the Semi-circle are also the centre and radius of the sphere.

Let a sphere of radius r has centre O. Now,

(i) $\quad \text{Surface Area} = 4\,\pi r^2$

(ii) $\quad \text{Volume of the Sphere} = \dfrac{4}{3}\pi r^3$

For a hemisphere of radius r.

(i) $\quad \text{Curved surface area} = 2\,\pi r^2$

(ii) $\quad \text{Total Surface area} = 2\,\pi r^2 + \pi r^2$

$\quad \text{Total Surface area} = 3\,\pi r^2$

(iii) $\quad \text{Volume} = \dfrac{4}{3}\pi r^3$

Let us consider a hollow spherical shell of external radius R and internal radius r.

$$\therefore \quad \text{Total Surface Area} = 4\pi (R^2 - r^2)$$

$$\therefore \quad \text{Volume of the Shell} = \frac{4}{3}\pi (R^3 - r^3)$$

Illustration 54.

Find the volume and surface area of a sphere of radius 2·1 metre.

Solution :

Surface area of the sphere of radius r is given by

$$S = 4\pi r^2$$

$$S = 4 \times \frac{22}{7} \times (2{\cdot}1)^2$$

$$= \frac{88}{7} \times 2{\cdot}1 \times 2{\cdot}1$$

$$= 88 \times 0{\cdot}63 \text{ metre}^2$$

Now, Volume of the sphere is given by

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (2{\cdot}1)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2{\cdot}1 \times 2{\cdot}1 \times 2{\cdot}1$$

$$= 88 \times 0{\cdot}441 \text{ metre}^3$$

Illustration 55.

A spherical lead ball of radius 10 cm is melted and small lead balls of radius 5 mm are made. Find the total number of possible lead balls.

Solution :

Since, small lead balls are melted from the spherical lead ball.

So, Volume of the spherical lead ball = Number of small lead balls × Volume of one small lead ball

$$\text{Or,} \quad \frac{4}{3}\pi (10 \text{ cm})^3 = n \times \frac{4}{3}\pi (5 \text{ mm})^3$$

$$\text{Or,} \quad \frac{4}{3}(10 \text{ cm})^3 = n \times \frac{4}{3}\pi \left(\frac{5}{10}\text{ cm}\right)^3$$

$$\text{Or,} \quad \frac{10 \times 10 \times 10 \times 10 \times 10}{5 \times 5 \times 5} \times 10 = n$$

$$\text{Or,} \quad n = 8000$$

Illustration 56.

A hemispherical bowl of internal diameter 24 cm contains a liquid. This liquid to be filled in cylindrical bottles of radius 3 cm and height 5 cm. How many bottles are required to empty the bowl ?

Solution :

Volume of the hemispherical bowl is given by $\dfrac{2}{3}\pi r^3$

Now, Volume of hemispherical bowl $= \dfrac{2}{3}\pi (24)^3$

$$\text{Volume of the cylindrical bottle} = \pi R^2 h$$

$$= \pi \times 3^2 \times 5$$

Now, n be the number of cylindrical bottle, then

$$\frac{2}{3}\pi (24)^3 = n \times \pi \times 3^2 \times 5$$

$$\text{Or,} \quad n = \frac{2}{3} \times \frac{24 \times 24 \times 24}{5 \times 3 \times 3}$$

$$n = \frac{2 \times 512}{5}$$

Illustration 57.

A spherical ball of radius 4 cm is melted and recast into three spherical balls. The radii of two of the balls 1·5 cm and 2 cm. Find the diameter of the third ball.

Solution :

Let R be radius of the initial spherical ball.

Now, r_1, r_2, r_3 be the radii of the three spherical balls.

$$\text{Now,} \quad \frac{4}{3}\pi R^3 = \frac{4}{3}(r_1)^3 + \frac{4}{3}(r_2)^3 + \frac{4}{3}(r_3)^3$$

$$\text{Or,} \quad R^3 = (r_1)^3 + (r_2)^3 + (r_3)^3$$

$$\text{Or,} \quad 4^3 = (1{\cdot}5)^3 + 2^3 + (r_3)^3$$

Or, $\quad 64 = 2{\cdot}25 + 4 + (r_3)^3$

Or, $\quad (r_3)^3 = 60 - 2{\cdot}25 = 57{\cdot}75$

$$r_3 = \sqrt[3]{57{\cdot}75} \text{ cm.}$$

Illustration 58.

A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4·2 cm and the total height of the toy is 10·2 cm. Find the volume of the wooden toy.

Solution :

Let R be the radius of solid wooden toy and h be the height of the cone.

$$R + h = 10.2$$
$$4{\cdot}2 + h = 10.2$$
$$h = 6 \text{ cm}$$

Volume of hemispherical part $= \frac{2}{3}\pi R^3$

$$= \frac{2}{3} \times \frac{22}{7} \times (4{\cdot}2)^3$$

Volume of conical part $= \frac{1}{3}\pi R^2 h$

$$= \frac{1}{3} \times \pi \times (4{\cdot}2)^2 \times 6$$

Volume of wooden toy $= \frac{44}{21} \times 4{\cdot}2 \times 4{\cdot}2 \times 4{\cdot}2$

$$+ \frac{1}{3} \times \frac{22}{7} \times 4{\cdot}2 \times 4{\cdot}2 \times 6$$
$$= 266{\cdot}11 \text{ cm}^3$$

Illustration 59.

Find the thickness of a hollow sphere whose inner diameter is 4 cm, if it weigh half as much as a solid ball of the same diameter.

Solution :

Let R and r be the external and inner radii of the hollow sphere.

Given $\quad r = 2$ cm.

Radius of solid sphere is also 2 cm.

If both are made of the same material, then density remains constant.

Now, $\quad$ Weight = Volume × Density

Volume of solid sphere $= \frac{4}{3}\pi \times 2^3$

Volume of hollow sphere $= \frac{4}{3}\pi \times (R^3 - 2^3)$

Now, According to question,

$$\frac{4}{3}\pi \times (R^3 - 2^3) \times d = \frac{4}{3}\pi \times 2^3 \times d$$

where d is the density of the material.

Now, $\quad R^3 - 2^3 = 2^3 \times \frac{3}{4}$

$$R^3 = 2^3 \times \frac{7}{4}$$
$$R = 2 \times \sqrt[3]{\frac{7}{4}}$$

Now, $\quad$ Thickness $= R - r$

$$= \left(2 \times \sqrt[3]{\frac{7}{4}} - 2\right) \text{cm.}$$

Illustration 60.

Find the whole surface of a hemisphere if—

(i) It is a solid one with a diameter 14 cm.

(ii) It is 1 cm in thickness and 14 cm in external diameter.

Solution :

(i) Radius of hemisphere = 7 cm.

Now, Total Surface area of hemisphere $= 3\pi r^2$

Total surface area $= 3 \times \frac{22}{7} \times 7^2$

$$= 66 \times 7$$
$$= 462 \text{ cm}^2$$

(ii) Thickness = 1 cm

External radius R = 7 cm

∴ Internal radius $= r = R -$ thickness

$$= (7 - 1) = 6 \text{ cm}$$

Now, Total surface area of the hemisphere = Internal surface area + External surface area + upper Surface area

$$= 2\pi r^2 + 2\pi R^2 + (R^2 - r^2)$$
$$= 2\pi \times 6^2 + 2\pi \times 7^2 + (7^2 - 6^2)$$
$$= 2 \times \frac{22}{7} \times 85 + \frac{22}{7} \times 13$$
$$= \frac{22}{7}(170 + 13) = \frac{22}{7} \times 183 \text{ cm}^2$$

Exercise A

1. The foot of a ladder 13 m long is 5 m from a house and its top reaches the upper part of a circular window. When the foot is drawn away to a distance of 7 m remote from the house, the top reaches the lower edge of the window is—

(A) 97 m^2 (B) $\frac{77}{2}$ m^2

(C) $9\frac{7}{14}$ m^2 (D) None of these

2. Ram by walking diametrically across a circular grass plot, finds that it has taken 30 seconds less than if he had kept to the path round the outside. If he walks at the rate of 60 m per minute, the diameter of the grass plot is—

(A) 27 m (B) 26·25 m

(C) 28 m (D) 35 m

3. A garden is 450 m long and 200 m broad. It has two roads each 5 m wide running in the middle of it, one parallel to the length and the other parallel to the breadth. The cost of gravelling them at Rs. 2·15 per square metre will be—

(A) Rs. 5998·50 (B) Rs. 6920·50

(C) Rs. 6934 (D) Rs. 6933·75

4. The length of a room is 15 m. The cost of carpeting it at Rs. 1·50 per square metre is Rs. 225 and the cost of repairing its walls at Rs. 6·00 per square metre is Rs. 3600. The breadth and height of the room are—

(A) 10 m, 12 m (B) 5 m, 9 m

(C) 15 m, 12 m (D) 8 m, 13 m

5. Within a rectangular courtyard of length 60 m, a gravelled path 32 m wide is laid down along the four sides. The cost of the gravel is Rs. 3 per square metre. If the path had been twice as wide, then the gravel would have cost Rs. 1200 more. The width of the courtyard is—

(A) 6 m (B) 12 m

(C) 5 m (D) 7 m

6. The area of the parallelogram is—

(A) 66 cm^2 (B) 135 cm^2

(C) 132 cm^2 (D) None of these

7. □ABCD is a parallelogram, then the value of $\angle C$ and $\angle B$ will be—

(A) 125°, 70° (B) 90°, 75°

(C) 55°, 125° (D) 110°, 98°

8. A plot of ground, 15 m by 12 m, has a flower-bed cut-out in the centre, 5 m by 4·5 m. What fraction of the whole is occupied by the flower-bed ?

(A) $\frac{3}{8}$ (B) $\frac{1}{8}$

(C) $\frac{1}{11}$ (D) $\frac{4}{11}$

9. If the fore-wheel of a carriage is 4 m and the hind wheel is 6 m in circumference, then how far will the carriage have gone when the fore-wheel has made 2500 revolutions more than the hind wheel ?

(A) 25 km (B) 27·75 km

(C) 15 km (D) 30 km

10. The volumes of spheres are proportional to the cubes of their radii. Two spheres of the same material weigh 9·8 kg and 1·4 kg and the radius of the smaller is 2 cm. If the two were melted down and formed into a single sphere, then what would be its radius ?

(A) 4 cm (b) 4·3 cm

(C) 5 cm (D) 15·75 cm

11. A railing encloses a rectangular field of 4500 m^2. The length of the field is to its breadth as 4 : 5. What is the whole length of the railing ?

(A) 250 m (B) 270 m

(C) 275 m (D) 345 m

12. A path 5·5 m wide, running all round a square park, has an area of 451 m^2. The cost of covering the area of the park enclosed by the path at Rs. 1·25 per square metre will be—

(A) Rs. 250 (B) Rs. 195·90

(C) Rs. 281·25 (D) Rs. 300

13. The area of a square field is 12100 m^2. How long will it take a man to walk round the outside of it at the rate of 3 km per hour ?

(A) 8·8 minutes (B) 7·5 minutes

(C) 9·0 minutes (D) 10·15 minutes

Directions (Q. 14 to 16)—Refer to the following information to answer the question that follow—

S (x) is the area of a square where, x is side of square.

P $(x)$is the perimeter of a square where, x is side of square.

R (x, y) is the area of a rectangle where, x is length and y is the breadth.

h (x, y) is the perimeter of a rectangle where, x is length and y is the breadth.

14. The value of $h[P\{R(2, 3)+ S(4)\}, 4]$ is equal to—

(A) 170 (B) 200

(C) 95 (D) 184

15. The value of $\frac{[S(4) - R(3, 4)]}{P(1)} \times \frac{h(2, 3)}{S(2)}$ is equal to—

(A) 2 (B) 2·5

(C) 5 (D) 9

16. The value of $\left[\frac{h(10, 20) - h(10, 20)}{S(10) - P(20)} - \frac{S(5)}{P(2{\cdot}5)}\right]$ is equal to—

(A) 5·5 (B) 4·5

(C) 3·25 (D) 5·0

17. A square contains 9 times the area of another square. If one side of the larger square be 10 cm greater than that of smaller square, then the perimeter of smaller square will be equal to—

(A) 24 cm (B) 20 cm

(C) 18 cm (D) 12 cm

18. Mr. Khanna walks in morning in a square enclosure of 900 m^2. A man walks at the rate of 60 kmph along one side, a diagonal, along another side, and returns along the other diagonals to the starting point. The time taken in walking the total distance is—

(A) 1·50 minutes (B) 1·45 minutes

(C) 2·50 minutes (D) 1·35 minutes

19. Three concentric circles, numbered a_1, a_2 and a_3 are drawn in such a way that the circumference of a_2 is the average of the circumferences of a_1 and a_3. The ratio of the area between a_1 and a_2 to that between a_2 and a_3 will be (given that the radii are as 10 : 11 : 12)—

(A) $\frac{17}{19}$ (B) $\frac{15}{17}$

(C) $\frac{21}{23}$ (D) None of these

Directions (Q. 20 to 21)—Refer to the following information to answer the question that follow.

A piece of wire 20 m long is cut into two pieces, one of which is bent into a circle and the other into the square enclosing it.

20. The area of square outside the circle is—

(A) 1·2 cm^2 (B) 1·68 cm^2

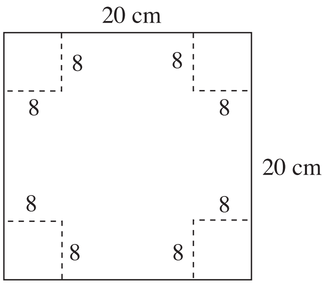
(C) 3 cm^2 (D) 1 cm^2

21. The ratio of the radius of circle to the perimeter of square is—

(A) 1 : 8 (B) 8 : 1

(C) 5 : 4 (D) 2 : 3

22. From a square plate of side 20 cm four squares each of side 4 cm are cut away as given in figure. The perimeter of the figure is—



(A) 75 cm (B) 82 cm

(C) 64 cm (D) 105 cm

23. A spherical ball of lead, 6 cm in diameter is melted and recast into three spherical balls. The diameter of two of these are 2 cm and 3 cm respectively. The diameter of the third ball is—

(A) 5·65 cm (B) 6·15 cm

(C) 7·80 cm (D) 9·05 cm

24. A carpet 20 m long by 15 m wide is so placed on the floor of a room that there is a border 30 cm wide all round the carpet. What is the area of the floor of the room ?

(A) 329·64 m^2 (B) 253·45 m^2

(C) 321·36 m^2 (D) 450 m^2

25. A rectangular park, with a side of 18 m long and 13 m wide is crossed centrally by two perpendicular concrete roads, each 2·2 m and 4·3 m wide respectively. The area of the park left is—

(A) 96·54 m^2 (B) 100·25 m^2

(C) 122·75 m^2 (D) 137·46 m^2

26. The dimensions of the floor of a hall are 12×48 m^2. If square tiles of largest possible dimensions are to be used to pave the floor such that none of the tiles need to be broken, then the number of tiles to be used is—

(A) 2 (B) 5

(C) 4 (D) 10

27. The length and breadth of a rectangular plot of a land are in the ratio of 7 : 5. The owner spent Rs. 2400 for surrounding it from all the sides at the rate of Rs. 2 per metre. What is the difference between the length and breadth of the plot ?

(A) 85 m (B) 100 m

(C) 127 m (D) 250 m

28. A 8 metres wide road is to be constructed surrounding a square plot of area 225 m^2. If the unit cost of construction is Rs. 310 per sq. m, then what is the total cost of construction ?

(A) Rs. 15,000 (B) Rs. 2,28,160

(C) Rs. 2,28,350 (D) None of the above

29. If the side of an equilateral triangle is increased by 25%, then its area will increase by—

(A) 45·15% (B) 56·25%

(C) 70% (D) 85%

30. In the figure drawn alongside, by how much would the shaded area increase if the radii of both the inner and outer circles get doubled ?

(A) 300% (B) 500%

(C) 525% (D) 800%

31. If the length, breadth and height of a cuboid are 5 m, 3 m and 2 m respectively, then its surface area is—

(A) 62 m^2 (B) 65 m^2

(C) 30 m^2 (D) None of these

32. The length, breadth and height of a rectangular box are 6 m, 5 m and 2 m respectively. How many cubic metres of sand will be needed to fill the box to a depth of 80 cm ?

(A) 9·85 m^3 (B) 9·40 m^3

(C) 30 m^3 (D) 12 m^3

33. The number of bricks, each measuring 25 cm × 12 cm × 45 cm, needed to construct a wall 10 m long, 5 m high and 50 cm thick, is—

(A) 1500 (B) 1852

(C) 1659 (D) 1955

34. Three cubes of copper are melted and formed into a single cube of edge 12 cm, if edge of one cube is 6 cm and edge of second cube is 10 cm then the edge of third cube ? The surface area of the new cube formed is—

(A) 8 cm, 44 cm^2 (B) 8 cm, 864 cm^2

(C) 6 cm, 144 cm^2 (D) 6 cm, 864 cm^2

35. A rectangular room measure 15 m × 12 m × 9 m. What is the maximum length of an iron sticks that it can accommodate ?

(A) 15 m (B) $15\sqrt{2}$ m

(C) 17 m (D) $18\sqrt{3}$ m

36. The volume of a right circular cylinder is 14850 cubic centimeters. If the vertical height of the cylinder is 21 cm, then what is its lateral area ?

(A) 1239 cm^2 (B) 1276 cm^2

(D) 2250 cm^2 (D) 2970 cm^2

37. A right circular solid cylinder of base radius 3 cm and vertical height 320 cm is melted to form 6 equal solid spheres. If there is a process loss of 40% during such formation, then what is the radius of each of the solid sphere so formed ?

(A) $4\sqrt{3}$ cm (B) 6 cm

(C) 7 cm (D) 7·5 cm

38. A room measures 12 m ×12 m × 12 m. What is the maximum length of sticks (in metres) which it can contain ?

(A) 12 m (B) $12\sqrt{2}$ m

(C) $15\sqrt{2}$ m (D) None of these

39. The base radius of the right circular cone is 5 cm and its vertical height is 42 cm. Its volume is—

(A) 1156 cm^3 (B) 1225 cm^3

(C) 1100 cm^3 (D) 1352 cm^3

40. A sphere circumscribes a cube of side '*a*'. How many times the volume of cube is the volume of the sphere ?

(A) 2 (B) π

(C) $\pi\sqrt{3}/2$ (D) $\sqrt{3}$

41. A circle of radius 5 cm is cut such that a sector PABC (sector angle 90°) is formed, where P is the centre of the circle and PA = PB = PC are the radii. Now the sector is folded to make PA and PC coincident. Thus, the volume of the figure now generated will be (cc)—

(A) 7·1 (B) 7·9

(C) 8·9 (D) 6·2

42. Each edge of a cube is increased by 25%. The percentage increase in the surface area is—

(A) 75% (B) 120%

(C) 56·25% (D) 85%

43 The percentage increase in the volume of a cuboid when three edges are increased by 200%, 120% and 350% respectively will be—

(A) 1650% (B) 670%

(C) 470% (D) 2870%

44. Keeping the volume of a wire the same as before, we decrease its diameter by 95%. The per cent change in its length is—

(A) – 125% (B) 300%

(C) 135·8% (D) – 12·5%

45. The volume of a sphere is changing @ 121 cc/min. The rate at which the surface area of the sphere area of the sphere is changing when the radius of the sphere = 11 cm, is—

(A) 32π cm^2/min (B) 22 cm^2/min

(C) 22π cm^2/min (D) 32 cm^2/min

46. A right circular cone and right circular cylinder have equal height and equal bases. Their curved surface are in the ratio 5 : 6. The ratio of their base radius to the height will be—

(A) 3 : 6 (B) 2 : 3

(C) 4 : 3 (D) 5 : 5

47. From a solid cylinder of height 15 cm and radius 10 cm, a cavity to a depth of 5 cm followed by a cylindrical bore of radius 3 cm is made. The volume of material in the solid is—

(A) 1500 π (B) 1650 π

(C) 1080 π (D) 1200 π

48. A rectangular courtyard of dimensions 15 m × 9 m is surrounded an all sides by a footpath 3·5 m wide. Since, the President of India is about to visit this site, we want to provide a proper foundation to this footpath and hence we need to dig it to a depth of 20 cm and fill it completely with gravel. The amount of gravel (filling material) needed will be—

(A) 20·08 m^3 (B) 43·4 m^3

(C) 50·5 m^3 (D) 65·9 m^3

49. The greatest possible sphere is turned from a cubical block of wood. If the volume of the block removed be 4410 c. in., the diameter of the sphere (π = 22/7) will be—

(A) 42 in. (B) 47 in.

(C) 41 in. (D) 21 in.

50. A sphere of 6 ft. radius resets on a table. The volume of right hollow cone which can just cover it will be—
[The section of the cone through the axis being an equilateral triangle (π = 22/7).]

(A) 2036·6 c.ft. (B) 2038 c.ft.

(C) 1539·9 c.ft. (D) None of these

Exercise B

1. A circular garden of radius 84 m is to be surrounded by a road 8·4 metre wide. What is the ratio of the area of the road to that of the garden ?

(A) 1·1 (B) 1·21

(C) 0·21 (D) 0·11

2. How many spherical balls. Each of radius 3 cm, can be made by melting a spherical ball of radius 9 cm ?

(A) 8 (B) 27

(C) 9 (D) 81

3. The rectangular floor of a room having dimension 40 feet 60 feet needs to be carpeted. If a 2 foot margin is kept on all sides, then what % of the area of the room would be occupied by the carpet ?

(A) 96% (B) 98%

(C) 60% (D) 84%

4. A hemisphere of radius 3 cm is cast into a right circular cone of height 150 cm. What is radius of the base of the cone ?

(A) 0·3 cm (B) 0·5 cm

(C) 0·8 cm (D) 0·6 cm

5. A wire in the form of a circle of radius 7 cm is bent to form a square. What is the area of the square ?

(A) 49 cm^2 (B) 121 cm^2

(C) 176 cm^2 (D) 144 cm^2

6. The volume of a right circular cone varies as square of the radius of the base when the height is constant, and as the height when the radius is constant. When the radius of the base is 7 cm and the height 30 cm, the volume is 1540 cubic cm. Find the height of a cone whose volume is 264 cubic cm and which stands on a base whose radius is 6 cm—

(A) 28 cm (B) 14 cm

(C) 21 cm (D) $\frac{5}{10}$ cm

7. Pipe A can fill a tank in 8 hours and pipe B can fill it in 6 hours. If A, B and an outlet which removes 10 litres of water in an hour, are opened together, the tank is filled in 5 hours. The volume of the tank is—

(A) 521 litres (B) Data inadequate

(C) 109 litres (D) None of these

8. The external dimensions of a closed wooden box are 50 cm, 45 cm, and 35 cm. The wood (of which it is made) is 2·5 cm thick. How many bricks of size 6 cm $\times$ 5 cm $\times$ 4 cm can be put in this box ?

(A) 190 (B) 230

(C) 375 (D) 450

9. There is an error of + 2.5% while measuring the radius of a sphere. What is the percentage error in calculating the volume of the sphere ?

(A) 7·7% (B) 7·5%

(C) 3·3% (D) 5·9%

10. The exact no. of cubes which can be made out of a cuboid of dimensions 65 m $\times$ 26 m $\times$ 13 m is—

(A) 15 (B) 13

(C) 9 (D) None of these

11. A closed box made of wood of uniform thickness has length, breadth and height 15 cm, 13 cm and 11 cm respectively. If the thickness of the wood is 2·5 cm, then the inner surface area is—

(A) 426 cm^2 (b) 376 cm^2

(C) 490 cm^2 (D) 150 cm^2

12. A cylindrical rod of iron whose height is four times its radius is melted and cast into spherical balls of the same radius. Then there are—

(A) 5 balls (B) 8 balls

(C) 9 balls (D) 12 balls

13. There is a cube of edge length equal to one linear unit. Then the distance (shortest) between any two of its vertices can only be—

(A) 1 or $\sqrt{2}$ linear units

(B) 1, $\sqrt{2}$ or $\sqrt{3}$ linear units

(C) 1 or $\sqrt{2}$, $\sqrt{3}$ or 2 linear units

(D) None of the above

14. The diameter of hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, then what portion of the ball will remain outside the cone ?

(A) 50% (B) Less than 50%

(C) More than 50% (D) 100%

15. How many lead balls of radius $\frac{1}{4}$ cm can be made out of a solid lead sphere of diameter 8 cm ?

(A) < 4000 (B) $> 3000, < 4000$

(C) Around 7000 (D) > 4000

16. A cylindrical container, used for holding petrol, had a diameter of 16 m and a height of 3 m. The owner wishes to increase the volume. However, he wishes to do it such that if X m are added to either the radius or the height, the increase in volume is the same. Thus, X will be—

(A) 16 m (B) 5·33 m

(C) 6·77 m (D) 3·56 m

17. Mr. Badriprasad has a cylinder and a sphere. The sphere is such that it perfectly fits in the cylinder with no parts of its outside. Thus, the ratio of curved surface area of the cylinder to the surface area of inscribed sphere will be—

(A) Equal to one (B) More than 1

(C) Less than 1 (D) Equal to two

18. One cone of height = diameter = $\frac{1}{2}$ and another cone of height = diameter = 1 are cut from the opposite sides of a unit cube with areas at the centre. The surface area (excluding the base) of a cone is $\frac{\pi l d}{2}$,

where is the base diameter and l is the lateral height . The surface area of the resultant cavity is—

(A) $35\sqrt{5}\frac{\pi}{192}$ (B) $19\sqrt{5}\frac{\pi}{64}$

(C) $5\sqrt{5}\frac{\pi}{16}$ (D) $9\sqrt{5}\frac{\pi}{32}$

Answers with Hints

Exercise A

1. (B) Let

$$\begin{aligned} AC &= 13 \text{ m} \\ BC &= 5 \text{ m} \\ \Rightarrow \quad AB &= \sqrt{(AC)^2 - (BC)^2} \\ &= \sqrt{(13)^2 - (5)^2} \\ &= \sqrt{169 - 25} \\ &= \sqrt{144} = 12 \text{ cm} \end{aligned}$$

In IInd case

$$\begin{aligned} \Rightarrow \quad DB &= \sqrt{(DE)^2 - (BE)^2} \\ &= \sqrt{(13)^2 - (12)^2} \\ &= \sqrt{169 - 144} \\ &= \sqrt{25} = 5 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{So,} \quad AD &= AB - DB \\ &= 12 - 5 = 7 \text{ m} \end{aligned}$$

Diameter of circular windows = 7 m

So, Radius of window $= \frac{7}{2} = 3{\cdot}5$ m

So, Area of window $= \pi r^2$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ m}^2.$$

2. (B) By the question, difference between circumference of semi-circle and diameter = Distance travelled in 30 seconds

$$\begin{aligned} \Rightarrow \quad \pi r - 2r &= 60 \times \frac{30}{60} \\ \Rightarrow \quad r(\pi - 2) &= 30 \\ \Rightarrow \quad r\left(\frac{22}{7} - 2\right) &= 30 \\ \Rightarrow \quad r\left(\frac{8}{7}\right) &= 30 \\ \Rightarrow \quad r &= \frac{30 \times 7}{8} = \frac{105}{4} = 26{\cdot}25 \text{ m.} \end{aligned}$$

3. (D) Area of roads

$$\begin{aligned} &= 5 \times 450 + 5 \times 200 - (5 \times 5) \\ &= 2250 + 1000 - 25 \\ &= 3250 - 25 = 3225 \text{ m}^2 \end{aligned}$$

Cost of graveling $= 2{\cdot}15 \times 3225$ m^2 $= 6933{\cdot}75$ m^2.

4. (A) Let Breadth = b and Height = h.

From question,

$$\begin{aligned} (15 \times x)\, 1{\cdot}50 &= 225 \\ x &= \frac{225}{1{\cdot}5 \times 15} = 10 \text{ m.} \\ \text{and} \quad [2h(l + b) \times 6] &= 3600 \\ 12h(l + b) &= 3600 \\ 12h(15 + 10) &= 3600 \\ h &= \frac{3600}{12 \times 25} = 12 \text{ m.} \end{aligned}$$

5. (A) Let width of the courtyard = x m.

Then, Area of courtyard with path = $60x$ m^2

Area of courtyard without path = $(60 - 4) \times (x - 4)$ m^2 $= 56(x - 4)$ m^2

So, Area of path $= [60x - 56(x - 4)]$ m^2

When width of path is twice, then area of courtyard without path

$$\begin{aligned} &= (60 - 12)(x - 12) \text{ m}^2 \\ &= 48(x - 12) \text{ m}^2 \end{aligned}$$

$\therefore$ Area of path $= [60x - 48(x - 120]$ m^2

By the question,

$$\begin{aligned} 3x[12x + 576] - 3x[4x + 224] &= 1200 \\ \Rightarrow \quad 12x + 576 - 4x - 224 &= 400 \\ \Rightarrow \quad 8x &= 400 - 352 \\ \Rightarrow \quad 8x &= 48 \\ \Rightarrow \quad x &= 6 \text{ m.} \end{aligned}$$

6. (C)

$$\begin{aligned} \text{Area of ABCD} &= \text{Area } [\Delta ABC + \Delta ACD] \\ ABCD &= 2[\Delta ABC] \\ S &= \frac{11 + 13 + 20}{2} = \frac{44}{2} = 22 \\ \text{Area} &= \sqrt{S(S - a)(S - b)(S - c)} \end{aligned}$$

where a, b, c are the sides length.

$$\begin{aligned} &= \sqrt{22 \times (22 - 11) \times (22 - 13) \times (22 - 20)} \\ &= \sqrt{22 \times 11 \times 9 \times 2} = 22 \times 3 = 66 \text{ cm}^2 \end{aligned}$$

So, area of $\square$ABCD $= 2 \times 66 = 132$ cm^2.

7. (C) By the property of parallelogram

AB || CD and DA || CB

$$\begin{aligned} \text{and} \quad \angle A &= \angle C \\ \angle B &= \angle D \\ \Rightarrow \quad \angle C &= 55 \\ \text{and} \quad 360° - 110° &= 250° \\ \text{So,} \quad \angle B + \angle D &= 250° \\ \text{Or} \quad \angle B + \angle B &= 250° \\ \Rightarrow \quad \angle B &= \frac{250}{2} = 125° \end{aligned}$$

8. (B) Area of the plot $= (12 \times 15)\ m^2$
$= 180\ m^2$
Area of the flower-bed $= (5 \times 4{\cdot}5)\ m^2 = 22{\cdot}5\ m^2$
So, Required fraction $= \frac{22{\cdot}5}{180} = \frac{225}{1800} = \frac{75}{600}$
$= \frac{3}{24} = \frac{1}{8}$

9. (D) Let the revolutions made by hind wheel $= x$
Then revolution made by fore wheel $= x + 2500$
Since, distance covered by both wheels is equal, then
$6x = 4(x + 2500)$
$\Rightarrow \quad 2x = 10000$
$\therefore \quad x = 5000$ m
So, Distance travelled by carriage $= 6x$
$= 6 \times 5000$ m $= 30{,}000$ m
$= 30$ km.

10. (A) Since, $\frac{W_1}{W_2} = \frac{V_1}{V_2} = \frac{r_1^3}{r_2^3}$
$\Rightarrow \quad \frac{9{\cdot}8}{1{\cdot}4} = \frac{r_1^3}{r_2^3}$
$\Rightarrow \quad r_1^3 = \frac{9{\cdot}8}{1{\cdot}4} r_2^3$
and $\quad r_2 = 2$ cm
$\Rightarrow \quad r_1^3 = \frac{9{\cdot}8}{1{\cdot}4} \times 2^3$
Let required radius $=$ R.
$\Rightarrow \quad R^3 = r_1^3 + r_2^3$
$\Rightarrow \quad = \frac{9{\cdot}8}{1{\cdot}4} \times 8 + 8$
$\Rightarrow \quad \left(\frac{98}{14} + 1\right) \times 8 = 8 \times 8 = 64$
$\Rightarrow \quad R^3 = 64$
$\Rightarrow \quad R = 4$ cm.

11. (B) $l : b = 4 : 5$
Length of the field $= 4x$
Breadth of the field $= 5x$
Area of field $= 4x \times 5x = 20x^2$
$\Rightarrow \quad 21x^2 = 4500$
$\Rightarrow \quad x^2 = 225\ m^2$
$\Rightarrow \quad x = 15$ m
So, Length $= 4 \times 15 = 60$ m,
Breadth $= 5 \times 15 = 75$ m
Length of railing $= 2[l + b]$
$= 2[60 + 75]$
$= 2[135] = 270$ m.

12. (C) Area of park $= 451\ m^2$
$\Rightarrow \quad (x + 11)^2 - x^2 = 451\ m^2$
$\Rightarrow \quad 22x = 451 - 121$
$\Rightarrow \quad 22x = 330$
$\Rightarrow \quad x = \frac{330}{22} = 15$ m.
So, Area of square $= 15 \times 15 = 225\ m^2$
Cost of covering $= 225 \times 1{\cdot}25 =$ Rs. $281{\cdot}25$

13. (A) Area of square field $= 12100\ m^2$
i.e., $\quad x^2 = 12100\ m^2$
$x^2 = (110)^2\ m^2$
Side of the field $= 110$ m
$\therefore$ Perimeter $= 4 \times 110 = 440$ m
Required time $= \frac{440}{3000}$ hrs $= 0{\cdot}1467$ hrs
$= 8{\cdot}799$ minutes
$= 8{\cdot}8$ minutes.

14. (D) $S(x) = x^2 \qquad P(x) = 4x$
$R(x, y) = xy \qquad h(x, y) = 2(x + y)$
So, $R(2, 3) = 6, \qquad S(4) = 16$
$P(16 + 6) = P(22) = 4 \times 22 = 88$
$H(88, 4) = 2(88 + 4)$
$= 2(92) = 184$

15. (B) $S(4) = 16$
$R(3, 4) = 12$
$P(1) = 4 \times 1 = 4, h(2, 3) = 2(2 + 3) = 10$
$\Rightarrow \quad \frac{16 - 12}{4} \times \frac{10}{4} = \frac{4}{4} \times \frac{10}{4} = 2{\cdot}5$

16. (B) $R(10, 20) = 200$
$h(10, 20) = 2(10 + 20) = 60$
$S(10) = 100, P(20) = 4 \times 20 = 80$
$\Rightarrow \quad \frac{200 - 60}{100 - 80} - \frac{25}{10}$
$\Rightarrow \quad \frac{140}{20} - 2{\cdot}5 = 7 - 2{\cdot}5 = 4{\cdot}5$

17. (B) Let, side of smaller square $= x$ cm.
$\therefore$ Side of larger square $= (x + 10)$ cm
and $\quad (x + 10)^2 = 9(x^2)$
$x^2 + 100 + 20x = 9x^2$
$8x^2 - 40x + 20x - 100 = 0$
$\Rightarrow \quad x = 5$ or $x = -\frac{20}{8}$
So, perimeter of smaller square $= 4x = 20$ cm

18. (B) Area of square park $= 900\ m^2$
One side $= 30$ m
$AB \rightarrow BD \rightarrow DC \rightarrow CA$
$AC = BD = 30\sqrt{2}$

$$\text{Total distance} = 2(30 + 30\sqrt{2})$$

$$= 60(1 + \sqrt{2}) = 60 \times 2.414$$

$$= 144.85$$

$$\text{Speed} = 6 \text{ kmph}$$

$$= \frac{6 \times 1000}{60} \text{ m/minutes}$$

$$= 100 \text{ m/minute.}$$

$$\therefore \quad \text{Time} = \frac{144{\cdot}85}{100} = 1{\cdot}448 \text{ minutes.}$$

19. (C) Let the radii are be r_1, r_2, r_3 respectively.

$$\Rightarrow \quad 2\pi r_2 = \frac{2\pi r_1 + 2\pi r_3}{2}$$

$$\Rightarrow \quad r_2 = \frac{r_1 + r_3}{2} \quad \ldots(1)$$

$$\text{Area between } a_1 \text{ \& } a_2 = \pi r_2^2 - \pi r_1^2$$

$$\text{Area between } a_2 \text{ \& } a_3 = \pi r_3^2 - \pi r_2^2$$

$$\Rightarrow \quad \frac{(r_2^2 - r_1^2)}{(r_3^2 - r_2^2)} = \frac{(r_2 + r_1)(r_2 - r_1)}{(r_3 + r_2)(r_3 - r_2)}$$

$$\Rightarrow \quad \frac{r_2 + r_1}{r_3 + r_2} = \frac{11 + 10}{11 + 12} = \frac{21}{23}$$

20. (B) Let side of square = x m = Let diameter of circle = 20

$$\Rightarrow \quad 4x + \pi x = 20$$

$$\Rightarrow \quad 4x + \frac{22}{7}x = 20$$

$$\Rightarrow \quad \frac{28x + 22x}{7} = 20$$

$$\Rightarrow \quad 50x = 20 \times 7$$

$$\Rightarrow \quad x = \frac{14}{5} \text{ m}$$

$$\text{Now, Area of square} = \left(\frac{14}{5}\right)^2 = \frac{196}{25} \text{ m}^2$$

$$\text{Area of circle} = \pi\left(\frac{r}{2}\right)^2 = \frac{22}{7} \times \frac{7}{5} \times \frac{7}{5} = \frac{154}{25} \text{ m}^2$$

$$\therefore \quad \text{Required area} = \text{Area of square} - \text{Area of circle}$$

$$= \frac{196}{25} - \frac{22 \times 7}{25} = \left(\frac{196 - 154}{25}\right) \text{m}^2$$

$$= \frac{42}{25} = 1{\cdot}68 \text{ m}^2.$$

21. (A)

$$\text{Radius of circle} = \frac{x}{2} = \left(\frac{14}{5}\right)\frac{1}{2} = \frac{7}{5}$$

$$\text{and Perimeter of square} = 4x$$

$$= 4 \times \frac{14}{5} = \frac{56}{5} \text{ m.}$$

$$\therefore \quad \text{Required ratio} = \frac{7}{5} : \frac{56}{5} = \frac{7 \times 5}{5 \times 56} = \frac{1}{8}.$$

22. (C)

$$\text{The perimeter} = 2[(20 - 8) + (12 - 8)] + 4 \times 8$$

$$= 2[12 + 4] + 32$$

$$= 2 \times 16 + 32 = 64 \text{ cm.}$$

23. (A) Volume of spherical ball = volume of three spherical balls.

$$\Rightarrow \quad \frac{4}{3}\pi\left(\frac{6}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{2}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 + \frac{4}{3}\pi x^3$$

$$\Rightarrow \quad 27 = 1 + \frac{27}{8} + x^3$$

$$\Rightarrow \quad \left(27 - \frac{27}{8}\right) = 1 + x^3$$

$$\Rightarrow \quad \frac{7 \times 27}{8} = x^3 + 1$$

$$\Rightarrow \quad x^3 = \frac{189 - 8}{8} = \div (\text{required diameter})^3$$

$$= (2x)^3$$

$$= 8x^3 = 8 \times \frac{181}{8} = 5{\cdot}65 \text{ cm.}$$

24. (C)

$$\text{Length of the floor} = (20 + 2 \times 0{\cdot}30)$$

$$= (20 + 0{\cdot}60) = 20{\cdot}60 \text{ m}$$

$$\text{Breadth of the floor} = (15 + 2 \times 0{\cdot}30)$$

$$= (15 + 0{\cdot}60) = 15{\cdot}60 \text{ m}$$

$$\therefore \text{ Area of floor} = (20{\cdot}60 \times 15{\cdot}60) \text{ m}^2 = 321{\cdot}36 \text{ m}^2.$$

25. (D)

$$\text{Area of the park} = 18 \times 13 = 234 \text{ m}^2$$

$$\text{Area of Roads} = 18 \times 4{\cdot}3 + 13 \times 2{\cdot}2 - 4{\cdot}3 \times 2{\cdot}2$$

$$= 77{\cdot}4 + 28{\cdot}6 - 9{\cdot}46$$

$$= 106{\cdot}0 - 9{\cdot}46 = 96{\cdot}54 \text{ m}^2$$

$$\therefore \quad \text{Required area of roads} = 234 - 96{\cdot}54$$

$$= 137{\cdot}46 \text{ m}^2.$$

26. (C) Each dimension of the tile must be the H.C.F of 12 and 48, *i.e.*, 12 m.

So, number of tiles required will be $\frac{12 \times 48}{12 \times 12} = 4$.

27. (B)

$$\text{The perimeter of the plot} = 2(7X + 5X)$$

$$= (24X) \text{ metres.}$$

$$\therefore \quad 24X \times 2 = 2400$$

$$\Rightarrow \quad X = \frac{1200}{24} = 50$$

$$\text{So, length of the plot} = 7 \times 50 = 350 \text{ m}$$

$$\text{and breadth of the plot} = 5 \times 50 = 250 \text{ m.}$$

$$\therefore \quad \text{Required difference} = 350 - 250 = 100 \text{ m.}$$

28. (B)

$$\text{Required area of road} = (15 + 16)^2 - 15^2$$

$$= (31)^2 - 15^2$$

$$= 961 - 225$$

$$= 736 \text{ sq.m.}$$

$\therefore$ Total Cost of construction

$$= 736 \times 310 = \text{Rs. } 2{,}28{,}160$$

29. (B) Area of equilateral $\Delta = \frac{\sqrt{3}}{2} a^2$

If a is increased by 25%.

i.e., $a \times \frac{25}{100} = \frac{a}{4}$

i.e., side $= a + \frac{a}{4} = \frac{5a}{4}$

New area $= \frac{\sqrt{3}}{2} \frac{25a^2}{16}$

Increment $= \frac{\sqrt{3}}{2} a^2 \left(\frac{25}{16} - 1\right)$

$= \frac{\sqrt{3}}{2} a^2 \left(\frac{25-16}{16}\right)$

$= \frac{\sqrt{3}}{2} a^2 \left(\frac{9}{16}\right)$

Increment in % $= \frac{\frac{\sqrt{3}}{2} a^2 \left(\frac{9}{16}\right)}{\frac{\sqrt{3}}{2} a^2} \times 100$

$= \frac{9}{16} \times 100 = 56{\cdot}25\%$.

30. (D) Shaded area $= A = \pi\,(r_2^2 - r_1^2)$

New shaded area $= A_1 = \pi\,[(3r_2)^2 - (3r_1)^2]$

$= 9\pi\,(r_2^2 - r_1^2)$

$\therefore$ Percentage Increase $= \frac{A_1 - A}{A} \times 100$

$= \frac{9A - A}{A} \times 100 = 800\%$.

31. (A) The surface area of cuboid whole length, breadth and height are l, b, and h respectively.

$= 2(lb + bh + hl)$

$= 2(5 \times 3 + 3 \times 2 + 2 \times 5)\ m^2$

$= 2(15 + 6 + 10)$

$= 2(21 + 10) = 31 \times 2 = 62\ m^2$.

32. (D) Volume of the speed $= (l \times b \times h)$

$= (6 \times 5 \times 0{\cdot}40)\ m^3$

$= (30 \times 0{\cdot}40) = 12\ m^3$.

33. (B) The number of bricks are

$= \frac{\text{Volume of wall}}{\text{Volume of bricks}} = \frac{1000 \times 500 \times 50}{25 \times 12 \times 45}$

$= \frac{2000 \times 500}{12 \times 45} = 1851{\cdot}85$

$= 1852$ (approx.).

34. (B) The volume of the new formed cube

$= \sum$[Volume of smaller cubes]

$\Rightarrow \quad 12^3 = 6^3 + 10^3 + x^3$

$\Rightarrow \quad 1728 = 216 + 1000 + x^3$

$\Rightarrow \quad x^3 = 1728 - 1216$

$\Rightarrow \quad x^3 = 512 = 8^3$

$\Rightarrow \quad x = 8$ cm.

Surface area $= 6 \times \text{Side}^2 = 6 \times 12^2$

$= 144 \times 6 = 864\ cm^2$.

35. (B) Length of longest rod $= \sqrt{(15^2 + 12^2 + 9^2)}$

$= \sqrt{(225 + 144 + 81)}$

$= \sqrt{450} = \sqrt{(15)^2 \times 2}$

$= 15\sqrt{2}$ m.

36. (D) Volume of cylinder $= \pi r^2 \times h = 14850\ cm^3$

$\Rightarrow \quad \frac{22}{7} \times r^2 \times 21 = 14850$

$\Rightarrow \quad 66 \times r^2 = 14850$

$\Rightarrow \quad r^2 = \frac{14850}{66} = 225$

$\Rightarrow \quad r = 15$ cm.

Lateral area $= 2\pi r \times h$

$= 2 \times \frac{22}{7} \times 15 \times 21$

$= 1980\ cm^2$.

37. (B) (Vol. of solid cylinder) $\times 0{\cdot}6 = 6 \times$ Vol. of each solid sphere

$\Rightarrow \quad \pi r^2 h \times 0{\cdot}6 = 6 \times \frac{4}{3} \times \pi r_1^3$

$\Rightarrow \quad \pi \times 9 \times 320 \times 0{\cdot}6 = 6 \times \pi \times \frac{4}{3} \times r_1^3$

$\Rightarrow \quad \frac{9 \times 32 \times 3}{4} = r_1^3$

$\Rightarrow \quad r_1^3 = 2^3 \times 3^3$

$\Rightarrow \quad r_1 = 6$ cm.

38. (D) Length $= \sqrt{12^2 + 12^2 + 12^2}$

$= \sqrt{144 + 144 + 144}$

$= 12\sqrt{3}$

39. (C) Volume of right circular cone $= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 25 \times 42$

$= 22 \times 50 = 1100\ cm^3$

40. (C) If edge of cube $= a$, diameter of sphere $= a\sqrt{3}$ and radius $= a\sqrt{3}/2$.

Now ratio $= a^3 : 4/3\pi\,(a\sqrt{3}/2)^3 = 1 : \pi\sqrt{3}/2$.

41. (B) ABC becomes the base circumference of the cone thus generated and P becomes the vertex. Thus, radius of cone can be found.

Circumference $= ABC = 1/4\,[2\pi R]$

$= 1/4\,[2\pi \times 5] = 5/2\pi$ cm.

$$\text{Circumference of base} = 2R = 5/2\pi$$

$$\Rightarrow \quad R \text{ (of cone)} = 5/4 \text{ cm} = OA.$$

$$\text{Height of cone OP} = \sqrt{\{AP^2 - OA^2\}}$$

$$= \sqrt{\{5^2 - (5/4)^2\}}$$

$$\Rightarrow \quad \text{Volume} = 1/3\, \pi R^2 H$$

$$= 1/3\, \pi (5/4)^2 \times \sqrt{\{25^2 - 25/16\}}$$

$$= 7{\cdot}9 \text{ cc.}$$

42. (C) $$\text{Old Surface area} = 6a^2$$

$$\text{New Surface area} = 6(1{\cdot}25a)^2$$

$$\text{Percentage Increase} = \frac{(\text{New} - \text{Old})}{\text{Old}} \times 100\%$$

$$= \frac{6(1{\cdot}25)^2\, a^2 - 6a^2}{6a^2} \times 100$$

$$= [(1{\cdot}25)^2 - 1] \times 100$$

$$= [1{\cdot}5625 - 1] \times 100$$

$$= 0{\cdot}5625 \times 100 = 56{\cdot}25\%.$$

43. (D) $$\text{Old volume} = lbh$$

$$\text{New Volume} = 2l \times 2{\cdot}2b \times 4{\cdot}5h$$

$$= (29.7)\, lbh$$

$$\Rightarrow \quad \%\text{ increase} = \frac{(\text{New} - \text{Old})}{\text{Old}} \times 100\%$$

$$= \frac{(29{\cdot}7 - 1)\text{ Old}}{\text{Old}} \times 100\%$$

$$= (29{\cdot}7 - 1) \times 100\%$$

$$= 28{\cdot}7 \times 100\% = 2870\%.$$

44. (B) $$\text{Wire} = \text{Cylinder}$$

$$\Rightarrow \quad \text{Volume} = \pi r^2 h = \pi \left(\frac{d}{2}\right)^2 h = \frac{\pi d^2 h}{4}$$

According to question,

$$\Rightarrow \quad \frac{\pi d^2 h}{4} = \frac{\pi (0{\cdot}5)^2\, h'}{4}$$

$$\Rightarrow \quad d^2 h = 0{\cdot}25 d^2 h'$$

$$\Rightarrow \quad \frac{h'}{h} = \frac{1}{0{\cdot}25} = 4 = h' = 4h$$

$$\%\text{ increase} = \frac{(\text{New} - \text{Old})}{\text{Old}} \times 100\%$$

$$= \frac{4h - h}{h} \times 100\%$$

$$= \frac{3h}{h} \times 100\% = 300\% \text{ (increment)}$$

45. (B) $$V = \frac{4}{3} \pi r^3 h$$

$$\Rightarrow \quad \frac{dV}{dt} = \frac{4}{3} \pi 3r^2 \frac{dr}{dt}$$

$$\Rightarrow \quad 121 = 4\pi (17)^2 \frac{dr}{dt}$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{1}{4\pi}$$

$$\text{Surface area } s = 4\pi r^2$$

$$\Rightarrow \quad \frac{ds}{dt} = 8\pi r \frac{dr}{dt}$$

$$\Rightarrow \quad \frac{ds}{dt} = 8\pi r \frac{1}{4\pi}$$

$$\Rightarrow \quad \frac{ds}{dt} = 2r = 2 \times 11 = 22 \text{ cm}^2/\text{min}$$

46. (B) $$\frac{\text{Surface area of cone}}{\text{Surface area of cylinder}} = \frac{5}{6}$$

$$\Rightarrow \quad \frac{\pi r l l}{2\pi r h} = \frac{5}{6}$$

$$\Rightarrow \quad \frac{l}{h} = \frac{5}{3}$$

$$\Rightarrow \quad l = \frac{5}{3} h$$

$$\text{and we know that} \quad l^2 = r^2 + h^2$$

$$\Rightarrow \quad \frac{25}{9} h^2 = r^2 + h^2$$

$$\Rightarrow \quad \frac{25 - 9}{9} h^2 = r^2$$

$$\Rightarrow \quad \frac{16}{9} h^2 = r^2$$

$$\Rightarrow \quad \frac{r^2}{h^2} = \frac{16}{9}$$

$$\Rightarrow \quad \frac{r}{h} = \frac{4}{3} \text{ or } 4 : 3.$$

47. (C) $$\text{Volume of solid cylinder} = \pi r^2 h$$

$$= \pi (10)^2 \times 15 = 1500\pi$$

$$\text{Volume of frustum} = \frac{\pi}{3} h\, (r_1{}^2 + r_1 r_2 + r_2{}^2)$$

Here, $r_1 = 4$ cm, $r_2 = 10$ cm, H = 5 cm

$$= \frac{\pi}{3} \times 5 \times (16 + 40 + 100)$$

$$= \frac{5\pi \times 156}{3} = 260\pi$$

and the volume of the cylindrical bore $= \pi r^2 h$

$$= \pi \times (4)^2 \times 10 = 160\pi$$

The volume of the remaining material

$$= (1500\pi - (260\pi + 160\pi))$$

$$= (1500 - 420)\pi$$

$$= 1080\pi.$$

48. (B) Area of the path = Outer rectangle – courtyard area

$$= 22 \times 16 - 15 \times 9$$

$$= 352 - 135$$

$$= 217 \text{ m}^2$$

$$\text{Amount of gravel} = (217 \times 0{\cdot}2) \text{ m}^3$$

$$= 43{\cdot}4 \text{ m}^3$$

49. (D) Let the cube side $= x$ in.

Diameter of the sphere $= x$ in.

Now, volume of wood removed = volume of cube – volume of sphere

$$= x^3 - \frac{4}{3}\pi\left(\frac{x}{2}\right)^3$$

$$= x^3 - \frac{4x^3}{6}$$

$$= x^3\left[1 - \frac{22}{7 \times 6}\right]$$

$$= x^3\left(\frac{20}{42}\right)$$

According to question,

$$\frac{x^3}{42} \times 20 = 4410$$

$$\Rightarrow \quad x^3 = \frac{4410 \times 42}{20}$$

$$\Rightarrow \quad x^3 = 21 \times 21 \times 21$$

$$\Rightarrow \quad x = 21 \text{ in.}$$

50. (A) The section of the cone through the axis cut the sphere in a circle which is the inscribed circle of the equilateral triangle in which it cuts the cone. Hence, the side of the equilateral is

$$a = 2\sqrt{3}.r$$

where r is the radius of the inscribed circle

$$= 2\sqrt{3} \times 6 \text{ ft} = 12\sqrt{3} \text{ ft.}$$

Hence, the height of the equilateral triangle is $= \frac{\sqrt{3}}{2} a$

$$= \frac{\sqrt{3}}{2} \times 12\sqrt{3} = 18 \text{ ft.}$$

Also, the radius of the base of the cone $= \frac{1}{2} a$

$$= \frac{a}{2} = 6\sqrt{3} \text{ ft.}$$

So, volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (6\sqrt{3})^2 \times 18$$

$$= \frac{22}{7} \times 36 \times 3 \times 6$$

$$= \frac{18 \times 36 \times 22}{7}$$

$= 2036{\cdot}57$ or $2036{\cdot}6$ c.ft (approx.)

Exercise B

1. (C) Area of garden $= \pi \times 84^2$

Area of garden $= (92{\cdot}4)^2 - \pi \times 84^2$

$$= \pi \times 8{\cdot}4 \times 176{\cdot}4$$

Required ratio $= \dfrac{\pi \times 8{\cdot}4 \times 176{\cdot}4}{\pi \times 84 \times 84}$

$$= \frac{176{\cdot}4}{84 \times 10} = 0{\cdot}21$$

2. (B) $\dfrac{\text{Volume of big ball}}{\text{Volume of small ball}} = \dfrac{\frac{4}{3}\pi \times 9^3}{\frac{4}{3}\pi \times 3^3}$

$$= 9 \times 3 = 27$$

3. (D) Area of floor $= 60 \times 40$

Area of carpet $= (60 - 4) \times (40 - 4)$

$$= 56 \times 36$$

% required $= \dfrac{56 \times 36}{40 \times 60} \times 100 = 84\%$

4. (D) $\dfrac{2}{3}\pi\, 3^3 = \dfrac{1}{3}\pi \times r^2 \times 150$

$$\frac{2 \times 27}{150} = r^2$$

$$\Rightarrow \quad r^2 = \frac{27}{75} = \frac{9}{25}$$

$$\Rightarrow \quad r = \frac{3}{5} = 0{\cdot}6 \text{ cm.}$$

5. (B) Circumference of circle

$$= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$$

Perimeter of square $= 4a = 44$

$$a = 11$$

Area of square $= 112 = 121$ cm^2.

6. (A) Let the height and radius of the base is n and r respectively in cm.

and let v be the volume in c.c.

Then, $v = mr^2h$ when m is constant by question

$$1540 = m \times 7^2 \times 30$$

$$m = \frac{154}{3 \times 49}$$

$$\Rightarrow \quad \frac{22}{21} r^2 h = v$$

By substituting $v = 264$ and $r = 6$, $h = ?$

$$\Rightarrow \quad 264 = \frac{22}{21} \times 6^2 \times h$$

$$\Rightarrow \quad h = \frac{264 \times 21}{22 \times 36} = 28 \text{ cm.}$$

7. (C) Let the volume of tank = V

A's one hour work $= \dfrac{V}{8}$

B's one hour work $= \dfrac{V}{6}$

According to the given condition the volume of the tank

$$= \left(\frac{V}{8} + \frac{V}{6} - 10\right) \times 5 = V$$

$$\Rightarrow \left(\frac{3V + 4V - 240}{24}\right) \times 5 = V$$

$$\Rightarrow 7V - 240 = \frac{24V}{5}$$

$$\Rightarrow \frac{7V}{1} - \frac{24V}{5} = 240$$

$$\Rightarrow 11V = 240 \times 5$$

$$\Rightarrow V = 109 \text{ litre.}$$

8. (D) Thickness of the wood = 2·5 cm

$\Rightarrow$ Internal length of box = 50 – (5) = 45 cm

Internal breadth of box = (45 – 5) cm = 40 cm

Internal height of box = (35 – 5) = 30 cm

$\Rightarrow$ Volume of the box = $(45 \times 40 \times 30)$ cm^3

Volume of one bricks = $(6 \times 5 \times 4)$ cm^3

Number of bricks = $\dfrac{45 \times 40 \times 30}{4 \times 5 \times 6} = 450.$

9. (A) $V = \frac{4}{3}\pi r^3$

$$V' = \frac{4}{3}\pi (1{\cdot}025r)^3$$

$$= \frac{4}{3}\pi r^3 (1{\cdot}07689)$$

$$V' = V(1{\cdot}0769)$$

$$\Rightarrow \frac{V'}{V} = 1{\cdot}0769$$

$\Rightarrow$ Error in volume of sphere

$$= (1 - 1{\cdot}0769) \times 100$$

$$= 0{\cdot}0769 \times 100 = 7{\cdot}67$$

$$= 7{\cdot}7\% \text{ (approx.)}$$

10. (D) The exact number cubes are got when side of cube is edge length is HCF of (65, 26, 13)

HCF (65, 26, 13) = 13

Then, exact number of cubes = $\dfrac{65 \times 26 \times 13}{(13)^3} = 10.$

11. (B) Thickness = 2·5 cm.

The inner length, inner breadth, and inner height

= (15 – 5), (13 – 5), (11 – 5) cm.

= 10 cm, 8 cm, 6 cm.

Then the inner Surface Area

$$= 2\,[lb + bh + hl]$$

$$= 2[10 \times 8 + 8 \times 6 + 6 \times 10]$$

$$= 2[80 + 48 + 60] = 2 \times 188$$

$$= 376 \text{ cm}^2.$$

12. (C)Volume of cylinder = $\pi r^2 h$

Here, $h = 12r$

$$\Rightarrow = 12\pi r^3$$

Volume of sphere = $\frac{4}{3}\pi r^3$

Number of sphere = $\dfrac{12\pi r^3}{\frac{4}{3}\pi r^3} = \dfrac{12 \times 3}{4} = 9$ balls.

13. (B) Obvious, Any of the adjacent vertex will have the distance as 1.

The vertex opposite on the surface will have $\sqrt{1^2 + 1^2} = \sqrt{2}$ units of distance. Also, the body diagonal will be $\sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$ units apart.

14. (C) Obviously more than 50% will remain outside.

15. (D) Volume of solid lead sphere = $\frac{4}{3}\pi\left(\frac{8}{2}\right)^3$

Volume of 1 lead ball = $\frac{4}{3}\pi\left(\frac{1}{4}\right)^3$

Number of balls = $\dfrac{\frac{4}{3}\pi 4^3}{\frac{4}{3}\pi\left(\frac{1}{4}\right)^3}$

$$= 4^3 \times 4^3 = 64 \times 64$$

$$= 4096 > 4000.$$

16. (B) Volume = $\pi R^2 H$

$\Rightarrow$ New volume = $\pi.8^2.(3 + X)$

$$= \pi.(8 + X)^2.3$$

$$\Rightarrow X = 16/3 = 5{\cdot}33 \text{ m.}$$

17. (A) Curved area of cylinder = $(2\pi R) \times (2R) = 4\pi R^2$, Curved area of sphere = $4\pi R^2 \Rightarrow$ Ratio $4\pi R^2 : 4\pi R^2 = 1 : 1$.

Note that the cylinder and sphere have equal radii R and height of cylinder = 2R = Diameter of sphere.

18. (D) For smaller cone, $r = \frac{1}{4}, h = \frac{1}{2}, l = \frac{\sqrt{5}}{4}$

For greater cone, $R = \frac{1}{2}$, H = 1, $L = \sqrt{R^2 + H^2} = \frac{\sqrt{5}}{4}$

Required surface area = surface area of greater cone $+ 2 \times \frac{1}{4} \cdot$ Surface area of smaller cone

$$= (\pi \times R \times L) + \left(2 \times \frac{1}{4} \times \pi r l\right)$$

$$= \left(\pi \times \frac{1}{2} \times \frac{\sqrt{5}}{2}\right) + \left(2 \times \frac{1}{4} \times \pi \times \frac{1}{4} \times \frac{\sqrt{5}}{4}\right)$$

$$= \frac{\sqrt{5}\pi}{4} + \frac{\sqrt{5}\pi}{32} = \sqrt{5}\pi\left(\frac{8 + 1}{32}\right) = \frac{9\sqrt{5}\pi}{32}.$$

14 Linear and Quadratic Equation

Pair of Linear Equations in Two Variables

1. An equation that can be written in the form ax + by = c, $a \neq 0$, $b \neq 0$ where a, b and c are real numbers, is known as a *linear equation in two variables x and y*.

2. A solution is a pair of values, one for each variable, which satisfies the equation. Every linear equation in two variables has infinitely many solutions.

3. Two linear equations, each containing the same two unknown variables, *e.g.*,

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

are said to form a system of simultaneous linear equations.

4. The solution to a system of linear equations is the values of x and y common to all lines in the system when the lines are drawn on a graph.

If a system of equations has at least one solution, it is said to be consistent.

5. When the system has a unique solution, it is called consistent and the solution has to be given in the form $x = \alpha, y = \beta$.

- When there is no common solution of the two given equations, it is called inconsistent.
- When the equations have infinitely many common solutions, it is called system of dependent equations.

6. Graphic Method for the solution of system of linear equations—

- When line intersect at a single point, the equations have a unique common solution.
- When the lines are parallel, the equations have no common solution.
- When the lines are coincident, the equations have infinitely many solutions.

7. For the given system of linear equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$.

8. Algebraic methods for the solution of system of linear equations

(a) Method by Substitution

Step 1. Find the value of one the variables in terms of the other from any one of the given equations.

Step 2. Substitute the value of the variable obtained in step 1 in the other equation.

Step 3. Solve the simple equation obtained in step 2 and find the value of one of the variables.

Step 4. Substitute the value of the variable obtained in step 3 in any one of the given equations and find the value of the other variable.

(b) Elimination by Equating the Coefficients

Step 1. Multiply the equations by the constant and non-zero numbers so as to make the coefficient of one variable to be eliminated equal.

Step 2. Now, if the coefficients of the variable to be eliminated are having same sign, then subtract the equation obtained in step (1).

Also, if the coefficients of the variable to be eliminated are having opposite signs, then add the equation obtained in step (1).

Step 3. Solve the equation obtained in step (2) to obtaine the value of one variable.

Step 4. Substitute the value of variable obtained in step 3 in any of the given equations to find the value of the other variable.

(c) Method of Comparison

Step 1. From each equation, find the value of same variable (say y) in terms of other.

Step 2. Equate the values thus found in step 1.

Step 3. Solve the equation obtained as linear equation in one variable.

	Consistent		**Inconsistent**
Condition →	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Graph →	**Intersecting Lines (Point of intersection)**	**Coinciding Lines (Overlapping)**	**Parallel Lines**
Solution →	**Unique (One) Solution**	**Many(infinite) Solution**	**No Solution**

Step 4. Substitute the value of variable obtained in step 3 in given equation to find the value of the other variable.

(d) Method of Cross Multiplication

In the given system of equations in the form of

$$a_1x + b_1y + c_1 = 0$$
$$a_1x + b_1y = c_1$$
$$\Rightarrow \quad a_1x + b_1y - c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$
$$a_2x + b_2y = c_2$$
$$\Rightarrow \quad a_2x + b_2y - c_2 = 0$$

Solution of variables x and y can be found with the help of following expressions—

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$
$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}$$

Quadratic Equations

1. Quadratic polynomial—A polynomial of the form $p(x) = ax^2 + bx + c$ when $a \neq 0$ and a, b, c are real numbers and x is a real variable, is called a quadratic polynomial.

2. Quadratic equation—An equation $p(x) = 0$ where $p(x)$ is a quadratic polynomial, is called zeros of quadratic equation *i.e.*, $ax^2 + bx + c = 0$, where $a \neq 0$.

3. Zeros of quadratic equations—Those values of x for which $ax^2 + bx + c = 0$ is satisfied are called zeros of quadratic polynomial. If $p(\alpha) = a\alpha^2 + b\alpha + c = 0$, then α is called the zero of quadratic polynomial.

4. Roots of quadratic equations—If α, β are the roots of a quadratic polynomial $ax^2 + bx + c$, then α, β are called roots (or solutions) of the corresponding equation $ax^2 + bx + c = 0$, which implies that $p(\alpha) = p(\beta) = 0$

i.e., $a\alpha^2 + b\alpha + c = 0$ and $a\beta^2 + b\beta + c = 0$.

5. Solution of a quadratic equation can be found by two methods—

(i) by factorization, (ii) by completion of square.

Note—By completing the square, we get roots as

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Where $(b^2 - 4ac)$ is called the discriminant, denoted by 'D'.

6. Nature of roots—

(i) When D = 0, the roots of quadratic equation are real and equal and each $= \frac{-b}{2a}$

(ii) When D > 0, the roots are real but unequal.

(iii) When D < 0, the no-real roots are possible.

Exercise A

1. Solve for x and y—
$$ax + \text{by} = a - b$$
$$bx - ay = a + b$$

2. Solve for x and y—
$$\frac{57}{x+y} + \frac{6}{x-y} = 5$$
$$\frac{38}{x+y} + \frac{21}{x-y} = 9$$

3. Solve for x and y—
$$\frac{44}{x+y} + \frac{30}{x-y} = 10$$
$$\frac{55}{x+y} + \frac{40}{x-y} = 13$$

4. Solve for x and y—
$$\frac{x}{10} + \frac{y}{5} + 1 = 15$$
$$\frac{x}{8} + \frac{y}{6} = 15$$

5. Solve for x and y—
$$\frac{2x}{a} + \frac{y}{b} = 2$$
and $$\frac{x}{a} - \frac{y}{b} = 4$$

6. Solve for x and y—
$$\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$$
and $$\frac{3}{x} + \frac{2}{y} = 0$$
Hence, find 'a' for which $y = ax - 4$.

7. Places A and B are 80 km apart from each other on a highway. A car starts from A and another car starts from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions they meet in 1 hour and 20 minutes. Find the speed of the cars.

8. The sum of two digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number. The new number is 4 more than 5 times the sum of digits in the first number. Find the number.

9. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

10. A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car. It takes him 6 hours and 30 minutes. But, if he travels 200 km. by train and the rest by car, he takes half an hour longer. Find the speed of the train and that of the car.

11. A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days, one has taken food in the mess. When a student A takes food for 20 days, he has to pay Rs. 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs. 1180 as hostel charges. Find the fixed charge and the cost of food per day.

12. A two digit number is 4 times the sum of its digits. If 18 is added to the number the digit is reversed. Find the numbers.

13. A two digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number the digits are reserved. Find the number.

14. A two digit number is 4 more than 6 time the sum of digits. If 18 is subtracted from the number the digits are reversed. Find the number.

15. The denominator of a fraction is 4 more than twice the numerator. When both the numerator and the denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the number.

16. A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days one has taken food in the mess. When a student X takes food for 25 days, he has to pay Rs. 1750 as hostel charges whereas a student Y, who takes food for 28 days, pays Rs. 1900 as hostel charges. Find the fixed charge and the cost of food per day.

17. The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km the charge paid is Rs. 75 and for a journey of 15 km the charge paid is Rs. 110. What will a person have to pay for traveling a distance of 25 km ?

18. The car hire charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 12 km, the charge paid is Rs. 89 and for a journey of 20 km the charge paid is Rs. 145. What will a person have to pay for traveling a distance of 30 km ?

19. Taxi charges consist of fixed charges and the remaining depending upon the distance traveled in kilometres. If a person travels 10 km he pay Rs. 68 and for traveling 15 km he pays Rs. 98. Express the above statements with the help of simultaneous equations and hence find the fixed charges and the rate per km.

20. The total expenditure per month of a household consists of a fixed rent of the house and the mess charges depending upon the number of people sharing the house. The total monthly expenditures is Rs. 3900 for 2 people and Rs. 7500 for 5 people. Find the rent of the house and mess charges per head per month.

21. Taxi charges consist of fixed charges per day and the remaining depending upon the distance travelled in kilometres. If a person travels 110 km he pays Rs. 690 and for travelling 200 km he pays Rs. 1050. Express the above statements in the form of simultaneous equations and hence find the fixed charges per day and the rate per km.

22. A number consisting of two digits is seven times the sum of its digits. When 27 is subtracted from the number, the digits are reversed. Find the number.

23. A number consists of two digits. When it is divided by the sum of the digits, the quotient if 6 with no remainder. When the number is diminished by 9, the digits are reversed. Find the number.

24. There are two classrooms A and B containing students. If 5 students are shifted from Room A to Room B, the resulting number of students in the two rooms become equal. If 5 students are shifted from Room B to Room A, the resulting number of students in Room A becomes double the number of students left in Room B. Find the original number of students in the two rooms separately.

25. The area of a rectangle gets reduced by 80 sq. units, if its length is reduced by 5 units and the breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, the areas is increased by 50 sq. units. Find the length and breadth of the rectangle.

26. For what value of k, does the quadratic equation $9x^2 + 8kx + 16 = 0$ have equal roots.

27. If roots of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal. Prove that $2a = b + c$.

28. For what value of k the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots.

29. Find the value of 'k' so that the equation $9x^2 - kx + 81 = 0$ has equal roots.

30. If the equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots. Prove that $c^2 = a^2(1 + m^2)$.

31. Find the value of c for which the quadratic equation $4x^2 - 2(c + 1)x + (c + 4) = 0$ has equal roots.

32. Find whether the quadratic equation $x^2 - x + 2 = 0$ has real roots. If yes, find the roots.

33. If one root of the quadratic equation $2x^2 + ax + 3 = 0$ is 1, find the other root, and the value of 'a'.

34. Find the value of α such that quadratic equation $(\alpha - 3)x^2 + 4(\alpha - 3)x + 4 = 0$ has equal roots.

35. Find the values of k so that quadratic equation $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$ has equal roots.

36. Find the values of p for which the $x^2 + p(4x + p - 1) + 2 = 0$ has equal roots.

37. One root of the equation $2x^2 - 8x - m = 0$ is $\frac{5}{2}$. Find the other root and value of m.

38. For what value of k the equation $2kx^2 - 40x + 25 = 0$ has equal roots ? Find the roots.

39. For what value of k the equation $9x^2 - 24x + k = 0$ has equal roots ? Find them.

40. If the list price of a book is reduced by Rs. 5, a person can buy 5 more books for Rs. 300. Find the original price of the book.

41. If the list price of a toy is reduced by Rs. 2, a person can buy 2 toys more for Rs. 360. Find the original price of the toy.

42. Two numbers differ by 3 and their product is 504. Find the numbers.

43. Two numbers differ by 4 and their product is 192, find the numbers.

44. Two numbers differ by 2 and their product is 360, find the numbers.

45. Find two consecutive numbers, whose squares have the sum 85.

46. The sum of two numbers is 15 and sum of their reciprocals is $\frac{3}{10}$. Find the numbers.

47. Rs. 9,000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs. 160 less ? Find the original number of persons.

48. A plane left 30 minutes later than the scheduled time and in order to reach the destination 1,500 km away in time. It has to increase the speed by 250 km/hr from the usual speed. Find its usual speed.

49. The length of the hypotenuse of a right angled triangle exceeds the length of the base by 2 cm and exceeds the length of the altitude by 1 cm. Find the length of each side of the triangle.

50. Some students planned a picnic. The budget for food was Rs. 500. But 5 of these failed to go and thus the cost of food for each member increased by Rs. 5. How many students attended the picnic?

51. Two pipes running together can fill a cistern in 6 minutes. If one pipe takes 5 minutes more than other to fill the cistern. Find the time in which each pipe would fill the cistern.

52. Two pipes running together can fill a cistern in $2\frac{8}{11}$ minutes. If one pipe takes 1 minute more than the other to fill the cistern. Find the time in which each pipe would fill the cistern.

53. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe take 3 minutes more than the other to fill the cistern. Find the time in which each pipe would fill the cistern.

54. In a flight of 2800 km an aircraft was slowed down due to bad weather, its average speed for the trip was reduced by 100 km/hr and time increased by 30 minutes. Find the original duration of flight.

55. In a flight of 3000 km an aircraft was slowed down due to bad weather. Its average speed for the bad weather. Its average speed for the trip was reduced by 100 km/hr and time increased by one hour. Find the original duration of flight.

56. In a flight of 6000 km an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 400 km/hr and time increased by 30 minutes. Find the original duration of flight.

57. A rectangular field is 16 m long and 10 m wide. There is a path of uniform width all around it, having an area of 120 sq.m. Find the width of the path.

58. X and Y are centers of circles of radius 9 cm and 2 cm respectively and XY = 17 cm Z is the centre of a circle of radius r cm which touches the above circles externally. Given that $\angle$ XZY = 90°, write an equation in r and solve it for r.

59. A person on tour has Rs. 360 for his daily expenses. If he extends his tour for 4 days. He has to cut down his daily expense by Rs. 3. Find the original duration of the tour.

60. A piece of cloth costs Rs. 200 if the piece was 5 m longer and each metre of cloth costs Rs. 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?

Exercise B

1. Ramesh travels 760 km to his home, partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by car. He takes 12 minutes more if he travels 240 km by train and the rest by car. Find the speed of the train and the car separately.

2. A person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of Rs. 130. But if he had interchanged the amounts invested, he would have received Rs. 4 more as interest. How much amount did he invest at different rates?

3. A part of monthly expenses of a family is constant and the remaining varies with the price of wheat. When the rate of wheat is Rs. 250 a quintal, the total monthly expenses of the family are Rs. 1000 and when it is Rs. 240 a quintal, the total monthly expenses are Rs. 980. Find the total monthly expenses of the family when the cost of wheat is Rs. 350 a quintal.

4. The sum of the numerator and the denominator of a fraction is 18. If the denominator is increased by 2, the fraction is reduced to $\frac{1}{3}$ · Find the fraction.

5. 2 tables and 3 chairs together cost Rs. 2000, whereas 3 tables and 2 chairs together cost Rs. 2500. Find the total cost of 1 table and 5 chairs.
6. Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.
7. A and B each have certain number of oranges. A says to B, "If you give me 10 of your oranges. I will have twice the number of oranges left with you." B replies, "If you give me 10 of your oranges, I will have the same number of oranges as left with you." Find the number of oranges with A and B separately.
8. Jai raj travels 300 km to his home partly by train and partly by bus. He takes 4 hours, if he travels 60 km by train and the remaining distance by bus. If he travels 100 km by train and the remaining distance by bus, he takes 10 minutes longer. Find the speeds of the train and the bus separately.
9. Ten years ago, a father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be. Find their present ages.
10. The present age of a father is three years more than three times the age of the son. Three years hence father's age will be 10 years more than twice the age of the son. Determine their present ages.
11. **Solve—**

$$\frac{2}{13}(2x+3y) = 3+\frac{x-y}{4}$$

$$\frac{4y+5x}{3} = 2x+7\frac{1}{6}$$

12. **Solve for x and y—**

$$bx + ay = a + b$$

$$ax\left(\frac{1}{a-b}-\frac{1}{a+b}\right) + by\left(\frac{1}{b-a}-\frac{1}{b+a}\right) = \frac{2a}{a+b}$$

13. Find real values of x and y which will make—

$$(2x-3y-13)^2+(3x+5y+9)^2 = 0$$

14. Solve for x and y—

$$2^x + 3^y = 17$$

and $$2^{(x+2)} - 3^{(y+1)} = 5$$

15. Solve for x and y—

$$\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x+y-12}{11}$$

16. If two liquids are mixed in the ratio 3 : 2, a mixture is obtained weighting 1.04 g. per c.c., while if they are mixed in the ratio 5 : 3, the resulting mixture weights 1.05 g per c.c. Find the weight of *a* c.c. of each of the original liquids.
17. A says to B, "I am three times old as you were, when I was as old as you are." The sum of their present ages is 64 years. Find their ages.
18. It takes me 8 hours to reach city A from city B. If I increase my speed by 6 km per hour. It takes me 1 hour 20 minutes less. Find the distance between the cities A and B.
19. Rs. 6500 were divided equally among a certain number of persons. Had there been 15 more persons, each would have got Rs. 30 less ? Find the original number of persons.
20. Two squares have sides x cm and $(x + 4)$ cm. The sum of their areas in 656 cm^2. Find the sides of the squares.
21. The side of a square exceeds the side of another square by 4 cm and the sum of the area of the two squares is 400 sq.m. Find the dimensions of the two squares.
22. The length of a rectangle exceeds its width by 8 cm and the area of the rectangle is 240 sq.cm. Find the dimensions of the rectangle.
23. The area of a right angled triangle is 600 sq.cm. If the base of the triangle exceeds the altitude by 10 cm. Find the dimensions of the triangle.
24. An express train makes a run of 240 km at a certain speed. Another train whose speed is 12 km/hr less takes an hour longer to cover the same distance. Find the speed of the express train in km/hr.
25. A train covers a distance of 90 km at a uniform speed. Had the speed be 15 km/hr more, it would have taken half an hour less for the journey ? Find the original speed of the train?
26. A train travels a distance of 300 km at a constant speed. If the speed of the train is increased by 5 km per hour, the journey would have taken 2 hours less. Find the original speed of the train.
27. The sum of the squares of two consecutive natural numbers is 313. Find the numbers.
28. Divide 29 into two parts so that the sum of the squares of the parts is 425.
29. The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.
30. The sum of two numbers is 48 and their product is 432. find the numbers.
31. The speed of a boat in still water is 15 km/hr it can go 30 km upstream and return downstream to the original point in 4 hours 30 minutes. Find the speed of the stream.
32. Find the whole number which when decreased by 20 is equal to 69 times the reciprocal of the number.
33. Find two consecutive natural numbers whose product is 20.
34. One year ago, the father was 8 times as old as his son. Now, his age is square of the son's age. Find their present ages.

35. A shopkeeper buys a number of books for Rs. 80. If he had bought 4 more for the same amount, each book would have cost Rs. 1 less. How many books did he buy?

36. If a integer is added to its square, the sum is 90. Find the integer with the help of a quadratic equation.

37. If the roots of the equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal. Prove that $2a = b + c$.

38. If the roots of $(a^2 + b^2)x^2 + 2(bc - ad)x + (c^2 + d^2) = 0$ are real and equal. Show that $ac + bd = 0$.

39. If the roots of the equation $p(q - r)x^2 + q(r - p)x + r(p - q) = 0$ be equal. Show that $\frac{1}{p} + \frac{1}{r} = \frac{2}{q}$.

40. If the equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots. Prove that $c^2 = a^2(1 + m^2)$.

41. A cyclist cycles non – stop from A to B a distance of 14 km at a certain average speed. If his average speed reduces by 1 km/hr he takes $\frac{1}{3}$ hrs. more to cover the same distance. Find his average original speed.

42. If I had walked 1 km/hr faster, I would have taken 10 minutes less to walk 2 km. Find the rate of my walking.

43. In a flight of 1600 km aircraft was slowed down by bad weather. Its average speed for the trip was reduced by 400 km/hr and the time to flight increased by 40 minutes. Find the actual time to flight.

44. In a group of children, each child gives a gift to every other child. If the number of gifts is 132. Find the number of children.

45. A man purchased number of books at Rs. 720. If the price of each book were Rs. 2 less, he would, then get 4 more books. How many books he had purchased.

46. Out of a group of swans, $\frac{7}{2}$ times the square root of the total number are playing on the share of a pond. The two remaining ones are swimming in water. Find the total number of swans.

Answers

Exercise A

1.
$$ax + by - (a - b) = 0$$
$$bx - ay - (a + b) = 0$$

Using cross multiplication method, we get—

$$\frac{x}{\begin{array}{cc} b & -(a-b) \\ -a & -(a+b) \end{array}} = \frac{y}{\begin{array}{cc} -(a-b) & b \\ -(a+b) & a \end{array}} = \frac{1}{\begin{array}{cc} a & b \\ b & -a \end{array}}$$

$$\frac{x}{-ab - b^2 - a^2 + ab} = \frac{y}{-ab - b^2 - a^2 + ab} = \frac{1}{-a^2 - b^2}$$

Or $$\frac{x}{(-a^2 + b^2)} = \frac{y}{(a^2 + b^2)} = \frac{1}{-(a^2 + b^2)}$$

Or $$x = \frac{-(a^2 + b^2)}{-(a^2 + b^2)}$$

and $$y = \frac{(a^2 + b^2)}{-(a^2 + b^2)}$$

Hence, $x = 1$ and $y = -1$

2.
$$\frac{57}{x+y} + \frac{6}{x-y} = 5 \quad \ldots\text{(i)}$$
$$\frac{38}{x+y} + \frac{21}{x-y} = 9 \quad \ldots\text{(ii)}$$

Multiplying (i) by 7 and (ii) by 2 and subtracting

$$\frac{399}{x+y} + \frac{42}{x-y} = 35$$
$$\frac{76}{x+y} + \frac{42}{x-y} = 18$$

$$\frac{323}{x+y} = \frac{17}{1}$$
$$17(x + y) = 323$$
$$x + y = 19 \quad \ldots\text{(iii)}$$

Putting the value of $(x + y)$ in (i), we get

$$\frac{57}{19} + \frac{6}{x-y} = 5$$
$$\Rightarrow \quad 3 + \frac{6}{x-y} = 5$$
$$\frac{6}{x-y} = 5 - 3 = 2$$
$$2(x - y) = 6$$
$$\Rightarrow \quad x - y = 3 \quad \ldots\text{(iv)}$$

Adding (iii) and (iv), we get

$$2x = 22$$
$$x = 11$$

Putting the value of x in (iii),

$$11 + y = 19$$
$$y = 19 - 11 = 8$$
$$\therefore \quad x = 11, y = 8$$

3.
$$\frac{44}{x+y} + \frac{30}{x-y} = 10 \quad \ldots\text{(i)}$$
$$\frac{55}{x+y} + \frac{40}{x-y} = 13 \quad \ldots\text{(ii)}$$

Multiplying (i) by 4 and (ii) by 3 and subtracting

$$\frac{176}{x+y} + \frac{120}{x-y} = 40$$
$$\frac{165}{x+y} + \frac{120}{x-y} = 39$$

$$\frac{11}{x+y} = 1$$
$$\Rightarrow \quad x + y = 11 \quad \ldots\text{(iii)}$$

Using the value of $(x + y)$ in (i)

$$\frac{44}{11}+\frac{30}{x-y} = 10$$

$$\frac{30}{x-y} = 10-\frac{44}{11}$$

$$\frac{30}{x-y} = \frac{110-44}{11}=\frac{66}{11}=6$$

$$6(x-y) = 30$$

$$\Rightarrow \quad x-y = 5 \quad \ldots(iv)$$

Adding (iii) and (iv), we get

$$2x = 16$$

$$x = 8$$

Putting the value of x in (iii)

$$8+y = 11$$

$$y = 11-8=3$$

4. $$\frac{x}{10}+\frac{y}{5}+1 = 15 \quad \ldots(i)$$

$$\frac{x}{8}+\frac{y}{6} = 15 \quad \ldots(ii)$$

Multiplying (i) by 10 and (ii) by 24, we get—

$$x+2y+10 = 150$$

or $$x+2y = 140 \quad \ldots(iii)$$

and $$3x+4y = 360 \quad \ldots(iv)$$

$$3\times(iii) \Rightarrow 3x+6y = 420 \quad \ldots(v)$$

(iv) – (v) $$-2y = -60$$

$$y = 30$$

$$x+2y = 140$$

and $$y = 30$$

$$\Rightarrow \quad x+60 = 140$$

$$\Rightarrow \quad x = 80$$

5. $$\frac{2x}{a}+\frac{y}{b} = 2 \quad \ldots(i)$$

$$\frac{x}{a}-\frac{y}{b} = 4 \quad \ldots(ii)$$

By adding $$\frac{3x}{a} = 6$$

$$\Rightarrow \quad 3x = 6a$$

$$\Rightarrow \quad x = 2a$$

Putting the value of $x = 2a$ in (i), we get

$$2\left(\frac{2a}{a}\right)+\frac{y}{b} = 2$$

$$\Rightarrow \quad 4+\frac{y}{b} = 2$$

$$\Rightarrow \quad \frac{y}{b} = 2-4$$

$$\Rightarrow \quad \frac{y}{b} = -2$$

$$\Rightarrow \quad y = -2b$$

$$\Rightarrow \quad x = 2a, y=-2b$$

You may also use cross multiplication method.

6. $$\frac{2}{x}+\frac{2}{3y} = \frac{1}{6} \quad \ldots(i)$$

$$\frac{3}{x}+\frac{2}{y} = 0 \quad \ldots(ii)$$

(i) $$\Rightarrow \frac{12}{x}+\frac{4}{y} = 1 \quad \ldots(iii)$$

(Multiplying by '6')

Now, (ii) $$\Rightarrow \frac{12}{x}+\frac{8}{y} = 0 \quad \ldots(iv)$$

(Multiplying by '4')

(iii) – (iv) $$\Rightarrow \frac{-4}{y} = 1 \text{ or } y=-4$$

Substitute $y = -4$ in (ii), we get

$$\frac{3}{x}+\frac{2}{-4} = 0$$

$$\Rightarrow \quad \frac{3}{x}-\frac{1}{2} = 0$$

or $$\frac{3}{x} = \frac{1}{2}$$

or $$x = 6$$

Also, $$y = ax-4$$

$$-4 = 6a-4$$

or $$6a = 0$$

or $$a = 0$$

7. Let the speed of car, starts from A = x km/hr
and the speed of car, starts from B = y km/hr
According to the question,

$$8x = 8y+80 \quad \text{(same direction)}$$

$$\{\because \text{Distance} = \text{Speed}\times\text{Time}\}$$

and $$\frac{4}{3}x+\frac{4}{3}y = 80 \quad \text{(opposite direction)}$$

$$\left[1 \text{ hr. } 20 \text{ min.} = 1\frac{20}{60}=\frac{4}{3}\text{ hr.}\right]$$

$$4x+4y = 240$$

$$4(y+10)+4y = 240 \quad \text{[From (i)]}$$

$$4y+40+4y = 240$$

$$4y+4y = 240-40$$

$$8y = 200$$

$$\Rightarrow \quad y = 25$$

Taking value of y in (i), we get

$$x = 25+10=35$$

$\therefore$ Speeds of cars are 35 km/hr and 25 km/hr.

8. Let unit's place digit be x
and ten's place digit be y.
Then, original numbers—

Ist number $= x+10y$

And Reversed number $= 10x+y$

According to the question,

$$\begin{aligned} x + 10y + 10x + y &= 110 \\ 11x + 11y &= 110 \\ x + y &= 10 \quad \text{...(i) [÷ by 11]} \\ x + 10y - 10 &= 5(x + y) + 4 \\ 5x + 5y + 4 - x - 10y + 10 &= 0 \\ 4x - 5y &= -14 \quad \text{...(ii)} \end{aligned}$$

Multiplying (i) by 5 and adding to (ii), we get

$$\begin{aligned} 5x + 5y &= 50 \\ 4x - 5y &= -14 \\ \hline 9x &= 36 \\ x &= 4 \end{aligned}$$

Putting the value of x in (i), we get

$$\begin{aligned} 4 + y &= 10 \\ y &= 10 - 4 = 6 \\ \text{Original number} &= x + 10y \\ &= 4 + 10(6) \\ &= 64 \end{aligned}$$

9. Let the speed of the train = x km/hr
and the speed of the car = y km/hr
According to the question,

$$\frac{250}{x} + \frac{120}{y} = 4 \quad \text{...(i)}$$

$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\begin{aligned} \frac{130}{x} + \frac{240}{y} &= 4 + \frac{18}{60} \\ &= \frac{43}{10} \quad \text{...(ii)} \end{aligned}$$

Multiplying (i) by 2 and subtracting (ii) from it.

$$\begin{aligned} \frac{500}{x} + \frac{240}{y} &= 8 \\ \frac{130}{x} + \frac{240}{y} &= \frac{43}{10} \\ \hline \frac{370}{x} &= \frac{37}{10} \\ \Rightarrow \quad 37x &= 3700 \\ \Rightarrow \quad x &= 100 \end{aligned}$$

Putting the value of x in (i)

$$\begin{aligned} \frac{250}{100} + \frac{120}{y} &= 4 \\ \frac{120}{y} &= 4 - 2{\cdot}5 \\ &= 1{\cdot}5 \\ 1{\cdot}5y &= 120 \\ \Rightarrow \quad y &= 80 \end{aligned}$$

$\therefore$ Speed of train = 100 km/hr
and Speed of car = 80 km/hr.

10. Let the speed of the train = x km/hr
and the speed of the car = y km/hr
Accroding to the question,

$$\Rightarrow \quad \frac{400}{x} + \frac{200}{y} = \frac{13}{2} \text{ hr.} \quad \text{...(i)}$$

$$\left[\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\frac{200}{x} + \frac{400}{y} = 7 \text{ hrs.} \quad \text{...(ii)}$$

Multiplying (i) by 2 and subtracting (ii) from it

$$\begin{aligned} \frac{800}{x} + \frac{400}{y} &= 13 \\ \frac{200}{x} + \frac{400}{y} &= 7 \\ \hline \frac{600}{x} &= 6 \\ \Rightarrow \quad 6x &= 600 \\ \Rightarrow \quad x &= 100 \end{aligned}$$

Putting the value of x in (ii), we get

$$\begin{aligned} \frac{200}{100} + \frac{400}{y} &= 7 \\ \frac{400}{y} &= 5 \\ 5y &= 400 \\ \Rightarrow \quad y &= \frac{400}{5} \\ \Rightarrow \quad y &= 80 \end{aligned}$$

Speed of the train = 100 km/hr
and Speed of the car = 80 km/hr.

11. Let the fixed hostel charges be Rs. x
and the cost of food per day be Rs. y
According to the question,

$$\begin{aligned} x + 20y &= 1000 \quad \text{...(i)} \\ x + 26y &= 1180 \quad \text{...(ii)} \\ \text{Subtracting,} \quad -6y &= -180 \\ \Rightarrow \quad y &= 30 \end{aligned}$$

Putting the value of y in (i)

$$\begin{aligned} x + 20(30) &= 1000 \\ x + 600 &= 1000 \\ x &= 400 \\ \text{The fixed charge} &= \text{Rs. } 400 \end{aligned}$$

and the cost of food per day = Rs. 30

12. Let ten's digit be = x
and Unit's digit be = y
Number formed = $10x + y$
If digits are reversed, then number formed
= $10y + x$

$$\text{Number} = 4(\text{sum of digits})$$
$$10x + y = 4(x + y)$$
$$10x + y = 4x + 4y$$
$$6x = 3y$$
$$y = 2x \quad \ldots(i)$$

Number + 18 = Number with reversed digits

$$10x + y + 18 = 10y + x$$
$$9x - 9y + 18 = 0$$
$$x - y + 2 = 0 \quad \ldots(ii)$$

From (i) taking the value of y in (ii), we get

$$x - 2x + 2 = 0$$
$$-x = -2$$
$$\Rightarrow \quad x = 2$$
$$y = 2x$$
$$= 2 \times 2 = 4$$

Hence, $\quad \text{Number} = 10 \times 2 + 4 = 24$

13. Let the digit at ten's place $= x$

and the digit at unit's place $= y$

Number formed $= 10x + y$

When digits are reversed, then new number $= 10y + x$

According to the question,

$$10x + y = 3 + 4(x + y)$$
$$10x + y = 3 + 4x + 4y$$
$$\Rightarrow \quad 6x = 3 + 3y$$
$$\Rightarrow \quad 2x = 1 + y$$
$$\Rightarrow \quad y = 2x - 1 \quad \ldots(i)$$

Also, $\quad 10x + y + 18 = 10y + x$

$$\Rightarrow \quad 9x + 18 = 9y$$
$$\Rightarrow \quad x + 2 = y \quad \ldots(ii)$$

Using the value of y in (ii), we get

$$x + 2 = 2x - 1$$
$$2 + 1 = 2x - x$$
$$3 = x$$
$$x = 3$$

Now, $\quad y = x + 2$

$$\Rightarrow \quad y = 3 + 2 = 5$$

So, $\quad$ the number $= 10x + y$

$$= 10 \times 3 + 5 = 30 + 5 = 35$$

14. Let ten's digit $= x$

Unit's digit $= y$

Number formed $= 10x + y$

Sum of digits $= x + y$

The number formed when digits are interchanged

$$= 10y + x$$

According to the conditions—

$$10x + y = 4 + 6(x + y)$$

and $\quad 10x + y - 18 = 10y + x$

or $\quad 4x = 4 + 5y$

and $\quad x - 2 = y$

15. Let the fraction be $= \dfrac{x}{y}$

According to the conditions—

$$y = 2x + 4 \text{ and } y - 6 = 12(x - 6)$$

So, $\quad 2x + 4 - 6 = 12(x - 6)$

(by putting the value of y)

$$\Rightarrow \quad 12x - 2x = 72 - 2 \Rightarrow x(\text{numerator}) = 7$$

Y(denominator) $= 18$ (by putting the value of x)

Hence, the fraction is $\dfrac{7}{18}$.

16. Let the fixed hostel charges be Rs. x

and the cost of food per day be Rs. y.

According to the question—

$$x + 25y = 1750 \quad \ldots(i)$$
$$x + 28y = 1900 \quad \ldots(ii)$$

Subtracting $\quad -3y = -150$

$$\Rightarrow \quad y = 50$$

Putting $\quad y = 50$ in (i), we get

$$x + 25\,(50) = 1750$$
$$x + 1250 = 1750$$
$$x = 500$$

Fixed charges $=$ Rs. 500

and Cost of food $=$ Rs. 50 per day

17. Let fixed charge of taxi = Rs. x

and running charges of taxi = Rs. y per km.

According to the question—

$$x + 10y = 75 \quad \ldots(i)$$
$$x + 15y = 110 \quad \ldots(ii)$$

Subtracting, $\quad -5y = -35$

$$\Rightarrow \quad y = 7$$

Putting the value of y in (i)

$$x + 70 = 75$$
$$x = 5$$

$\therefore$ Person has to pay for traveling a distance of 25 km.

$$= x + 25y$$
$$= 5 + 25\,(7)$$
$$= 5 + 175$$
$$= \text{Rs. } 180$$

18. Let fixed charges of taxi = Rs. x

and running charges of taxi = Rs. y per km

According to the question—

$$x + 12y = 89 \quad \ldots(i)$$
$$x + 20y = 145 \quad \ldots(ii)$$

Subtracting, $-8y = -56$

$$y = 7$$

Putting the value of y in (i)

$$x + 84 = 89$$

$$\Rightarrow \quad x = 5$$

Now, person has to pay for traveling a distance of 30 km.

$$= x + 30y$$
$$= 5 + 30(7)$$
$$= 5 + 210$$
$$= \text{Rs. } 215$$

19. Let fixed charges of taxi = Rs. x

and running charges of taxi = Rs. y per km.

According to the question—

$$x + 10y = 68 \quad \ldots\text{(i)}$$
$$x + 15y = 98 \quad \ldots\text{(ii)}$$

Subtracting, $-5y = -30$

$$y = 6$$

Putting the value of y in (i), we get

$$x + 10y = 68$$
$$x = 8$$

20. Let the monthly rent of the house = Rs. x

and the mess charges per head per month = Rs. y

According to the question—

$$x + 2y = 3900 \quad \ldots\text{(i)}$$
$$x + 5y = 7500 \quad \ldots\text{(ii)}$$

By subtracting, $-3y = -3600$

$$y = 1200$$

Putting the value of y in (i)

$$x + 2400 = 3900$$
$$x = 3900 - 2400$$
$$x = \text{Rs. } 1500$$

$\therefore$ Monthly rent (x) = Rs. 1500

Mess charges per head per month (y) = Rs. 1200

21. Let the fixed charges of taxi per day = Rs. x

and the running expenses of taxi = Rs. y per km.

According to the question—

$$x + 110y = 690 \quad \ldots\text{(i)}$$
$$x + 200y = 1050 \quad \ldots\text{(ii)}$$

Subtracting, $-90y = -360$

$$\Rightarrow \quad y = 4$$

Putting the value of y in (i)

$$x + 110(4) = 690$$
$$x + 440 = 690$$
$$x = 690 - 440 = \text{Rs. } 250$$

$\therefore$ Fixed charges (x) = Rs. 250

and Rate per km (y) = Rs. 4

22. Let unit's place digit = x

and ten's place digit = y

Original Number = $x + 10y$

Reversed Number = $10x + y$

According to the question—

$$x + 10y = 7(x + y)$$
$$\Rightarrow \quad x + 10y = 7x + 7y$$
$$\Rightarrow \quad 10y - 7y = 7x - x$$
$$\Rightarrow \quad 3y = 6x$$
$$\Rightarrow \quad y = 2x \quad \ldots\text{(i)}$$

and $x + 10y - 27 = 10x + y$

$$x + 10y - 10x - y = 27$$
$$-9x + 9y = 27$$
$$\Rightarrow \quad -x + y = +3 \quad \text{(Dividing by 9)}$$
$$\Rightarrow \quad -x + 2x = 3 \quad [\because y = 2x \text{ from (i)}]$$
$$\Rightarrow \quad x = 3$$

Putting $x = 3$ in (i), we get

$$y = 2x$$
$$y = 2 \times 3 = 6$$

$\therefore$ Original number = $3 + 60 = 63$

23. Let unit's place digit = x

and ten's place digit = y

Original Number = $x + 10y$

Reversed Number = $10x + y$

According to the question—

$$\frac{x + 10y}{x + y} = \frac{6}{1}$$
$$\Rightarrow \quad 6x + 6y = x + 10y$$
$$\Rightarrow \quad 6x - x + 6y - 10y = 0$$
$$\Rightarrow \quad 5x - 4y = 0$$
$$\Rightarrow \quad -4y = -5x$$
$$\Rightarrow \quad y = \frac{5x}{4} \quad \ldots\text{(i)}$$
$$x + 10y - 9 = 10x + y$$
$$x + 10y - 10x - y = 9$$
$$\Rightarrow \quad -9x + 9y = 9$$
$$\Rightarrow \quad -x + y = 1 \quad \text{(Dividing by 9)}$$
$$\Rightarrow \quad -x + \frac{5x}{4} = 1 \quad \left[\because y = \frac{5x}{4} \text{ from (i)}\right]$$
$$\Rightarrow \quad x = 4$$

Putting $x = 4$ in (i), we get

$$y = \frac{5x}{4}$$
$$y = \frac{5 \times 4}{4} = 5$$

$\therefore$ Original number = $4 + 10(5)$

$$= 4 + 50 = 54$$

24. Let the number of students in class A = x and in class B = y

According to the Ist condition—

$$x - 5 = y + 5$$

$$\Rightarrow \quad x - y = 10 \quad \ldots(i)$$

According to the IInd condition—

$$x + 5 = 2(y - 5)$$

$$\Rightarrow \quad x - 2y = -15 \quad \ldots(ii)$$

$$(10 + y) - 2y = -15 \quad \text{(by putting } x = 10 + y\text{)}$$

$$y = 25 \text{ and } x = 35$$

In room A original number of students = 35 : in room B original number of students = 25

25. Let length = x units and Breadth = y units

Then area of rectangle = $x\,y$ sq. units

According to the Ist condition—

$$xy - (x - 5)(y + 2) = 80$$

$$5y - 2x = 70 \quad \ldots(i)$$

According to the IInd condition—

$$(x + 10)(y - 5) - xy = 50$$

$$xy - 5x + 10y - 50 - xy = 50$$

$$\text{or} \quad 2y - x = 20 \quad \ldots(ii)$$

Multiplying (ii) by 2

$\Rightarrow \quad 4y - 2x = 40$ and subtract from eq. (i)

We get $\quad y = 30$

Putting $\quad y = 30$ in equation (ii), So, $x = 40$

Hence, length is 40 units and breadth is 30 units.

26. $a = 9, b = 8k, c = 16$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (8k)^2 - 4(9) \times (16) \\ &= 64k^2 - 64 \times 9 \end{aligned}$$

For equal roots. $\quad D = 0$

$$64k^2 - 64 \times 9 = 0$$

$$64k^2 = 64 \times 9$$

$$k^2 = 9 \Rightarrow k = \pm 3$$

27. A = $a - b$, B = $b - c$, C = $c - a$

$$\begin{aligned} D &= B^2 - 4AC \\ &= (b - c)^2 - 4\times (a - b) \times (c - a) \\ &= b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) \\ &= b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab \\ &= b^2 + c^2 + 4a^2 + 2bc - 4ac - 4ab \\ &= (2a - b - c)^2 \end{aligned}$$

For equal roots. D = 0

$$(2a - b - c)^2 = 0$$

$$2a - b - c = 0$$

$$2a = b + c$$

28. $a = k + 4, b = k + 1, c = 1$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (k + 1)^2 - 4 \times (k + 4) \\ &= k^2 + 1 + 2k - 4k - 16 \\ &= k^2 - 2k - 15 \end{aligned}$$

For equal roots. D = 0

$$k^2 - 2k - 15 = 0$$

$$(k - 5)(k + 3) = 0$$

$$k = 5, -3$$

29. $a = 9, b = -k, c = 81$

$$\begin{aligned} D &= b^2 - 4ac \\ &= k^2 - 4 \times 9 \times 81 \\ &= k^2 - 2916 \end{aligned}$$

For equal roots. D = 0

$$k^2 - 2916 = 0$$

$$k = \pm \sqrt{2916}$$

$$k = \pm 54$$

30. $a = 1 + m^2, b = 2mc, c = c^2 - a^2$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (2mc)^2 - 4 \times (1 + m^2)(c^2 - a^2) \\ &= 4m^2c^2 - 4 \times (c^2 - a^2 + m^2c^2 - m^2a^2) \\ &= 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 \\ &= 4a^2 + 4m^2a^2 - 4c^2 \end{aligned}$$

For equal roots. D = 0

$$\Rightarrow \quad 4a^2 + 4m^2a^2 - 4c^2 = 0$$

$$\Rightarrow \quad a^2 + m^2a^2 - c^2 = 0 \quad \text{[Dividing by 4]}$$

$$\Rightarrow \quad a^2 + m^2a^2 = c^2$$

$$\Rightarrow \quad a^2(1 + m^2) = c^2$$

31. $a = 4, b = -2(c + 1), c = c + 4$

$$\begin{aligned} D &= b^2 - 4ac \\ &= [-2(c + 1)]^2 - 4 \times 4 \times (c + 4) \\ &= 4(c^2 + 1 + 2c) - 16c - 64 \\ &= 4c^2 + 4 + 8c - 16c - 64 \\ &= 4c^2 - 8c - 60 = 4(c^2 - 2c - 15) \end{aligned}$$

For equal roots. D = 0

$$4(c^2 - 2c - 15) = 0$$

$$(c - 5)(c + 3) = 0$$

$$c = 5, -3$$

32. $a = 1, b = -1, c = 2$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-1)^2 - 4 \times 1 \times 2 \\ &= 1 - 8 \\ &= -7 \end{aligned}$$

As $D < 0$, Roots are not real.

33. $$2x^2 + ax + 3 = 0 \quad \ldots(i)$$

As '1' is a root, therefore it must satisfies the equation.

$$2(1)^2 + a(1) + 3 = 0$$
$$a + 5 = 0$$
$$a = -5$$

Put $a = -5$ in (1)

$$2x^2 - 5x + 3 = 0$$
$$2x^2 - 3x - 2x + 3 = 0$$
$$x(2x-3) - 1(2x-3) = 0$$
$$(x-1)(2x-3) = 0$$
$$x = 1, \frac{3}{2}$$

Other root is $\frac{3}{2}$.

34. Here, $a = \alpha - 3$, $b = 4(\alpha - 3)$.

The quadratic equation will have equal roots.

If $b^2 = 4ac$

$\therefore$ $[4(\alpha-3)^2] = 4(\alpha-3)\times 4$

$\Rightarrow$ $(4)^2 \times (\alpha-3)^2 = (4)^2 \times (\alpha - 3)$

$\Rightarrow$ $(\alpha-3)^2 = \alpha - 3$

$\Rightarrow$ $(\alpha-3)^2 - (\alpha-3) = 0$

$\Rightarrow$ $(\alpha-3)(\alpha-3-1) = 0$

$\Rightarrow$ $(\alpha-3)(\alpha-4) = 0$

$\Rightarrow$ $\alpha = 3$

or $\alpha = 4$

But $\alpha = 3$ is not possible since it reduces the equation to a constant. Hence, $\alpha = 4$.

35. Equation is $x^2 - 2x(1+3k) + 7(3+2k) = 0$

Here, $a = 1$,

$b = -2(1+3k)$,

$c = 7(3+2k)$

Given that the equation has equal roots.

$\therefore$ $b^2 - 4ac = 0$

$\Rightarrow$ $4(1+3k)^2 - 4\times 1 \times 7 \times (3+2k) = 0$

$\Rightarrow$ $4(1+9k^2+6k) - 28(3+2k) = 0$

$\Rightarrow$ $4 + 36k^2 - 24k - 84 - 56k = 0$

$\Rightarrow$ $36k^2 - 32k - 80 = 0$

$\Rightarrow$ $9k^2 - 8k - 20 = 0$ [Dividing by 4]

$\Rightarrow$ $9k^2 - 18k + 10k - 20 = 0$

$\Rightarrow$ $9k(k-2) + 10(k-2) = 0$

$\Rightarrow$ $(9k+10)(k-2) = 0$

$\Rightarrow$ $9k = -10$

or $k = 2$

$\Rightarrow$ $k = \frac{-10}{9}$

or, $k = 2$

36. Here, $a = 1$, $b = 4p$, $c = p^2 - p + 2$

$$D = b^2 - 4ac$$
$$= (4p)^2 - 4 \times (p^2 - p + 2)$$
$$= 16p^2 - 4p^2 + 4p - 8$$
$$= 12p^2 + 4p - 8$$

For equal roots. D = 0

$$12p^2 + 4p - 8 = 0$$
$$12p^2 + 12p - 8p - 8 = 0$$
$$12p(p+1) - 8(p+1) = 0$$
$$(12p-8)(p+1) = 0$$
$$p = -1, \frac{2}{3}$$

37. The given equation is $2x^2 - 8x - m = 0$.

As $\frac{5}{2}$ is a root of the equation, therefore it must satisfies the equation.

$\Rightarrow$ $2\times\left(\frac{5}{2}\right)^2 - 8\times\left(\frac{5}{2}\right) - m = 0$

$\Rightarrow$ $\frac{25}{2} - 20 - m = 0$

$\Rightarrow$ $25 - 40 - 2m = 0$

$\Rightarrow$ $-15 - 2m = 0$

$\Rightarrow$ $-2m = 15$

$m = \frac{-15}{2}$

The equation is $2x^2 - 8x + \frac{15}{2}$

$$4x^2 - 16x + 15 = 0$$
$$4x^2 - 10x - 6x + 15 = 0$$
$$2x(2x-5) - 3(2x-5) = 0$$
$$(2x-3)(2x-5) = 0$$
$$x = \frac{3}{2}, \frac{5}{2}$$

Other root is $\frac{3}{2}$.

38. Here, $a = 2k$, $b = -40$, $c = 25$

$$D = b^2 - 4ac$$
$$= (-40)^2 - 4 \times 2k \times 25$$
$$= 1600 - 200k$$

For equal roots. $D = 0$

$$1600 - 200k = 0$$
$$k = 8$$

$\therefore$ Equation is $16x^2 - 40x + 25 = 0$

$$(4x-5)^2 = 0$$
$$x = \frac{5}{4}, \frac{5}{4}$$

Roots are $\frac{5}{4}, \frac{5}{4}$.

39. $a = 9, b = -24, c = k$

$$D = (-24)^2 - 4 \times 9 \times k$$
$$= 576 - 36k$$

For equal roots. D = 0

$$576 - 36k = 0$$
$$k = 16$$

Equation is $9x^2 - 24x + 16 = 0$

$$(3x - 4)^2 = 0$$
$$x = \frac{4}{3}, \frac{4}{3}$$

Roots are $\frac{4}{3}, \frac{4}{3}$.

40. Let the original price of book = Rs. x

No. of books in Rs. 300 = $\frac{300}{x}$

Reduced price of book = Rs. $(x - 5)$

Then, No. of books = $\frac{300}{x-5}$

Given $\frac{300}{x-5} - \frac{300}{x} = 5$

$\Rightarrow \frac{300x - 300(x-5)}{x(x-5)} = 5$

$\Rightarrow \frac{300x - 300x + 1500}{x(x-5)} = 5$

$$1500 = 5(x^2 - 5x)$$
$$x^2 - 5x - 300 = 0$$
$$x^2 - 20x + 15x - 300 = 0$$
$$(x - 20)(x + 15) = 0$$
$$x = 20, -15$$

Price can never be – ve

Price = Rs. 20.

41. Let the original price of the toy = Rs. x

No. of toys in Rs. 360 = $\frac{360}{x}$

Reduced price of toy = Rs. $(x - 2)$

Then, No. of toys = $\frac{360}{x-2}$

Given $\frac{360}{x-2} - \frac{360}{x} = 2$

$\Rightarrow \frac{360x - 360(x-2)}{x(x-2)} = 2$

$$360x - 360x + 720 = 2x^2 - 4x$$
$$x^2 - 2x - 360 = 0$$
$$(x + 18)(x - 20) = 0$$
$$x = -18, 20$$

Price can never be – ve

Original price of the toy = Rs. 20.

42. Let the numbers be x and y.

$x - y = 3$ ...(i)

and $xy = 504$ ...(ii)

From (i), $x = 3 + y$ Put in (ii)

$$(3 + y)y = 504$$
$$y^2 + 3y - 504 = 0$$
$$y^2 + 24y - 21y - 504 = 0$$
$$(y - 21)(y + 24) = 0$$
$$y = 21, -24$$

When $y = 21, x = 24$ and

When $y = -24, x = -21$

43. Let the numbers be x and y (Let $x > y$).

$x - y = 4$ ...(i)

and $xy = 192$ ...(ii)

From (i) $x = 4 + y$

Put $x = (4 + y)$ in (ii)

$$(4 + y)y = 192$$
$$y^2 + 4y - 192 = 0$$
$$y^2 + 16y - 12y - 192 = 0$$
$$(y + 16)(y - 12) = 0$$
$$y = -16, 12$$

When $y = -16, x = -12$

and $y = 12, x = 16$

44. Let the numbers be x and y.

Given $x - y = 2$ ...(i)

and $xy = 360$ ...(ii)

From (i) $x = 2 + y$

Put $x = (2 + y)$ in (ii), we get

$$(2 + y)y = 360$$
$$2y + y^2 = 360$$
$$y^2 + 2y - 360 = 0$$
$$(y + 20)(y - 18) = 0$$
$$y = 18, -20$$

When $y = 18, x = 20$

When $y = -20, -18$

45. Let two consecutive numbers be x and $x + 1$.

According to the question—

$$x^2 + (x + 1)^2 = 85$$
$$x^2 + x^2 + 2x + 1 - 85 = 0$$
$$2x^2 + 2x - 84 = 0$$

$x^2 + x - 42 = 0$ [Dividing by 2]

$$x^2 + 7x - 6x - 42 = 0$$
$$x(x + 7) - 6(x + 7) = 0$$
$$(x - 6)(x + 7) = 0$$
$$x - 6 = 0$$

or $x + 7 = 0$

$x = 6$

or $x = -7$

When $x = 6$, numbers are 6 and 7

When $x = -7$, numbers are -7 and -6

46. Let two numbers be x and $(15 - x)$.

According to the question—

$$\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10}$$

$$\Rightarrow \frac{15 - x + x}{x(15-x)} = \frac{3}{10}$$

$$\Rightarrow 150 = 3x(15 - x)$$

$$\Rightarrow 150 = 45x - 3x^2$$

$$\Rightarrow 3x^2 - 45x + 150 = 0$$

$$\Rightarrow x^2 - 15x + 50 = 0 \quad \text{[Dividing by 3]}$$

$$\Rightarrow x^2 - 5x - 10x + 50 = 0$$

$$\Rightarrow x(x - 5) - 10(x - 5) = 0$$

$$\Rightarrow (x - 5)(x - 10) = 0$$

$$\therefore x = 5 \text{ or } x = 10$$

When $x = 5$, numbers are 5 and 10

When $x = 10$, numbers are 10 and 5.

47. Let original number of persons = x

The increased number of persons = $x + 20$

Total Amount = Rs. 9,000

According to the question—

$$\frac{9000}{x} - \frac{9000}{x+20} = 160 \quad \left[\text{Each got} = \frac{\text{Total Amount}}{\text{No. of Persons}}\right]$$

$$\frac{9000x + 180000 - 9000x}{(x+20)} = \frac{160}{1}$$

$$\Rightarrow 160(x + 20) = 180000$$

$$\Rightarrow x(x + 20) = 1125$$

[Dividing by 160]

$$\Rightarrow x^2 + 20x - 1125 = 0$$

$$\Rightarrow x^2 + 45x - 25x - 1125 = 0$$

$$\Rightarrow x(x + 45) - 25(x + 45) = 0$$

$$\Rightarrow (x - 25)(x + 45) = 0$$

$$\Rightarrow x = 25$$

or $x = -45$

But number of persons can't be – ve

$\therefore$ The original number of persons = 25

48. Let the usual speed of plane = x km/hr

The increased speed of plane = $(x + 250)$ km/hr

Distance = 1500 km

According to the question—

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$$\left[\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\frac{1500x + 375000 - 1500x}{x(x+250)} = \frac{1}{2}$$

$$\Rightarrow x(x + 250) = 750000$$

$$\Rightarrow x^2 + 250x - 750000 = 0$$

$$\Rightarrow x^2 + 1000x - 750x - 750000 = 0$$

$$\Rightarrow x(x + 1000) - 750(x + 1000) = 0$$

$$\Rightarrow (x - 750)(x + 1000) = 0$$

$$\Rightarrow x = 750 \text{ or } x = -1000$$

But speed can never be – ve

Usual speed = 750 km/hr.

49. Let length of base = x cm

Hypotenuse = $(x + 2)$ cm

Altitude = $(x + 1)$ cm

According to Pyth. Theorem

$$P^2 + B^2 = H^2$$

$$x^2 + (x + 1)^2 = (x + 2)^2$$

$$x^2 + x^2 + 1 + 2x = x^2 + 4 + 4x$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3, -1$$

Length can never be – ve.

$x = 3$ cm

Here, Base = 3 cm

Altitude = $(x + 1) = (3 + 1) = 4$ cm

Hypotenuse = 5 cm.

50. Let the no. of students who attended picnic = x

Nos of students who planned picnic = $(x + 5)$

Total Budget = Rs. 500

According to the question—

$$\frac{500}{x} - \frac{500}{x+5} = \text{Rs. } 5$$

$$\left[\text{Each contribution} = \frac{\text{Total Budget}}{\text{No. of Students}}\right]$$

$$\frac{500x + 2500 - 500x}{x(x+5)} = 5$$

$$\Rightarrow 5x(x + 5) = 2500$$

$$\Rightarrow 5x^2 + 25x - 2500 = 0$$

$$\Rightarrow x^2 + 5x - 500 = 0 \quad \text{[Dividing by 5]}$$

$$\Rightarrow x^2 + 25x - 20x - 500 = 0$$

$$\Rightarrow x(x + 25) - 20(x + 25) = 0$$

$$\Rightarrow (x + 25)(x - 20) = 0$$

$$\Rightarrow x + 25 = 0$$

or $x - 20 = 0$

$$\Rightarrow x = -25$$

or $x = 20$

But number of students cannot be – ve.

$\therefore x = 20$

Nos. of students who attended picnic = 20

51. Let the two pipes fill the cistern = x and $x + 5$ minutes.

Then, $$\frac{1}{x}+\frac{1}{x+5} = \frac{1}{6}$$

$\Rightarrow$ $6(x + 5) + 6x = x(x + 5)$

$\Rightarrow$ $6x + 30 + 6x = x^2 + 5x$

$\Rightarrow$ $x^2 + 5x - 6x - 30 - 6x = 0$

$\Rightarrow$ $x^2 - 7x - 30 = 0$

$\Rightarrow$ $x^2 - 10x + 3x - 30 = 0$

$\Rightarrow$ $x(x - 10) + 3(x - 10) = 0$

$\Rightarrow$ $(x - 10)(x + 3) = 0$

$\Rightarrow$ $x = 10$ or $x = -3$

Since, time cannot be – ve.

$\therefore$ The pipes would fill the cistern in 10 and 10 + 5 *i.e.*, 15 minutes.

52. Let the time taken by one pipe = x minutes

Time taken by other pipe = $x + 1$ minutes

Then, $$\frac{1}{x}+\frac{1}{x+1} = \frac{11}{30} \qquad \left[\because 2\frac{8}{11}=\frac{30}{11}\right]$$

$\Rightarrow$ $30(x + 1) + 30x = 11x(x + 1)$

$\Rightarrow$ $30x + 30 + 30x = 11x^2 + 11x$

$\Rightarrow$ $11x^2 - 30x - 30x + 11x - 30 = 0$

$\Rightarrow$ $11x^2 - 49x - 30 = 0$

$$\Rightarrow \quad x = \frac{49 \pm \sqrt{(49)^2 + 4 \times 11 \times 30}}{22}$$

$$= \frac{49 \pm \sqrt{2401 + 1320}}{22} = \frac{49 \pm \sqrt{3721}}{22}$$

$$= \frac{49 \pm 61}{22} = \frac{110}{22} \text{ or } -\frac{12}{22} = 5 \text{ or } -\frac{6}{11}$$

Since, time cannot be – ve.

$\therefore$ Time taken by Ist pipe = 5 minutes

and time taken by IInd pipe = 5 + 1 = 6 minutes

53. Let the two pipes be A and B.

Let the time taken by pipe A = x minutes

and time taken by pipe B to fill = $x + 3$ minutes

As per the question,

$$\frac{1}{x}+\frac{1}{x+3} = \frac{13}{40} \qquad \left[\because 3\frac{1}{13}=\frac{40}{13}\right]$$

$\Rightarrow$ $40(x + 3) + 40x = 13x(x + 3)$

$\Rightarrow$ $40x + 120 + 40x = 13x^2 + 39x$

$\Rightarrow$ $13x^2 + 39x - 40x - 120 - 40x = 0$

$\Rightarrow$ $13x^2 - 41x - 120 = 0$

$$\Rightarrow \quad x = \frac{41 \pm \sqrt{(41)^2 + 4 \times 13 \times 120}}{26}$$

$$= \frac{41 \pm \sqrt{1681 + 6240}}{26}$$

$$= \frac{41 \pm \sqrt{7921}}{26}$$

$$= \frac{41 \pm 89}{26}$$

$$= \frac{130}{26} \text{ or } \frac{48}{26}$$

$$= 5 \text{ or } -\frac{24}{13}$$

Since, time cannot be – ve.

$\therefore$ $x = 5$ minutes

$\therefore$ Time taken by Ist pipe = 5 minutes

and time taken by IInd pipe = 5 + 3 = 8 minutes

54. Let usual speed of aircraft = x km/hr

Reduced speed of aircraft = $(x - 100)$ km/hr

Distance = 2800 km

According to the question—

$$\frac{2800}{x-100} - \frac{2800}{x} = 30 \text{ minutes} \qquad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\frac{2800x - 2800x + 280000}{x(x-100)} = \frac{1}{2} \text{ hr.}$$

$\Rightarrow$ $x^2 - 100x - 560000 = 0$

$\Rightarrow$ $x^2 - 800x + 700x - 560000 = 0$

$\Rightarrow$ $x(x - 800) + 700(x - 800) = 0$

$\Rightarrow$ $(x - 800)(x + 700) = 0$

$\Rightarrow$ $x = 800$ or $x = -700$

$\because$ Speed can never be – ve.

$\because$ Speed = 800 km/hr

$$\text{Time} = \frac{D}{S} = \frac{2800}{800} = \frac{7}{2} = 3\frac{1}{2} \text{ hours.}$$

55. Let usual speed of aircraft = x km/hr

Reduced speed of aircraft = $(x - 100)$ km/hr

Distance = 3,000 km

According to the question—

$$\frac{3000}{x-100} - \frac{3000}{x} = 1 \text{ hr} \qquad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\frac{3000x - 3000x + 300000}{x(x-100)} = \frac{1}{1}$$

$\Rightarrow$ $x(x - 100) = 300000$

$\Rightarrow$ $x^2 - 100x - 300000 = 0$

$$\Rightarrow \quad x^2 - 600x + 500x - 300000 = 0$$
$$\Rightarrow \quad x(x - 600) + 500(x - 600) = 0$$
$$\Rightarrow \quad (x - 600)(x + 500) = 0$$
$$\Rightarrow \quad x = 600$$
$$\text{or} \quad x = -500$$

$\therefore$ Speed can never be – ve.

$\therefore$ Speed = 600 km/hr

$$\text{Time} = \frac{D}{S} = \frac{3000}{600} = 5 \text{ hours.}$$

56. Let usual speed of aircraft = x km/hr

Reduced speed of aircraft = $(x - 400)$ km/hr

Distance = 6,000 km

According to the question—

$$\frac{6000}{x-400} - \frac{6000}{x} = 30 \text{ minutes}$$
$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$
$$\frac{6000x - 6000x + 2400000}{x(x-400)} = \frac{1}{2}\text{hr}$$
$$\Rightarrow \quad x^2 - 400x = 4800000$$
$$\Rightarrow \quad x^2 - 400x - 4800000 = 0$$
$$\Rightarrow \quad x^2 - 2400x + 2000x - 4800000 = 0$$
$$\Rightarrow \quad x(x - 2400) + 2000(x - 2400) = 0$$
$$\Rightarrow \quad (x - 2400)(x + 2000) = 0$$
$$\Rightarrow \quad x = 2400 \text{ or } x = -2000$$

$\therefore$ Speed can never be – ve.

$\therefore$ Speed = 2400 km/hr

$$\text{Time} = \frac{D}{S} = \frac{6000}{2400} = \frac{5}{2} = 2\frac{1}{2} \text{ hours.}$$

57. Let width of the path = x m

$$\text{Area of outer rectangle} = (16 + 2x)(10 + 2x)$$
$$= 160 + 32x + 20x + 4x^2$$
$$= 4x^2 + 52x + 160$$
$$\text{Area of the inner rectangle} = 16 \times 10 = 160 \text{ m}^2$$
$$\text{Area of the path} = 4x^2 + 52x + 160 - 160$$
$$= 120$$
$$\Rightarrow \quad 4x^2 + 52x - 120 = 0$$
$$\Rightarrow \quad x^2 + 13x - 30 = 0$$
$$\Rightarrow \quad x^2 + 15x - 2x - 30 = 0$$
$$\Rightarrow \quad (x + 15)(x - 2) = 0$$
$$\Rightarrow \quad x = -15 \text{ or } x = 2$$

Since, the width cannot be negative.

Hence, width of the path = 2 m.

58. From Δ XZY, we get XZ = $9 + r$ cm

ZY = $2 + r$ cm

$$\text{Now,} \quad XZ^2 + ZY^2 = XY^2$$
$$\Rightarrow \quad (9 + r)^2 + (2 + r)^2 = (17)^2$$
$$\Rightarrow \quad 81 + r^2 + 18r + 4 + r^2 + 4r = 289$$
$$\Rightarrow \quad 2r^2 + 22r + 85 = 289$$
$$\Rightarrow \quad 2r^2 + 22r = 289 - 85 = 204$$
$$\Rightarrow \quad r^2 + 11r = 102$$
$$\Rightarrow \quad r^2 + 11r - 102 = 0$$
$$\Rightarrow \quad r^2 - 6r + 17r - 102 = 0$$
$$\Rightarrow \quad r(r - 6) + 17(r - 6) = 0$$
$$\Rightarrow \quad (r - 6)(r + 17) = 0$$
$$\Rightarrow \quad r = 6 \text{ or } r = -17$$

Rejecting the – ve value, we get

$$r = 6 \text{ cm.}$$

59. Let usual number of days of tour = x

After increasing number of days = $(x + 4)$

Total amount to be spent = Rs. 360

According to the question—

$$\frac{360}{x} - \frac{360}{x+4} = \text{Rs. } 3$$
$$\frac{360x + 1440 - 360x}{x(x+4)} = \frac{3}{1}$$
$$\Rightarrow \quad 1440 = 3x^2 + 12x$$
$$\Rightarrow \quad 3x^2 + 12x - 1440 = 0$$
$$\Rightarrow \quad x^2 + 4x - 480 = 0 \quad \text{[Dividing by 3]}$$
$$\Rightarrow \quad x^2 + 24x - 20x - 480 = 0$$
$$\Rightarrow \quad x(x + 24) - 20(x + 24) = 0$$
$$\Rightarrow \quad (x + 24)(x - 20) = 0$$
$$\Rightarrow \quad x = -24 \text{ or } x = 20$$

But the number of days cannot be – ve.

Original duration of tour = 20 days.

60. Let the length of piece of cloth = x m

Increased length of piece of cloth = $(x + 5)$m

Total cost = Rs. 200

According to the question—

$$\frac{200}{x} - \frac{200}{x+5} = \text{Rs. } 2$$
$$\left[\because \text{Rate per metre} = \frac{\text{Total Cost}}{\text{Length}}\right]$$
$$\frac{200x + 1000 - 200x}{x(x+5)} = \text{Rs. } 2$$
$$\Rightarrow \quad 1000 = 2x^2 + 10x$$
$$\Rightarrow \quad 2x^2 + 10x - 1000 = 0$$
$$\Rightarrow \quad x^2 + 5x - 500 = 0 \quad \text{[Dividing by 2]}$$
$$\Rightarrow \quad x^2 + 25x - 20x - 500 = 0$$
$$\Rightarrow \quad x(x + 25) - 20(x + 25) = 0$$
$$\Rightarrow \quad (x + 25)(x - 20) = 0$$
$$\Rightarrow \quad x = -25 \text{ or } x = 20$$

Length of can never be – ve.

$\therefore$ Length of cloth $=$ 20 metre

and Rate per metre $= \frac{200}{20} =$ Rs. 10

Exercise B

1. Let the speed of the train $= x$ km/hr

Speed of the car $= y$ km/hr

Total Distance $=$ 760 km

Distance travelled by train $=$ 160 km

Distance travelled by car $= 760 - 160 = 600$ km

According to the question—

$$\frac{160}{x} + \frac{600}{y} = 8 \text{ hours} \quad \ldots(i)$$

and

$$\frac{240}{x} + \frac{520}{y} = 8\frac{1}{5} = \frac{41}{5} \text{ hours} \quad \ldots(ii)$$

Multiplying (i) by 3 and (ii) by 2, we get

$$\frac{480}{x} + \frac{1800}{y} = 24 \quad \ldots(iii)$$

$$\frac{480}{x} + \frac{1040}{y} = \frac{82}{5} \quad \ldots(iv)$$

Subtracting (iv) from (iii), we get

$$\frac{760}{y} = \frac{38}{5}$$

$\Rightarrow$ $38y = 760 \times 5$

$\Rightarrow$ $y = \frac{760 \times 5}{38} = 100$ km/hr

Substituting the value of y in (i), we get

$$\frac{160}{x} + \frac{600}{100} = 8$$

$\Rightarrow$ $\frac{160}{x} + 6 = 8$

$\Rightarrow$ $\frac{160}{x} = 8 - 6$

$\Rightarrow$ $\frac{160}{x} = 2$

$\Rightarrow$ $2x = 160$

$\Rightarrow$ $x = 80$ km/hr

Speed of the train $=$ 80 km/hr

and Speed of the car $=$ 100 km/hr.

2. Let invested amount @ 12% be Rs. x and invested amount @ 10% be Rs. y.

$$\text{Interest} = \frac{p \times r \times t}{100}$$

$$\text{The yearly interest} = \frac{x \times 12 \times 1}{100} + \frac{y \times 10 \times 1}{100} = 130$$

$$\frac{12x}{100} + \frac{10y}{100} = 130$$

$\Rightarrow$ $12x + 10y = 13000$

$\Rightarrow$ $6x + 5y = 6500 \quad \ldots(i)$

If the amounts invested are interchanged, then yearly interest

$$= \frac{10x}{100} + \frac{12y}{100} = 134$$

$\Rightarrow$ $10x + 12y = 13400$

or $5x + 6y = 6700 \quad \ldots(ii)$

Multiplying (i) by 5 and (ii) by 6, we get

$$30x + 25y = 32500 \quad \ldots(iii)$$

$$30x + 36y = 40200 \quad \ldots(iv)$$

Subtracting (iii) from (iv), we get

$11y = 7700$

$\Rightarrow$ $y = 700$

Putting $y = 700$ in (i), we get

$6x + 5 \times 700 = 6500$

$\Rightarrow$ $x = 500$

3. Let the constant expenditure = Rs. x

and consumption of wheat = Rs. y quintals.

Then, total expenditure = $x + y \times$ Rate per quintal

$$1000 = x + 250y \quad \ldots(i)$$

and $980 = x + 240y \quad \ldots(ii)$

Subtracting, $20 = 10y$

$\Rightarrow$ $y = 2$

Putting $y = 2$ in (i), we get

$1000 = x + 2 \times 250$

$\Rightarrow$ $1000 = x + 500$

$\Rightarrow$ $x = 1000 - 500 = 500$

Total expenses when the price of wheat is Rs. 350 per quintal $= 500 + 350 \times 2 = 500 + 700 =$ Rs. 1200

4. Let the fraction be $\frac{x}{y}$.

Then, $x + y = 18 \quad \ldots(i)$

and by IInd condition $\frac{x}{y+2} = \frac{1}{3} \quad \ldots(ii)$

$y + 2 = 3x$

$\Rightarrow$ $y = 3x - 2$

Putting value of y in equation (i), we get

$x + 3x - 2 = 18$

$\Rightarrow$ $x = 5$

Putting $x = 5$ in in equation (i), we get

$y = 13$

Hence, required fraction is $\frac{5}{13}$.

5. Let the cost of a table be Rs. x and that of a chair be Rs. y.

Then, $2x + 3y = 2000$ …(i)

and $3x + 2y = 2500$ …(ii)

Multiplying (i) by 3 and (ii) by 2, we get

$$\begin{aligned} 6x + 9y &= 6000 \\ 6x + 4y &= 5000 \\ - \quad - \quad & \quad - \\ \hline 5y &= 1000 \end{aligned}$$

$\Rightarrow \quad y = 200$

Putting value of y in equation (i), we get

$$2x + 600 = 2000$$

$\Rightarrow \quad x = 700$

and hence the value of one table ($1x$) and 5 chair ($5y$) is Rs 1700.

6.

	Father	Son
Let the present age of	x years	y years
5 years ago,	$(x - 5)$ years	$(y - 5)$ years
After 5 years,	$(x + 5)$ years	$(y + 5)$ years

As per the question

After 5 years— $(x + 5) = 3(y + 5)$

$\Rightarrow \quad x + 5 = 3y + 15$

$$x - 3y - 15 + 5 = 0$$

$$x - 3y - 10 = 0 \quad \text{…(i)}$$

5 Years ago— $(x - 5) = 7\,(y - 5)$

$$x - 5 = 7y - 35$$

$$x - 7y + 35 - 5 = 0$$

$$x - 7y + 30 = 0 \quad \text{…(ii)}$$

Subtracting (ii) from (i)

$$x - 3y = 10 \quad \text{…(i)}$$

$$x - 7y = -30 \quad \text{…(ii)}$$

$$4y = 40$$

$$y = 10$$

Putting the value of y in (i), we get

$$x - 3\,(10) = 10$$

$$x - 30 = 10$$

$$x = 40$$

$\therefore$ Present age of the father $=$ 40 years

and Present age of the son $=$ 10 years

7. Let the number of oranges with A be x

and number of oranges with B be y.

Then, $x + 10 = 2(y - 10)$ …(i)

and $x - 10 = y + 10$ …(ii)

From (i),

$$2y - 20 - x - 10 = 0$$

$$2y - x - 30 = 0 \quad \text{…(iii)}$$

From (ii),

$$x - 10 - y - 10 = 0$$

$$x - y - 20 = 0 \quad \text{…(iv)}$$

Adding (iii) and (iv), we get

$$2y - x - 30 = 0$$

$$-y + x - 20 = 0$$

$\Rightarrow \quad y = 50$

Putting the value of y in (iv), we get

$$x - 50 - 20 = 0$$

$$x - 70 = 0$$

$$x = 70$$

Number of oranges with A = 70

and Number of oranges with B = 50

8. Let the speed of the train $= x$ km/hr

and speed of the bus $= y$ km/hr

Total distance $=$ 300 km

Distance travelled by train $=$ 60 km

$\therefore$ Distance travelled by bus $= 300 - 60 = 240$ km

$$\frac{60}{x} + \frac{240}{y} = 4 \text{ hours} \quad \text{…(i)}$$

II-Distance travelled by train $=$ 100 km

$\therefore$ Distance travelled by bus $= 300 - 100$

$= 200$ km

$$\therefore \quad \frac{100}{x} + \frac{200}{y} = 4 \text{ hours} + \frac{10}{60} \text{ hrs}$$

$$= \frac{25}{6} \text{ hr} \quad \text{…(ii)}$$

Multiplying (i) by 5 and (ii) by 3, we get

$$\frac{300}{x} + \frac{1200}{y} = 20 \quad \text{…(iii)}$$

$$\frac{300}{x} + \frac{600}{y} = \frac{75}{6} \quad \text{…(iv)}$$

By subtracting (iv) from (iii), we get

$$\frac{600}{y} = \frac{45}{6}$$

$$45y = 3600$$

$\Rightarrow \quad y = 80$

Using the value of y in (i), we get

$$\frac{60}{x} + \frac{240}{80} = 4$$

$$\frac{60}{x} = 4 - 3$$

$$\frac{60}{x} = 1$$

$\Rightarrow \quad x = 60$

$\therefore$ Speed of the train $=$ 60 km/hr

and Speed of the bus $=$ 80 km/hr.

9.

	Father	Son
Let the present age of	x years	y years
10 years ago,	$(x-10)$ years	$(y-10)$ years
After 10 years,	$(x+10)$ years	$(y+10)$ years

As per the question,

Ist condition,

$$x - 10 = 12(y - 10)$$

$$\Rightarrow \quad x - 10 = 12y - 120$$

$$\Rightarrow \quad x - 12y = -120 + 10 = -110 \qquad \ldots\text{(i)}$$

IInd condition,

$$x + 10 = 2(y + 10)$$

$$x + 10 = 2y + 20$$

$$x - 2y = 20 - 10 = 10 \qquad \ldots\text{(ii)}$$

Subtracting (ii) from (i), we get

$$x - 12y = -110 \qquad \ldots\text{(i)}$$

$$x - 2y = 10 \qquad \ldots\text{(ii)}$$

$$-10y = -120$$

$$\Rightarrow \quad y = 12$$

Now, Putting the value of y in (i), we get

$$x - 2(12) = 10$$

$$\Rightarrow \quad x - 24 = 10$$

$$\Rightarrow \quad x = 24 + 10 = 34$$

$\therefore$ Father's age = 34 years and Son's age = 12 years

10.

	Father	Son
Let the present age of	x years	y years
After 3 years,	$(x+3)$ years	$(y+3)$ years

As per the question,

$$x = 3(y) + 3$$

$$x = 3y + 3 \qquad \ldots\text{(i)}$$

According to the question—

$$x + 3 = 2(y + 3) + 10$$

$$x + 3 = 2y + 6 + 10$$

$$x - 2y = 6 + 10 - 3$$

$$x - 2y = 13 \qquad \ldots\text{(ii)}$$

Putting x value in equation (ii), we get

$$3y + 3 - 2y = 13$$

$$\Rightarrow \quad y = 10$$

Putting this y value in equation (i), we get

$$x = 33$$

11. Multiplying both sides of equation (i) by 52, the L.C.M. of 13 and 4, we get

$$8(2x + 3y) = 156 + 13(x - y)$$

$$16x + 24y = 156 + 13x - 13y$$

$$\text{or} \quad 3x + 37y = 156$$

Multiplying both sides of equation (ii) by 6, the L.C.M. of 3 and 6, we get

$$8y + 10x = 12x + 43$$

$$\text{or} \quad -2x + 8y = 43$$

12. Multiplying both sides of equation (ii) by $(a^2 - b^2)$, we get

$$ax\{(a + b) - (a - b)\} + by\{-(a + b) - (a - b)\} = 2a(a - b)$$

$$\text{or} \quad ax(2b) + by(-2a) = 2a(a - b)$$

$$\text{or} \quad bx - by = a - b$$

13. $(2x - 3y - 13)^2 + (3x + 5y + 9)^2 = 0$

$$\Rightarrow \quad 2x - 3y - 13 = 0$$

$$3x + 5y + 9 = 0$$

($\because$ For any real numbers a and $b - a^2 + b^2 = 0 \Rightarrow a = 0$ and $b = 0$)

14. Let $2^x = u$ and $3^y = v$

Then, $u + v = 17$ and $4u - 3v = 5$

15.
$$\frac{x + y - 8}{2} = \frac{x + 2y - 14}{3} = \frac{3x + y - 12}{11}$$

$$\frac{x + y - 8}{2} = \frac{x + 2y - 14}{3}$$

$$\text{and} \quad \frac{x + 2y - 14}{3} = \frac{3x + y - 12}{11}$$

$$\Rightarrow \quad 3x + 3y - 24 = 2x + 4y - 28$$

$$\text{and} \quad 11x + 22y - 154 = 9x + 3y - 36$$

$$\Rightarrow \quad x - y = -4$$

$$\text{and} \quad 2x + 19y = 118$$

16. Let the weights per c.c. be A and B, then

$$\frac{3}{5}\text{A} + \frac{2}{5}\text{B} = 1{\cdot}04,$$

$$\frac{5}{8}\text{A} + \frac{3}{8}\text{B} = 1{\cdot}05$$

17. Let the present age of A be x years and the present age of B be y years.

$\therefore$ According to the question, we have

$$x + y = 64 \qquad \ldots\text{(i)}$$

$$\text{and} \quad [x - (x - y)] = 3[y - (x - y)]$$

$$\textit{i.e.,} \quad y = 3(2y - x)$$

$$\Rightarrow \quad 3x - 5y = 0 \qquad \ldots\text{(ii)}$$

Multiplying (i) by 3, we get

$$3x + 3y = 192 \qquad \ldots\text{(iii)}$$

Subtracting (ii) from (iii), we get

$$8y = 192$$

$$\Rightarrow \quad y = 192 \div 8 = 24$$

Putting $y = 24$ in (i), we get

$$x + 24 = 64$$

$$x = 64 - 24 = 40$$

Hence, the present age of A is 40 years and the present age of B is 24 years.

18. Let the original speed be x km/hr and the distance between two cities A and B by 'y' km.

Now, Distance = Speed × Time

$$\therefore \quad y = x \times 8$$

$$\Rightarrow \quad y = 8x \quad \ldots\text{(i)}$$

If I increase my speed by 6 km/hr it takes me 1 hour 20 minutes.

i.e., 80 minutes less

$$(x+6)\left(8-\frac{80}{60}\right) = y \Rightarrow (x+6)\left(8-\frac{4}{3}\right)$$

$$\frac{20(x+6)}{3} = y$$

$$\Rightarrow \quad \frac{20(x+6)}{3} = 8x \quad \text{[Using (i)]}$$

$$\Rightarrow \quad 20x + 120 = 24x$$

$$\Rightarrow \quad 4x = 120;$$

$$x = 30$$

Putting $x = 30$ in (i), we get $y = 8 \times 30 = 240$

Hence, the distance between the two cities A and B is 240 km.

19. Let the number of persons in Ist condition = x

and the 2nd condition no. of persons = $(x + 15)$

Amount divided = Rs. 6500

According to the question—

$$\frac{6500}{x} - \frac{6500}{x+15} = \text{Rs. } 30$$

$$\frac{6500x + 97500 - 6500x}{x(x+15)} = \frac{30}{1}$$

$$\Rightarrow \quad 30x^2 + 450x = 97500$$

$$\Rightarrow \quad 30x^2 + 450x - 97500 = 0$$

$$\Rightarrow \quad x^2 + 15x - 3250 = 0 \quad \text{[Dividing by 30]}$$

$$\Rightarrow \quad x^2 + 65x - 50x - 3250 = 0$$

$$\Rightarrow \quad x(x+65) - 50(x+65) = 0$$

$$\Rightarrow \quad (x+65)(x-50) = 0$$

$$\Rightarrow \quad x = -65$$

$$\text{or} \quad x = 50$$

Since, the number of persons can never be – ve.

$\therefore$ Number of persons = 50

20. Area of square having side x cm = x^2 cm^2

Area of square having side $(x + 4)$ cm = $(x + 4)^2$ cm^2

Given

$$x^2 + (x+4)^2 = 656$$

$$\Rightarrow \quad x^2 + x^2 + 16 + 8x = 656$$

$$\Rightarrow \quad 2x^2 + 8x - 640 = 0$$

$$\Rightarrow \quad x^2 + 4x - 320 = 0 \quad \text{[Dividing by 2]}$$

$$\Rightarrow \quad x^2 + 20x - 16x - 320 = 0$$

$$\Rightarrow \quad (x+20)(x-16) = 0$$

$$\Rightarrow \quad x = 16, -20$$

Side can never be – ve.

Side of one square = 16 cm

$\therefore$ Side of another square = 16 + 4 = 20 cm.

21. Let the side of the square be x cm.

$\therefore$ Side of the another square be $(x + 4)$ cm

Equation as per the question—

$$x^2 + (x+4)^2 = 400$$

$$\Rightarrow \quad x^2 + (x^2 + 8x + 16) = 400$$

$$\Rightarrow \quad x^2 + x^2 + 8x + 16 - 400 = 0$$

$$\Rightarrow \quad 2x^2 + 8x - 384 = 0$$

$$\Rightarrow \quad x^2 + 4x - 192 = 0 \quad \text{[Dividing by 2]}$$

$$\Rightarrow \quad x^2 + 16x - 12x - 192 = 0$$

$$\Rightarrow \quad x(x+16) - 12(x+16) = 0$$

$$\Rightarrow \quad (x+16)(x-12) = 0$$

$$\Rightarrow \quad x = -16$$

$$\text{or} \quad x = 12$$

As the length of the square cannot be – ve.

$$\therefore \quad x = 12$$

$\therefore$ Side of the first square = 12 cm

and Side of the second square = 12 + 4 = 16 cm.

22. Let the Breadth of the rectangle be x cm.

Then, Length = $(x + 8)$ cm

Area = Length × Breadth

$$x(x+8) = 240 \quad \text{(given)}$$

$$x^2 + 8x - 240 = 0$$

$$\Rightarrow \quad x^2 + 20x - 12x - 240 = 0$$

$$\Rightarrow \quad x(x+20) - 12(x+20) = 0$$

$$\Rightarrow \quad (x-12)(x+20) = 0$$

$$\Rightarrow \quad x = 12,$$

$$\text{or} \quad x = -20$$

Since, Breadth cannot be – ve.

x = 12 cm = Breadth

and Length = 12 + 8 = 20 cm.

23. Let the altitude of triangle = x cm

Then, Base = $(x + 10)$ cm

$$\text{Area of triangle D} = \frac{1}{2} \times \text{B} \times \text{H}$$

$$600 = \frac{1}{2} \times x \times (x+10)$$

$$1200 = x^2 + 10x$$

$$x^2 + 10x - 1200 = 0$$

$$x^2 + 40x - 30x - 1200 = 0$$

$$(x+40)(x-30) = 0$$

$$x = -40, 30$$

Side can never be – ve.

$\therefore$ Altitude of triangle $= 30$ cm

and Base $= 30 + 10$

$= 40$ cm.

24. Let the speed of Ist train be = x km/hr

and speed of the another train be = $(x - 12)$ km/hr

Distance = 240 km

According to the question—

$$\frac{240}{x-12} - \frac{240}{x} = 1 \text{ hour} \quad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\frac{240x - 240x + 2880}{x(x-12)} = \frac{1}{1}$$

$$\Rightarrow x(x-12) = 2880$$

$$\Rightarrow x^2 - 12x = 2880$$

$$\Rightarrow x^2 - 60x + 48x - 2880 = 0$$

$$\Rightarrow x(x-60) + 48(x-60) = 0$$

$$\Rightarrow (x-60)(x+48) = 0$$

$$\Rightarrow x = 60$$

$$\text{or} \quad x = -48$$

$\therefore$ Speed of Ist train = 60 km/hr

$\therefore$ Speed of IInd train = 48 km/hr.

25. Let the original speed of the train be = x km/hr

and increased speed of the train be = $(x + 15)$ km/hr

Distance = 90 km

According to the question—

$$\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2} \text{hr} \quad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\frac{90x + 1350 - 90x}{x(x+15)} = \frac{1}{2}$$

$$\Rightarrow 2700 = x^2 + 15x$$

$$\Rightarrow x^2 + 15x - 2700 = 0$$

$$\Rightarrow x^2 + 60x - 45x - 2700 = 0$$

$$\Rightarrow x(x+60) - 45(x+60) = 0$$

$$\Rightarrow (x+60)(x-45) = 0$$

$$\Rightarrow x = -60$$

$$\text{or} \quad x = 45$$

$\therefore$ Since, the speed cannot be – ve.

$\therefore$ Speed = 45 km/hr.

26. Let the usual speed of the train be = x km/hr

and increased speed of the train be = $(x + 5)$ km/hr

Distance = 300 km

According to the question—

$$\frac{300}{x} - \frac{300}{x+5} = 2 \text{ hr} \quad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\frac{300x + 1500 - 300x}{x(x+5)} = \frac{2}{1}$$

$$\Rightarrow 1500 = 2x(x+5)$$

$$\Rightarrow 2x^2 + 10x - 1500 = 0$$

$$\Rightarrow x^2 + 5x - 750 = 0 \quad \text{[Dividing by 2]}$$

$$\Rightarrow x^2 + 30x - 25x - 750 = 0$$

$$\Rightarrow x(x+30) - 25(x+30) = 0$$

$$\Rightarrow (x-25)(x+30) = 0$$

$$\Rightarrow x = 25$$

$$\text{or} \quad x = -30$$

$\because$ Since the speed cannot be – ve.

$\therefore$ Usual Speed = 25 km/hr.

27. Let the consecutive natural numbers be n and $(n + 1)$.

As per the question,

$$n^2 + (n+1)^2 = 313$$

$$\Rightarrow n^2 + n^2 + 1 + 2n = 313$$

$$\Rightarrow 2n^2 + 2n - 313 + 1 = 0$$

$$\Rightarrow 2n^2 + 2n - 312 = 0$$

$$\Rightarrow n^2 + n - 156 = 0 \quad \text{[Dividing by 2]}$$

$$\Rightarrow n^2 + 13n - 12n - 156 = 0$$

$$\Rightarrow n(n+13) - 12(n+13) = 0$$

$$\Rightarrow (n-12)(n+13) = 0$$

$$\Rightarrow n - 12 = 0$$

$$\text{or} \quad n + 13 = 0$$

$$\Rightarrow n = 12$$

$$\text{or} \quad n = -13$$

Since – 13 is not a natural number.

Hence, the numbers are 12, and 12 + 1 *i.e.*, 13.

28. Let the parts be x and $(29 - x)$.

As per the question,

$$(x)^2 + (29-x)^2 = 425$$

$$x^2 + 841 + x^2 - 58x = 425$$

$$\Rightarrow 2x^2 - 58x + 841 - 425 = 0$$

$$\Rightarrow 2x^2 - 58x + 416 = 0$$

$$\Rightarrow x^2 - 29x + 208 = 0 \quad \text{[Dividing by 2]}$$

$$\Rightarrow x^2 - 16x - 13x + 208 = 0$$

$$\Rightarrow x(x-16) - 13(x-16) = 0$$

$$\Rightarrow (x-16)(x-13) = 0$$

$$\Rightarrow x = 16$$

$$\text{or} \quad x = 13$$

Hence, the parts are 16 and 13.

29. Speed of boat in still water = 8 km/hr

Let the speed of stream in still water = x km/hr

$\therefore$ Speed upstream = $(8 - x)$ km/hr

and speed of downstream = $(8 + x)$ km/hr

According to the question—

$$\frac{15}{8-x} + \frac{22}{8+x} = 5$$

$$\frac{15(8+x) + 22(8-x)}{(8-x)(8+x)} = 5$$

$$\Rightarrow \frac{120 + 15x + 176 - 22x}{64 - x^2} = 5$$

$$\Rightarrow 120 + 15x + 176 - 22x = 320 - 5x^2$$

$$\Rightarrow 5x^2 + 15x - 22x + 120 + 176 - 320 = 0$$

$$\Rightarrow 5x^2 - 7x - 24 = 0$$

$$\Rightarrow 5x^2 - 15x + 8x - 24 = 0$$

$$\Rightarrow 5x(x-3) + 8(x-3) = 0$$

$$\Rightarrow (x-3)(5x+8) = 0$$

$$\Rightarrow x - 3 = 0$$

or $5x + 8 = 0$

$$\Rightarrow x = 3$$

or $5x = -8$

$$\Rightarrow x = 3$$

or $x = -\frac{8}{5}$

$\because$ Speed cannot be – ve.

$\therefore$ Speed of stream = 3 km/hr.

30. Let one number = x

Then, other number = $(48 - x)$

Product = $x(48 - x) = 432$

$$48x - x^2 = 432$$

$$\Rightarrow x^2 - 48x + 432 = 0$$

$$\Rightarrow x^2 - 36x - 12x + 432 = 0$$

$$\Rightarrow x(x-36) - 12(x-36) = 0$$

$$\Rightarrow (x-36)(x-12) = 0$$

$$\Rightarrow x - 36 = 0$$

or $x - 12 = 0$

$$\Rightarrow x = 36$$

or $x = 12$

$\therefore$ One number = 36, 12

and other number = 12, 36

$\therefore$ Two numbers are 36 and 12.

31. Let the speed of stream = x km/hr

Speed of the boat in still water = 15 km/hr (given)

Speed of boat in downstream = $(15 + x)$ km/hr

Speed of boat in upstream = $(15 - x)$ km/hr

Distance = 30 km

Time taken by boat in upstream = $\frac{30}{15-x}$ hour

Time taken by boat in downstream = $\frac{30}{15+x}$ hour

$$\text{Total time taken} = \frac{30}{15-x} + \frac{30}{15+x}$$

$$= 4 \text{ hours } 30 \text{ minutes (given)}$$

$$\frac{30(15+x) + 30(15-x)}{(15-x)(15+x)} = 4\frac{1}{2}$$

$$\Rightarrow \frac{450 + 30x + 450 - 30x}{225 - x^2} = \frac{9}{2}$$

$$\Rightarrow \frac{900}{225 - x^2} = \frac{9}{2}$$

$$\Rightarrow 1800 = 2025 - 9x^2$$

$$\Rightarrow 9x^2 = 2025 - 1800$$

$$\Rightarrow 9x^2 = 225$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = \sqrt{25} = \pm 5$$

$\therefore$ Speed of stream = 5 km/hr.

32. Let one number be = x

$$x - 20 = 69 \times \frac{1}{x}$$

$$x^2 - 20x - 69 = 0$$

$$\Rightarrow x^2 - 23x + 3x - 69 = 0$$

$$\Rightarrow x(x-23) + 3(x-23) = 0$$

$$\Rightarrow (x-23)(x+3) = 0$$

$$\Rightarrow x - 23 = 0$$

or $x + 3 = 0$

$$\Rightarrow x = 23$$

or $x = -3$

But – 3 is not a whole number

$$x = 23.$$

33. Suppose one natural number = x

IInd natural number = $x + 1$

According to the question—

$$x(x+1) = 20$$

$$x^2 + x = 20$$

$$\Rightarrow x^2 + x - 20 = 0$$

$$\Rightarrow x^2 - 5x - 4x - 20 = 0$$

$$\Rightarrow x(x+5) - 4(x+5) = 0$$

$$\Rightarrow (x+5)(x-4) = 0$$

$$\Rightarrow x + 5 = 0$$

or $x - 4 = 0$

$$\Rightarrow x = -5$$

or $x = 4$

But $x = -5$ is not a natural number

Hence, two consecutive natural numbers are 4, 5.

34. Let the present age of son = x years

Present age of father = x^2 years

One year ago, age of son = $(x - 1)$ year

One year ago, age of father = $(x^2 - 1)$ year

According to the question—

$$8(x-1) = (x^2-1)$$
$$8x - 8 = x^2 - 1$$
$$x^2 - 8x + 7 = 0$$
$$(x-7)(x-1) = 0$$
$$x = 7, 1$$
$x = 1$ is not suitable for the equation
$$x = 7$$

Hence, the age of son = 7 years

Age of father = 49 years.

35. Let number of books = x

Cost of each book = Rs. $\frac{80}{x}$

If number of books = $x + 4$

Then, cost of each book = $\frac{80}{x+4}$

According to the question—

$$\frac{80}{x} = \frac{80}{x+4} + 1$$
$$\frac{80}{x} = \frac{80 + x + 4}{x+4}$$
$$80x + 320 = 80x + x^2 + 4x$$
$$x^2 + 4x - 320 = 0$$
$$x^2 + 20x - 16x - 320 = 0$$
$$(x+20)(x-16) = 0$$
$$x = 16, -20$$

Number of books can never be – ve.

$$x = 16$$

Hence, number of books = 16

36. Let the integer = x

According to the question—

$$x^2 + x = 90$$
$$x^2 + x - 90 = 0$$
$$x^2 + 10x - 9x - 90 = 0$$
$$(x+10)(x-9) = 0$$
$$x = -10, 9$$

37. The equation is $(a-b)x^2 + (b-c)x + (c-a) = 0$

Since, the roots are real, therefore Discriminate = 0

$\Rightarrow \quad B^2 - 4AC = 0$

$\Rightarrow \quad (b-c)^2 - 4(a-b)(c-a) = 0$

$\Rightarrow \quad b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$

$\Rightarrow \quad 4a^2 + b^2 + c^2 - 4ac - 4ab + 2bc = 0$

$\Rightarrow \quad (2a - b - c)^2 = 0$

$\Rightarrow \quad 2a - (b + c) = 0$

$\Rightarrow \quad 2a = b + c$

Hence, proved.

38. Roots are real and equal.

$\Rightarrow \quad B^2 - 4AC = 0$

$\Rightarrow \quad 4(bc - ad)^2 = 4(a^2 + b^2)(c^2 + d^2)$

$\Rightarrow \quad b^2c^2 + a^2d^2 - 2abcd = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$

$\Rightarrow \quad b^2d^2 + a^2c^2 + 2abcd = 0$

$\Rightarrow \quad (ac + bd)^2 = 0$

$\Rightarrow \quad ac + bd = 0$

39. The equation is $p(q-r)x^2 + q(r-p)x + r(p-q) = 0$

Since, the roots are real.

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad q^2(r-p)^2 - 4pr(q-r)(p-q) = 0$

$\Rightarrow \quad q^2(r^2 + p^2 - 2rp) - 4pr(pq - pr - q^2 + qr) = 0$

$\Rightarrow \quad q^2r^2 + p^2q^2 - 2prq^2 - 4p^2qr + 4p^2r^2 + 4pq^2r^2 - 4\,pqr^2 = 0$

$\Rightarrow \quad p^2q^2 + q^2r^2 + 4\,r^2p^2 + 2\,pq^2r - 4\,p^2rq = 0$

$\Rightarrow \quad (pq + qr - 2rp)^2 = 0$

$\Rightarrow \quad pq + qr - 2rp = 0$

$\Rightarrow \quad \frac{1}{p} + \frac{1}{r} = \frac{2}{q}$

40. The equation is $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$

The roots are real and equal. $D = 0$

$\Rightarrow \quad D = B^2 - 4\,AC$

$\Rightarrow \quad (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow \quad 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$\Rightarrow \quad 4(-c^2 + a^2 + m^2a^2) = 0$

$\Rightarrow \quad -c^2 + a^2 + m^2a^2 = 0$

$\Rightarrow \quad a^2(1 + m^2) = c^2$

41. Let the original average speed of the cyclist be x km/hr.

Distance from A and B = 14 km

$\therefore$ Time taken to cover a distance of 14 km at original speed = $\frac{14}{x}$ hrs.

When speed is decreased by 1 km/hr new speed = $(x - 1)$ km/hr.

Time taken to cover 14 km at reduced speed = $\frac{14}{(x-1)}$ hr.

According to the question—

$$\frac{14}{(x-1)} - \frac{14}{x} = \frac{1}{3}$$

$\Rightarrow \quad x^2 - x = 42$

$\Rightarrow \quad x^2 - x - 42 = 0$

$$\Rightarrow \quad (x-7)(x+6) = 0$$

$$\Rightarrow \quad x = 7, -6$$

$$\text{Original average speed} = 7 \text{ km/hr.}$$

42. Let the rate of walking = x km/hr

Distance = 2 km

$$\text{Time} = \frac{2}{x} \text{ hr}$$

$$\text{Again, Speed} = (x+1) \text{ km/hr}$$

$$\text{Time} = \frac{2}{(x+1)} \text{ hr}$$

According to the question—

$$\frac{2}{x} - \frac{2}{(x+1)} = \frac{1}{6}$$

$$\frac{2}{x^2+x} = \frac{1}{6}$$

$$x^2 + x - 12 = 0$$

$$(x-3)(x+4) = 0$$

$$x = 3, -4, \text{Rejecting } x = -4$$

$$\text{Rate of walking} = 3 \text{ km/hr.}$$

43. Let the average speed of the aircraft = x km/hr

Time to cover 1600 km = $\frac{1600}{x}$ hrs

On decreasing the speed by 400 km, its speed will be = $(x - 400)$ km/hr

Time taken to cover a distance of 1600 km at reduced speed = $\frac{1600}{(x-400)}$ hr

According to the question,

$$\frac{1600}{(x-400)} - \frac{1600}{x} = \frac{2}{3}$$

$$\Rightarrow \quad 1600\left[\frac{400}{x^2-400x}\right] = \frac{2}{3}$$

$$\Rightarrow \quad x^2 - 400x - 960000 = 0$$

$$\Rightarrow \quad x = \frac{400 \pm \sqrt{(400)^2 + 4 \times 960000}}{2 \times 1}$$

$$= \frac{400 \pm \sqrt{4000000}}{2} = \frac{400 \pm 2000}{2}$$

$$= \frac{2400}{2}, \frac{-1600}{2}$$

$$= 1200, -800, \text{Rejecting } x = -800$$

$\therefore$ Average speed = 1200 km/hr

Actual time = $\frac{1600}{1200} = \frac{4}{3}$ hr $= 1\frac{1}{3}$ hr.

44. Let the number of children in a group = x

$\therefore$ Each child give a gift to every other child.

$\therefore$ One child give $(x-1)$ gift to other children.

$\therefore$ x children gives $x(x-1)$ gifts.

According to the question—

$$x(x-1) = 132$$

$$\text{or} \quad x^2 - x - 132 = 0$$

$$(x+11)(x-12) = 0$$

$$x = 12, -11,$$

$$\text{Rejecting } x = -11$$

$$\text{Number of children} = 12$$

45. Let a man purchased x books at Rs. 720.

$$\therefore \quad \text{Cost of one book} = \text{Rs. } \frac{720}{x}$$

If each book cost Rs. 2 less, he would get 4 more books.

$\therefore$ Now, with Rs. 720, he can purchase $(x+4)$ books.

$$\therefore \quad \text{Cost of each book} = \text{Rs. } \frac{720}{x+4}$$

According to the question,

$$\frac{720}{x} - \frac{720}{x+4} = 2$$

$$\Rightarrow \quad 720\left[\frac{4}{x^2+4x}\right] = 2$$

$$\Rightarrow \quad x^2 + 4x - 1440 = 0$$

$$\Rightarrow \quad (x-36)(x+40) = 0$$

$$x = 36, -40$$

$$\text{Rejecting x} = -40$$

$$\therefore \quad \text{Number of books} = 36$$

46. Let original number of swans = n

Number of swans playing on the shore of the pond $= \frac{7}{2}\sqrt{n}$

According to the question—

$$\frac{7}{2}\sqrt{n} + 2 = n$$

$$\Rightarrow \quad 7\sqrt{n} + 4 = 2n$$

$$\Rightarrow \quad 7\sqrt{n} = 2n - 4$$

$$\Rightarrow \quad 49\,n = 4n^2 + 16 - 16n$$

$$\Rightarrow \quad 4n^2 - 65n + 16 = 0$$

$$n = \frac{65 \pm \sqrt{(65)^2 - 4\,(4)\,(16)}}{2 \times 4}$$

$$n = \frac{65 \pm \sqrt{3969}}{8}$$

$$n = \frac{128}{8}, \frac{2}{8}$$

$$n = 16, \frac{1}{4}, \text{But } n \neq \text{become } n \in \text{N}$$

$\therefore$ Total number of swans = 16.

15 Progression and Sequence

Progression implies sequence of numbers. It consists of a set of numbers arranged in order. The numbers in the sequence are termed as terms of the sequence or series.

Examples—

3, 5, 7, 9, 11, 13, ...

2, 4, 6, 8, 10, 12, ...

The General form of representing a series is

X_1, X_2, X_3, X_4 ....

There are three types of progression—

(i) Arithmetic Progression (ii) Geometric Progression (iii) Harmonic Progression.

(A) Arithmetic Progression (A.P.)

A sequence of numbers is said to be in Arithmetic progression if the difference between any two consecutive numbers is always the same. If first term of the progression is 'a' and the difference between two consecutive terms of the series is termed as its common difference 'd'. Then, the nth term of the series is T_n.

$$T_n = a + (n-1)d$$

Illustration 1.

Let an A.P. is 3, 6, 9, 12, 15, 18, ... Find its 15th term.

Solution :

Let the first term of the progression = a

Common difference = d

n – represent nth term of the series.

$$
\begin{aligned}
\text{Ist term} &= a = 3 \\
\text{Common difference} &= d = 6 - 3 = 9 - 6 = 3 \\
t_2 = \text{2nd term} &= 3 + 3 = 6 = a + d \\
t_3 = \text{3rd term} &= 3 + 6 = 9 = a + 2d \\
t_4 = \text{4th term} &= 3 + 9 = 12 = a + 3d \\
t_5 = \text{5th term} &= 3 + 12 = 15 = a + 4d \\
\ldots & \quad \ldots \\
\ldots & \quad \ldots \\
\ldots & \quad \ldots \\
t_n = n\text{th term} &= a + (n-1)\, d
\end{aligned}
$$

$$
\begin{aligned}
\text{So, 15th term} &= a + (15 - 1).d \\
&= a + 14\, d = 3 + 14 \times 3 = 45.
\end{aligned}
$$

Illustration 2.

Given A.P. is – 4, – 7, – 10, – 13, – 16, ... Find the common difference and the 10th term.

Solution :

$$
\begin{aligned}
\text{Ist term} &= a = -4 \\
\text{Common difference} &= -7 - (-4) = -10\ (-7) = -3 \\
\text{10th term is given by} &= a + (10 - 1).d \\
&= -4 + 9 \times (-3) = -31.
\end{aligned}
$$

Illustration 3.

Given A.P. is $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$, ... Find its 115th term.

Solution :

$$
\begin{aligned}
\text{Ist term} &= a = \frac{1}{2} \\
\text{Common difference} &= 1 - \frac{1}{2} = \frac{3}{2} - 1 = \frac{1}{2} \\
\text{Now,} \quad n\text{th term} &= t_n = a + (n-1).d \\
t_{115} &= \frac{1}{2} + (115 - 1) \times \frac{1}{2} \\
t_{115} &= \frac{115}{2}
\end{aligned}
$$

Illustration 4.

Find the Sum of n natural numbers.

Solution :

We known that

$$k^2 - (k-1)^2 = 2\,k - 1$$

$$
\begin{aligned}
\text{Now, when } k &= 1 \quad 1^2 - 0^2 = 2 \times 1 - 1 \\
k &= 2 \quad 2^2 - 1^2 = 2 \times 2 - 1 \\
k &= 3 \quad 3^2 - 2^2 = 2 \times 3 - 1 \\
k &= 4 \quad 4^2 - 3^2 = 2 \times 4 - 1 \\
k &= 5 \quad 5^2 - 4^2 = 2 \times 5 - 1 \\
& \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
& \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
k &= n \quad n^2 - (n-1)^2 = 2 \times n - 1 \\
n^2 &= 2(1 + 2 + 3 + 4 + \ldots + n) - n \\
n^2 &= 2(\text{Sum of natural numbers}) - n
\end{aligned}
$$

$$\text{Sum of } n \text{ natural numbers} = \frac{n(n+1)}{2}$$

Illustration 5.

Find the sum of n terms of an A.P. whose first number is 'a' and common difference is 'd'.

Solution :

According to given condition—

Arithmetic progression is—$a, a + d, a + 2d, a + 3d, a + 4d$, ... $a + (n-1)d$

If S_n be the sum of the above numbers, then

$$S_n = a + a + d + a + 2d + a + 3d + \ldots\ldots + a + (n-1)\,d$$

$$S_n = na + d[1 + 2 + 3 + \ldots\ldots (n-1) \text{ term}]$$

$$S_n = na + d \times \frac{n(n-1)}{2}$$

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

Illustration 6.

Find the sum of 20 terms of an A.P. which is – 5, – 3, – 1, 1, 5, 7, ……

Solution :

According to question—

Ist term = a = – 5; Common difference = d = – 3 (– 5) = 2; n = 20

Since, $S_{20} = \frac{20}{2}[2a + (20-1)d]$

$= 10[2 \times (-5) + 19 \times 2]$

$= 280$

Illustration 7.

Find 20th term of the following A.P.

(i) 430, 415, 400, 385 …………

(ii) $(4x + 8y), (6x + 7y), (8x + 11y), (10x + 15y)$……

Solution :

(i) First term = a = 430

Common difference = d = 415 – 430 = – 15

Now, $t_n = a + (n-1)d$

$t_{20} = 430 + (20-1)(-15)$

$= 430 + 19(-15)$

$= 430 - 285 = 145$

(ii) Given A.P. is $(4x + 3y), (6x + 7y), (8x + 11y), (10x+15y)$ ……

We consider above A.P. in two separated A.P.'s $4x$, $6x$, $8x$, $10x$, ……

and $3y$, $7y$, $11y$, $15y$, ……

Now, $xt_{20} = 4x + (20-1)2x = 4x + 38x = 42x$

and $yt_{20} = 3y + (20-1)4y = 3y + 19 \times 4y = 3y + 76y = 79y$

Now, $t_{20} = 42x + 79y$

Illustration 8.

If Ist term of an A.P. is 10 and common difference is 5, write down the A.P.

Solution :

Ist term = a = 10

Common difference = d = 5

So, Arithmetic Progression is 10, 15, 20, 25, 30, 35, 40, ……

Illustration 9.

The first and 7th term of an A.P. are 7 and 49 respectively. Write the A.P.

Solution :

Since, we know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[a + a + (n-1)d]$$

$$= \frac{n}{2}[a + n\text{th term}]$$

$$S_n = \frac{n}{2}[\text{Ist term} + n\text{th term}]$$

If last term = nth term

So, $S_n = \frac{n}{2}[\text{Ist term} + \text{Last term}]$

Illustration 10.

The first and last term of an A.P. consisting of 12 terms are 20 and 245 respectively. Write the A.P.

Solution :

Given a = 20

Last term = 245

n = 12

From formula—

$$S_{12} = \frac{12}{2}[2a + (n-1)d]$$

$$= \frac{12}{2}[\text{Ist term} + \text{Last term}]$$

$$= 6 \times (20 + 245)$$

$$= 6 \times (265)$$

$$= 1590$$

Illustration 11.

If the first and last terms of an A.P. are 5 and 1025 and the Sum of terms is 2060, then find the value of n.

Solution :

a = 5

Last term = $a + (n-1)d = 1025$

$$S_n = \frac{n}{2}[\text{Ist term} + \text{Last term}]$$

$$2060 = \frac{n}{2}[5 + 1025]$$

n = 4

Illustration 12.

Find the Sum of first n even natural numbers.

Solution :

The Ist n odd natural number start with 2.

An A.P. is 2, 4, 6, 8, 10,

So, $a = 2, d = 2$

$$S_n = \frac{n}{2}[2.a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 2 + (n-1) \times 2]$$

$$= \frac{n}{2}(2n + 2)$$

$$\text{Even} = n(n+1)$$

Illustration 13.

Find the Sum of first n odd natural numbers.

Solution :

Given A.P. is 1, 3, 5, 7, 9, 11, ……

$$a = 1, d = 2$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n-1) \times 2]$$

$$S_{odd} = n^2$$

Illustration 14.

Find the sum of the squares of the first n natural numbers.

Solution :

Given series is $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + \ldots\ldots + n^2$

Now, we have $k^3 - (k-1)^3 = 3k^2 - 3k + 1$

Putting $k = 1$ $\quad 1^3 - 0^3 = 3 \times 1^2 - 3 \times 1 + 1$

Putting $k = 2$ $\quad 2^3 - 1^3 = 3 \times 2^2 - 3 \times 2 + 1$

Putting $k = 3$ $\quad 3^3 - 2^3 = 3 \times 3^2 - 3 \times 3 + 1$

……………………………………………………

……………………………………………………

Putting $k = n$ $\quad n^3 - (n-1)^3 = 3 \times n^2 - 3n + 1$

Now, $n^3 = 3(1^2 + 2^2 + 3^2 + \ldots\ldots + n^2) - 3(1 + 2 + 3 + 4 + \ldots\ldots + n) + n$

or $n^3 = 3(S_n) - 3\dfrac{n(n+1)}{2} + n$

or $n^3 = 3S_n - \dfrac{3}{2}(n^2 + n) + n$

or $3S_n = n^3 + \dfrac{3}{2}(n^2 + n) - n$

$$= \frac{1}{2}[2n^3 + 3n^2 + 3n - 2n]$$

$$= \frac{n}{2}[2n^2 + 3n + 1]$$

$$S_n = \frac{n(2n+1)(n+1)}{6}$$

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

The Arithmetic Mean of *n* Quantities is Equal to the Sum of the *n* Quantities Divided by *n*

$$\therefore \quad \text{A.M.} = 8 = \frac{X_1 + X_2 + X_3 + \ldots\ldots + X_n}{n}$$

Extra Shot— Several kinds of mean exist, and the method of calculating a mean depends upon the relationship known or assumed to govern the other members. The arithmetic mean, denoted 8, of a set of n numbers $X_1 + X_2 + X_3 + \ldots\ldots + X_n$ is defined as the sum of the numbers divided by n—

$$\therefore \quad 8 = \frac{X_1 + X_2 + X_3 + \ldots\ldots + X_n}{n}$$

The arithmetic mean represents a point about which the numbers balance. For example, if unit masses are placed on a line at points with co-ordinates $X_1, X_2, X_3, \ldots\ldots, X_n$, then the arithmetic mean is the co-ordinate of the centre of gravity of the system. In statistics, the arithmetic mean is commonly used as the single value typical of a set of data. For a system of particles having unequal masses, the centre of gravity is determined by a more general average, the weighted arithmetic mean. If each number x_i is assigned a positive weight w_i, the weighted arithmetic mean is defined as the sum of the products $w_i x_i$ divided by the sum of the weights. In this case,

$$8 = \frac{X_1 w_1 + X_2 w_2 + X_3 w_3 + \ldots\ldots + X_n w_n}{w_1 + w_2 + w_3 + \ldots\ldots + w_n}$$

The weighted arithmetic mean also is used in statistical analysis of grouped data; each number x_i is the mid-point of an interval; and each corresponding value of w_i is the number of data points within that interval.

For a given set of data, many possible means can be defined, depending on which features of the data are of interest. For example, suppose five squares are given, with sides 1, 1, 2, 5 and 7 inches. Their average area is $(1^2 + 1^2 + 2^2 + 5^2 + 7^2)/5$, or 16 square inches, the area of a square of side 4 inches. The number 4 is the quadratic mean (or root mean square) of the numbers 1, 1, 2, 5, 7 and differs from their arithmetic mean, which is $3\frac{1}{5}$. In general, the quadratic mean of n numbers $X_1 + X_2 + X_3 + \ldots\ldots + X_n$ is the square root of the arithmetic mean of their squares,

$$= \sqrt{\frac{X_1^2 + X_2^2 + X_3^2 + \ldots\ldots X_n^2}{n}}$$

The arithmetic mean gives no indication of how widely the data are spread or dispersed about the mean. Measures of the dispersion are provided by the arithmetic and quadratic means of the n differences $X_1 - 8, X_2 - 8, X_3 - 8 \ldots, X_n - 8$. These are called the variance and the standard deviation of $X_1, X_2, X_3, \ldots\ldots, X_n$.

The arithmetic and quadratic means are the special cases $p = 1$ and $p = 2$ of the pth–power mean, Mp, defined by the formula

$$\text{Mp} = \left(\frac{X_1^p + X_2^p + X_3^p + \ldots + X_n^p}{n}\right)^{\frac{1}{p}}$$

where p may be any real number except zero. The case $p = -1$ is also called the harmonic mean. Weighted pth–power means are defined by

$$\text{Mp} = \left(\frac{w_1X_1^p + w_2X_2^p + w_3X_3^p + \ldots + w_nX_n^p}{w_1 + w_2 + w_3 + \ldots\ldots + w_n}\right)^{\frac{1}{p}}$$

If a is the arithmetic mean of x_1 and x_2, the three numbers x_1, a, x_2 are in arithmetic progression. If h is the harmonic mean of x_1 and x_2, the numbers x_1, h, x_2 are in harmonic progression. A number g such that x_1, g, x_2 are in geometric progression is defined by the condition that $\frac{x_1}{g} = \frac{g}{x_2}$, or $g_2 = x_1x_2$; hence $g = \sqrt{x_1x_2}$ This g is called the geometric mean of x_1 and x_2. The geometric mean of n numbers $x_1, x_2, \ldots, x_n$ is defined to be the nth root of their product; $g = \sqrt[n]{x_1x_2x_3\ldots\ldots x_n}$.

All the means discussed are special cases of a more general mean. If f is a function having an inverse f^{-1}, the number

$$= f^{-1}\left(\frac{f(x_1) + f(x_2) + f(x_3) + \ldots\ldots + f(x_n)}{n}\right)$$

is called the mean value of $x_1, x_2, \ldots, x_n$ associated with f. When $f(x) = x^p$ the inverse is $f^{-1}(x) = x^{1/p}$ and the mean value is the pth–power mean, Mp. When $f(x) = \log_e x$, the inverse is $f^{-1}(x) = e^x$ and the mean value is the geometric mean.

Illustration 15.

Find the arithmetic mean of the following—

(i) 3, 9, 12, 15, 21, 24

(ii) $a, 4a, 9a, 6a, 20a$

Solution :

(i) Arithmetic mean $= \dfrac{3 + 9 + 12 + 15 + 21 + 24}{6}$

$= \dfrac{84}{6} = 14$

(ii) Arithmetic mean $= \dfrac{a + 4a + 9a + 6a + 20a}{5}$

$= \dfrac{40}{5}a = 8a$

Illustration 16.

Insert n arithmetic means between a and b with n arithmetic means and the two given quantities a and b, the total number of terms will be $(n + 2)$. Further all the terms will be in A.P.

Solution :

$$\text{Ist term} = a$$

$$(n + 2)\text{th term} = b$$

If d be the common difference.

$$\text{Now, } (n + 2)\text{th term} = a + (n + 2 - 1)d$$

$$b = a + (n - 1)d$$

$$d = \frac{b - a}{n + 1}$$

So, 2nd term $= a + d$

$$= a + \frac{b - a}{n + 1} = \frac{an + a + b - a}{n + 1} = \frac{b + an}{n + 1}$$

$$\text{3rd term} = a + 2d = a + 2\left(\frac{b - a}{n + 1}\right)$$

$$= \frac{an + a + 2b - 2a}{n + 1} = \frac{2b + (n - 1)a}{n + 1}$$

So, the required arithmetic mean to be inserted is

$$= \frac{b + an}{n + 1}, \frac{2b + (n - 1)a}{n + 1}, \frac{3b + (n - 2)a}{n + 1}$$

Illustration 17.

Find the sum of cubes of first n natural numbers ?

Solution :

Given $S_n = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \ldots\ldots + n^3$

$$k^4 - (k - 1)^4 = 4k^3 - 6k^2 + 4k - 1$$

Putting $k = 1$ $\quad 1^4 - 0 = 4 \times 1^3 - 6 \times 1^2 + 4 \times 1 - 1$

Putting $k = 2$ $\quad 2^4 - 1^4 = 4 \times 2^3 - 6 \times 2^2 + 4 \times 2 - 1$

Putting $k = 3$ $\quad 3^4 - 2^4 = 4 \times 3^3 - 6 \times 3^2 + 4 \times 3 - 1$

....

Putting $k = n$ $\quad n^4 - (n - 1)^4 = 4 \times n^3 - 6 \times n^2 + 4n - 1$

$$n_4 = 4(1^3 + 2^3 + 3^3 + \ldots\ldots + n^3) - 6(1^2 + 2^2 + 3^2 + \ldots\ldots + n^2) + 4(1 + 2 + 3 + 4 + \ldots\ldots + n) - n$$

or $\quad n_4 = 4S_n - 6\dfrac{n(n + 1)(2n + 1)}{6} + 4 \times \dfrac{n(n + 1)}{2} - n$

or $\quad n_4 = n^4 + n(2n^3 + 3n + 1) - (2n^2 - 2n) + n$

or $\quad = n^4 + 2n^3 + 3n^2 + n - 2n^2 - 2n + n$

So, $\quad 4S_n = n^4 + 2n^3 + n^2$

$$4S_n = n^2(n^2 + 2n + 1)$$

$$\therefore \quad S_n = \left[\frac{n(n + 1)}{2}\right]^2$$

Illustration 18.

Find the sum of the series of n terms—

$(4^2 - 1^2) + (5^2 - 2^2) + (6^2 - 3^2) + (7^2 - 4^2) + \ldots\ldots$

Solution :

The given series is combination of two series $4^2, 5^2, 6^2, 7^2 \ldots\ldots$ and $1^2, 2^2, 3^2, 4^2, \ldots\ldots$

$$n\text{th term} = a + (n-1)d = 4 + (n-1)1 = n + 3$$

$$n\text{th term of series} = 1^2, 2^2, 3^2, 4^2, \ldots\ldots$$

$$n\text{th term} = n^2$$

$$\begin{aligned} \text{Now, } n\text{th term of the given series} &= (n+3)^2 - n^2 \\ &= n^2 + 6n + 9 - n^2 \\ &= 6n + 9 \end{aligned}$$

$$\begin{aligned} \text{So,} \quad S_n &= {}^{n}\Sigma_{n=1} (6n + 9) \\ &= 6 \times \frac{n(n+1)}{2} + 9n \\ &= 3n + 3n^2 + 9n \\ &= 3n^2 + 12n = 3n(n+4) \end{aligned}$$

(B) Geometric Progression (G.P.)

A sequence of numbers is said to be in geometric progression. If the ratio of two consecutive numbers is constant, the constant ratio is known as common ratio and represented by r. Then nth term of the series is $T_n = ar^{n-1}$. So, series is $a, ar, ar^2, ar^3, \ldots\ldots ar^{n-1}$

Here, First term $= a$; and common ratio $= r$.

Example—

(i) $3, 6, 12, 24, 48, \ldots\ldots$

(ii) $2, 4, 8, 16, 32, 64, \ldots\ldots$

(i) First term $= a = 3$

$$\text{Common ratio} = r = \frac{12}{6} = 2$$

$$T_n = 3.(2)^{n-1}$$

(ii) First term $= a = 2$

$$\text{Common ratio} = r = \frac{8}{4} = 2$$

$$T_n = 2.(2)^{n-1} = 2^n$$

Illustration 19.

Find the common ratio of the following G.P.

(i) $15a^2, 45a^4, 135a^6, 405a^8, \ldots\ldots$

(ii) $-6, -42, -234, -2058, \ldots\ldots$

Solution :

(i) Ist term $= 15a^2$

$$\text{Common ratio} = \frac{\text{Second term}}{\text{First term}} = \frac{45a^4}{15a^2} = 3a^2$$

(ii) Ist term $= -6$

$$\text{Common ratio} = \frac{\text{Second term}}{\text{First term}} = \frac{-42}{-6} = 7$$

Illustration 20.

Find the nth term of a G.P. whose Ist term is a and common ratio be r.

Solution :

$$\begin{aligned} t_1 &= \text{Ist term} = a \\ t_2 &= \text{2nd term} = t_1 \times r = a \times r = a\,r^{2-1} \\ t_3 &= \text{3rd term} = t_2 \times r = a\,r \times r = a\,r^2 = a\,r^{3-1} \\ t_4 &= \text{4th term} = t_3 \times r = a\,r^2 \times r = a\,r^3 = a\,r^{4-1} \\ t_5 &= \text{5th term} = t_4 \times r = a\,r^3 \times r = a\,r^4 = a\,r^{5-1} \end{aligned}$$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$t_n = n\text{th term} = a\,r^{n-1}$$

Illustration 21.

Find the sum of n terms of a G.P.

Solution :

Let Ist term $= a$

Common ratio $= r$

If S_n be the sum of n terms.

$$\text{So,} \quad S_n = a + ar + ar^2 + ar^3 + ar^4 + \ldots + ar^{n-1} \ldots (1)$$

Now, multiplying equation (1) with r, we get

$$rS_n = ar + ar^2 + ar^3 + \ldots + ar^n$$

Now, $S_n - r\,S_n$, then we get

$$S_n = a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1}$$

$$-r\,S_n = -ar - ar^2 - ar^3 - \ldots\ldots - ar^{n-1} - ar^n$$

$$(1-r)S_n = a - ar^n$$

$$S_n = a \times \frac{(1-r^n)}{1-r} \qquad \text{when } r < 1$$

When $r > 1$

$$\therefore \quad S_n = a \times \left(\frac{r^n - 1}{r - 1}\right)$$

Illustration 22.

A finite G.P. consists of 6 terms. If its first and last terms are 5 and 84035. Find the G.P.

Solution :

$$\begin{aligned} \text{Ist term} &= a = 5 \\ n &= 6 \\ \text{Last term } T_6 &= 84035 = a.\,r^5 \\ 84035 &= 5 \times r^5 \\ 16807 &= r^5 \\ 7^5 &= r^5 \\ r &= 7 \end{aligned}$$

So, G.P. is $5, 35, 245, 1715, \ldots\ldots$

Illustration 23.

Find the sum of first 5 terms of G.P. whose first term is 2 and common ratio is 3.

Solution :

$n = 5, a = 2, r = 3$

Using Formula—

$$S_5 = a \times \left(\frac{r^n - 1}{r - 1}\right)$$

$$= 2 \times \left(\frac{3^5 - 1}{3 - 1}\right) = 243 - 1 = 242$$

Illustration 24.

Find the sum of 6 terms of a G.P. whose first term is 5 and the common ratio is $\frac{1}{2}$.

Solution :

$$a = 5 = \text{Ist term}$$

$$\text{Common ratio } (r) = 2$$

$$\text{Sum} = S_n = a \times \left(\frac{r^n - 1}{r - 1}\right) \qquad r < 1$$

$$S_6 = 5 \times \left(\frac{1 - \frac{1}{2^6}}{1 - \frac{1}{2}}\right)$$

$$= 5 \times \left(\frac{2^6 - 1}{2^5}\right) = 5 \times \frac{63}{32} = \frac{315}{32}$$

Geometric Mean (G.M.)

The square root of the product of two quantities is equal to their Geometric mean. When three quantities are in G.P., the middle quantity is the geometric mean of the other two quantities.

If a and b are in G.P. So,

$$\text{G.M.} = \sqrt{ab}$$

The Geometric mean of n quantities is equal to the nth root of the product of n quantities.

$$\text{G.M.} = \sqrt[n]{X_1.X_2.X_3.X_4. \ldots\ldots X_n}$$

$$\text{G.M.} = (X_1.X_2.X_3.X_4. \ldots\ldots X_n)^{1/n}$$

Illustration 24.

Find the Geometric mean, the common ratio and write G.P. for each of the following paris.

(i) $5x, 20\,x^3$

(ii) $(a + b), (a^2 - b^2), (a - b)$

Solution :

(i) $5x, 20\,x^3$

$$\text{G.M.} = \sqrt{5x.\,20x^3} = 10x^2$$

Clearly, $5x, 10x^2, 20x^3$ are in G.P.

$$\text{Common ratio} = \frac{10x^2}{5x} = 2x$$

Now, G.P. is $5x, 10x^2, 20x^3, 40x^4, \ldots\ldots$

(ii) $$\text{G.M.} = \sqrt{(a + b)(a^2 - b^2)(a - b)}$$

$$= \pm (a^2 - b^2)$$

Now, there are two G.M. $= a^2 - b^2$ and $-(a^2 - b^2)$

$$\text{So, Ist common ratio} = \frac{(a^2 - b^2)}{(a + b)} = a - b$$

$$\text{2nd common ratio} = -\frac{(a^2 - b^2)}{(a + b)} = -(a - b)$$

Illustration 25.

Find the missing terms in the following G.P. –

96, –, –, –, 486

Solution :

Clearly, $a = 96, \quad t_5 = ar^4$

$$\therefore \quad 486 = 96 \times r^4$$

$$r^4 = \frac{81}{16} = \left(\frac{3}{2}\right)^4$$

$$\therefore \quad r = \frac{3}{2}$$

$$\text{Second term} = 96 \times \frac{3}{2} = 48 \times 3 = 144$$

$$\text{3rd term} = 144 \times \left(\frac{3}{2}\right)^2 = 72 \times 3 = 216$$

$$\text{Fourth term} = 216 \times \frac{3}{2} = 324$$

Illustration 26.

The sum of three numbers in G.P. is 63 and their product is 1728. Find the numbers.

Solution :

Let the three numbers in G.P. be $a, \frac{a}{r}, \frac{a}{r^2}$.

According to question—

$$a + \frac{a}{r} + \frac{a}{r^2} = 63 \qquad \ldots(1)$$

or $$a \cdot \frac{a}{r} \cdot \frac{a}{r^2} = 1728 \qquad \ldots(2)$$

or $$\frac{a^3}{r^3} = 1728 = (4 \times 3)^3$$

or $$\frac{a}{r} = 12$$

or $$a\left(1 + \frac{1}{r} + \frac{1}{r^2}\right) = 63$$

or $$12.r\left(1 + \frac{1}{r} + \frac{1}{r^2}\right) = 63$$

or $$12.r\left(\frac{r^2 + r + 1}{r^2}\right) = 63$$

or $$12.r^2 + 12.r + 12 = 63.r$$

or $$12.r^2 - 51.r + 12 = 0$$

or $$12.r^2 - 48.r - 3.r + 12 = 0$$

or $\quad 12.r(r-4)-3(r-4) = 0$

$r = 4, \quad r = \frac{1}{4} \quad a = 48, 3$

Clearly, there are two G.P. which are –

$3, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}, \ldots\ldots$

and $48, 48 \times 4, 48 \times 4^2, 48 \times 4^3, \ldots\ldots$

Illustration 27.

The sum of first five terms of a G.P. is 93 and the sum of first ten term is 3069. Find the G.P.

Solution :

Now, $\quad S_n = a \times \left(\frac{r^n - 1}{r-1}\right)$

So, $\quad S_5 = a \times \left(\frac{r^5 - 1}{r-1}\right)$

and $\quad S_{10} = a \times \left(\frac{r^{10} - 1}{r-1}\right)$

$$\frac{S_{10}}{S_5} = \frac{r^{10}-1}{r^5-1} = r^5 + 1 = \frac{3069}{93}$$

$$r^5 = \frac{2976}{93} = 2^5$$

$$r = 2$$

$$S_5 = a \times \left(\frac{2^5-1}{2-1}\right) = a \times 31$$

$$93 = a \times 31$$

$$a = 3$$

Illustration 28.

If A and G are A.M. and G.M. of a, b, then find the equation having a, b as its roots.

Solution :

Since, a, b are two numbers

$$\text{A.M.} = \frac{a+b}{2} = \text{A}$$

$$a + b = 2.\text{A} \quad \ldots(1)$$

Again, $\quad \text{G.M.} = \text{G} = \sqrt{ab}$

$$\text{G} = \sqrt{ab}$$

$$a.b = \text{G}^2 \quad \ldots(2)$$

We know that if a, b are the roots of a quadratic equation, then it will be

$$X^2 - (a+b)\,X + ab = 0$$

$$X^2 - 2A.X + G^2 = 0$$

(C) Harmonic Progression (H.P.)

A sequence of numbers is said to be in H.P. if reciprocals of its term from an arithmetic progression (A.P.) conversely. If terms of a sequence are in A.P., their reciprocals form a H. P.

If H. P. is $X_1, X_2, X_3, X_4, \ldots\ldots$

Then, $\frac{1}{X_1}, \frac{1}{X_2}, \frac{1}{X_3}, \frac{1}{X_4} \ldots\ldots$ are in A.P.

Illustration 29.

Show that $\left(\frac{2}{11}, \frac{1}{7}, \frac{2}{17}, \frac{1}{10}, \frac{2}{23}, \ldots\ldots\right)$ is a H.P.

Solution :

The new sequence formed by reciprocals of the terms of the given sequence is $\frac{11}{2}, 7, \frac{17}{2}, 10, \frac{23}{2}$.

Now, $\quad 7 - \frac{11}{2} = \frac{3}{2}$

$$\frac{17}{2} - 7 = \frac{3}{2}$$

$$10 - \frac{17}{2} = \frac{3}{2}$$

$$\frac{23}{2} - 10 = \frac{3}{2}$$

Since, the difference between two consecutive terms of the new sequence is always same.

So, they form an A.P. and Given sequence is in H.P.

Illustration 30.

The first and 16th terms of a H.P. are $\frac{3}{2}$ and $\frac{1}{2}$ respectively. Find its 6th term.

Solution :

We know that nth term of an A.P. is

$$t_n = a + (n-1)d$$

where a = first term ; d = common difference

$\therefore \quad$ 16th term $= a + (16-1)d = a + 15d$

According to question—

$$\frac{1}{a} = \frac{3}{2} \quad \ldots(1)$$

$$\frac{1}{a + 15d} = \frac{1}{2} \quad \ldots(2)$$

Putting value of a in equation (2), we get

$$\frac{1}{\frac{3}{2} + 5d} = \frac{1}{2}$$

or $\quad \frac{2}{3 + 10d} = \frac{1}{2}$

or $\quad 4 = 3 + 10d$

or $\quad d = \frac{1}{10}$

Now, $\quad$ 6th term $= a + 5d$

$$= \frac{2}{3} + 5 \times \frac{1}{10}$$

$$= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$$

$\therefore \quad$ 6th term of a H.P. $= \frac{6}{7}$

Harmonic Mean (H.M.)

The Harmonic mean (H) of two given quantities a and b is given by

$$H = \frac{2ab}{a+b}$$

where a, H, b form a H.P.

Illustration 31.

Find the harmonic mean of 4 and 12.

Solution :

Given two numbers are 4 and 12.

So, $\quad \text{H.M.} = \frac{2ab}{a+b} = \frac{2 \times 4 \times 12}{16} = 6$

SOME IMPORTANT POINTS IN A.P.

1. If a is the first and d be the common difference of an A.P. and if AP having m terms, then nth term from the end is equal to the $(m - n + 1)$th term from the beginning. So, nth term

   from the end $= t_{m-n+1} = a + (m-n)d$

2. If a constant is added to or subtracted from an A.P., then the resulting sequence is also an A.P. with some common difference.

3. In a finite A.P., the sum of terms equidistant from the beginning and end and is always equal to the sum of first and last term of the series.

   $t_m + t_{n-(m-1)} = t_1 + t_n$ for all $n = 1, 2, 3, \ldots\ldots$

4. If between two given quantities a and b, we have to insert n quantities $a, A_1, A_2, A_3, \ldots\ldots A_n, b$ such that $a, A_1, A_2, A_3, \ldots\ldots A_n, b$ are in A.P., then we say A_1, A_2, A_3, ... A_n are n arithmetic means between a and b.

SOME IMPORTANT POINTS IN G.P.

1. The nth terms from the end of a finite G.P. consisting of m terms is ar^{m-n}

   where $\quad a$ = Ist term

   $\quad r$ = Common ratio

2. The nth term from the end of G.P. with last term l and common ratio r is

$$l \times \left(\frac{l}{r}\right)^{n-1}$$

3. a, b, c three non–zero numbers are in G.P. if $b^2 = ac$.

4. If all the terms of a G.P. be multiplied or divided by the same non–zero constant, then it remains a G.P. with the same common ratio.

5. In a finite G.P., the product of the terms equidistant from the beginning and the end is always the same and is equal to the product of first and the last term.

6. If a_1, a_2, a_3, ... a_n is a G.P. of non–zero, non–negative terms.

   Log a_1, Log a_2, Log a_3, ... Log a_n is an A.P. and *vice–versa*.

7. If a $G_1, G_2, G_3, \ldots\ldots G_n$ b are in G.P., where a and b two given numbers, then G_1, G_2, G_3 ...... G_n are known as n geometric mean (G.M.) between a and b.

Short Cuts

(a) If A.M., G.M. and H.M. between a and b are in A, G and H respectively, then

$$A = \frac{a+b}{2}, \; G = \sqrt{ab}, \; H = \frac{2ab}{a+b}$$

Therefore, $A \times H = \frac{a+b}{2} \times \frac{2ab}{a+b}$

$= ab = G^2 = A \times H$

$G^2 = A \times H$

(b) Always $\quad$ A.M. $\geq$ G.M. $\geq$ H.M.

Hence, $\quad A \geq G \geq H$

(c) The way of assuming terms in A.P.

(i) $\alpha - \beta, \alpha, \alpha + \beta$

(ii) $(\alpha - 3\beta), (\alpha - \beta), (\alpha + \beta), (\alpha + 3\beta)$

(iii) $(\alpha - 5\beta), (\alpha - 3\beta), (\alpha - \beta), (\alpha + \beta), (\alpha + 3\beta), (\alpha + 5\beta)$

(d) The way of selecting terms in G.P. –

(i) $\frac{\alpha}{r}, \alpha, \alpha r$

(ii) $\frac{\alpha}{r^3}, \frac{\alpha}{r}, \alpha r, \alpha r^3$

(iii) $\frac{\alpha}{r^2}, \frac{\alpha}{r}, \alpha, \alpha r, \alpha r^2$

where α = Ist term, r = common ratio

Exercise A

1. A.M. between the roots of a quadratic equation is 5 and G.M. is 4, then find the quadratic equation.

2. If A, G, H are A.M., G.M. and H.M. between three given numbers a, b and c, then the equation having a, b, c as its roots is $X^3 - 3\,AX^2 + \frac{3G^3}{H} - G^3 = 0$.

3. If in an A.P., the sum of m terms is equal to n and the sum of n terms is equal to m, then find the sum of $(m + n)$ terms.

4. If $(b + c), (c + a), (a + b)$ are in H.P., then what is the series of a^2, b^2, c^2 ?

5. If the ath, bth and cth terms of a G.P. are P, Q, R respectively, then show that

$$P^{b-c} \times Q^{c-a} \times R^{a-b} = 1$$

6. If for a G.P. if $T_{m+n} = P$ and $T_{m-n} = Q$, then find out T_m.

7. The ratio of the sum of m and n terms of an A.P. is $m^2 : n^2$, then find the ratio of mth and nth term.
8. If H be the H.M. between a, b, then prove that
   (i) $(H - 2a)(H - 2b) = H^2$
   (ii) $\frac{1}{H-a} + \frac{1}{H-2b} = \frac{1}{a} + \frac{1}{b}$
   (iii) $\frac{H+a}{H-a} + \frac{H+b}{H-b} = 2$
9. Find the value of ${}^{8}\Sigma_{r=1}(3n^3 + 2n)$.
10. Find the value of ${}^{10}\Sigma_{r=1}(r+1)^2$.
11. Express the following summations in Σ sign
    (i) $3b + 12b^2 + 48b^3 + 192b^4 + 768b^5$
    (ii) $1\cdot2 + 2\cdot5 + 3\cdot10 + 4\cdot17 + 5\cdot26 + \ldots\ldots + n(n^2+1)$
12. Find the sum of the series of n terms
    $(4^2 - 1^2) + (5^2 - 2^2) + (6^2 - 3^2) + (7^2 - 4^2) + \ldots\ldots$
13. Find the sum of first n terms of the series—
    $(1^2 + 3^2 + 3^2) + (2^2 + 4^2 + 5^2) + (3^2 + 5^2 + 7^2) + (4^2 + 6^2 + 9^2) + \ldots\ldots$
14. Find the sum of first n terms of the series
    $(5^3 + 3^3) + (7^3 + 5^3) + (9^3 + 7^3) + (11^3 + 9^3) + \ldots\ldots$
15. Find the sum of first n terms of the following series
    $\frac{1}{3\cdot4} + \frac{1}{4\cdot5} + \frac{1}{5\cdot6} + \frac{1}{6\cdot7} + \frac{1}{7\cdot8} + \ldots\ldots$
16. Find the sum of first n terms of the series
    $3 - 5 + 7 - 9 + 11 - 13 + 15 - 17 + \ldots\ldots$
17. Find the number of terms in the series $20, 19\frac{1}{3}, 18\frac{2}{3}, \ldots\ldots$ of which the sum is 300.
18. The third term of an A.P. is 7 and its 7th term is 2 more than thrice of its 3rd term. Find the first term, common difference and the sum of its first 20 terms.
19. If the sum of first 8 and 19 terms of an A.P. are 64 and 361 respectively. Find the common difference and sum of its n terms.
20. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120^0 and the common difference is 5^0. Find the number of sides of the polygon.
21. Find the sum of n terms of the series
    $\log a + \log\frac{a^2}{b} + \log\frac{a^3}{b^2} + \log\frac{a^4}{b^3} + \ldots\ldots$ to n terms.
22. A class consists of number of boys whose ages are in A.P. The common difference being four months. If the youngest boy is just eight years old and if the sum of the ages is 168 years. Find the number of boys.
23. The mth term of an A.P. is n and its nth term is m. Find the $(m+n)$th term.
24. Between 1 and 31 are inserted m arithmetic means so that ratio of the 7th and $(m-1)$th means is 5 : 9. Find the value of m.
25. The sum of three numbers in A.P. is 15, whereas sum of their squares is 83. Find the numbers.
26. There are two A.P.'s whose common differences differ by unity but sum of three consecutives terms in each is 15. If P and P_1 be the products of these terms such that $\frac{P}{P_1} = \frac{7}{8}$, then find two A.P.'s.
27. The pth term of A.P. is a and qth term is b, then find the sum of $(p+q)$ term.
28. There are two A.P.'s each of n terms
    $a, a+d, a+2d, \ldots\ldots\ldots\ldots L$
    $p, p+q, a+2q, \ldots\ldots\ldots\ldots L^1$
    These A.P.'s satisfy the following conditions—
    $$\frac{L}{p} = \frac{L^1}{a} = 4, \qquad \frac{S_n}{S^1{}_n} = 2$$
    Find out $\frac{d}{q}$ and $\frac{L}{L_1}$.
29 There are n A.P.'s whose common differences are 1, 2, 3, ... n respectively, the first term of each being unity. Find sum of their nth term.
30. If $S_1, S_2, S_3, \ldots\ldots S_m$ are the sums of n terms of m A.P.'s whose first terms are 1, 2, 3, ... m and common difference are 1, 3, 5, ... $2m - 1$ respectively. Show that
    $S_1 + S_2 + S_3 + \ldots\ldots + S_m = \frac{1}{2}mn(mn+1)$.
31. Find the first term and common ratio for a G.P. whose sum to infinity is $3\frac{1}{8}$ and whose second term is $\frac{1}{2}$.
32. Find the sum of all odd numbers between 1 and 1000 which are divisible by 3.
33. Find four numbers in A.P. whose
    (a) Sum is 20 and Sum of their squares is 120.
    (b) Sum is 32 and Sum of squares is 276.
34. Divide 28 into four parts in A.P., so that ratio of the product of first and third with the product of second and fourth is 8 : 15.

Exercise B

1. Find the Sum of series—
   (i) $(a+b) + (a^2 + 2b) + (a^3 + 3b) + \ldots\ldots$ to n terms.
   (ii) $(x+y)(x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots + n$ terms.
2. The fifth term of a G.P. is 81, whereas its second term is 24. Find the series and sum of its first eight terms.
3. The first term of an infinite G.P. is 1 and any term is equal to the sum of all the succeeding terms. Find the series.

4. If $x = 1 + a + a^2 + a^3 + a^4 + \ldots\ldots$ to ∞ $\quad (|a| < 1)$

$y = 1 + b + b^2 + b^3 + \ldots\ldots$ to ∞ $\quad (|b| < 1)$

Now, find out $1 + a \cdot b + a^2 . b^2 + a^3 \cdot b^3 + \ldots\ldots \infty$

5. In an increasing G.P., the sum of the First and the last term is 66. The products of the second and the last but one term is 128 and the sum of all the terms is 126. How many terms are there in the progression ?

6. The Sum of three numbers in G.P. is 14. If the first two terms are each increased by 1 and the third term decreased by 1, the resulting numbers are in A.P. Find the numbers.

7. In a set of four numbers the first three are in G.P. and the last three in A.P. with common difference 6. If the first number is the same as the fourth. Find the four numbers.

8. Three numbers from a G.P. If the 3rd term is decreased by 64, then the three numbers thus obtained will consecutive on A.P. If the second term of this A.P. is decreased by 8 a G.P. will be formed again determine the numbers.

9. The consecutive digits of a three digit number are in G.P. If the middle digit be increased by 2, then they form an A.P. If 792 is subtracted from this number the we get the number consisting of same three digits but in reverse order. Find the number.

10. If p, q, r, s, t form an A.P. Find the values of $p - 4q + 6r - 4s + t$.

11. If the sum of n terms of an A.P. is $(2n^2 + 3n)$. Find the t_n term and common difference.

12. If one G.M. G and two arithmetic means A and B be inserted between any two given numbers, then show that $G^2 = (2A - B)(2B - A)$.

13. In a G.P. of real numbers, it is given that $T_1 + T_2 + T_3 + T_4 = 30$ and

$T_1^2 + T_2^2 + T_3^2 + T_4^2 = 340$ determine G.P.

14. Find the sum of n terms of the series

$$\frac{3}{1^2} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + \frac{9}{1^2 + 2^2 + 3^2 + 4^2} + \ldots\ldots = \frac{6n}{n+1}$$

15. If a, b and l be the first, second and the last term of a G.P. respectively, then find the sum of the G.P.

16. If S_n, S_{2n} and S_{3n} are the sum of first n, $2n$ and $3n$ terms respectively of a G.P., then show that

$S_n \times (S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$

17. If a, b and c are in A.P., then

$10^{ax + 10}, 10^{bx + 10}, 10^{cx + 10} \quad [x \neq 0]$

Find the series of given numbers.

18. Find the value of $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots\ldots \infty$.

19. Find the sum of series whose mth term is $6m(m + 2)$ where $m = 1, 2, 3, \ldots\ldots$ and series has total number to terms $= n$.

20. Find the sum of the series of n terms

$(6^2 - 3^2) + (7^2 - 4^2) + (8^2 - 5^2) + (9^2 - 6^2) \ldots\ldots n$th

21. Sum of p, q, r terms of an A.P. be a, b and c respectively, then prove that

$$\frac{a}{p}(q - r) + \frac{b}{q}(r - p) + \frac{c}{r}(p - q) = 0$$

22. Find the sum of first n terms of the series

$(2^2 + 4^2 + 5^2) + (3^2 + 5^2 + 7^2) + (4^2 + 6^2 + 9^2) + \ldots$

23. Find the sum of first n terms of the series

$(5^3 + 3^3) + (7^3 + 5^3) + (9^3 + 7^3) + (11^3 + 9^3) + \ldots\ldots$

24. Find the sum of first n terms of the following series

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \ldots\ldots$$

25. Find the sum of n terms of the series

$7 + 77 + 777 + 7777 +$ ———— to n terms

26. Find the sum of n terms of the series—

$1^2 - 2^2 + 3^2 - 4^3 + 5^2 - 6^2 + \ldots\ldots$

27. Find the sum of the series $31^3 + 32^3 + \ldots\ldots + 50^3$

28. Find the sum of n terms of the series

$3 + 8 + 22 + 72 + 266 + 1036 + \ldots\ldots$

29. Find the sum of all possible products of the first n natural numbers taken two by two.

30. Find sum of the series

$n \cdot 1 + (n - 1) \cdot 2 + (n - 2) \cdot 3 + \ldots\ldots + 1 \cdot n$

31. Find the value of the expression

$1 \cdot (2 - w)(2 - w^2) + 2(3 - w)(3 - w^2) + \ldots + (n - 1)(n - w)(n - w^2) \ldots\ldots$

where w is an imaginary cube root of unity.

32. Find the sum S_n of cubes of the first n terms of an A.P. and show that the sum of first n terms of the A.P. is a factor of S_n.

33. 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more workers dropped the third day and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.

34. Along a road lay an odd number of stones placed at intervals of 10 metres. These stones had to be assembled around the middle stone. A person could carry only one stone at a time. A man started the job with one of the end stones by carrying all the stones he covered a distance of 3 km. Find the number of stones.

35. Find the sum of integers from 1 to 100, which are divisible by 2 or 5.

36. Find the sum of all odd integers between 2 and 100 divisible by 3.

37. Find the sum of all natural numbers which are multiples of 7 or 3 or both and lie between 200 and 500.

Answers

Exercise A

1. Let roots are a and b.

According to question—

$$\text{A.M.} = \frac{a+b}{2}$$

$$5 = \frac{a+b}{2}$$

$$a+b = 10 \qquad \ldots(1)$$

$$\text{G.M.} = \sqrt{ab}$$

$$4 = \sqrt{ab}$$

$$ab = 16 \qquad \ldots(2)$$

If a, b are the roots of a quadratic equation, then

$$X^2 - (a+b)X + ab = 0$$

or $\quad X^2 - 10X + 16 = 0$

2. According to question—

$$3A = a+b+c \qquad \ldots(1)$$

$$G^3 = a \times b \times c \qquad \ldots(2)$$

$$\frac{1}{H} = \frac{1}{3}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \qquad \ldots(3)$$

$$a \times b + b \times c + c \times a = \frac{3G^3}{H}$$

If l, m, n be the roots of a cubic equation, then

$$X^3 - (l+m+n)X^2 + (lm+ln+nm)X - lmn = 0$$

So, $\quad X^3 - 3AX^2 + \frac{3G^3}{H}X - G^3 = 0$

3. Let a be the Ist term and d is the common difference.

According to question—

$$S_m = \text{Sum of } m \text{ terms}$$

$$= \frac{m}{2}\{2a + (m-1)d\} = n \qquad \ldots(1)$$

$$S_n = \frac{n}{2}\{2a + (n-1)d\} = m \qquad \ldots(2)$$

From equation (1) $\frac{2n}{m} = \{2a + (m-1)d\}$

From equation (2) $\frac{2m}{n} = \{2a + (n-1)d\}$

Now, equation (1) – equation (2), we get

$$\frac{2(n^2-m^2)}{mn} = (m-n)d$$

$$d = -\frac{2(m+n)}{mn}$$

and now, From equation (1) $2a = \frac{2n}{m} - (m-1)d$

$$2a = \frac{2n}{m} + (m-1) \times \frac{2(m+n)}{mn}$$

$$a = \frac{n^2+m^2+mn-m-n}{mn}$$

$$S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)d\}$$

By putting the value of a and

$$S_{m+n} = \frac{m+n}{2}\left[2 \times \frac{m^2+n^2+mn-m-n}{mn} - \frac{2(m+n-1)(m+n)}{mn}\right]$$

$$= 2 \times \frac{m+n}{2} \times \frac{1}{mn}[m^2+n^2+mn-m-n - \{m^2+2mn+n^2-m-n\}]$$

$$= \frac{m+n}{mn}(-mn)$$

$$S_{m+n} = -(m+n)$$

4. Since, $(b+c), (c+a), (a+b)$ are in H.P.

$\therefore \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

Now, common difference remains constant.

So, $\quad \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

$$\frac{2}{c+a} = \frac{1}{a+b} + \frac{1}{b+c} = \frac{2b+a+c}{(a+b)(b+c)}$$

or $\quad 2(ab+b^2+bc+ac) = (2b+a+c)(c+a)$

or $\quad 2ab+2b^2+2bc+2ac = 2bc+2ab+a^2+c^2 + 2ac \; a^2+c^2 = 2b^2$

That's mean a^2, b^2, c^2 are in AP.

5. Let $\quad$ Ist term $= X$

Common ratio $= Y$

Now, $\quad P = X \times Y^{a-1}$

$$Q = X \times Y^{b-1}$$

$$R = X \times Y^{c-1}$$

Now, $\quad P^{b-c} = (X \cdot Y^{a-1})^{(b-c)} = X^{b-c} \times Y^{(a-1)(b-c)}$

$$Q^{c-a} = (X \cdot Y^{b-1})^{(c-a)} = X^{c-a} \times Y^{(b-1)(c-a)}$$

$$R^{a-b} = (X \cdot Y^{c-1})^{(a-b)} = X^{a-b} \times Y^{(c-1)(a-b)}$$

$$P^{b-c} \times Q^{c-a} \times R^{a-b} = X^0 \times Y^0 = 1$$

Hence, proved $P^{b-c} \times Q^{c-a} \times R^{a-b} = 1$

6. Let a be the Ist term

$$r = \text{common ratio}$$

Now, $\quad T_{m+n} = P = a \times r^{m+n-1} \qquad \ldots(1)$

$$T_{m-n} = Q = a \times r^{m-n-1} \qquad \ldots(2)$$

From equation (1) and equation (2), we get

$$\frac{P}{Q} = r^{2n} \Rightarrow r = \left(\frac{P}{Q}\right)^{\frac{1}{2n}} \qquad \ldots(3)$$

Now, $\quad P = a \times r^{m+n-1}$

$$P = a \times \left(\frac{P}{Q}\right)^{\frac{m+n-1}{2n}}$$

$$a = P \times \left(\frac{Q}{P}\right)^{\frac{m+n-1}{2n}} \qquad \ldots(4)$$

$$T_m = a \times r^{m-1}$$

$$= P \times \left(\frac{Q}{P}\right)^{\frac{m+n-1}{2n}} \left[\left(\frac{P}{Q}\right)^{\frac{1}{2n}}\right]^{m-1}$$

$$= P \times \left(\frac{Q}{P}\right)^{\frac{m+n-1}{2n}} \left[\left(\frac{Q}{P}\right)^{\frac{m-1}{2n}}\right]$$

$$= P \times \left(\frac{Q}{P}\right)^{\frac{m+n-1}{2n} - \frac{m-1}{2n}}$$

$$T_m = \sqrt{P \cdot Q}$$

7. Let a = Ist term of an A.P.

d = Common difference

Then, $S_m = \frac{m}{2}\{2a + (m-1)d\}$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

Given $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

or $\frac{m}{n} \times \frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m^2}{n^2}$

or $\frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n}$

Now, replaying m by $(2m - 1)$ and n by $(2n - 1)$, we get

$$\frac{2a + 2(m-1)d}{2a + 2(n-1)d} = \frac{2m-1}{2n-1}$$

or $\frac{a + (m-1)d}{a + (n-1)d} = \frac{2m-1}{2n-1}$

$\therefore$ $\frac{t_m}{t_n} = \frac{2m-1}{2n-1}$

8. (i) Since, H be H.M. of a, b.

$$H = \frac{2ab}{a+b}$$

$$H - 2a = 2a \times \left(\frac{b-a-b}{a+b}\right) = \frac{-2a^2}{a+b}$$

$$H - 2b = \frac{-2a^2}{a+b}$$

$$(H-2a)(H-2b) = \frac{4a^2b^2}{(a+b)^2} = \left(\frac{2ab}{a+b}\right)^2$$

$$= H^2$$

(ii) $H - a = \frac{2ab}{a+b} - a = a \times \left(\frac{2b-a-b}{a+b}\right)$

$$= \frac{2ab - a^2 - ab}{a+b} = -a \times \left(\frac{a-b}{a+b}\right)$$

$$H - b = b \times \left(\frac{a-b}{a+b}\right) = \frac{2ab - b^2 - ab}{a+b}$$

$$= -b \times \left(\frac{a-b}{a+b}\right)$$

$$\frac{1}{H-a} + \frac{1}{H-b} = \left[\frac{1}{-a\left(\frac{a-b}{a+b}\right)} + \frac{1}{b\left(\frac{a-b}{a+b}\right)}\right]$$

$$= \left(\frac{a+b}{a-b}\right)\left[\frac{1}{-a} + \frac{1}{b}\right]$$

$$\left[\frac{(a+b)(a-b)}{(a-b)ab}\right] = \frac{1}{a} + \frac{1}{b}$$

(iii) $H + a = \frac{2ab}{a+b} + a = a \times \left(\frac{2b+a+b}{a+b}\right)$

$$= a \times \left(\frac{3b+a}{a+b}\right)$$

$$H + b = \frac{2ab}{a+b} + b = b \times \left(\frac{2b+a+b}{a+b}\right)$$

$$= b \times \left(\frac{3a+b}{a+b}\right)$$

$$\frac{H+a}{H-a} = \frac{a(3b+a)}{a(b-a)} = \left(\frac{3b+a}{b-a}\right)$$

$$\frac{H+b}{H-b} = \left(\frac{3a+b}{a-b}\right)$$

$$\frac{H+a}{H-a} + \frac{H+b}{H-b} = -\left(\frac{3a+b}{b-a}\right) + \left(\frac{3b+a}{b-a}\right)$$

$$= \frac{-2a+2b}{b-a} = 2$$

$$\frac{H+a}{H-a} + \frac{H+b}{H-b} = 2$$

9. We know that

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Now, ${}^{8}\Sigma_{r=1}(3n^3 + 2n) = 3(1^3 + 2^3 + 3^3 + \ldots + 8^3) + 2(1^2 + 2^2 + 3^2 + \ldots + 8^2)$

$${}^{8}\Sigma_{r=1}(3n^3 + 2n) = 3\left(\frac{8(8+1)}{2}\right)^2 + 2 \times \frac{8(8+1)(17)}{6}$$

$$= 3 \times 16 \times 81 + \frac{8}{3} \times 9 \times 17$$

$$= 3888 + 408 = 4296$$

10. ${}^{10}\Sigma_{r=1}(r+1)^2 = {}^{10}\Sigma_{r=1}(r^2 + 2r + 1)$

$= (1^2 + 2^2 + 3^2 + \ldots + 10^2) + (2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3 + \ldots\ldots + 2 \cdot 10) + (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1)$

$= \frac{10(10+1)(21)}{6} + 2 \times \frac{10 \times 11}{2} + 10$

$= \frac{5}{3} \times 11 \times 21 + 55 \times 2 + 10$

$= 35 \times 11 + 110 + 10 = 385 + 120 = 505$

11. (i) $3b + 12b^2 + 48b^3 + 192b^4 + 768b^5$

1st term $= 3b$

2nd term $= 12b^2$

Common ratio $= 4b$

$\therefore 3b + 12b^2 + 48b^3 + 192b^4 + 768b^5$

$= 3b + 3\cdot(4b)\cdot b + 3b\cdot(4b)^2 + 3b\cdot(4b)^3 + \ldots\ldots$

$= {}^{n}\Sigma_{r=1}\, 3b(4b)^{n-1}$

We have 5 term, so $n = 5$

${}^{5}\Sigma_{r=1}\, 3b(4b)^{n-1}$

(ii) $1\cdot2 + 2\cdot5 + 3\cdot10 + 4\cdot17 + 5\cdot26 + \ldots\ldots + n(n^2+1)$

Ist digits of each term forms an A.P., so its rth term $= r$

2nd digits of each term are 2, 5, 10, 17, 26,

$2, 2^2+1, 3^2+1, 4^2+1, 5^2+1, \ldots\ldots,$

$1^2+1, 2^2+1, 3^2+1, 4^2+1, 5^2+1, \ldots\ldots$

$\therefore$ rth term $= r^2 + 1$

Now, rth term of above series $= r(r^2+1)$

Now, ${}^{n}\Sigma_{r=1}\, r(r^2+1)$

12. Clearly, rth term $= \frac{(r+3)^2 - r^2}{r^2} = 6r + 9$

Now, $S_n = {}^{n}\Sigma_{r=1}\,(6r+9)$

$= 6 \times \frac{n(n+1)}{2} + 9n$

$= 3n(n+1+3)$

$= 3n(n+4)$

13. We can write the above series as

$(1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots)$ …(1)

$+ (3^2 + 4^2 + 5^2 + 6^2 + \ldots\ldots)$ …(2)

$+ (3^2 + 5^2 + 7^2 + 9^2 + \ldots\ldots)$ …(3)

Now, rth of 1st $= r^2$

rth of 2nd $= (r+1)^2$

rth of 3rd $= (2r+1)^2$

Now, rth term $= r^2 + (r+1)^2 + (2r+1)^2$

t_r term $= r^2 + r^2 + 4r + 4 + 4r^2 + 4r + 1$

$= 6r^2 + 8r + 5$

Now, ${}^{n}\Sigma_{r=1}\, t_r = {}^{n}\Sigma_{r=1}\,(6r^2 + 8r + 5)$

$= 6 \times \frac{n(n+1)(2n+1)}{6} + 8 \times \frac{n(n+1)}{2} + 5n$

$= (n^2+n)(2n+1) + 4n^2 + 4n + 5n$

$= 2n^3 + n^2 + 2n^2 + n + 4n^2 + 9n$

$= 2n^3 + 7n^2 + 10n$

14. $(5^3 + 3^3) + (7^3 + 5^3) + (9^3 + 7^3) + (11^3 + 9^3) + \ldots$

$= (5^3 + 7^3 + 9^3 + \ldots) =$ its rth term $= (2r+3)^3$

$+ (3^3 + 5^3 + 7^3 + \ldots) =$ it rth term $= (2r+1)^3$

Now, rth term of the series $= (2r+1)^3 + (2r+3)^3$

$t_r = 16r^3 + 48r^2 + 60r + 28$

$S_n = {}^{n}\Sigma_{r=1}\,(16r^3 + 48r^2 + 60r + 28)$

$= 16 \times \left(\frac{n(n+1)}{2}\right)^2 + 48 \times \left(\frac{n(n+1)(2n+1)}{6}\right) + 60 \times \left(\frac{n(n+1)}{2}\right) + 28n$

$= 4 \times n^2(n^2 + 2n + 1) + 8 \times (n^2+n)(2n+1) + 30(n^2+n) + 28n$

$= 2n(2n^3 + 12n^2 + 29n + 33)$

15. rth term of the given series is represented by $\frac{1}{(r+2)(r+3)}$

$t_r = \frac{1}{r+2} - \frac{1}{r+3}$

$t_1 = \frac{1}{3} - \frac{1}{4}$

$t_2 = \frac{1}{4} - \frac{1}{5}$

$t_3 = \frac{1}{5} - \frac{1}{6}$

………… = …………

………… = …………

………… = …………

$t_n = \frac{1}{n+2} - \frac{1}{n+3}$

$S_n = t_1 + t_2 + t_3 + \ldots\ldots + t_n = \frac{1}{3} - \frac{1}{n-3}$

$S_n = \frac{n}{3(n+3)}$

16. When n is even $n = 2m$

Then, $3 - 5 + 7 - 9 + 11 - 13 + 15 - 17 + \ldots\ldots$

$S_n = (-2) + (-2) + (-2) + \ldots\ldots$ m times

$S_n = -2m$

$S_n = -2 \times \frac{n}{2} = -n$ when n is even.

When n is odd $n = 2m - 1$

$3 - 5 + 7 - 9 + 11 - 13 + 15 - 17 + \ldots\ldots$

$= (3 + 7 + 11 + 15 + \ldots\ldots + m \text{ terms}) - (5 + 9 + 13 + 17 + \ldots\ldots (m-1) \text{ terms})$

$S_n = 3 + (-5 + 7) + (-9 + 11) + \ldots\ldots \frac{2m-2}{2}$

$= 3 + 2 + 2 + 2 + \ldots\ldots (m-1)$ times

$= 3 + 2(m-1)$

$= 2m + 1$

$S_n = n + 1$ where n is odd

17. Given Ist term $(a) = 20$

Common difference $(d) = 19\frac{1}{3} - 20 = -\frac{2}{3}$

Let n be the number of terms.

Using formula—

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\text{or} \quad 300 = \frac{n}{2}\left[40 + (n-1)\left(-\frac{2}{3}\right)\right]$$

$$\text{or} \quad 300 = \frac{n}{2} \times \left[\frac{120 - 2n + 2}{3}\right]$$

$$\text{or} \quad 300 = \frac{n}{2} \times \frac{122 - 2n}{3} = \frac{n(61-n)}{3}$$

$$\text{or} \quad n^2 - 61n + 900 = 0$$

$$\text{or} \quad n^2 - 36n - 25n + 900 = 0$$

$$\text{or} \quad n(n-36) - 25(n-36) = 0$$

$$n = 25, \; n = 36$$

It has double answers.

Because, the last eleven terms (36 – 25) are present whose sum is zero.

18. Let Ist term $= a$

Common difference $= d$

According to question—

$$t_3 = a + 2d = 7 \quad \ldots(1)$$

$$t_7 = a + 6d = 3 \times 7 + 2 = 23 \quad \ldots(2)$$

Now, from equations (1) and (2), we get

$$4d = 16$$

$$d = 4$$

$$a = 7 - 2d = 7 - 8 = -1$$

$$\text{Now,} \quad S_{20} = \frac{20}{2} \times [2a + (n-1)d]$$

$$= 10 \times [2(-1) + 19(4)]$$

$$= 740$$

19. Let Ist term $= a$

Common difference $= d$

$$\text{Now,} \quad S_8 = \frac{8}{2} \times [2a + (8-1)d]$$

$$= 4 \times [2a + 7d]$$

$$64 = 8a + 28d$$

$$16 = 2a + 7d \quad \ldots(1)$$

$$S_{19} = \frac{19}{2}[2a + (19-1)d]$$

$$361 = \frac{19}{2} \times [2a + 18d]$$

$$19 = a + 9d \quad \ldots(2)$$

Now, equation (1) – equation (2), we get

$$38 - 16 = 18d - 7d$$

$$\text{or} \quad 22 = 11d$$

$$\text{or} \quad d = 2 \text{ and } a = 1$$

$$\text{So,} \quad S_n = \frac{n}{2}[2 \times 1 + (n-1) \times 2]$$

$$= n^2$$

20. Let the number of the sides of the polygon $= n$

Ist angle of the polygon $= 120°$

$$d = 5°$$

Now, Sum of all the interior angles $= (n - 2) \times 180 \ldots(1)$

$$\text{Sum of angles} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[240 + (n-1) \times 5] \quad \ldots(2)$$

Since, equation (1) and equation (2) are equal.

$$(n-2) \times 180 = \frac{n}{2}[235 + 5n]$$

$$\text{or} \quad (n-2) \times 180 \times 2 = 5n(47 + n)$$

$$\text{or} \quad n^2 + 47n = (n-2) \times 72$$

$$\text{or} \quad n^2 - 25n + 144 = 0$$

$$\text{or} \quad n = 9, 16$$

$$\text{If } n = 16 \quad T_n = a + (16-1)d$$

$$= 120 + 15 \times 5 = 195°$$

Since, interior angle can not be greater than 180°.

Hence, only $n = 9$ acceptable.

21. Ist term of the series $= \log a$

Common difference $= \log \frac{a^2}{b} - \log a = \log \frac{a}{b}$ = Constant

$$\text{So,} \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}\left[2 \log a + (n-1) \times \log \frac{a}{b}\right]$$

$$= \frac{n}{2}\left[n \log \frac{a}{b} + \log ab\right]$$

22. According to question—

First term $= a = 8$ years

Common difference $= d = \frac{4}{12} = \frac{1}{3}$ years

Let n be the number of boys in the class.

$$\text{So,} \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\text{or} \quad 168 = \frac{n}{2}\left[2 \times 8 + (n-1) \times \frac{1}{3}\right]$$

$$\text{or} \quad 168 \times 2 = n \times \frac{(48 + n - 1)}{3}$$

$$\text{or} \quad (n + 47) \times n = 168 \times 2 \times 3$$

or $n^2 + 47n - 168 \times 2 \times 3 = 0$

or $n^2 + 63n - 16n - 168 \times 6 = 0$

or $n(n + 63) - 16(n + 63) = 0$

$n = 16$

So, 16 boys are present in class.

23. Let Ist term $= a$

Common difference $= d$

$$t_m = a + (m - 1)d = n \quad \ldots(1)$$

$$t_n = a + (n - 1)d = m \quad \ldots(2)$$

From equations (1) and (2), we get

$$(m - 1)d - (n - 1)d = n - m$$

or $(m - n)d = n - m$

$$d = -1 \quad \ldots(3)$$

$$a + (m - 1)(-1) = n$$

or $a = n + m - 1 \quad \ldots(4)$

Now, $t_{m+n} = a + (m + n - 1)d$

$= (m + n - 1) + (m + n - 1)(-1)$

$t_{m+n} = 0$

24. Let d be the common difference.

First term $= 1$

Last term $= 31$

Since, m terms are inserted between 1 and 31 so, 31 is the $(m + 2)$th term of A.P.

Now, $31 = t_{m+2} = a + (m + 2 - 1)d$

$$31 = a + (m + 1)d \quad \ldots(1)$$

$$\frac{a + 7d}{a + (m - 1)d} = \frac{5}{9}$$

$$4a = 5md - 5d - 63d$$

or $4a = 5md - 68d$

or $(5m - 68)d = 4$

or $(5m - 68) \times \dfrac{30}{m + 1} = 4$

or $(5m - 68) \times 30 = 4m + 4$

or $75m - 68 \times 15 = 2m + 2$

or $73m = 1022$

or $m = \dfrac{1022}{73} = 14$

25. Let the three numbers of A.P. are $\alpha - \beta, \alpha, \alpha + \beta$.

According to question—

$$\alpha - \beta + \alpha + \alpha + \beta = 15$$

$$3\alpha = 15 \Rightarrow \alpha = 5$$

and $(\alpha - \beta)^2 + \alpha^2 + (\alpha + \beta)^2 = 83$

or $(5 - \beta)^2 + 25 + (5 + \beta)^2 = 83$

or $2(25 + \beta^2) = 83 - 25 = 58$

or $25 + \beta^2 = 29$

or $\beta^2 = 4 = 2^2$

or $\beta = \pm 2$

Now, the number are when $\beta = 2$, $\quad 3, 5, 7, \ldots\ldots$

When $\beta = -2$, $\quad 7, 5, 3, \ldots\ldots$

26. Let the three consecutive terms of Ist series are $a - d$, a, $a + d$

and 2nd series has $A - b$, A, $A + b$

Given $b = d + 1 \quad \ldots(1)$

$$\frac{P}{P_1} = \frac{(a^2 - d^2)a}{(A^2 - b^2)A}$$

or $\dfrac{7}{8} = \dfrac{(a^2 - d^2)a}{[A^2 - (b + 1)^2]A} \quad \ldots(2)$

Since, $a - d + a + a + d = S = 15$

$3a = 15, \ a = 5$

and $A - b + A + A + b = 15, \ A = 5$

Now, putting the value of a and A in equation (2), we get

$$\frac{7}{8} = \frac{(25 - d^2)5}{[25 - (d + 1)^2]5}$$

or $\dfrac{7}{8} = \dfrac{25 - d^2}{25 - (d + 1)^2}$

or $7 \times 25 - 7(d + 1)^2 = 25 \times 8 - 8d^2$

or $8d^2 - 7(d + 1)^2 = 25$

or $d^2 - 14d - 7 = 25$

or $d^2 - 14d - 32 = 0$

or $d^2 - 16d + 2d - 32 = 0$

or $d(d - 16) + 2(d - 16) = 0$

$d = 16$

Now, Ist series is $-11, 5, 21, \ldots\ldots$

2nd series is $-12, 5, 22, \ldots\ldots$

27. A be the Ist term and d be the common difference.

So, $t_p = A + (p - 1)d = a \quad \ldots(1)$

$t_q = A + (q - 1)d = b \quad \ldots(2)$

Now, equation (1) – equation (2), we get

$$(p - q)d = a - b$$

$$d = \frac{a - b}{p - q} \quad \ldots(3)$$

$$A + (p - 1)d = a$$

$$A = a - (p - 1)\left(\frac{a - b}{p - q}\right)$$

Now, $A = \dfrac{a(p - q) - (p - 1)(a - b)}{p - q}$

or $A = \dfrac{-aq + pb + a - b}{p - q}$

Now, $S_{p+q} = \dfrac{p + q}{2}[2A + (p + q - 1)d]$

$$= \frac{p + q}{2}\left[2 \times \frac{a(1 - q) + b(p - 1)}{p - q} + (p + q - 1)\frac{a - b}{p - q}\right]$$

$= \frac{p+q}{2} \times \frac{1}{p-q}$ $[2a(1-q) + 2b(p-1) + (p+q-1)$
$(a-b)]$

$= \frac{p+q}{2}\left[a + b + \frac{a-b}{p-q}\right]$

28. From Ist series

$$L = a + (n-1)d \qquad \ldots(1)$$

$$L^1 = p + (n-1)q \qquad \ldots(2)$$

Given $\frac{L}{p} = \frac{L^1}{a} = 4$

From First two $\frac{L}{L^1} = \frac{p}{a} \qquad \ldots(3)$

$$a + (n-1)d = 4p$$

$$p + (n-1)q = 4a$$

$$\frac{4p-a}{4a-p} = \frac{d}{q}$$

and $\frac{S_n}{S^1{}_n} = \frac{\frac{n}{2}(a+b)}{\frac{n}{2}(p+L^1)} = \frac{a+4p}{p+4a} = 2$

$$2p = 7a$$

Or $\frac{a}{2} = \frac{p}{7} = k$

$$\frac{d}{q} = \frac{28k-2k}{8k-7k} = 26$$

$\therefore$ $\frac{L}{L^1} = \frac{p}{a} = \frac{7}{2}$

29. Ist term of all A.P.'s = 1

Now, nth term of 1st A.P. = $1 + (n-1)\, 1 = n$

nth term of 2nd A.P. = $1 + (n-1)\, 2 = 2n - 1$

nth term of 3rd A.P. = $1 + (n-1)\, 3 = 3n - 2$

Similarly,

nth term of nth A.P. = $1 + (n-1)n = n^2 - (n-1)$

Now, their sum is

$$S_n = [n + 2n - 1 + 2n - 2 + \ldots\ldots + n^2 - (n-1)]$$

$$S_n = n + (n-1) \times \frac{n(n+1)}{2} = \frac{n}{2}(n^2+1)$$

30. Now, $S_1 = \frac{n}{2}[2 \times 1 + (n-1)1] = \frac{n}{2}[2 \times 1 + (n-1)1]$

$$S_2 = \frac{n}{2}[2 \times 2 + (n-1)3]$$

$$S_3 = \frac{n}{2}[2 \times 3 + (n-1)5]$$

Now, $S_1 + S_2 + S_3 + \ldots\ldots + S_m$

$= \frac{n}{2}[2 \times 1 + (n-1)1] + \frac{n}{2}[2 \times 2 + (n-1)3]$

$+ \frac{n}{2}[2 \times 3 + (n-1)5] + \ldots\ldots + \frac{n}{2}[2 \times m$
$+ (n-1)(2m-1)]$

$= \frac{n}{2}[(2 + 2 \times 2 + 2 \times 3 + 2 \times 4 + \ldots\ldots + 2 \times m)$
$+ (n-1)(1 + 3 + 5 + 7 + \ldots\ldots + (2m-1)]$

$= \frac{n}{2}\left[2 \times \frac{m(m+1)}{2} + (n-1)m^2\right]$

$= \frac{n}{2}(m^2 + m + n \times m^2 - m^2)$

$= \frac{n}{2}m(mn+1)$

31. Let 1st term = a

Common ratio = r

Here, $r < 1$

Now, Sum of infinity terms = $\frac{a}{1-r}$

$$3\frac{1}{8} = \frac{a}{1-r}$$

or $\frac{25}{8} = \frac{a}{1-r} \qquad \ldots(1)$

and $ar = \frac{1}{2} \Rightarrow a = \frac{1}{2r}$

$$\frac{25}{8} = \frac{\frac{1}{2r}}{1-r}$$

or $25(1-r)r = 4$

or $25r^2 - 25r + 4 = 0$

or $25r^2 - 20r - 5r + 4 = 0$

or $5r(5r-4) - r(5r-4) = 0$

or $r = \frac{4}{5}, \frac{1}{5}$

or $ar = \frac{1}{2}$

where $r = \frac{4}{5} \qquad a = \frac{5}{8}$

When $r = \frac{1}{5} \qquad a = \frac{5}{2}$

32. Odd numbers between 1 and 1000 are

3, 5, 7, 9 11, 13, 15, 17, 19, …… 999

Now, odd numbers divisible by 3 are 3, 9, 15, 21, …… 993, 999

Now, Ist term = 3

Common difference = 6

Last term = 999

If nth term be 999

Then, $3 + (n-1)6 = 999$

or $6n - 3 = 999$

or $6n = 1002$

or $n = 167$

$$S_n = \frac{167}{2}(\text{first} + \text{Last terms})$$

$$S_n = \frac{167}{2}(3+999) = \frac{167}{2} \times 501$$
$$S_n = 501 \times 167$$
$$S_n = 83667$$

33. (a) Let the four terms of A. P. are $\alpha - 3\beta, \alpha - \beta, \alpha + \beta, \alpha + 3\beta$.

According to questions—

$$\alpha - 3\beta + \alpha - \beta + \alpha + \beta + \alpha + 3\beta = 20$$
$$4\alpha = 20$$
$$\alpha = 5$$

and $(\alpha - 3\beta)^2 + (\alpha - \beta)^2 + (\alpha + \beta)^2 + (\alpha + 3\beta)^2 = 120$

or $2(\alpha + 5\beta^2) + 2(\alpha^2 + \beta^2) = 120$

or $\alpha^2 + 5\beta^2 = 30$

or $25 + 5\beta^2 = 30$

$\beta = \pm 1$

When $\beta = 1$

Numbers are 2, 4, 6, 8

When $\beta = -1$

Numbers are 8, 6, 4, 2

(b) Let the four terms of A. P. are $\alpha - 3\beta, \alpha - \beta, \alpha + \beta, \alpha + 3\beta$

$$\alpha - 3\beta + \alpha + \beta + \alpha - \beta + \alpha + 3\beta = 32$$
$$4\alpha = 32$$
$$\alpha = 8$$

and $(\alpha - 3\beta)^2 + (\alpha - \beta)^2 + (\alpha + \beta)^2 + (\alpha + 3\beta)^2 = 276$

or $4(2 + 5\beta^2) = 276$

or $\alpha^2 + 5\beta^2 = 69$

or $64 + 5\beta^2 = 69$

or $5\beta^2 = 5$

or $\beta = \pm 1$

When $\beta = 1$

Numbers are 5, 7, 9, 11

When $\beta = -1$

Numbers are 11, 9, 7, 5.

34. Let four consecutive terms of A.P. are $\alpha - 3\beta, \alpha - \beta, \alpha + \beta, \alpha + 3\beta$.

According to question—

$$\alpha + \beta + \alpha - 3\beta + \alpha - \beta + \alpha + 3\beta = 28$$

or $4\alpha = 28$

or $= 7$ …(1)

$$\frac{(\alpha - 3\beta)(\alpha + \beta)}{(\alpha - \beta)(\alpha + 3\beta)} = \frac{8}{15}$$

or $15(\alpha^2 - 2\alpha\beta - 3\beta^2) = 8(\alpha^2 + 2\alpha\beta - 3\beta^2)$

or $7\alpha^2 - 46\alpha\beta - 21\beta^2 = 0$

or $7 \times 49 - 46 \times 7\beta - 21\beta^2 = 0$

or $3\beta^2 + 46\beta - 49 = 0$

or $\beta = 1, -\frac{49}{3}$

Suitable value of $\beta = 1$

Now, numbers are 4, 6, 8, 10.

Exercise B

1. Given series can be written as

$$(a + a^2 + a^3 + \ldots + n \text{ terms}) + (b + 2b + 3b + \ldots + n \text{ terms})$$

First Series Second Series

First Series—Is in G.P. whose First term $= a$

Common ratio $= a$

Now, its Sum $= a\left(\frac{r^n - 1}{r - 1}\right) = a\left(\frac{a^n - 1}{a - 1}\right)$

Second Series—$b + 2b + 3b + 4b + \ldots\ldots + n$ terms

$b(1 + 2 + 3 + 4 + \ldots\ldots + n \text{ terms})$

$b \times$ Sum of n first natural numbers

$$= b \times \frac{n(n+1)}{2}$$

So, Net Sum $= a \times \left(\frac{a^n - 1}{a - 1}\right) + \frac{b}{2} n(n+1)$

Dividing each term by $(x - y)$ and multiplying each by $(x - y)$, we get

$$S_n = \frac{x^2 - y^2}{x - y} + \frac{x^3 - y^3}{x - y} + \frac{x^4 - y^4}{x - y} + \ldots\ldots + n \text{ terms}$$

$$= \frac{1}{x - y}[(x^2 + x^3 + x^4 + \ldots\ldots n \text{ terms}) - (y^2 + y^3 + y^4 + \ldots\ldots n \text{ terms})]$$

$$= \frac{1}{x - y}\left[\frac{x^2(1 - x^n)}{1 - x} - \frac{y^2(1 - y^n)}{1 - y}\right]$$

2. Let First term $= a$

Common ratio $= r$

Fifth term $= ar^4 = 81$

Now, $r^3 = \frac{81}{24} = \frac{27}{8} = \left(\frac{3}{2}\right)^3$

Now, $a = \frac{24}{3} \times 2 = 16$

Series is 16, 24, 36, 54, ……

Sum of its first eight terms

$$= a\left(\frac{r^8 - 1}{r - 1}\right)$$

$$= 16 \times \left[\frac{\left(\frac{3}{2}\right)^8 - 1}{\frac{3}{2} - 1}\right]$$

$$= 16 \times \frac{3^8 - 2^8}{2^8} \times 2$$

$$= 16 \times \frac{81 \times 81 - 32 \times 8}{32 \times 8} \times 2$$

$$= \frac{6561 - 256}{16} \times 2$$

$$= \frac{1}{8} \times 6305$$

3. Let First term $= a = 1$

Common ratio $= r$

According to question—

$$T_n = T_{n+1} + T_{n+2} + T_{n+3} + \ldots + \infty$$

or $a \times r^{n-1} = a \times r^n + a \times r^{n+1} + \ldots + \infty$

or $a \times r^{n-1} = \dfrac{ar^n}{1-r}$

or $(1-r) \times r^{n-1} = r^n$

or $r^{n-1} - r^n = r^n$

or $\dfrac{r^n}{r} = 2 \times r^n$

$$r = \frac{1}{2}$$

So, series is $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots\ldots$

4. $x = 1 + a + a^2 + a^3 + a^4 + \ldots\ldots \infty$

= Sum of infinity series whose common ratio is less than 1.

$$= \frac{1}{-a+1}$$

$$y = \frac{1}{1-b} \text{ or } y\,(1-b) = 1$$

$$b = 1 - \frac{1}{y} = \frac{y-1}{y}$$

Now, $1 + a.b + a^2.b^2 + a^3.b^3 + \ldots\ldots$ to ∞

$$= \frac{1}{1-a.b} = \frac{1}{1 - \frac{y-1}{y} \times \frac{x-1}{x}}$$

$$= \frac{x.y}{x+y-1}$$

5. Let First term $= a$

Common ratio $= r \quad (r > 1)$

Number of terms $= n$

Now, according to question—

$$a + a \times r^{n-1} = 66$$

$$a \times r \times a \times r^{n-2} = a^2 \times r^{n-1} = 128$$

$$a \times r^{n-1} = \frac{128}{a}$$

$$66 - a = \frac{128}{a}$$

or $a^2 - 66a + 128 = 0$

or $(a-2)\,(a-64) = 0$

or $a = 2, 64$

$$r^{n-1} = 32, \frac{1}{32}$$

$$\text{Sum} = a \times \frac{r^n - 1}{r-1} = 126$$

or $\dfrac{2(32r-1)}{r-1} = 126$

or $32r - 1 = 63 \quad (r-1)$

or $31r = 63 - 1 = 62$

$$r = 2 \quad (r > 1)$$

So, $r^{n-1} = 32, 2^{n-1} = 32 = 2^5$

$$n = 6;\ r = 2;\ a = 2$$

6. Let First term of G.P. $= a$

Common ratio $= r$

Now, according to question—

$$a + a.r + a.r^2 = 14$$

or $a \times (1 + r + r^2) = 14$

Now, $(a+1), (ar+1), (a.r^2 - 1)$ these are in A.P.

So, $2(ar+1) = (a+1) + (a.r^2 - 1)$

or $2(ar+1) = a(1+r^2)$

or $a(1 + r^2 - 2r) = 2$

$$\frac{1+r+r^2}{1+r^2-2r} = \frac{14}{2} = 7$$

$$1 + r^2 + r = 7 + 7 \times r^2 - 14r$$

$$6r^2 - 15r + 6 = 0$$

$$2r^2 - 5r + 2 = 0$$

$$r = 2, \frac{1}{2}$$

So, numbers are 2, 4, 8 or 8, 4, 2.

7. Since, the last three are in A.P.

So, these numbers are $(a-d), a, (a+d)$.

Since, these number are also in G.P.

Since, first number is same as that of last.

So, numbers in G.P. are $(a+d), (a-d), a$

$$(a-d)^2 = a(a+d) \quad (d = 6)$$

$$(a-6)^2 = a(a+6)$$

$$36 - 12a = 6a$$

$$18a = 36$$

$$a = 2$$

So, Four numbers are 8, – 4, 2, 8.

8. Let three numbers a, a, r, ar^2 are in G.P. and $a, a, r, (ar^2 - 64)$ are in A.P.

Now, $2ar = a + ar^2 - 64$

$$a \times (r^2 - 2r + 1) = 64 \qquad \ldots(1)$$

Again, $a, (ar-8), (ar^2 - 64)$ are in G.P.

$$(ar-8)^2 = a(ar^2 - 64)$$

$$-16ar + 64 = -64a$$

$$a \times (16r - 64) = 64 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{r^2 - 2r + 1}{16r - 64} = 1$$

or $\quad r^2 - 2r + 1 = 16r - 64$

or $\quad r^2 - 18r + 65 = 0$

or $\quad (r - 13)(r - 5) = 0$

$$r = 13, 5$$

So, numbers are 4, 20, 100 are in G.P.

4, 20, 36 are in A.P.

9. Let digits of the three digit number are x, y, z.

Three numbers in G.P. are $a, a.b, ab^2$.

According to question—

$a, (ab + 2), (ab^2)$ are in A.P.

So, $\quad 2(ab + 2) = a + ab^2$

Or $\quad 2ab + 4 = a(1 + b^2)$

$$a(b - 1)^2 = 4 \quad \ldots(1)$$

Again, $100\,a + 10\,ab + ab^2 - 792 = 100\,ab^2 + 10\,ab + a$

Or $\quad 99(b^2 - 1)a = -792$

Or $\quad (b^2 - 1)(b + 1)a = -8 \quad \ldots(2)$

From equations (1) and (2), we get

$$\frac{b+1}{b-1} = -2$$

$$b = \frac{1}{3}$$

$\therefore \quad a = 9$

Hence, numbers are 9, 3, 1 or 931.

10. Let d be common difference of the given A.P.

Now, $p - 4q + 6r - 4s + t$

or $= p - 4(p + d) + 6(p + 2d) - 4(p + 3d) + (p + 4d)$

$= p - 4p + 6p - 4p + p - 4d + 12d - 12d + 4d$

$= 0$

11. Given $\quad S_n$ = sum of nth term = S_n

$$S_n = 2n^2 + 3n \quad \ldots(1)$$

$$S_{n-1} = 2(n - 1)^2 + 3(n - 1) \quad \ldots(2)$$

nth term $= S_n - S_{n-1}$

$= 2(n - 1)^2 + 3(n - 1) - (2n^2 + 3n)$

$= 2(2n - 1)(-1) + 3(-1)$

$= -4n + 2 - 3n$

$t_n = 2 - 7n$

where we put $n = 1$

First term $= 2 - 7 \times 1 = 2 - 7 = -5$

12. Let G be G.M. of X and Y, then

$$G^2 = X.Y \quad \ldots(1)$$

Since, A and B be two arithmetic means between X and Y.

Then, X, A, B, Y are in A.P.

So, $\quad 2A = X + B$

$2B = A + Y$

$2A - B = X \quad \ldots(3)$

$2B - A = Y \quad \ldots(2)$

From equations (1), (2) and (3), we get

$$G^2 = (2A - B)(2B - A)$$

13. Let First term $= a$

Common ratio $= b$

$$a + ab + ab^2 + ab^3 = 30$$

$$a(1 + b + b^2 + b^3) = 30$$

Similarly, $\quad a^2(1 + b^2 + b^4 + b^6) = 340$

$$a \times \left(\frac{1 - b^4}{1 - b}\right) = 30$$

$$a^2 \times \left(\frac{1 - b^8}{1 - b^2}\right) = 340$$

or $\quad \dfrac{a^2(1 - b^8)}{1 - b^2} \times \left[\dfrac{1 - b}{a(1 - b^4)}\right]^2 = \dfrac{340}{30 \times 30}$

or $\quad \dfrac{(1 + b^4)}{1 - b^4} \times \dfrac{1 - b}{1 + b} = \dfrac{17}{45}$

or $\quad 17(1 + b + b^2 + b^3)(1 + b) = 45(1 + r^4)$

or $\quad 14b^4 - 17b^3 - 17b^2 - 17b + 14 = 0$

or $\quad 14 \times \left(b^2 + \dfrac{1}{b^2}\right) - 17\left(b + \dfrac{1}{b}\right) - 17 = 0$

Put $\quad \left(b + \dfrac{1}{b}\right) = X$

or $\quad 14(X^2 - 2) - 17X - 17 = 0$

or $\quad X = 2, \dfrac{1}{2}$

$$a = 2$$

Required G.P.'s are 2, 4, 8, 16, 32

and $\quad$ 16, 8, 4, 2, 1, ……

14. rth term of the given series is

$$t_r = \frac{3 + (r - 1)^2}{1^2 + 2^2 + 3^2 + \ldots\ldots + r^2}$$

$$t_r = \frac{2r + 1}{\frac{r(r + 1)(2r + 1)}{6}}$$

$$= \frac{6(2r + 1)}{r(r + 1)(2r + 1)} = \frac{6}{r(r + 1)}$$

$$= 6 \times \left[\frac{r + 1 - r}{r(r + 1)}\right] = 6 \times \left(\frac{1}{r} - \frac{1}{r + 1}\right)$$

$$t_1 = 6 \times \left(\frac{1}{1} - \frac{1}{2}\right)$$

$$t_2 = 6 \times \left(\frac{1}{2} - \frac{1}{3}\right)$$

$$t_3 = 6 \times \left(\frac{1}{3} - \frac{1}{4}\right)$$

....

....

$$t_n = 6 \times \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$t_1 + t_2 + t_3 + \ldots\ldots + t_n = 6 \times \left(1 - \frac{1}{n+1}\right)$$

$$= \frac{6n}{n+1}$$

15. Let last term will be nth term so $l = ar^{n-1}$

Since, b is second term so r = common difference $= \frac{b}{a}$

$$l = a \times \left(\frac{b}{a}\right)^{n-1}$$

$$\text{Sum} = a \times \left(\frac{r^n - 1}{r - 1}\right) = a \times \left[\frac{\left(\frac{b}{a}\right)^n - 1}{\frac{b}{a} - 1}\right]$$

$$= \frac{a(bl - a^2)}{a^2(b - a)} \times a = \frac{bl - a^2}{b - a}$$

16. Let First term $= a$

Common ratio $= r$

Now, sum is given by $S_n = a\left(\frac{r^n - 1}{r - 1}\right)$

Now, $S_n(S_{3n} - S_{2n})$

$$= a \times \left(\frac{r^n - 1}{r - 1}\right)\left[a \times \frac{r^{3n} - 1}{r - 1} - a \times \frac{r^{2n} - 1}{r - 1}\right]$$

$$= a^2 \times \left(\frac{r^n - 1}{r - 1}\right)\left[\frac{r^{3n} - 1 - r^{2n} + 1}{r - 1}\right]$$

$$= a^2 \times \left(\frac{r^n - 1}{r - 1}\right)^2 \times r^{2n}$$

$$= \left[a \times \frac{r^n - 1}{r - 1} \times r^n\right]^2$$

$$= \left[a \times \frac{r^{2n} - r^n}{r - 1}\right]^2$$

$$= \left[a \times \frac{r^{2n} - 1}{r - 1} - a \times \frac{r^n - 1}{r - 1}\right]^2$$

$$= [S_{2n} - S_n]^2$$

17. Since, a, b, c are in A.P.

So, $2b = c + a$

$2bx = (c + a)\,x$

Now, $10^{2(bx + 10)} = 10^{2bx + 20}$

$= 10^{(c + a)x + 20}$

$= 10^{cx + 10} \times 10^{ax + 10}$

Clearly, $10^{ax + 10}, 10^{bx + 10}, 10^{cx + 10}$ are in G.P.

18. Given $9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}} \times \ldots\ldots \infty$

$$= 9^{\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\ldots\right)}$$

$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots\ldots$ are in G.P.

$$= \frac{1}{3} \times \frac{1}{1 - \frac{1}{3}} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

$$S_n = 9^{1/2} = 3$$

19. Given $t_m = 6m(m + 2)$

$$S_n = \Sigma^n_{m=1}\, t_m = \Sigma^n_{m=1}\, 6m(m + 2)$$

$$= \Sigma^n_{m=1}\,(6m^2 + 12m)$$

$$= 6 \times \frac{n(n + 1)\,(2n + 1)}{6} + 12 \times \frac{n(n + 1)}{2}$$

$$= n(n + 1)\,(2n + 7)$$

20. We divide the series in two parts such as

$6^2 + 7^2 + 8^2 + 9^2 + \ldots\ldots$ nth ...(1)

and $3^2 + 4^2 + 5^2 + 6^2 + \ldots\ldots$ nth ...(2)

rth term of First series

$= [6 + (r - 1) \times 1]^2$

$= (r + 5)^2$

rth term of Second series

$= [3 + (r - 1) \times 1]^2$

$= (r + 2)^2$

Now, rth term the net series

$= (r + 5)^2 - (r + 2)^2$

$= (2r + 7)\,(3)$

$= 6r + 21$

Now, $S_n = \Sigma^n_{r=1}\, t_r = \Sigma^n_{r=1}\,(6r + 21)$

$$= 6 \times \frac{n(n + 1)}{2} + 21n$$

$= 3(n^2 + n) + 21n$

$S_n = 3n^2 + 24n$

21. Let First term = X

Common difference = Y

$$a = \frac{p}{2} \times [2X + (p - 1)Y]$$

or $$\frac{a}{p} = X + \frac{1}{2} \times (p - 1)Y$$

$$b = \frac{q}{2} \times [2X + (q - 1)Y]$$

or $$\frac{b}{q} = X + \frac{1}{2} \times (q - 1)Y$$

$$c = \frac{r}{2} \times [2X + (r-1)Y]$$

or $$\frac{c}{r} = X + \frac{1}{2}(r-1)Y$$

$$= \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$$

$$= \left[X + \frac{1}{2} \times (p-1)Y\right] + \left[X + \frac{1}{2} \times (q-1)Y\right] + \left[X + \frac{1}{2} \times (r-1)Y\right]$$

$$= 0$$

22. Given series is divided into three different parts, such as

$2^2 + 3^2 + 4^2 + \ldots\ldots\ldots\ldots$ …(1)

$4^2 + 5^2 + 6^2 + \ldots\ldots\ldots\ldots$ …(2)

$5^2 + 7^2 + 9^2 + \ldots\ldots\ldots\ldots$ …(3)

rth term of series first

$$= [2 + (r-1) \times 1]^2 = (r+1)^2$$

rth term of series second

$$= [4 + (r-1) \times 1]^2 = (r+3)^2$$

rth term of series third

$$= [5 + (r-1) \times 1]^2 = (r+4)^2$$

Now, rth term of net series

$$= t_r = (r+1)^2 + (r+3)^2 + (r+4)^2$$

$$S_n = \Sigma^n_{r=1} t_r$$

$$S_n = \Sigma^n_{r=1} [(r+1)^2 + (r+3)^2 + (r+4)^2]$$

$$= \Sigma^n_{r=1} (3r^2 + 16r + 26)$$

$$= 3 \times \frac{n(n+1)(2n+1)}{6} + 16 \times \frac{n(n+1)}{6} + 26n$$

$$= \frac{n(n+1)(2n+1)}{2} + 8n(n+1) + 26n$$

$$= n[2n^2 + 3n + 1 + 16n + 16 + 52]$$

$$= n(2n^2 + 19n + 69)$$

23. Given series can be written as

$(5^3 + 7^3 + 9^3 + 11^3 + \ldots) + (3^3 + 5^3 + 7^3 + 9^3 + \ldots)$

Its rth term of first series

$= [5 + (r-1) \times 2]^3$

$= (2r + 3)^3$

Its rth term of second series

$= [3 + (r-1) \times 2]^3$

$= (2r + 1)^3$

Now, rth term of the given series

$= (2r + 3)^3 + (2r + 1)^3$

$= 8r^3 + 27 + 36r^2 + 54r + 8r^2 + 1 + 12r^2 + 6r$

$= 16r^3 + 48r^2 + 60r + 28$

Now, Sum $= S_n = \Sigma^n_{r=1} t_r$

$= \Sigma^n_{r=1} (16r^3 + 48r^2 + 60r + 28)$

$$= 16 \times \left(\frac{n(n+1)}{2}\right)^2 + 48 \times \frac{n(n+1)(2n+1)}{6} + 60 \times \frac{n(n+1)}{2} + 28n$$

$$= 4 \times n^2(n+1)^2 + 8n(n+1)(2n+1) + 30n(n+1) + 28n$$

$$= 4 \times n^2(n^2 + 2n + 1) + 8n(2n^2 + 3n + 1) + 30n^2 + 30n + 28n$$

$$= 2n[2n(n^2 + 2n + 1) + 4(2n^2 + 3n) + 15n + 33]$$

$$= 2n[2n^3 + 4n^2 + 2n + 8n^2 + 12n + 15n + 33]$$

$$= 2n[2n^3 + 12n^2 + 29n + 33]$$

24. Now, rth term of the given series is

$$t_r = \frac{1}{r(r+1)} = \frac{r+1-r}{r(r+1)}$$

$$= \frac{1}{r} - \frac{1}{r+1}$$

Now, $$t_1 = \frac{1}{1} - \frac{1}{2}$$

$$t_2 = \frac{1}{2} - \frac{1}{3}$$

$$t_3 = \frac{1}{3} - \frac{1}{4}$$

$$t_4 = \frac{1}{4} - \frac{1}{5}$$

………………

………………

$$t_n = \frac{1}{n} - \frac{1}{n+1}$$

$$t_1 + t_2 + t_3 + \ldots\ldots t_n = 1 - \frac{1}{n+1} = \frac{1}{n+1}$$

$$S_n = \frac{1}{n+1}$$

25. Given series is $7 + 77 + 777 + 7777 + \ldots\ldots$ n terms

$= 7[1 + 11 + 111 + 1111 + \ldots\ldots$ n terms$]$

$= \frac{7}{9}[9 + 99 + 999 + 9999 + \ldots\ldots$ n terms$]$

$= \frac{7}{9}[10 - 1 + 10^2 - 1 + 10^3 - 1 + 10^4 - 1 + \ldots$ n terms$]$

$$= \frac{7}{9}[(10 + 10^2 + 10^3 + 10^4 + \ldots\ldots + 10^n) - n]$$

$$= \frac{7}{9}\left[10 \times \left(\frac{10^n - 1}{10 - 1}\right) - n\right]$$

$$= \frac{7}{9}\left[\frac{10}{9} \times (10^n - 1) - n\right]$$

26. Given series can be written as

$(1^2 + 3^2 + 5^2 + \ldots\ldots) - (2^2 + 4^2 + 6^2 + \ldots\ldots)$

Now, First series $= S_1 = 1^2 + 3^2 + 5^2 + \ldots\ldots$

and second series $= S_2 = 2^2 + 4^2 + 6^2 + \ldots\ldots$

$$S_n = S_1 - S_2$$

Now, rth term of S_1 series

$$= [1 + (r - 1) \times 2]^2 = (2r - 1)^2$$

$$S_n = \Sigma^m_{r=1} (2r - 1)^2 = \Sigma^m_{r=1} (4r^2 - 4r + 1)$$

$$S_1 = 4 \times \frac{m(m+1)(2m+1)}{6} - 4 \times \frac{m(m+1)}{2} + m$$

$$= \frac{2}{3} \times m(2m^2 + 3m + 1) - 2m(m + 1) + m$$

$$= \frac{m}{3}[4m^2 + 6m + 2 - 6m - 6 + 3] = \frac{m}{3}(4m^2 - 1)$$

$$= \frac{m}{3}(2m - 1)(2m + 1)$$

and, rth term of S_2 series

$$= [2 + (r - 1) \times 2]^2 = (2r)^2$$

$$= 4r^2$$

It's Sum

$$= \Sigma^m_{r=1} 4r^2 = 4 \times \frac{m(m+1)(2m+1)}{6}$$

$$= \frac{2}{3} m(m + 1)(2m + 1)$$

Now, Sum of the given series

$$= S_1 - S_2$$

$$= \frac{m}{3}(2m - 1)(2m + 1) - \frac{2}{3} m(m + 1)(2m + 1)$$

$$\text{Sum} = -m(2m + 1)$$

Case-I—When n is odd.

Let $n = 2m + 1$

where $m = 0, 1, 2, 3$

$$\text{Sum} = -m(2m + 1)$$

$$= -\left(\frac{n-1}{2}\right)(n) = -\frac{n}{2}(n - 1)$$

Case-II—n is even.

Let $n = 2m$ $(m = 1, 2, 3, \ldots\ldots)$

$$\text{Sum} = -m(2m + 1)$$

$$= -\frac{n}{2}(n + 1)$$

27. We can write $31^3 + 32^3 + \ldots\ldots + 50^3$ as

$$(1^3 + 2^3 + 4^3 + \ldots\ldots + 30^3 + 31^3 + 32^3 + \ldots\ldots + 50^3) - (1^3 + 2^3 + \ldots\ldots + 30^3)$$

$$= \left[\frac{50(50+1)}{2}\right]^2 - \left[\frac{30(30+1)}{2}\right]^2$$

$$= \left(\frac{50 \times 51}{2}\right)^2 - \left(\frac{30 \times 31}{2}\right)^2$$

$$= (25 \times 51)^2 - (15 \times 31)^2$$

$$= 25[(255)^2 - (93)^2]$$

$$= 25 \times 348 \times 162$$

28. Given series is $3 + 8 + 22 + 72 + 266 + 1036 + \ldots\ldots$

First difference 5, 14, 50, 194, 770

Second difference 9, 36, 144, 576

They are in G.P. where nth term is

$$a.r^{n-1} = a.4^{n-1}$$

Now, T_n term of the given series will be of the form

$$T_n = a\,4^{n-1} + b\,n + c$$

When $n = 1$ $T_1 = a + b + c = 3$

$n = 2$ $T_2 = 4a + 2b + c = 8$

$n = 3$ $T_3 = 16a + 3b + c = 22$

Solving them in First equation, we get

$$a = 1,\ b = 2, c = 0$$

$$T_n = 4^{n-1} + 2n$$

$$S_n = \Sigma 4^{n-1} + \Sigma 2n$$

$$S_n = 1 \times \frac{4^{n-1}}{4-1} + 2 \times \frac{n(n+1)}{2}$$

$$= \frac{1}{3} \times (4^{n-1}) + n^2 + n$$

29. We know that

$$= (X_1 + X_2 + X_3 + \ldots\ldots + X_n)^2$$

$$= (X_1^2 + X_2^2 + X_3^2 + \ldots\ldots + X_n^2) + 2[X_1X_2 + X_2X_3 + X_3X_4 + \ldots\ldots + X_1X_n]$$

$$= \Sigma X_i^2 + 2\Sigma X_iX_j$$

Now, We put $X_1 = 1, X_2 = 2, X_3 = 3, X_4 = 4,$

$X_n = n$, we get

$$= (1 + 2 + 3 + \ldots\ldots + n)^2$$

$$= 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 + 2\Sigma\, X_iX_j$$

or
$$\left[\frac{n(n+1)}{2}\right]^2 = \frac{n(n+1)(2n+1)}{6} + 2\Sigma\, X_iX_j$$

or
$$2\Sigma\, X_iX_j = \frac{n(n+1)}{2} \times \left[\frac{n(n+1)}{2} - \frac{(2n+1)}{3}\right]$$

$$= \frac{n(n+1)}{2} \times \frac{3n^2 + 3n - 2n - 1}{6}$$

$$= \frac{n(n+1)}{2} \times \frac{3n^2 + n - 1}{6}$$

30. rth term of the given series is

$$= [n + (r - 1) \times 1]r$$

$$= (n + r - 1)r$$

$$= nr + r^2 - r$$

$$\text{Now, Sum} = \Sigma\,(nr + r^2 - r)$$

$$= n \times \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[n + \frac{2n+1}{3} - 1\right]$$

$$= \frac{n(n+1)}{2} \times \left(\frac{3n+2n+1-3}{3}\right)$$

$$= \frac{n(n+1)}{2} \times \frac{5n-2}{3}$$

31. $(r–1)$th term of the given series is

$$t_{r-1} = (r-1)(r-w)(r-w^2)$$

Now, $S = \sum_{r=1}^{n} t_r = \sum_{r=1}^{n} [(r-1)(r-w)(r-w^2)]$

$$= \sum_{r=1}^{n} (r-1)[r^2 - r(w+w^2) + w^3]$$

$$= \sum_{r=1}^{n} (r-1)(r^2+r+1) \qquad (w^3 = 1, w^2 + w = 1)$$

$$= \sum_{r=1}^{n} (r^3 - 1)$$

$$= \frac{1}{4} \times n^2 (n+1)^2 - n$$

32. S_n = Sum of cubes of the first n terms of an A.P.

$$= 1^3 + 2^3 + 3^3 + \ldots\ldots + n^3$$

$$S_n = \left[\frac{n(n+1)}{2}\right]^2$$

Let First term of an A.P. $= a + d$

Common difference $= d$

$$S^1_n = (a+d) + (a+2d) + (a+3d) + \ldots\ldots + (a+nd)$$

$$= \frac{n}{2} \times [2a + (n-1)d]$$

$$S_n = (a+d)^3 + (a+2d)^3 + \ldots\ldots + (a+nd)^3$$

$$= na^3 + 3a^2 d\,\Sigma n + 3ad^2\,\Sigma n^2 + d^3\,\Sigma n^3$$

$$= na^3 + 3a^2 d \times \frac{n(n+1)}{2} + 3ad^2 \times \frac{n(n+1)(2n+1)}{6} + d^3 \times \left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{n}{4} \times [4a^3 + 6a^2 d(n+1) + 2ad^2(n+1)(2n+1) + d^3 n(n+1)^2]$$

$$= \frac{1}{2} \times \frac{n}{2} \times [2a + (n+1)d]\,[2a^2 + 2ad(n+1) + d^2 n(n+1)]$$

$$= \frac{S^1}{2} \times [2a^2 + 2ad(n+1) + d^2 n(n+1)]$$

33. Let the work finish in n days when the workers started dropping, so that the total number of workers who worked all these days is the sum of A.P.

$$= 150 + 146 + 142 + \ldots\ldots + n \text{ terms}$$

$$= \frac{n}{2} \times [2 \times 150 + (n-1) \times (-4)]$$

$$= n(152 - 2n)$$

Had the workers not dropped then the work would have finished in $(n – 8)$ days with 150 workers working on each day

i.e. $150\,(n – 8)$

$$n(152 - 2n) = 150\,(n-8)$$

$$n^2 - n - 600 = 0$$

$$n = 25$$

34. Let the number of stones be $(2n + 1)$ so that there is one mid–stone and n stones each on either side of it. If P be mid stone and A, B be last stones on the left and right of P respectively. There will be (n) stones on the left and (n) stones on the right side of P or n intervals each of 10 metres both on the right and left side of mid–stone. Now, he starts from one of the end stones, picks it up, goes to mid–stone, drops it and goes to last stone on the other side, picks it and comes back to mid–stone.

In all the travels n intervals of 10 metres each 3 times. Now, from centre he will go to 2nd stone on L.H.S. then comes back and again go to 2nd last on R.H.S. and again come back.

Thus, he will travel $(n – 1)$ intervals of 10 metres each 4 times.

Similarly, $(n – 2)$ intervals of 10 metres each times for 3rd and so on for the last.

Hence, the total distance covered as given = 3 km = 3000 metre

$$3 \times 10n + 4 \times [10\,(n-1) + 10\,(n-2) + \ldots\ldots + 10]$$

or $30\,n + 40 \times [1 + 2 + 3 + \ldots\ldots + (n-1)] = 3000$

or $2n^2 + n - 300 = 0$

or $(n-12)(2n+25) = 0$

or $n = 12$

Hence, the number of stones $= 2n + 1$

$= 25$

35. L.C.M. of 2 and 5 is 10

Numbers divisible by 2 will contain numbers which are also divisible by 10.

Similarly,

Numbers divisible by 5 will contain numbers which are also divisible by 10.

Thus, the number divisible by 10 will occur twice.

So, Hence we can write to

$$S = S_2 + S_5 - S_{10}$$

S_2 = Number divisible by 2 between 1 to 100

$$= 2 + 4 + 6 + 8 + \ldots\ldots + 100$$

$$= 2\,(1 + 2 + 3 + 4 + \ldots\ldots + 50)$$

$$= 2 \times \frac{50 \times 51}{2} = 2550$$

S_5 = Numbers divisible by 5 between 1 to 100

$$= 5 + 10 + 15 + 20 + \ldots\ldots + 100$$

$$= 5\,(1 + 2 + 3 + 4 + \ldots\ldots + 20)$$

$$= 5 \times \frac{20 \times 21}{2} = 1050$$

$$S_{10} = \text{Numbers divisible by 10 between 1 to 100}$$
$$= 10 + 20 + 30 + \ldots\ldots + 100$$
$$= 10\,(1 + 2 + 3 + \ldots\ldots + 10)$$
$$= 10 \times \frac{10 \times 11}{2} = 550$$

Now, $S = 2550 + 1050 - 550$
$$= 3050$$

36. Odd integers between 2 and 100 are 3, 5, 7, 9, 11, 13, …… 97, 99

So, those odd numbers which are divisible by 3 are 3, 9, 15, 21, …… 99

Now, First term $= 3$

2nd term $= 9$

Common difference $= 6$

If n term will be 99.

So, $t_n = 3 + (n-1) \times 6 = 5n - 3$
$$99 = 6n - 3$$
$$102 = 6n$$
$$n = 17$$

Now, $\text{Sum} = \frac{n}{2} \times [2a + (n-1) \times d]$
$$= \frac{17}{2} \times [2 \times 3 + (17-1) \times 6]$$
$$= 17\,[3 + 16 \times 3]$$
$$= 17 \times 51$$
$$= 867$$

37. L.C.M. of 7 and 3 is 21.

Between 200 and 500 numbers divisible by 7 are 203, 217, …… 497

If nth term of series is 497.
$$t_n = 203 + (n-1) \times 7$$
$$497 = 203 + 7n - 7$$
or $301 = 7n$
$$n = 43$$

S_7 = Sum of numbers divisible by 7 between 200 and 500
$$= \frac{43}{2} \times [2 \times 203 + (43-1) \times 7]$$
$$= \frac{43}{2} \times [406 + 42 \times 7]$$
$$= \frac{43}{2} \times [406 + 294]$$
$$= \frac{43}{2} \times 700 = 43 \times 350$$
$$= 15050$$

Numbers divisible by 3 are 201, 204, 207, …… 498

First term $= 201$

Common difference $= 3$

$n\text{th term} = a + (n-1) \times d$
$$498 = 201 + (n-1) \times 3$$
or $297 + 3 = 3n$

or $n = 100$
$$S_3 = \frac{100}{2} \times [2 \times 201 + 99 \times 3]$$
$$= \frac{100}{2} \times [402 + 297]$$
$$= 50 \times 699 = 34950$$

Numbers divisible 7 and 3 are 210, 231, 252, … 483
$$483 = 210 + (n-1) \times 21$$
or $23 = 10 + n - 1$

or $n = 14$
$$S_{21} = \frac{n}{2} \times [2 \times 210 + (n-1) \times 21]$$
$$S_{21} = \frac{14}{2} \times [2 \times 210 + 13 \times 21]$$
$$= 7 \times (420 + 273)$$
$$= 7 \times 693 = 4851$$

Now, numbers divisible by 7 or 3.
$$= S_7 + S_3 - S_{21}$$
$$= 15050 + 34950 - 4851$$
$$= 45149$$

ONE DAY CAPSULE OF NUMERICAL APTITUDE

1. Average

The numerical result obtained by dividing the sum of two or more quantities by the number of quantities is called **Average.**

An arithmetic mean of given observations is called **Average.**

Average is defined in so many ways. We can say average means Usual or Normal Kind, amount, quality, rate, etc. Hence, normal or ordinary capability of student makes him an **average** student. An above the **Average,** is called intelligent.

Average is a number or value of a set of values carefully defined to typify the set, as a median or mode.

Average refers to the result obtained by dividing a sum by the number of quantities added. For example the **average** of 7, 9, 17 is $\frac{7+9+17}{3} = \frac{33}{3} = 11$ and in extended use is applied to the usual or ordinary kind, instance, etc.

Average is different from mean and median.

The **Average** of a given set of numbers is a measure of the central tendency of the set . In other words, it is the mean value of a set of numbers or values. Therefore, average of a set of numbers is given by :

$$\text{Average} = \frac{(x_1 + x_2 + x_3 + \ldots\ldots + x_n)}{n}$$

Or in other words average of some observations :

$$\text{Average} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Mean commonly designates a figure intermediate between two extremes; for example the *mean* temperature for a day with a high of 34°C and a low of 18°C is

$\frac{34+18}{2} = 26°C$ and the median is the middle number or *m* point in a series arranged in order of size *i.e.,* the *median* grade in the group 50, 55, 85, 88, 92 is 85; the **average** is 74.

Norm implies a standard of **average** performance for a given group *i.e.,* a child below the *norm* for his age in reading comprehension.

$$\text{Average} = \frac{(x_1 + x_2 + x_3 + \ldots\ldots + x_n)}{n}$$

Or in other words average of some observations :

$$= \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Example 1. In a class, the age of four students are 18 years,20 years , 22 years ,and 24 years , then what is the average age of the students in the class ?

Solution : By the above definition, average age $= \frac{20+22+18+24}{4} = \frac{84}{4} = 21$years.

Therefore, average age of students = 21 years.

Weighted Average

The concept of weighted Average is used when we have two or more groups whose individual averages are known.

Suppose in a class, there are 2 students each of 20 years, 3 of 21 years, 4 of 22 years and 5 of 23 years, then their average age is given by :

$$\frac{(2 \times 20) + (3 \times 21) + (4 \times 22) + (5 + 23)}{2+3+4+5}$$

$$= \frac{2}{14} \times 20 + \frac{3}{14} \times 21 + \frac{4}{14} \times 22 + \frac{5}{14} \times 23$$

$$= \frac{306}{14} \text{ years.}$$

Here, $\frac{2}{14}$, $\frac{3}{14}$, $\frac{4}{14}$ and $\frac{5}{14}$ are called the weights of each category of students.

Example 2. What is the average concentration of a mixture if 3 L of 36 % sulphuric acid is added to 9 L of 24% sulphuric acid solution ?

Solution : The average concentration of the combined mixture is the weighted average

$$= \left(\frac{3}{12}\right) \times 36 + \left(\frac{9}{12}\right) \times 24$$

$$= 9 + 18 = 27\%$$

In other words, weights are the fraction of the number in that category with respect to the total students in that class. This average is also called the weighted average of that class.

Average Speed

If a (body) certain distance is covered in parts at different speeds, the average speed is given by :

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

As, if a body travels $d_1, d_2, d_3 \ldots, d_n$ distances, with speeds $s_1, s_2, s_3 \ldots, s_n$ in time $t_1, t_2, t_3 \ldots, t_n$ respectively, then the average speed of the body through the total distance is given by :

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

Always remember that, Average speed

$$\neq \frac{\text{Sum of speeds}}{\text{Number of different speeds}} \neq \frac{s_1 + s_2 + s_3 + \dots + s_n}{n}$$

$$\text{Average speed} = \frac{d_1 + d_2 + \dots + d_n}{t_1 + t_2 + t_n}$$

$$= \frac{s_1 t_1 + s_2 t_2 + \dots s_n t_n}{t_1 + t_2 + t_3 + t_4 + t_n}$$

$$= \frac{d_1 + d_2 + d_3 + \dots + d_n}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + \dots + \frac{d_n}{s_n}}$$

Short Cut

If you travel equal distances with speeds u and v, then the average speed over the entire journey is $\frac{2uv}{(u+v)}$.

If a man changes his speed in the ratio $m : n$, then the ratio of times taken becomes $n : m$.

Example 3. Let the distance between two points A and B be d and speed in travelling from point A to B be x km/hr and from point B to A be y km/hr.

Solution :

$$\text{Then, average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{2d}{\frac{d}{x} + \frac{d}{y}}$$

If two speeds are given as x km/hr an y km/hr, then

Average speed (distance being same)

$$= \frac{2xy}{x + y}$$

Example 4. If a person travels two equal distances at 10 km/hr and 30 km/hr .What is the average speed for the entire journey ?

Solution :

$$\text{Average speed} = \frac{2 \times 30 \times 10}{30 + 10} = \frac{600}{40} = 15 \text{ km/hr.}$$

Age and Average

If the average age of n persons decreases by x years. Then, the sum of age of n persons decreases by $(n \times x)$ years. Also, if the average age of n persons increases by x years. Then, the sum of age of n persons increases by $(n \times x)$ years.

Example 5. The average weight of 6 men decrease by 3 kg when one of them weighing 80 kg is replaced by a new man. Calculate the weight of the new man.

Solution : Total weight reduced of 6 men = 6×3 = 18 kg.

This weight of the group is reduced because the man weighing 80 kg is replaced by a man who is 18 kg lighter than him. Therefore, weight of new man = $(80 - 18)$ = 62 kg.

Runs and Average

Example 6. A cricketer has a certain average of 9 innings. In the tenth inning he scores 100 runs, thereby increasing his average by 8 runs. Calculate his new average.

Solution : Let the average of 9 innings be x runs, hence new average will be $(x + 8)$ runs.

Total runs scored for 9 innings = $9x$

Total runs scored for 10 innings = $(9x + 100)$

$$\text{Average for 10 innings} = \frac{\text{Total runs}}{10}$$

$$\Rightarrow \quad (x + 8) = \frac{(9x + 100)}{10}$$

$$\Rightarrow \quad x = 20$$

Therefore, new average = (20 + 8) = 28 runs.

Average of Some Important Series of Numbers

(a) The average of **odd numbers** from 1 to n is $\frac{(n+1)}{2}$, when n = last odd number.

(b) The average of **even numbers** from 2 to n is $\frac{(n+2)}{2}$, when n = last even number.

(c) The average of **square of natural numbers** till n is $\frac{n(n+1)(2n+1)}{6n}$.

$$\Rightarrow \frac{(n+1)(2n+1)}{6}$$

(d) The average of **cubes of natural numbers** till n is = $\frac{n^2 (n+1)^2}{4n}$.

$$\Rightarrow \frac{n(n+1)^2}{4}$$

(e) The average of **first n consecutive even numbers** is $(n + 1)$.

(f) The average of **first n consecutive odd numbers** is n.

(g) The average of **squares of first n consecutive even numbers** is $\frac{2(n+1)(2n+1)}{3}$.

(h) The average of **squares of consecutive even numbers till n** is $\frac{(n+1)(n+2)}{3}$.

(i) The average of **squares of consecutive odd numbers till n** is $\frac{n(n+2)}{3}$.

Example 7. What is the average of odd numbers from 1 to 25 ?

Solution : Average $= \frac{25+1}{2} = 13$

Example 8. What is the average of even numbers from 1 to 40 ?

Solution : Average $= \frac{40+2}{2} = 21$

Example 9. What is the average of square of natural numbers from 1 to 20 ?

Solution : Average $= \frac{(20+1)(40+1)}{6} = 143{\cdot}5$

Example 10. What is the average of cubes of natural numbers from 1 to 5 ?

Solution : Average $= \frac{5(5+1)^2}{4} = 45$

Example 11. What is the average of first 49 consecutive even numbers ?

Solution : Average $= 49 + 1$

Example 12. What is the average of first 19 consecutive odd numbers ?

Solution : Average $= 19$

Example 13. What is the average of square of first 10 consecutive even numbers ?

Solution : Average $= \frac{2(10+1)(20\ +1)}{3}$

$$= \frac{2 \times 11 \times 21}{3} = 154$$

Example 14. What is the average of square of consecutive even numbers till 10?

Solution : Average $= \frac{(10+1)(10+2)}{3}$

$$= \frac{11 \times 12}{3} = 44$$

Example 15. What is the average of square of consecutive odd numbers till 12?

Solution : Average $= \frac{12(12+2)}{3}$

$$= \frac{12 \times 14}{3} = 56$$

2. Number System

Number system is the key concept in every branch of mathematics. The use and scope of number system is unlimited. The system deals with the nomenclature, use and properties of number. The chapter is a brief introduction of number and its application in different competitive questions. Its scope in this book is limited keeping in view our domain of competitive examination. The number system that we use in over every day life is called decimal system. This is because there are 10 digits (0, 1, 2,…, 9,) .

We are giving here under the number chart, which is self—explanatory in its nature and use for the practical application.

I. Complex Number : Complex number is also referred as imaginary. The form in which complex number is written as $a + ib$, where a and b are real number and i is the imaginary unit whose value is $\sqrt{-1}$. In real number system the square root of negative number does not exist.

II. Real Number : Set of all numbers that can be represented on the number line is called real numbers.

For example : 4, – 8, 0, 3·92, $2 + \sqrt{11}, \frac{9}{11}$, etc .

A number line is a straight line with an arbitrary defined point zero. To the right of this point lie all positive numbers and to the left, all negative numbers.

Real Number line : Now, real numbers can be divided into two categories, rational numbers and irrational number .

(a) Rational Number : If a number can be expressed in the form of $\frac{p}{q}$ where $q \neq 0$ and where p and q are integers, then the number is called rational number *e.g.,* $\frac{9}{25}, \frac{16}{7}, \frac{8}{1}, -\frac{27}{51}$ etc.

All integers are also rational numbers. Every terminating decimal or a repeating decimal is also a rational number, *e. g.* 3·1, 7·323232…., etc.

(b) Irrational Number : If a numbers cannot be expressed in the form of $\frac{p}{q}$, $q \neq 0$, then the number is called irrational number.

In other words, non-repeating as well as non-terminating type of decimals are called irrational numbers.

e.g., $\sqrt{2}, 3\sqrt{4}$,… 4·965896…, 3·14592 …

Rational numbers can be further sub-divided into two parts-integers and fractions.

(i) Integers : Integers are the set of all non–fractional numbers lying between – ∞ and + ∞. Hence, integers include negative as well as positive non–fractional numbers. Integer is denoted by Z or I.

I = {– ∞ …, – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4,…, + ∞}.

(Note that 0 is neither a positive nor a negative integer)

(ii) Fractions : A fraction includes two parts, numerator and denominator $-\frac{3}{7}, \frac{9}{5}, \frac{11}{7}$ etc.

Integers can be further subdivided into negative number and whole numbers have two sections zero and positive numbers popularly called as natural number.

Natural Numbers : Set of natural numbers is denoted by N

N = {1, 2, 3, 4, 5,…, ∞}

1. **Even numbers :** All numbers that are divisible by 2 are called even numbers *e.g.* {2, 4, 6, 8, 10, 12,..., ∞}.

2. **Odd numbers :** All number that are not divisible by 2 are called odd numbers *e.g.* {1, 3, 5, 7, 9, 11,..., ∞}.

3. **Prime numbers :** The numbers that have only two factors, 1 and the number itself, are called prime numbers *e.g.*, {2, 3, 5, 7, 11, 13, 17, 19,...}

Note—

- Number 1 is not a prime number .
- There are 25 prime numbers from 1 to 100 *i.e.*, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ..., 97.

SOME IMPORTANT POINTS ABOUT PRIME NUMBERS

- The smallest prime number is 2, which is the only even prime number .
- All prime numbers can be written in the form of (6N – 1) or (6N + 1) .The converse is not necessarily true . This means any number of the form (6N – 1) or (6N + 1) is not necessarily a prime number .
- The remainder when a prime number P ≥ 5 is divided by 6 is 1 or however, if a number on being divided by 6 gives a remainder of 1 or 5 . The number need not be prime.
- The remainder when the square of prime number P≥ 5 divided by 24 is 1.
- For prime number > 3 , $P^2 - 1$ is divisible by 24.
- The reminder of the division of the square of a prime P≥ 5 divided by 12 is 1.

Operation of Numbers

(a) Finding a unit digit in a product :

Example 1. Find the unit digit in the product (289 × 156 × 439 × 151).

Solution : Product of unit's digits in given numbers

$$= (9 \times 6 \times 9 \times 1)$$
$$= 486$$

∴ Unit digit in the given product is 6.

Example 2. Find the unit digit in the product $(3^{66} \times 6^{41} \times 7^{53})$.

Solution : We know, unit digit in 3^4 is 1.

∴ unit digit in 3^{64} is1

Hence, unit digit in $3^{66} = 1 \times 3 \times 3 = 9$

Unit digit in every power of 6 is 6.

Therefore, unit digit in $6^{41} = 6$

Unit digit, in 7^4 is 1

∴ Unit digit in 7^{52} is1

Hence, Unit digit in $7^{53} = 1 \times 7 = 7$

Therefore, product of unit's digit in the given numbers

$$(3^{66} \times 6^{41} \times 7^{53}) = (9 \times 6 \times 7) = 378.$$

∴ Unit digit in the given product = 8.

(b) Representation of rational numbers : Rational numbers when converted into decimal form can be either a recurring and non-terminating or a terminating decimal. For example, terminating decimal = 2·6, non-terminating and recurring decimal = 2.636363...

(c) Remainder theorem :

(I) Dividend = (Divisor × Quotient) + Remainder

If a number when divided by 5 leaves a remainder 3, the number can be written as 5x+3, where x is a whole number . This is also written as number or N = 3 mod 5 . It means the number when divided by 5 leaves the remainder 3 .

Example 1. There are two positive numbers x, y. Each of them when divided by 6 leaves the remainders 2 and 3 respectively. Find the remainder when $(x + y)$ is divided by 6.

Solution : In case of number x,

$$x = 6a + 2 \qquad \text{...(i)}$$

In case of number y,

$$y = 6b + 3 \qquad \text{...(ii)}$$

Adding equation (i) and (ii)

We get is $x + y = (6a + 6b) + 5$

$$\Rightarrow \quad x + y = 6(a + b) + 5$$

So, it is clear that on dividing numbers $(x + y)$ by 6 it leaves a reminder 5.

Example 2. A number when divided by 256 give a reminder 77. when the same number is divided by 16, what would be the remainder ?

Solution : We know that if a number N is divided by 256, leaves the remainder 77.

Then, N = 256x + 77 where x is the quotient

$$= (16 \times 16x + 16 \times 4 + 13)$$
$$N = 16\,(16x + 4) + 13$$

Therefore, number N leaves remainder 13, when it is divided by number 16.

Example 3 : If N ≡ 3 mod 6, what is the remainder when N^2 is divided by 6 ?

Solution : N ≡ 3 mod 6, means that number divided by 6 leaves a remainder 3.

In other words, N = 6x + 3

$$N^2 = (6x + 3)^2 = (6x)^2 + 2\,(6x)\,(3) + (3)$$
$$= 36x^2 + 36x + 9 = 36x^2 + 36x + (6 + 3)$$
$$= 6(6x^2 + 6x + 1) + 3$$
$$= 6M + 3 \quad \text{(where M is a natural number)}$$

(II) The expression $\left[\frac{0 \times P \times q}{n}\right]$ will give the same remainders as $\left[\frac{0_r \times P_r \times q_r}{n}\right]$ where 0_r, P_r and q_r are the

remainders when O, P and q are respectively divided by n.

Example 4. Find the remainder for $[(85 \times 73 \times 69)]/12$.

Solution : According to the theorem , the remainder for the expression $[(85 \times 73 \times 69]/12$ will be the same as the remainder for $[(1 \times 1 \times 9)]/12 = \frac{9}{12} \Rightarrow$ remainder $= 9$.

Example 5. What is the remainder if 8^{25} is divided by 7 ?

Solution :

Method I. 8^{25} can be written as $(7+1)^{25}$. In this binomial expansion there are total 26 terms. All the first 25 terms will have 6 as a multiple in it .

The 26th term is $(1)^{25} = 1$. Hence, the expansion can be written as $7x + 1$(where $7x$ is the sum of all first 25 terms). It is now clear $(7x + 1)$, if divided by 6, leaves a reminder 1 .

Method II. Using the remainder theorem we find that reminder for $\frac{8^{25}}{7}$ is the same as the reminder for $\frac{(1)^{25}}{7}$, *i.e.*, reminder 1. Therefore, remainder for $\frac{8^{25}}{7}$ will be 1.

Factorisation

It is the process of splitting any number into the form of its basic prime factors.

For example, $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

24 is expressed in the factorised from in terms of its basic prime factors. This is the factorisation form of 24.

Example 6. Calculate the total number of factors of 12.

Solution : 12 can be expressed as $2^n \times 3$

Here, the powers of 2 can be one of (0, 1, 2) and that of 3 can be one of (0, 1) . So, the total possibilities, if you take the two combinations are $3 \times 2 = 6$.

In general, for any composite number C, which can be expressed as $C = a^m \times b^n \times c^p \times \ldots$ where $a, b, c, \ldots$ are all prime factors and m, n, p, are positive integers, the number of factors is equal to $(m + 1)(n + 1)(p + 1)\ldots$

Example 7. Find the total number of factors of 576 .

Solution : The factorised form of 576 is

$$(24 \times 24) = (2^3 \times 3)(2^3 \times 3) = (2^6 \times 3^2)$$

Therefore , the total number of factors is

$$(6+1)(2+1) = 21.$$

Important Result

A number having odd factors is always a perfect square.

e.g.,

$$64 = 2^6 = (6 + 1) = 7$$

$$144 = 2^4 \times 3^2 = (4 + 1)(2 + 1) = 15$$

$$36 = 2^2 \times 3^2 = (2 + 1)(2 + 1) = 9$$

Some Important Formulae

- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$
- $(a + b)^2 - (a - b)^2 = 4ab$
- $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
- $(a^2 - b^2) = (a + b)(a - b)$
- $(a^3 + b^3) = (a - b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- $a^2 + b^2 + c^2 - ab - bc - ca$
  $= \frac{1}{2}(a - b)^2 + \frac{1}{2}(b - c)^2 + \frac{1}{2}(c - a)^2$
- $a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$
- $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ac - bc - ab)$
  If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$
- $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$
- $a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 - 2c^2a^2$
  $= -(a + b + c)(a + b - c)(b + c - a)(c + a - b)$

3. Partnership

Partnership : When two or more than two persons run a business jointly, they are called partners and the deal is known as partnerships. It can be simple or compound type.

Simple Partnership : When investments of all the partners are for the same time, the profit or loss is divided among them in the ratio of their investments.

Compound Partnership : When investments are for different times, then equivalent capitals are calculated for a unit of time by multiplying the capital with the number of units of time.

1. If A and B are two partners and A invest M_A money for T_A time and B invest M_B money for T_B time , the ratio of share of profit of A and share of profit of B is $\mathbf{T_A \times M_A : T_B \times M_B}$

$$\text{Share of Profit of A} = \frac{T_AM_A}{T_AM_A + T_BM_B}$$

$$\text{Share of Profit of B} = \frac{T_BM_B}{T_AM_A + T_BM_B}$$

2. If A, B and C are three partners and A invests M_A money for T_A time, B invests M_B money for T_B time and C invests M_C money for T_C time , the ratio of share of profit of A, share of profit of B and share of profit of C is $\mathbf{T_A \times M_A : T_B \times M_B : T_C \times M_C}$

$$\text{Share of Profit of A} = \frac{T_AM_A}{T_AM_A + T_BM_B + T_CM_C}$$

$$\text{Share of Profit of B} = \frac{T_BM_B}{T_AM_A + T_BM_B + T_CM_C}$$

$$\text{Share of Profit of C} = \frac{T_CM_C}{T_AM_A + T_BM_B + T_CM_C}$$

A partner who participates in the working and manages the business, is called a working partner while the one only invests capital but does not participate in the working of the business, is called a sleeping partner.

A working partner gets either monthly payment or a part in the profit for his contribution in the management of the business.

This payment is deduced from the total profit before its distribution.

4. Divisibility

Rules of Divisibility

Divisibility by 2 : A number is divisible by 2, if its last digit (unit's place) is either 0, 2, 4, 6, or 8 *e.g.*, each of the numbers 21674 , 31856, 20018 , 43560 is divisible by 2. We note that all even numbers are divisible by 2.

Divisibility by 3 : A number is divisible by 3, if the sum of its digits is divisible by 3 .

Example 1. Which of the following numbers is divisible by 3 ?

(i) 98730527 (ii) 17096528 (iii) 93476388.

Solution : Sum of digit in (i) is 41, since 41 is not divisible by 3, hence the given number is not divisible by 3.

Sum of digit in (ii) is 38 , since 38 is not divisible by 3, hence the number (ii) is not divisible by 3.

Sum of digit in (iii) is 48, since 48 is divisible by 3 , hence number (iii) is divisible by 3.

Divisibility by 4 : A number is divisible by 4, if the number formed by the last two digits is divisible by 4.

Example 2. Which of the following numbers is divisible by 4?

(i) 6897956 (ii) 6893573

Solution : It is clear that number formed by the last two digits in (i) is 56 , which is divisible by 4, hence the entire number is divisible by 4. In (ii) the last two digits is 73, which is not divisible by 4 hence the number is not divisible by 4.

Example 3. If $abc3d$ is divisible by 4, find the value of d.

Solution : For the number to be divisible by 4, the number formed by the last two digit *i.e.*, $3d$ should be divisible by 4. Hence, d should be replaced by 2 or 6.

Divisibility by 5 : A number is divisible by 5, if the last digit is either 0 or 5. For example, 2635, 12970, 38525, …..

Example 4. What is the remainder if a number $3ab9$ is divisible by 5 ?

Solution : For the number $3\,ab\,9$ to be divisible by 5, the last digit should be either 0 or 5. Now, since the last digit 9 exceeds 5. Therefore, $(9 - 5) = 4$ would be the remainder .

Divisibility by 6 : A number is divisible by 6, if the number is divisible by both 2 and 3 simultaneously.

Note—

- A number is divisible by ab only when it is divisible by co-prime of that number .
- CO–Prime : Two number s are said to be co-prime, if their HCF is 1

e.g., (2, 3) (8, 11) , (7, 9) (2, 7) etc .

Example 5. What least number should be added to 345670 in order to make it divisible by 6 ?

Solution : The number will be divisible by 6, if it is divisible by 2 and 3 both .

To make it divisible by 3 the sum of digits should be divisible by 3 .The sum of digits is 25 . If two is added to the number it becomes divisible 3 and satisfies the condition of divisibility by 2 as well .

Example 6 : If N is divisible by 2 but not 3, what is the remainder when N is divided by 6 ?

Solution : Since, the number is divisible by 2, it has to be an even number .

$$N = 6a + x$$

x (remainder) will be even and can take the value either 2 or 4 .

Divisibility by 8 : A number is divisible by 8 , if the last 3 digits taken together, is divisible by 8 .

Divisibility by 9 : A number is divisible by 9, if the sum of the digits of given number is divisible by 9.

Divisibility by 11 : A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

Example 7. Is number 96858256 divisible by 11 ?

Solution : The divisibility rule of 11

= (sum of digits at even place)

– (sum of digits at odd places)

$= (9 + 8 + 8 + 5) - (6 + 5 + 2 + 6)$

$= (30) - (19) = 11$

$\therefore$ 96858256 is divisible by 11.

5. Area

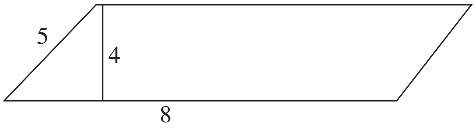
1. Area

1. Rectangle = bh

Area = $4 \cdot 7 = 28$

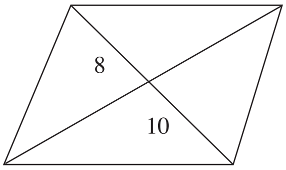
2. Parallelogram = bh

Area = $8 \cdot 4 = 32$



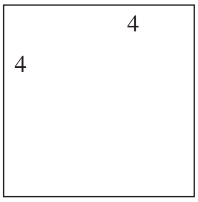
3. Rhombus = $\frac{1}{2} d_1 d_2$

Area = $\frac{1}{2} . 8 \cdot 10 = 40$



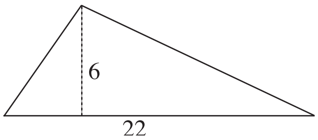
4. Square = side × side

Area = $4 \times 4 = 16$



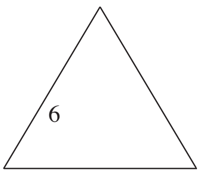
5. Triangle = $\frac{1}{2} bh$

Area = $\frac{1}{2}(6)(22) = 66$



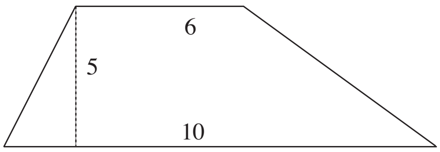
6. Equilateral Triangle = $\frac{\sqrt{3}}{4} \times a^2$

Area = $\frac{\sqrt{3}}{4} \times 6^2 = 9\sqrt{3}$



7. Trapezoid = $\frac{1}{2} h (b_1 + b_2)$

Area = $\frac{1}{2} \times 5 \times (6 + 10) = 40$

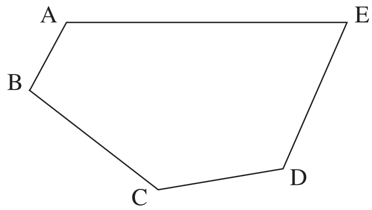


Polygons

1. The sum of the measures of the angles of a polygon of n sides is $(n - 2) \times 18°$.

Since, ABCDE has 5 sides.

$m \angle A + m \angle B + m \angle C + m \angle D + m \angle E = (5 - 2)$ $180 = 540$



2. In a parallelogram :

(a) Opposite sides are parallel.

(b) Opposite sides are congruent.

(c) Opposite angles are congruent.

(d) Consecutive angles are supplementary.

(e) Diagonals bisect each other.

(f) Each diagonal bisects the parallelogram into two congruent triangles.

3. In a rectangle, in addition to the properties listed in (2) above;

(a) All angle are right angles.

(b) Diagonals are congruent.

4. In a rhombus, in addition to the properties listed in (2), above;

(a) All sides are congruent.

(b) Diagonals are perpendicular.

(c) Diagonals bisect the angles.

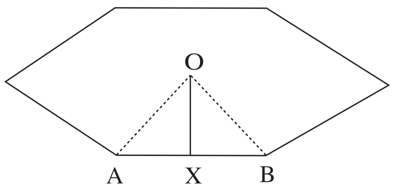
5. A square has all of the properties listed in (2) (3) and (4) above.

6. The apothem of a regular polygon is perpendicular to a side, bisect that side, and also bisect a central angle.

OX is an apothem.

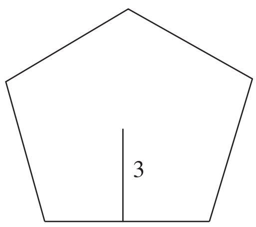
It bisects AB, is perpendicular to AB,

and bisect angle AOB



7. The area of a regular polygon is equal to one-half the product of its apothem and perimeter.

$$A = \frac{1}{2}(3)(6 \times 5) = 45$$



6. Simplification

Fractions

A fraction is a number which represents a ratio or division of two whole number (0, 1, 2, 3, 4……) (integers). A fraction is written in the form of $\left(\frac{p}{q}\right)$ $\frac{p(\text{ numerator})}{q \text{ (denominator)}}$.

For example, $\frac{6}{7}$ is a fraction, it represents taking 6 of 7 equal parts, or dividing 6 into 7.

A fraction with 1 as the denominator is the same as the whole number which its numerator.

For Example, $\frac{0}{1} = 0$ or $\frac{23}{1} = 23$

Note—q cannot be equal to zero.

Fraction are primarily of five types—

1. Proper Fraction : A rational number in the form of $\frac{p}{q}$, where $q \neq 0$, where the numerator is less than the denominator, *e.g.*, $\frac{3}{7}$.

2. Improper Fraction : A rational number in the form of $\frac{p}{q}$, where $q \neq 0$, where the numerator is more than the denominator, *e.g.*, $\frac{7}{3}$.

3. Mixed Fraction : Mixed fraction consist of integral as well as the fractional part, *e.g.*, $2\frac{3}{7} = 2 + \frac{3}{7} = \frac{17}{7} \Rightarrow$ It means that a mixed fraction is always an improper fraction.

4. Compound Fraction : A fraction of a fraction is known as compound fraction, *e.g.*, $\frac{4}{5}$ of $\frac{9}{11} = \frac{4}{5} \times \frac{9}{11}$.

5. Complex Fraction : Any complicated combination of the other type of fractions, *e.g.*,

$$2\frac{1}{3} \text{ of } \frac{3}{1+\frac{2}{3}}; \frac{4}{7} \text{ of } \frac{3}{3+\frac{2}{2+\frac{1}{1+3}}}$$

Mixed Number

A mixed number consists of a whole number and a fraction.

For example, $11\frac{3}{4}$ is a mixed number. It means $11 + \frac{3}{4}$

Any mixed number can be changed into a fraction.

For example, write $11\frac{3}{4}$ as a fraction

$11 \times 4 = 44$ and $44 + 3 = 47$ so, fraction is $\frac{47}{4}$.

Note—$2\frac{1}{3} + 3\frac{1}{6} + 4\frac{1}{2} + 23\frac{1}{2} + 68 + 22\frac{1}{3} + 34\frac{5}{6} = ?$

$(2 + 3 + 4 + 23 + 68 + 22 + 34 = 156)$

$$+ \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + \frac{5}{6} = 3\right) = 159$$

Operation with Fractions

Adding and Subtracting : For adding and subtracting, the number must have the same (common) denominator.

For example, $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{cb}{bd} = \frac{ad + cb}{bd}$

Multiplying : To multiply fractions, always try to divide common factors from any numerators and any denominators where possible before actually multiplying. In multiplying mixed numbers, always rename them as improper fraction first.

For example, Multiply $-\frac{2}{7} \times \frac{21}{16} \times \frac{88}{3} = 11$ or Multiply $-4\frac{1}{2} \times 3\frac{2}{3} \times \frac{6}{9} = 11$

In word problems, *of,* usually indicates multiplication.

For example, Dr. Dim, director *of* Institute of Perfection donates $\frac{1}{3}$ of \$ 690.

It means $\frac{1}{3} \times \frac{690}{1} = \frac{690}{3}$ = \$ 230 is the amount Dr. Dim donates.

Note—Multiply the numerator and denominator of a fraction by the same non-zero number, the fraction remains the same.

Dividing : To divide fractions or mixed numbers, remember to multiply by the reciprocal of the divisor (the number after the division sign).

For example, $7\frac{1}{3} \div \frac{2}{3} = \frac{22}{3} \times \frac{3}{2} = 11$

To divide one fraction (dividend) by another fraction (divisor), invert divisor and multiply.

For example, $\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{5}{6}$

Note—Dividing the numerator and denominator of a fraction by the same non-zero number the fraction remains the same.

Reducing a fraction to lowest term: A fraction has been reduced to lowest terms when the numerator and denominator have no common factors.

For example, $\frac{4}{5}$ is reduced to lowest terms but $\frac{6}{21}$ is not because 3 is a common factor of 6 and 21. Hence, the lowest term of fraction is $\frac{3 \times 2}{3 \times 7} = \frac{2}{7}$.

7. Ratio and Proportion

Ratio

If a and b are two quantities of the same kind, then $\frac{a}{b}$ is known as the ratio of a and b. Therefore, the ratio of two quantities in the same units is a fraction that one quantity is to the other.

Thus, a to b is a ratio $\left(\frac{a}{b}\right)$, written as $a : b$.

The first term of the ratio is called antecedent, while the second term is called consequent.

Ratio between 60 kg and 100 kg is 3 : 5.

The multiplication or division of each term of ratio by a same non-zero number does not affect the ratio. Hence, 3 : 5 is the same as 6 : 10 or 9 : 15 or 12 : 20 etc.

Ratio can be expressed as percentages. To express the value of a ratio as a Percentage, we multiply the ratio by 100 .

Therefore, $\frac{3}{5} = 0{\cdot}6 = 60\%$.

Proportion

The equality of two ratios is called proportion. a, b, c, d are said to be in proportion if $a : b = c : d$

or $a : b :: c : d$.

In a proportion , the first and fourth terms are known as extremes, while second and third terms are known as means. Hence, a and d are extremes and b and c are means of the proportion $a : b :: c : d$.

In a proportion we always have :

Product of extremes = Product of means

$$a \times d = b \times c$$

Continued Proportion

Four quantities : a, b, c, d, are said to be in a continued proportion, if

$$a : b = b : c = c : d \text{ or } \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Three quantities are said to be in continued proportion, if

$$a : b = b : c \text{ or, } ac = b^2$$

In this relationship, b is said to be the mean proportional between a and c and c is said to be a third proportional to a and b.

Example 1. An object 1·6 m long casts a shadow 1·4 m long. At the same time another object kept nearby casts a shadow 6·2 m long. Find the length of the second object.

Solution : Ratio of length of the object to its shadow would be same.

$$\therefore \quad 1{\cdot}6 : 1{\cdot}4 = x : 6{\cdot}2$$

$$\text{or} \quad x = \frac{1{\cdot}6 \times 6{\cdot}2}{1{\cdot}4} = 7{\cdot}08 \text{ m}$$

Some Results on Ratio and Proportion

1. **Invertendo**—If $a : b :: c : d$, then $b : a :: d : c$
2. **Alternendo**—If $a : b :: c : d$, then $a : c :: b : d$.
3. **Componendo**—If $a : b :: c : d$, then $(a + b) : b :: (c + d) : d$.
4. **Dividendo**—If $a : b :: c : d$, then $(a - b) : b :: (c - d) : d$.
5. **Componendo and Dividendo**—If $a : b :: c : d$, then $(a + b) : (a - b) :: (c + d) : (c - d)$.

Direct Proportion

If A is directly proportional to B then as A increases, B also increases proportionally. For example, the relation between speed, distance and time, speed is directly proportional to distance when time is kept constant.

It is therefore important to note here that the variation is direct and proportional. If one quantity is doubled the related quantity will also be doubled.

Other examples of direct proportion are :

(a) Simple Interest *Vs* Time (principal and rate being constant).

(b) Density *Vs* Mass (volume being constant).

(c) Force *Vs* Acceleration (mass being constant).

Direct Variation

If A is said to vary directly as B , then as A increases B also increases but not proportionally. This variation is denoted by $A \propto B$ or $A = KB$, where K is a constant.

For example, the total cost of production is directly related to the number of items being produced.

Here, the variation is direct but not proportional .

Inverse Proportion

A is inversely proportional to B means if A increases B decreases proportionally. If speed is doubled, time taken to cover the same distance is reduced to half.

Other example of inverse proportion are :

(a) Density *Vs* volume (mass being constant).

(b) Number of person *Vs* time taken to complete the work (work being same).

Inverse Variation

If A is inversely related to (or) varies inversely as B, then if B increases as A decreases but not proportionally . This relation can be expressed mathematically as $A \propto \frac{1}{B}$

$\Rightarrow A = K \times \frac{1}{B}$, where K is a constant.

Here, the variation is inverse but not proportional.

8. Progression

A sequence of numbers, each of which is obtained from its predecessor by the same rule.

Arithmetic Progression (AP)—A sequence of terms each of which, after the first, is derived by adding to the preceding one, a common difference (Ex. 5, 9, 13, 17, etc.) form an arithmetic progression.

And also, a number series which progresses in such a way that the difference between two consecutive numbers is common, is called the arithmetic progression.

In the above Ex (series) the difference between two consecutive number is 4.

(a) The nth term of the AP is given by

$$T_n = a + (n - 1)\, d$$

where a = First term of the series

n = Number of the terms in the series

d = Common difference of the series

(b) The sum of n terms of the AP is given by :

$$S_n = \frac{n}{2}[2a + (n - 11d] = \frac{n}{2}\text{(first term + last term)}$$

Example 1. Find the sum of even numbers between 1 and 100 (including 100).

Solution : The series of even numbers between 1 and 100 will be 2, 4, 6, 8,……., 100.

Since, the difference between two consecutive terms is common, it is arithmetic progression.

$$S_n = \frac{n}{2}\text{[first name + last name]}$$

There will be 50 even numbers between 1 and 100 .

$$n = 50$$

$$\therefore \quad S_n = \frac{50}{2}[2 + 100] = 25 \times 102 = 2550.$$

Example 2. The sum of first natural number from one to fifty, is divisible by

(a) 3 (b) 51 (c) 5 (d) 25 (e) all

Solution :

$$\text{Sum } S_n = \frac{n}{2}\text{[first name + last name]}$$

$$= \frac{50}{2}[1 + 50] = 25 \times 51 = 1275$$

Now, 1275 is divisible by each one of 3, 51, 5 and 25.

Hence option (e) is correct.

Example 3. Find the sum, if all 2 digit numbers are divisible by 3.

Solution : All 2 digit numbers divisible by 3 are :

12, 15, 18, 21, 24,………….99.

The above series is an AP, where a = 12 and d = 3.

To find the number of terms in the above series

$$T_n = a + (n - 1)d$$

$$99 = 12 + (n - 1)3$$

$$\frac{99 - 12}{3} = (n - 1)$$

$$n = 30$$

$\therefore$ The required sum

$$S_n = \frac{n}{2}(12 + 99) = 1665$$

Important Result of an AP

(i) Sum of n consecutive natural numbers

$$(1 + 2 + 3 + 4 + \ldots\ldots\ldots. + n) = \frac{n(n+1)}{2}$$

(ii) Sum of squares of n consecutive, natural numbers

$$(1^2 + 2^2 + 3^2 + \ldots\ldots. + n^2) = \frac{n\,(n+1)(2n+1)}{6}$$

(iii) Sum of cubes of n consecutive natural numbers

$$(1^3 + 2^3 + 3^3 + 4^3 + \ldots\ldots + n^3) = \frac{n^2\,(n+1)^2}{4}$$

Geometric Progression (GP)—A sequence of terms in which the ratio of each term to the preceding one is the same throughout the sequence (Ex. : 1, 2, 4, 8, 16, 32).

And also, a progression of numbers in which every term bears a constant ratio with its preceding term, is called geometrical progression.

The constant ratio is called the common ratio of the GP.

Therefore, a ar, ar^2 is a GP

where a = first term, r = common ratio

The nth term of GP is given by $T_n = ar^{n-1}$

And sum of n terms of GP is given by :

$$S_n = \frac{a\,(1 - r^n)}{(1 - r)}$$

Example 4. How many terms are there in 2, 4 8, 16, …, 1024?

Solution : $a = 2, r = 2$

Then, nth term is given by ar^{n-1}

$$1024 = 2 \times 2^{n-1}$$

$$\text{or,} \quad 2^{n-1} = \frac{1024}{2} = 512$$

$$\Rightarrow \quad 2^n \times 2^{-1} = 512$$

$$\Rightarrow \quad 2^n = 512 \times 2 = 1024 = 2^{10}$$

$$\therefore \quad 2^n = 2^{10} \text{ or } n = 10.$$

9. Percentage

The word percentage means per 100 or for each hundred. A fraction whose denominator is 100 is called a percentage and the numerator of the fraction is called the rate per cent. It is denoted by the symbol %.

$$19\% = \frac{19}{100} = 0{\cdot}19$$

$$\frac{12}{7}\% = \frac{\frac{12}{7}}{100} = \frac{12}{7} \times \frac{1}{100} = \frac{3}{175} = 0{\cdot}017$$

Questions based on Percentage

(1) To find the % equivalent of a fraction or as a decimal, add the % sign and multiply by 100.

Example : To rename a fraction $\frac{4}{25}$ as *a* %.

Solution : $\frac{4}{25} \times 100\% = 16\%$

Example : To rename a decimal 0·074 as *a* %.

Solution : This has the effect of moving the decimal point two places to the right.

$$0{\cdot}074 = 7{\cdot}4\%$$

(2) To find the fraction or number or decimal equivalent of a %, remove the % sign and divide by 100.

Example : To rename a 25% as a fraction or decimal.

Solution : $25\% = \frac{25}{100} = \frac{1}{4} = 0{\cdot}25$

Most percentage problems can be solved by using the following relation :

$$\frac{\%}{100} = \frac{\text{Part}}{\text{Whole}}$$

Example : Find 27% of 92.

Solution : $\frac{27}{100} = \frac{x}{92}$

$$X = 24{\cdot}84$$

Example : 7 is 5 % of what number?

Solution : $\frac{5}{100} = \frac{7}{x}$

$$X = 140$$

Example : 90 is what % of 1500?

Solution : $\frac{x}{100} = \frac{90}{1500}$

$$X = 6$$

Example : Find 125% of 16.

Solution : $\frac{125}{100} = \frac{x}{16}$

$$X = 20$$

(3) To increase a number by a given %, multiply the number by the factor $\left[\frac{100 + \text{rate}}{100}\right]$.

Example : Find out what is new number after increasing 22 by 200%.

Solution :

$$22 \times \left[\frac{100 + 200}{100}\right] = 22 \times 3 = 66$$

(4) To decrease a number by a given %, multiply the number by the factor $\left[\frac{100 - \text{rate}}{100}\right]$

Example : Find out what is new number after decreasing 40 by 50%.

Solution :

$$40 \times \left[\frac{100 - 50}{100}\right] = 40 \times \frac{1}{2} = 20$$

(5) To find the % increase of a number, use formula

$$\% \text{ increase} = \frac{\text{Total Increase}}{\text{Initial Value}} \times 100$$

Example : Over a five year period, the enrollment at Institute of Perfection, Haridwar; increases from 800 to 1000. Find the per cent of increase.

Solution :

$$\% \text{ increase} = \frac{1000 - 800}{800} \times 100\%$$

$$= \frac{200}{800} \times 100\% = \frac{1}{4} \times 100\% = 25\%$$

(6) To find the % decrease of a number, use formula

$$\% \text{ decrease} = \frac{\text{Total Decrease}}{\text{Initial Value}} \times 100$$

Example : Over a five year period, the enrollment at Institute of Perfection, Haridwar; decreases from 1000 to 800. Find the per cent of decrease.

Solution :

$$\% \text{ decrease} = \frac{1000 - 800}{100} \times 100\%$$

$$= \frac{200}{1000} \times 100\% = \frac{1}{5} \times 100\% = 20\%$$

Growth Rate

Growth is normally in absolute values whereas growth rate is expressed in percentage terms. It could be positive as well as negative. It is always with respect to the previous value unless mentioned otherwise.

$$\text{Growth } \% = \frac{\text{Change in Growth}}{\text{Initial Value}} \times 100$$

Rule of Successive Changes

Let $a\%$ and $b\%$ are the first and second changes respectively than the net change could be expressed as a single percentage.

$$\text{Net Change} = \left(a + b + \frac{ab}{100}\right)\%$$

Note—You can use this formula, where the product of two equal to one.

For example, Area = Length × Breadth, Revenue = Price × Sale, Expenditure = Consumption × Rate of Commodity etc.

Example 1. Institute of Perfection, Haridwar normally employs 100 people. During a slow spell, it fired 20% if its employees. By what per cent must it now increase its staff to return to full capacity ?

Solution : 20% means $\frac{1}{5}$ of 100 = 20

IOP now has 100 – 20 = 80 employees.

If it then increases by 20, the percent of increase is $\frac{20}{80} \times 100\% = \frac{1}{4} \times 100\% = 25\%$.

Example 2. The production of a company is increased from 200 crores to 350 in a financial years 2005-06 and decreased by 35 crore in the subsequent year. What is percentage increase and decrease in production in the respective years ?

Solution : Increase in production in 2005-2006

$$= (350\text{-}200) = 150 \text{ crores}$$

% increase in production

$$= \frac{\text{Increase}}{\text{Base value}} \times 100 = \frac{150}{200} \times 100$$

$$= 75\%$$

Decrease in production in 2006-07 = 35 crores

% decrease in production during 2006-07

$$= \frac{\text{Decrease}}{\text{Base value}} \times 100$$

$$= \frac{35}{350} \times 100 = 10\%$$

Percentage increase / decrease

$$= \frac{\text{Increase/decrease}}{\text{Base value}} \times 100$$

Or, $$\% = \left[\frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} \times 100\right]$$

Example 3. If Vivek's income is 20% more than that of Tony then what per cent is Tony's income less than that of Vivek ?

Solution : Let the income of Tony be Rs. 100,

Income of Vivek = Rs. 120

In the question Tony's income is being compared with that of Vivek's and hence base value to find the % decrease will be the income of Vivek.

$$\% \text{ Decrease} = \frac{\text{Decrease}}{\text{Base value}} \times 100$$

$$= \frac{(120 - 100)}{120} \times 100 = \frac{20}{120} \times 100$$

$$= \frac{50}{3}\% \text{ or } 16\frac{2}{3}\%$$

Example 4. If the price of petrol increases successively by 20 % and then by 10% , what is the net change in percentage terms ?

Solution :

$$\text{Net change in } \% = \left[20 + 10 + \frac{200}{100}\right]\% = 32\%$$

It means that the successive increase of 20% and 10% are equal to a single increase of 32% .

Example 5. The length of a rectangle is increased by 20% and breath is decreased by 10%. Calculate the percentage change in the area .

Solution :

$$\text{Percentage change in area} = \left(a + b + \frac{ab}{100}\right)\%$$

$$= 20 - 10 - \frac{200}{100} = 8\%$$

Example 6. The price of a Maruti car rises by 30% , while the sales of the car goes down by 20% . What is the percentage change in the total revenue ?

Solution : Revenue = Price × Sale

Hence, Percentage change in revenue

$$= \left(30 - 20 - \frac{30 \times 20}{100}\right)\% = 4\%$$

Therefore, revenue will increase by 4%.

Example 7. When the price of sugar was increased by 32%, a family reduced its consumption in such a way that the expenditure on sugar was only 10% more than before. If 30 kg were consumed per month before, find the new monthly consumption.

Solution : Let the original price be Rs. x per kg.

We know that Price × Consumption = Expenses

$\therefore$ Original expenses = $x \times 30$ = Rs. $30x$

After increases, let the new consumption be y kg.

Given, new price = $1{\cdot}32x$

New expenses = $1{\cdot}1 \times 30x$

$$\therefore \quad 1{\cdot}32x \times y = 1{\cdot}1 \times 30x \Rightarrow y = \frac{1{\cdot}1 \times 30x}{1{\cdot}32x} = 25 \text{ kg.}$$

Hence, new monthly consumption = 25 kg .

Example 8. From a man's salary, 10% is deducted on tax , 20% of the rest is spent on education and 25% of the rest is spent on food. After all these expenditures, he is left with Rs. 2700. Find his salary .

Solution : Let the salary of the man be Rs. 100.

Then , after all the expenses he is left with :

$100 \times 0{\cdot}9 \times 0{\cdot}8 \times 0{\cdot}75$ = Rs. 54.

If he is left with Rs. 54, his salary = Rs. 100

If he is left with Rs. 2700, his salary would be

$$= \frac{100}{54} \times 2700 = \text{Rs. } 5000.$$

Some Short Cuts

If, A's income is $r\%$ more than that of B, then :

B's income is $\left(\frac{r}{100 + r} \times 100\right)\%$ less than that of A.

If A's income is $r\%$ less than that of B, then :

B's income is $\left(\frac{r}{100 - r} \times 100\right)\%$ more than that of A.

If, the price of a commodity increases by $r\%$, then reduction in consumption, so as not to increase the expenditure :

$$\left(\frac{r}{100 + r} \times 100\right)\%$$

If, the price of a commodity decreases by $r\%$, then increase in consumption,

$$\left(\frac{r}{100-r}\times 100\right)\%$$

Let, the per cent population of a town be P and let there be an increase of R% per annum, then

(a) Population after n years $= P\left(1+\frac{R}{100}\right)^n$

(b) Population n years ago $= \frac{P}{\left(1+\frac{R}{100}\right)^n}$

Let, the present value of a machine be P and let it depreciate at R% per annum, then

(a) Value of machine after n years $= P\left(1-\frac{R}{100}\right)^n$

(c) Value of machine n years ago $= \frac{P}{\left(1-\frac{E}{100}\right)^n}$

Let, Maximum marks = M, Marks obtained = $x\%$ and Pass marks = P%

If a student obtains x % marks and fails by y marks,

$$M = \frac{y}{P-x}\times 100$$

If marks obtained is given as z and fails by y marks ,

$$M = \frac{z+y}{P}\times 100$$

A scores $x\%$ marks but fails by y marks. B scores $z\%$ marks which is w marks more than pass marks . Then

$$M = \left(\frac{w+y}{z-x}\right)\times 100$$

In a class test $x\%$ student failed in English, $y\%$ failed in Hindi. If $z\%$ failed in both subjects the percentage of students who passed in both subjects is given by :

$[100-(x+y-z]\%$

10. Profit and Loss

Definition of Some Very Important Words

Cost Price (CP) : The price for which an article is bought, is called its cost price.

Selling Price (SP) : The price at which an article is sold, is called its selling price.

Profit or Gain : The difference between the selling price and cost price, is called the profit (If SP > CP). Otherwise it is called the loss. (if CP > SP)

Profit and loss are generally represented as a per cent of the cost price, unless otherwise stated.

Overhead Charges : If an individual has to spend some money on transportation etc., then this extra expenditure, is called overhead charges, *i.e.*, which is not directly connected with production.

Marked Price (MP) : The price on the label is called the marked price or list price .

Discount : The reduction made on the 'Marked Price' of an article is called discount. When no discount is given, 'Selling Price' is same as 'Marked Price'.

Commission : Many sales people earn money on a commission basis. In order to encourage sales, they are paid a percentage of the value of goods sold. This amount is called commission.

List of important short cuts and formulae—

1. Profit = SP – CP (if SP > CP otherwise it is loss)
2. $\%\text{ profit} = \frac{\text{Profit}}{\text{CP}}\times 100 = \frac{\text{SP}-\text{CP}}{\text{CP}}\times 100$
3. $\text{Profit} = \frac{\%\text{ Profit}\times\text{CP}}{100}$ or $\text{Loss} \ \frac{\%\text{ Loss}\times\text{CP}}{100}$
4. $\text{SP} = \frac{100+\text{Profit }\%}{100}\times\text{CP}$
5. $\text{SP} = \frac{100-\text{Loss }\%}{100}\times\text{CP}$
6. SP = Marked price – Discount
7. $\text{CP} = \text{SP}\times\left(\frac{100}{100+\text{Profit }\%}\right)$

   or $\text{SP}\times\left(\frac{100}{100-\text{Loss }\%}\right)$
8. $\%\text{ Discount} = \frac{\text{Discount}}{\text{Marked Price}}\times 100$

Some Short Cuts

In case of successive discount $a\%$ and $b\%$, the effective discount is $\left(a+b-\frac{ab}{100}\right)\%$.

If two items are sold, each at Rs. X, one at a profit of $p\%$ and another at a loss of $p\%$, there is an overall loss

(i) In percentage $= \frac{p^2}{100}\%$

(ii) In value $= \frac{2p^2}{100^2-p^2}X$

Buy A and get B free, *i.e.*, if A + B items are sold at cost price of A items then the percentage discount is calculated by $= \frac{B}{A+B}\times 100$.

If (i) CP of two items is the same and (ii) % loss and % profit on the two articles are equal then net loss or net profit is zero.

If someone (i) buys a table for Rs. l and (ii) sells b tables for Rs. m

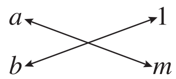
Then net profit $= \left(\frac{m}{b}a\right)-1$ ...(i)

In percentage profit $= \frac{\left[\frac{ma}{bl}-l\right]}{l}\times 100\%$

or $= \left[\frac{ma}{bl}-1\right]\times 100\%$

N.B. If the result is negative ,it represents loss.

Visualization : Quantity price (CP or SP)



$$\% \text{ Profit} = \left(\frac{am}{bl} - 1\right) \times 100\%$$

Example 1. A person sells an article for a price which gives him a profit of 15% on cost price of Rs. 600. Calculate the selling price of the article.

Solution :

$$\begin{aligned} SP &= CP + \text{Profit} \\ &= CP + 15\% \text{ of } CP \\ &= CP\left(1 + \frac{15}{100}\right) \\ &= \frac{CP \times 115}{100} \\ &= \frac{115}{100} \times 600 = 690 \end{aligned}$$

∴ Selling price of the article = Rs. 690 .

Hence, we conclude that , if gain percentage is given alongwith the CP, then

$$\begin{aligned} SP &= \left[1 + \frac{\text{Gain }\%}{100}\right] \times CP \\ &= \frac{(100 + \text{Gain }\%) \times CP}{100} \end{aligned}$$

In case of loss the following formula are applicable—

$$\begin{aligned} \text{Loss} &= CP - SP \\ \text{Loss percentage} &= \frac{\text{Loss}}{CP} \times 100 \end{aligned}$$

Like Profit, Loss percentage is also calculated on CP unless specified.

Example 2. A person buys an article for Rs. 600 and sells the same at a loss of 20%. Find the selling price of the article .

Solution :

$$\begin{aligned} SP &= CP - \text{loss} \\ &= CP - 20\% \text{ of } CP \\ &= CP\left(1 - \frac{20}{100}\right) \\ &= \frac{4}{5} \times 600 = 480 \end{aligned}$$

Therefore, the selling price of the article is Rs. 480.

Hence, we conclude that if loss percentage is given alongwith the CP, then

$$\begin{aligned} SP &= \left[1 - \frac{\text{Loss }\%}{100}\right] \times CP \\ &= \left[\frac{100 - \text{Loss }\%}{100}\right] \times CP \end{aligned}$$

Marked Price or LIST Price or PRINT Price

In a sale transaction , the seller marks the goods more than the cost price in order to earn a profit . This addition to the cost price is called the mark up price and this mark up value added to cost price is called the marked price or list price.

Therefore, Print price = CP + Mark up price

Now, the seller may sell the product on the marked price. In such case,

$$\text{Print price} = \text{Selling price}$$

He may also sell the product after allowing a discount on the marked price. In such case

$$\text{Selling price} = \text{Print price} - \text{Discount}$$

Example 3. A person marks his goods 30% more than the cost price and allows some discount on it. He still makes a profit of 10%. Find the discount percentage.

Solution : Let the CP of the article be Rs. 100.

Then, MP of the article = Rs. 130

Since, profit = 10%, *i.e.,* Rs. 10 (Profit is always on CP hence profit is 10% of Rs. 100 = Rs. 10).

$$\begin{aligned} \therefore \quad SP &= CP + \text{Profit} \\ &= 100 + 10 = \text{Rs. } 110. \end{aligned}$$

The difference between marked price and selling price is the discount.

$$\begin{aligned} \therefore \quad \text{Discount} &= MP - SP \\ &= 130 - 110 = \text{Rs. } 20 \\ \text{Discount }\% &= \frac{\text{Discount}}{\text{Marked Price}} \times 100 \\ &= \frac{20}{130} \times 100 = \frac{200}{13} = 15{\cdot}38 \end{aligned}$$

Note : Discount percentage is always calculated on print price unless specified .

False Weight

Example 4. A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 g for a kg weight. Find his gain per cent.

Solution : Suppose the cost price of 1kg of goods is Rs. 100.

Therefore, cost price of 960 g of goods will be Rs. 96.

Since, he is selling 960 g (Using a false weight instead of 1 kg), the selling price of 1 kg would be, *i.e.,* Rs. 100.

$$\begin{aligned} \text{Therefore,} \quad \text{profit} &= SP - CP \\ &= (100 - 96) = \text{Rs. } 4 \\ \text{Hence,} \quad \text{Profit }\% &= \frac{\text{Profit}}{CP} \times 100 \\ &= \frac{4}{96} \times 100 = 4\frac{1}{6}\% \end{aligned}$$

II Method

$$\text{Gain \%} = \left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100\right]$$

This formula can be used in questions of false weight.

$$\therefore \quad \text{Gain \%} = \left(\frac{40}{960} \times 100\right)\% = 4\frac{1}{6}\%$$

Example 5. By selling two articles for Rs. 180 each, a shopkeeper gains 20% on one and losses 20% on the other, find the percentage profit / loss.

Solution :

$$\begin{aligned}
\text{SP} &= \text{CP} + 20\% \text{ (profit)} \\
\text{and} \quad \text{SP} &= \text{CP} - 20\% \text{ (loss)} \\
\therefore \quad 180 &= 1{\cdot}2 \times \text{CP} \\
\text{and} \quad 180 &= 0{\cdot}8 \times \text{CP} \\
\therefore \quad \text{CP} &= \frac{180}{1{\cdot}2} = \text{Rs. } 150 \\
\text{and} \quad \text{CP} &= \frac{180}{0{\cdot}8} = \text{Rs. } 225 \\
\text{Total cost price} &= (150 + 225) = \text{Rs. } 375 \\
\text{Total selling price} &= (180 + 180) = \text{Rs. } 360 \\
\text{Hence, loss} &= \text{CP} - \text{SP} \\
&= (375 - 360) = \text{Rs. } 15 \\
\text{and} \quad \text{Loss \%} &= \frac{15}{375} \times 100 = 4\%
\end{aligned}$$

In case, where the selling price of two articles is same and one is sold at the loss of x % and another is sold at a profit of x% . Or in other words, the profit % and loss % is same and selling price is same. This transaction always has a loss and such loss % is

$$= \left[a + (-a) + \frac{(a) \times (-a)}{100}\right]$$

$$= \left(\frac{\text{Common loss or gain \%}}{10}\right)^2$$

In the above case loss % $= \left(\frac{20}{10}\right)^2 = 4\%$

Example 6. A Shopkeeper sells two items at the same price. If he sells one of them at a profit of 10%. Find the percentage profit/loss.

Solution : In such transaction, there will always be a loss and Loss %

$$= \left(\frac{\text{Common loss or grain \%}}{10}\right)^2 \%$$

$$= 1\% \text{ loss.}$$

Example 7. By selling 20 posters, a person recovers the cost price of 25 posters. Find the gain or loss percentage.

Solution : Given, SP of 20 poster = CP of 25 poster

Now, suppose SP of 20 poster = CP of 25 poster

= Rs. 100

$$\therefore \quad \text{SP of 1 poster} = \text{Rs. } \frac{100}{20} = \text{Rs. } 5$$

$$\text{and} \quad \text{SP of 1 poster} = \text{Rs. } \frac{100}{25} = \text{Rs. } 4$$

Since, SP > CP hence this transaction will yield a profit.

$$\begin{aligned}
\therefore \quad \text{Profit} &= \text{Rs. } (5 - 4) \\
&= \text{Re. 1 and profit \%} \\
&= \frac{1}{4} \times 100 = 25\%
\end{aligned}$$

Example 8. Toffees are bought at 12 for a rupee and are sold 10 for a rupee. Find the profit or loss percentage.

Solution : CP of 12 toffees = SP of 10 toffees = Re. 1

$$\therefore \quad \text{CP of 1 toffee} = \text{Re. } \frac{1}{12}$$

$$\text{and} \quad \text{SP of 1 toffee} = \text{Re. } \frac{1}{10}$$

Since, SP > CP, hence there is a gain in this transaction.

$$\begin{aligned}
\text{Gain} &= \text{SP} - \text{CP} \\
&= \text{Re. } \left(\frac{1}{10} - \frac{1}{12}\right) = \text{Re. } \frac{1}{60} \\
\text{Gain \%} &= \frac{\frac{1}{60}}{\frac{1}{12}} \times 100 = 20\%
\end{aligned}$$

Example 9. If a commission of 20% is given on retail price, the profit is 60%. Find the profit percentage when the commission is increased by 5% of the retail price.

Solution : Let the retail price be Rs. 100 .

$$\begin{aligned}
\text{Then,} \quad \text{SP} &= \text{Rs. 80 (after a discount of 20\%)} \\
\text{Since,} \quad \text{Profit} &= 60\% \\
\therefore \quad \text{CP} &= \frac{\text{SP}}{1{\cdot}6} = \frac{80}{1{\cdot}6} = \text{Rs. } 50 \\
\text{New} \quad \text{SP} &= \text{Rs. } 75
\end{aligned}$$

(after a commission of 25%)

$$\begin{aligned}
\therefore \quad \text{Profit} &= \text{Rs. } (75 - 50) = \text{Rs. } 25 \\
\text{Hence, Profit \%} &= \frac{25}{50} \times 100 = 50\%
\end{aligned}$$

11. Mixture of Alligations

This Topic deals with a specific type of questions which can be solved quickly using the method of Alligations. Hence, it is very necessary to understand and identify the pattern of such questions so as to apply the rules of alligations.

Alligation is the rule to find the proportion in which the two or more quantities at the given price must be mixed to produce a mixture at a given price.

Rule of Alligation

If two quantities are mixed in a ratio, then

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{CP of dearer} - \text{Mean price}}{\text{Mean price} - \text{CP of cheaper}}$$

Some Short Cuts

(1) m gm of sugar solution has $x\%$ sugar in it. To increase the sugar content in the solution of y %

$$\text{Quantity of sugar needs to be added} = \frac{m\,(y-x)}{100-y}$$

(2) The ingredients of a mixture in its pure form has percentage value 100% and fraction value 1.

(3) When X_1 quantity of ingredient A of cost C_1 and X_2 quantity of ingredient B of cost C_2 are mixed, cost of the mixture C_m is given by

$$C_m = \frac{C_1X_1 + C_2X_2}{X_1 + X_2}$$

(4) Similarly, when more than two ....than

$$C_m = \frac{C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4 + C_5X_5 + \dots}{X_1 + X_2 + X_3 + X_4 + X_5 + \dots}$$

(5) When two mixtures M_1 and M_2, each containing ingredient A and B in the ratio $a : b$ and $x : y$, respectively are mixed, the proportion of the ingredients A and B, *i.e.*, $Q_A : Q_B$ in the compound mixture is given by—

$$\frac{Q_A}{Q_B} = \frac{M_1 \times \left(\frac{a}{a+b}\right) + M_2 \times \left(\frac{x}{x+y}\right)}{M_1 \times \left(\frac{b}{a+b}\right) + M_2 \times \left(\frac{y}{x+y}\right)}$$

and, the quantity in which M_1 and M_2 is to be mixed when the quantity of A and B, *i.e.*, $Q_A : Q_B$ in the compound mixture is given by

$$\frac{\text{Quantity of } M_1}{\text{Quantity of } M_2} = \frac{\left(\frac{x}{x+y}\right) - \left(\frac{Q_A}{Q_A + Q_B}\right)}{\left(\frac{Q_A}{Q_A + Q_B}\right) - \left(\frac{a}{a+b}\right)}$$

If a container originally contain x units of liquid and y unit of liquid is taken out. If this operation is repeated n times. Then, final quantity of the liquid in the container left is :

$$x\left(1 - \frac{y}{x}\right)^n \text{ units}$$

12. Time, Speed and Distance

The speed of a body is the defined as the distance covered by it in unit time. Or the speed of a body is the rate at which it is moving .

(i) More distance, more time, at same speed.

(ii) More speed, less time, for same distance.

(iii) More speed, more distance for same time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{or Distance} = \text{Speed} \times \text{Time}$$

The expression shows that :

1. Distance and time are directly proportional. If distances to be travelled were doubled, then the time taken would also be doubled at the same speed.

2. Time is inversely proportional to speed. If the distance remains the same and speed is doubled, then time taken to travel the same distance becomes half of the original time taken at the original speed.

3. Speed is directly proportional to distance. If the speed is doubled, then distance travelled in the same time, will also be doubled.

Conversion of Units or Standard Conversion Factors

In general : Time is measured in seconds, minutes or hours.

Distance is usually measured in metres, kilometres, miles or feet .

(i) 1 hr = 60 minutes = 60 × 60 seconds

(ii) 1 km = 1000 m or 1km = 0·6214 mile

or 1 mile = 1·609 km

i.e., 8 km ≅ 5 mile (aprox.)

(iii) $x \text{ km/hr} = \left(x \times \frac{5}{8}\right) \text{ m/sec}$

or $y \text{ m/s} = \frac{18}{5}\, y \text{ km/hr}$

(iv) $x \text{ km/hr} = x \times \left(x \times \frac{5}{8}\right) \text{ mile /hr}$

(v) $x \text{ mile/hr} = \left(x \times \frac{22}{15}\right) \text{ ft/s}$

Example 1. If a man walks at the rate of 5 km/hr, he misses a train by 7 minutes. However, if he walks at the rate of 6 km/hr, he reaches the station 5 minutes before the arrival of the train. Find the distance covered by him to reach the station.

Solution : Difference in the times taken at two speeds

$$= (7 + 5) = 12 \text{ min} = \frac{1}{5} \text{ hr}$$

Now, let the required distance be x km.

$$\therefore \quad \frac{x}{5} - \frac{x}{6} = \frac{1}{5} \text{ or } 6x - 5x = 6$$

$$x = 6 \text{ km} \qquad \left(\because \text{time} = \frac{\text{distance}}{\text{speed}}\right)$$

Example 2. If a student walks from his house to school at 5 km/hr, he is late by 30 minutes. However, if he walks at 6 km/hr, he is late by 5 minutes only. Find the distance of his school from his house.

Solution : In both case the student is late.

Therefore, difference in timings :

$$= (30 - 5) = 25 \text{ min}$$

$$= \frac{25}{60} \text{ hr} = \frac{5}{12} \text{ hr}$$

Let the distance be x km.

$$\therefore \quad \frac{x}{5} - \frac{x}{6} = \frac{5}{12}$$

$$6x - 5x = \frac{5 \times 30}{12}$$

$$\Rightarrow \quad x = 12 \cdot 5 \text{ km.}$$

Example 3. A and B are two stations 350 km apart. A train starts from A at 7 a.m. and travels towards B at 40 km/hr. Another train starts from B at 8 a.m. and travels towards A at 60 km/hr. At what time do they meet ?

Solution : Let both the trains meet x hours after 7 a.m.

Then,

Distance moved by first train in x hours + Distance moved by other train in $(x - 1)$ hours = 350 km.

$$40x + 60(x - 1) = 350$$
$$100x = 350 + 60$$
$$x = 4 \cdot 1 \text{ hours}$$

Hence, both the trains will meet at 6 minute past 11.

Example 4. Mr. X arrives at his office 30 minutes late everyday. On a particular day, he reduces his speed by 25% and hence arrives 50 min late instead. Find how much time would he take to travel to his office if he decides to be on time on a particular day ?

Solution :

$$\text{New speed} = \frac{3}{4}\text{S}$$

$$\therefore \quad \text{New time} = \frac{4}{3}\text{T}$$

$$\frac{4}{3}\text{T} - \text{T} = 50 - 30$$

$$\text{or} \quad \frac{\text{T}}{3} = 20$$

$$\text{T} = 60 \text{ minutes.}$$

It was the time taken by X when he was 30 min late. So, if he decides to come on time, he would take 30 minutes to travel.

Example 5. Excluding stoppages, the speed of bus is 54 km/hr and including stoppages, it is 45 km/hr. For how many minutes does the bus stops per hour?

Solution : Due to his stoppages the bus travels 9 km less.

Hence, time taken by the bus to cover 9 km is the time used up at stoppages and time taken to cover 9 km $= \frac{9}{54} \times 60 = 10$ minutes.

Therefore, the bus stops 10 minutes per hour.

Example 6. A is twice as fast as B and B is thrice as fast as C. The journey covered by B in—

(a) 18 min (b) 27 min (c) 38 min (d) 9 min

Solution : Let the speed of C be x km/hr, then speed of B = $3x$ km/hr and speed of A = $6x$ km/hr. Therefore, the ratio of speed of A, B and C = 6 : 3 : 1 and ratio of time taken by A, B and C to cover the same distance $= \frac{1}{6} : \frac{1}{3} : 1 = 1 : 2 : 6$.

Therefore, if C takes 54 minute to cover a distance, then time taken by B to cover the same distance $= \frac{54}{6} \times 2$ = 18 min.

Average Speed

\# If a body convered certain distance in parts at different speeds, the average speed is given by :

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

\# As, if a body travels $d_1, d_2, d_3, \ldots, d_n$ distance, with speed s_1 in time $t_1, t_2, t_3, \ldots, t_n$ respectively then the average speed of the body through the total distance is given by :

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

\# Always remember that , Average speed

$$\neq \frac{\text{Sum of speeds}}{\text{Number of different speeds}} \neq \frac{s_1 + s_2 + \ldots + s_n}{n}$$

\# $$\text{Average speed} = \frac{d_1 + d_2 + \ldots + d_n}{t_1 + t_2 + \ldots + t_n}$$

$$= \frac{s_1 t_1 + s_2 t_2 + \ldots + s_n t_n}{t_1 + t_2 + t_3 + t_4 \ldots + t_n}$$

$$= \frac{d_1 + d_2 + d_3 + \ldots + d_n}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + \ldots + \frac{d_n}{s_n}}$$

Short cut

(1) If you travel equal distance with speeds u and v, then the average speed over the entire journey is $\frac{2uv}{(u + v)}$.

(2) If a man changes his speed in the ratio $m : n$, then the ratio of times taken becomes $n : m$.

Example 7. A person goes to Mansha Devi from Har ki Pauri at the speed of 40 km/hr and comes back at the speed of 60 km/hr. Calculate the average speed of the person for the entire trip.

Solution : It is given that distance travelled by the person at different speeds are the same.

Hence, average speed calculated by $\frac{60 + 40}{2} = 50$ km/hr does not give the correct average speed.

$$\text{Thus, average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

In travelling equal distance with speeds u and v, the average speed is expressed as $\frac{2uv}{(u + v)}$.

$$\text{Hence, Average speed} = \frac{2 \times 60 \times 40}{60 + 40} = 48 \text{ km/hr.}$$

Example 8. What is the average speed if a person travels at the speed of 20 km/hr and 30 km/hr ?

(a) For the equal interval of time.

(b) For equal distance.

Solution :

(a) Average speed $= \frac{20 + 30}{2} = 25$ km/hr

However, it can also be solved by the basic formula of average speed.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{20 \text{ km} + 30 \text{ km}}{1 \text{ hr} + 1 \text{ hr}}$$

$$= \frac{50}{2} = 25 \text{ km/hr.}$$

(b) $$\text{Average speed} = \frac{2uv}{(u+v)} = \frac{2 \times 20 \times 30}{20 + 30}$$

$$= \frac{1200}{50} = 24 \text{ km/hr.}$$

Example 9. A car travelled for 30% of time at a speed of 20 km/hr, 40% of time at a speed of 30 km/hr and rest of the journey at a speed of 40 km/hr. What is the average speed of the car for the entire journey ?

Solution : Let the total time taken in the entire journey be 1 hr.

Then distance travelled in 30% of time at 20 km/hr

$$= 0{\cdot}3 \times 20 = 6 \text{ km}$$

Distance travelled in 40% of time at 30 km/hr

$$= 0{\cdot}4 \times 30 = 12 \text{ km}$$

Distance travelled in 30 % of time at 40 km/hr

$$= 0{\cdot}3 \times 40 = 12 \text{ km}$$

$$\therefore \text{ Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{6 + 12 + 12}{1} = 30 \text{ km/hr.}$$

Example 10. A car covers four successive 3 km stretches at speeds of 10 km/hr, 20 km/hr, 30 km/hr and 60 km/hr respectively. What is the average speed of the car for the entire journey ?

Solution :

$$\text{Total distance} = (3 + 3 + 3 + 3) = 12 \text{ km.}$$

$$\text{Total time} = \frac{3}{10} + \frac{3}{20} + \frac{3}{30} + \frac{3}{60}$$

$$= 3\left(\frac{6 + 3 + 2 - 1}{60}\right)$$

$$= \frac{12 \times 3}{60} = \frac{3}{5} \text{ hr}$$

$$\therefore \text{ Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{12}{3/5} = 20 \text{ km/hr}$$

Relative Speed

The word 'relative' means one with respect to another. Basically relative speed is the phenomenon that we use (observe) every day. Relative speed means the speed of an object A with respect to another object B, which may be stationary, moving in the same direction as A or in the opposite direction as A.

It is often seen, while sitting in a moving train, speed of the train seems to increase when another train moving in the opposite direction crosses it. The speed appears decreasing when another train moving in the parallel track in the same direction passes it. Hence, this feeling of change in speed of train is nothing but its relative speed in relation to the another moving train .

Case I : When one object is stationary and the other is moving

For example : If one Ambulance (A) is standing and second Ambulance (B) passing by, the relative speed of the Ambulance A and the B will be the speed of B. If B's speed is 60 km/hr, then the relative speed is also 60 km/hr.

Case II : When both moving in the same direction

As both Ambulances (A and B) move in the same direction at speeds of 40 km/hr and 60 km/hr respectively, the relative speed of B with respect to A is (60 – 40) = 20 km/hr .

Case III : When both moving in the opposite direction

As both A and B move in the opposite direction, relative speed of B with respect to A is (60 + 40) = 100 km/hr .

Example 11. A person starts from city A towards B at 8·00 a.m. at a speed of 40 km/hr. Another person starts from city B towards A at 8·30 a.m. at a speed of 50 km/hr. At what time will they meet, if the distance between A and B is 200 km.

Solution : It is clear from the above question that both persons are simultaneously in motion at 8·30 a.m. Hence, concept of relative speed will apply only when both the bodies are in motion *i.e.*, at 8·30 a.m. In half hour, person starting from A would have travelled 20 km. Hence, at 8·30 a.m. the distance between A and B is (200 – 20) = 180 km.

$$\text{Distance} = 180 \text{ km,}$$

$$\text{Relative speed} = (50 + 40) = 90 \text{ km/hr}$$

$$\text{Hence, time when they meet} = \frac{108}{90} = 2\text{hr.}$$

Therefore, 2 hours after 8·30 a.m. *i.e.*, 10·30 a.m. both the person will meet.

Example 12. A thief steals a car at 2·30 p.m. and drives it at 60 km/hr. The theft is discovered at 3 p.m. and the owner sets off in another car at 75 km/hr. When will he overtake the thief ?

Solution : Both the persons are in motion at 3 p.m., hence distance between the two persons at 3 p.m. = 30 km (because the thief has travelled 30 km in half an hour).

$$\text{Relative speed} = (75 - 60) = 15 \text{ km/hr}$$

Distance to be covered = 30 km

Hence, time taken by the owner in overtaking the thief

$$= \frac{30}{15} = 2 \text{ hrs.}$$

Therefore, owner will overtake the thief at 2 hours after 3 p.m., *i.e.,* at 5 p.m.

Example 13. Mohit and Ajay are two friends whose homes are 20 km apart. Both of them decide to meet somewhere between their houses. Mohit rides at 8 km/hr and Ajay at 10 km/hr. Mohit leaves his house at 8·00 a.m. and Ajay leaves his house at 9·00 a.m.

(a) At what time they meet ?

(b) At what distance from Ajay's house ?

Solution : Ajay leaves one hour after Mohit has started. Hence at 9.00 a.m. Mohit would have travelled 8 km at a speed of 8 km/hr. Now, at 9.00 a.m. the distance between Mohit and Ajay =(20-8)=12 km. which is to be covered at a relative speed of (8+10)=18 km/hr.

$$\therefore \quad \text{Time taken} = \frac{12}{18} \times 60 = 40 \text{ min.}$$

Thus, both of them will meet 40 minutes after 9·00 a.m., *i.e.*, at 9·40 a.m.

(b) Distance travelled by Ajay in 40 minutes

$$= 10 \times \frac{40}{60} = \frac{400}{60} = \frac{20}{3} \text{ km or } 6\frac{2}{3} \text{ km}$$

Concept of Relative Speed in Motion of Trains

1. Train

(i) Time taken (t) by a train x metres long to cross a stationary person or pole is equal to time taken by the train to cover the distance x metres (equal to its length) with its own speed (v)

$$t = \frac{x}{v}$$

(ii) Time taken by a train x metres long to cross a stationary objects (Like Railway station , bridge, tunnel, another standing train etc.) y metres long is equal to time taken by the train to cover a total distance ($x + y$) meters as its own speeds (v)

$$t = \frac{x+y}{v}$$

(iii) If two trains of length 'x' and 'y' metres move in the same direction at 'a' and 'b' m/sec, then the time taken to cross each other from the time they meet

$$t = \frac{\text{Sum of their length}}{\text{Relative speed}} = \frac{x+y}{a-b}$$

N.B. If above trains move in the opposite direction, then

$$T = \frac{\text{Sum of their length}}{\text{Relative speed}} = \frac{x+y}{a+b}$$

(iv) Two trains A and B start from two points p and q and move towards each other, after crossing they take time a and b in reaching q and p respectively. Then the ratio of their speed is given by :

$$\text{A's speed : B's speed} = \sqrt{b} : \sqrt{a} = \sqrt{\frac{b}{a}}$$

Example 14. A train 140 m long is running at 60 km/hr. In how much time will it pass a platform 260 m long ?

Solution :

$$\text{Speed of train} = 60 \times \frac{5}{18} = \frac{50}{3} \text{ m/s}$$

Distance covered by train in crossing the platform

$$= (140 + 260) = 400 \text{ m}$$

$$\therefore \quad \text{Time taken} = 400 \times \frac{3}{50} = 24 \text{ s}$$

Example 15. Two trains are running on parallel lines in the same direction at speeds of 40 km/hr and 20 km/hr respectively. The faster train crosses a man in the second train in 36 seconds. Find the length of faster train.

Solution : Let the length of the faster train be x metre. Since, train crosses a man sitting in another train, hence we are concerned with the length of one train only.

$$\text{Speed} = (40 - 20)$$

$$= 20 \times \frac{5}{18} \text{ m/s}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{x \times 18}{20 \times 5}$$

$$\text{or} \quad 36 = \frac{18x}{100}$$

$$\therefore \quad x = 200 \text{ m}$$

Example 16. A train speeds past a pole in 15 second and a platform 100 m long in 25 seconds. Find the length of train.

Solution : Let the length of train be x metre and speed be y m/s.

$$\therefore \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{x}{y} \text{ or } 15 = \frac{x}{y}$$

$$\therefore \quad \text{Speed } (y) = \frac{x}{15} \text{ m/s}$$

The train passes the platform 100 m long in 25 seconds.

Therefore, train will have to cover a distance of (x + 100) metre with a speed of $\frac{x}{15}$ m/s.

$$\text{Speed} = \frac{(x + 100)}{25}$$

$$\frac{x}{15} = \frac{(x+100)}{25}$$

$$x = 150 \text{ m}$$

Example 17. A train running at 54 km/hr takes 20 seconds to pass a platform. Next it takes 12 seconds to pass man walking at 6 km/hr in the same direction in which the train is going. Find the length of the train and the length of the platform.

Solution :

Speed of train in relation to man = (54 – 6) km/hr

$$= 48 \times \frac{5}{18}$$

$$= \frac{40}{3} \text{ m/sec.}$$

Length of train (T) = Speed of Train (Relative) × Time to pass a man

$$= \frac{40}{3} \times 12 = 160 \text{ m}$$

Train + Platform (T + P)

$$= \frac{54 \times 5}{18} \times 20$$

$$= 300 \text{ m}$$

Hence, length of platform

$$= 300 - 160$$

$$= 140 \text{ m}$$

2. Boats and Streams

Upstream : If a boat (or person) moves against the stream (in opposite direction) it is called upstream $(\rightarrow \leftarrow)$.

Downstream : If a boat (or person) moves with the stream (in same direction) it is called downstream $\left(\begin{matrix}\rightarrow\\\rightarrow\end{matrix}\right)$.

(i) If the speed of a boat in still water is B km/hr, and the speed of the stream is S km/hr.

Speed of boat in downstream $\left(\begin{matrix}\rightarrow\\\rightarrow\end{matrix}\right)$ = (B + S) km/hr.

Speed of boat in upstream $(\rightarrow \leftarrow)$ = (B – S) km/hr.

(ii) Let boat's speed downstream $\left(\begin{matrix}\rightarrow\\\rightarrow\end{matrix}\right) = x$ km/hr and boat's speed upstream $(\rightarrow \leftarrow) = y$ km/hr. Then,

(a) rate of boat in still water $= \frac{1}{2}(x + y)$ km/hr.

(b) rate of stream $= \frac{1}{2}(x - y)$ km/hr.

Example 18. A man can row 6 km/hr in still water and the river is running at 4 km/hr. If a man takes $1\frac{1}{2}$ hour to row to a place and back, how far is the place?

Solution : Downstream Speed = (6 + 4) = 10 km/hr

Upstream Speed = (6 – 4) = 2 km/hr

Let the distance be x, then

$$\frac{x}{10} + \frac{x}{2} = \frac{3}{2}$$

$$\Rightarrow \quad 6x = \frac{30}{2}$$

Hence, $x = 2{\cdot}5$ km.

13. Work and Time

Some Basic Relations

1. If A can do a piece of work in 'm' number of days, then in one day $\left(\frac{1}{m}\right)$th of a work is done. Conversely, if a man does $\left(\frac{1}{m}\right)$th of a work in 1 day, then he can complete the work in $1 \div \frac{1}{m} = m$ days.

2. If A is 'm' times as good a workman as B, then he (A) will take $\left(\frac{1}{m}\right)$th of the time taken by B to do the same work .

3. Amount of work done, basically depends on various variables, like number of persons, time period, their efficiency etc.

$$\frac{L_1T_1}{W_1} = \frac{L_2T_2}{W_2} = k \text{ (constant)}$$

[L means number of Labour doing the work (W) in Time T]

4. If A and B can do a piece of work in 'l' and 'm' days respectively, then working together, they will take $\left(\frac{lm}{l+m}\right)$ days to finish the work and in one day, they will finish $\left(\frac{l+m}{lm}\right)$th part to the work.

5. If A, B and C can do a certain piece of work in l, m and n days respectively, then they can together do the same work in $\left(\frac{l.m.n}{lm + mn + nl}\right)$ days.

6. A and B together can do a work in l days, B and C together in m days, C and A together in n days, then the same work can be done :

By A alone in $\left(\frac{2lmn}{lm + mn - nl}\right)$ days.

By B alone in $\left(\frac{2lmn}{mn + nl - lm}\right)$ days.

and by (A + B + C) together in $\left(\frac{2lmn}{lm + mn + nl}\right)$ days.

7. A group of 'm' persons can do a work in 'd' days. If the group had 'n' more persons the work could be finished in 't' days less. Then, the number of persons initially present in the group m :

$$= \frac{n \times (d - t)}{t}$$

8. 'm' persons start to do a work in 'd' days. After 't' days, n more men had to join in order to complete the work in scheduled time. If 'n' men had not joined them, the number of additional days needed to complete the work is given by :

$$= \frac{n \times (d - t)}{m}$$

14. Simple Interest and Compound Interest

Simple interest is the interest accrued on a certain sum at a certain rate of interest on flat basis irrespective of any time. It means for first and subsequent years the amount on which the interest is calculated remains the same and hence no benefit on the interest calculated on the previous years is given in the subsequent years.

Simple interest is given by the following formula.

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$\therefore \quad P = \frac{100 \times \text{S.I.}}{R \times T}, R = \frac{100 \times \text{S.I.}}{P \times T},$$

$$T = \frac{100 \times \text{S.I.}}{P \times R}$$

where P = Principal amount or amount on which the interest is calculated .

R = Rate of interest (per annum)

T = Time period for which the interest is calculated .

$$\text{S.I.} \propto P \times R \times T$$

Simple interest is directly proportional to principal, rate and time. It means if the interest on a sum at a given rate of interest for 1 year is Rs. 100, then interest for 2 years, 3 years and 4 years (other things being same) will be Rs. 200, Rs. 300 and Rs. 400 respectively.

Example 1. Calculate the simple interest on Rs. 7200 at $12\frac{3}{4}\%$ per annum for 9 months.

Solution : $\text{S.I.} = \frac{P \times R \times T}{100} \left[R = \frac{51}{4}\%, t = \frac{9}{12}\text{ years}\right]$

$$= \frac{7200 \times 51 \times 9}{4 \times 12 \times 100} = \text{Rs. } 688{\cdot}50.$$

Example 2. At what rate percent per annum will a sum of money double in 8 years ?

Solution : Let $P = x$, then $a = 2x$, S.I. $= (2x - x) = x$, $t = 8$ years

$$\therefore \quad R = \frac{\text{S.I.} \times 100}{P \times T} = \frac{x \times 100}{x \times 8} = 12{\cdot}5\%$$

Example 3. The simple interest on a sum of money is 25% of the principal and the rate per annum is equal to number of years. Find the rate per cent.

Solution : Let the principal be Rs. x, then S.I.= Rs. $\frac{x}{4}$

$$T = R, \text{ if } R = \text{rate per annum}$$

$$\therefore \quad \frac{x}{4} = \frac{x \times R \times R}{100}$$

$$\Rightarrow \quad R^2 = 25 \text{ or } R = 5\%$$

Example 4. A certain sum of money amounts to Rs.1560 in 2 years and Rs. 2100 in 5 years. Find the rate percentage per annum.

Solution : Amount (Principal + Interest) for 2 years = Rs. 1560

Amount (Principal + Interest) for 5 years = Rs. 2100

Hence, interest for 3 years = (2100 –1560) = Rs.540

Simple interest for 2 years = Rs .360

$$\therefore \quad \text{Principal} = (1560 - 360) = \text{Rs. } 1200$$

$$R = \frac{360 \times 100}{1200 \times 2} = 15\%$$

$$R = 15\%$$

Example 5. A certain sum of money amounts to Rs. 1008 in 2 years and to Rs. 1164 in $3\frac{1}{2}$ years. Find the sum and the rate of interest.

Solution :

$$\text{SI for } \left(3\tfrac{1}{2} - 2\right) = 1\tfrac{1}{2}\text{ years}$$

$$= \text{Rs. } (1164 - 1008) = \text{Rs. } 156$$

$$\text{SI for 2 year} = \text{Rs. } 208$$

$$\therefore \quad \text{Principal} = \text{Rs. } (1008 - 208) = \text{Rs. } 800$$

Now, P = 800, T = 2 years and SI = Rs. 208

$$\therefore \quad \text{Rate} = \frac{100 \times \text{SI}}{P \times T} = \frac{100 \times 208}{800 \times 2} = 13\%$$

The difference in amount for two different time periods is equal to the simple interest for the difference in two different time periods .

Example 6. A man borrowed Rs. 24000 from two money lenders. For one loan he paid 15% annum and for the other 18% per annum. At the end of one year, he paid Rs. 4050. How much did he borrow at each rate ?

Solution : Let the sum borrowed at 15% be Rs. x then sum borrowed at 18% will be Rs. $(24000 - x)$.

$$\text{Therefore, } \frac{x \times 15 \times 1}{100} + \frac{(24000 - x) \times 18 \times 1}{100} = 4050$$

$$\text{Or, } \quad 15x + 432000 - 18x = 405000 \text{ or } x = 9000$$

$$\therefore \quad \text{Money borrowed at 15\%} = \text{Rs. } 9000$$

$$\text{and money borrowed at 18\%} = \text{Rs. } (24000 - 9000)$$

$$= \text{Rs. } 15000$$

Example 7. Rs.800 amounts to Rs. 920 in 3 years at simple interest. If the interest is increased by 3%, it would amount to how much ?

Solution : $\text{SI} = (920 - 800) = \text{Rs. } 120$

$$120 = \frac{800 \times r \times 3}{100}$$

$$\therefore \quad r = 5\%$$

$$\text{Interest at 8\%} = \frac{800 \times 8 \times 3}{100}$$

$$= \text{Rs. } 192$$

$$\therefore \quad \text{Amount} = (800 + 192)$$

$$= \text{Rs. } 992$$

Example 8. What annual installment will discharge a debt of Rs. 1092 due in 3 years at 12% simple interest ?

Solution : Let each installment be Rs. x, then first installment paid after 1 year will earn an interest for 2 years at 12% and second installment paid after 2 year will earn an interest for 1 year at the same rate as the debt has to be squared off in 3 years.

$$\therefore \quad \left[x + \frac{x \times 12 \times 1}{100}\right] + \left[x + \frac{x \times 12 \times 2}{100}\right] + x = \text{Rs. } 1092$$

$$\frac{28x}{25} + \frac{31x}{25} + x = 1092$$

$$\text{or} \quad 28x + 31x + 25x = 1092 \times 25$$

$$\therefore \quad x = \text{Rs. } 325$$

Short cut : The annual payment that will discharge a debt of Rs. A due in t years at r% rate of interest per annum is :

$$\frac{100A}{\left[100t + \frac{\text{Rt}\,(t-1)}{2}\right]} = \left[\frac{100 \times 1092}{100 \times 3 + \frac{3 \times 12\,(3-1)}{2}}\right]$$

$$= \frac{109200}{336} = \text{Rs. } 325$$

Example 9. A sum was put at simple interest at a certain rate for 2 years. Had it been put 3% higher rate, it would have fetched Rs. 72 more ? Calculate the sum.

Solution : Let the sum be Rs. x and rate be r%.

$$\text{Then,} \quad 1280 = \frac{x \times (r+3) \times 2}{100} - \frac{x \times r - 2}{100}$$

$$= 72$$

$$\text{Or,} \quad 2rx + 6x - 2rx = 7200$$

$$\Rightarrow \quad x = \text{Rs. } 1200$$

Example 10. The rate of interest on a sum of money is 4% per annum for the first 2 years, 6% per annum for the next 3 years and 8% per annum for the period beyond 5 years. If the simple interest accrued by the sum for a total period of 8 years is Rs. 1280, what is the sum ?

Solution :

Rate of interest for first 2 years = $4 \times 2 = 8\%$

Rate of interest for next 3 years = $6 \times 3 = 18\%$

Rate of interest for last 3 years = $8 \times 3 = 24\%$

Total rate of interest for 8 years = 50%

$$\text{Then,} \quad 1280 = \frac{\text{P} \times 50}{100}$$

$$\therefore \quad \text{P} = \text{Rs. } 2560$$

(Time of 8 years is already adjusted in the total rate of interest calculation)

Compound Interest

Compound Interest : The interest charged every year on the amount of last year is called compound interest.

$$\text{Amount} = \text{Principal}\left(1 + \frac{\text{R}}{100}\right)^n$$

$$\text{or} \quad \text{A} = \text{P}\,(1 + \text{R} \times 0{\cdot}01)$$

$$\text{Compound interest} = \text{Amount} - \text{Principal.}$$

Conversion Period : The time period after which the interest is added each time to form a new principal, is called conversion period. It may be one year, six months or three months, *i.e.*, annually, half-yearly or quarterly etc.

The following table will illustrate the conceptual working of simple interest and compound interest.

Rate of interest per annum is 10%.

For the Year	Simple Interest		Compound Interest	
	Principal	**SI**	**Principal**	**CI**
1	1000	100	1000	100
2	1000	100	1000 + 100 = 1100	110
3	1000	100	1100 + 110 = 1210	121

On the basis of above calculation, it is clear that :

(1) Simple interest for each year is constant.

(2) Compound interest calculated for each year includes—Simple interest on principal and simple interest on interest calculated for previous year.

Let principal = p, rate = R% per annum, time = n years and amount = A

(i) When interest is compounded annually :

$$\text{Amount} = p\left[1 + \frac{\text{R}}{100}\right]^n$$

(ii) When interest is compounded half- yearly :

Half-yearly : n = 2 × (given time in years) and R = 1/2 (given rate of interest per annum)

$$\text{Amount} = p\left[1 + \frac{\frac{\text{R}}{2}}{100}\right]^{2n}$$

(iii) When interest is compounded quarterly :

Quarterly : $n = 4 \times$ (given time in years) and R = $\frac{1}{4}$ (given rate of interest per annum)

$$\text{Amount} = p\left[1 + \frac{\frac{\text{R}}{4}}{100}\right]^{4n}$$

(iv) When interest is compounded annually but time is in fraction , say $3\frac{2}{5}$ years

$$\text{then,} \quad \text{Amount} = p\left[1 + \frac{\text{R}}{100}\right]^3 \times \left[1 + \frac{\frac{2}{5}\text{R}}{100}\right]$$

(v) Present worth of Rs. x due in n years, hence is given by :

$$\text{Present worth} = \frac{x}{\left[1 + \frac{R}{100}\right]^n}$$

Arrangement for Different Conversion Periods

The ratio of change in an object over a particular period to the measurement of that object for that particular Period .

Rate of growth of objects can be positive in some cases as in case of population; and in some cases it may be negative as in the case of depreciation of machinery over a period of time .

In compound interest , the amount at rate R and n time can be calculated by the formula

$$A = P(1 + R \times 0{\cdot}01)^n$$

If the rate of growth is negative, in that case formula becomes

$$A = P(1 - R \times 0{\cdot}01)^n$$

(where, n is the number of conversion periods)

The rate of growth may vary from one to the other conversion period . In that case the formula becomes :

$$A = P(1 + R_1 \times 0{\cdot}01)(1 + R_2 \times 0{\cdot}01)(1 + R_3 \times 0{\cdot}01) \ldots (1 + R_n \times 0{\cdot}01)$$

To find rate (R), principal (P), time (n) ,

$$P = A \div (1 + R \times 0{\cdot}01)^n$$

$$\text{Or} \quad (1 + R \times 0{\cdot}01)^n = \frac{A}{P}$$

$$\text{Or} \quad C.I. = P\{(1 + R \times 0{\cdot}0)^n - 1\}$$

Example 11. Calculate the population of a town after 2 years, if it grows at a rate of 10% per annum. The present population is 1200000.

Solution : Population of the town after 2 years

$$= 12{,}00{,}000 \times 1{\cdot}1 \times 1{\cdot}1$$

$$= 1452000 \qquad \left(\because 1{\cdot}1 = \frac{100 + 10}{100}\right)$$

The same question can be calculated by the method of compound interest .

$$\text{Amount} = P\left(1 + \frac{r}{100}\right)^n$$

$$= 1200000\left(1 + \frac{10}{100}\right)^2 = 1452000$$

Example 12. Find the compound interest on Rs. 30500 at 15% per annum for 2 years compounded annually.

Solution :

$$A = 30500\left(1 + \frac{15}{100}\right)^2$$

$$= 30500 \times 1{\cdot}15 \times 1{\cdot}15$$

$$= 40336{\cdot}25$$

$$\therefore \quad CI = (40336{\cdot}25 - 30500)$$

$$= \text{Rs. } 9836{\cdot}25$$

Example 13. Find the compound interest on Rs. 12000 at 12% per annum for 1 years, compounded quarterly.

Solution : $R = \frac{12}{4} = 3\%$ per quarterly, P = 12000

$$\therefore \quad \text{Amount} = 12000\left(1 + \frac{3}{100}\right)^4$$

$$= 12000\,(1{\cdot}03)^4$$

$$= \text{Rs. } 13506{\cdot}11$$

$$\therefore \quad CI = (13506{\cdot}11 - 12000) = 1506{\cdot}11$$

Example 14. Find the compound interest on Rs. 16000 at 20% per annum for 9 months compounded quarterly .

Solution : R = 5% per quarter, T = 9 months or 3 quarterly

$$\therefore \quad \text{Amount} = 16000 \times \left(1 + \frac{5}{100}\right)^3$$

$$= 16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$= \text{Rs. } 18522$$

$$\therefore \quad C.I. = (18522 - 16000) = \text{Rs. } 2522$$

Example 15. The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is Rs. 631. Find the sum.

Solution : Let the sum be Rs. x.

$$C.I. = x\left(1 + \frac{10}{100}\right) - x = \frac{21x}{100}$$

$$SI = \frac{x \times 10 \times 2}{100} = \frac{x}{5}$$

$$\text{Given,} \quad \frac{21x}{100} - \frac{x}{5} = 631$$

$$\Rightarrow \quad x = \text{Rs. } 63100$$

Concept : Simple interest and compound interest for the first year is the same. Difference in the second year's interest is due to the fact that compound interest is calculated over the first year's interest also .

Hence, Rs. 631 is the interest on the interest of first year at 10% . Hence , interest on first year = Rs. 6310.

Now, if interest for first year is Rs. 6310 at 10% then principal = Rs. 63100.

Example 16. Rs. 25000 is borrowed at CI at the rate of 3% for the first year, 4% for the second year and 5% for the third year. Find the amount to be paid after 3 years.

Solution :

$$\text{Amount} = 25000 \times 1{\cdot}03 \times 1{\cdot}04 \times 1{\cdot}05$$

$$= \text{Rs. } 28119$$

Example 17. At what rate per cent per annum will Rs. 1000 amount to Rs. 1331 in 3 years ? The interest is compounded yearly.

Solution :

$$\frac{1331}{1000} = \left(1 + \frac{R}{100}\right)^3$$

$$\left(\frac{11}{10}\right)^3 = \left(1 + \frac{R}{100}\right)^3$$

$$\Rightarrow \quad \frac{11}{10} = 1 + \frac{R}{100}$$

$$\Rightarrow \quad R = 10\%$$

Example 18. A sum of money placed at compound interest doubles itself in 4 years. In how many years will it amount to eight times itself ?

Solution : Given, $2P = P\left(1 + \frac{r}{100}\right)^4$

or, $\left(1 + \frac{r}{100}\right)^4 = 2$

or, $\left[\left(1 + \frac{r}{100}\right)^4\right]^3 = (2)^3 = 8$

or, $P\left(1 + \frac{r}{100}\right)^{12} = 8P$

Hence, required time is 12 years.

Remember : If a sum becomes n times in t years at compound interest then it will be $(n)^m$ times in mt years.

Thus, if a sum of money doubles itself in 4 years , then it will become $(2)^3$. Times in $4 \times 3 = 12$ years..

Example 19. If the compound interest on a certain sum for 2 years at 3% be Rs. 101·50, what would be the simple interest ?

Solution :

$$\text{CI on 1 rupee} = \left(1 + \frac{3}{100}\right)^2 - 1 = \frac{609}{1000}$$

$$\text{SI on 1 rupee} = \frac{2 \times 3}{100} = \frac{6}{100}$$

$$\frac{\text{SI}}{\text{CI}} = \frac{6}{100} \times \frac{10000}{609} = \frac{200}{203}$$

$$\therefore \quad \text{SI} = \frac{200}{203} \times \text{CI} = \frac{200}{203} \times 101{\cdot}50$$

$$= \text{Rs. } 100$$

If on a certain sum of money , the SI of 2 years at the rate r% per annum is Rs. X, then the difference in the compound interest and simple interest is given by Rs. $\left(\frac{Xr}{200}\right)$. This formula is applicable only for 2 years .

Example 20. On a certain sum of money, the simple interest for 2 years is Rs . 50 at the rate of 5% per annum. Find the difference in CI and SI.

Solution : Using above formula :

$$\text{Difference in CI and SI} = \frac{50 \times 5}{200} = \text{Rs. } 1{\cdot}25$$

$$\text{Concept :} \quad 50 = \frac{P \times 2 \times 5}{100}$$

$$\therefore \quad P = \text{Rs. } 500$$

$$A = 500 \times \frac{105 \times 105}{100 \times 100} = \text{Rs. } 551{\cdot}25$$

$$\therefore \quad \text{CI} = (551{\cdot}25 - 500) = \text{Rs. } 51{\cdot}25$$

$$\text{Hence,} \quad \text{CI} - \text{SI} = 51{\cdot}25 - 50 = \text{Rs. } 1{\cdot}25$$

When difference between the CI and SI on a certain sum of money for 2 years at r% rate is x, then the sum is given by :

$$\text{Sum} = \frac{\text{Difference} \times 100 \times 100}{\text{Rate} \times \text{Rate}} = x \times \left(\frac{100}{r}\right)^2$$

Example 21. The difference between the compound interest and the simple interest on a certain sum of money at 5% per annum for 2 years is Rs. 1·50. Find the sum.

Solution : Using the above formula :

$$\text{Sum} = 1{\cdot}5\left(\frac{100}{5}\right)^2 = 1{\cdot}5 \times 400 = \text{Rs. } 600$$

On a certain sum of money, the difference between compound interest and simple interest for 2 years at r% rate is given by sum $\left(\frac{r}{100}\right)^2$.

Example 22. Find the difference between the compound interest and simple interest for Rs. 2500 at 10% per annum for 2 years.

Solution : Using the above formula :

$$\text{Difference} = \text{sum}\left(\frac{R}{100}\right)^2$$

$$= 2500 \times \left(\frac{10}{100}\right)^2 = \text{Rs. } 25$$

If the difference between CI an SI on a certain sum for 3 years at r% is Rs x, the sum is given by:

$$Sum = \frac{\text{Difference} \times (100)^3}{R^2\,(300 + R)}$$

Example 23. If the difference between CI and SI on a certain sum of money for 3 years at 5% per annum is. 122, find the sum.

Solution : Using the above formula :

$$\text{Sum} = \frac{122 \times 100 \times 100 \times 100}{5^2\,(300 + 5)}$$

On a certain sum of money, the difference between compound interest and simple interest for 3 years at *r*% per annum is given by :

$$\text{Difference} = \frac{\text{Sum} \times R^2\,(300 + R)}{(100)^3}$$

Example 24. Find the difference between CI and SI on Rs. 8000 for 3 years at 2·5 % per annum.

Solution : Using above formula

$$\text{Difference} = \frac{8000 \times (2{\cdot}5)^2 \times (300 + 2{\cdot}5)}{100 \times 100 \times 100}$$

$$= \frac{8 \times 25 \times 25 \times 3025}{100 \times 100 \times 100}$$

$$= \frac{121}{8} = \text{Rs. } 15{\cdot}125$$

Hire Purchase : In a hire purchase plan, a customer can make use of the goods while paying for them. The amount paid at the time of purchase is called the down payment. The remainder is paid in equal installments and

each is the monthly installment. The difference between the total amount to be paid and the cash price is called the installment charges.

$$\text{Monthly Installment} = \frac{\text{Amount to be Paid} - \text{Down Payment}}{\text{Number of Instalments}}$$

Example 25. If a LG Refrigerator is available at Rs 4,000 cash or Rs. 1,000 down payment and Rs. 700 per month for 5 months. Find : (i) Total Amount paid for it, (ii) The Installment Charge.

Solution :

(i) Amount paid $= 1000 + 700 \times 5 = 4500$

(ii) Installment Charge $= 4500 - 4000 = 500$

15. Pipe and Cistern

Problems on pipes and cisterns are closely related to problems on work.

A pipe connected with a cistern is called an inlet, if it fills the cistern.

A pipe connected with a cistern is called an outlet, if it empties the cistern.

In fact, filling or emptying a cistern can be considered as work done.

Some Useful Points :

1. If an inlet pipe fills a cistern in 'a' hours, then $\frac{1}{a}$th part is filled in 1 hour.

Similarly, if an outlet pipe empties a cistern in 'a' hour, then $\frac{1}{a}$th part is emptied in 1 hour.

2. If pipe A is 'x' time bigger than pipe B, then pipe A will take $\frac{1}{x}$th of the time taken by pipe B to fill the cistern.

3. If A and B fill a cistern in 'm' and 'n' hours, respectively then together they will take $\frac{mn}{m+n}$ hours to fill the cistern and in one hour $\left(\frac{m+n}{mn}\right)^{\text{th}}$ part of the cistern will be filled.

4. If an inlet pipe fills a cistern in 'm' hours and an outlet pipe empties the cistern in 'n' hours, then the net part filled in 1 hour when both the pipes are opened, is $\left(\frac{1}{m} - \frac{1}{n}\right)$ *i.e.,* $\frac{n-m}{mn}$ and the cistern will get filled in $\left(\frac{mn}{m-n}\right)$ hours.

If more then one pipe is working for inlet or as outlet pipes then you can use following formula.

Net part filled of a cistern = (Sum of work done by inlets pipes) – (Sum of work done by outlet pipes)

Some Short Cuts

1. If an inlet pipe fills a cistern in 'a' minutes, takes 'x' minutes longer to fill the cistern due to a leak in the cistern, then the time in which the leak will empty the cistern is given by $a \times \left(1 + \frac{a}{x}\right)$.

2. If two pipes A and B can fill a cistern in 'x' minutes and if A alone can fill it in 'a' minutes more than 'x' minutes and B alone can fill it in 'b' minutes more than 'x' minutes, then $x = \sqrt{ab}$.

Part B

ADVANCE TOPICS OF MBA MATHEMATICS

1 Numbers

Numbers are collection of certain symbols or figures called digits. The common number system is use in Decimal system. In this system we use ten symbols each representing a digit.

These are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. A combination or single of these figures representing a number is called a natural number.

Number system is the key concept in every branch of mathematics. The use and scope of number system is unlimited. The system deals with the nomenclature, use and properties of numbers.

CLASSIFICATION OF NUMBERS

1. Natural Number—These are also called counting numbers as these numbers are the ones which we use for counting purpose. It is represented by

N : { 1, 2, 3, 4, …………10,000 …………}

2. Whole Number—It include all Natural numbers plus zero and we can denote it by

W : { 0, 1, 2, 3, ………10256, …………}

3. Integer—It includes all whole numbers alongwith negative numbers. It is represented by

I : {………, -5, -4, -1, 0, 1, 2, 3, 4, …………}

Natural numbers are categorized into the following numbers.

(a) Even Number—A number which is completely divisible by 2 is called an even number.

Example—{ 2, 4, 6, 8, 10, ………,102056,………}

(b) Odd Number—A number which is not completely divisible by 2 is called odd number.

Example—{ 1, 3, 5, 7, ………,10001, ………}

(c) Prime Number—The numbers that have only two factors 1 and number itself are called prime numbers.

Example—{ 2, 3, 5, 7, 11, 13, 17, 19, ………}

(d) Twin Prime Number—If the difference between two consecutive prime number is 2 then

Both prime numbers are known as twin prime numbers.

Example—{ 5, 7 },{ 17, 19 }

(e) Composite Number—A composite number is one which has other factors besides itself and unity.

Example—4, 6, 9, 14, 15, ………etc.

SOME IMPORTANT POINTS ABOUT PRIME NUMBERS

1. 1 is not a prime number.
2. There is only one even prime number *i.e.*, 2.
3. A composite number may be even or odd.
4. All prime number can be written in the form (6N – 1) or (6N + 1). The converse is not necessarily true. This means any number of the form (6N – 1) or (6N + 1) is not necessarily a prime number.
5. The remainder when a prime number $p \geq 5$ is divided by 6 or 5.

4. Fractional Number—A number which represents a ratio or division of two numbers is called fractional number.

Example— $-\frac{3}{2}, \frac{5}{2}, \frac{1}{2}, \frac{7}{2}$

A fractional number has two parts Numerator and Denominator

$\frac{3}{2}$ = 3 is = Numerator and 2 is denominator

(a) Equivalent Fraction—Two fractions are said to be equivalent if they represent the same ratio or number.

So if we multiply or divide the numerator and denominator of a fraction by the same non-zero, integer, the result obtained will be equivalent to the original fraction.

Example— $\frac{20}{25}$ is equivalent to $\frac{4}{5}$

$\frac{1}{3}$ is equivalent to $\frac{3}{9}$

(b) Proper Fraction—When denominator is greater than numerator then such fraction number is known as proper fraction.

Example— $\frac{1}{3}, \frac{1}{5}, \frac{3}{4}, \frac{7}{9}$

(c) Improper Fraction—Those fractional number whose numerator is greater than denominator are called improper fraction.

Example— $\frac{7}{2}, \frac{5}{2}, \frac{7}{3}, \frac{17}{4}$

(d) Mixed Fraction—It consists of integral as well as the fractional part.

Example— $2\frac{3}{4} = \frac{11}{4}$

where 2 is integral part and $\frac{3}{4}$ is fractional part.

In general, for any composite number C, which can be expressed as

$$C = a^m \times b^n \times c^p \ \ldots\ldots$$

where a, b, c ……… are all prime factors and m, n, p are positive integers.

So, the total number of factor is given by $= (m + 1)(n + 1)(p + 1)$………

(e) Compound Fraction—A fraction of a fraction is known as compound fraction.

Example— $\frac{4}{5}$ or $\frac{9}{11}$

$\frac{4}{5} \times \frac{9}{11} = \frac{36}{55}$

$\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

(f) Complex Fraction—Any complicated combination of other type of fractions.

Example— $2\frac{1}{3}$ or $\cfrac{3}{1 + \cfrac{2}{3}}$

$\frac{4}{3}$ of $\cfrac{3}{3 + \cfrac{2}{1 + \cfrac{1}{2+3}}}$

5. Rational Number—If a number can be expressed in the form of $\frac{p}{q}$ where $q \neq 0$ and where p and q are integers, then the number is called rational number.

Example— $\frac{9}{25}, \frac{1}{3}, \frac{5}{2}, -\frac{27}{51}, -\frac{3}{5}$

Important points—Fractional numbers and integers are the part of rational number.

6. Irrational Number— If a number can not be expressed in the form of $\frac{p}{q}$, where $q \neq 0$, then the number is called irrational number.

In other words—

Non-repeating and non-terminating type of decimals are called irrational numbers.

Example— $\sqrt{2}, \sqrt[3]{4}, 4{\cdot}3767, 0{\cdot}3333$

7. Real Number— Set of all numbers that can be represented on the number line are called real numbers. It includes all number such as whole, fractional, integer, rational, natural numbers.

8. Complex Number—It is also known as imaginary number.

Imaginary number is written in the form $a + ib$, where a, b are real numbers.

i = imaginary unit whose value is $\sqrt{-1}$.

Those numbers which are inside root symbol with negative sign are known as complex number.

Example— $\sqrt{-3}, 2 + 3i, 3 - i\sqrt{3}$

Complex Number

Example—$2i, 3i, 1 + \sqrt{-3}, 1 + 2i$

Real

Example—(4, – 8, 1·32)

Imaginary

Example—$\sqrt{-1}, \sqrt{-5}$

Rational

Example—$1, 2, \frac{3}{4}, \frac{2}{9}, \frac{10}{9}$

Irrational

Example—$\sqrt{2}, 1{\cdot}33, \frac{10}{3}$

Fractions

Example—$\frac{1}{3}, \frac{2}{4}, \frac{5}{9}$

→ Equivalent

→ Proper

→ Improper

→ Mixed Fraction

Integers

Example—∞, -1, 0, 1, 2, ∞

Negative Number

Ex.—–1, – 2,… – ∞

Whole Number

Ex.—W = 0, 1, 2,....∞

Natural Numbers

Example—N =1,2, 3, … ∞

Even Numbers

Example—2,4,6,…

→ Prime Number
Example—2 only

→ Composite Numbers
Example—4, 6, 12

Odd Numbers

Example—1,3,5,….

Prime Numbers
Example—3, 5, 11, 13

Composite Number
Example—9, 15, 21

Illustration 1. Find the unit digit in the product $439 \times 151 \times 289 \times 156$.

Solution : Product of units digits in given numbers

$= 9 \times 1 \times 9 \times 6$

$= 486$

∴ Unit digit in the given product is 6.

Illustration 2. Find the unit digit in the product $3^{66} \times 6^{41} \times 7^{53}$.

Solution : We know, unit digit in $3^1 = 3$; $3^2 = 9$; $3^3 = 7$; $3^4 = 1$.

So, unit digit of 3^{64} is 1.

Then unit digit in 3^{66} is $3^{64+2} = 1 \times 3 \times 3 = 9$

Unit digit in $6^{41} = 6$

Unit digit in $7^1 = 7$; $7^2 = 9$; $7^3 = 3$; $7^4 = 1$

So, unit digit in 7^{52} is 1

Hence, unit digit in 7^{53} is $7^{52+1} = 1 \times 7 = 7$

Therefore, product of unit digit in the given numbers $3^{66} \times 6^{41} \times 7^{53}$

$= 9 \times 6 \times 7 = 378$

∴ Unit digit in the given product = 8

Illustration 3. Convert 1.33333………into a rational number.

Solution : Let X = 1.33333 ………

10 X = 13.33333

$$10X - X = (13.3333\ldots\ldots) - (1.3333\ldots\ldots\ldots)$$
$$9X = 12$$
$$X = \frac{4}{3}$$

Inverse proportion : A is inversely proportion to B means if A increases B decreases proportionally.

Example : Speed $= \frac{\text{Distance}}{\text{Time}}$ · If speed is doubled, time taken to cover the same distance is reduced to half.

Inverse Proportion : If $A \propto \frac{1}{B}$

It means A is inversely related to B.

$$A = k \times \frac{1}{B} \quad \text{Where } k = \text{constant}$$

Here, variation is inverse but not proportional.

Illustration 4. Ram can do a piece of work in 24 days, Shyam is 60% more efficient than Ram. Find the number of days that Shyam takes to do the same piece of work.

Solution : Ratio of efficiencies of A and B = 100 : 160

$$= 5 : 8$$

Since, efficiency is inversely proportional to the number of days.

Hence, ratio of days taken to complete to the job is 8 : 5

So, number of days taken by Shyam

$$= \frac{5}{8} \times 24 = 15 \text{ days}$$

Commensurable Ratio : It is the ratio of two fractions or any two quantities which can be expressed exactly by the ratio of two integers.

Example : The ratio of 10 m to 40 m. is 10 : 40 *i.e.*, 1 : 4 which is the ratio of two integers, So these are commensurable quantities.

Incommensurable Ratio : It is the ratio of two fractions or any two quantities in which one or both the terms is a surd quantity. No integers can be found which will exactly measure their ratio *i.e.*, cannot be exactly expressed by any two integers.

Example : The ratio of a side of a square to its diagonal is $1 : \sqrt{2}$ which can not be expressed as a ratio of two integers. Thus, 1 and $\sqrt{2}$ are incommensurable quantities.

Duplicate Ratio : It is compounded ratio of two equal ratios. Thus, duplicate ratio of $a : b$ is $\frac{a^2}{b^2}$ or $a^2 : b^2$.

Illustration 5. Find the duplicate ratio of 4 : 5.

Solution : The duplicate ratio of 4 : 5 is 16 : 25.

Triplicate Ratio : It is the compounded ratio of three equal ratios. Thus, the triplicate ratio of $a : b$ is $\frac{a^3}{b^3}$ or $a^3 : b^3$.

Illustration 6. Find the triplicate ratio of 4 : 5.

Solution : The triplicate ratio of 4 : 5 is 64 : 125.

Sub-Duplicate Ratio : For any ratio $a : b$, its sub-duplicate ratio is defined as $\sqrt{a} : \sqrt{b}$.

Illustration 7. What is the sub-duplicate ratio of 16 : 25 ?

Solution : The sub-duplicate ratio of 16 : 25 is

$$\sqrt{16} : \sqrt{25} = 4 : 5$$

Sub-Triplicate Ratio : For any ratio $a : b$, its sub-triplicate ratio is defined as $\sqrt[3]{a} : \sqrt[3]{b}$.

Illustration 8. Find the sub-triplicate ratio of 27 : 64.

Solution : The sub-triplicate ratio of 27 : 64 is

$$\sqrt[3]{27} : \sqrt[3]{64} = 3 : 4$$

Illustration 9. If $a : b$ is the duplicate ratio of $(a + y) : (b + y)$ show that $y^2 = a.b$.

Solution : Given $\frac{a}{b} = \left(\frac{a+y}{b+y}\right)^2$

Or $\frac{a}{b} = \frac{a^2 + y^2 + 2ay}{b^2 + y^2 + 2by}$

Or, $a.b^2 + a.y^2 + 2\,a.b.y$

$$= a.y^2 + 2\,a.b.y + b.a^2$$

Or, $y^2(a - b) = a.b(-b + a)$

Or, $y^2 = a.b$

Illustration 10. Find the compound ratio of (a + y) : $(a - y)$, $(a^2 + y^2) : (a + y)^2$ and $(a^2 - y^2)^2 : (a^4 - y^4)$.

Solution : $\frac{a+y}{a-y} \times \frac{a^2+y^2}{(a+y)^2} \times \frac{(a^2-y^2)^2}{a^4-y^4}$

$$= \frac{(a^2 - y^2)^2}{(a-y)(a+y)^2(a^2-y^2)}$$

$$= 1$$

Illustration 11. If $ax + cy + bz = 0$, $cx + by + az = 0$ and $bx + ay + cz = 0$, then show that $a^3 + b^3 + c^3 = 3\,abc$.

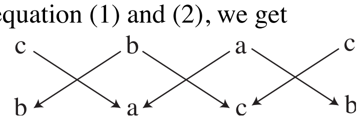
Solution :

Given $ax + cy + bz = 0$ ...(1)

$cx + by + az = 0$ ...(2)

$bx + ay + cz = 0$ ...(3)

From equation (1) and (2), we get



$$ac - b^2 : bc - a^2 : ab - c^2$$

Let $\frac{x}{ac - b^2} = \frac{y}{bc - a^2} = \frac{z}{ab - c^2} = k$

Then, $x = k(ac - b^2)$

$y = k(bc - a^2)$

$z = k(ab - c^2)$

Put them in equation (3) we get,

$$bk(ac - b^2) + ak(bc - a^2) + ck(ab - c^2) = 0$$

$$3\,abc = a^3 + b^3 + c^3$$

Test of Divisibility

(A) Divisibility by 2 : A number is divisible by 2 when its unit digit is either even or zero.

Ex. : 4, 6, 8, 112, 13256 are divisible by 2.

3, 5, 7, 111, 11567 are not divisible by 2.

(B) Divisibility by 3 : A number is divisible by 3, when the sum of its digits is divisible by 3.

Ex. : 426 : 4 + 2 + 6 = 12 which is divisible by 3.

Hence, 426 is the divisible by 3.

5436 : 5 + 4 + 3 + 6 = 18 which is divisible by 3 Hence, 5436 is divisible by 3.

(C) Divisibility by 4 : A number is divisible by 4 when the number formed by its two extreme right digit is either divisible by 4 or both these digits are zero.

Ex. : 524; 1032; 111524; 200; 400 are divisible by 4.

(D) Divisibility test by 5 : A number is divisible by 5, when its unit digit is either 5 or zero.

Ex. : 50,4500, 5155, 735 are divisible by 5.

(E) Divisibility test by 6 : A number is divisible by 6, when it is divisible by 2 as well as 3.

Ex. : 72; 840 are divisible by 6.

(F) Divisibility test by 8 : A number is divisible by 8, when the number formed by its three extreme right digits is divisible by 8 or when these last three digits are zeroes.

Ex. : 13248;11600;1000 are divisible by 8.

(G) Divisibility test by 9 : A number is divisible by 9 when the sum of its digits is divisible by 9.

Ex. : 53973 : 5 + 3 + 9 + 7 + 3 = 27 is divisible by 9.

(H) Divisibility test by 11 : A number is divisible by 11 when the difference between the sum of the digits at odd places and the sum of the digits at even places is either 0 or divisible by 11.

Ex. : 3**4**3**8**1**5**6**7** sum of digits at odd places = 3 + 3 + 1 + 6 = 13

Sum of digits at even places = 4 + 8 + 5 + 7 = 24

Difference = 24 – 13 =11

Difference is multiple of 11 So, the given number is divisible by 11.

(I) Divisibility test by 12 : A number is divisible by 12, when it is divisible by 3 or 4.

Ex. : 4716 ; 4 + 7 + 1 + 6 = 18 divisible by 3 and last two digits 16 also divisible by 4.

3108 last digit 08 is divisible 4 and sum 12 is divisible by 3.

So 4716, 3108 are divisible by 12.

(J) Divisibility test by 14 : A number is divisible by 14 when it is simultaneously divisible by both 2 as well as 7.

Ex. : 460992, 2352 are divisible by 14.

(K) Divisibility test by 15 : A number is divisible by 15 when it is simultaneously divisibly by 3 as well as 5.

Ex. : 4875; 3840 are divisible by 15.

(L) Divisibility test by 16 : A number is divisible by 16, when the number formed by its four extreme right digits is divisible by 16 or these last four digits are zeroes.

Ex. : 50248, 200768,10000 are divisible by 16.

(M) Divisibility by 18 : A number is divisible by 18 when it is divisible by 9 as well as by 2 that is the sum of its digits is divisible by 9 and also the number has either zero or even digit at its unit's place.

Ex. : 2610, 781812.

BINARY NUMBER SYSTEM

Binary Number system contains only 0 and 1. Every decimal number can be represented by binary numbers.

Our practical numbers such as 10, 12, 13, 14, …… are called the decimal number system. This is because there are 10 digits in the system 0 – 9.

(A) Conversion of a decimal Number to a binary system:

1. Divide the decimal number by 2.
2. Keep dividing the quotient by 2 still the quotient is 0.
3. Write down the remainders on the right side after each of the above division.
4. Arrange the remainders in the reverse order to get the required equivalent binary number.

Conversion of $(31)_{10}$ to binary number

2	31	
2	15	1
2	7	1
2	3	1
	1	0

So $(31)_{10} = (10111)_2$

(B) Conversion of binary number to decimal number :

1. Write down the binary number in a series.
2. Multiply the extreme right digit by 0 power of 2 *i.e.*, 20.
3. Multiply the digit next to extreme right by 1 power of 2 *i.e.*, 2^1.
4. Multiply the third digit from right by 2 power of 2 *i.e.*, 2^2 or 4.
5. Follow the same procedure till the extreme left digit.
6. Add all the number so obtained to find the equivalent decimal number

For Example : Conversion of $(110001)_2$ equivalent to decimal number :

$$\begin{aligned}(110001)_2 &= 1 \times 2^5 + 1 \times 2^4 + 0 \times 23 \\ &\quad + 0 \times 2^2 + 0 \times 2^1 + 2^0 \\ &= 32 + 16 + 1 \\ &= 49\end{aligned}$$

SOME IMPORTANT FORMULA

1. $(X + Y)^2 = X^2 + Y^2 + 2.X.Y$
2. $(X - Y)^2 = X^2 + Y^2 - 2.X.Y$
3. $(X + Y)^2 - (X - Y)^2 = 4.X.Y$
4. $(X + Y)^2 + (X - Y)^2 = 2(X^2 + Y^2)$
5. $X^2 - Y^2 = (X + Y)(X - Y)$
6. $X^3 - Y^3 = (X + Y)(X^2 + Y^2 - X.Y)$
   $= (X + Y)^3 - 3.X.Y(X + Y)$
7. $X^3 - Y^3 = (X - Y)(X^2 + Y^2 + X.Y)$
   $= (X - Y)^3 + 3.X.Y(X - Y)$
8. $X^3 + Y^3 + Z^3 = (X + Y + Z)(X^2 + Y^2 + Z^2 - X.Y - Y.Z - X.Z)$
9. $X^2 + Y^2 + Z^2 - X.Y - Y.Z - X.Z = \frac{1}{2}[(X - Y)^2 + (Y - Z)^2 + (Z - X)^2]$
10. $(X + Y + Z)^3 - X^3 - Y^3 - Z^3 = 3(X + Y)(Y + Z)(Z + X)$
11. $(X^2 + XY + Y^2)(X^2 - XY + Y^2) = X^4 + X^2.Y^2 + Y^4$

Perfect Number : If the sum of the divisions of N excluding N itself is equal to N^1, then N is called a perfect number.

Example : 6, 28, 496, 8128

6 = 1 + 2 + 3 where 1, 2, 3 are the divisors of 6.

28 = 1 + 2 + 4 + 7 + 14, where 1, 2, 4, 7, 14 are divisors of 28.

The sum of the reciprocals of the divisors of a perfect numbers including that of its own is always = 2.

Example : For the perfect number 28; $\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28} = 2$

Every even perfect number is of the form $2^{n-1}(2^n - 1)$ where $2^n - 1$ is a prime number.

Example : N = 2, 3, 5, 7, 11, 13, 17, 19, ...

Largest prime number $2^{132048}(2^{132049} - 1)$ which consists of 39,751 digits.

Fibonacci Number : Form a sequence $\{a_n\}$ where $a_{n+2} = a_{n+1} + a_n$

where $a_1 = 1$, $a_2 = 1$

e.g., 1, 1, 2, 3, 5, 8, 13, 21, ...

n^{th} Fibonacci number F_n is given by

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Given any Fibonacci number greater than 1, you can calculate the next Fibonacci number.

Call the given number A, the next Fibonacci number is

$$\frac{A + 1 + A\sqrt{5}}{2}$$

where bracket indicate rounding down to the nearest integer if A = 13 the next Fibonacci number is 21.54 ≈ 21

IMPORTANT POINTS

1. In a Generalized Fibonacci sequence, the sum of the first n terms is F_{n+2} minus the second term of the series.
2. The square of any Fibonacci number differs by 1 from the product of the two Fibonacci numbers on each side.
3. $(F_n)^2 + (F_{n+1})^2 = F_{2n+1}$
4. For any four consecutive Fibonacci number A, B, C, D

$$C^2 - B^2 = A \times D$$

Golden Ratio : It is obtained by having the sum of 1 and square root of 5

i.e., Golden Ratio = $\frac{1+\sqrt{5}}{2}$

Reverse Century : Now, using all the non-zero digits *i.e.*, 1, 2, 3, 4, 5, 6, 7, and 9 in reverse sequence, place plus and minus signs between them that the result of the arithmetic operation will be 100.

(i) 98 – 76 + 54 + 3 + 21 = 100

(ii) 98 + 7 + 6 – 5 – 4 – 3 + 2 – 1 = 100

(iii) 9 – 8 + 76 + 54 – 32 + 1 = 100

(iv) 98 – 7 + 6 + 5 – 4 + 3 – 2 + 1 = 100

Similarly, we can do and get 100.

BODMAS : When we have to perform a series of mathematical operations, there is a rule regarding the order in which we should perform these operation. This rule is BODMAS rule.

B = Bracket ( ), { }, [ ]
O = of
D = Division
M = Multiplication
A = Addition
S = Subtraction

Order of performing the operations

Exercise

1. If $X = \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{3 + \ldots\ldots\ldots\infty}}}}$

What the value of X ?

(A) $\frac{-6 \pm \sqrt{15}}{6}$ (B) $\frac{-3 \pm \sqrt{15}}{4}$

(C) $\frac{-3 \pm \sqrt{15}}{2}$ (D) $\frac{-6 \pm \sqrt{15}}{4}$

(E) 1

2. The smallest number which, when divided by 4, 6 or 7 leave a remainder of 2 is ?

(A) 84 (B) 86

(C) 68 (D) 88

(E) 48

3. Three bells chime at intervals of 18 min., 24 min. and 32 min. respectively. At a certain time, they begin together. What length of time will elapse before they chime together again ?

(A) 4 hr. and 24 min. (B) 2 hr. and 24 min.

(C) 4 hr. and 48 min. (D) 2 hr. and 48 min.

(E) None of these

4. A number when divided by the sum of 555 and 445 gives two times their difference as quotient and 30 as the remainder. Find the number?

(A) 220040 (B) 220080

(C) 220015 (D) 220030

(E) 220055

5. A number when divided by sum of 55 and 45 gives two times their difference as quotient and 11 as the remainder. Find the number?

(A) 2011 (B) 2022

(C) 2033 (D) 2044

(E) 2055

6. The largest number which exactly divides 522, 1276 and 1624 is ?

(A) 116 (B) 232

(C) 29 (D) 58

(E) 64

7. Find the unit's digit in the product $152 \times 169 \times 171 \times 144$?

(A) 2 (B) 9

(C) 1 (D) 4

(E) 3

8. Find the unit's digit in the product $(3^6 \times 6^{41} \times 7^{59})$?

(A) 3 (B) 6

(C) 7 (D) 2

(E) 1

9. Find the total number of factors 512 ?

(A) 10 (B) 11

(C) 8 (D) 9

(E) 2

10. A number when divided by the sum of 255 and 345 gives two times their difference as quotient and 20 as the remainder. Find the number ?

(A) 108060 (B) 108020

(C) 108000 (D) 106020

(E) 106080

11. $\left[\frac{1}{4} \text{ of } \frac{4}{3} \{(2 \times 3) + (4 \times 5)\} + \frac{1}{3}\right]$

(A) 7 (B) 9

(C) 11 (D) 13

(E) 15

12. $3 - [2 - \{7 - (6 - 3 - 2)\}]$

(A) 1 (B) 3

(C) 5 (D) 7

(E) 9

13. Simplify $\overline{0 \cdot 63} + \overline{0 \cdot 37}$

(A) 1.00 (B) 1.05

(C) 1.01 (D) 1.10

(E) 1.21

14. What is the smallest number which must be added to 1953701 to obtain such which is

(i) Divisible by 3 or multiple of 3.

(ii) Divisible by 11 or multiple of 11.

(A) 1; 6 (B) 2; 7

(C) 2; 8 (D) 1; 9

(E) 1; 2

15. What is the minimum value of the expression $(X^2 + X + 1)(Y^2 + Y + 1)(Z^2 + Z + 1)$, where X, Y, Z are all positive integer ?

(A) 1 (B) 8

(C) 27 (D) Data insufficient

(E) Cannot be determine

16. If a, b, c are real numbers such that $a + b + c = 5$ and $ab + bc + ac = 3$, then which of the following equations best describes the largest value of a ?

(A) $\frac{13}{3}$ (B) $\frac{1}{3}$

(C) 13 (D) 1

(E) 3

17. We have

$$A(a, b) = a + b$$
$$B(a, b) = a \times b$$
$$C(a, b) = a - b$$

For convenience A (a, b) is represented as A and so on …………, Now

Which of the following is equal to a ?

(A) $\left[\frac{A^3 + C^3 + 3B(C - A)}{2}\right]^{\frac{1}{3}}$

(B) $[A^3 + C^3 + 3B(C - A)]^{1/3}$

(C) $\left[\frac{A^3 + C^3 + 3B(C + A)}{2}\right]^{\frac{1}{3}}$

(D) 1

(E) None of these

18. If $a^2 + b^2 = 1$ the value of $2(a^6 + b^6) - 3(a^4 + b^4) + 1$?

(A) 0 (B) 1

(C) – 1 (D) 2

(E) – 2

19. A number $(22222222)_{11}$ is written in a number system which uses 11 as its base. What is the remainder when this number is divided by number 11 in that number system?

(A) 0 (B) 1
(C) 2 (D) – 1
(E) – 2

20. If a number 774958A96B is to be divisible by 8 and 9. The respective value of A and B will be?

(A) 8; 0 (B) 1; 4
(C) 2; 8 (D) 8; 4
(E) 0; 2

21. If the unit digit in the product (459 × 46 × 28 * × 594) is 2. Find the digit in place of *?

(i) 2 only (ii) 7 only
(iii) 3 only (iv) 2 & 7 both
(v) 3 & 7 both (vi) 2 & 3 both

(A) (i) & (v) (B) (iv) only
(C) (ii) & (vi) (D) (v) only
(E) (i), (ii) & (iii)

22. Find the unit digit in the expression $(25^{6252} + 36^{529} + 73^{52})$?

(A) 1 (B) 2
(C) 3 (D) 4
(E) 5

23. Find the unit digit in the expression $7^1 + 8^2 + 9^3 + 10^4 + 11^5$?

(A) 2 (B) 3
(C) 1 (D) 4
(E) 5

24. Find the value of $\frac{1}{(2^2-1)} + \frac{1}{(4^2-1)} + \frac{1}{(6^2-1)} + \ldots + \frac{1}{(20^2-1)}$?

(A) $\frac{2}{21}$ (B) $\frac{20}{21}$
(C) $\frac{1}{2}$ (D) $\frac{3}{21}$
(E) $\frac{6}{7}$

25. Two different numbers when divided by a certain divisors leave remainder 47 and 59 respectively. When the sum of the two numbers is divided by the same divisor remainder is 19. Find the divisor?

(A) 2 (B) 19
(C) 59 (D) 87
(E) 47

26. In a division sum, the divisor is 16 times the quotient and five times the remainder if the remainder is 16. Determine the dividend?

(A) 256 (B) 5
(C) 16 (D) 80
(E) 416

Solution

1. (A) $X = \cfrac{1}{2 + \cfrac{1}{3 + X}}$

Or, $= \frac{3 + X}{6 + 2X + 1}$

Or, $6X + 2X^2 + X = 3 + X$

Or, $2X^2 + 6X - 3 = 0$

Or, $X = \frac{-6 \pm \sqrt{6^2 + 4 \times 3 \times 2}}{2 \times 2}$

$\left[\because \text{If equation is } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$

Or, $X = \frac{-6 \pm 2\sqrt{15}}{2 \times 2} = \frac{-3 \pm \sqrt{15}}{2}$

2. (B) L.C.M. of 4, 6 or 7 is

4, 6, 7	2
2, 3, 7	

L.C.M. = 2 × 3 × 7 × 2 = 84

Required number = 84 + 2 = 86

3. (C) First of all we find the L.C.M. of 18, 24, 32

18, 24, 32	2
9, 12, 16	2
9, 6, 8	3
3, 2, 8	2
3, 1, 4	

L.C.M. = 2 × 2 × 3 × 2 × 4 × 1 × 3 = 288

The bells will chime together again after 288 min.

= 4 hr. and 48 min.

4. (D) According to question :

Divisor = 555 + 445 = 1000

Divident = ?

Quotient = (555 – 445) × 2

= 110 × 2 = 220

Remainder = 30

Divident = (Divisor × Quotient) + Remainder

= (1000 × 220) + 30

= 220000 + 30

= 220030

5. (A) According to question :

Divisor = 55 + 45 = 100

Quotient = (55 – 45) × 2 = 20

Remainder = 11

$$\text{Divident} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$
$$= 20 \times 100 + 11$$
$$= 2000 + 11$$
$$= 2011$$

6. (D)
$$\begin{array}{l} 522\,\overline{)\,1276}\,(\,2 \\ \quad 1044 \\ \quad\ \ 232\,)\,522\,(\,2 \\ \qquad\quad 464 \\ \qquad\ \ 58\,\overline{)\,232}\,(\,4 \\ \qquad\qquad 232 \\ \qquad\qquad \times \end{array} \qquad \begin{array}{l} 58\,\overline{)\,1624}\,(\,28 \\ \quad 116 \\ \qquad 464 \\ \qquad 464 \\ \qquad \times \end{array}$$

7. (A) Product of any digits in given numbers
$$= 2 \times 9 \times 1 \times 4$$
$$= 72$$
$\therefore$ Unit digit in the given product = 2

8. (B) We know the unit's digit in $3^4 = 1$
So, unit's digit of $3^6 = 1 \times 3^2 = 9$
Unit's digit of $6^n = 6$
So, unit's digit of $6^{41} = 6$
Unit's digit of 7^4 is 1
Unit's digit of $(7^4)^{14} = 1$
Now, unit's digit of $7^{59} = 1 \times 7^3 = 3$
Now, product of unit's digit in the given numbers
$$= 3^6 \times 6^{41} \times 7^{59}$$
$$= 9 \times 6 \times 3$$
$$= 54 \times 3$$
$$= 162$$
So, required unit's digit = 2

9. (A) Factorization of 512 is given by

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

$\therefore$ $512 = 2^9$
Total number of factors $= 9 + 1 = 10$

10. (B) According to question :
Divisor $= 255 + 345 = 600$
We have to find dividend
Now, Quotient $= (345 - 255) \times 2$
$$= 90 \times 2$$
$$= 180$$
Since, Remainder = 20
So, Dividend = Divisor $\times$ Quotient + Remainder
$$= 600 \times 180 + 20$$
$$= 108000 + 20$$
Number = 108020

11. (B) $\left[\frac{1}{4} \text{ of } \frac{4}{3} \{6 + 20\} + \frac{1}{3}\right]$
$$= \left[\frac{1}{4} \text{ of } \frac{4}{3} \times 26 + \frac{1}{3}\right]$$
$$= \left[\frac{1}{3} \times 26 + \frac{1}{3}\right]$$
$$= \frac{26}{3} + \frac{1}{3}$$
$$= \frac{27}{3} = 9$$

12. (B) $3 - [2 - \{7 - (6 - 1)\}]$
$$= 3 - [2 - \{7 - 5\}]$$
$$= 3 - [2 - 2]$$
$$= 3 - 0$$
$$= 3$$

13. (C) Let $X = \overline{0{\cdot}63}$
$$100\,X = 63.63636363 \quad \ldots(1)$$
$$X = 0.63636363 \quad \ldots(2)$$
Subtracting equation (2) from equation (1), we get
$$99.X = 63$$
$$\therefore \quad X = \frac{63}{99}$$
Similarly, $\overline{0{\cdot}37} = \frac{37}{99}$
$$\therefore \quad \overline{0{\cdot}63} + \overline{0{\cdot}37} = \frac{63}{99} + \frac{37}{99}$$
$$= \frac{100}{99} = 1 + \frac{1}{99}$$
$$= 1 + 0.01$$
$$= 1.01$$

14. (D) Since, divisible by *a* and multiple of *a* are equivalent expression *i.e.,* if a number is divisible by a then that number is a multiple of *a*.

We can also say that X is a factor or sub-multiple of the number.

(i) For divisibility by 3,
Sum of digits must be divisible by 3.
So, $1 + 9 + 5 + 3 + 7 + 0 + 1 = 26$
Since, we have to added smallest number
So, required number = 1

(ii) For divisibility by 11, the difference of sums of digits at odd and even places must be either zero or multiple of 11.
So, Difference $= (1 + 5 + 7 + 1) - (9 + 3 + 0)$
$$= 2$$
Since, Here unit's place is at even place.
So, we have $11 - 2 = 9$
and we add 9 to the number.
$1953701 + 9 = 1953710$
Clearly, 1953710 is divided by 11.

15. (C) Given Question is-

$$(X^2 + X + 1)(Y^2 + Y + 1)(Z^2 + Z + 1)$$

$$= \left[\left(X+\frac{1}{2}\right)^2+\frac{3}{4}\right] \times \left[\left(Y+\frac{1}{2}\right)^2+\frac{3}{4}\right] \times \left[\left(Z+\frac{1}{2}\right)^2+\frac{3}{4}\right]$$

= minimum values of X, Y, Z are 1, 1, 1 respectively.

So, value $= 3 \times 3 \times 3 = 27$

16. (A) We know that

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

Or, $5^2 = a^2+b^2+c^2+2\times 3$

Or, $a^2+b^2+c^2 = 25-6 = 19$

For the maximum value of a, the value of $(b^2 + c^2)$ must be minimum.

Now, $b^2+c^2 = (b+c)^2 - 2bc$

Hence, b must be maximum for which $b = c$

Now, $a+b+c = 5$

$a + 2b = 5$

$b = \frac{5-a}{2}$ ...(1)

$ab + bc + ac = 3$

Or, $ab + b^2 + ab = 3$

Or, $b^2 + 2ab = 3$ ...(2)

From equation (1) and equation (2), we get

$$\left(\frac{5-a}{2}\right)^2 + 2ac \times \frac{5-a}{2} = 3$$

Or, $25 - 10a + a^2 + 4(5a - a^2) = 12$

Or, $-3a^2 + 10a + 25 = 12$

Or, $3a^2 - 13a + 3a - 13 = 0$

$a(3a-13) + 1(3a-13) = 0$

Or, $(a+1)(3a-13) = 0$

Or, $a = -1, \frac{13}{3}$

17. (A) $A^3 = (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$A^3 = a^3 + b^3 + 2BA$

$C^3 = (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$= a^3 - b^3 - 3\,BC$

$A^3 + C^3 = 2a^3 + 3B(A-C)$

$2a^3 = A^3 + C^3 - 3B(A-C)$

$$a^3 = \left[\frac{A^3 + C^3 + 3B(C-A)}{2}\right]$$

$$a = \left[\frac{A^3 + C^3 + 3B(C-A)}{2}\right]^{\frac{1}{3}}$$

18. (A) $2(a^6 + b^6) - 3(a^4 + b^4) + 1$

$= 2[(a^2)^3 + (b^2)^3] - 3[(a^2)^2 + (b^2)^2] + 1$

$= 2[(a^2+b^2)^3 - 3a^2b^2(a^2+b^2)] - 3[(a^2+b^2)^2 - 2a^2b^2] + 1$

$= 2[1 - 3a^2b^2] - 3(1 - 2a^2b^2) + 1$

$= 2 - 6a^2b^2 - 3 + 6a^2b^2 + 1$

$= 0$

19. (A) The number $(22222222)_{11}$ can be written as

$(2 \times 11^0 + 2 \times 11^1 + 2 \times 11^2 + \dots\dots\dots + 2 \times 11^7)$ in decimal system

$= 2(11^0 + 11^1 + 11^2 + 11^3 + \dots\dots\dots + 11^7)$

$$= 2 \times 1\left[\frac{11^8 - 1}{11 - 1}\right]$$

Now, $(11^8 - 1)$ is divisible by $(11 - 1)$ and $(11 + 1)$ *i.e.*, 10 and 12 both, thus the number is divisible by 12 in the decimal system which is written as 11 in the system with base 11.

$\therefore$ Ans. 0.

20. (A) 774958A96B is divisible by 8 if 96B is divisible by 8 and 96B is divisible by 8 if B is either 0 or 8.

Now, to be become the same.

Divisible by 9 sum of all the digits should be divisible by 9.

If (A + B) is 8.

$\Rightarrow$ more either A = 0 or B = 8, or A = 8 or B = 0

Since, the number is divisible by both A and B, hence A and B may take either values *i.e.*, 8 or 0.

21. (B) Unit digit in the product of $9 \times 6 \times 4 = 6$

Now, to get a digit 2 in the unit place 6 should be multiplied by either 2 or 7. So, Answer is 2 or 7.

22. (B) We know that unit digit in the product of any number with 5 at unit place is always 5. $5^n = 5$

So, unit place of $(25)^{6252} = 5$

Similarly, unit digit in the product of any number in the product of any number with 6 at unit place is always. $6^n = 6$

Unit digit in $(36)^{529} = 6$

and Unit digit in $3^4 = 1$

So, unit digit in $[(73)^4]^{13} = 1$

Now, Unit digit for the expression

$(25)^{6252} + (36)^{529} + (73)^{52} = 5 + 6 + 1 = 12$

Required unit digit = 2

23. (E) Unit digit of $11^5 = 1$

Unit digit of $10^4 = 0$

Unit digit of $9^3 = 9$

Unit digit of $8^2 = 4$

and Unit digit of $7^1 = 7$

Now, Unit digit in the expression

$7^1 + 8^2 + 9^3 + 10^4 + 11^5 = 7 + 4 + 9 + 0 + 1 = 21$

Required unit digit = 1

24. (B) We have $T_n \frac{1}{(n^2-1)} = \frac{1}{(n-1)(n+1)}$

$$= \frac{(n+1)-(n-1)}{(n-1)(n+1)}$$

$$T_n = \frac{1}{(n-1)} - \frac{1}{(n+1)}$$

where n is even number.

Now, we can write $T_1 = \frac{1}{(2^2-1)} = \frac{1}{1} - \frac{1}{3}$

and $T_2 = \frac{1}{4^2 - 1} = \frac{1}{3} - \frac{1}{5}$

Similarly, $T_3 = \frac{1}{(6^2 - 1)} = \frac{1}{5} - \frac{1}{7}$

$$T_4 = \frac{1}{(8^2 - 1)} = \frac{1}{7} - \frac{1}{9}$$

....

....

$$T_{20} = \frac{1}{(20^2 - 1)} = \frac{1}{19} - \frac{1}{21}$$

$$\Sigma T_{1-20} = \frac{1}{(2^2 - 1)} + \frac{1}{4^2 - 1} + \frac{1}{(6^2 - 1)} + \ldots + \frac{1}{(20^2 - 1)}$$

$$= 1 - \frac{1}{21} = \frac{20}{21}$$

25. (D) Let the two numbers be X and Y.

According to question—

Let certain divisor $= z$

$\therefore$ X – 47 is exactly divisible by the divisor z.

Y – 59 is completely divided by z and

X + Y – 19 is completely divided by z.

Now, X + Y – 47 – 59 = X + Y –106 must be divided by the same divisor.

So, Divisor $= (X + Y - 19) - (X + Y - 106)$

$= 87$

26. (E) Let a be the quotient and b be the remainder.

$\therefore$ Divisor $= 16a = 5b$

$$a = \frac{5}{16} \times b = \frac{5}{16} \times 16 = 5$$

Divisor $= 16a = 16 \times 5 = 80$

Dividend $=$ Divisor $\times$ Quotient $+$ Remainder

$= 80 \times 5 + 16$

$= 400 + 16$

$= 416$

2 Set Theory

Sets : A set is collection of well defined objects or elements. By well defined, means that the object follow a given definition or rule.

For example : The collection of short students in a class is not defined – hence is not a set.

But collection of students whose height is less than 5 feet is a set.

Between the years 1874 and 1897, the German mathematician and logician Georg Cantor created a theory of abstract sets of entities and made it into a mathematical discipline. This theory grew out of his investigations of certain concrete problems regarding certain types of infinite sets of real numbers. A set, wrote Cantor, is a collection of definite, distinguishable objects of perception or thought conceived as a whole. The objects are called elements or members of the set.

Fundamental Set Concepts

If the elements and sets to be considered are restricted to some fixed class of objects, such as the letters of the alphabet, the universal set (or the universe), which is commonly denoted by U, can then be defined as that which includes all of the elements—in this case, the set of all of the 26 letters. Thus, if A is one of the sets being considered, it will be understood that A is a subset of U. Another set may now be defined that includes all of the elements of U that are not elements of A. This set, which is called the complement of A, is denoted by A´. (Some writers, employing the convention of "difference sets," speak of "the complement of A with respect to U," which they denote by "U - A.")

A set can be represented in two forms :

(a) Tabular Form : A set is described by listing all its elements enclosed in curly brackets the elements are separated by commas and each element is written only once.

Example : A set of even natural numbers less than 10 is represented by $\{2, 4, 6, 8\}$.

(b) Set Builder Form : A set is defined by specifying the property which determines the elements of the set uniquely.

Example : Set of odd natural numbers more than 1 is represented by

$\{x : x = 2n + 1\}$ where $n \geq 1$ $n \in$ N

Types of Sets

(1) Finite Set : Finite set means number of element is fixed or constant.

Example : Set of vowels of English alphabets $\{a, e, i, o, u\}$.

(2) Infinite Set : The number of elements is infinite.

Example : Set of natural numbers $\{1, 2, 3, 4, 5 \ldots\ldots 10{,}000 \ldots\ldots\}$.

(3) Empty Set or Null Set : The set which contains no elements at all, is called an empty set or Null set. The empty set is written as { } or ϕ.

Example : Set of even prime number greater than 5 is null set.

(4) Singleton Set : A set containing only one element is a singleton set.

Example : Even number between 2 and 6 is 4. {4}.

(5) Equal Sets : Two sets are said to be equal if every elements of one set is in the other set and *vice-versa*. So, two elements A and B are equal if $x \in A \Rightarrow x \in B$

$\Rightarrow \qquad A = B$

(6) Equivalent Sets : Two sets A and B are equivalent if the elements of A can be paired with elements of B, so that to each element of A there corresponds exactly one element of B.

In other words number of elements in both the sets are equal.

Example : $A = \{a, b, c\}$,
$B = \{1, 3, 5\}$

Then, A and B are equivalent.

(7) Subsets : Let $A = \{1, 3, 5\}$,
$B = \{1, 3, 5, 7, 9, 11\}$

Since, each elements of set A is present in the set B. So, A is subset of B.

Clearly, A set X is said to be subset of Y if and only if each element of X is an element of set Y.

Clearly, X is subset of Y. And Y is super set of X.

It has some important properties :

(a) Every set is a subset of itself.

(b) A set having n elements has 2^n subsets.

(c) The empty set is a subset of every set $\phi \subset A, \phi \subset \phi$.

(d) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

(8) Universal Sets (U) : A set which contains all the set in a given context is a universal sets.

Example : Set of natural number is a universal sets.

Note : Universal set contains more than one subsets.

SOME OPERATION APPLIED ON SETS

(1) Difference of Sets : If we have two sets X and Y then the set of all elements which belong to A but do not belong to B is called the difference of sets A and B denoted by A – B.

Example : $A = \{1, 3, 5, 7, 4\}$,
$B = \{1, 3, 5, 6, 7, 8, 9, 10\}$

Clearly, $A - B = \{4\}$

Some Important Points on Difference :

(i) $A - B \neq B - A$

(ii) If $A \subset B$, $A - B = \phi$

(iii) $(A - B) \cup B = A \cup B$

(iv) $A - B = A \cap B^1$

(v) $A - B \subseteq A$

(vi) The sets $(A - B)$, $(A \cap B)$ and $(B - A)$ are mutually disjoint.

(2) Cardinal Number : The number of elements in a set is called the cardinal number of the set. The cardinal number is represented as $n(A)$.

If $A = \{a, b, c, d, e\}$

$n(A) = 5$

(3) Complement of a Set :

Let $A = \{1, 3, 5\}$,
$B = \{1, 3, 5, 7, 9, 10, 11, 12\}$

Now, complement of set $A = \{7, 9, 10, 11, 12\}$

where B is universal set and $A \subseteq B$.

Complement of a set is determined relative to other when Ist set is subset of 2nd set.

Some Important Points :

(i) Complement of the universal set is the null set ϕ and *vice-versa*.

(ii) $(A^1)^1 = A$

(iii) If $A \subseteq B$, then $B^1 \subseteq A^1$

A^1 represent the complement of set A.

(4) Union of A set : The union of two sets A and B is a set which contains all the elements of A and the elements of B.

Symbol $\cup$ denotes union

$$A \cup B = \{x/x \in A \text{ or } x \in B\}$$

Example : $A = \{1, 3, 5\}$,
$B = \{2, 4, 6\}$
$A \cup B = \{1, 2, 3, 4, 5, 6\}$

Some Points :

(i) $A \cup B = B \cup A$

(ii) $A \cup A^1 =$ Universal set

(iii) $A \cup \phi = A$

(5) Intersection of Sets : The intersection of the two sets A and B is the set of common elements of A and B. The Symbol $\cap$ denotes intersection.

Example : $A = \{1, 3, 5, 4\}$,
$B = \{1, 3, 5, 8, 9, 10, 12, 13\}$

Now, $A \cap B = \{1, 3, 5\}$

(6) Disjoint Sets : Two sets are said to be disjoint sets if they have no common elements. That is $A \cap B = \phi$

Example : $A = \{1, 3, 5, 9, 11\}$,
$B = \{2, 4, 6, 8, 10\}$

Now, $A \cap B = \phi$

Some Important Formula :

If A, B, C are three infinite sets, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$
if A and B are disjoint.

(iii) $n(A - B) = n(A) - n(A \cap B)$

(iv) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(v) $n(A^1 \cup B^1) = n\{(A \cup B)^1\}$
$= n(\cup) - n(A \cap B)$

Some Important Laws :

(1) Commutative Laws : If A and B are two sets, then

$$A \cup B = B \cup A$$
and
$$A \cap B = B \cap A$$

(2) Associative Laws : If A, B and C are three sets, then

$$(A \cup B) \cup C = A \cup (B \cup C)$$
and
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(3) Distributive Laws : If A, B and C are three sets, then

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(4) Identity Laws : $A \cup \phi$
$= A$ where ϕ is null set.
$A \cap U = A$ U is universal.

(5) De-Margan's Laws : If A and B are two sets

$$(A \cup B)^1 = A^1 \cap B^1$$
$$(A \cap B)^1 = A^1 \cup B^1$$

The notion of ***set*** belongs to primary mathematical concepts. Here, are some examples of sets: the sets of tables in a classroom, the set of residential houses in a certain town, the set of whole numbers, the set of all triangles which can be inscribed in a given circle, and so on.

Objects contained in a set are ***elements*** of this set. For instance, the set of natural numbers less than 7 consists of the following elements : 1, 2, 3, 4, 5 and 6.

We distinguish between ***finite and infinite sets***. The set of table in a classroom and the set of residential houses in a town are finite: the tables and houses can be

counted; there is a definite number of them, whereas the set of whole numbers and the set of all triangles inscribed in a given circle are infinite.

We shall denote sets by capital Latin letters and the elements belonging to them by lower- case letters.

For instance, the set notation A = $\{a, b, c, d\}$ means that the set A consists of the elements a, b, c and d.

If the elements x is an element of the set E, then we use the notions x $\in$ E (read : "x belongs to the set E") , where $\in$ is the membership symbol. And if x is not a member of the set E, then we write $x \notin$ E (read : "x does not belong to the set E").

For example : If N is the set of natural numbers, then $1 \in N, 2 \in N, \frac{1}{3} \notin N, -2 \notin N$, and $\pi \notin$ N.

A set that contains no elements is called the ***empty set (or null set)*** and is denoted by ϕ.

Here, are some example of empty sets : the set of natural roots of the equation $x + 1 = -2$, the set of real solutions of the inequality $x^2 + 1 < 0$, the set of common points of two distinct parallel straight lines, and so on.

Sometimes, we have to consider not the entire set but only its part. For instance, we consider not the entire set of natural numbers but only the set of prime numbers. Instead of the words "a part of a set" we often say "***a subset***".

A and B is a subset of a set A if the set B has the property that each element of B is also an element of A. In this case we write B $\subset$ A (read : "B is included in A").

This definition implies that any set is a subset of itself : A $\subset$ A. And the empty set is regarded as a subset of any set : $\phi \subset$ A.

If for two sets A and B the statements A $\subset$ B and B $\subset$ A hold simultaneously, then this means that these two sets have precisely the same elements. Such sets are said to be ***equal sets***, and we write A = B.

For instance, $\{a, b, c\} = \{b, c, a\} = \{c, a, b\}$ and so on. Finding all the subsets of the set M = $\{a, b, c\}$; we obtain $\{a\}, \{b\}, \{c\}, \{a, b\}$ $\{a, c\}, \{b, c\}, \{a, b, c\}, \phi$.

We can perform various operations with sets, intersection and union being the simplest of them.

The ***intersection*** of two sets is defined as the set which consists of the elements that both sets have in common, if C is the intersection of the sets A and B, then we write C = A $\cap$ B ($\cap$ being the intersection symbol).

For instance, the intersection of the set of even natural numbers and the set of prime numbers is the set consisting of one elements, *i.e.,* of the number 2.

If the set B is a subset of the set A, then the intersection of the sets A and B is the set B, *i.e.,* if B $\subset$ A, then A $\cap$ B = B.

For instance, if A = $\{a, b, c, d\}$ and B = $\{ab, d\}$, then B$\subset$ A, and therefore A $\cap$ B = B.

The ***union*** of two sets is the set of elements that are members of at least one of the given sets. If C is the union of the sets A and B then we write C = A $\cup$ B ($\cup$ *being the union symbol*).

For instance, the union of the set of even natural numbers and the set of odd natural numbers is the set of all natural numbers.

If the set B is a subset of the set A, then the union of the sets A and B is the set A, *i.e.,* if B $\subset$ A, then A $\cup$ B = A.

For instance, if A = $\{a, b, c, d, e\}$ and B = $\{a, b, c, d\}$ then B $\subset$ A, therefore A $\cup$ B = A.

For sake of obviousness, we shall regard sets as certain sets of points in the plane. Figure 1 shows an intersection and a union of two sets.

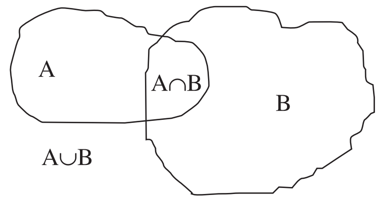


Fig. 1

In arithmetic and algebra use is made of ***number sets***, that is, of sets whose elements are numbers.

First the natural numbers1, 2, 3, 4,.... are considered. The addition and multiplication of natural numbers always yield natural numbers. But the subtraction of two natural numbers does not always yield a natural number. Therefore, negative whole numbers (integers) and the number zero were introduced, thus extending the set of natural numbers to the set of whole numbers $0, \pm 1, \pm 2, \pm 3, \pm 4$.......

The addition, multiplication and subtraction of whole numbers always yield whole numbers. But the result of division of one whole number by another is not always a whole number, therefore fractions were introduced. There appears the notion of a ***rational number*** *i.e.,* of a number of the form $\frac{a}{b}$, where a and b are whole numbers, and $b \neq 0$. The set of whole numbers has thus extended to the set of rational numbers. The set of rational numbers is the union of whole numbers and fractions. The four arithmetic operations (except for division by zero) performed on rational numbers always yield rational numbers. But when taking a square root of a rational number we do not always obtain a rational number. Therefore, new numbers are added to rational numbers which are called ***irrational***.

Rational and irrational numbers from the set of real numbers.

The ***modulus*** (or ***absolute value***) of a real number a is defined as the number a itself if a is positive; as zero if $a = 0$; as the number $(-a)$ if a is negative. The modulus of a real number of a is denoted by $|a|$ thus,

$$A = \begin{Bmatrix} a \text{ if } a \geq 0, \\ -a \text{ if } a \geq 0. \end{Bmatrix}$$

For instance $|4| = 4$, $|0| = 0$, $|-6| = -(-6) = 6$

Fig. 2

Now, we proceed to the geometric representation of numbers. Let us take a straight line and a point O on it as the initial point (the origin) for reckoning lengths. We also choose a scale that is unit length, and a positive direction along the line **(Fig. 2).** Then every real number x will be associated with a definite point M, the abscissa of which is equal to x.

In analysis, numbers are depicted in this way (for greater pictorialness) by points.

A straight line with origin, scale, and positive direction for reckoning lengths is called a ***number axis (or a number line)***.

As is known from geometry any line segment OM has a length expressed by a rational or irrational number. Therefore, to every point M on the number axis there corresponds a quite definite real number x, which is positive if M lies on the right of O and negative if M lies on the left of O. The modulus of the number x is equal to the length of the line segment OM.

Conversely, to every real number x there corresponds a definite point M which lies on the number axis at a distance equal to $|x|$ from the point O and is found on the right of O if $x > O$ and on the left of O if $x < 0$. For $x = 0$ the point M coincides with the point O. Thus, one-to-one correspondence has been established between real numbers and points of the number axis.

Consider the following number sets. If $a < b$, then the set of real numbers x satisfying the inequalities $a \leq x \leq b$ is called ***a number interval (or simply interval)*** and is denoted by the symbol $[a, b]$.

If $a < b$, then the set of real numbers x satisfying the inequalities $a < x < b$ is spoken of as ***an open (or non closed)*** interval, its commonly used notation being (a, b).

If one of the end points is included into the interval while the other is not, the resultant set is specified by the inequalities $a \leq x < b$ (if the end point a is added to the interval) or by $a < x \leq b$ (if the end point b is joint to the interval). Respectively, the **half- interval** thus obtained are denoted as $[a, b)$ and $(a, b]$.

When there is no need in distinguishing whether or not an end point is included into the interval in question we simply speak of ***an interval***.

For instance, the closed interval $[-2, 1]$ is the set of numbers x satisfying the inequalities $-2 \leq x \leq 1$, and the half- interval $(2, 5]$ is the set off number x satisfying the inequalities $2 < x \leq 5$. **(Fig 3)**

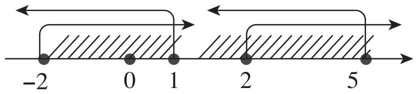


Fig. 3

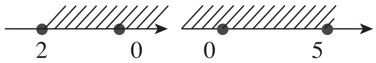


Fig. 4

Besides finite interval discussed above, we often deal with infinite intervals. For instance $[a, +\infty]$ is the set of numbers x satisfying the inequalities $x \geq a$, $(a, +\infty)$ is the set of numbers x satisfying the inequality $x > a$ and so on; $(-\infty, +\infty)$ is the set of all real numbers.

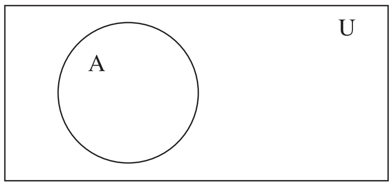
Infinite intervals are respected by rays on the numbers axis.

For example, $[-2, +\infty)$ is the set of numbers x satisfying the condition $x \geq -2$; $(-\infty, 5)$ is the set of numbers x satisfying the condition $x < 5$. **(Fig. 4)**

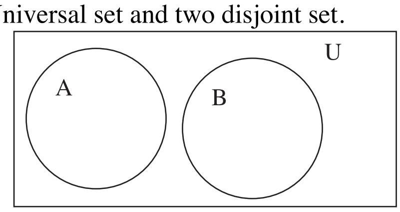
Venn Diagram—Venn diagrams are the pictorial representation of the inner-relationship among two or more than two sets.

- The universal set is represented by a rectangle.
- The subset of the universal set is shown by circles.

(i) Universal set and one subset.



(ii) Universal set and two disjoint set.



(iii) Universal set and two intersecting sets :

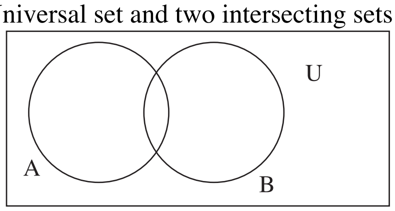


Illustration 1. In a coaching institute, 80 students are selected in a banking exam coaching, 50 students selected in staff selection exam coaching and 40 students selected in both the examination coaching. How many students are there in the institute?

How many students are selected ?

(i) In banking exam coaching only.

(ii) In staff selection exam coaching only.

Solution : The number of students in banking exam

$$= n(B) = 80$$

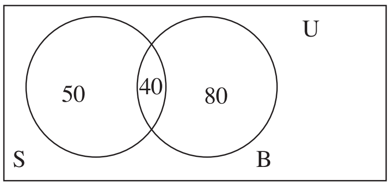
The number of students in staff selection exam.

$$= n(S) = 50$$

Now, $n(B \cap S) = 40$

Total students in the institute

$$= n(B \cup S)$$
$$= n(B) + n(S) - n(B \cap S)$$
$$= 80 + 50 - 40 = 90$$



(i) Number of students selected in banking exam.

$$= n(B) - n(B \cap S)$$
$$= 80 - 40 = 40$$

(ii) Number of students selected in staff selection exam.

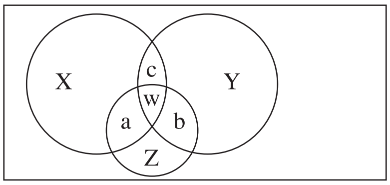
$$= n(S) - n(B \cap S)$$
$$= 50 - 40 = 10$$

Illustration 2. 65% of students in a class like cartoon movies, 75% like horror movies and 85% like war movies. What is the smallest percent of student liking all the three types of movies?

Solution : Total number of students

$$= X + Y + Z + a + b + c + w$$
$$= 100$$

From the Venn diagram it is clear that k will be minimum if $a = b = c = 0$



$\therefore \quad X + Y + Z + w = 100 \quad \ldots(1)$

Now, $X + a + c + w = 65 \quad \ldots(2)$

$Y + b + c + w = 75 \quad \ldots(3)$

$Z + a + b + w = 85 \quad \ldots(4)$

Adding equations (2), (3) and (4), we get

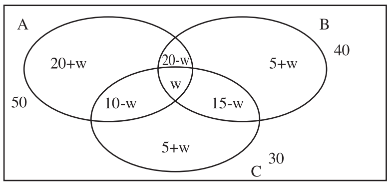
$$X + Y + Z + 3w = 225$$
$$2 \times w + 100 = 225$$
$$w = 125/2 = 62{\cdot}5$$

Hence, at least 62.5% of students like all the three type of movies.

Illustration 3. In a survey of 100 students it was found that 50 used the college library, 40 had their own library and 30 borrowed books of these, 20 used both the college library and their own, 15 used their own library and borrowed books and 10 used the college library books and 10 used the college library and borrowed books. How many students used all the three sources of books ?

Solution : Let college library, own library, borrow books are represented by A, B and C respectively.

So, Let w = All the three sources



Now given, $100 = (20 + w) + (5 + w) + (5 + w) + (20 - w) + (10 - w) + (15 - w) + w$

Or, $w = 25$

Illustration 4. In group of 1000 people, there are 750 people who can speak Hindi and 400 who can speak English. How many people can speak only Hindi ?

(1) 600 (2) 500

(3) 650 (4) 700

Solution : (1) Here $n(H \cup E)$

$$= 1000, n(H) = 750, n(E) = 400$$

Using $n(H \cup E) = n(H) + n(E) - n(H \cap E)$;

$$1000 = 750 + 400 - n(H \cap E)$$
$$\Rightarrow \quad n(H \cap E) = 1150 - 1000 = 150.$$

Number of people who can speak Hindi only

$$= n(H) - n(H \cap E)$$
$$= 750 - 150 = 600.$$

Short-cut : by Venn diagram.

Now, $750 - x + x + 400 - x = 1000$.

$\therefore \quad x = 150$. Only Hindi

$$= 750 - 150 = 600.$$

Illustration 5 & 6 : Read of the information given and answer the question that follow—

In a school with 727 students, 600 students offer Mathematics and 173 students offer both Mathematics and Physics. Each student is enrolled in at least one of the two subjects.

5. How many students are enrolled in Physics ?

(1) 250 (2) 300

(3) 280 (4) 290

Solution : (2) Let M be the set of students offering Mathematics and P the set of students offering Physics. We are given that :

$$n(M \cup P) = 727,$$
$$n(M) = 600, n(M \cap P) = 173$$

Using $n(M \cup P) = n(M) + n(P) - n(M \cap P)$

$$727 = 600 + n(P) - 173$$
$$\Rightarrow \quad 727 = 427 + n(P)$$
$$\Rightarrow \quad n(P) = 727 - 427 = 300$$

$\Rightarrow$ Number of students enrolled in Physics

$$= n(P) = 300.$$

6. The students who enrolled in only Physics are :

(1) 300 (2) 280

(3) 127 (4) 173

Solution : (3) Number of students enrolled in Physics only

$$= n(P) - n(P \cap M)$$
$$= 300 - 173 = 127.$$

Illustration 7. In a school, 21 students are on the basket ball team, 26 students are on the hockey team, 29 students are on the football team. 14 students play hockey and basketball, 15 students play hockey and football, 12 students play football and basketball & 8 students are on all the three teams. How many members are there altogether ?

(1) 38 (2) 47

(3) 51 (4) 43

Solution : (4) Let B, H, F denote the sets of members who are on the basketball team, hockey team and football team respectively.

Then, $n(B) = 21, n(H) = 26,$

$n(F) = 29, n(H \cap B) = 14,$

$n(H \cap F) = 15, n(F \cap B) = 12$

and $n(B \cap H \cap F) = 8.$

We have to find $n(B \cup H \cup F)$.

Using the result $n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$.

We get $n(B \cap H \cap F)$

$$= 21 + 26 + 29 - 14 - 15 - 12 + 8$$
$$= 84 - 41 = 43.$$

Short-cut : by Venn diagram.

Total numbers $= 3 + 6 + 5 + 4 + 8 + 7 + 10$
$= 43.$

For Q. 8 & 9 : Read of the information given and answer the question that follow—

A survey regarding the complaints of students was conducted. All complaints of students fell into three categories :

Complaint about :

Mess (M), Food (F) and Service (S).

The total number of complaints received was 173 and were as follows :

$$n(M) = 110, n(F) = 55, n(S) = 67,$$
$$n(M \cap F \cap S'') = 20,$$
$$n(M \cap S \cap F') = 11, n(F \cap S \cap M') = 16.$$

Illustration 8. The number of complaints about all three categories is :

(1) 8 (2) 5

(3) 4 (4) 6

Solution : (4) $M \cap F \cap S'$ means complaints about both M and F but not about S.

$$M \cap F \cap S'' = 20, M \cap F \cap S = ?,$$
$$F \cap S \cap M' = 16$$

Let $M \cap S \cap F = X,$

Complaints about M alone

$$= 110 - 20 - X - 11 = 79 - X$$

Complaints about F alone

$$= 55 - 20 - X - 16 = 19 - X$$

Complaints about S alone

$$= 67 - 16 - X - 11 = 40 - X$$

Total numbers of complaints

$$= 79 - X + 20 + 19 - X + 16 + X + 11 + 40 - X$$
$$= 185 - 2X.$$

But the total number of complaints = 173.

$$185 - 2X = 173$$
$$\Rightarrow \quad -2X = 173 - 185 = -12$$
$$\Rightarrow \quad 2X = 12 \Rightarrow X = 6.$$

There are 6 complaints about all the three.

Illustration 9. The total number of complaints about two or more than two categories is :

(1) 48 (2) 51

(3) 53 (4) 63

Solution : (3) Numbers of complaints about 2 or more than two $= 20 + 6 + 16 + 11 = 53.$

$$79 - x + 20 + 19 - x + 11 + x + 16 + 40 - x = 173.$$
$$\Rightarrow \quad 185 - 2x = 173.$$
$$\therefore \quad x = 6.$$

Illustration 10. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea ?

(1) 19 (2) 21

(3) 17 (4) 15

Solution : (1) Let A be the set of people who like coffee and B be the set of people who like tea. Then, $n(A \cup B) = 70, n(A) = 37$ and $n(B) = 52.$

We know that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$70 = 37 + 52 - n(A \cap B)$$
$$\text{or} \quad (A \cap B) = 37 + 52 - 70 = 89 - 70 = 19$$

$\Rightarrow$ 19 people like both coffee and tea.

Illustration 11. In a group of 65, people, 40 like cricket and 10 like both cricket and tennis. Each one likes at least one of the two games. How many people like only tennis ?

(1) 35 (2) 25

(3) 30 (4) 20

Solution : (2) Let A be the set of people who like cricket and B be the set of people who like tennis. Then, $n(A \cup B) = 65, n(A) = 40$ and $n(A \cap B) = 10.$

$$\Rightarrow \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$\Rightarrow \quad 65 = 40 + n(B) - 10$$
$$\Rightarrow \quad n(B) = 65 - 40 + 10 = 35.$$

Number of people who like tennis

$= n(B) - n(A \cap B)$

$= 35 - 10 = 25$

$\Rightarrow$ Number of people who like tennis only and not cricket = 25.

Illustration 12. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many people speak at least one of these two languages ?

(1) 55 (2) 50

(3) 70 (4) 60

Sol. (4) Let A be the set of people who speak French and B be the set of people who speak Spanish. Then $n(A) = 50$, $n(B) = 20$.

$$n(A \cap B) = 10$$

$$\Rightarrow \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 50 + 20 - 10 = 70 - 10 = 60.$$

Hence, 60 people speak at least one of these two languages.

People speak at least one of these two languages

$= 10 + 10 + 40 = 60.$

Short-cut : by Venn diagram.

Exercise

1. If A and B are two sets then $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to—
   (A) $A \cup B$ (B) A B
   (C) A (D) B'

2. Let $\cup$ be the universal set and $A \cup B \cup C = \cup$. Then $\{(A - B) \cup (B - C) \cup (C - A)\}'$ is equal to—
   (A) $A \cup B \cup C$ (B) $A \cup (B \cap C)$
   (C) $A \cap B \cap C$ (D) $A \cap (B \cup C)$

3. Let A and B be two sets, then $(A \cup B)' \cup (A' \cap B)$ is equal to—
   (A) A' (B) A
   (C) B' (D) None of these

4. Let A and B be two sets such that $A \cup B = A$. Then $A \cap B$ is equal to—
   (A) ϕ (B) B
   (C) A (D) None of these

5. 20 teachers of a school either teach mathematics or physics. 12 of them teach mathematics while 4 teach both the subjects. Then the number of teachers teaching physics only is—
   (A) 12 (B) 8
   (C) 16 (D) None of these

6. Of the numbers of three athletic teams in a school 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and football, and 12 play football and cricket. Eight play all the three games. The total number of members in the three athletic teams is—
   (A) 43 (B) 76
   (C) 49 (D) None of these

7. The relation "congruence modulo m" is—
   (A) Reflexive only (B) Transitive only
   (C) Symmetric only (D) An equivalence relation

8. R is a relation over the set of real number and it is given by $mn \geq 0$. Then R is—
   (A) Symmetric and transitive
   (B) Reflexive and symmetric
   (C) A partial order relation
   (D) An equivalence relation

9. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. If f is a function from A to B and g is a one-one function from A to B then the maximum number of definitions of—
   (A) f is 9 (B) g is 9
   (C) f is 27 (D) g is 6

10. Let r be a relation over the set NxN and it is defined by $(a, b)\ r(c, d) \Rightarrow a + b = b + c$. Then r is—
    (A) Reflexive only (B) Symmetric only
    (C) Transitive only (D) An equivalence relation

11. Let $A = \{1,2,3\}$. The total number of distinct relations that can be defined over A is—
    (A) 2^9 (B) 6
    (C) 8 (D) None of these

12. Let R = set of real numbers and R_c = set of real angles in radian measure. If f: $R_c \rightarrow R$ be a mapping such that $f(x) = \sin x, x \in R_c$, then f is—
    (A) One-one and into
    (B) One-one and onto
    (C) Many-one and onto
    (D) Many-one and into

13. Let $f : R \rightarrow R$ such that $f(x) = \frac{1}{1 + x^2}, x \in R$. Then f is—
    (A) Injective (B) Surjective
    (C) Bijective (D) None of these

14. $f : R\ x\ R \rightarrow R$ such that $f(x + iy) = +\sqrt{x^2 + y^2}$. Then f is—
    (A) Many-one and into
    (B) One-one and onto
    (C) Many-one and onto
    (D) One-one and into

15. Let $A = \{x : -1 < x < 1\} = B$. If $f : A \rightarrow B$ be bijective, then a possible definition of $f(x)$ is—
    (A) $|x|$ (B) $x\,|x|$
    (C) $\sin \pi x$ (D) None of these

16. Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$. Which of the following subsets of A x B is a mapping from A to B ?
    (A) $\{(1, a), (3, b), (2, a), (2, b)\}$
    (B) $\{(1, b), (2, a), (3, a)\}$

(C) $\{(1, a), (2, b)\}$

(D) None of these

Directions—From Question 17 to 21 one or more option may be correct.

17. Let R be the relation over the set of straight lines of a plane such that—

$l_1 R l_2 \Leftrightarrow l_1 \perp l_2$. Then R is

(A) Symmetric (B) Reflexive

(C) Transitive (D) An equivalence relation

18. Let R be the relation over the set of integers such that mRn if and only if m is a multiple of n. Then R is—

(A) Reflexive (B) Symmetric

(C) Transitive (D) An equivalence relation

19. Let A = {1,2,3,4} and R be a relation in A given by

R = {(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(3,1),(1,3)}

Then R is—

(A) Reflexive (B) Symmetric

(C) Transitive (D) An equivalence relation

20. Let $f : R \to R$ be a mapping such that $f(x) = \dfrac{x^2}{1+x^2}$.

Then f is—

(A) Many – one (B) One-one

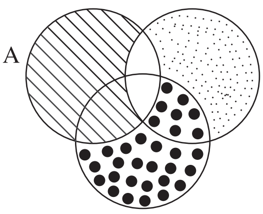
(C) Into (D) Onto

Solutions

1. (A) Draw the Venn diagram. From the Venn diagram,

$(A - B) \cup (B - A) \cup (A \cap B) = A \cup B$

2. (C) $(A - B) \cup (B - C) \cup (C - A)$ is represented by the shaded portion in the figure. The unshaded portion is $A \cap B \cap C$.



$\therefore \{(A - B) \cup (B - C) \cup (C - A)\}' = A \cap B \cap C$

3. (A) $(A \cup B)' \cup (A' \cap B) = (A' \cap B') \cup (A' \cap B)$

$= (A' \cup A') \cap (A' \cup B) \cap (B' \cup A') \cap (B' \cup B)$

$= A' \cap \{A' \cup (B \cap B')\} \cap U = A' \cap (A' \cup \phi) \cap U$

$= A' \cap A' \cap U = A' \cap = A'$

4. (B) $A \cup B = A$

$\Rightarrow B \subseteq A \Rightarrow A \cap B = B$

5. (B) $n(M \cup P) = 20, n(M) = 12, n(M \cap P) = 4$

$n(M \cup P) = n(M) + n(P) - n(M \cap P)$

$\Rightarrow 20 = 12 + n(P) - 4$

$\therefore n(P) = 12.$

So, the required number

$= n(P) - n(M \cap P) = 12 - 4 = 8.$

6. (A) $n(C) = 21, n(H) = 26, n(F) = 29,$

$n(H \cap C) = 14, n(H \cap F) = 15, n(F \cap C)$

$= 12, n(C \cap H \cap F) = 8.$

$n(C \cap H \cap F) = n(C) + n(H) + n(F) - n(H \cap C)$

$- n(H \cap F) - n(F \cap C) + n(C \cap H \cap F)$

$= 21 + 26 + 29 - 14 - 15 - 12 + 8$

$= 43$

7. (D) If R be the relation, $xRy \Leftrightarrow x - y$ is divisible by m.

xRx because $x - x$ is divisible by m. So, R is reflexive.

$xRy \Rightarrow y Rx$. So, R is symmetric.

xRy and yRz

$\Rightarrow x - y = k_1 m, y - z = k_2 m$

$\therefore x - z = (k_1 + k_2)m$. So, R is transitive.

As R is reflexive, symmetric and transitive, it is an equivalence relation.

8. (D) R is reflexive, symmetric and transitive. So, the most appropriate option is (D).

9. (C, D) Every element of A can have image in B in 3 ways. So, the total number of ways in which 3 elements of A can have images in B = maximum number of definitions of $f = 3 \times 3 \times 3$.

The number of ways of arranging 1, 2, 3 in places of a, b, c is 3! = the maximum number of definitions of one-one function g.

10. (D) $(a, b)\ r(a, b)$ because $a + b = b + a$. So, r is reflexive.

$(a, b)\ r(c, d)$

$\Rightarrow a + d = b + c$

$\Rightarrow c + b = d + a$

$\Rightarrow (c, d)\ r(a, b)$

So, r is symmetric.

$(a, b)\ r(c, d)$ and $(c, d)\ r(e, f)$

$\Rightarrow a + d = b + c, c + f = d + e$

Adding , $a + b + c + f = b + c + d + e$

$\Rightarrow a + f = b + e$

$\Rightarrow (a, b)\ r(e, f)$.

$\therefore$ R is transitive

11. (A) $n(AxA) = n(A).n(A)$

$= 3^2 = 9.$

So, the total number of subsets of AxA is 2^9 and a subset of AxA is a relation over the set A.

12. (D) 13. (D) 14. (A)

15. (D) $f(x) = |x|$ is many-one and into.

$f(x) = x\,|x|$ is one-one but into as $f(x)$ will have only rational values.

$f(x) = \sin \pi x$ is onto but many – one

$\left[\because f\left(\frac{3}{4}\right) = f\left(\frac{1}{4}\right)\right]$

16. (B) 17. (A) 18. (A, C) 19. (A, B) 20. (A, C)

3 Logarithms

When studying logarithms it is important to note that all the properties of logarithms are consequences of the corresponding properties of powers, which means that the aspirant should have a good working knowledge of power as a foundation for tackling logarithms. This close relationship between logarithms and powers stems from the definition of a logarithm in terms of the concept of a power.

Here is a definition taken from a commonly used textbook : "The logarithm of a given number to a given base is the exponent of the power to which the base must be raised in order to obtain the given number". Thus, a number x is the logarithm of a number 'N' to the base '**a**' if $a^x = N$.

There is one very essential detail in this definition: no restrictions are imposed on the phrases "to a given base", and so if we are to follow this definition literally (and a definition must always be followed literally), then we will have to concede that 3 is the logarithms of – 8 to the base-2 (since $(-2)^3 = 8$), 2 is the logarithms of – 4 to the base-2 (since $(-2)^2 = 4$), and so forth. As for the base 1, the situation is stranger still : any number x is the logarithm of 1 to the base 1 because $1^x = 1$ for every x.

Any person acquainted with the school course of mathematics will say that these examples are meaningless since we have to consider only logarithms to a positive base different from 1. True enough, that is the convention, but it is much better to impose this restriction on the base **directly** in the definition. And so the **definition** should read :

Let there be a number $a > 0$ and $a \neq 1$. A number x is called the logarithm of a number N to the base a if $a^x = N$.

The more attentive readers have perhaps noticed that we have not once written $x = \log_a N$ but have always stated : x is the logarithms of N to the base a. The explanation is very simple. Until we are sure that no number can have two distinct logarithms to a given base, we have no right to use the equal sign. Indeed, imagine for a moment that some number N has two distinct logarithms to the same base a : then, using the equals sign, we should be able to write $\alpha = \log_a N$ and $\beta = \log_a N$, whence $\alpha = \beta$.

For this reason we will introduce a notation for logarithms only when we are convinced that no number can have two distinct logarithms to the same base. Indeed, If two distinct numbers α and β were logarithms of the number N to the base a, then by definition, the following equations would hold true :

$$a^\alpha = N \text{ and } a^\beta = N$$

whence $a^\alpha = a^\beta$. But then, by the properties of powers with positive base different from unity, we should arrive at the equation $\alpha = \beta$. Thus, if the number N has a logarithms to a base a, then this logarithm is unique; we denote it by the symbol $\log_a N$.

Thus, by definition,

$$X = \log_a N \qquad \text{if } a^x = N$$

Consequently, the equation $x = \log_a N$ and $a_x = N$ (provided the restriction imposed earlier on hold true) express one and same relationship between the numbers x, a, N: in logarithmic form in the former case and in equivalent exponential form in the latter.

It is easy to prove that ***negative numbers and zero do not have logarithms*** to any base a (with the usual provision that $a > 0$ and $a \neq 1$). Indeed, if $N \leq 0$ and $x = \log_a N$, then $a^x = N \leq 0$, which contradicts the property of powers having a positive base.

As for positive numbers, we assume without proof that any positive number to any base has a logarithm. This assertion is taken in school to be self- evident and is not even stated. Although, it is no easy job to establish its validity (this would require invoking a highly developed theory of real numbers and the theory of limits).

Quite naturally, the aspirant must have a through knowledge of the definition and of the properties of logarithms and must be able to prove them.

First of all, note the so-called *fundamental logarithmic identity*

$$a^{\log_a N} = N$$

which is valid for every N and a such that $a > 0, a \neq 1, N > 0$. This identity follows directly from the equation.

Here are some formulas that are frequently used in problem solving (we stress once aging that, according to the definition of a logarithm, all bases are positive and different from unity).

I. $\log_a MN = \log_a M + \log_a N \qquad (M > 0, N > 0)$

II. $\log_a \frac{M}{N} = \log_a M - \log_a N \qquad (M > 0, N > 0)$

III. $\log_a N^\alpha = \alpha \log_a N \qquad (N > 0, \alpha \text{ any number})$

IV. $\log_{a^\beta} N^\alpha = \frac{\alpha}{\beta} \log_a N \qquad (N > 0, \alpha \neq 0, \beta \neq 0)$

V. $\log_b N = \frac{\log_a N}{\log_a b} (N > 0)$

VI. $\log_b a . \log_a b = 1$.

Let us prove formula I. raise a to the power of $\log_a M + \log_a N$. By the property of powers and by the fundamental logarithmic identity, we have

$$a^{\log_a m + \log_a N} = a^{\log_a M} \cdot a^{\log_a N} = MN$$

The resulting equation

$$a^{\log_a M + \log_a N} = MN$$

May be rewritten in logarithmic thus: $\log_a M + \text{Log}_a N = \log_a MN$ which signifies the validity of formula I.

Formula II is proved similarly.

To prove equation III, raise a to the power $\alpha \log_a N$ and utilize the properties of powers :

$$\alpha^{a \log_a N} = (\alpha^{\log_a N})\, \alpha = N^\alpha$$

From this, by the definition of a logarithm, we obtain the required equation

Equation IV follows from the manipulations

$$(\alpha^\beta)^{\frac{\alpha}{\beta} \log_a N} = a^{\alpha \log_a N} = (a^{\log_a N})^\alpha = N^\alpha$$

It will prove useful to memorize the following two special cases of formula IV :

IV. (a) $\log_{a^\beta} N = \dfrac{1}{\beta} \log_a N \qquad (N > 0, \beta \neq 0)$

V. (b) $\log_{a^\alpha} N^\alpha = \log_\alpha N \qquad (N > 0, \alpha \neq 0)$

To prove V, let us first write it in the form $\log_\alpha N = \log_a b . \log_b N$. The proof is similar to that of the preceding case :

$$\alpha^{\log_\alpha b . \log_b N} = (\alpha^{\log_\alpha b})^{\log_b N} = b^{\log_b N} = N$$

We can reason differently. Writing the fundamental logarithmic identity

$$b^{\log_b N} = N$$

we derive from it the equation

$$\log_a (b^{\log_b N}) = \log_a N$$

(equal number have the same logarithms!). Now, using property III, we convince ourselves of the validity of formula V.

Formula VI is a special case of the preceding one obtained for $N = b$. Equation V is usually called the rule for changing the base of a logarithm. This rule makes different tables of logarithms to various bases unnecessary; it suffices to have, say, tables of common logarithms (base 10). For instance, suppose it is required to compute $\log_5 13$. On the basis of property V, we can write $\log_5 13 = \dfrac{\log_{10} 13}{\log_{10} 5}$ · Using logarithmic tables, we find $\log_{10} 13 \approx 1{\cdot}1139$ and $\log_{10} 5 \approx 0{\cdot}6990$, and thus $\log_5 13 \approx 1{\cdot}5937$. (Using Logarithmic Table)

Some other properties of logarithms that are absolutely necessary in the solution of inequalities are :

VII. If $a > 1$, then from $0 < x_1 < x_2$ it follows that $\log_a x_1 < \log_a x_2$ and from $\log_a x_1 < \log_a x_2$ it follows that $0 < x_1 < x_2$. In other words, for $a > 1$ the inequalities $0 < x_1 < x_2$ and $\log_a x_1 < \log_a x_2$ are equivalent.

VIII. If $0 < a < 1$, then from $0 < x_1 < x_2$ it follows that $\log_a x_1 > \log_a x_2$ and from $\log_a x_1 > \log_a x_2$ it follows that $0 < x_1 < x_2$. In other words, when $a < 1$ the inequalities $0 < x_1 < x_2$ and $\log_a x_1 > \log_a x_2$ are equivalent.

These two properties are proved in exactly the same way, and so we confine ourselves to proving property VIII.

Let a number a be positive and less than unity. If the inequality $0 < x_1 < x_2$ holds, then there exist numbers $\log_a x_1$ and $\log_a x_2$. Using the fundamental logarithmic identity, rewrite the inequality $x_1 < x_2$ in the form

$$a^{\log_a x_1} < a^{\log_a x_2}$$

whence, by the properties of a power to a base less than unity, we conclude that $\log_a x_1 > \log_a x_2$.

Conversely, if the inequality $\log_a x_1 > \log_a x_2$ is true, then, firstly, both numbers x_1 and x_2 are positive. Secondly, raising the number a, $0 < a < 1$, to the powers $\log_a x_1$ and $\log_a x_2$ we get (again by the properties of powers to a base less than 1) the inequality

$$a^{\log_a x_1} < a^{\log_a x_2}$$

or $x_1 < x_2$. Now since, as we have already mentioned, the numbers x_1 and x_2 are positive, it follows that $0 < x_1 < x_2$ which completes the proof.

The following statements are consequences of the properties that have just been proved :

VII (a) If $a > 1$, then the inequalities $\log_a x < \alpha$ and $0 < x < a^\alpha$ are equivalent.

VIIL (b) If $a > 1$, then the inequalities $\log_a x > \alpha$ and $x > a^\alpha$ are equivalent.

VIII (a) If $0 < a < 1$, then the inequalities $\log_a x < \alpha$ and $x > a^\alpha$ are equivalent.

VIII (b) If $0 < a < 1$, then the inequalities $\log_a x > \alpha$ and $0 < x < a^\alpha$ are equivalent.

To prove this it suffices to note that $\alpha = \log_\alpha \alpha^\alpha$.

From these statements it is easy to drive that logarithms of numbers exceeding 1 to bases exceeding 1 are positive and logarithm of numbers less than 1 (but positive !) are negative; and, conversely, logarithms to bases less than 1 are negative for numbers exceeding 1 and positive for numbers less than 1.

Let us now solve some illustration involving the basic properties of logarithms.

Illustration 1. Compute $\log_{3\sqrt{3}} 27$.

By formula IV, we have

$$\log_{3\sqrt{3}} 27 = \log_{3^{3/2}} 3^3 = \frac{3}{3/2} \log_3 3 = 2$$

Illustration 2. Compute $2^{\log_{2\sqrt{2}} 15}$

By formula IV (a), we have

$$\text{Log}_{2\sqrt{2}} 15 = \log_{2^{3/2}} 15 = \frac{2}{3} \log_2 15$$

Applying the fundamental logarithmic identity, we get

$$2^{\log_{2\sqrt{2}}15} = 2^{\frac{2}{3}\log_2 15}$$

$$= (2^{\log_2 15})^{2/3} = 15^{2/3} = \sqrt[3]{225}$$

Illustration 3. Compute $\log_3 5.\log_{25} 27$.

By formula IV, we have

$$\text{Log}_3 5.\log_{25} 27 = \log_3 5.\log_{5^2} 3^3 = \frac{3}{2}\log_3 5.\log_5 3$$

And since, by formula VI, $\log_3 5.\log_5 3 = 1$, it follows that $\log_3 5.\log_{25} 27 = 3/2$.

Illustration 4. Compute $\left(\sqrt[3]{9}\right)^{\frac{1}{5\log_5 3}}$.

By formula VI, we have

$$\frac{1}{5\log_5 3} = \frac{1}{5}\log_3 5$$

It then only remains to take advantage of the fundamental logarithmic identity and the laws of exponents :

$$\left(\sqrt[3]{9}\right)^{\frac{1}{5\log_5 3}} = \left(9^{\frac{1}{3}}\right)^{\frac{1}{5}\log_3 5} = (3^{2/3})^{\frac{1}{5}\log_3 5}$$

$$(3^{\log_3 5})^{2/3\cdot 1/5} = 5^{2/15} = \sqrt[15]{25}$$

Illustration 5. Compute $\sqrt{\left(\frac{1}{\sqrt{27}}\right)^{2-\frac{\log_5 13}{2\log_5 9}}}$.

Using in succession the laws of logarithms and exponents we compute the radicand :

$$\left(\frac{1}{\sqrt{27}}\right)^{2-\frac{\log_5 13}{2\log_5 9}} = \frac{1}{27}\cdot\left(\sqrt{27}\right)^{1/2\log_9 13}$$

$$= \frac{1}{27}\cdot(3^{\log_3 13})^{3/8} = 3^{-3}\cdot 13^{3/8}$$

whence it is clear that the given number is equal to $3^{-3/2}\cdot 13^{3/16}$.

Illustration 6. Which is greater, $\log_4 5$ or $\log_{1/16}\frac{1}{25}$?

By formula IV b, we have

$$\text{Log}_{1/16}\frac{1}{25} = \log_{4^{-2}} 5^{-2} = \log_4 5$$

So, that the two numbers are equal.

Illustration 7. Compute $\log_3 2.\log_4 3 \ldots\ldots\ldots\ldots \log_{10} 9.\log_{11} 10$.

By formula V,

$$\text{Log}_3 2 = \frac{\log_{11} 2}{\log_{11} 3};\ \log_{4^{-2}}$$

$$= \frac{\log_{11} 3}{\log_{11} 4};\ \ldots \log_{10} 9 = \frac{\log_{11} 9}{\log_{11} 10}$$

whence $\text{Log}_3 2.\log_4 3\ldots\ldots\log_{11} 10$

$$= \frac{\log_{11} 2}{\log_{11} 3}\cdot\frac{\log_{11} 3}{\log_{11} 4}\cdots\frac{\log_{11} 9}{\log_{11} 10}\cdot\log_{11} 10$$

$$= \log_{11} 2$$

Illustration 8. Prove that the ratio of the logarithms of the two numbers is not dependent on the base; that is,

$$\frac{\log_a N_1}{\log_a N_2} = \frac{\log_b N_1}{\log_b N_2} \quad (N_1 > 0, N_2 > 0, N_2 \neq 1)$$

By formula V, we have

$$\frac{\log_a N_1}{\log_a N_2} = \log_{N_2} N_1 \text{ and } \frac{\log_b N_1}{\log_b N_2} = \log N_2 N_1$$

whence it is clear that our equation holds true.

Illustration 9. Which is greater, $\log_2 3$ or $\log_{1/4} 5$?

Since, $\log_2 3 > 0$ and $\log_{1/4} 5 < 0$, it follows that $\log_2 3 > \log_{1/4} 5$.

Illustration 10. Which is greater, $\log_5 7$ or $\log_8 3$?

Since, $\log_5 7 > 1$ and $\log_8 3 < 1$, it follows that $\log_5 7 > \log_8 3$.

Illustration 11. Compute $\log_{ab}\frac{\sqrt[3]{a}}{\sqrt{b}}$ if $\log_{ab} a = 4$.

By the laws of logarithms, we have

$$\log_{ab}\frac{\sqrt[3]{a}}{\sqrt{16}} = \frac{1}{3}\log_{ab} a - \frac{1}{2}\log_{ab} b = \frac{4}{3} - \frac{1}{2}\log_{ab} b$$

It remains to find the quality $\log_{ab} b$. Since,

$$1 = \log_{ab} ab = \log_{ab} a + \log_{ab} b = 4 + \log_{ab} b$$

It follows that $\log_{ab} b = -3$ and so

$$\log_{ab}\frac{\sqrt[3]{a}}{\sqrt{b}} = \frac{4}{3} - \frac{1}{2}(-3) = \frac{17}{6}$$

Illustration 12. Compute $\log_6 16$ if $\log_{12} 27 = a$.

The chain of transformations

$$\text{Log}_6 16 = 4\log_6 2$$

$$= \frac{4}{\log_2 6} = \frac{4}{1 + \log_2 3}$$

Shows us that we have to know $\log_2 3$ in order to find $\log_6 16$. We find it from the condition $\log_{12} 27 = a$:

$$a = \log_{12} 27 = 3\log_{12} 3 = \frac{3}{\log_3 12} = \frac{3}{1 + 2\log_3 2}$$

$$= \frac{3}{1 + \frac{2}{\log_2 3}} = \frac{3\log_2 3}{2 + \log_2 3}$$

which means that $\log_2 3 = \frac{2a}{3-a}$ (note that, obviously, $a \neq 3$). we finally have $\log_6 16 = \frac{4(3-a)}{3+a}$.

Illustration 13. Compute $\log_{25} 24$ if $\log_6 15 = \alpha$ and $\log_{12} 18 = \beta$.

We have the equation

$$\text{Log}_{25} 24 = \frac{1}{2}(\log_5 3 + 3\log_5 2)$$

$$= \frac{3}{2}\log_5 2 + \frac{1}{2}\log_5 3$$

which shows us that we have to determine $\log_5 2$ and $\log_5 3$. The equation $\log_6 15 = \alpha$ yields.

$$\alpha = \log_6 15 = \log_6 3 + \log_6 5$$
$$= \frac{1}{1+\log_3 2} + \frac{1}{\log_5 2 + \log_5 3}$$

and the equation $\log_{12} 18 = \beta$ yields

$$\beta = \log_{12} 18 = \log_{12} 2 + 2\log_{12} 3$$
$$= \frac{1}{2+\log_2 3} + \frac{1}{1+2\log_3 2}$$

Taking logs to base 5 in all cases we find, by formula V,

$$\alpha = \frac{1}{1+\log_3 2} + \frac{1}{\log_5 2 + \log_5 3}$$
$$= \frac{1}{1+\dfrac{\log_5 2}{\log_5 3}} + \frac{1}{\log_5 2+\log_5 3}$$
$$= \frac{1+\log_5 3}{\log_5 2+\log_5 3},$$
$$\beta = \frac{1}{2+\log_2 3} + \frac{2}{1+2\log_3 2}$$
$$= \frac{1}{2+\dfrac{\log_5 3}{\log_5 2}} + \frac{2}{1+2\dfrac{\log_5 2}{\log_5 3}}$$
$$= \frac{\log_5 2+2\log_5 3}{\log_5 3+2\log_5 2}$$

The last two equations may be regarded as a system of equations for determining $\log_5 2$ and $\log_5 3$:

$\alpha \log_5 2 + (\alpha - 1)\log_5 3 = 1$

$(2\beta - 1)\log_5 2 + (\beta - 2)\log_5 3 = 0$

If $\alpha(\beta-2) - (\alpha-1)(2\beta-1) = -\alpha - \alpha\beta + 2\beta - 1 \neq 0$ then this system has the solution

$$\log_5 2 = \frac{2-\beta}{\alpha+\alpha\beta-2\beta+1}, \log_5 3 = \frac{2\beta-1}{\alpha+\alpha\beta-2\beta+1}$$

We finally get

$$\log_{25} 24 = \frac{5-\beta}{2\alpha+2\alpha\beta-4\beta+2}$$

Now, let us verify that the expression $\alpha + \alpha\beta - 2\beta + 1$ is indeed different from zero. Thus, we have

$$\alpha + \alpha\beta - 2\beta + 1 = \log_6 15 + \log_6 15 . \log_{12} 18 - 2\log_{12} 18 + 1$$
$$= (\log_6 15 - \log_{12} 18 + 1) + \log_{12} 18 . (\log_6 15 - 1)$$

The second summand here is positive since $\log_{12} 18 > 0$ and $\log_6 15 > 1$. As to the first summand, using the properties of logarithms, we can write $\log_6 15 > 1$, $\log_{12} 18 < 2$ and so $\log_6 15 - \log_{12} 18 + 1 > 0$. Thus, the expression $\alpha + \alpha\beta - 2\beta + 1$ is positive.

The properties of logarithms, among them the properties I to VIII given above, are widely used in solving a broad range of problems such as logarithmic equations and system logarithmic inequality and so on.

Illustration 14. Solve the equation $x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$.

Transposing $x \log_{10} 5$ to the left member of the equation and utilizing the laws of logarithms, we get

$x + \log_{10}(1+2^x) - x\log_{10} 5 = x\log_{10} 10 - x\log_{10} 5 + \log_{10}(1+2^x) = \log_{10} 2^x(1+2^x)$.

The equation can thus be rewritten as $\log_{10} 2^x(1 + 2^x) = \log_{10} 6$, whence

$$(2^x) + 2^x - 6 = 0$$

Denoting $z = 2^x$, we arrive at the quadratic equation $z^2 + z - 6 = 0$ which has the roots $z_1 = -3$, $z_2 = 2$. Since the equation $2^x = -3$ is impossible (because 2^x is positive for all values of x), it remains to solve the equation $2^x = 2$. It has the root $x = 1$, which is the sole root of the original equation.

Illustration 15. Solve the equation

$$\log_a(ax).\log_x(ax) = \log_{a^2}\frac{1}{a}, \text{ where } a > 0, a \neq 1.$$

Clearly, the roots must satisfy the condition $x > 0$, $x \neq 1$. Using the properties of logarithms, transform the expression that enter into this equation :

$$\log_x(ax) = 1 + \log_x a = 1 + \frac{1}{\log_a x} = \frac{\log_a x + 1}{\log_a x},$$
$$\log_{a^2}\frac{1}{a} = -\frac{1}{2}\log_a a = -\frac{1}{2}, \log_a(ax) = 1 + \log_a x$$

Our equation can now be rewritten as

$$\frac{(\log_a x + 1)^2}{\log_a x} = -\frac{1}{2}$$

whence $(\log_a x)^2 + \frac{5}{2}\log_a x + 1 = 0$ solving this equation, we get

$$x_1 = \frac{1}{a^2}, x_2 = \frac{1}{\sqrt{a}}$$

Illustration 16. Solve the system of equations

$$5(\log_y x + \log_x y) = 26$$
$$xy = 64$$

It is clear that it must be true that $x > 0$, $y > 0$, $x \neq 1$, $y \neq 1$. Denoting $z = \log_x y$ and using Formula VI, we find that the first equation of the system can be rewritten as $5(z + 1/z) = 26$, whence $z_1 = 5$, $z_2 = 1/5$. This means that the solutions of the original system must be sought among the solutions of the system

$$\log_x y = 5$$
$$xy = 64$$

and of the system

$$\log_x y = 1/5$$
$$xy = 64$$

Solving these systems and choosing those solutions which satisfy the conditions $x > 0$, $x \neq 1$, $y > 0$ $y \neq 1$, we obtain the answer. The original system has two solutions : $x_1 = 2$, $y_1 = 32$, $x_2 = 32$, $y_2 = 2$.

Illustration 17. What can be said about the number x if it is known that for every real $a \neq 0$

$$\log_x (a^2 + 1) < 0 \ ?$$

For every $a \neq 0$ the number $1 + a^2 > 1$. But since the logarithm of a number greater than unity is negative only to a base less than unity, it follows that $x < 1$. Further more, since logarithms are only considered to a positive base, $x > 0$. And so finally we see that the number x of our problem is taken in the interval $0 < x < 1$.

Illustration 18. Find all x such that $\log_{1/2} x > \log_{1/3} x$.

From Formula V, we have

$$\log_{1/3} x = \frac{\log_{1/2} x}{\log_{1/2} \frac{1}{3}} = \log_{1/3} \cdot \frac{1}{2} \cdot \log_{1/2} x$$

and so our inequality can be rewritten as

$$\log_{1/2} x \left(1 - \log_{1/3} \frac{1}{2}\right) > 0$$

Since, $1 - \log_{1/3} \frac{1}{2} > 0$, from the letter inequality we obtain $\log_{1/2} x > 0$, whence $x < 1$. But the original inequality is meaningful only when $x > 0$. Therefore, all x that satisfy the original inequality lie in the interval $0 < x < 1$.

Illustration 19. Solve the inequality $\dfrac{1}{\log_a x} > 1, a > 1$.

The fraction $1/p$ is greater than unity if its denominator p lies between zero and unity. Thus, our task is to find values of x such that their logarithms (to the base $a > 1$) lie between zero and unity, that is to say, so that the following two conditions hold true simultaneously : $0 < \log_a x$ and $\log_a x < 1$. The first states that the values of x must exceed unity, the second that they must be less than a. Hence, the solution of the original inequality is the interval $1 < x < a$.

We can also reason differently. The left member of the proposed inequality is meaningful only for positive values of x different from unity, and so the inequality may be rewritten as $\log_x a > 1$. This inequality holds true only for values of x which are greater than unity (since for $0 < x < 1$ we have $\log_x a < 0$ when $a > 1$) but less than a (since for $x > a > 1$ we have, by the logarithmic laws, $\log_x a < 1$).

In the foregoing examples, formulas I to VI were used successfully to transform a variety of expressions both with concrete number and literal data. Such manipulations are necessary primarily in the solution of equations and inequalities.

But in many such cases these formulas are not sufficient. First of all, this is due to the fact that the letters in the formulas have to satisfy very stringent restrictions. A still greater drawback of formulas I to IV is that the right and left members are meaningful for different restrictions on the values of the literal elements that enter into them.

For example, in formula I, $\log_a$ MN has meaning when the numbers M and N are both positive as well as when they are both negative. By contrast, the right-hand member of this formula is meaningful only in the first instance. But this mean that if we transform an equation and replace the logarithm of a product of two expression M and N containing the unknown by the sum of the logarithm of these expression, then for values of the unknowns which make M and N negative numbers, we change the meaningful expression $\log_a$ MN into a meaningless expression $\log_a \text{M} + \log_a \text{N}$. The very same goes for formulas II and III.

For these reasons, formulas of a more general nature are used in solving problems containing unknowns :

I* $\quad \log_a \text{MN} = \log_a |\text{M}| + \log_a |\text{N}| \qquad (\text{MN} > 0)$

II* $\quad \log_a \dfrac{\text{M}}{\text{N}} = \log_a |\text{M}| - \log_a |\text{N}| \qquad (\text{MN} > 0)$

III* $\quad \log_a \text{N}^{2k} = 2k \log_a |\text{N}| \qquad (\text{N} \neq 0, k \text{ an integer})$

IV* $\quad \log_{x^{2k}} \text{N} = \dfrac{1}{2} \log_{|x|^k} \text{N}$

$(\text{N} > 0, k \text{ an integer}, x \neq 0, |x| \neq 1)$

It should be noted that formula I* and II* also have drawbacks stated above: their left and right members are meaningful for different restrictions on the values of the letters that enter into them. Namely, the right-hand members have meaning for arbitrary M and N different from zero, while the left-hand members are only meaningful for M and N having the same sign, which means that they are subjects to more stringent restrictions. For this reason, replacing $\log_a \text{MN} = \log_a |\text{M}| + \log_a |\text{N}|$ when solving equations can lead to extraneous solutions but not to the loss of solutions, as can happen when using formulas I-IV. Since, acquiring extraneous solutions of an equation is preferable to losing solutions (superfluous solutions may be discarded by verification, but lost solution cannot be found), one should use formulas I* to IV * when manipulating literal expressions.

Here, are some problems which illustrate the importance of utilizing these properties.

Illustration 20. Simplify the expression

$\log_4 \dfrac{x^2}{4} - 2\log_4 4x^4$ and then compute its value for $x = -2$.

It is quite evident here that computations by formulas I and III, that is,

$$\log_4 \frac{x^2}{4} - 2 \log_4 4x^4 = 2 \log_4 x - \log_4 4 - 2 \log_4 4 - 8 \log_4 x = -3 - 6 \log_4 x$$

are erroneous because the letter expression for $x = -2$ is meaningless, whereas the original one is meaningful and is equal to -6.

This paradoxical result is due to the fact that formulas I and III are only applicable to positive values of the letters. Now, if we use formulas I* and III* in which the values of the letters may be negative as well, we get

$\log_4 \frac{x^2}{4} - 2\log_4 4x^4 = 2\log_4 |x| - 1 - 2 - 8\log_4 |x|$

$= -3 - 6\log_4 |x|$

It is clear that for $x = -2$ this expression is equal to -6.

Illustration 21. Solve the system of equations

$$\text{Log}_2\, xy = 5$$

$$\log_{1/2} \frac{x}{y} = 1$$

Using formulas I* and II* rewrite the system as

$$\log_2 |x| + \log_2 |y| = 5$$

$$\log_{1/2} |x| - \log_{1/2} |y| = 1$$

Denoting $z_1 = \log_a |x|$, $z_2 = \log_2 |y|$, we get

$$z_1 + z_2 = 5$$

$$z_1 - z_2 = -1$$

whence $z_1 = 2$, $z_2 = 3$, and so $|x| = 4$, $|y| = 8$.

But this does not mean that the original system has four solutions :

$x_1 = 4$, $y_1 = 8$, $x_2 = -4$, $y_2 = -8$

$x_3 = 4$, $y_3 = -8$. $x_4 = -4$ $y_4 = 8$

Because it is required that the expressions $\log_2 xy$ and $\log_{1/22} \frac{x}{y}$ be meaningful. They will clearly have meaning only for x and y having the same signs. And so our system will only have two solutions : $x_1 = 4$, $y_1 = 8$, and $x_2 = -4$, $y_2 = -8$.

Thus, using formulas I* and II* we acquired extraneous solutions which were readily discarded in a verification; now if we had used formulas I and II and had rewritten the system as

$$\log_2 x + \log_2 y = 5$$

$$\log_{1/1} x - \log_{1/2} y = 1$$

we would have lost the solutions $x_2 = -4$, $y_2 = -8$.

Note also that the original system may be solved in a different way by reducing it directly to the system

$$xy = 32, \frac{x}{y} = \frac{1}{2}$$

whence the required answer is obtained.

Exercise

1. If $\log_7 \log_5 \left(\sqrt{x+5} + \sqrt{x}\right) = 0$, find the value of x—
(A) 1 (B) 0
(C) 2 (D) None of these

2. If $\log_3 2, \log_3 (2^x - 5), \log_3 (2^x - 7/2)$ are in arithmetic progression, then the value of x is equal to—
(A) 5 (B) 4
(C) 2 (D) 3

3. What is the sum of 'n' terms in the series: $\log m + \log(m^2/n) + \log m^3/n^2) + \log (m^4/n^3) + \ldots\ldots$
(A) $\log \left[\frac{n^{(n-1)}}{m^{(n+1)}}\right]^{n/2}$ (B) $\log \left[\frac{m^m}{n^n}\right]^{n/2}$
(C) $\log \left[\frac{m^{(1-n)}}{n^{(1-m)}}\right]^{n/2}$ (D) $\log \left[\frac{m^{(1+n)}}{n^{(n-1)}}\right]^{n/2}$

4. If $\frac{1}{3}\log 3\text{M} + 3\log 3\text{N} = 1 + \log_{0\cdot 008} 5$, then—
(A) $\text{M}^9 = \frac{9}{\text{N}}$ (B) $\text{N}^9 = \frac{9}{\text{M}}$
(C) $\text{M}^3 = \frac{3}{\text{N}}$ (D) $\text{N}^9 = \frac{3}{\text{M}}$

5. If $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$, then a possible value of x is given by—
(A) 10 (B) $\frac{1}{100}$
(C) $\frac{1}{1000}$ (D) None of these

6. Let $u = (\log_2 x)^2 - 6\log_2 x + 12$, where x is a real number. Then the equation $x^n = 256$, has—
(A) No solution for x
(B) Exactly one solution for x
(C) Exactly two distinct solutions for x
(D) Exactly three distinct solutions for x

7. If $\log_4 5 = a$ and $\log_5 6 = b$, then $\log_3 2$ is equal to—
(A) $\frac{1}{2a+1}$ (B) $\frac{1}{2b+1}$
(C) $2ab + 1$ (D) $\frac{1}{2ab-1}$

8. If $\log_{ax} x, \log_{bx} x, \log_{cx} x$ are in HP, where a, b, c, x belong to $(1, +\infty)$, then a, b, c are in—
(A) AP (B) GP
(C) HP (D) None of these

9. If $\log_5 a . \log_a x = 2$, then x is equal to—
(A) 125 (B) a^2
(C) 25 (D) None of these

10. The value of $3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15}$ is—
(A) log 2 (B) log 3
(C) 1 (D) 0

11. Let $f(x) = \sqrt{\log_{10} x^2}$. The set of all values of x for which $f(x)$ is real, is—
(A) $[-1, 1]$ (B) $[1, +\infty)$
(C) $(-\infty, 1]$ (D) $(-\infty, -1] \cup [1, +\infty)$

12. The set of real values of x for which $2^{\log_{\sqrt{2}}(x-1)} > x + 5$ is—
(A) $(-\infty, -1) \cup (4, +\infty)$
(B) $(4, +\infty)$
(C) $(-1, 4)$
(D) None of these

13. If $a_1, a_2, a_3, \ldots\ldots.$ are positive numbers in GP, then $\log a_n, \log a_{n+1}, \log a_{n+2}$ are in—
(A) AP (B) GP
(C) HP (D) None of these

14. If $x = \log_a(bc)$, $y = \log_b(ca)$ and $z = \log_c(ab)$, then which of the following is equal to 1?
(A) $x + y + z$
(B) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$

(C) xyz

(D) None of these

15. If $x_n > x_{n-1} > \ldots\ldots\ldots\ldots. > x_2 > x_1 > 1$, then the value of

$\log_{x_1} \log_{x_2} \log_{x_3} \ldots\ldots \log_{x_n} x_n^{x_{n-1}}$ is equal to—

(A) 0 (B) 1

(C) 2 (D) None of these

16. If $\log x : \log y : \log z = (y - z) : (z - x) : (x - y)$, then—

(A) $x^y.y^z.z^x = 1$ (B) $x^x y^y z^z = 1$

(C) $\sqrt[x]{x} \cdot \sqrt[y]{y} \cdot \sqrt[z]{z} = 1$ (D) None of these

17. $x^{\log_x a \times \log_a y \times \log_y z}$ is equal to—

(A) x (B) y

(C) z (D) None of these

18. The number of zeroes coming immediately after the decimal point in the value of $(0{\cdot}2)^{25}$ is (given $\log 10^2 = 0{\cdot}30103$)—

(A) 16 (B) 17

(C) 18 (D) None of these

19. If $[x]$ = the greatest integer less than or equal to x, then $[\log_{10} 6730{\cdot}4]$has the value—

(A) 6 (B) 4

(C) 5 (D) None of these

20. The number of solutions of $\log_2 (x + 5) = 6 - x$ is—

(A) 2 (B) 0

(C) 3 (D) None of these

21. The number of real values of the parameter k for which $(\log_{16} x)^2 - \log_{16} x + \log_{16} k = 0$ with real coefficient will have exactly one solution is—

(A) 2 (B) 1

(C) 4 (D) None of these

22. If $\log_{0.5} \sin x = 1 - \log_{0.5} \cos x$, then the number of solutions of $x \in [-2\pi, 2\pi]$ is—

(A) 3 (B) 2

(C) 1 (D) 4

23. If $\log_{\cos x} \tan x + \log_{\sin x} \cot x = 0$, then the most general solutions of x are—

(A) $n\pi + \frac{\pi}{4}, n \in \mathbb{Z}$ (B) $2n\pi + \frac{\pi}{4}, n \in \mathbb{Z}$

(C) $2n\pi - \frac{3\pi}{4}, n \in \mathbb{Z}$ (D) None of these

24. The number of values of $x \in [0, n\pi], n \in \mathbb{Z}$, that satisfy $\log_{|\sin x|} (1 + \cos x) = 2$, is—

(A) 0 (B) n

(C) $2n$ (D) None of these

25. The value of $\sum_{r=1}^{89} \log_{10} \tan \frac{\pi r}{180}$ is equal to—

(A) 10 (B) 1

(C) 0 (D) None of these

26. The solution set of $\log_2 |4 - 5x| > 2$ is—

(A) $\left(\frac{8}{5}, +\infty\right)$

(B) $\left(\frac{4}{5}, \frac{8}{5}\right)$

(C) $(-\infty, 0) \cup \left(\frac{8}{5}, +\infty\right)$

(D) None of these

27. The set of real values of x for which $\log_{0.2} \frac{x+2}{x} \leq 1$ is—

(A) $\left(-\infty, -\frac{5}{2}\right) \cup (0, +\infty)$

(B) $[\frac{5}{2}, +\infty)$

(C) $(-\infty, -2) \cup (0, +\infty)$

(D) None of these

28. The set of real values of x satisfying $\log_{1/2} (x^2 - 6x + 12) \geq -2$ is—

(A) $(-\infty, 2]$ (B) $[2,4]$

(C) $[4, +\infty)$ (D) None of these

29. If $\log_{0.04} (x - 1) \geq \log_{0.2} (x - 1)$, then x belongs to the interval—

(A) $[2, 1]$ (B) $(2, 1)$

(C) $[2, 1)$ (D) $(2, 1]$

30. If $\log_{1/\sqrt{2}} \sin x > 0, x \in [0, 4\pi]$, then the number of values of x which are integral multiples of $\frac{\pi}{4}$, is—

(A) 6 (B) 12

(C) 3 (D) None of these

31. If $\log_{\cos x} \sin x \geq 2$ and $0 \leq x \leq 3\pi$, then $\sin x$ lies in the interval—

(A) $\left[\frac{\sqrt{5}-1}{2}, 1\right]$ (B) $\left[0, \frac{\sqrt{5}-1}{2}\right]$

(C) $[0, 1/2]$ (D) None of these

32. If $\log_{\sqrt{3}} (\sin x + 2\sqrt{2} \cos x) \geq 2, -2\pi \leq x \leq 2\pi$, then the number of solutions of x is—

(A) 0 (B) Infinite

(C) 3 (D) None of these

Directions—(Q. 33 to 36) Choose the correct options. One or more options may be correct.

33. If $\log_k x.\log_5 k = \log_x 5, k \neq 1, k > 0$, then x is equal to—

(A) k (B) 1/5

(C) 5 (D) None of these

34. If $x^2 + 4y^2 = 12xy,\ x \in [1, 4], y \in [1, 4]$, then—

(A) The greatest value of $\log_2 (x + 2y)$ is 4

(B) The least value of $\log_2 (x + 2y)$ is 3

(C) The range of value of $\log_2 (x + 2y)$ is $[2, 4]$

(D) The number of integral values of (x, y) is 2 such that $\log_2 (x + 2y)$ is equal to 3

35. If $\frac{1}{2} \le \log_{0\cdot 1} x \le 2$, then—

(A) The maximum value of x is $\frac{1}{\sqrt{10}}$

(B) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

(C) x does not lie between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

(D) The minimum value of x is $\frac{1}{100}$

36. If $x^{3/4 (\log_3 x)^2 + \log_3 x - 5/4} = \sqrt{3}$, then x has—

(A) One positive integral value

(B) One irrational value

(C) Two positive rational values

(D) None of these

Solutions

1. (B) $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$

$$\log_5 (\sqrt{x+5} + \sqrt{x}) = 7^0 = 1$$

$$\sqrt{x} + 5 + \sqrt{x} = 5^1 = 5$$

$\Rightarrow$ $2\sqrt{x} = 0$

$\therefore$ $x = 0$

2. (D) In an AP, the three terms a, b, c are related as $2b = a + c$

Hence, $2 [\log_3 (2^x - 5)] = \log_3 2 + \log_3 (2^x - 7/2)$

$$\log_3 (2^x - 5)^2 = (2^{(x+1)} - 7)$$

Substitute the choices, only $x = 3$ satisfies the conditions.

3. (D) $S = \log m + \log \frac{m^2}{n} + \log \frac{m^3}{n^2} + \ldots.. n$ terms

$$= \log \left(m \cdot \frac{m^2}{n} \cdot \frac{m^3}{n^2} \cdots \frac{m^n}{n^{n-1}}\right)$$

$$= \log \left[\frac{m^{\frac{n(n+1)}{2}}}{n^{\frac{n(n-1)}{2}}}\right] = \log \left(\frac{m^{(n+1)}}{n^{(n-1)}}\right)^{n/2}$$

4. (B) $\frac{1}{3} \log_3 M + 3 \log_3 N = 1 + \log_{0\cdot 008} 5$

$\Rightarrow$ $\log_3 (M.N^9)^{1/3} = 1 + \dfrac{\log_3 5}{\log_3 \cdot \frac{8}{1000}}$

$$= 1 - 1/3 = 2/3$$

$\Rightarrow$ $(M.N^9)^{1/3} = 3^{2/3}$

$\Rightarrow$ $N^9 = 9/M$

5. (B) $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$

$\Rightarrow$ $1/2 \log_{10} x = 2 \log_x 10$

$\Rightarrow$ $\log_{10} x / \log_x 10 = 4$

$\Rightarrow$ $\log_{10} x = 2$

$\Rightarrow$ $x = 1/100$

6. (B) $u = (\log_2 x)^2 - 6(\log_2 x) + 12$

Let $(\log_2 x) = p$ …(1)

$\Rightarrow$ $u = p^2 - 6p + 12$

$$x^u = 256 = 2^8$$

Applying log to base 2 on both sides, we get $u \log_2 x = \log_2 2^8$

$$u\log_2 x = 8 \quad \ldots(2)$$

Dividing eq. 2 by eq. 1, we get

$$u = 8/p$$

$\Rightarrow$ $8/p = p^2 - 6p + 12$

$\Rightarrow$ $8 - p^3 - 6p^2 + 12p$

$\Rightarrow$ $(p-2)^3 = 0$

$\Rightarrow$ $p = 2$

$\Rightarrow$ $\log_2 x = 2 \Rightarrow x = 4$

Thus, we have exactly one solution.

7. (D) Here, $5 = 4^a$ and $6 = 5^b$

Let $\log 3^2 = x$. Then, $2 = 3^x$

Now, $6 = 5^b = (4^a)^b = 4^{ab}$ or $3 = 2^{2ab-1}$

$\therefore$ $2 = (2^{2ab-1})^x = 2^{x(2ab-1)}$

$\Rightarrow$ $x(2ab - 1) = 1$

8. (B) Clearly, $\log_x (ax), \log_x (bx), \log_x (cx)$ are in AP

$\Rightarrow 1 + \log_x a, 1 + \log_x b, 1 + \log_x c$ are in AP

$\Rightarrow \log_x a, \log_x b, \log_x c$ are in AP

$\Rightarrow$ $\frac{\log a}{\log x} + \frac{\log c}{\log x} = 2 \frac{\log b}{\log x}$

$\Rightarrow$ $\log a + \log c = 2\log b$

$\Rightarrow$ $ac = b^2$

9. (C) 10. (A)

11. (D) $\log_{10} x^2 \ge 0 \Rightarrow \log_{10} x^2 \ge \log_{10} 1$

$\Rightarrow x^2 \ge 1 \Rightarrow x \ge 1$ or $x \le -1$

12. (A) $\{(\sqrt{2})^2\}^{\log \sqrt{2} (x-1)} > x + 5$

or $(\sqrt{2})^{\log \sqrt{2} (x-1)^2} > x + 5$

or $(x-1)^2 > x + 5$ or $(x - 4)(x + 1) > 0$

13. (A) 14. (B)

15. (B) $\log_{x_1} \log_{x_2} \log_{x_3} \ldots\ldots \log_{x_n} (x_{n-1}^{x_{n-2}^{\cdot^{\cdot^{x_1}}}} \log_{x_n} x_n)$

$= \log_{x_1} \log_{x_2} \log_{x_3} \ldots\ldots \log_{x_n} x_{n-1}^{x_{n-2}^{\cdot^{\cdot^{x_1}}}}$

= ………

$= \log_{x_1} x_1 = 1.$

16. (B) 17. (C) 18. (B)

19. (D) $\log_{10} 7630{\cdot}4 = 3.xxxx$

$\Rightarrow$ $[\log_{10} 6730] = 3$

20. (D) Here, $x + 5 = 2^{6-x}$.

Clearly, there cannot be more than one solution and by trial, the solution is $x = 3$.

21. (B) $\log_{16} x = \dfrac{1 \pm \sqrt{1 - 4 \log_{16} k}}{2}$ · For exactly one solution, $4\log_{16} k = 1$

$\therefore K^4 = 16$

$\therefore k = 2, -2, 2i, -2i$. But k is positive and real.

22. (B) $\log_{0.5} \sin x + \log_{0.5} \cos x = 1$

$\Rightarrow$ $\log (\sin x . \cos x) = 1/2$

$\Rightarrow$ $\sin 2x = 1$

$\therefore$ $2x = \frac{\pi}{2}, \pm 2\pi + \frac{\pi}{2}, \pm 4\pi + \frac{\pi}{2}, \ldots\ldots$

$\therefore \quad x = \frac{\pi}{4}, \pm\pi + \frac{\pi}{4}, \pm 2\pi + \frac{\pi}{4}, \ldots$

But $\log_{0.5} \sin x$, $\log_{0.5} \cos x$ have to be real at the same time. So, angles in the first quadrant only can be considered.

$\therefore \quad x = \frac{\pi}{4}, -2\pi + \frac{\pi}{4}.$

23. (B) $\log_{\cos x} \sin x - 1 + \log_{\sin x} \cos x - 1 = 0$

Or $\quad y + 1/y = 2$, where $y = \log_{\cos x} \sin x$

$\therefore (y-1)^2 = 0$ or $\log_{\cos x} \sin x = 1$ or $\sin x = \cos x$

Also, $\sin x$, $\cos x$, $\tan x$ must be positive and $\sin x \neq 1$, $\cos x \neq 1$.

24. (A) $\quad 1 + \cos x = |\sin x|^2$

$= \sin^2 x$ or $\cos x\,(1 + \cos x) = 0$

But $1 + \cos x \neq 0$ for the value of the logarithm to be real.

$\cos x = 0$

$\Rightarrow \sin x = 1$ (not possible, because the base of the logarithm cannot be 1)

25. (C) Value $= \log_{10}\{\tan 1°.\ \tan 2°.\ \tan 3° \ldots .. \tan 89°)$

$= \log_{10}\{(\tan 1°.\ \tan 89°) \ldots \ldots$

$(\tan 44°.\tan 46°).\ \tan 45°\}$

$= \log_{10} 1 = 0$

26. (C) $\quad |4 - 5x| >^2 = 4$

$\Rightarrow \quad \left|\frac{5x}{4} - 1\right| > 1$

$\Rightarrow \quad \frac{5x}{4} - 1 > 1$

or $\quad \frac{5x}{4} - 1 < -1$

$\therefore x > 8/5$ or $x < 0$. So, the solution set

$= (-\infty, 0) \cup \left(\frac{8}{5}, +\infty\right)$

27. (A) $\frac{x+2}{x} \geq (0{\cdot}2)^1$ or $\frac{x+2}{x} \geq \frac{1}{5}$

Multiplying by $5x^2$

$5x\,(x + 2) \geq x^2$ or $4x^2 + 10x \geq 0$

$\therefore x \geq 0$ or $x \leq -\frac{5}{2}$

Also, $\frac{x+2}{x} > 0$

$\Rightarrow x\,(x + 2) > 0$

$\therefore x < -2$ or $x > 0$

$\therefore$ The solution set is $(-\infty, -\frac{5}{2}] \cup (0, +\infty)$.

28. (B) $x^2 - 6x + 12 \leq \left(\frac{1}{2}\right)^{-2}$ or $x^2 - 6x + 8 \leq 0$

or $(x-2)(x-4) \leq 0$

29. (C) $\frac{\log_{10}(x-1)}{\log_{10} 0{\cdot}04} \geq \frac{\log_{10}(x-1)}{\log_{10}(0{\cdot}2)}$

$\Rightarrow \log_{10}(x-1) \cdot \left\{\frac{1}{2\log_{10}(0{\cdot}2)} - \frac{1}{\log_{10}(0{\cdot}2)}\right\} \geq 0$

$\Rightarrow \log_{10}(x-1) \cdot \frac{-1}{2\log_{10}(0{\cdot}2)} \geq 0$

$\Rightarrow \log_{10}(x-1) \geq 0$ because $\log_{10}(0.2) < 0$

$\therefore x - 1 \geq 10^0 \ \therefore x \geq 2$

Also $x - 1 > 0$, *i.e.*, $x > 1$.

30. (A) $\sin x < \left(\frac{1}{\sqrt{2}}\right)^0 = 1$ and $\sin x > 0$

$\therefore \sin x$ has all values in $(0, \pi)$, $(2\pi, 3\pi)$, $(4\pi, 5\pi)$,…

$\therefore \quad$ required $x = \frac{\pi}{4}, 2 \times \frac{\pi}{4}, 3 \times \frac{\pi}{4}, 9 \times \frac{\pi}{4},$

$10 \times \frac{\pi}{4}, 11 \times \frac{\pi}{4}$

31. (B) $\sin x \leq \cos^2 x$, because $\cos x$ must be a positive proper fraction.

Or $\sin^2 x + \sin x - 1 \leq 0$ or $\left(\sin x + \frac{1}{2}\right)^2 - \frac{5}{4} \leq 0$

From the definition of logarithm, $\sin x > 0$, $\cos x > 0$, $\cos x \neq 1$

$\therefore \sin x + 1/2 \leq \frac{\sqrt{5}}{2}$ $\therefore 0 < \sin x \leq \frac{\sqrt{5}-1}{2}$

32. (D) $\quad \sin x + 2\sqrt{2}\cos x \geq \left(\sqrt{3}\right)^2$

Or $\quad \sin x + 2\sqrt{2}\cos x \geq 3$

Or $\quad \sin\left(x + \cos^{-1}\frac{1}{3}\right) \geq 1$

$\Rightarrow \quad \sin\left(x + \cos^{-1}\frac{1}{3}\right) = 1$

$\therefore \quad x = n\pi + (-1)^n \frac{\pi}{2} - \cos^{-1}\frac{1}{3}$

For solutions in $[-2\pi, 2\pi]$, $n = 0, 1, -1, -2$

33. (B, C) $\quad \log_5 x = \log_x 5$

$\Rightarrow \quad (\log_5 x)^2 = 1$

$\Rightarrow \quad \log_5 x = \pm 1$

$\Rightarrow \quad x = 5, 5^{-1}$

34. (A, C, D)

$(x + 2y)^2 = 16xy$

or $\quad 2\log_2(x + 2y) = 4 + \log_2 x + \log_2 y$

$\therefore \quad \log_2(x + 2y) = 2 + \frac{1}{2}(\log_2 x + \log_2 y)$

But $1 \leq x \leq 4$, $1 \leq y \leq 4$.

$\therefore \max\log_2(x + 2y) = 2 + \frac{1}{2}(\log_2 4 + \log_2 4)$

$\min\log_2(x + 2y) = 2 + \frac{1}{2}(\log_2 1 + \log_2 1)$

Also $\log_2(x + 2y) = 3$

$\Rightarrow \quad x + 2y = 8$

This is satisfied by $x = 2, y = 3$; $x = 4, y = 2$.

35. (A, B, D) $\frac{1}{2} \leq \log_{1/10} x \leq 2 \Rightarrow \left(\frac{1}{10}\right)^{1/2} \geq x, \left(\frac{1}{10}\right)^2 \leq x$

So, $\frac{1}{100} \leq x \leq \frac{1}{\sqrt{10}}$

36. (A, B, C)

Taking logarithm, $\left\{\frac{3}{4}(\log_3 x)^2 + \log_3 x - \frac{5}{4}\right\}\log_3 x$

$= \log_3 \sqrt{3}$

Or, $\quad 3/4y^3 + y^2 - 5/4y = 1/2$ (let $\log_3 x = y$)

Or $\quad (y-1)(3y^2 + 7y + 2) = 0$

Or $(y-1)(3y+1)(y+2) = 0$

Hence, $\quad \log_3 x = 1, -2, -1/3$

$\Rightarrow \quad x = 3, 3^{-1/3}, 3^{-2}$.

4 Functions and Graphs

CONCEPT OF A FUNCTION

Methods of Representing Functions

When studying various phenomena of nature and in our everyday practical activity, we come across quantities of various character, such as length, area, volume, mass, temperature, time, and so on. Depending on concrete condition, some quantities have ***constant*** and ***variable***, respectively.

Mathematics studies the dependence between variable quantities (or simply, variables) in the process of their change.

For instance, with a change in the radius of a circle, its area also changes, and we consider the question of how the circle changes depending on the change in its radius.

Let the variable x take on numerical values from this set E.

Consider the concept of a function. A ***function*** is a rule which attributes to every number x from E one definite number y.

Here x is called the ***independent variable***, or the ***argument*** of the function, and y is called the ***dependent variable,*** the set E is spoken of as the ***domain of definition*** of the function. The set of all values attained by the variable y is called the ***range*** of the function.

The above given definition can be formulated in other words : A variable y is said to be a function of a variable x in the domain of definition E if to each value of x belonging to this domain there corresponds a definite value of the variable y.

The notation $y = f(x)$ or $y(x)$ means that y depends on x. The letter f symbolizes the rule according to which we obtain the value of y corresponding to a given value of x from the set E.

Instead of the letter x, E,Y, $f(x)$ any other letters and notations are also used. To represent or, which is the same, to specify a function $y = f(x)$ on the set E means to indicate the rule according to which for every x from E the corresponding value of y is found.

Consider basic methods of representing functions :

(1) Analytical representation (by means of formula).

A function can be given by a single formula in the entire domain of its definition or by several formulas, different for different parts of the domain of its definition. For instance,

$$Y = 3x, y = \begin{Bmatrix} x \ \ if\, x \le 0 \\ x^2 \ if\, x > c \end{Bmatrix}$$

Such a method of representing a function is called ***analytical.***

In the general case, when a function is specified analytically, its domain of definition is usually understood (provided those are no additional conditions) as the maximum set of values of x for which the formula representing the function makes sense.

For instance, the function $y = x^2$ is defined throughout the number axis, the same as the analytical expression which represents it. But if this function express the dependence of the area of a square on the length of its side, then the function $y = x^2$ is specified for any $x > 0$.

(2) Tabular method : When specifying a function by means of a table, we simply write down a sequence of valuable of the argument $x_1, x_2, \ldots, x_n$ and the corresponding values of the function $y_1, y_2, \ldots\ldots\ldots y_n$. This method of representing a function is called ***tabular.***

This way of representing functions is widely used; for instance, the reader is undoubtedly familiar with tables of logarithms, tables of trigonometric functions and their logarithms, etc.

The tabular method is particularly often used in natural sciences and technology. The numerical results obtained in a sequence of observations (measurements) of a process are usually complied in a table, which thus shows relation between the quantities under investigations.

An advantage of a tabular representation of a function is that for any value of the independent variable included into the table the corresponding value of the function is immediately found without any additional measurement or calculation. But it also has an essential demerit: usually it is impossible to specify a function by a table in a complete manner, since there are some values of the independent variable that do not enter into the table.

(3) Graphical representation : On the co-ordinate xy-plane for every value of x from the set E (that is, from the domain of definition of a function) a point M(x, y) is constructed whose abscissa is equal to x and whose obtained to the corresponding value of the function $y(x)$. Points thus constructed plot a certain line which is called the ***graph*** of a given function.

In general, the graph of a function $y = f(x)$ specified on the set E is the set of points $M(x, f(x))$, where $x \in E$, belonging to the co-ordinate plane.

In general, the graph of a function $y = f(x)$ specified on the set E is the set of points $M(x, f(x)$ where $x \in E$ belonging to the co-ordinate plane.

The method of representation of a function with the aid of a graph is called ***graphical.*** To find the value of a function $y(x)$ for a definite value of x by a given graph of the function, let us proceed as follows. Through the point x on the axis of abscissas erect a perpendicular to this axis and find the point of intersection of this perpendicular with the graph of the given function. The ordinate of the point of intersection just yields the corresponding value of the function.

The graphical method of representation a function is widely used in scientific investigations as well as in modern production. Graphs of change of various quantities are automatically drawn by recording instrument (recorders).

Illustration 1. Find the domain of definition of the following functions :

(a) $y = \dfrac{1}{x^2 - 3x + 2}$; **(b)** $y = \sqrt{4 - x^2}$; **(c)** $y = \dfrac{1}{\sqrt{4 - x^2}}$.

Solution : (a) The function is defined for all values of x, except for those for which $x^2 - 3x + 2 = 0$. Solving this quadratic equation, we find $x = 1$ and $x = 2$.

Thus, the domain of definition of the given function consists of three intervals : $(-\infty, 1), (1, 2)$ and $(2, +\infty)$.

(b) The domain of definition of the given function is determined from the condition $4 - x^2 \geq 0$. Solving this inequality we obtain $-2 \leq x \leq 2$. Thus, the domain of definition of the given function is the interval $[-2, 2]$.

(c) The domain of definition of the function is determined from the condition $4 - x^2 > 0$, whence $-2 < x < 2$. Consequently, the domain of definition of the given function is the interval $(-2, 2)$.

We shall most often consider function represented analytically with an interval or half-interval as the domain of their definition.

Properties of Functions

1. Even and Odd Functions : Let a function $y = f(x)$ be given in a certain interval symmetric with respect to the point O (in particular, throughout the entire x-axis). The function $y = f(x)$ is said to be ***even*** if for any

$$f(-x) = f(x).$$

Examples of even functions : $y = x^2$, $y = x^2 + 3$, $y = -3x^2 + 4$, $y = |x|$, $y = 4$.

Indeed, $(-x)^2 = x^2$, $(-x)^2 + 3 = x^2 + 3$, $-3(-x)^2 + 1 = -3x^2 + 1$, $|-x| = |x|$, $y = 4$ for any x.

The sum difference, product and quotient of an even function is again an even function.

A function $y = f(x)$ is said to be ***odd*** if for any x

$$f(-x) = -f(x).$$

Examples of odd functions : $y = x^3$, $y = x^3 + x$, $y = \dfrac{x}{x^2 + 1}$

Indeed, $(-x)^3 = -x^3$, $(-x)^3 + (-x) = -(x^3 + x)$, $\dfrac{(-x)}{(-x)^2 + 1} = -\dfrac{x}{x^2 + 1}$ of any x.

The sum and difference of an odd function is again an odd function, but the product and quotient of an odd function is an even function.

It should not be regarded that every function is even or odd. Most functions do not posses the property to be even or odd.

For instance such is the function $y = x^3 + x^2$

Indeed $(-x)^3 + (-x)^2 = -x^3 + x^2$, *i.e.,* $(-x)^3 + (-x)^2 \neq x^3 + x^2$ and also $(-x)^3 + (-x)^2 \neq -(x^3 + x^2)$.

The definition of even and odd functions implies that the graph of an even function is symmetric about the axis of ordinates, and that of an odd function about the origin.

Really let the point $M(x_0, y_0)$ be a point of the graph of an even function $y = f(x)$, *i.e.,* $y_0 = f(x_0)$.

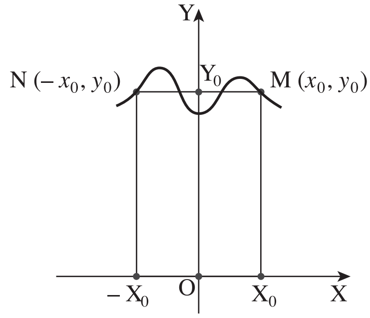


Fig. 1

Consider he point $N(-x_0, y_0)$ which is symmetric to the point $M(x_0 y_0)$ about the y-axis (Fig. 1). Since, the given function is even, we have

$$f(-x_0) = f(x_0) = y_0$$

which means that the point $N(-x_0, y_0)$ also belongs to the graph of the functions $y = f(x)$.

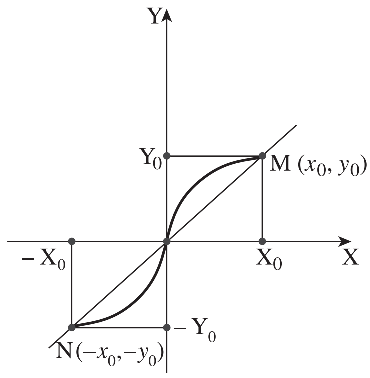


Fig. 2

The symmetry of the graph of an odd function about the origin follows from the fact that along with the point $M(x_0, y_0)$ of the graph of an odd function, there also belongs to this graph the point $N(-x_0, -y_0)$ which is

symmetric to the point M(x_0, y_0) about the origin. The point O bisects the line segment MN. **(Fig. 2)**

2. Monotone Functions : A function $y = f(x)$ is said to be ***increasing*** on a certain interval if for any two values of x from this interval to greater values of the argument there correspond greater values of the function, that is, the condition $x_1, < x_2$ implies that $f(x_1) < f(x_2)$ for any x_1 and x_2 from the given interval.

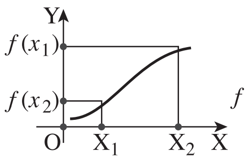


Fig. 3

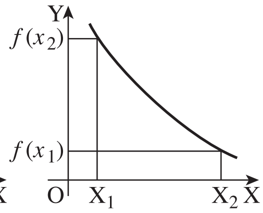


Fig. 4

The ordinate of the graph of an increasing function increase with an increase in x. **(Fig. 3)**

Similarly, a function $y = f(x)$ is called ***decreasing*** on some interval if for any two values of x from this interval to greater values of the argument there correspond smaller values of the function, that is, the condition $x_1 < x_2$ implies that $f(x_1) >> f(x_2)$ for any x_1 and x_2 from this interval.

The ordinate of the graph of a decreasing function decreases with an increases in x. **(Fig. 4)**

Increasing and decreasing functions are referred to as ***monotone functions***.

3. Interval of constant sign and roots of a function : Interval within which a function keeps its sign unchanged (that is, remain positive or negative) are called ***intervals of constant sign*** of the function.

For instance, the function $y = x^2 + 1$ is positive throughout the x-axis; the functions $y = x^3$ is positive for $x > 0$ and negative for $x < 0$, its intervals of constant sign are $(0, +\infty)$ and $(-\infty, 0)$ consequently the graph of the function $y = x^3$ is situated above the x-axis for $x > 0$ and below the x-axis for $x < 0$.

The values of the argument x for which $f(x) = 0$ are called the ***roots*** (or ***zeros***) of the function $f(x)$. Thus, the root of the function $f(x)$ is the same as the root of the equation $f(x) = 0$. The roots of a function are the points of intersection of its graph with the x-axis.

The root of the function $y = x^3$ is $x = 0$ the function $y = x^2 + 1$ has no real root.

Inverse of a Function and its Graph

Let there be given a function $y = f(x)$, and let E be its domain of definition and D the set of its values (or the range). Then to every value x_0 from E there will correspond one definite value $y_0 = f(x_0)$ from D.

Let us take an arbitrary number y_0 from D. In the domain E there is necessarily at least one number x_0 for which $f(x_0) = y_0$.

In general, to every value y_0 from D there will correspond one or several values of x_0 from E for which $f(x_0) = y_0$. To obtain these values of x_0, we may draw through the point y_0 on the axis of ordinate a straight line parallel to the axis of abscissas. This straight line will intersect the graph of the function $y = f(x)$ at one or several points. The abscissas of these points just yield the desired values of x (one of them being x_0) for which the function is equal to y_0. **(Fig. 5)**

Suppose that the function $y = f(x)$ is such that to every value y_0 from D there corresponds one value x_0 from E for which $f(x_0) = y_0$. In this way the function $x = g(y)$ is defined in the domain D and is called the ***inverse*** of the function $y = f(x)$.

If E is an interval and the function $y = f(x)$ is monotone (increasing or decreasing), then it has an inverse function $x = g(y)$, increasing or decreasing, respectively.

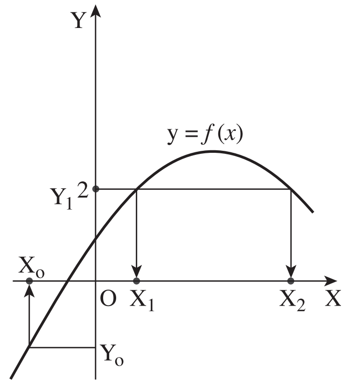


Fig. 5

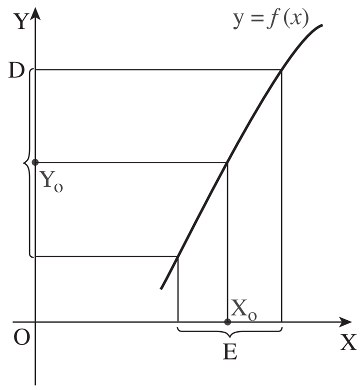


Fig. 6. Gives an example of an increasing function. The function graphed in Fig. 5. has no inverse.

In a function given by a formula has an inverse, then to find the formula defining the inverse, we have to express x in terms of y from the given formula.

For instance, derive the formula specifying in the inverse of the given function $y = 2x - 1$, we have $x = \dfrac{y + 1}{2}$ which is the inverse.

The graphs of the functions $y = 2x - 1$ and $x = \dfrac{y + 1}{2}$ coincide, since both function express one and the same relation between the variables x and y. In general, the

graph of the function $y = f(x)$ and the graph of its inverse $x = g(y)$ is one and the same curve. Usually, when studying the inverse function, its argument is denoted by x and the dependent variable by y, *i.e.*, instead of $x = g(y)$, we write $y = g(x)$. In such a notation the inverse of the function $y = 2x - 1$ will be $y = \frac{x+1}{2}$.

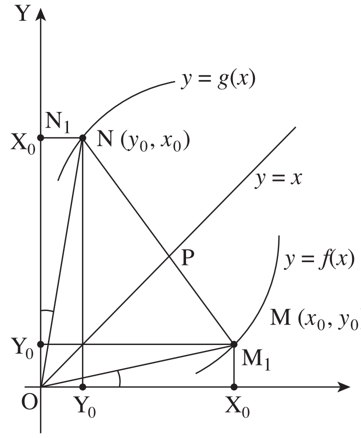


Fig. 7

The graph of two inverse (or reciprocal) functions $y = f(x)$ and $y = g(x)$ are symmetric about the bisector $y = x$ of the first and third quadrants. **(Fig. 7)**

Proof : Let M (x_0, y_0) be a point belonging to the graph of the function $y = f(x)$. Then, by the definition of the inverse of a function, the point N(y_0, x_0) will belong to the graph of the inverse function $y = g(x)$. We have to prove that the points M (x_0, y_0) and N (y_0, x_0) are symmetric with respect to the straight line $y = x$. For this purpose, let us consider the triangle MON. In this triangle, $|OM| = |ON|$, the line segment OP being the bisector.

Indeed, $|OM| = |ON| = \sqrt{x^2{}_0 + y^2{}_0}$ by the formula of the distance between two points on a co-ordinate plane, the equality $|OM| = |ON|$ can be obtained from the triangle MPM_1 and NON_1. From the same triangle we have : $MOM_1 = NON_1$. The bisector OP in the isosceles triangle MON also serve as the median and altitude *i.e.*, $|MP| = |NP|$ and $|OP| \perp |MN|$, which was required to be proved.

Properties and Graph of Certain Simplest Functions

In the general case, a function $y = f(x)$ is investigated according to the following plan :

1. Find the domain of definition of the function and the set of its values.

2. Check to see whether the function is even or odd.

3. Find the intervals of monotonicity and the intervals of constant sign of the function.

4. Determine the points of intersection of the graph of the function with the co-ordinate axes and so on.

Then we can plot the graph of the function. Sometime, it is simpler to construct the graph of the function and then by its shape, to find out the properties of the function.

1. Linear Function $y = kx + b$ and its Graph : A linear function is defined as a function of the from

$$Y = kx + b,$$

where k and b are given numbers,

(1) consider a particular case when $k = 0$, then

$$Y = b$$

This function is defined throughout the entire x-axis and attains one and the same value b for all x's. Consequently, its graph is a straight line parallel to the x-axis and passing at a distance of $|b|$ units from it (if $b > 0$ and below it if $b < 0$) **(Fig. 8).** If $b = 0$, then the graph of the function $y = 0$ is a straight the coinciding with the x-axis.

(2) If $b = 0$, then $y = kx$. For $k \neq 0$ the function $y = kx$ is called the ***direct proportionality relation*** (or ***dependence***). This function is defined everywhere. It increase monotonically for $k > 0$ and decrease for $k < 0$. Let us prove that the function $y = kx$ is monotone.

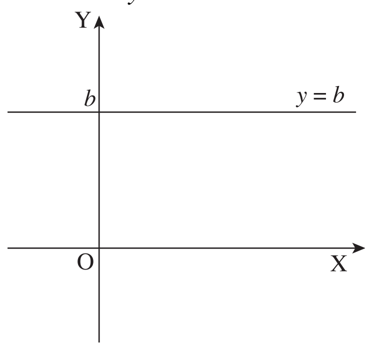


Fig. 8

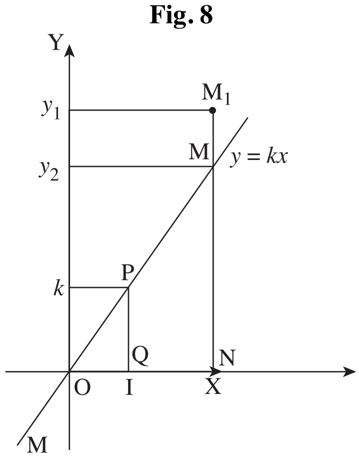


Fig. 9

We take two arbitrary values x_1 and x_2 and find the corresponding values y_1 and y_2 :

$$y_1 = kx_1, \quad y_2 = kx_2.$$

Subtracting y_1 from y_2 ,we obtain

$$y_2 - y_1 = k(x_2 - x_1).$$

If $x_2 > x_1$ and $k > 0$ then $y_2 - y_1 > 0$; then $y_2 > y_1$ and the function $y = kx$ increases.

If $x_2 > x_1$ and $k < 0$ then $y_2 - y_1 < 0$; then $y_2 < y_1$ and the function $y = kx$ decrease.

Consequently, the function $y = kx$ is monotone.

If $x = 0$, then the value of the function $y = kx$ is also equal to zero, hence, the point (0, 0) belongs to the function. For $k > 0$ the sign of x and y coincide; for $k < 0$ the signs of x and y are opposite.

Hence, we conclude that for $k > 0$ the points of the graph of the function $y = kx$ belongs to the first and third quadrants, and for $k < 0$ to the second and forth quadrants.

Let us now prove that the graph of direct proportionality is a straight line passing through the origin.

Let us take $x = 1$. then $y = k$. the straight line passing through the point $P(1, k)$ and the origin (0, 0) is the graph of the function $y = kx$ indeed let $k > 0$.

The triangle MON and POQ are similar for any position of the point M on the constructed straight line. The similarity implies that

$$\frac{|MN|}{|ON|} = \frac{|PQ|}{|OQ|} \quad \text{or} \quad \frac{y}{x} = \frac{k}{1}, \text{ i.e., } y = kx$$

The result is also retained for any point M which lies on the straight line under consideration situated in the third quadrant (in this case, its distances from the x and y-axis are respectively equal to $|y| = -y$ and $|x| = -x$ since $y < 0$ and $x < 0$.

Thus, it has been proved that any point situated on the straight line passing through the points $P(1, k)$ and $O(0, 0)$ belongs to the graph of the function $y = kx$. No other points M_1 situated outside this straight line can belong to the graph $y = kx$.**(See Fig. 9)** If we assume that the point $M_1(x, y_1)$ belongs to this graph then it must be $y_1 = kx$. At the same time, the point $M(x, y)$ obtained as the intersection of the straight line drawn from the point M_1 parallel to the y-axis and the straight line OP belongs (as it was proved) to the sought for graph. Hence, $y = kx$, which contradicts the equality $y_1 = kx$: their right-hand members are equal, while the left hand members are different, since $y \neq y_1$. Thus, the graph of the function $y = kx$ is the straight line OP.

The case $k < 0$ is considered analogously.

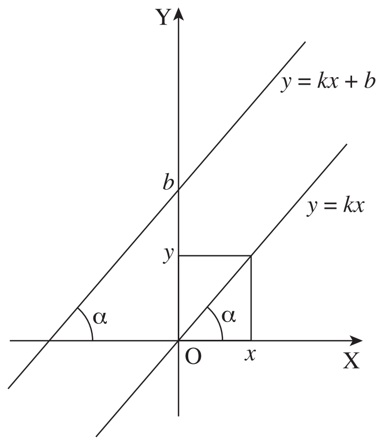


Fig. 10

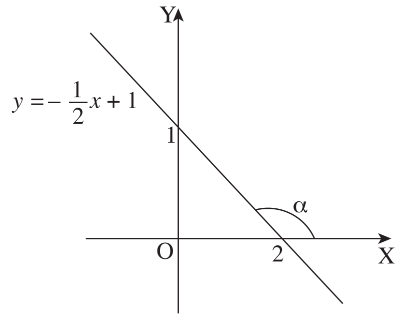


Fig. 11

(3) General case; $y = kx + b$. Every point of the graph of this function is obtained by shifting (or translating) the corresponding point of the graph of the function $y = kx$ by $|b|$ units along the axis of ordinates (upward if $b > 0$ and downward if $b < 0$). Therefore, the graph of a linear function is a straight line parallel to the straight line $y = kx$. **(Fig. 10)**

The coefficient k is called the ***slope*** of the straight line $y = kx$. The slope determines the angle of inclination α of this straight line to the x-axis : $k = \tan \alpha$. If $k > 0$, then the angle α is acute; If $k < 0$, then the angle α is obtuse. The ordinate of the point of intersection of the straight line and the y-axis equal to b.

Thus, the location of the straight line $y = kx + b$ on the co-ordinate plane depends on the values of k and b.

To construct the graph of a linear function, one has to plot two points belonging to this graph and then to draw a straight line through these points.

For instance, let us construct the graph of the function $y = -\frac{1}{2}x + 1$.

For $x = 0$, $y = 1$; for $y = 0$, $x = 2$. joining the found points by a straight line, we obtain the graph of the given function. **(Fig. 11)** here $k = -\frac{1}{2}$ and $\tan \alpha = -\frac{1}{2}$.

2. The Function $y = \frac{k}{x}$ and its Graph : A function of the form $y = \frac{k}{x}$, where $k \neq 0$ is a given number is called the ***inverse proportionality relation.***

Consider the case $k > 0$:

(1) The function is defined everywhere, except for $x = 0$, the domain of its definition being the intervals $(-\infty, 0)$ and $(0, \infty)$:

(2) The function is odd, since

$$f(-x) = \frac{k}{-x} = -\frac{k}{x} = -f(x);$$

Consequently, the graph of the function $y = \frac{k}{x}$ is symmetric about the origin and, therefore, the further investigation is conducted for $x > 0$;

(3) The sign of y coincide with the sign of x;

(4) The function is decreasing, since for $0 < x_1 < x_2$ we have

$$\frac{k}{x_2} - \frac{k}{x_1} = \frac{k(x_1 - x_2)}{x_1 x_2} < 0 \; (k > 0)$$

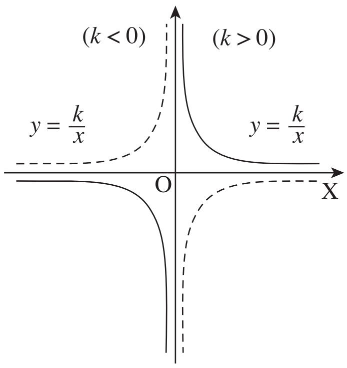


Fig. 12

(it is obvious that for $k > 0$ the function also decrease on the interval $(-\infty, 0)$)

Using these properties, we construct the graph of the function $y = \frac{k}{x}$ for $k > 0$ **(Fig. 12).** The obtained curve is called the ***hyperbola***. It consists of two branches situated in the first and third quadrants. Similarly, it is proved that if $k < 0$, then the function $y = \frac{k}{x}$ is ***monotone***: it increases on each of the intervals $(-\infty, 0)$ and $(0, +\infty)$ the graph is also a hyperbola. Its branches are situated in the second and fourth quadrants. **(See Fig. 12)**

Thus, the graph of the inverse proportionality $y = \frac{k}{x}$ $(k \neq 0)$ is a hyperbola whose location on the co-ordinate plane depends on the values of k.

For instance, **Fig. 13** depicts two hyperbolas; $y = -\frac{1}{x}$ and $y = -\frac{2}{x}$, the origin being the centre of symmetry of these hyperbolas.

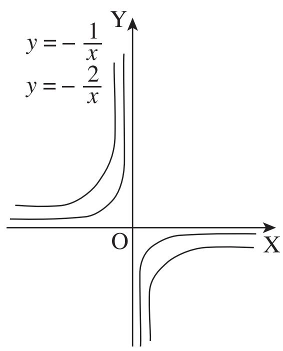


Fig. 13

3. Quadratic Trinomial and Its Graph : A quadratic trinomial is defined as a function of the form

$$Y = ax^2 + bx + c$$

where a, b and c are given numbers and $a \neq 0$ sometimes, the function $y = ax^2 + bx + c$ where $a \neq 0$, is called a ***quadratic function***.

Let us first consider particular cases of functional depends $y = x^2 + bx + c, a \neq 0$.

(1) Quadratic function $y = ax^2$

For $a = 1$ we have $y = x^2$. To construct the graph of the function $y = x^2$ let us compile a table of its values :

X	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3
$Y = X^2$	9	4	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1	4	9

The graph of the function $y = x^2$ is shown in **Fig. 14** and is called the ***parabola***. For $x = 0$ the value of the function $y = x^2$ equals zero. For $x \neq 0$ the values off the function are positive. This means that the parabola $y = x^2$ touches the x-axis at the origin, its remaining points lying above the x-axis.

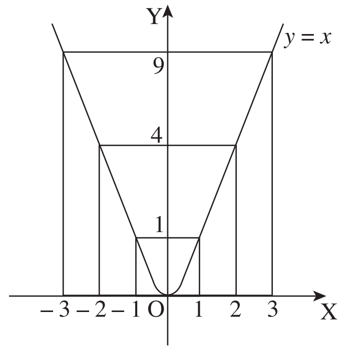


Fig. 14

The parabola $y = x^2$ is symmetric about the y-axis. The point of intersection of the parabola with its axis of symmetry is called the ***vertex*** of the parabola. The vertex of parabola $y = x^2$ is the origin.

Let us now compare the functions $y = 2x^2$ and $y = x^2$. For one and the same x the value of the function $y = 2x^2$ is twice the value of the function $y = x^2$ consequently, the graph of the function $y = 2x^2$ can be obtained by two – fold extension of the parabola $y = x^2$ along the y-axis.

In general, the graph of the function $y = ax^2$ for $a > 0$ can be obtained by stretching the parabola $y = x^2a$ times along the y-axis (more precisely by stretching for $a > 1$ and by compressing for $0 < a < 1$). Note the following properties of the function $y = ax^2$ for $a > 0$;

(a) The function is defined for any x, and also $y = ax^2 \geq 0$; consequently, the least value of the function is equal to zero and is attained for $x = 0$;

(b) The function is even, since $f(-x) = a(-x)^2 = ax^2 = f(x)$. Therefore, the y-axis is the axis of symmetry of the graph;

(c) The function is increase on the interval $(0, +\infty)$ and decrease on the interval $(-\infty, 0)$. Let us prove an increase in the function for $x > 0$. For $0 < x_1 < x_2$ we have $ax^2{}_1 < ax^2{}_2$ $(a > 0)$ (by the property of inequalities) and, hence, the function $y = ax^2$ is increasing for $a > 0$ on the interval $(0, +\infty)$. A decreases in the function for $x < 0$ follows from the evenness of the function and its increases for $x > 0$.

Compare the function $y = -x^2$ and $y = x^2$. For one and the same x the values of these functions are equal by modulus and opposite by sign. Consequently, the graph of the function $y = -x^2$ can be obtained by the symmetry

of the parabola $y = x^2$ about the x-axis. The branches of the parabola $y = x^2$ are said to be directed upward, and those of the parabola $y = -x^2$ downward.

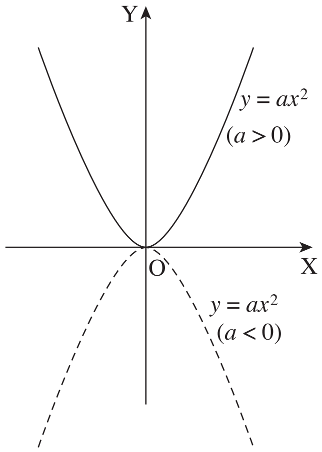


Fig. 15

The graph of the function $y = ax^2$ for any $a \neq 0$ is also a parabola with the y-axis as the axis of symmetry and the origin as its vertex for $a > 0$ the branches of the parabola are directed upward and $a < 0$ downward

(2) Quadratic function $y = a(x - x_0)^2$.

Compare the functions $y = 2(x - 1)^2$ and $y = 2x^2$. The function $y = 2(x - 1)^2$ takes on the same value as the function $y = 2x^2$, but with the corresponding value of the argument increased by unity. Consequently, the graph of the function $y = 2(x - 1)^2$ can be obtained by displacing (or shifting) the parabola $y = 2x^2$ along the x-axis rightward by unity. As a result, we shall get the parabola $y = 2(x - 1)^2$ whose axis of symmetry is parallel to the y-axis and whose vertex is the point $(1, 0)$.

Proceeding in a similar way, that is shifting the parabola $y = 2(x)^2$ along the x-axis leftward by unity, we obtain the parabola $y = 2(x + 1)^2$ whose axis of symmetry is parallel to the y-axis and whose vertex is the point $(-1, 0)$.

In general, the graph of the function $y = a(x - x_0)^2$ is a parabola with the vertex $(x_0, 0)$ whose axis of symmetry is a straight line passing through the vertex parallel to the y-axis.

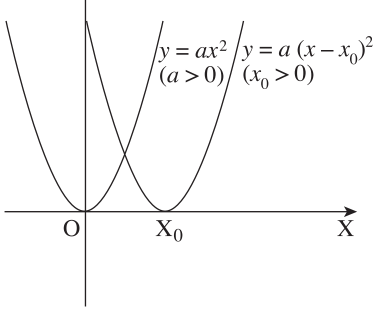


Fig. 16

This parabola can be obtained by shifting the parabola $y = ax^2$ along the x-axis by $|x_0|$ units (rightward if $x_0 > 0$ and leftward if $x_0 < 0$). **(Fig. 16)**

(3) Quadratic function $y = ax^2 + c$

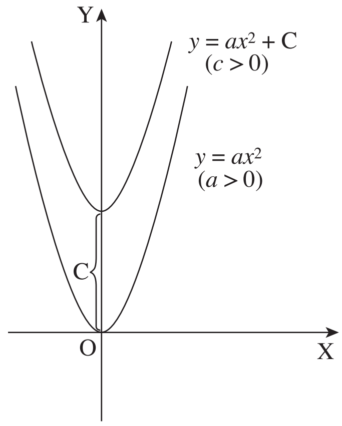


Fig. 17

The graph of the function $y = ax^2 + c$ is a parabola with the vertex $(0, c)$ and the y-axis as its axis of symmetry. This parabola can be obtained by shifting the parabola $y = ax^2$ along the y-axis by $|c|$ units (upward if $c > 0$ and downward if $c < 0$) **(Fig. 17)**

(4) General case : $y = ax^2 + bx + c$ $(a \neq 0)$.

Isolating a perfect square in the trinomial $ax^2 + bx + c$ we rewrite the function $y = ax^2 + bx + c$ as follows :

$$Y = a(x - x_0)^2 + y_0.$$

From the above considered particular cases it follows that the graph of a quadratic trinomial is a parabola with vertex at the point $C(x_0, y_0)$ whose axis is a straight line passing through its vertex parallel to the y-axis.

The branches of the parabola $y = ax^2 + bx + c$ are directed upward if $a > 0$ and downward if $a < 0$. Note that the abscissa x_0 of the parabola $y = ax^2 + bx + c$ can be found by the formula

$$X_0 = -\frac{b}{2a}$$

The ordinate of the vertex of the parabola

$$Y_0 = ax_0^2 + bx_0 + c.$$

The graph of a quadratic trinomial can be constructed with the aid of the following technique:

1. Reduce the quadratic trinomial to the form $y = a(x - x_0)^2 + y_0$ by isolating a perfect square.

2. Construct the vertex of the parabola *i.e.,* the point $C(x_0, y_0)$ and draw through it a straight line parallel to the y-axis which will be the axis of symmetry of the parabola.

3. Construct the point if intersection of the parabola and the y-axis.

4. Find the real root of the quadratic trinomial, if any, and plot the corresponding points of the parabola on the x-axis.

5. Join the constructed points to get a parabola.

Remark. It is easy to check that $X_0 = \dfrac{x_1 + x_2}{2}$,

where x_1 and x_2 are the roots of the quadratic trinomial.

Since, $y = ax^2 + bx + c = a(x - x_0)^2 + y_0$ we have:

(1) If $a > 0$, then for $x = x_0$ the quadratic trinomial takes on the least value equal to y_0 indeed, if $a > 0$, then for any x

$$a(x - x_0)^2 \geq 0.$$

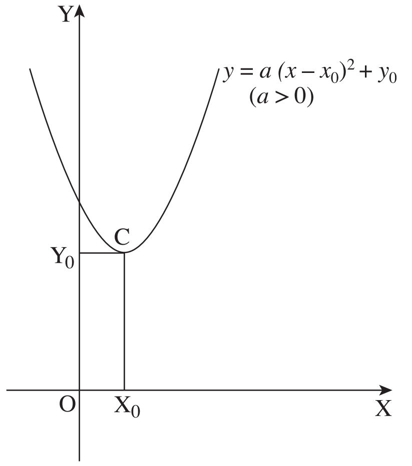


Fig. 18

Therefore, $y \geq y_0$, where $y = y_0$ only for $x = x_0$. Graphically, this means that of all points of the parabola $y = ax^2 + bx + c$ for $a > 0$ the least ordinate is possessed by the point C (x_0, y_0) *i.e.*, by the vertex of the parabola. **(Fig.18)**

(2) If $a < 0$, then for $x = x_0$ the quadratic trinomial attains the greatest value equal to v_0 Indeed, if $a < 0$, then for any x

$$a(x - x_0)^2 \leq 0.$$

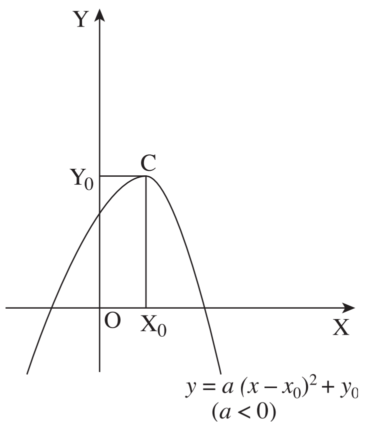


Fig. 19

Therefore, $y \leq Y_0$, where $y = y_0$ only for $x = x_0$. Graphically this means that of all points of the parabola $y = ax^2 + bx + c$ for $a < 0$ the point $C(x_0, y_0)$ i.*e.*, the vertex of the parabola has the greatest ordinate. **(Fig. 19)**

Consider Several Examples

Illustration 2.

Represent a given positive number a in the form of the sum of two addends so that their products is the greatest possible.

Solution : Let us denote one of the required addends by x. Then the other addends will be equal to $(a - x)$. Their product $x(a - x)$ is a quadratic trinomial. We now transform the trinomial by isolating a perfect square :

$$x(a - x) = -x^2 + ax \quad = -\left(x - \frac{a}{2}\right)^2 + \frac{a^2}{4}$$

Hence, it is seen that for $x = \dfrac{a}{2}$ the quadratic trinomial takes on the greatest value equal to $\dfrac{a^2}{4}$, thus each of the desired addends is equal to $\dfrac{a}{2}$.

Illustration 3.

Construct the graphs of the following functions :

(a) $y = x^2 + 2x + 3$; (b) $y = -2x^2 + 4x + 1$; (c) $y = -2(x - 1)(x + 3)$.

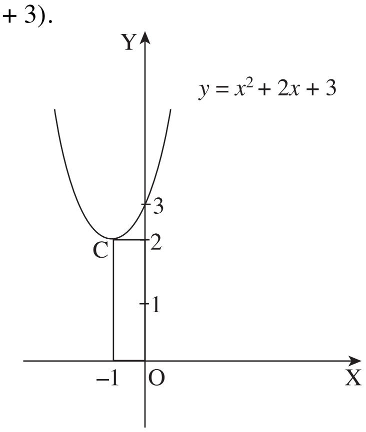


Fig. 20

Solution : (a) Separate a perfect square $y = x^2 + 2x + 3 = (x + 1)^2 + 2$. Consequently, the vertex of the parabola is C(– 1, 2); (0, 3) is the point intersection of the parabola and the y-axis; the branches of the parabola are directed upward. **(Fig. 20)**

(b) Transform of the trinomial; $y = -2x^2 + 4x + 1 = (-2x^2 + 4x - 2) + 3 = -2(x - 1)^2 + 3$. Hence, the vertex of the parabola is C(1, 3) **(Fig. 21).**

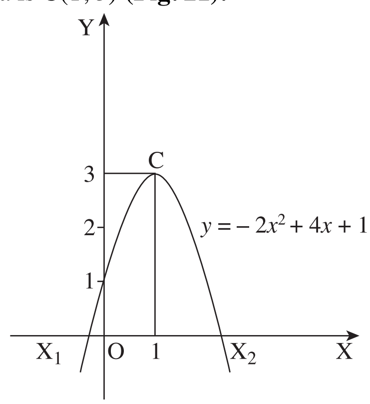


Fig. 21

The roots of the trinomial are $x_1 = 1 - \sqrt{\frac{3}{2}}$ and

$$x^2 = 1 + \sqrt{\frac{3}{2}}.$$

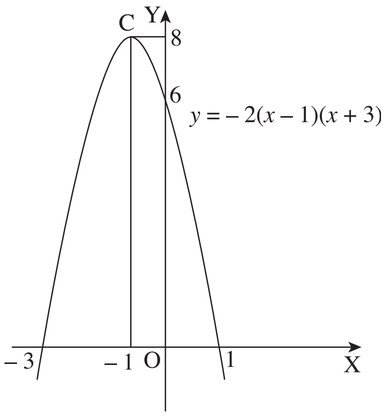


Fig. 22

(c) The roots of the trinomial $y = -2(x-1)(x+3)$ are $x_1 = 1$ and $x_2 = -3$, and, consequently, $x_0 = \frac{1-3}{2} = -1$ is the abscissa of the vertex of the parabola C. Finally, we find its ordinate y_0 : $y_0 = -2(-1-1)(-1+3) = 8$. Thus, the vertex of the parabola is C(– 1, 8) **(Fig. 22)**

Illustration 4.

Construct the graph of the following functions :

(a) $y = |x^2 - 1 :|$ (b) $y = x^2 + 2|x|$.

Solution : (a) First construct the parabola $y = x^2 - 1$. since $|x^2 - 1 :| = x^2 - 1$ for $x^2 - 1 \geq 0$ and $|x^2 - 1 :| = -(x^2 - 1)$ for $x^2 - 1 < 0$, we shall proceed as follows : we map the part parabola situated below the x-axis symmetrically about this axis. The graph of the function $y = |x^2 - 1 :|$ is depicted in **(Fig. 23).**

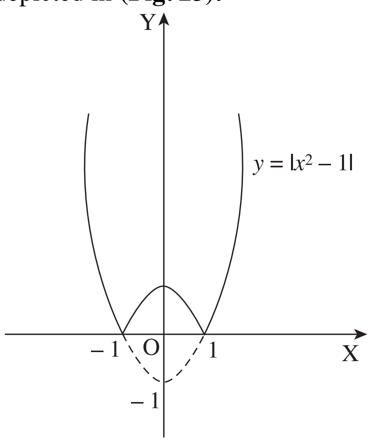


Fig. 23

(b) The given function is even :

$$(-x)^2 + 2|-x| = x^2 + 2|x|.$$

Consequently, its graph is symmetric about the y-axis. For $x \geq 0$ we obtain $y = x^2 + 2x = (x+1)^2 - 1$ which is a parabola with the vertex (– 1, – 1). Its points with the abscissa $x \geq 0$ are also points of the graph of the function $y = x^2 + 2|x|$. **(Fig. 24)**

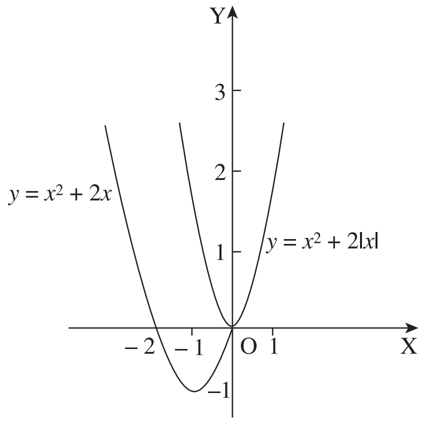


Fig. 24

4. Power Function with Integral Exponent and its Graph : A *power function with an integral exponent* is defined as a function of the form

$$Y = x^n,$$

where $n \neq 0$ is an arbitrary integer.

The function is defined for any x (except $x = 0$ for $n < 0$).

For $n = 1$, $n = 2$, and $n = -1$ we have $y = x$, $y = x^2$ and $y = x^{-1} = \frac{1}{x}$,

Respectively, their graphs are : a straight line (the bisector of the first and third quadrants), a parabola and a hyperbola, respectively.

If n is an even number, then the power function $y = x^n$ is an even function : $(-x)^n = (x^n)$ for any x. If n is an odd number, then the function $y = x^n$ is an odd function : $(-x)^n = (x^n)$. Consequently the graph of the function $y = x^n$ is symmetric about the y-axis for an even n and it is symmetric about the origin for an odd n.

If n is a positive integer, then the graph of the function $y = x^n$ is a parabola. For $n = 2$ this is simply a parabola for $n = 3$ a cubical parabola and so on. If $n > 0$, then by the property of inequalities, the condition $0 < x_1 < x_2$ implies $x_1^n < x_2^n <$ *i.e.*, the function $y = x^n$ where n is a natural number, increase on the interval $(0, +\infty)$; consequently, for an even n it decrease on the interval $(-\infty, 0)$ and for an odd n increase on the interval $(-\infty, 0)$ and, hence, throughout the x-axis. The graphs of the function $y = x^n$ for $n = 2k$ and $n = 2k + 1$ are given in Fig. 25 and 26, respectively.

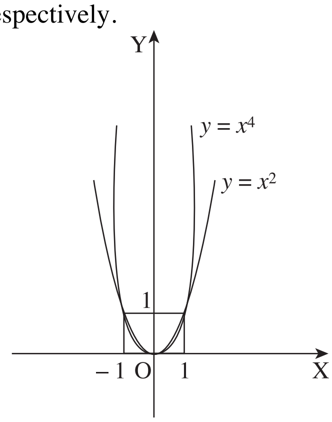


Fig. 25

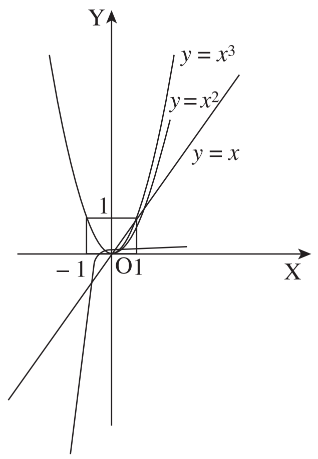


Fig. 26

In general, the graph of the power function $y = x^n$ with an integer positive exponent has for an even n the same shape as the graph of the function $y = x^2$, and for an odd n as the graph of the function $y = x^3$.

Consider now the power function $y = x^n$ with an integral negative exponent. If $n = -1$ then we have a hyperbola $y = \frac{1}{x}$ **(Fig. 27)**. If $n = -2$, then we have the function $y = \frac{1}{x^2}$ whose graph is shown is **Fig. 28** for an odd n the graph of the Function $y = x^n$ looks like the graph of the function $y = x^{-1}$ and for an even n like the graph of the function $y = x^{-2}$.

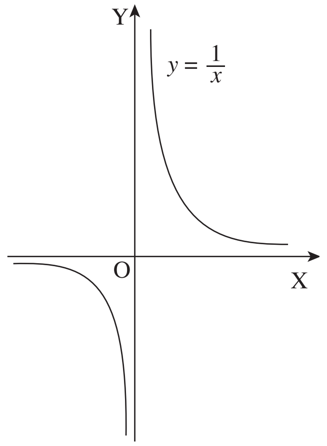


Fig. 27

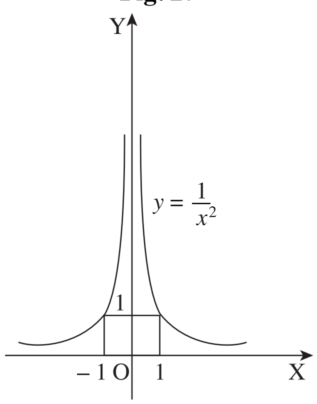


Fig. 28

5. The Graph of the Function $y = \sqrt{x}$. The Function $y = \sqrt[n]{x}$ and its Graph : We know the prosperities of the arithmetical square root. Hence, we have the following properties of the function $y = \sqrt{x}$:

(1) The function is defined for all $x \geq 0$.

(2) The value of the function is equal to zero only for $x = 0$ and is positive for any $x > 0$.

(3) The function is monotone, it increases in the entire domain of its definition. The graph of the function $y = \sqrt{x}$ is shown in **Fig 29**. It is the inverse of the function $y = x^2$ on the interval $(0, +\infty)$. Therefore, its graph is symmetric to the parabola $y = x^2$ about the bisector of the first quadrant. The inverse of the function $y = x^2$ on the interval $(-\infty, 0)$ is $y = -\sqrt{x}$.

Due to the properties of the arithmetical n^{th} root. Hence, we conclude that the function $y = \sqrt[n]{x}$, where $n \geq 2$ is a natural number, has the same properties as the function $y = \sqrt{x}$.

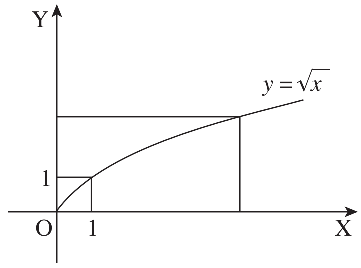


Fig. 29

The function $y = \sqrt[n]{x}$ can be understood in the following way.

The power function $y = x^n$ with an integer positive exponent $n \geq 2$ increases on the interval $(0, +\infty)$. Consequently, on this interval the function $y = x^n$ has an inverse which is also an increasing function. This function is specified by the formula $y = \sqrt[n]{x}$, where $x \geq 0$, and its graph is symmetric to the graph of the function $y = x^n$ about the bisector of the first quadrant. **(Fig. 30)**

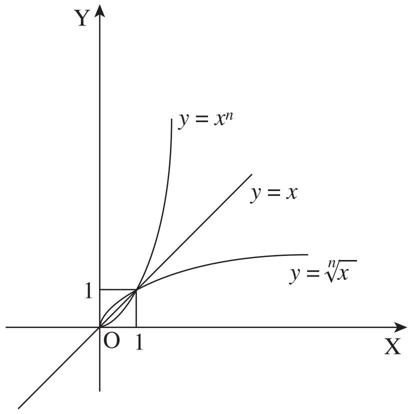


Fig. 30

6. Exponential Function its Properties and Graph : An ***exponential function*** is defined as a function of the form

$$Y = a^x,$$

where a is a given positive number not equal to unity.

The exponential function is considered only $a > 0$, since for $a < 0$ and $a = 0$ the expression a^x loses sense for some values of the variable x.

For instance, for $a = -4$ the expression $(-4)^x$ has no sense for $x = \frac{1}{2}$ (among the real numbers). If $a = 1$, then a^x $=1$ for any x. The case $a = 1$ is not considered since it is not interesting.

The properties of the exponential function are as follows :

(1) The exponential function $Y = a^x$, is defined for all real values of the argument x, that is, its domain of definition is the entire number axis $(-\infty, +\infty)$. This follows from the definition of the power of a positive real number with any real exponent.

(2) $a^0 = 1$ for any base $a \neq 0$.

(3) The exponential function $Y = a^x$ is positive in the entire domain of definition and attains all positive values. The latter means that for any $y > 0$ there exists a value of x such that $a^x = y$.

The first part of the statement follows from the properties of a power with a rational exponent (for a power with an irrational exponent the proof is omitted).

(4) For $a > 1$ we have $a^x > 1$ for $x > 0$ and $a^x < 1$ for $x < 0$; for $0 < a < 1$, *vice versa*.

Let $a > 1$. Consider the case of a rational x.

If $x = n$, where n is natural, then it is obvious that $a^n > 1$ for $a > 1$.

If $x = \frac{p}{q}$, where p and q are natural then $a^{\frac{p}{q}} = \frac{a^{p/q}}{\sqrt[q]{a^p}}$ > 1, since $a^p > 1$ for $a > 1$, thus $a^x > 1$ for $x > 0$ and > 1.

If $x = -n$, where n is natural then $a^{-n} = \frac{1}{a^n} < 1$.

If $x = -\frac{p}{q}$, where p and q are natural then $a^{\frac{p}{q}} = \frac{1}{\sqrt[q]{a^p}}$

< 1. $\sqrt[q]{a^p} > 1$ for $a > 1$. a^{x_1} a^{x_2} $a^{x_2 - x_1}$.

The case $0 < a < 1$ is proved in a similar way.

(5) The exponential function $Y = a^x$ is monotone, it increasing for $a > 1$ and decreases for $0 < a < 1$.

Let us prove that the function $y = a^x$ is increasing for any $a > 1$. We take two arbitrary values x_1 and x_2 ($x_1 < x_2$). Then $a^{x_2} - a^{x_1} = a^{x_1}(a^{x_2 - x_1} - 1)$. By the property of positiveness of an exponential function, $a^{x_1} > 0$, by property (4), $-a^{x_2 - x_1} - 1 > 0$, since $a > 1$ and $x_2 - x_1 > 0$. Consequently, $a^{x_2 - x_1} > 0$ or $a^{x_2} > a^{x_1}$ for $x_1 < x_2$, that is, the function $Y = a^x$ increase if $a > 1$.

(6) If $a < b$, then $a^x < b^x$ for $x > 0$ and $a^x > b^x$ for $x < 0$. for $x = 0$ $a^x = b^x = 1$.

Indeed, by the property of a power, $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$, where $\frac{a}{b} < 1$. Therefore, $\frac{a^x}{b^x} < 1$ for $x > 0$ and $\frac{a^x}{b^x} > 1$ for $x < 0$ (by property 4).

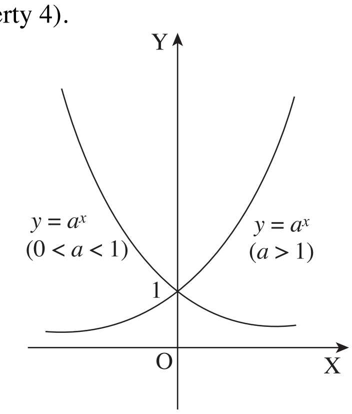


Fig. 31

The graph of the exponential function $Y = a^x$ is represented in **Fig. 31**. In the computation practice an important role is played by the exponential function $y = 10^x$. Let us show that, knowing its values for x from the interval [0, 1], it is easy to compute its values for any x.

Let there be required to find an approximate value of the expression $10^{2.36}$. We write the exponent 2.36 in the form of the sum of the integral and fractional parts $2.36 = 2 + 0.36$, then

$$10^{2.36} = 10^{2 + 0.36} = 10^2 . 10^{0.36} = 100.10^{0.36}$$

Consequently, to find an approximate value of $10^{2.36}$, it remains to find the value of $10^{0.36}$.

Suppose we have to find an approximate value of $10^{-3.24}$ to this end we write the exponent -3.24 in the form of the sum of the integral and fractional parts : $-3.24 = -4 + 0.76$, then

$$10^{-3.24} = 10^{-4 + 0.76} = 10^{-4} . 10^{0.76}.$$

It remains to find the value of $10^{0.76}$.

7. Logarithmic Function and its Graph :

Definition : The *logarithm of a number b to a base* $a(a > 0$ and $a \neq 1)$ is the exponential indicating the power to which a must be raised to obtain b.

Notation : $\log_a b$ (read : "The logarithm of the number b to the base a").

For the logarithm of a number b to the base $a = 10$ we use the symbol $\log b$ to mean $\log_{10} b$.

For instance, instead of $\log_{10} 27$ we write $\log 27$.

By the definition of logarithm, $\log_2 16$ is the exponent indicating the power to which 2 must be raised to obtain 16, that is $\log_2 16 = 4$, since $2^4 = 16$.

Similarly, $\log_3 27 = 3$, since $3^3 = 27$; $\log_2 \frac{1}{4} = -2$, since $2^{-2} = \frac{1}{4}$; $\log_5 1 = 0$,

Since, $5^0 = 1$; $\log 10 = 1$, $\log \frac{1}{100} = -2$. Note that such expression as $\log_3 (-27)$ and $6^x = 0$ have no roots.

In general the expression $\log_a b$, where $a > 0$ and $a \neq 1$, has sense only for $b > 0$.

From the definition of the logarithm it follows that

$$a^{\log_a b} = b \ (a > 0, a \neq 1)$$

For any $b > 0$. This equality is an identity on the set of positive numbers.

For instance, $10^{\log x} = x$ for $x > 0$.

The exponential function $y = a^x (a > 0$ and $a \neq 1)$ is monotone throughout the entire x-axis. Consequently, it has the inverse.

To specify by a formula the inverse of an exponential function, let us express from the formula $Y = a^x$ the variable x in terms of y :

$$X = \log_a y.$$

Passing over to the customary notation we obtain

$$Y = \log_a x.$$

A logarithmic function is a function of the from

$$Y = \log_a x. (a > 0, a \neq 1)$$

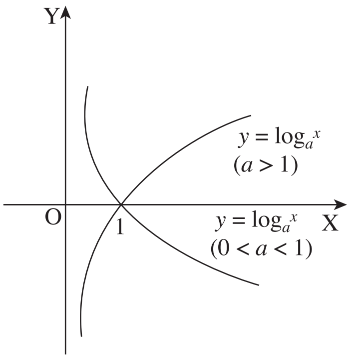


Fig. 32

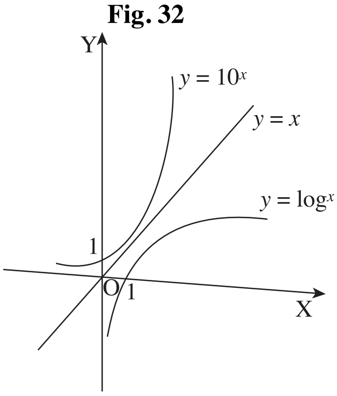


Fig. 33

This is the inverse of the exponential function. Therefore, their graphs are symmetric to about the straight line $y = x$. Knowing the graph of an exponential function, we obtain the graph of a logarithmic function. **(Fig. 32)**

In particular, the graph of the function $y = \log x$ is symmetric to the graph of the function $y = 10^x$ about the straight line $y = x$ **(Fig. 33)**. The property of the logarithmic function $y = \log x$ can be obtained from its graph. They are listed below :

(1) The function $y = \log x$ is defined for all positive numbers (therefore, all negative numbers and zero are said to have no logarithms).

(2) Log 1 = 0, that is, the graph intersects the x-axis at the point (1, 0).

(3) The function $y = \log x$ is a monotone increasing function (a greatest logarithm corresponds to a greatest number).

(4) The function $y = \log x$ attains all real value, that is, the range of values of the function is the set of all real numbers.

8. Graph of Function Containing a Modulus :

1. $y = |x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

We know that the straight line $y = x$ is the bisector of the first and third quadrants, and the straight line $y = -x$ is the bisector of the second and fourth quadrants. Hence, we obtain the graph of the function $y = |x|$ (represented in **Fig. 34**).

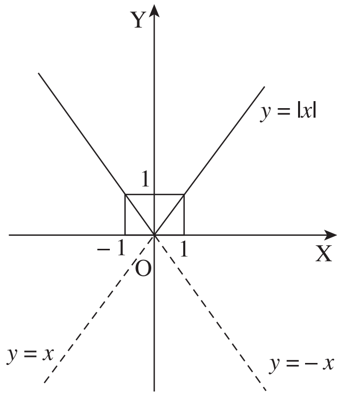


Fig. 34

2. $y = |2x + 1| = \begin{cases} 2x + 1 \text{ if } x \geq -\dfrac{1}{2} \\ -(2x + 1) \text{ if } x < -\dfrac{1}{2} \end{cases}$

Let us first construct the straight lines $y = 2x + 1$ and $y = -2x - 1$, by determining their points of intersection with the co-ordinate axes. On the straight line $y = 2x + 1$ we take only points with abscissa $x \geq -\frac{1}{2}$, and on the line $y = -2x - 1$ points with abscissa $x < -\frac{1}{2}$. Thus, we obtain the graph of the function $y = |2x + 1|$ **(Fig. 35).**

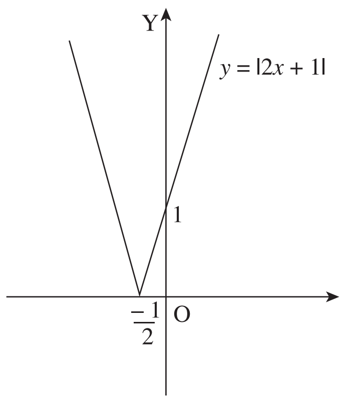


Fig. 35

3. $y = x\,|x| = \begin{cases} x^2 \text{ if } x \geq 0 \\ -x^2 \text{ if } x < 0 \end{cases}$

The function is odd; the graph is symmetric about the point O **(Fig. 36).**

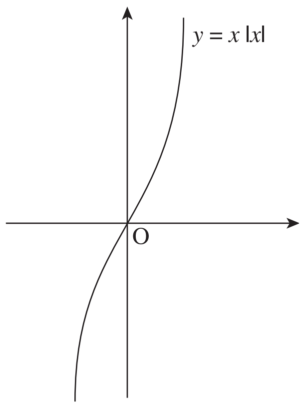


Fig. 36

4. $y = \sqrt{|x|}$. This function is defined for any x. The function is even; for $x \geq 0$; $y = \sqrt{x}$. Hence, the method of plotting the graph **(Fig. 37).**

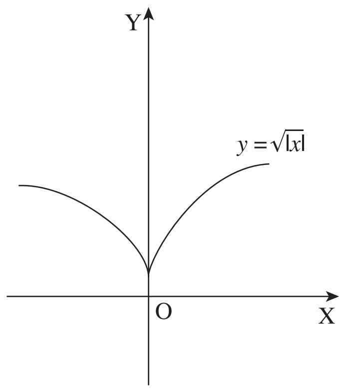


Fig. 37

5. $y = 2^{|x|}$. The function is even; for $x \geq 0$, $y = 2^x$. Its graph is shown in **(Fig. 38).**

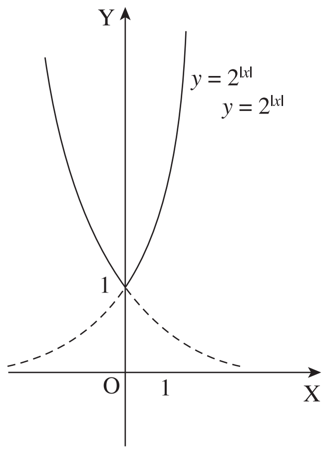


Fig. 38

6. $y = |\log x| = \begin{cases} \log x \text{ if } \log x \geq 0 \\ -\log x \text{ if } \log x < 0 \end{cases}$

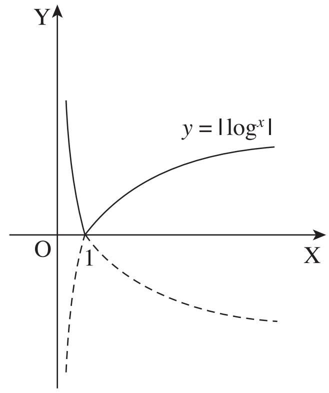


Fig. 39

Knowing the graph $y = \log x$, we obtain the graph $y = |\log x|$ **(Fig. 39).**

7. $y = |x + 1| - |x - 2|$. By the definition of the modulus,

$$|x + 1| = \begin{cases} x + 1 \text{ if } x \geq -1 \\ -(x + 1) \text{ if } x < -1 \end{cases}$$

$$|x - 2| = \begin{cases} x - 2 \text{ if } x \geq 2 \\ -(x - 2) \text{ if } x < 2 \end{cases}$$

The points $x = -1$ and $x = 2$ divided the entire number axis into three intervals : $(-\infty, -1)$, $(-1, 2)$ and $(2, +\infty)$.

Consider the function on each of the intervals.

Let $x \leq -1$, then $y = -(x + 1) + (x - 2) = -3$.

If $-1 \leq x \leq 2$, then $y = x + 1 + (x - 2) = 2x - 1$.

For $x \geq 2$, $y = x + 1 - (x - 2) = 3$.

Consequently, the given function can be written in the form

$$Y = \begin{cases} -3 \text{ if } x \leq -1 \\ 2x - 1 \text{ if } -1 \leq x \leq 2 \\ 3 \text{ if } x \geq 2 \end{cases}$$

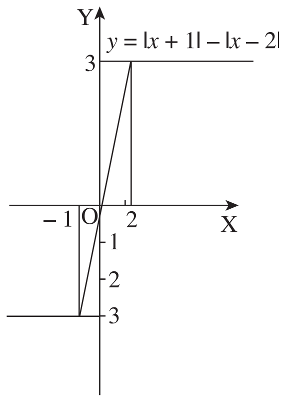


Fig. 40

Hence, it is clear that on each of the intervals under consideration the graph of the given function is a straight line **(Fig. 40).**

Graphical Method of Solving Equations and Systems of Equations. Equation of a Circle

Consider the equation in one unknown

$$f(x) = 0,$$

where $f(x)$ is a function of the variable x.

For a graphical solution of an equation it is necessary to construct the graph of the function $y = f(x)$ and find its points of intersection with the axis of abscissa. The abscissa of these points yield the values of the real roots of the equation $f(x) = 0$. In particular, the graphical method can be applied to solving the linear equation $ax + b = 0$ and quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$. In certain cases it is possible to transform the equation $f(x) = 0$ to an equivalent equation of the form $g(x) = h(x)$. In such cases we construct the graphs of the functions $y = g(x)$ and $y = h(x)$ and find the abscissa of their points of intersection.

Illustration 5.

Solve graphically the equation $x^2 + x - 2 = 0$.

Solution :

It is possible to construct the parabola $y = x^2 + x - 2$ and find the abscissa of the points at which it intersects the x-axis. But it is simpler to proceed in a different way. Let us rewrite the given quadratic equation as follows :

$$x^2 = 2 - x$$

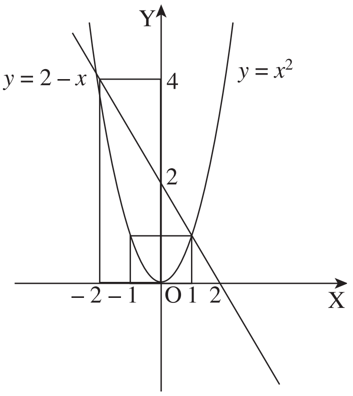


Fig. 41

We then construct the parabola $y = x^2$ and the straight line $y = 2 - x$ and find the abscissa of their points of intersection : $x = -2$ and $x = 1$ (**Fig. 41**). Hence, the given equation has the roots : $x_1 = -2$ and $x_2 = 1$.

Consider an equation in two unknowns x and y. The graph of an equation in two unknowns is the set of points of a plane whose co-ordinates reduce the given equation to a true equality.

Let, for instance there be given the equation $2x - 3y = 6$. We transform it to the form $y = \frac{2}{3}x - 2$ and construct the graph of the linear function $y = \frac{2}{3}x - 2$, which is a straight line.

Consider now as arbitrary linear equation

$$ax + by = c,$$

where a, b and c are given real numbers, where at least one of the numbers a and b is not equal to zero. Let $b \neq 0$. Then the equation can be transformed to the form

$$Y = -\frac{a}{b}x + \frac{c}{b}$$

The graph of the linear function $y = -\frac{a}{b}x + \frac{c}{b}$ is a straight line, which will just be the graph of the equation $ax + by = c$ if $b \neq 0$.

Let now $b = 0$. Then the equation takes the form $ax = c$ or $x = -\frac{c}{a}$ (if $b = 0$, then it follows that $a \neq 0$). The set of points in the plane whose co-ordinate satisfy the equation $x = \frac{c}{a}$ is a straight line parallel to the y-axis **(Fig. 42).**

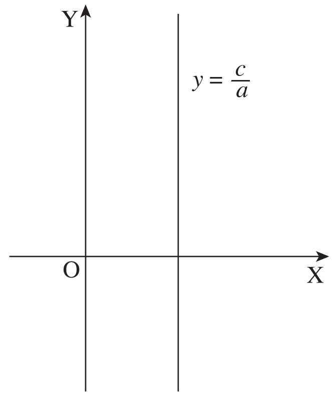


Fig. 42

Thus, the graph of any linear equation $ax + by = c$ is a straight line.

Let us construct the graph of the equation $xy = -1$. We transform the equation to the form $y = -\frac{1}{x}$ and construct the graph of the function $y = -\frac{1}{x}$ **(Fig. 43).**

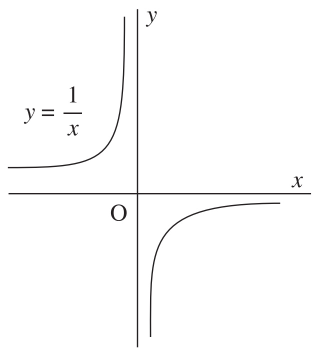


Fig. 43

The constructed hyperbola is thus the graph of the equation $xy = -1$. In the above considered example we were seeking for the graph of an equation to obtain in the plane the corresponding line- a straight line or hyperbola.

There arises an inverse problem : for a line in the plane set-up the equation whose graph is this line.

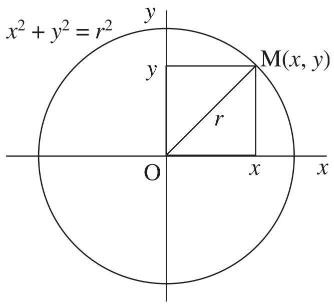


Fig. 44

Consider the circle of radius r with centre at the origin **(Fig. 44)**. Let us write the equation whose graph is represented by this circle.

Let $M(x, y)$ be an arbitrary points of the given circle. Its distance from the centre of the circle O is equal to the radius of the circle: $|OM| = r$.

On the other hand, by the formula for the distance of a point in the plane from the origin, we have

$$|OM| = \sqrt{x^2 + y^2}$$

Therefore,

$$\sqrt{x^2 + y^2} = r \text{ or } x^2 + y^2 = r^2$$

Thus, the co-ordinate of any point of a circle satisfy the equation $x^2 + y^2 = r^2$. If a point does not belong to the given circle, then its co-ordinates do not satisfy the equation $x^2 + y^2 = r^2$.

Indeed, then $|OM| \neq r$, $\sqrt{x^2 + y^2} \neq r$, and $x^2 + y^2 \neq r^2$.

Conclusion : A circle with centre at the origin whose radius is equal to r is the graph of the equation $x^2 + y^2 = r^2$, where x and y are variables and r is a given positive number.

To solve graphically a system of two equations in two unknowns we have to construct (in one and same co-ordinate system) the graphs of the given equations and find the co-ordinate of the points of intersection of these graphs.

Illustration 6.

Solve graphically the system of equation

$$\begin{cases} x^2 + y^2 = 25 \\ x + y = 5 \end{cases}$$

Solution :

Construct (in one and the same co-ordinate system) the graph of the equations $x^2 + y^2 = 25$ and $x + y = 5$ **(Fig. 45).**

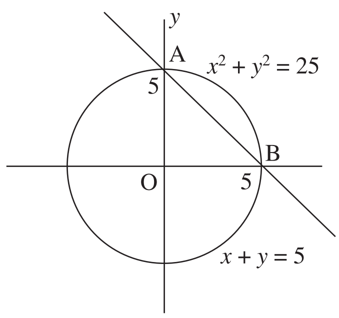


Fig. 45

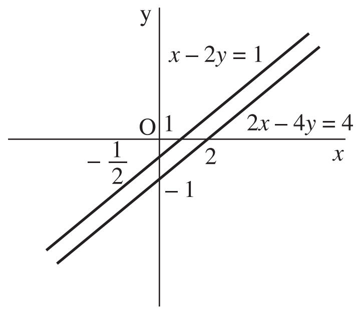


Fig. 46

The graph of the equation $x^2 + y^2 = 25$ is a circle of radius 5 centered at the origin, and the graph of the equation $x + y = 5$ is a straight line. The circle and the straight line intersect at the points A(0, 5) and B (5, 0). Consequently, the given system has two solutions: (0, 5) and (5, 0).

Consider the system of two linear equations in two unknowns x and y :

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

where a_1, b_1, c_1, a_2, b_2 and c_2 are given real numbers and also at least one of the numbers a_1 and b_1 and at least one of the numbers a_2 and b_2 are not equal to zero to solve this system graphically, we have to construct two straight lines which are the graphs of the equations entering into the system. The solution of the given system depends on the mutual position of two straight lines in the plane. The following three cases are possible here :

(1) The lines intersect; in the case, the system will unique solution.

(2) The lines are distinct and parallel; in this case, the system will have no solution;

(3) The lines coincide; in this case, the system will have an infinite set of solutions, since the co-ordinate of any point on the coincident lines are a solution of the system.

Such is the geometrical interpretation of the solution of a system of two linear equations in two unknowns.

Illustration 7.

Solve graphically the following system of equations :

$$\begin{cases} x - 2y = 1 \\ 2x - 4y = 4 \end{cases}$$

Solution :

The straight lines $x - 2y = 1$ and $2x - 4y = 4$ are parallel **(Fig. 46)** and hence, the system has no solution.

Constructing Graph (Solving Problems)

Consider the below given examples on constructing the graphs of functions and equations.

Illustration 8.

Construct the graph of the function

$$Y = \begin{cases} x, & x \le 0 \\ x^2, & x > 0 \end{cases}$$

Solution :

The graph of this function given by different formulas on different intervals of variation of the argument consists of the bisector of the third quadrant and a branch of a parabola **(Fig. 47).**

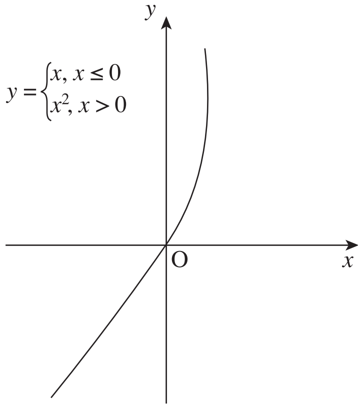


Fig. 47

Illustration 9.

Construct the graph of the function $y = \frac{|x|}{x}$.

Solution :

The given function is defined for any $x \neq 0$. Here,

$$Y = \begin{cases} 1 \text{ if } x > 0. \\ -1 \text{ if } x < 0. \end{cases}$$

The desired graph is represented in **Fig. 48**.

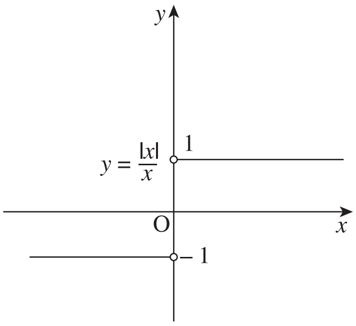


Fig. 48

Illustration 10.

Construct the graph of the function

$$y = -\sqrt{x^2 - 4x + 4}.$$

Solution :

Since, $x^2 - 4x + 4 = (x-2)^2$, $y = -|x-2|$, we have

$$|x - 2| = \begin{cases} x - 2 \text{ if } x \ge 2. \\ -2\,(x-2) \text{ if } x < 2. \end{cases}$$

Consequently $\quad Y = \sqrt{x^2 - 4x + 4}.$

$$= (-(x-2) \text{ if } x < 2)$$

$$(x - 2 \text{ if } x \ge 2).$$

Construct the straight lines $y = -(x-2)$ and $y = x - 2$. Taking the points with the abscissa $x \ge 2$, on the first of them and with the abscissa $x < 2$ on the second, we obtain the graph of the given function **(Fig. 49).**

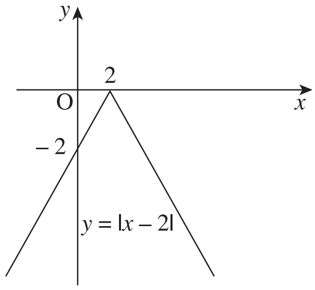


Fig. 49

Note that the graph of $y = -|x-2|$ can be obtained by shifting the graph of $y = |x|$ by 2 along the x-axis rightward and then mapping it symmetrically about the x-axis.

Illustration 11.

Construct the graph of the equation $|y| = |x|$.

Solution :

The given equation decomposes into two equalities : $y = x$ and $y = -x$, since if two numbers are equal modulus, then the numbers are either equal or differ only in sign. The graph of the equation $|y| = |x|$ consists of the bisectors of the quadrants **(Fig. 50).**

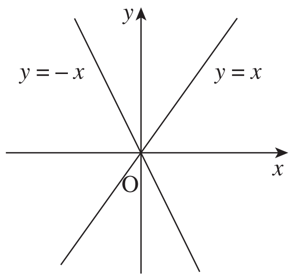


Fig. 50

Illustration 12.

Construct the graph of the equation $|x| + |y| = 1$.

Solution :

Since, $|-x| = |x|$, then if (x, y) is a point of the graph, the point $(-x, y)$ will also be a point belonging to the graph. Hence, the graph is symmetric about the y-axis. The given equation contains y only under the modulus sign and, consequently, along with the point (x, y) of the graph, the point $(x, -y)$ will also belong to the graph, that is, the graph is also symmetric about the x-axis.

Let $x \ge 0$ and $y \ge 0$. Then for the points of the first quadrant the equation takes the form $x + y = 1$. Construct the straight line $x + y = 1$ and take on it only the points situated in the first quadrant. Then map the obtained line segment symmetrically about the co-ordinate axes. The graph of the equation $|x| + |y| = 1$ is the contour of a square **(Fig. 51).**

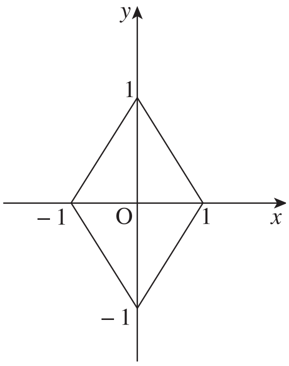


Fig. 51

Illustration 13.

Construct the graph of the equation $|x| + |y| = 0$.

Solution :

Since, $|x| \geq 0$ and $|y| \geq 0$, the given equation is satisfied only by the numbers $x = 0$ and $y = 0$. The graph consists of only one point- the origin.

Illustration 14.

Construct the graph of the function $y = \frac{x-1}{x}$.

Solution :

We have $y = 1 - \frac{1}{x}$. Therefore, the graph of the given function can be obtained by shifting the hyperbola $y = -\frac{1}{x}$ along the y-axis by unity upward **(Fig. 52).**

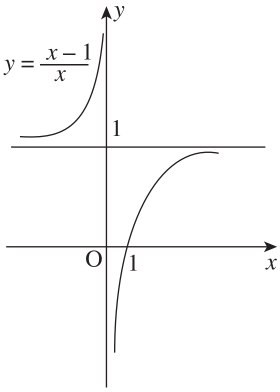


Fig. 52

Illustration 15.

Construct the graph of the function $y = x^3 - x$.

Solution :

The given function is odd :

$$(-x)^3 - (-x) = -(x^3 - x).$$

Consequently, the graph is symmetric about the origin.

Let $x \geq 0$. Since, $y = x^3 - x = x(x + 1)(x - 1)$, we have for $0 \leq x \leq 1$, $y \leq 0$; for $x \geq 1$, $y \geq 0$ and at the point $x = 0$, $x = 1$, and $x = -1$ the graph will intersect the x-axis. Taking into account the oddness of given function and intervals of constant sign, we construct the graph of the function **(Fig. 53).**

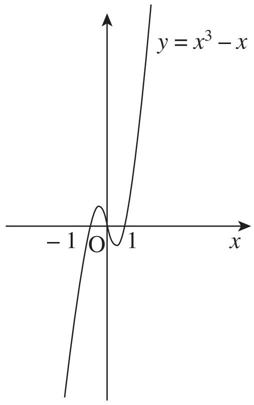


Fig. 53

Illustration 16.

Construct the graph of the function $y = \sqrt{1 - x^2}$.

Solution :

Squaring both sides, we obtain

$$Y^2 = 1 - x^2, \text{ or } x^2 + y^2 = 1.$$

Since, $y = \sqrt{1 - x^2} \geq 0$, for constructing the graph of the equation $x^2 + y^2 = 1$ it is necessary to leave only the points with the ordinate $y \geq 0$. The graph of equation $x^2 + y^2 = 1$ is a circle of radius 1 centered at the origin. Consequently, the graph of the function $y = \sqrt{1 - x^2}$ is the upper semi-circle **(Fig. 54).**

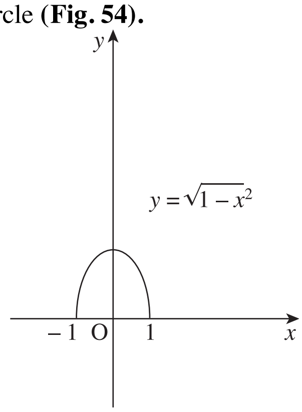


Fig. 54

Illustration 17.

Construct the graph of the function $y = |x^2 + 2|x| - 3|$.

Solution :

The given function is even; its graph is symmetric about the y-axis. For $x \geq 0$, we have $y = |x^2 + 2x - 3|$.

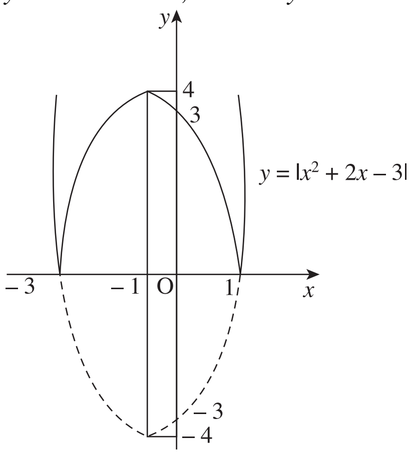


Fig. 55

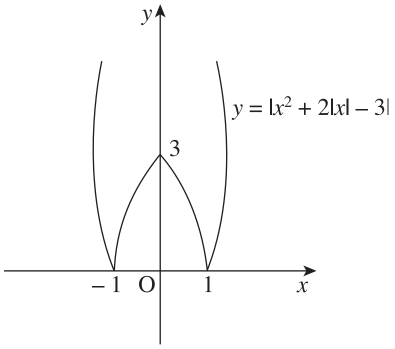


Fig. 56

Hence, the method of constructing the graph of the given function : first construct the parabola $y = x^2 + 2x - 3$, then the graph of the function $y = |x^2 + 2x - 3|$ **(Fig. 55)** and finally, the graph of the function $y = |x^2 + 2|x| - 3|$ **(Fig. 56).** The graph $y = x^2 + 2x - 3 = (x + 1)^2 - 4$ is a parabola with the vertex $(-1, -4)$ and branches directed upward, the ordinate of the point of intersection of the parabola and the y-axis being (-3). Solving the quadratic equation $x^2 + 2x - 3 = 0$, we find its roots : $x_1 = 1$ and $x_2 = -3$, which are the abscissa of the points at which the parabola intersects the x-axis.

Illustration 18.

Construct the graph of the function

$$y = \left| \left(\frac{1}{2}\right)^x - 1 \right| .$$

Solution :

Construct the graph of the exponential function $y = \left(\frac{1}{2}\right)^x$ with the base less than 1; shift the obtained graph along the y-axis by 1 downward, and, finally, taking into consideration the modulus, we have the graph of the given function **(Fig. 57).**

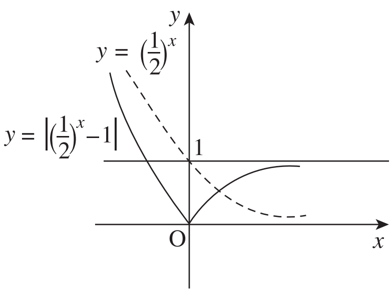


Fig. 57

Illustration 19.

Construct the graph $y = \log(x - 1)$.

Solution :

The desired graph can be obtained by displacing the known graph $y = \log x$ along the x-axis by 1 rightwards **(Fig. 58).**

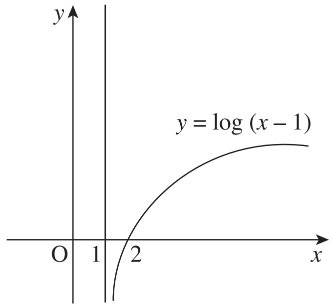


Fig. 58

Illustration 20.

Construct the graph of the function $y = 10^{-\log x}$.

Solution :

We use the basic logarithmic identity

$$10^{\log x} = x \text{ if } x > 0.$$

Then, $10^{-\log x} = (10^{\log x})^{-1} = x^{-1} = \frac{1}{x}$, where $x > 0$.

Therefore, the graph of the function $y = 10^{-\log x}$ is the branch of the hyperbola $y = \frac{1}{x}$ situated in the first quadrant **(Fig. 59).**

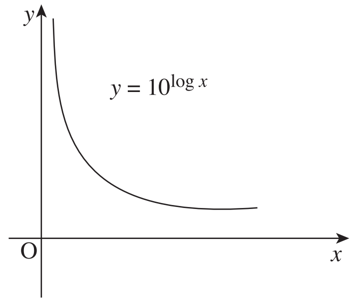


Fig. 59

Illustration 21.

Construct the graph of the equation $|y| = \log x$.

Solution :

Note that :

(1) Since, $|y| \geq 0$, $\log x \geq 0$, *i.e.,* $x \geq 1$.

(2) If (x, y) is a point belonging to the graph, then $(x, -y)$ will also be its point, that is, the graph is symmetric about the x-axis. Therefore, using the graph $y = \log x$, we obtain the graph of the equation $|y| = \log x$. **(Fig. 60).**

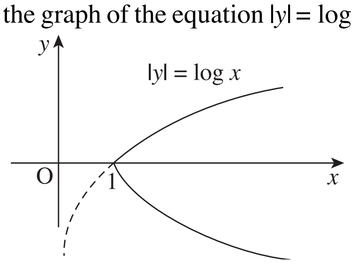


Fig. 60

Application of Graph to Solving Inequalities

The knowledge of how to construct a parabola (the graph of a quadratic trinomial) can be used for the graphical method solving quadratic inequalities.

Illustration 22.

Solve graphically the inequalities $-3x^2 - 5x + 2 > 0$.

Solution :

The graph of the trinomial $y = -3x^2 - 5x + 2$ is a parabola whose branches are directed downward. We find the roots of the trinomial : $x_1 = -2$ and $x_2 = \frac{1}{3}$. Therefore, the parabola intersects the x-axis at these point **(Fig. 61)**.

The inequalities $-3x^2 - 5x + 2 > 0$ is satisfied by those values of x for which the points of the parabola lie above the x-axis, that is, numbers x are such that $-2 < x < \frac{1}{3}$.

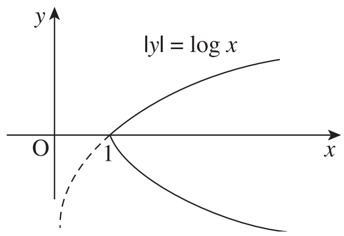


Fig. 61

A system of inequalities in one unknown can also be solved graphically.

Illustration 23.

Solve graphically the system of inequalities

$$\begin{cases} x - 1 > 0, \\ 3 - x > 0. \end{cases}$$

Solution :

Let us construct the graphs of the functions $y = x - 1$ and $y = 3 - x$ in one and the same co-ordinate system **(Fig. 62)**. Both graphs lie above the x-axis for the value of x from the interval $(1, 3)$.

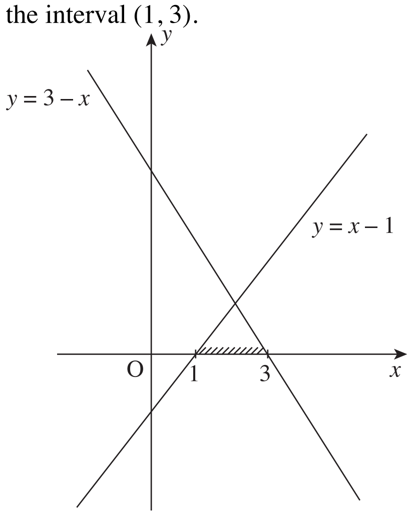


Fig. 62

We are now going to show how graphs are applied to solving inequalities and system of inequalities unknowns.

Illustration 24.

Solve graphically the inequalities $x + 2y - 1 > 0$.

Solution :

To solve graphically the inequalities $x + 2y - 1 > 0$ or $y > -\frac{1}{2} - \frac{1}{2}x$, first construct the graph of the linear function $y = \frac{1}{2} - \frac{1}{2}x$. The set of solution of the inequality $x + 2y - 1 > 0$ consists of the points in the plane lying above the straight line $y = \frac{1}{2} - \frac{1}{2}x$. **(Fig. 63).**

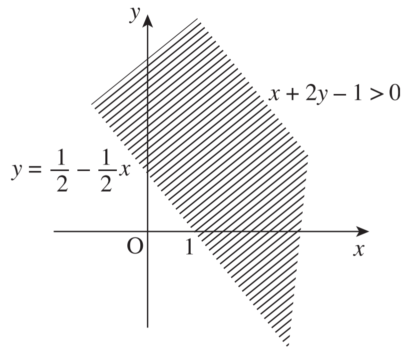


Fig. 63

Illustration 26.

Represent the set of points specified by the system of inequalities

$$\begin{cases} x^2 + y \leq 1, \\ y - x \geq -1 \end{cases}$$

Solution :

We have the inequalities $y \leq 1 - x^2$ and $y \geq x - 1$.

Let us construct the parabola $y = 1 - x^2$ and the straight line $y = x - 1$. The set given by the system of inequalities consist of the points lying on the parabola $y = 1 - x^2$ or below it and, simultaneously, on the straight line $y = x - 1$ or above it **(Fig. 64).**

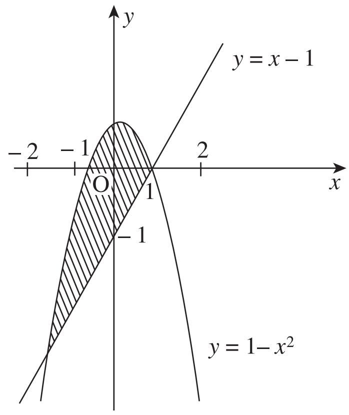


Fig. 64

Illustration 27.

Represented the set of points in the plane defined by the system of inequalities

$$\begin{cases} x + y < 1, \\ 2x - y < 2 \end{cases}$$

Solution :

Since, $x + y < 1$, we get $y < 1 - x$; since $2x - y < 2$, we obtain $y > 2x - 2$. The set specified by the given system of inequalities consist of the points lying below the straight line $y = 1 - x$ and, simultaneously, above the straight line $y = 2x - 2$ **(Fig. 65).**

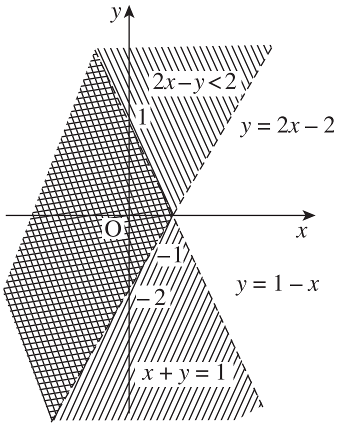


Fig. 65

i.e., the set of solution of each of these linear inequalities is a half-plane. The set defined by the system of these inequalities is the intersection of the two half-planes.

Exercise – A

Q. 1. Find the domain of definition (or, simply, domain) of the function $y = \log_x \cos x$.

Q. 2. Find the domain of the function

$$Y = \frac{\cot x}{\sqrt{\sin x - \cos x}} \qquad \ldots(1)$$

Q. 3. Find the domain of the function

$$Y = \sqrt{\cos(\cos x)} + \arcsin \frac{1 + x^2}{2x} \qquad \ldots(3)$$

Q. 4. Construct the graph of the function $y = 2 - 1/x$.

Q. 5. Construct the graph of the function $y = \frac{3}{x + 4}$.

Q. 6. Construct the graph of the function $y = \frac{-x + 5}{3x - 2}$.

Q. 7. Draw the graph of the function $y = \log_4 (-x)$.

Q. 8. Construct in a single drawing the graphs of the functions

$$y_1 = \sin x, Y_2 = \sin 2x, Y_3 = -2 \sin x$$

Q. 9. Construct the graph of the function

$$y = \sin [2x - (\pi/3n)].$$

Q. 10. Construct the graph of the function

$$y = \sin^2 x.$$

Q. 11. Draw the graph of the function

$$Y = \frac{1}{x \log_{10} x} \qquad \ldots(4)$$

Q. 12. Sketch the graph of the function

$$Y = \log_{1/2} \left(x - \frac{1}{2}\right) + \log_2 \sqrt{4x^2 - 4x + 1} \qquad \ldots(5)$$

Q. 13. Construct the graph of the function $y = |2 - 2^x|$.

Q. 14. Construct the graph of the function

$$y = ||x + 1| - 2|.$$

Q. 15. Construct the graph of the function

$$y = x^2 - 2|x| - 3.$$

Q. 16. Construct the graph of the function

$$y = (|x + 1| + 1)(x - 3) \qquad \ldots(6)$$

Q. 17. Construct the graph of the function

$$Y = \frac{|x - 3| + |x + 1|}{|x + 3| + |x - 1|} \qquad \ldots(7)$$

Q. 18. Construct the graph of the function

$$y\ |\sin x| + |\cos x|.$$

Q. 19. Graph the function

$$Y = \frac{\sin x}{\sqrt{1 + \tan^2 x}} + \frac{\cos x}{\sqrt{1 + \cot^2 x}} \qquad \ldots(8)$$

Q. 20. Graph the function $y = \frac{x^2 + 1}{x}$.

Q. 21. Construct the graph of the function $y = x \sin x$.

Q. 22. Sketch the graph of the function $y = 2^{1/x}$.

Q. 23. Construct the graph of the function

$$y = 1 - 2^{1 + \sin(x + 1)}$$

Q. 24. Construct the graph of the function

$$y = \log_2 (1 - x^2).$$

Q. 25. Construct the graph of the function

$$y = \log_{\sin x} 1/2.$$

Q. 26. Construct the graph of the function $y = \sin x^2$.

Q. 27. Find a set of points, in the plane, whose co-ordinate x and y satisfy the system of inequalities

$$5x + 3y \geq 0$$
$$y - 2x > 2 \qquad \ldots(11)$$

Q. 28. Determine the set of prints, in a plane, whose co-ordinate x and y satisfy the relation

$$|x + y| = |y| - x \qquad \ldots(12)$$

Q. 29. A system of Cartesian co-ordinates is given in a plane. Represent the region of this plane filled with all the points whose co-ordinates satisfy the inequality

$$\text{Log}_x \log_y x > 0 \qquad \ldots(13)$$

Q. 30. Find all the points in the plane, whose co-ordinate x and y satisfy the inequality

$$\cos x - \cos y > 0$$

Exercise – B

Directions (Q. 1–2) : Read the information given below and answer the questions that follows—

If $md(x) = |x|$, $mn(x, y)$ = minimum of x and y and Ma $(a, b, c, \ldots)$ = minimum of $a, b, c \ldots$

1. Value of Ma $[md(a), mn(md(b), a), mn(ab, md(ac))]$, where $a = -2, b = -3, c = 4$ is—
   (A) 2 (B) 6
   (C) 8 (D) –2
2. Give that $a > b$ then the relation Ma $[md(a). mn(a, b)] = mn[a, md(\text{Ma}(a, b))]$ does not hold if—
   (A) $a < 0, b < 0$
   (b) $a > 0, b > 0$
   (C) $a > 0, b < 0, |a| < |b|$
   (D) $a > 0, b < 0\ |a| > |b|$

Directions (Q. 3–6) : Read the information given below and answer the questions that follows—

If $f(x) = 2x + 3$ and $g(x) = \frac{x-3}{2}$, then

3. fog (x) =
   (A) 1 (B) go $f(x)$
   (C) $\frac{15x+9}{16x-5}$ (D) $\frac{1}{x}$
4. For what value of x; $f(x) = g(x-3)$?
   (A) -3 (B) 1/4
   (C) -4 (D) None of these
5. What is value of (gofofogogof) (x) (fogofog) (x) the ?
   (A) x (B) x^2
   (C) $\frac{5x+3}{4x-1}$ (D) $\frac{(x+3)(5x+3)}{(4x-5)(4x-1)}$
6. What is the value of fo (fog) 0 (gof) (x) ?
   (A) x (B) x^2
   (C) $2x+3$ (D) $\frac{x+3}{4x-5}$

Directions (Q. 7–10) : Read the information given below and answer the questions that follows—

Le (x, y) = least of (x, y), mo $(x) = |x|$, me (x, y) = maximum of (x, y)

7. Find the value of me $(a + mo$ $(le$ $(a, b))$; mo $(a + me$ $(mo$ (a) mo $(b))))$, at $a = -2$ and $b = -3$—
   (A) 1 (B) 0
   (C) 5 (D) 3
8. Which of the following must always be correct for a, $b > 0$?
   (A) mo $(le$ $(a, b)) \geq (me$ $(a), mo$ $(b))$
   (B) mo $(le$ $(a, b)) > (me$ $(mo$ $(a), mo$ $(b))$
   (C) mo $(le$ $(a, b)) < (le$ $(mo$ $(a)), mo$ $(b))$
   (D) mo $(le$ $(a, b)) = le$ $(mo$ $(a), mo$ $(b))$
9. For what value of a is me $(a^2 - 3a, a - 3) > 0$?
   (A) $1 < a < 3$ (B) $0 < a < 3$
   (C) $a < 0$ and $a < 3$ (D) $a < 0$ or $a < 3$
10. For what values of a le $(a^2 - 3a, a - 3) > 0$?
    (A) $1 < a < 3$ (B) $0 < a < 3$
    (C) $a < 0$ and $a < 3$ (D) $a < 0$ or $a < 3$

Directions (Q. 11) : Answer the questions independently.

11. Largest value of min $(2 + x^2, 6 - 3x)$, when $x > 0$ is—
    (A) 1 (B) 2
    (C) 3 (D) 4

Directions (Q. 12–13) : Read the information given below and answer the question that follows—

A, S, M and D are functions of x and y and they are defined as follows :

$$A(x, y) = x + y$$
$$S(x, y) = x - y$$
$$M(x, y) = xy$$
$$D(x, y) = x/y \quad \text{where } y \neq 0.$$

12. What is the value of M (M (A(M (x, y), S(y, x)) A(y, x)) for $x = 2, y = 3$?
    (A) 50 (B) 140
    (C) 25 (D) 70
13. What is the value of S [M (D (A (a, b), 2)), D (A (a, b), 2), M (D(S (a, b), 2), D (S (a, b), 2))] ?
    (A) $a^2 + b^2$ (B) ab
    (C) $a^2 - b^2$ (D) a/b

Directions (Q. 14–16) : Read the information given below and answer the question that follows—

The following functions have been defined

$$la(x, y, z) = min(x + y, y + z)$$
$$le(x, y, z) = max(x - y, y - z)$$
$$ma(x, y, z) = (1/2)[le(x, y, z) + la(x, y, z)]$$

14. Given that $x > y > z > 0$, which of the following is necessarily true?
    (A) la $(x, y, z) < le$ (x, y, z)
    (B) ma $(x, y, z) < la$ (x, y, z)
    (C) ma $(x, y, z) < le$ (x, y, z)
    (D) None of these
15. What is the value of ma (10, 4, le (la (10, 5, 3), 5, 3)) ?
    (A) 7.0 (B) 6.5
    (C) 8.0 (D) 7.5
16. For $x = 15$, $y = 10$ and $z = 9$, find the value of : le $(x$, min $(y, x - z)$, le (9, 8, ma (x, y, z))—
    (A) 5 (B) 12
    (C) 9 (D) 4

Directions (Q. 17–19) : Read the information given below and answer the questions that follows—

The following operations are defined for real number $a \# b = a + b$ if a and b both are positive else $a \# b = 1$. $a \nabla b = (a + b)^{a+b}$ if ab is positive else $a \nabla b = 1$.

17. $(2 \# 1)/(1 \nabla 2)$—
    (A) 1/8 (B) 1
    (C) 3/8 (D) 3
18. $\{((1 \# 1) \# 2) - (10^{1.3} \nabla \log_{10} 0.1)\}/(1 \nabla 2)$—
    (A) 3/8 (B) $4 \log_{10} 0.1/8$
    (C) $(4 + 10^{1.3})/8$ (D) None of these
19. $((x \# -y)/(-x \nabla Y)) = 3/8$, then which of the following must be true?
    (A) $x = 2, y = 1$ (B) $x > 0, y < 0$
    (C) x, y both positive (D) x, y both negative

Directions (Q. 20–22) : Read the information given below and answer the questions that follows—

If x and y are real numbers, the functions are defined as $f(x, y) = |x, y|$, F $(x, y) = -f(x, y)$ and G $(x, y) = -$ F $(x,$

y). Now, with the help of this information answer the following questions—

20. Which of the following will be necessarily true ?
    (A) G(f(x, y), F(x, y)) > F (f(x, y), G(x, y))
    (B) F (F(x, y), F(x, y)) = F (G(x, y), G(x, y))
    (C) F(G(x, y), (x + y) ≠ G(F(x, y), (x – y)))
    (D) f(f(x, y), F(x – y)) = G (F(x, y), f(x – y))

21. If y = which of the following will give x^2 as the final value ?
    (A) f(x, y) G(x, y)4
    (B) G(f(x, y)f(x, y))F (x, y)/8
    (C) – F(x, y)G(x, y)/$\log_2 16$
    (D) – f(x, y) G(x, y) F(x, y)/F(3x, 3y)

22. What will be the final value given by the function G(f(G(F(f(2, – 3),0) – 2).0)) ?
    (A) 2 (B) – 2
    (C) 1 (D) – 1

Directions (Q. 23–26) : Read the information given below and answer the questions that follows—

Any function has been defined for a variable x, where range of $x \in (-2, 2)$.

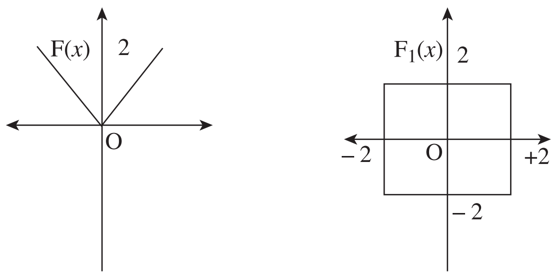
Mark (a) if $F_1(x) = -F(x)$

Mark (b) if $F_1(x) = F(-x)$

Mark (c) if $F_1(x) = -F(-x)$

Otherwise mark (d).

23.



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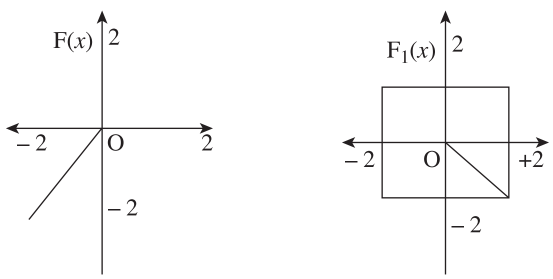

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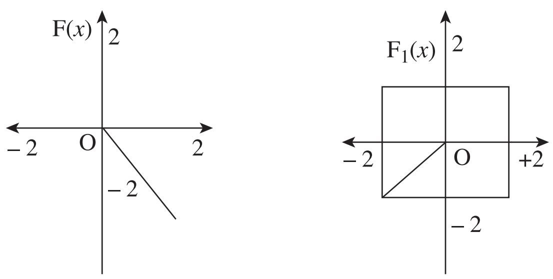

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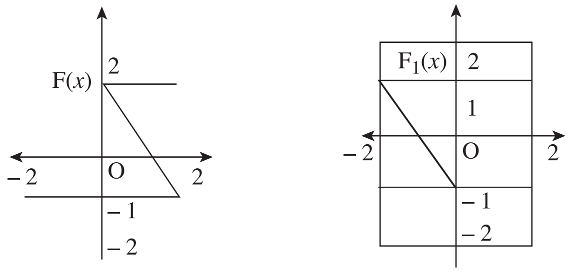


Directions (Q. 27–28) : Read the information given below and answer the questions that follows—

Certain relation is defined among variable A and B
Using the relation answer the questions given below
@ (A, B) = average of A and B
∴ (A, B) = product of A and B,
X (A, B) = the result when A is divided by B

27. The sum of A and B is given by—
    (A) /(@ (A, B), 2) (B) @(/(A, B), 2)
    (C) @(X(A, B), 2) (D) None of these

28. The average of A, B and C is given by—
    (A) @(× (/(@(A, B), 2), C), 3)
    (B) /(x (/(@(A, B)), C2))
    (C) X(@(/(@A, B), 2), C, 3))
    (D) X(/(@(/(@(A, B), 2), C), 2), 3)

Directions (Q. 29–31) : Read the information given below and answer the questions that follows—

x and y are non-zero real numbers
$f(x, y) = +(x + y)^{0.5}$, if $(x + y)^{0.5}$ is real otherwise $= (x + y)^2$
$g(x, y) = (x + y)^2$ if $(x + y)^{0.5}$ is real, otherwise $= -(x + y)$

29. For which of the following is F (x, y) necessarily greater than g (x, y) ?
    (A) x and y are positive
    (B) x and y are negative
    (C) x and y are greater than–1
    (D) None of these

30. Which of the following is necessarily false ?
    (A) $f(x, y) \geq g(x, y)$ for $0 \leq x, y < 0.5$
    (B) $f(x, y) > g(x, y)$ when $x, y < -1$
    (C) $f(x, y) > g(x, y)$ for $x, y > 1$
    (D) None of these

31. If $f(x, y) = g(x, y)$ then—
    (A) $x = y$ (B) $x + y = 1$
    (C) $x + y = -2$ (D) Both (B) and (C)

Directions (Q. 32–33) : Answer the questions independent of each other—

32. Which of the following question will be best fit for above data ?

X	1	2	3	4	5	6
Y	4	8	14	22	32	44

    (A) $y = ax + b$ (B) $y = a + bx + cx^2$
    (C) $y = e^{ax + b}$ (D) None of these

33. If $f(0, y) = y + 1$, and $f(x + 1, y) = f(x, f(x, y))$. Then what is the value of $f(1, 2)$?

(A) 1 (B) 2

(C) 3 (D) 4

Directions (Q. 34–36) : Read the information given below and answer the questions that follows—

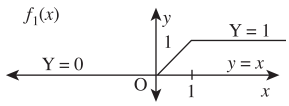
Graph of some functions are given mark the options.

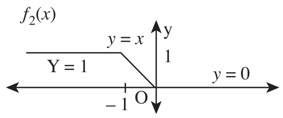
(A) If $f(x) = 3f(-x)$

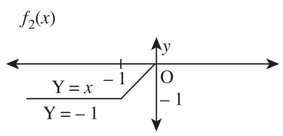
(B) If $f(x) = f(-x)$

(C) If $f(x) = -f(-x)$

(D) If $3f(x) = 6f(-x)$ for $x > 0$

34. 

35. 

36. 

Directions (Q. 37–39) : Read the information given below and answers the questions that follows—

Follows m and M are defined as follows :

$$m\,(a, b, c) = min\,(a + b, c, a)$$

$$M\,(a, b, c) = max\,(a + b, c, a)$$

37. If $a = -2$, $b = -3$ and $c = 2$ what is the maximum between $[m(a, b, c) + M\,(a, b, c)]/2$ and $[m\,(a, b, c) - M\,(a, b, c)]/2$?

(A) 3/2 (B) 7/2

(C) – 3/2 (D) – 7/2

38. If a and b, c are negative, than what gives the minimum of a and b?

(A) $m(a, b, c)$ (B) $-M\,(-a, a, -b)$

(C) $m\,(a + b, bc)$ (D) None of these

39. What is $m\,(M\,(a - b, b, c))\; m\,(a + b, c, b), - M(a, b, c)$ for $a = 2$, $b = 4$, $c = 3$?

(A) – 4 (B) 0

(C) – 6 (D) 3

Directions (Q. 40–41) : Read the information given below and answer the question that follows—

$f(x) = 1/(1 + x)$ if x is positive $= 1 + x$ is negative or zero $f^n(x) = f(f^{n-1}(x))$

40. If $x = 1$ find $f^1(x) f^2(x) f^3(x) f^4(x) \ldots f^9(x)$—

(A) 1/5 (B) 1/6

(C) 1/7 (D) 1/8

41. If $x = -1$ what will $f(x)$ be ?

(A) 2/3 (B) 1/2

(C) 3/5 (D) 1/8

Directions (Q. 42–43) : Read the information given below and answer the question that follows—

The batting average (BA) of a test batsman is computed from runs scored and inning played-completed inning and incomplete inning (not out) in the following manner :

r_1 = number of runs scored in completed innings

n_1 = number of completed innings

r_2 = number of runs scored in incomplete innings

n_2 = number of incomplete innings

$$BA = \frac{r_1 + r_2}{n_1}$$

To better assess a batsman's accomplishment, the ICC is considering two other measures MB A_1 and MB A_2 defined as follows :

$$MB\ A_1 = \frac{r_1}{n_1} + \frac{r_2}{n_2}\,max\left[0, \left(\frac{r_2}{n_2} - \frac{r_1}{n_1}\right)\right] : MB\ A_2 = \frac{r_1 + r_2}{n_1 + n_2}$$

42. Based on the information provided which of the following is true?

(A) MB $A_1 \le$ BA $\le$ MBA_2

(B) BA $\le$ MB $A_2 \le$ MB A_1

(C) MB $A_2 \le$ BA $\le$ MB A_1

(D) None of these

43. An experienced cricketer with no incomplete innings has a BA of 50. The next time he bats, the innings is incomplete and he scores 45 runs. It can be inferred that—

(A) BA and MB A_1 will both increase

(B) BA will increase and MB A_2 will increases

(C) BA will increase and not enough data is available to assess change in MB A_1 and MB A_2

(D) None of these

Directions (Q. 44–49) : Answer the questions independent of each other.

44. If $f(x) = \log\left\{\frac{1 + x}{1 - x}\right\}$, then $f(x) + f(y)$ is :

(A) $f(x + y)$ (B) $f\left\{\frac{x + y}{1 + xy}\right\}$

(C) $(x + y) f\frac{1}{1 + xy}$ (D) $\frac{f(x) + f(y)}{1 + xy}$

45. Suppose. For any real number x, $[x]$ denotes the greatest integer less than or equal to x. Let $L\,(x, y) = [x] + [y] + [x + y]$ and $R\,(x, y) = [2x] + [2y]$. Then it's impossible to find any two positive real numbers x and y for which—

(A) $L\,(x, y) = R\,(x, y)$ (B) $L\,(x, y) \ne R\,(x, y)$

(C) $L\,(x, y) < R\,(x, y)$ (D) $L\,(x, y) > (R\,x, y)$

46. Let $g(x) = max\,(50x, x + 2)$. The smallest possible value of $g(x)$ is—

(A) 4.0 (B) 4.5

(C) 1.5 (D) None of these

47. Let $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$, where x is real number attains a minimum at—

(A) $x = 2.3$ (B) $x = 2.5$

(C) $x = 2.7$ (D) None of these

48. When the curves $y = \log_{10} x$ and $y = x^{-1}$ are drawn in the $x - y$ plane, how many times do they intersects for values $x \geq 1$?

(A) Never (B) Once

(C) Twice (D) More than twice

49. Consider the following two curves in the $x - y$ plane; $y = x^3 + x^2 + 5$; $y = x^2 + x + 5$ which of the following statement is true for $-2 \leq x \leq 2$?

(A) The two curves intersect once

(B) The two curves intersect twice

(C) The two curves do not intersect

(D) The two curves intersect thrice

Directions (Q. 50–52) : Answer the question on the basis of the table given below—

Two binary operations $\oplus$ and * are defined over the set (a, e, f, g, h) as per the following tables :

	a	e	f	g	h
a	a	e	f	g	h
e	e	f	g	h	a
f	f	g	h	a	e
g	g	h	a	e	f
h	h	a	e	f	g

*	a	e	f	g	h
a	a	a	a	a	a
e	a	e	f	g	h
f	a	f	h	e	g
g	a	g	e	h	f
h	a	h	g	f	e

Thus, according to the first $f \oplus g–a$, while according to the second table $g * h = f$, and so on.

Also, let $f^2 = f * f$, $g^3 = g * g * G$, and so on.

50. What is the smallest positive integer n such that $g^n = e$?

(A) 4 (B) 5

(C) 2 (D) 3

51. Upon simplification, $f \oplus [f * \{f \oplus (f * f)\}]$ equals—

(A) e (B) f

(C) g (D) h

52. Upon simplification, $\{a^{10} * (f^{10} \oplus g^9)\} \oplus e^8$ equals—

(A) e (B) f

(C) g (D) h

53. Let $f(x) = ax^2 - b\,|x|$, where a and b are constants. Then at $x = 0$, $f(x)$ is—

(A) Maximized whenever $a > 0, b > 0$

(B) Maximized whenever $a > 0, b < 0$

(C) Minimized whenever $a > 0, b > 0$

(D) Minimized whenever $a > 0, b < 0$.

54. If $f(x) = x^3 - 4x + p$, and $f(0)$ and $f(1)$ are of opposite signs, then which of the following is necessarily true?

(A) $-1 < p < 2$ (B) $0 < p < 3$

(C) $-2 < p < 1$ (D) $-3 < p < 0$

Directions (Q. 55–56) : Answer the questions on the basis of the information given below—

$$\begin{aligned} f_1(x) &= x && 0 \leq x \leq 1 \\ &= 1 && x \geq 1 \\ &= 0 && \text{otherwise} \\ f_2(x) &= f_1(-x) && \text{for all } x \\ f_3(x) &= -f_2(x) && \text{for all } x \\ f_4(x) &= f_3(-x) && \text{for all } x \end{aligned}$$

55. How many of the following products are necessarily zero for every x :

$$f_1(x) f_2(x), f_2(x) f_3(x), f_2(x) f_4(x)?$$

(A) 0 (B) 1

(C) 2 (D) 3

56. Which of the following is necessarily true?

(A) $f_4(x) = f_1(x)$ for all x

(B) $f_1(x) = -f_3(-x)$ for all x

(C) $f_2(-x) = f_4(x)$ for all x

(D) $f_1(x) + f_3(x) = 0$ for all x

Solutions

Exercise – A

Ans. 1. It is obvious that the domain of this function includes only those values of x for which the following conditions are simultaneously valid: (a) $x > 0, x \neq 1$ (since the logarithmic base must be positive and non-zero); (b) $\cos x > 0$ (since negative numbers and zero do not have logarithms). '

Solving this system of inequalities, we find that the domain of the function at hand is the following set of numbers :

$$0 < x < 1, 1 < x < \frac{\pi}{2}, -\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$$

where $k = 1, 2, 3, \ldots$(represent it on the number line).

Ans. 2. This function is not defined for those values of x for which $\sin x$-$\cos x = 0$ (the denominator of the fraction must be different from zero), and, besides, for those x for which $\sin x$-$\cos x < 0$ (because for these values of x the denominator assumes imaginary values). Thus, the domain of function (1) consists only of those values of x for which the inequality $\sin x$-$\cos x > 0$ is valid; solving this inequality , we find

$$\frac{\pi}{4} + 2k\pi < x < \frac{5\pi}{4} + 2k\pi, k = 0, \pm 1, \pm 2 \qquad \ldots(2)$$

However, it must be further noted that $\cot x$ is not defined for $x = n\pi$, Where n is any integer. And so all the

values of $x = n\pi, n = 0, \pm 1$ .... likewise do not belong to the domain of the function and must be excluded from the resulting sequence of intervals (2). Thus, for the domain of function (1) we finally get the following set of real numbers :

$$\frac{\pi}{4} + 2k\pi < x < \pi + 2k\pi, \qquad \pi + 2k\pi < x < \frac{5\pi}{4} + 2k\pi,$$

$$K = 0, \pm 1, \pm 2, ...$$

Ans. 3. We consider each summand separately. The domain of this function can only embrace those values of the argument for which the first term assumes real values, *i.e.,* those values of x for which the radicand cos (cos x) is non-negative: cos (cos x) ≥ 0. It is easy to see that this inequality holds true for all real values of x.

Now, let us examine the second summand. By definition, the expression arcsin a is meaningful only for $|a| \leq 1$; in other words, only those values of x belong to the domain of function (3) for which $|(1 + x^2)/2x| \leq 1$. However, it may be proved directly that the inequality $|(1 + x^2)/2x| \geq 1$. holds for all non-zero real values of x, equality being achieved only when $x = 1$ and $x = -1$.

Consequently, the domain of (3) consists of two points only : $x = -1$ and $x = 1$.

The foregoing examples show that in finding the domain of definition of a function one has to invoke various branches of algebra and trigonometry. Only when these sections are fully mastered can the aspirant tackle such problems with ease.

The aspirant should have a firm knowledge of the definitions and be able to investigate such general properties of functions as bounded ness, monotonicity (intervals over which a function is increasing or decreasing), evenness and oddness, periodicity, and be able to find the range of a function, its zeroes, external values, and the like.

The investigation of the properties of functions is carried out without invoking the concept of a derivative, which is an element of mathematical analysis and is outside the school curriculum.

The aspirant should have a clear idea of a system of co-ordinates in the plane and be able to sketch, by memory, the graphs of the basic functions $y = kx + b$ (straight line); $y = ax^2 + bx + c$ (parabola); $y = k/x$ (hyperbola); $y = |x - a|$; $y = x^3$; $y = \sqrt{x}$; $y = 1/x^2$; $y = a^x$ ($a > 0, a \neq 1$); $y = \log_a x (a > 0, a \neq 1)$; $y = \sin x$ (sine curve); $y = \cos x$; $y = \tan x$; $y = \cot x$. The aspirant should be able to sketch the graphs of these functions in each concrete case, giving a general picture and the characteristic peculiarities of behaviour of the curve and not be forced to compute each time a table of values and plot the curve.

The aspirant should also be able to illustrate geometrically on the graph the properties of a function. When relating some property (say, the oddness of the sine), the aspirant sometimes sketches the appropriate graph (sine curve) and then makes the mistake of saying "this property is evident from the drawing." Such reasoning is faulty because it is precisely by using the property of the function that one can more or less accurately sketch its graph. For this reason, all the properties of a function must be demonstrated by rigorous analysis as is done in the standard text book.

Let us examine some problems in which graphs are constructed by translation or a specific deformation of the graphs of basic functions.

Ans. 4. The domain of this function consists of all real values of x except $x = 0$. If we consider the function $Y_1 = -1/x$ (which is a hyperbola whose branches are located in the second and fourth quadrants), it is obvious that for each value $x = x_0$ the value of y is greater by 2 than the value of y_1 for the same value x_0 of the argument. It therefore suffices to translate the graph of the function y_1 2 units upwards along the axis of ordinates to get the desired graph of the function y **(Fig. 1).**

It is easy to see that this device enables us to construct the graph of the function $y = a + f(x)$, where a is a given number, if we have already constructed the graph of the function $y_1 = f(x)$: it is sufficient to translate the graph of the function y_1 a units upwards if $a > 0$ or $|a|$ units down if $a < 0$.

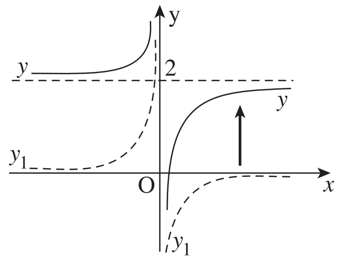


Fig. 1

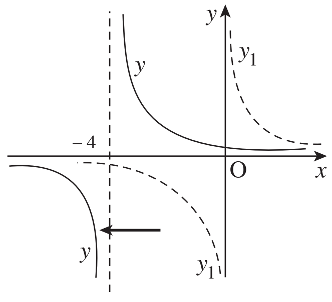


Fig. 2

Ans. 5. Evidently, x can assume all values except -4. Compare this function with the function $y_1 = 3/x$. It is clear that the value of the function y corresponding to some value $x = x_0$ coincides with the value of y_1, which corresponds to the value of its argument equal to $x_0 + 4$. For example, the function $y = 3/(x + 4)$ when, $x_0 = 1$, takes the value $y = 3/5$, and the function $y_1 = 3/x$ assumes this

very same value for the value of its argument equal to 5 = $x_0 + 4$. And so if we translate the graph of the function y_1 four units to the left along the x-axis, we get the graph of the function y that we want **(Fig. 2)**.

It is not so hard to figure out that this same method enables one to draw the graph of the function $y = f(x + b)$, where b is a given number, if we already have the graph of the function $y_1 = f(x)$: it suffices to translate the graph of y_1 b units leftwards if $b > 0$ or $|b|$ units rightwards if $b < 0$.

Ans. 6. To draw the graph, first transform the fraction and represent the function as

$$Y = -\frac{1}{3} + \frac{13/9}{x - (2/3)}$$

Arguing as in Problems 4, and 5, we see that the graph of the proposed function is an "ordinary" hyperbola $y = (13/9)/x$ translated 2/3 unit rightwards along the x-axis and 1/3 unit down along the y-axis (make the drawing!).

A similar device permits drawing the graph of any function

$$Y = \frac{ax + b}{cx + d}$$

This is called *a* linear fractional function. Indeed, a simple transformation permits writing this function as

$$Y = \frac{a}{c} + \frac{\frac{ad - bc}{c^2}}{x + \frac{d}{c}}$$

It then remains to invoke the arguments given above in solving Problems 4 and 5.

Note that in the very same way, by combining remarks pertaining to Problems 4 and 5, we can readily represent the graph of a function $y = a + f(x + b)$, where a and b are specified numbers, if the graph of the function $y_1 = f(x)$ has already been constructed.

Ans. 7. Sometimes aspirant gives answer like this : "The graph of the function does not exist since negative numbers do not have logarithms." The mistake here is the failure to grasp the fact that $-x$ does not by any, means always represents a negative number.

The domain of the function y under consideration is the set $x < 0$. It is immediately clear that the value of this function, when $x = -x_0$, $x_0 > 0$, coincides with that of the function $y_1 = \log_4 x$ for the value x_0 of its argument. Hence, to obtain the graph of the function y it is sufficient to reflect the graph of the function y_1 about the y-axis **(Fig. 3).**

It will be noted that this same device permits constructing the graph of the function $y = f(-x)$ if we have the graph of the function $Y_1 = f(x)$; it suffices to reflect the graph of y_1 about the y-axis.

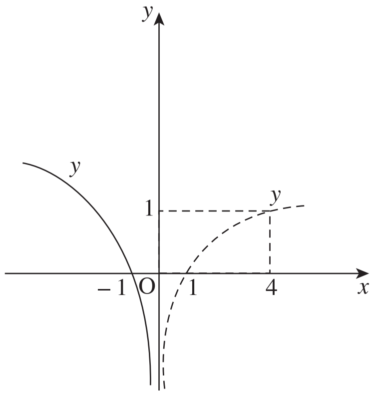


Fig. 3

Ans. 8. The aspirant is not always able to give a proper representation of all three curves in a single drawing and correctly to indicate their mutual positions **(Fig. 4)**, to indicate the peculiarities of each of the sine curves and to explain how they are obtained one from another.

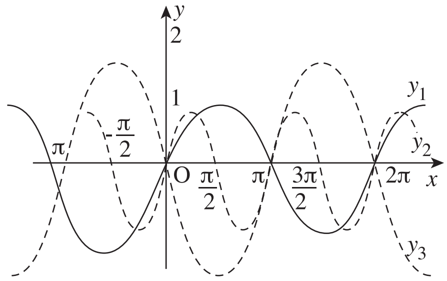


Fig. 4

For one thing, it is useful to remember that the smallest positive period of the function $y = A \sin \omega x$, where $\omega \neq 0$ and $A \neq 0$ are given numbers, is equal to $2\pi/|\omega|$ (for instance the smallest positive period of the function $y = -3 \sin \pi x$ is the number $2\pi/\pi = 2$, and for the function $y = 1/4 \sin(-x/3)$ is the number $2\pi/|-1/3| = 6\pi$), while its "amplitude" is equal to $|A|$ (thus, the "amplitude" of the function $y = -1/2 \sin 3x$ is 1/2).

The foregoing of course refers also to all the other trigonometric functions. It is important to stress the fact that it is possible to construct the graph of the function $y = A f(\omega x)$ where $\omega \neq 0$ and $A \neq 0$ are given numbers, if we know the graph of the function $y_1 = f(x)$. First compress the graph of y_1 along the x-axis ω times if $\omega > 0$; but if $\omega < 0$, then compress the graph of the function y_1 $|\omega|$ times along the x-axis and perform a reflection with respect to the y-axis (see the solution of problem 7). Then take the resulting curve and stretch it along the y-axis A times if $A > 0$; but if $A < 0$, then perform an $|A|$ -fold stretching along the y-axis and a reflection about the x-axis. Of course, if $|\omega| < 1$, then the compression along the x-axis is actually a stretching; in the same way, the $|A|$ -fold stretching along the y-axis for $|A| < 1$ is actually a compression.

Note particularly an important special case : if the graph of the function $y_1 = f(x)$ has been sketched, the

graph of the function $y = -f(x)$ can be obtained from it by a reflection about the x-axis.

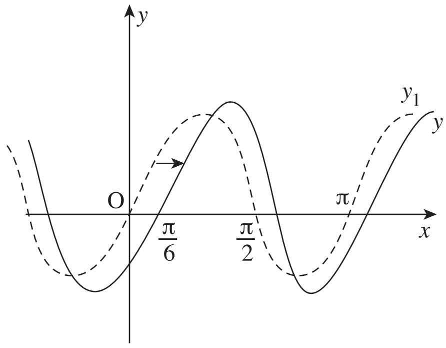


Fig. 5

Ans. 9. Representing the given function in the form $y = \sin 2[x - (n/6)]$, we see immediately that for every value $X = X_0$ the value of y coincides with the value of $Y_1 = \sin 2x$, which corresponds to the value $X_0 - (\pi/6)$ of its argument. And so to construct the graph of y, draw the graph of y_1 and then translated it $\pi/6$ units rightwards along the x-axis **(Fig. 5).**

A very common mistake in constructing the graph of the function y is as follows : the graph is drawn of the function y_1 and it is then translated rightwards by $\pi/3$ units along the x-axis.

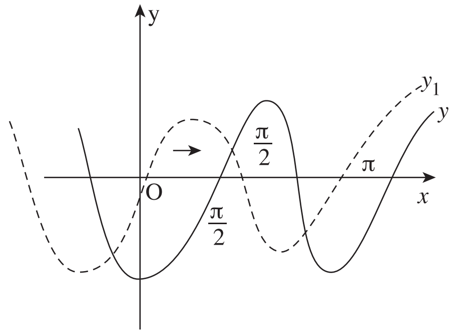


Fig. 6

It is easy to see that this construction is incorrect, because the graph crosses the x-axis at the point $\pi/3$ (since the graph of the function y_1 cuts this axis at the origin and is then translated $\pi/3$ units rightwards!). Yet the value of the function y is clearly non-zero for the value of the argument $x = \pi/3$.

The technique used in this specific instance enables one to construct the graph of any function of the form $y = A \sin(\omega x + \varphi)$, $y = A \cos(\omega x + \varphi)$ etc., and also $y = a \sin \omega x + b \cos \omega x$..

This technique is of a general nature and permits obtaining the graph of a function $y = f(\omega x + \varphi)$, where $\omega \neq 0$ and φ are specified numbers, if the graph of the function $y_1 = f(x)$ has already been drawn : it is sufficient to draw the graph of the function $y_2 = f(\omega x)$ (it may be obtained the method indicated in the solution of Problem 8) and then to translate it along the x-axis rightwards by an amount $|\varphi/\omega|$ if $\varphi/\omega < 0$ leftwards by φ/ω if $\varphi/\omega < 0$ (see Problem 5).

It is sometimes useful to first transform the formula defining the functional relation; then the graph is readily drawn. In particular, it is always desirable to represent a complicated functional relationship as an easily surveyable combination of elementary functions, the graph of which combination is obtainable by familiar techniques (that was precisely how we constructed the graph in Problem 6).

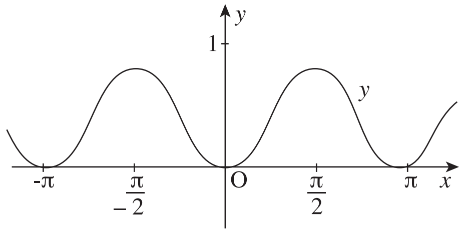


Fig. 7

Ans. 10. Since, this function may be written as $Y = 1/2 - 1/2 \cos 2x$, the graph of the function Y is obtained by familiar techniques: the cosine curve $y_1 = -1/2 \cos 2x$, which is constructed by the technique described in the solution of Problem 8, must be translated 1/2 unit upwards **(Fig. 7).**

Ans. 11. Employing familiar formulas involving logarithms, we see that $x^{1/\log_{10}x} = x^{\log_x 10} = 10$, whence aspirants often conclude immediately that the graph of the function (4) is the straight line $Y = 10$.

This conclusion is incorrect however. It is necessary to take into account the domain of definition of the function and the conditions under which the transformations that are carried out are legitimate.

The domain of the function (4) consists of the real numbers which satisfies the conditions : $x > 0$, $x \neq 1$. Under these conditions, it is legitimate to carry out the transformation indicated above. And so graph of the function (4) is the half-line $y = 10$, $x > 0$ with the point $(1 \cdot 10)$ deleted **(Fig. 8).**

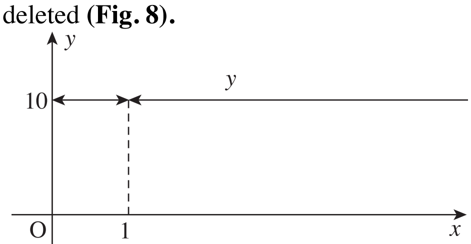


Fig. 8

The arrowhead at any point indicates, that point does not belong to the graph).

Ans. 12. First of all, perform an identity transformation of the second summand:

$$\begin{aligned} \log_2 \sqrt{4x^2 - 4x + 1} &= \log_2 \sqrt{(2x-1)^2} \\ &= \log_2 |2x - 1| \\ &= 1 + \log_2 \left| x - \frac{1}{2} \right| \end{aligned}$$

It is now clear that the domain of the function y is the set $x > 1/2$ (because the second term in the formula

defining this function is **(Fig. 8)** meaningful for all $x \neq 1/2$, while the first is meaningful only for $x > 1/2$).

However, the equation $\log_{1/2}(x - 1/2) = -\log_2(x - 1/2)$ is true for $x > 1/2$, and, hence, in its domain $(x > 1/2)$ function (5) can be written $y = 1$.

Thus the graph of the function y is the ray $y = 1$, $x > 1/2$ **(Fig. 9)** the arrowhead at the point (1/2, 1) signifies that this point does not belong to the graph of the function (5)].

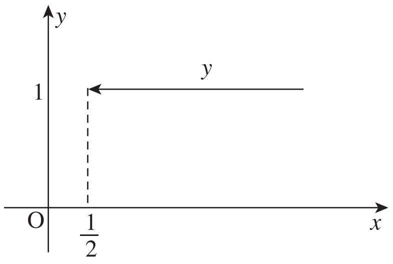


Fig. 9

Aspirant often find it difficult to construct graphs of functions whose analytic expressions involve the absolute-value sign. The next few illustrations illustrate how the graphs of such functions are constructed.

Ans. 13 First note that the proposed function can obviously be written in the form $y = |2^x - 2|$

Consider the auxiliary function $y_1 = 2^x - 2$, the graph of which is readily drawn (by the technique described in the solution of Problem 4). How does the graph of the function y differ?

Recall the definition of absolute value; from this definition it follows that

$$Y = \begin{cases} 2^x - 2 \text{ for values of } x \text{ for which} \\ \quad 2^x - 2 \geq 0 \text{ that is, for } x \geq 1, \\ -(2^x - 2) \text{ for values of } x \text{ for which} \\ \quad 2^x - 2 < 0, \text{ that is, for } x < 1 \end{cases}$$

It is then clear that the graph of the function y, for $x \geq 1$, coincides with the graph of the function y_1 and, for $x < 1$, is a curve symmetric to the graph of the function y_1 with respect to the x-axis **(Fig. 10).**

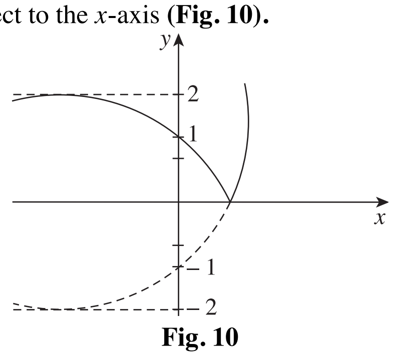


Fig. 10

In precisely the same way we can obtain the graph of the function $y = |f(x)|$ if the graph of the function $y_1 = f(x)$ has been drawn. It suffices **(Fig. 10)** to replace the portions of the graph of y_1 lying below the x-axis by corresponding portions symmetric with respect to the x-axis [to find these portions we have to solve the inequality $f(x) < 0$].

Ans. 14 Here, without dropping the absolute-value signs, we can carry out the construction using the techniques indicated in the solution of problems 6 and 13. Indeed, taking the graph of the function $y_1 = |x|$ **(Fig. 11),**

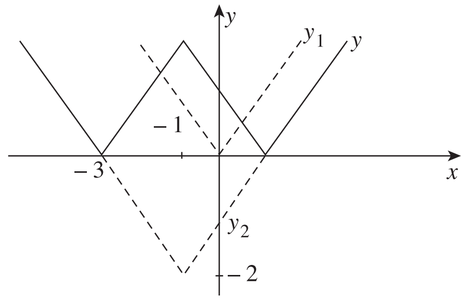


Fig. 11

translate it one unit leftwards along the x-axis and two units Downwards along the y-axis. This yields the graph of the function $y_2 = |x + 1| - 2$. Then replace the portion of the graph below the x-axis corresponding to $-3 \leq x \leq 1$, by the portion symmetric to it about the x-axis. The resulting polygonal line is the graph of the function y.

The general technique for constructing the graph of a function whose analytical expression contains an absolute-value sign consists in rewriting the expression of the functional relationship without using the absolute value sign. In this case, the functional relationship on different portions of variation of the argument is, as a rule, described by different formulas. Quite naturally, on each of these portions, the graph must be constructed on the basis of the appropriate formula.

Ans. 15. To get rid of the absolute-value sign, consider separately two cases. $x \geq 0$ and $x < 0$. If $x \geq 0$, then $y = x^2 - 2x - 3$. It is easy to draw this parabola, then we take that portion which corresponds to non-negative values of x. But if $x < 0$, then $y = x^2 + 2x - 3$. Draw this parabola and take that portion which corresponds to negative values of x. Taken together, the two pieces of the parabolas constitute the graph that interests us **(Fig. 12).**

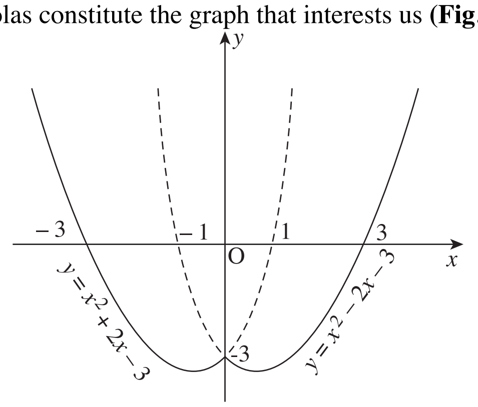


Fig. 12

Ans. 16. By the definition of absolute value we can represent this function in the form

$$y = \begin{cases} [(x + 1) + 1](x - 3) = (x + 2)(x - 3) \text{ if } x \geq -1, \\ [-(x + 1) + 1](x - 3) = -x(x - 3) \text{ if } x > -1 \end{cases}.$$

It now remains simply to sketch the curve, using the appropriate formula, for each of the indicated intervals ($x \geq -1$ and $x < -1$). Together, the two curves yield the graph of function (6).

Let us first consider the function $y_1 = (x + 2)(x - 3)$. Ordinarily, aspirants remove the brackets and perform a rather lengthy procedure of isolating a perfect square, whereas it is better not to remove the brackets because it is at once clear that this is a parabola, the graph of a quadratic trinomial; the parabola intersects the x-axis at the points A $(-2, 0)$ and B $(3, 0)$ (because -2 and 3 are the roots of the trinomial) and its branches are directed upwards (since the leading coefficient is positive). Substituting the value $x = 0$ into the formula for the function y_1, we get the co-ordinates of the point C of intersection of this parabola with the axis of ordinates (y-axis): C $(0, -6)$. It is also easy to find the co-ordinates of the vertex D of this parabola. Since, the parabola is symmetric about the vertical straight line passing through the vertex, its axis of symmetry bisects the line segment AB. It is therefore clear that the abscissa of the vertex is equal to 1/2; the ordinate is computed directly and we get D $(1/2, -25/4)$.

Having constructed the parabola—the graph of the function y_1—we must isolate that portion which corresponds to the values $x \geq 1$ of the argument **(Fig. 13)**.

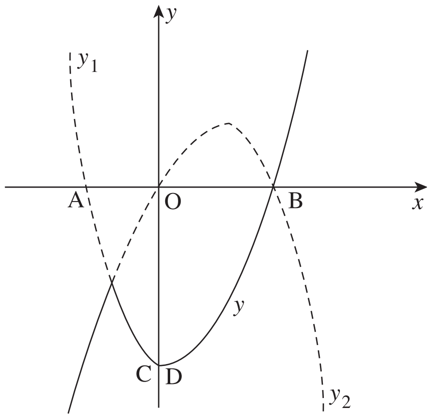


Fig. 13

The construction of the graph of the function $y_2 = -x(x - 3)$ is similar. Take only that portion of the parabola which corresponds to the values $x < -1$ of the argument. The graph of function (6) is shown in **Fig. 13** by the solid line.

Ans. 17. We first find the values of x for which each of the expressions under the absolute-value sign vanishes; they are $-3, -1, 1, 3$. By considering function (7) on each of the five intervals into which these values partition the number line, we obtain the following notation:

$$Y = \begin{cases} 1 - \dfrac{2}{x+1} & \text{if} \quad x < -3 \\ -\dfrac{x}{2} + \dfrac{1}{2} & \text{if} \quad -3 \leq x < -1, \\ 1 & \text{if} \quad -1 \leq x < 1, \\ \dfrac{2}{x+1} & \text{if} \quad 1 \leq x < 3 \\ 1 - \dfrac{2}{x+1} & \text{if} \quad 3 \leq x \end{cases}$$

The subsequent construction now follows familiar techniques (**Fig. 14**).

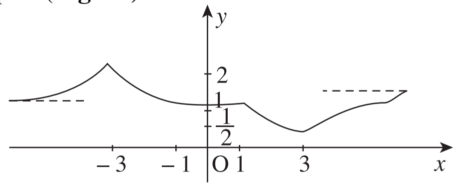


Fig. 14

It will be noted that if x increases without bound, the graph of the function (7) approaches without bound the straight line $y = 1$, remaining all the time below it; but if x decreases without bound, then the graph approaches the same line without bound, remaining all the time above it.

Ans. 18. When drawing the graph of a periodic function it is often helpful to know that all values of such a function are repeated in every period. Thus, if a periodic function is given with period T, then it is sufficient to construct the graph on some segment of length T; for $0 \leq x \leq T$, the portions of the graph on the intervals $T \leq x \leq 2T$, $2T \leq x \leq 3T$, $-T \leq x \leq 0$, etc., will have the very same shape.

It is clear that the number 2π is the period of the function y under consideration so that we can confine ourselves to the interval $0 \leq x \leq 2n$. Partitioning this interval into four parts in each of which both sin x and cos x preserve sign, we get

$$Y = \begin{bmatrix} \sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right) & \text{if } 0 \leq x \leq \dfrac{\pi}{4}, \\ \sqrt{2}\sin\left(x - \dfrac{\pi}{4}\right) & \text{if } \dfrac{\pi}{2} \leq x \leq \pi, \\ -\sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right) & \text{if } \pi \leq x \leq \dfrac{3\pi}{2}, \\ -\sqrt{2}\sin\left(x - \dfrac{\pi}{4}\right) & \text{if } \dfrac{3\pi}{2} \leq x \leq 2\pi \end{bmatrix}$$

We now construct the graphs of $y_1 = \sqrt{2}$ sin $[x + (\pi/4)]$ and $y_2 = \sqrt{2} \times$ sin $[x - (\pi/4)]$ and sin $[x - (\pi/4)]$ and then we take the portion of the curve y_1 on the interval from 0 to $\pi/2$, the portion of the curve y_2 on the interval from $\pi/2$ to π; and on the intervals from π to $3\pi/2$ and from $3\pi/2$ to 2π, we take the curves that are symmetric, about the x-axis, to the corresponding portions of the curves y_1 and y_2. Then, taking advantage of periodicity, we extend the resulting curve beyond the interval $0 \leq x \leq 2\pi$ (the solid line in **Fig. 15**).

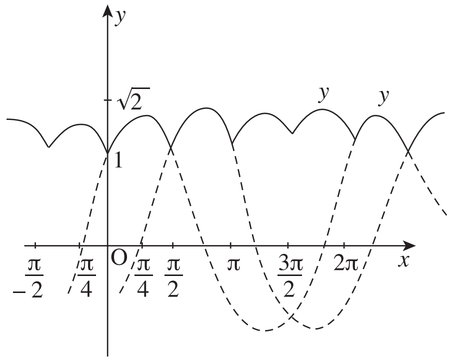


Fig. 15

It is clear from this graph that $\pi/2$ is the period of the given function so that we were overcautious in considering the interval from 0 to 2π. If we had realized from the very start that $\pi/2$ is the period of this function-and this is easy to demonstrate :

$$\left|\sin\left(x+\frac{\pi}{2}\right)+\cos\left(x+\frac{\pi}{2}\right)\right| = |\cos x| + |\sin x|$$

then the graph could be constructed much faster. This example shows that a careful preliminary analysis of the properties of a given function very often appreciably simplifies the construction of the graph.

Ans. 19. At first glance this function might appear to be very complicated. However, by transforming the formula defining the given function we obtain a simpler notation for (8) that will permit drawing the required graph with comparative ease.

First of all, note that the domain of the function (8) is the entire number line with the exception of the points $x = n\pi/2$, where n is any integer (at each of these points, either tan x or cot x becomes meaningless).

Since for $x \neq n\pi/2$ the equations

$$\sqrt{1+\tan^2 x} = \frac{1}{|\cos x|}, \sqrt{1+\cot^2 x} = \frac{1}{|\sin x|}$$

are true, it is clear that the function (8) can, in its domain of definition, be written as

$$y = \sin x . |\cos x| + \cos x . |\sin x|$$

This is a periodic function with period 2π. The graph can be constructed as was done In Problem 18. It is shown in **Fig. 16.**

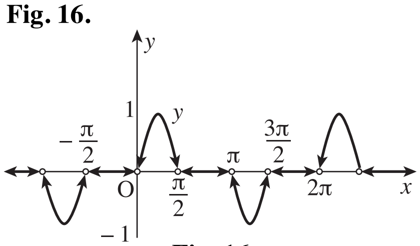


Fig. 16

Note once again, however, that the function (8) is not defined at the points $x = n\pi/2, n = 0, \pm 1, \pm 2, ...$ In the figure this is indicated by arrowheads the endpoints of the segments of the curve. We will now consider some instances of the construction of complicated in which the foregoing elementary devices do not suffice. Each of these examples has its peculiarities that must be taken into account when sketching the graph. In solving problems like those given below, one often has to reason in quite an unorthodox manner.

So to speak. The approach should be to get onto an item that will give some clue to the construction.

Ans. 20. Representing the given function as $y = x + (1/x)$, we apply a technique called addition of graphs, which means that the desired graph is constructed by "combining" two auxiliary graphs, $y_1 = x$ and $y_2 = 1/x$. In other words, for each admissible value of the argument (that is, for every $x \neq 0$) the corresponding ordinate y is built up as an algebraic sum of the ordinates y_1 and y_2 corresponding to the same value of the argument **(Fig. 17).**

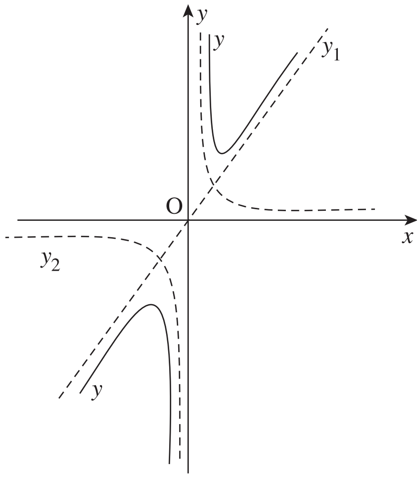


Fig. 17

It is easy to figure out the shape of the graph of the function on the positive x-axis: for each value $x > 0$, the corresponding ordinate of the straight line $y_1 = x$ has to be increased by an ordinate of the hyperbola $y_2 = 1/x$ corresponding to the same value of x. It is quite obvious (Fig. 45) that for a positive x tending to zero, the expression $x + (1/x)$ tends to $+\infty$ (increases without bound), and for x tending to $+\infty$, the desired graph approaches the bisector $y_1 = x$ without bound, since the summand $1/x$ becomes smaller and smaller. It is easy in this case to determine the smallest value of the function y (recall that so far we are only considering positive values of x) : indeed, when $x > 0$ the inequality $x + (1/x) \geq 2$ holds true, which is to say the smallest value is equal to 2 and is reached when $x = 1$.

Construction of the graph is similar on the negative x-axis as well. Incidentally, we could take advantage of the fact that the function y is odd and, hence, its graph is symmetric about the origin.

Ans. 21 Take advantage of the fact that the formula defining this function is a product and we apply a technique called multiplication of graphs. The required graph will be constructed by "multiplying" two auxiliary graphs $y_1 = x$ and $y_2 = \sin x$. In other words, for each value of the argument, the corresponding ordinate y is constructed as a product of the ordinates y_1 and y_2 which correspond to the same value of the argument **(Fig. 18)**.

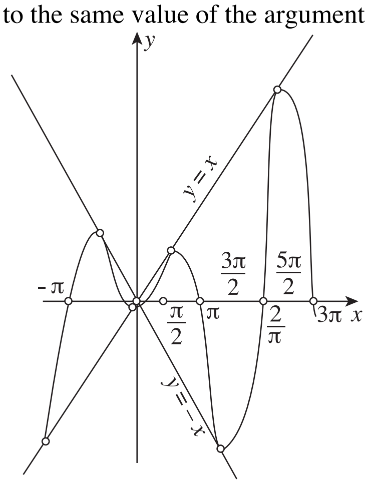


Fig. 18

We first construct the graph of the function y for non-negative value; of the argument. For each value of x we multiply the value of the corresponding ordinate of the straight line $y_1 = x$ and the value of the ordinate of the sine curve $y_2 = \sin x$, and are thus able to construct a smooth curve that gives an approximate idea of the behaviour of the graph of the function y on the non-negative x-axis .The aspect of the we can be improved somewhat by plotting a few characteristic points. First of all, it is clear that $y = 0$ for those values of x for which since $x = 0$, and so the graph of the function y crosses the positive x-axis at the points $x = k\pi$, $k = 0, 1, 2,\ldots$ Furthermore, for $x >$ o the obvious inequality $-x \le x \sin x \le x$ holds true; this means that for **Fig. 18.** Positive values of the argument the graph of the function y does not extend above the straight line $y = x$ or below the straight line $y = x$. or the below the straight line $y = -x$. In this case, the points of the graph of the function y that correspond to the values of $x > 0$ for which $\sin x = 1$ *i.e.*, to the values $x = (\pi/2) + 2k\pi$, $k = 0, 1, 2,\ldots\ldots$ lie on the straight line $y = x$, and the points corresponding to the values of $x > 0$ for which $\sin x = -1$, *i.e.*, to the values $x = (3\pi/2) + 2k\pi, k = 0, 1, 2,\ldots$, lie on the straight line $y = -x$.

Ans. 22. Here we have to construct the graph of a function. To graph such a function, the aspirant must know the properties of the basic elementary functions and have a clear-cut idea of the consequent properties of combinations of these functions.

The domain of the function y consists of all real numbers except $x = 0$. Since, for $x > 0$ the exponent $1/x > 0$, it follows that $y > 1$ for all positive values of the argument, by the property of an exponential function.

Note that $y = 2$ when $x = 1$. If x increases without bound, then the expression $1/x$ decreases to zero monotonically, remaining, and so $2^{1/x}$ decreases to unity monotonically, remaining greater than unity (by the property of an exponential function). When x is positive and tends to zero, the exponent $1/x$ increases without bound and, hence, $2^{1/x}$ also increases without bound. This enables us to sketch a graph of the function y when $x > 0$.

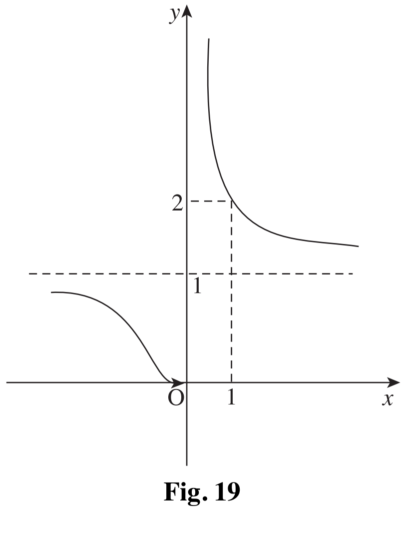


Fig. 19

It is easy to demonstrate that the inequality $0 < y < 1$ is true on the negative x-axis. Using similar reasoning, we construct the graph of the function y for $x < 0$ as well **(Fig. 19).** The arrowhead at the origin indicates that the origin does not belong to the graph).

Ans. 23. If we represent this function as

$$y = 1 + (-2).\, 2^{\sin(x+1)} \qquad \ldots(9)$$

then it is clear that having the graph of the function $y_1 = 2^{\sin x}$ it is easy to obtain the graph of the function y by means of techniques discussed in the solution of Problems 4, 5, and 8.

And so let us first tackle the graph of the function Y_1. This is a periodic function with period 2π; therefore, all we need to do is draw the graph on the interval $0 \le x \le 2n$ (see Problem 18).

For $x = 0$ the function y_1 assumes the value 1. If x is increased from 0 to $\pi/2$, then $\sin x$ monotonically increases from 0 to 1 and $2^{\sin x}$ monotonically increases from 1 to 2.If x then increases from $\pi/2$ to $3\pi/2$, then $\sin x$ monotonically decreases from 1 to -1 and $2^{\sin x}$ monotonically decreases from 2 to 1/2; in particular, for $x = \pi$ the function y_1 takes on the value 1**.** Finally, if x increases from $3\pi/2$ to 2π, then $\sin x$ monotonically increases from -1 to 0 and $2^{\sin x}$ monotonically increases from 1/2 to 1: the function Y_1 has the value 1 for $x = 2\pi$. All these statements about the behaviour of the function y_1 follow from the properties of a sine function and an exponential function (it is left to the reader to put a rigorous foundation under these statements). They permit determining the approximate behaviour of the graph of the function y_1 when $0 \le x \le 2\pi$, it then remains to extend the resulting curve periodically over the entire x-axis (in **Fig. 20**, the graph of y_1 is shown as a dashed line).

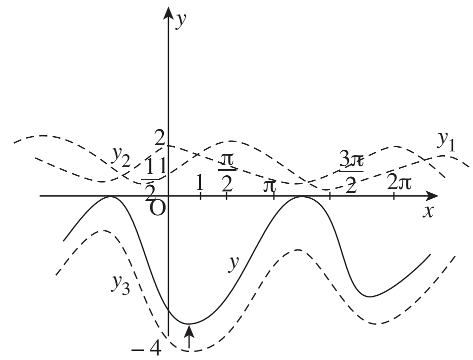


Fig. 20

Everything is now ready for the construction, first translate the graph of the function Y_1 one unit leftwards along the x-axis; this yield a curve which is the graph of the function $y_2 = 2^{\sin(x+1)}$ (see Problem 5). It is also a periodic function (with period 2π). It has a maximal value of 2 which it assumes at the points $x = (\pi/2) - 1 + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$, it has a minimal value equal to 1/2 which it takes on at the points $x = -(\pi/2) - 1 + 2\,k\pi$, $k = 0, +1, \pm 1, \pm 2, \ldots\ldots$ **(Fig. 20)**. Stretching the curve y_2 by a factor of 2 along the y-axis and then reflecting it about

the x-axis, we construct the graph of the function $y_3 = (-2). 2^{\sin(x+1)}$ (See Problem 8). Note that this periodic function has a maximal value of – 1 and a minimal value of – 4. **(Fig. 20).** Finally, the graph of the function y is obtained by translating the curve of Y_3 up one unit on y-axis (see Problem 4).

The graph (solid line in **Fig. 20**) conveys the basic features of the behaviour of the function y. This is a periodic function (with period 2π) which vanishes at the points $x = -(\pi/2) - 1 + 2k\pi$, $k = 0, \pm 1, \pm 2\ldots$ (with 0 the maximal value) and assumes a minimal value of – 3 at the points $x = (\pi/2) - 1 + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$ The function y varies monotonically in between the external values. When $x = 0$ the function y is equal to $1 - 2^{1 + \sin 1}$ (note that sin 1 is the sine of an angle of one radian!)

Of course, **Fig. 20** gives only a rough idea of the graph of the function y, but that is as much as is ordinarily required at an examination.

Ans. 24. First draw the graph of the auxiliary function $y_1 = 1 - x^2$. This parabola is shown in **Fig. 21**

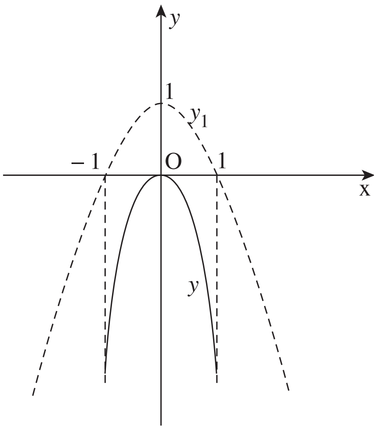


Fig. 21

by the dashed line. It is then necessary to construct the graph of the logarithm of this function.

For $x = 0$ we have $y = \log_2 1 = 0$. If x is increased from 0 to 1, then as may be seen from the graph of the auxiliary function, $1 - x^2$ decrease from 1 to 0 and so $\log_2 (1 - x_2)$ decreases from 0 to $-\infty$.

Similarly, if x decreases from 0 to – 1, then $1 - x^2$ decreases from 1 to 0 and $\log_2 (1 - x^2)$ decreases from 0 to $-\infty$. For the remaining values x, that is, for $x \le -1$ and $x \ge 1$, we have $1 - x_2 \le 0$ so that $\log_2 (1 - x^2)$ is meaningless. The graph of the function y is shown in **Fig. 21** as a solid line.

Note that in the construction of this graph we did not start out by finding domain of the function, which was obtained almost automatically. A preliminary determination of the domain of a function is frequently very useful, however.

Ans. 25. The domain of this function is the collection of all values of x for which, simultaneously, $\sin x > 0$ and $\sin x \ne 1$, that is, the set

$$2k\pi < x < \frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi < x < (2k+1)\pi$$

$$k = 0, \pm 1, \pm 2, \ldots\ldots$$

The function y is clearly periodic with period 2π. And so we can confine ourselves to an interval of length 2π, say, the interval $0 \le x \le 2\pi$. **(Fig. 21)**

But not the whole of this interval lies in the domain of the function : the function is meaningful (over this interval) only for $0 < x < \pi/2$, $\pi/2 < x < \pi$. It is precisely on these intervals that we first of all have to construct the graph (then we can simply extend it over the entire domain because of its periodicity).

It will be seen that the function y can, in its domain, be rewritten as

$$Y = \frac{1}{\log_{1/2} \sin x} \qquad \ldots(10)$$

We first of all construct the graph of the auxiliary function $y_1 = \log_{1/2} \sin x$. It will only interest us over the interval $0 < x < \pi$. Taking the piece of the sine curve $y_2 = \sin x$ corresponding to this interval, we can use the same method as in the preceding problem (don't forget that the base of the logarithm $1/2 < 1$) to obtain the graph of the composite function y_1 **(Fig. 22),**

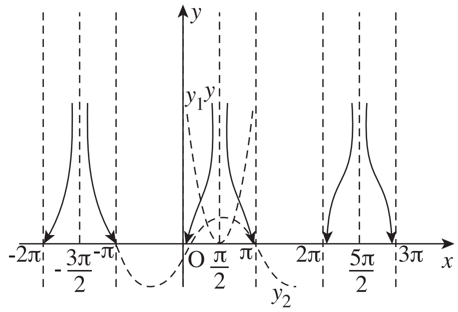


Fig. 22

the auxiliary graphs y_1 and y_2 are depicted by dashed lines).

We now consider the interval $0 < x < \pi/2$. Since for any value of x in this interval, the corresponding value of the function y is the reciprocal of the value of y_1 corresponding to the same value of the argument [see (10)], it is easy to obtain a rough sketch of the graph of y for $0 < x < \pi/2$ (the solid line in **Fig 22**; the arrowhead on the curve at the origin indicated that this point does not to the graph).

It is easy to prove that by using familiar properties of elementary function the function y monotonically increases when x varies from 0 to $\pi/2$; if x increases from 0 to $\pi/2$, then $\sin x$ increases monotonically from 0 to 1, and then $\log_{1/2} \sin x$ decreases monotonically from $+\infty$ to 0; and hence, [see (10)], the value of y increases monotonically from 0 to $+\infty$. Let us stress that if x approaches $\pi/2$, remaining all the time less than this value, then the value of the function y_1 tends to zero, remaining all the time positive, and therefore the value of y increases without bound. But if x approaches zero and

remains positive , then the value of y_1 increases without bound and so the value of y tends to zero (although it does not take on the value 0).

The construction is similar for the graph of the function at hand when $\pi/2 < x < \pi$.

It is useful to note that just about the same reasoning as is used, in construction of the graph of the function y on the basis of the Graph of the auxiliary function y_1 enables us to construct the graph of $y = 1/f(x)$ if the graph of the function $y_1 = f(x)$ is known.

Considering **Fig. 22** in more detail, we note that we did not obtain a complete description of the behaviour of the graph of the given function in the foregoing solution (for instance, the fact that this graph is bent in the specific way as it is shown in the figure was not even discussed). But this is not required since it goes beyond the scope of the elementary means at the disposal of the aspirant: And a rough Sketch of the graph can, as we have seen, be made with relative ease.

True, the shape of the curve could be improved a bit by computing a table of values of the function for "convenient" values of the argument and taking these into account when drawing the curve. As a rule, examination questions do not require such improvement. The important thing is to be able to sketch a curve that conveys the general aspect and characteristic features of the graph.

Ans. 26. First of all, do not confuse the notation sin x^2 with $\sin^2 x$: the former means $\sin(x^2)$, the latter means $(\sin x)^2$.

Let us first consider the non-negative values of the argument and partition the semi-axis $x \geq 0$ into intervals over which the function y increases or decreases. If x^2 increases from 0 to $\pi/2$ (which is to say that x increases from 0 to $\sqrt{\pi/2}$, then sin x^2 increases from 0 to 1; if x^2 increases from $\pi/2$ to $3\pi/2$ (*i.e.*, x increases from $\sqrt{\pi/2}$ to $\sqrt{3\pi/2}$), then sin x^2 decreases from 1 to – 1; if x^2 increases from $3\pi/2$ to $5\pi/2$ (*i.e.*, x increases from $\sqrt{3\pi/2}$ to $\sqrt{5\pi/2}$), then sin x^2 increases from – 1 to 1, and so on. The graph of the function y therefore is of a wavelike nature with amplitude of 1 **(Fig. 23)**. It is easy to obtain the x-intercept of this graph : all you have to do is solve the equation sin $x^2 = 0$. It is clear that the nonnegative roots of this equation are the numbers $x = \sqrt{k\pi}, k = 0, 1, 2, \ldots\ldots$

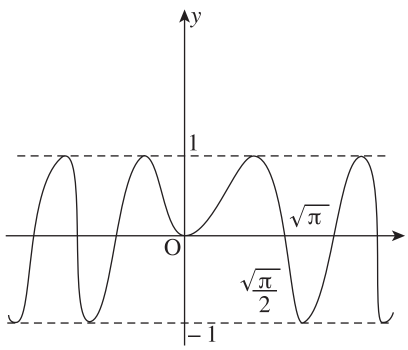


Fig. 23

On the negative x-axis, the graph is drawn at once since the function y is even.

We conclude this section with some problems of a different nature, but also connected with graphical construction in the plane with a specified co-ordinate system.

Ans. 27. From the first inequality we have $y \geq -5x/3$. We first of all graph the function $y = -5x/3$ **(Fig. 24).**

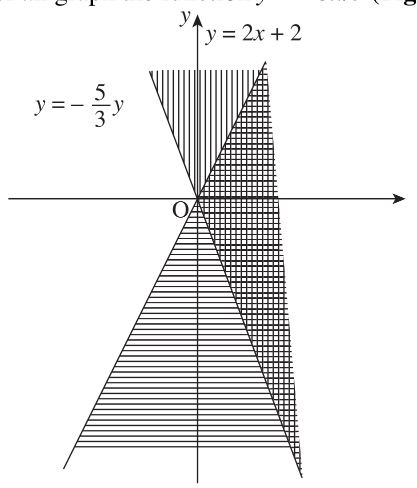


Fig. 24

Then the points whose co-ordinates satisfy the equation $y = -5x/3$ lie on the constructed straight line and the points whose co-ordinate y exceeds $-5x/3$ will lie above this line. Thus, the set of points whose co-ordinates satisfy the first inequality of (11) will constitute the half-Plane lying above the straight line $y = -5x/3$ (the straight line included; in **Fig. 24** this region is denoted by vertical hatching).

Similarly, from the second inequality of (11) we have $y < 2x + 2$ so that the set of points whose co-ordinates satisfy the second inequality of (11) will constitute the half-plane lying below the straight line $y = 2x + 2$ (the line itself is not included; in **Fig. 24** this region is indicated by horizontal hatching).

Hence, the points of the plane whose co-ordinates x and y satisfy of inequalities (11) lie in the common portion (intersection) of the two resulting half-planes; this is an angular region (in **Fig. 24** the desired set is indicated by double cross-hatching). Here, one of the bounding rays of the region-a piece of the straight line $y = -5x/3$ – is included in the sought for set, while the other—a piece of the straight line $y = 2x + 2$ – is not included (the vertex A of the angular region, the intersection point of the straight lines $y = -5x/3$ and $y = 2x + 2$, does not belong to this set either).

Ans. 28. As in the solution of other problems involving absolute values, it is useful first of all to attempt to get rid of the absolute-value sign.

To do this (see **Fig. 25**), construct straight lines in the

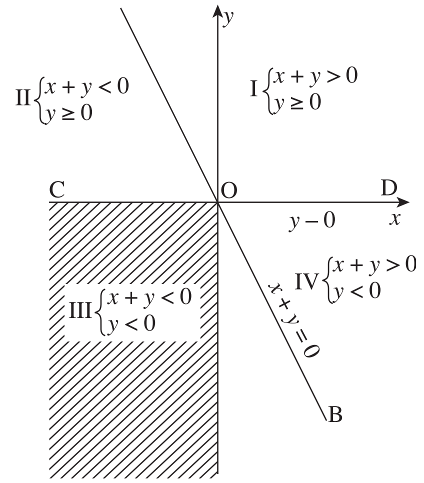


Fig. 25

plane: $x + y = 0$ (the bisector AOB of the second and fourth quadrants) and $y = 0$ (the axis COD of abscissas). Clearly, the co-ordinates x and y of any point above the straight line $x + y = 0$ satisfy the inequality $x + y > 0$, and for any below this line the inequality $x + y < 0$ holds. Similarly, any point in the upper (relative to the x-axis) half-plane has a positive ordinate, and any point in the lower half-plane has a negative ordinate.

These straight lines partition the plane into four regions **(Fig. 25)** and it is clear that in each one of these regions the expressions $x + y$ and y preserve sign for all points (x, y). It is therefore advisable, in each of these regions, to seek separately the points whose co-ordinates x and y satisfy the relation (12).

For any point (x, y) of Region I (the angular region DOA including the bounding rays as well) we have the inequalities $x + y \ge 0$, $y \ge 0$. Hence, in Region I relation (12) takes the form $x + y = y - x$, or $x = 0$. But this last equation is satisfied by the co-ordinates of the point of the positive y-axis (by no means all the points of the y-axis, since we are only interested in those points which lie in Region I, and the negative y-axis does not belong to this region).

For any point of Region II (the angular region AOC; of the boundary rays only the ray CO is included) we have the inequalities $x + y < 0$, $y \ge 0$, and for this reason relation (12) takes the form $-(x + y) = y - x$, or $y = 0$, in Region II. This latter equation is satisfied by the points of the negative x-axis (the other points the x-axis do not lie in Region II).

The inequalities $x + y < 0$, $y < 0$ hold for any point of Region III (the angular region COB excluding the bounding rays), and so relation (12) in Region III assumes the form $-(x + y) = -y - x$, or $0 = 0$. This means that the co-ordinates of any point of Region III satisfy relation (12).

Finally, for any point of Region IV (the angular region BOD including only the ray BO) we have $x + y \ge 0$, $y < 0$, and so relation (12) in Region IV assumes the form $x + y = -y - x$, or $x + y = 0$. This latter equation is clearly satisfied by those points of Region IV which lie on the bisector of the fourth quadrant.

Thus, in the plane, the set of points whose co-ordinates x and y satisfy relation (12) is the angular region between the negative x-axis and the bisector of the fourth quadrant (including the bounding rays) and the positive y-axis **(Fig. 25)**.

Ans. 29. Note right off that x and y which satisfy condition (13) are such that $x > 0, y > 0, x \neq 1$ and $y \neq 1$. Since the properties of logarithms are different for bases that exceed unity or are less than unity, it is natural to consider two cases.

(a) Let $x > 1$. then by the properties of logarithms, the inequality (13) will hold true if the inequality $\log_y x > 1$ is valid. It will be recalled the logarithms of numbers greater than unity to a base less than unity are negative. And so the inequality $\log_y x > 1$ cannot be valid for y in the interval $0 < y < 1$.

Thus, the inequality $\log_y x > 1$ can hold only when $y > 1$. But if $y > 1$, then all $x > y$ will be solutions to the inequality $\log_y x > 1$.

Thus, if $x > 1$, then for inequality (13) to hold, y must be greater than unity : $y > 1$, and the original inequality will be satisfied by those points for whose co-ordinates the condition $x > y$ is also valid.

If we depict the set of these points in a drawing, it will be seen that the set is the interior of the angular region CBD **(Fig. 26)**.

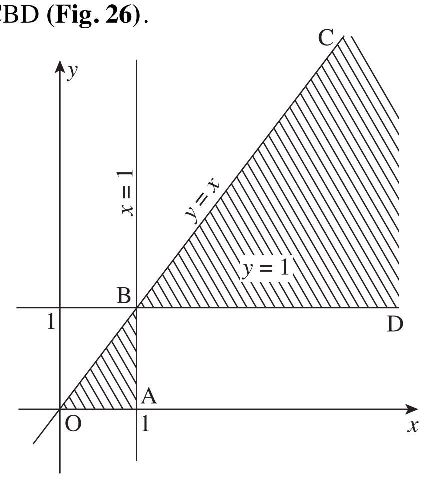


Fig. 26

(b) Now let $0 < x < 1$. Reasoning in similar fashion, we find that condition of the problem is satisfied by points for whose co-ordinates the conditions $0 < y < 1$ and $y < x$ are fulfilled. The set of these points is the interior of the triangle AOB **(Fig. 26)**.

Consequently points whose co-ordinates satisfy inequality (13) from the region cross- lined in **Fig. 26** [the co-ordinates of the boundary points of this region do not satisfy relation (13)].

Ans. 30. Using a familiar trigonometric identity, rewrite the given inequality as

$$\sin \frac{x+y}{2} \sin \frac{y-x}{2} > 0$$

This inequality holds true for all points whose co-ordinates x and y are such that the expressions A = sin $[(x + y)/2]$ and B = sin $[(y - x)/2]$ have the same signs.

Let us first that the expression A. solving the equation sin $[(x + y)/2] = 0$, we find $x + y = 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots\ldots$ Geometrically, this signifies that the expression A reduces to zero only the co-ordinates x and y of points in the plane which lie on one of the straight lines $y = -x + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots\ldots$ (in **Fig. 27** these

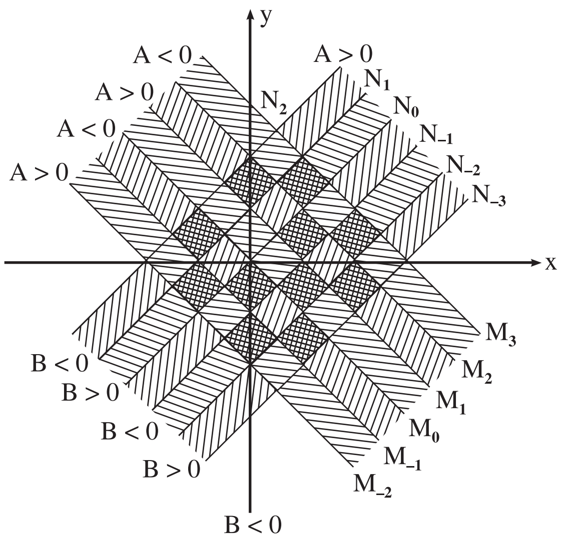


Fig. 27

straight lines are indicated by solid lines). For the sake of brevity, we denote by Mk the straight line $y = -x + 2k\pi$ for integral k (thus, the straight line M_0 is the bisector of the second and fourth quadrants, M_{-1} has equation $y = -x - 2\pi$ etc.).

All the straight lines M_k are parallel and partition the plane into strips. Let us agree to call the strip between two adjacent lines M_k and M_{k+1} the strip $\{M_k, M_{k+1}\}$, the lines M_k and M_{k+1} themselves not being included in this strip. For example, $\{M_0, M_1\}$ is the strip between the straight lines $y = -x$ and $y = -x + 2\pi$, that is, the set of points whose co-ordinates x and y satisfy the inequality

$0 < x + y < 2\pi.$

Analogously, in the general case, the strip $\{M_k, M_{k+1}\}$ is the set of points whose co-ordinates x and y satisfy the inequality $2k\pi < x + y < 2\,(k + 1)\pi$.

Now, let us determine the set of points whose coordinates x and y satisfy the inequality sin $[(x + y)/2] > 0$. This inequality can easily be solved; it is valid for $2.2n\pi < x + y < 2\,(2n + 1)\pi$, $n = 0, \pm 1, \pm 2 \ldots\ldots$

Geometrically, this signifies that the expression A is positive for it coordinates x and y of all points lying in each of the strips $\{M_{2n}, M_{2n+1}\}$, $n = 0, \pm 1, \pm 2, \ldots\ldots$ *i.e.*, in each strip bounded from below by the straight line M_{2n} with even index and from above by M_{2n+1}.

In the same way, by solving the inequality sin $[(x + y)/2] < 0$ we convince ourselves that expression A is negative for the co-ordinates x and y of all points lying in each of the strips $\{M_{2.n-1}, M_{2n}\}$, $n = 0. \pm 1, \pm 2 \ldots\ldots$ In each of the strips $\{M_k, M_{k+1}\}$ of **Fig. 27** is indicated the sign of expression A: the strips where A > 0 are marked with horizontal lines, the strips where A < 0 are marked with vertical lines.

Let us now examine expression B. Similar reasoning shows that expression B is made to vanish by the co-ordinates x and y of point lying on the straight lines $y = x + 2m\pi$, $m = 0 \pm 1, \pm 2 \ldots\ldots$ (these are shown dashed in **Fig. 27**). For integral m we denote the straight line $y = x + 2m\pi$, N_m, and we will agree to call the strip between adjacent lines N_m and N_{m+1} the strip $\{N_m, N_{m+1}\}$ (the straight lines N_m and N_{m+1} themselves do not belong to this strip). It is easy to verify that the strip $\{N_m, N_{m+1}\}$ is a set of points whose co-ordinate x and y satisfy the inequality $2m\pi < y - x < 2\,(m + 1)\pi$.

Solving the inequalities B> 0 and B < 0, we see that B is positive for the co-ordinates x and y of all points lying in each of the strips $\{N_{2p}, N_{2p+1}\}$, $p = 0, \pm 1, \pm 2 \ldots\ldots$, which is to say in each strip bounded from below by the line N_{2p} with even index and from above by N_{2p+1}. Furthermore, the expression B is negative for the co-ordinates x and y of all points lying in each of the strip $\{N_m, N_{m+1}\}$, $p = 0, \pm 1, \pm 2 \ldots\ldots$ In **Fig. 27**, in each of the strip $\{N_m, N_{m+1}\}$ is indicated the sign of the expression B : strips with B > 0 are marked with vertical lines, those with B < 0 are marked with horizontal lines.

It is now easy to describe the set of points, in the plane, whose co-ordinates x and y satisfy the inequality A.B > 0 : it includes all rectangles (excluding their contours) which are double cross-hatched in **Fig. 27**).

Exercise – B

1. (B) $Ma\,[md\,(a), mn\,(md(b), a), mn\,(ab, md\,(ac))]$

   $Ma\left[|-2|,\ mn\,(|-3|, -2)\ mn\,(6, |-8|)\right]$

   $Ma\,[2, mn\,(3, -2), mn(6, 8)]$

   $Ma\,[2, -2, 6] = 6$

2. (A) $Ma\,[md(a), mn(a, b)] = mn\,[a, md(Ma(a, b)]$

   $Ma\,[2, -3] = mn\,[-2, md(-2)]$

   $2 = mn\,(-2, 2)$

   $2 = -2$

   Relation does not hold for $a = -2$ and $b = -3$ or $a < 0$, $b < 0$

3. (B)

$$\begin{aligned} \text{fog}\,(x) &= f\{g(x)\} = f\left(\frac{x-3}{2}\right) \\ &= 2\left(\frac{x-3}{2}\right) + 3 = x \\ \text{gof}\,(x) &= g\,\{f(x)\} = g(2x + 3) \\ &= \frac{2x + 3 - 3}{2} = x \\ \therefore \quad \text{fog}\,(x) &= \text{gof}\,(x) \end{aligned}$$

4. (C)
$$\begin{aligned} f(x) &= g(x-3) \\ 2x+3 &= \frac{x-3-3}{2} = \frac{x-6}{2} \\ 4x+6 &= x-6 \\ 3x &= -12 \\ x &= -4 \end{aligned}$$

5. (B) {go fo fo go go $f(x)$} {fo go $g(x)$} from Q.3, we have $\text{fog}(x) = \text{go}f(x) = x$.

Therefore, above expression becomes $(x).(x) = x^2$

6. (C) fo (fog) o (gof) (x)

We have, fog (x) = gof $(x) = x$

So, given expression reduces to $f(x)$ that is $2x + 3$

7. (A) $me\ (a + mo\ (le\ (a, b)), mo\ (a + me\ (mo\ (a), mo\ (b)))$

Given $a = -2, b = -3$

$$\begin{aligned} a + mo\ (le\ (a, b)) &= -2 + mo(\text{le}(-2, -3)) \\ &= -2 + mo\ (-3) \\ &= -2 + 3 = 1 \end{aligned}$$

$mo\ (a + me\ (mo(a), mo(b)))$

$= mo(-2 + me\ (mo\ (-2), mo(-3)))$

$= mo(-2 + me\ (2, 3)) = mo(-2, +3) = mo(1) = 1$

$me(1,1) = 1$

8. (D, A) $mo(le(a, b)) \geq me(mo(a), mo(b))$

$= le\ (a, b) > me(a,b)$ as $a, b > 0$ which is false.

(B) $mo\ (le(a, b)) > me\ (mo(a), mo(b))$ which is again false.

Can be true only for $a = b$.

(C) $mo\ (le(a, b)) < le(mo(a), mo(b))$

Or $le\ (a, b) < le(a, b)$ which is false.

(D) $mo\ (le(a, b) = le\ (mo(a), mo(b))$

Or $le\ (a, b)) = le\ (a, b)$ **TRUE**

9. (B) $me(a^2 - 3a, a - 3) < 0$ or $me[a(a-3), a-3] < 0$

Case I. $a < 0, a^3 - 3a > a - 3$

$\Rightarrow$ $a(a-3) < 0$ or $0 < a < 3$ which is not true.

Case II. $0 < a < 3$, $a(a-3) < 0$ or $0 < a < 3$ which is true.

Case III. $a = 3$, $me\ (0,0) <$ not true.

Case IV. $a > 3$, $a(a-3) < 0$ or $0 < a < 3$ not true.

Alternatively, it can also be found by putting some values of a, say $a = -1$ in case I. $a = 1$ in case II and $a = 4$ in case IV.

10. (B) $le\ (a(a-3), (a-3)) < 0$

Again in case I, $a < 0$; $a - 3 < 0$ or $a < 3$ (from last question) can be true

In case II, $0 < a < 3$; $a - 3 < 0$ or $a < 3$ can be true

In case III, $a = 3$, $\text{le}(0,0) = 0 < 0$, not true

In case IV, $a > 3$, $a - 3 < 0$ or $a < 3$ not true. Hence (B) and (C) are correct.

11. (C) Equating $2 + x^2 = 6 - 3x$

$$\begin{aligned} \Rightarrow \quad x^2 + 3x - 4 &= 0 \\ \Rightarrow \quad x^2 + 4x - x - 4 &= 0 \\ \text{Or} \quad (x+4)(x-1) &= 0 \\ \Rightarrow \quad x &= -4 \text{ or } 1 \end{aligned}$$

But $x > 0$ so $x = 1$, so LHS = RHS = 2 + 1 = 3. It means the largest value of function min $(2 + x^2, 6 - 3x)$ is 3.

12. (D) M (M(A(M(x, y),S(y, x)), x),A(y, x)

M(M(A(6, 1), 2), A(3, 2))

M(M(7, 2), A(3, 2))

M(14 ,5) = 70.

13. (B) S[M(D(A(a, b), 2), D(A(a, b), 2)), M(D(S(a, b), 2), D(S(a, b), 2))]

$\Rightarrow$ S[M(D$(a + b, 2)$, D$(a + b, 2)$),

M(D$(a - b, 2)$), D$(a - b, 2)$)]

$$\Rightarrow \text{S}\left[\text{M}\left(\left(\frac{a+b}{2}\right)\left(\frac{a+b}{2}\right)\right), \text{M}\left(\frac{a-b}{2}, \frac{a-b}{2}\right)\right]$$

$$\Rightarrow \text{S}\left[\left(\frac{a+b}{2}\right)^2, \left(\frac{a-b}{2}\right)^2\right]$$

$$= \frac{(a+b)^2 - (a-b)^2}{2^2}$$

$$= \frac{(2a)(2b)}{4} = ab$$

14. (D) Since, $x > y > z > 0$

$$\begin{aligned} \therefore \quad la(x, y, z) &= y + z \\ \text{and} \quad le &= \max(x - y, y - z) \end{aligned}$$

We cannot find the value of le. Therefore we can't say whether $la > le$ or le $> la$.

Hence, we can't comment, as data is insufficient.

15. (B)
$$\begin{aligned} la\ (10, 5, 3) &= 8 \\ le\ (8,5,3) &= 3 \\ ma\ (10, 4, 3) &= \frac{1}{2}[7 + 6] = \frac{13}{2} = 6{\cdot}5 \end{aligned}$$

16. (C)
$$\begin{aligned} ma\ (15,10,9) &= \frac{1}{2}[19 + 5] = 12 \\ \min(10,6) &= 6 \\ le\ (9,8,12) &= 1 \\ le\ (15,6,1) &= 9 \end{aligned}$$

17. (C) $(2 \# 1)/(1\ \Delta\ 2) = \dfrac{2+1}{2^{2+1}} = \dfrac{3}{8}$

18. (A)
$$\begin{aligned} \text{Numerator} &= 4 - [(10^{1{\cdot}3}\ \Delta \log_{10})\ 0{\cdot}1] \\ &= 4 - (10^{1{\cdot}3}\ \Delta\ (-1) = 4 - 1 = 3 \\ \text{Denominator} &= 1 \nabla 2 = 2^{1+2} \\ &= 8 \text{ hence answer } \frac{3}{8} \end{aligned}$$

19. (B) Try for (A), (C) and (D) all give numerator and denominators as 1 *i.e.*, $\dfrac{\text{Num}}{\text{Den}} = \dfrac{1}{2} = 1$.

Hence, (B) is the answer.

20. (B) Going by option elimination.

(A) will be invalid when $x + y = 0$

(B) is the correct option as both sided gives $-2\,|x + y|$ as the result.

(C) will be equal when $(x + y) = 0$.

(D) is not necessarily equal (plug values and check)

21. (C) consider option (C) as $-F(x, y)$. $G(x, y) = [-|x + y|.|x + y|] = 4x^2$ for $x = y$.

And $\log_2 16 = \log_2 2^4 = 4$, which gives values of option (C) as x^2.

22. (B) Solve sequentially from innermost bracket to get the answer is (B).

23. (D) From the graph $F_1(x) = F(x)$ for $x \in (-2, 0)$ but, $F_1(x) = -F(x)$ for $x \in (0, 2)$.

24. (D) From the graphs, $F_1(x) = -F(x)$ and also $F_1(x) = F(-x)$. So, both (A) and (B) are satisfied which is not given in any of the option.

25. (D) By observation $F_1(x) = -F(x)$ and also $F_1(x) = F(-x)$. So, both (A) and (B) are satisfied. Since no option is given mark (D) as the answer.

26. (C) By observation $F_1(x) = -F(-x)$. This can be checked by taking any value of x say 1, 2 so answer is (C).

27. (A)
$$@(A, B) = \frac{A + B}{2}$$
$$/(@(A, B), 2) = \left(\frac{A + B}{2}\right) \times 2 = A + B$$

28. (D) X(/(@(/(@ (A, B), 2), C), 2), 3)
$$= \left(\left(\left(\left(\frac{(A + B)}{2} * 2\right) + C\right) /2\right) *2\right) /3$$
$$= \frac{A + B + C}{3}$$
= average of A, B and C.

29. (D)
$$\begin{Bmatrix} x^2 < x, \ 0 < x < 1 & f(x, y) = (x + y)^{0\cdot 5} \\ x^2 > x \quad\quad 1 < x & g(x, y) = (x + y)^2 \end{Bmatrix}$$
when x and y are positive. Thus for $x + y > 1$, $(x + y)^{0.5} < (x + y)^2$ therefore, $f(x, y) < g(x, y)$. We can therefore eliminate answer option a if x and y are both negative then $f(x, y) = (x + y)^2$ and $g(x, y) = -(x + y)$ now for $-1 < x + y < 0$, then $(x + y)^2 < -1x + y$

Therefore, $f(x, y) < g(x, y)$ thus answer option (B) is eliminate. As in evident from the above discussion, for x and $y > 1$, we cannot again guarantee that $f(x, y) > g(x, y)$.

30. (C) When $0 \le x, y < 0\cdot 5$, $x + y$ may be < 1 or 1, so given statement (A) cannot be true or false. when $x, y < -1$, again statement (B) can be true or false.

When $x, y > 1$, $x + y > 1$, hence $f(x, y) < g(x, y)$. $f(x, y) > g(x, y)$.

Thus, statement (C) given is necessarily false.

31. (B) when $x + y = 1$, we have $(x + y)^2 = (x + y)^{0\cdot 5}$ *i.e.,* $f(x, y) = g(x, y)$. Thus answer is (B).

32. (B) It is not linear in x and y, that's why option (a) is neglected. It also can't be exponential. By substituting X and Y in $y = a + bx + cx^2$ we see that it gets satisfied.

33. (D) $f(x + 1, y) = f[f, f(x, y)]$

Put $x = 0, f(1, y) = f[0, f(0, y)] = f[0, y + 1]$
$$= y + 1 + 1 = y + 2$$
Put $y = 2, f(1, 2) = 4$.

34. (B) As graph is symmetrical about y-axis, we can say function is even so $f(x) = f(-x)$.

35. (D) We see from the graph. Value of $f(x)$ in the left region is twice the value of $f(x)$ in the right region. So, $2f(x) = f(-x)$ or $6f(x) = 3f(-x)$.

36. (C) $f(-x)$ is replication of $f(x)$ about y-axis $-f(x)$ is replication of $f(x)$ about x-axis and $-f(-x)$ is replication of $f(x)$ about y-axis followed by replication about x-axis. Thus, given graph is of $f(x) = -f(-x)$.

37. (C) putting the actual values in the functions, we get the required answers.
$$m(a, b, c) = -5, M(a, b, c) = 2$$
So, $[m(a, b, c) + M(a, b, c)]/2$ is maximum.

38. (C) $m(a, b, c) = \min(a + b, c, a)$; $-M(-a, a, -b) = -\max(0, -b, -a)$; $m(a + b, b, c) = \min(a + 2b, c, a + b)$

39. (C) $m(M(a - b, b, c), m(a + b, c, b), -M(a, b, c)) = m(3, 4, -6) = -6$.

40. (D)
$$f(1) = \frac{1}{1 + 1} = \frac{1}{2} \text{ as } x \text{ is positive.}$$
$$f^2(1) = f(f(1)) = \frac{1}{1 + \frac{1}{2}} = \frac{2}{3},$$
$$f^3(1) = f(f^2(1)) = f[2/3] = \frac{3}{5},$$
$$f^4(1) = \frac{5}{8} \text{ thus } f^1(1) f^2(x) f^3(1)$$
$$\ldots f^9(1) = \frac{1}{8}$$

41. (C) When x is negative, $f(x) = 1 + x$
$$f(-1) = 1 - 1 = 0;$$
$$f^2(-1) = f(f(-1)) = f(0) = 1;$$
$$f^3(-1) = f(f^2(-1)) f(1) = \frac{1}{1 + 1} = \frac{1}{2};$$
$$f^4(-1) = f(f^3(-1)) f(1/2)$$
$$= 2/3 \text{ and } f^5(-1) = 3/5.$$

42. (D) Clearly, BA $\ge$ MB A_1 and MB $A_2 \le$ BA as $n_1 \ge n_2 + n_2$.

So option (A), (B) and (C) are neglected.

See $BA = \frac{r_1}{n_1} + \frac{r_2}{n_1} \ge \frac{r_1}{n_1} + \frac{n_2}{n_1} \max\left[0, \frac{r_2}{n_2} - \frac{r_1}{n_1}\right]$

because $\frac{r_2}{n_1} \geq 0$ and $\frac{r_2}{n_1} \geq \left(\frac{n_2}{n_1} \times \frac{r_2}{n_2} - \frac{n_2}{n_1} \times \frac{r_1}{n_1}\right)$

or $\frac{r_2}{n_1} \geq \frac{r_2}{n_1} - \frac{n_2 r_1}{{n_1}^2}$

So, none of the answers match.

43. (B) initial BA = 50, BA increases as numerator increases with denominator remaining the same.

MB $A_2 = \frac{r_1 + r_2}{n_1 + n_2}$ decrease as average of total runs decrease from 50, as runs scored in this inning are less than 50.

44. (B) $f(x) = \log\left(\frac{1+x}{1-x}\right)$

and $f(y) = \log\left(\frac{1+y}{1-y}\right)$

$$\therefore \quad f(x) + f(y) = \log\left(\frac{1+x}{1-x}\right) + \log\left(\frac{1+y}{1-y}\right)$$

$$= \log\left\{\left(\frac{1+x}{1-x}\right)\left(\frac{1+y}{1-y}\right)\right\}$$

$$= \log\left(\frac{1+x+y+xy}{1-x-y-xy}\right)$$

$$= \log\frac{(1+xy)\left(1+\frac{x+y}{1+xy}\right)}{(1+xy)\left(1-\frac{x+y}{1+xy}\right)}$$

[divided the Nr and Dr by (1+xy)]

$$= \log\frac{1+\frac{x+y}{1+xy}}{1-\frac{x+y}{1+xy}} = f\left(\frac{x+y}{1+xy}\right)$$

45. (D) $[x]$ means if $x = 5{\cdot}5$, then $[x] = 5$

$$L\,[x, y] = [x] + [y] + [x + y]$$

$$R\,(x, y) = [2x] + [2y]$$

Relationship between $L(x, y)$ and $R(x, y)$ can be found by putting various values of x and y. Put $x = 1{\cdot}6$ and $y = 1{\cdot}8$; $L(x, y) = 1 + 1 + 3 = 5$ and $R(x, y) = 3 + 3 = 6$, so (B) and (C) are wrong.

If $x = 1{\cdot}2$ and $y = 2{\cdot}3$

$L(x, y) = 1 + 2 + 3 = 6$ and $R(x, y) = 2 + 4 = 6$ or $R(x, y) = L(x, y)$, so (A) is not true.

We see that (D) will never be possible.

46. (D) $g(x) = \max(5 - x, x + 2)$. Drawing the graph.

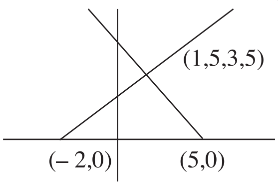


Fig. 28

The dark lines represent the function $g(x)$. It clearly shows the smallest value of $g(x) = 3{\cdot}5$

47. (B) $f(x) = |x - 2| + |2{\cdot}5 - x| + |3{\cdot}6 - x|$ can attain minimum value when either of the terms = 0.

Case I : when $|x - 2| = 0 \Rightarrow x = 2$, value of $f(x)$

$= 0{\cdot}5 + 1{\cdot}6 = 2{\cdot}1$

Case II : when $|2{\cdot}5 - x| = 0 \Rightarrow x = 2{\cdot}5$ value of $f(x)$

$= 0{\cdot}5 + 0 + 1{\cdot}1 = 1{\cdot}6$

Case III : when $|3{\cdot}6 - x| = 0 \Rightarrow x = 3{\cdot}6$

$\Rightarrow \quad f(x) = 1{\cdot}6 + 1{\cdot}1 + 0 = 2{\cdot}7$

Hence, the minimum value of $f(x)$ is $1{\cdot}6$ at $x = 2{\cdot}5$.

48. (B) The curves can be plotted as follows :

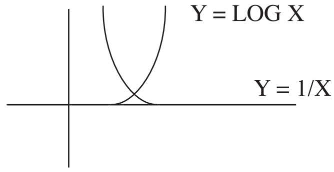


Fig. 29

We see that they meet once.

49. (D) substitute values $-2 \leq x \leq 2$ in the given curves, we find the curves will intersect at $x = 0$, 1 and – 1.

50. (A) From the table, we have $g*g = h$ (this is g squared) $h*g = f$ (this is g cubed) $f*g = e$.(this is g to the power 4)

51. (D) $f \oplus [f * \{f \oplus (f * f)\}]$ is to be simplified so we start from the innermost bracket

$$f*f = h$$
$$f \oplus h = e$$
$$f*e = f$$
$$f \oplus f = h.$$

52. (A) $\{a^{10}*(f^{10} \oplus g^9)\} \oplus e^8$

$f*f = hg*g = ha*a = ae*e = e$

$h*f = gh*g = fa^{10} = a\,e^8 = e$

$g*f = ef*g = e$

$e*f = fe*g = g$

$f^5 = fg^5 = g$

So, $f^{10} = f^5$ & $f^5 = f*f = h$ so, $g^9 = g^{5} * g^4 = g*e = g$

$\because \quad a^{10}*(f^{10} \oplus g^9)\} \oplus e^8$

$\{a*(h \oplus g)\} \oplus e$

$\{a*f\} \oplus e \Rightarrow e$.

53. (D) $y = ax^2 - b\,|x|$

As the options (A) and (C) include $a > 0$, $b > 0$ we take $a = b = 1$.

Accordingly the equation becomes $y = x^2 - |x|$. A quick plot gives us.

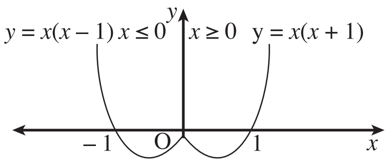


Fig. 30

So, at $x = 0$, we neither a maxima nor a minima As the options (B) and (D) include $a > 0$, $b < 0$ we take $a = 1$, $b = -1$

Accordingly the equation becomes $y = x^2 + |x|$

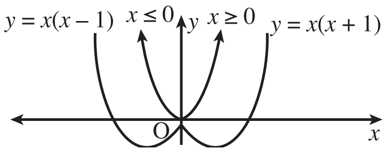


Fig. 31

so at $r = 0$, we have a minima.

54. (B) $f(x) = x^3 - 4x + p$

$f(0) = p, f(1) = p - 3$

Given $f(0)$ and $f(1)$ are of opposite signs,

$p(p-3) < 0$

If $p < 0$ then $p - 3$ is also less than 0.

$\therefore$ P$(p-3)$>0 *i.e.*, p cannot be negative

$\therefore$ Choices (A), (C) and (D) are eliminated $0 < p < 3$.

55. (C) Consider the product $f_1(x) f_2(x)$; for $x \ge 0, f_2(x) = 0$, hence $f_1(x) f_2(x) = 0$ and for $x < f_1(x) = 0$, hence $f_1(x) f_2(x) = 0$ consider the product $f_2(x) f_3(x)$;

for $x \ge 0, f_2(x) = 0, f_3(x) = 0$, hence $f_2(x) f_3(x) = 0$

for $x < 0, f_2(x) > 0, f_3(x) < 0$, hence $f_2(x) f_3(x) < 0$

Consider the product $f_2(x) f_4(x)$

for $x \ge 0, f_2(x) = 0$, hence $f_2(x) f_4(x) = 0$

for $x < 0, f_4(x) = 0$, hence $f_2 . f_4(x) = 4$

$\therefore$ $f_1(x) . f_2(x)$ and $f_3(x) . f_4(x)$ always take a zero value.

56. (B) **Choice (A) :** from the graph it can be observed that $f_1(x) = f_4(x)$, for $x \le 0$ but $f_1(x) \ne f_4(x)$ for $x > 0$.

Choice (B) : The graph of $f_3(x)$ is to be reflected in x-axis followed by a reflection in y-axis (in either order), to obtain the graph of $-f_3(-x)$ this would give the graph of $f_1(x)$.

Choice (C) : The graph of $f_2(-x)$ is obtained by the reflection of the graph of $f_2(x)$ in y-axis, which gives us the graph of $f_1(x)$ and not $f_4(x)$, hence option 3 is ruled out.

Choice (D) : For $x > 0$, $f_1(x) > 0$ and $f_3 = 0$, hence $f_1(x) + f_3(x) > 0$.

5 Inequalities

It is quite common, at examinations, for the aspirant to be asked to solve certain numerical or literal inequalities. This section is devoted to an analysis of proof of literal and numerical inequalities. It would be nice of course if there were some unified method for providing all inequalities. Unfortunately, no such method exists. But we will give below a number of techniques that are of use in proving a rather large number of inequalities.

First we take up some inequalities that are frequently used in problem solving, such as the inequalities between an arithmetic mean and a geometric mean, a consequences of this inequalities concerning the sum of reciprocal quantities, and also the following trigonometric inequality :

$$-\sqrt{a^2+b^2} \le a \sin x + b \cos x \le \sqrt{a^2+b^2} \qquad \ldots(1)$$

The relation between an arithmetic mean and a geometric mean of two numbers reads :

For any two non-negative numbers a and b the inequality

$$\sqrt{ab} \le \frac{a+b}{2} \qquad \ldots(2)$$

holds true; equality occurs only, when $a = b$

A special case of (2) is the inequality

$$x + \frac{1}{x} \ge 2$$

which is valid for all $x > 0$. In this inequality, the equals sign holds for $x = 1$ only. It is useful to remember the verbal statement of this inequality.

The sum of two positive reciprocals does not exceed two, and is equal to two only when both numbers are equal to unity.

Also, note that for any $x \ne 0$ the inequality holds true.

$$\left|x + \frac{1}{x}\right| \ge 2$$

$$\left|\frac{1+x^2}{2x}\right| \ge 1 \qquad \ldots(3)$$

Illustration 1. Prove the inequality

$$\frac{1}{\log_2 \pi} + \frac{1}{\log_\pi 2} > 2.$$

By the properties of logarithms, $1/\log_\pi 2 = \log_2 \pi > 0$, which means that the left member of our inequality is the sum of two positive reciprocal different from unity ($\log_2 \pi \ne 1$).

Such a sum is greater than two. Hence, the original inequality holds true.

Illustration 2. Prove that if $a > 0, b > 0, c > 0$, then

$$\frac{bc}{a} + \frac{ac}{b} + \frac{ab}{c} \ge a + b + c$$

We take advantage of the following inequalities :

$$\frac{1}{2}\left(\frac{bc}{a} + \frac{ac}{b}\right) \ge c, \frac{1}{2}\left(\frac{ac}{b} + \frac{ab}{c}\right) \ge a, \frac{1}{2}\left(\frac{bc}{a} + \frac{ab}{c}\right) \ge b$$

(these inequalities are valid because the left members are arithmetic means and the right members are geometric means of positive numbers). Combining them term by term, we get inequality that we wish to prove.

Illustration 3. Prove that if $a > 0, b > 0, c > 0$, then

$$(a + b)(b + c)(a + c) \ge 8\,abc$$

Taking the following inequalities (see formula [2])

$$a + b \ge 2\sqrt{ab}, b + c \ge 2\sqrt{bc}, a + c \ge 2\sqrt{ac}$$

and multiplying them term wise, we get the desired inequality.

This inequality may be proved in a different manner by using the inequality between the arithmetic mean and the geometric mean for 8 positive numbers (see formula (5)). Indeed, removing brackets in the left member of our inequality, we find that it can be rewritten as follows :

$$\frac{a^2b + b^2c + c^2b + a^2c + b^2a + c^2a + abc + abc}{8} \ge abc$$

On the left side we have the arithmetical mean of 8 positive numbers; on the right, as can readily be verified, we have their geometric mean, which completes the proof of the original inequality.

Before going on to the next problem, let us dwell on a typical mistake that is rather often made in proving inequality. It is this. The aspirant writes the inequality to be proved, then performs certain (quite legitimate) manipulations and finally arrives at an obviously valid inequality (say $1 < 2$ or $(a - b)_2 \ge 0$) and then concludes : "hence, the inequality is proved." This is a crude logical error : from the fact that a true inequality has been obtained, we can by no means conclude that the original inequality was true! To be more exact, we proved the following : if one assumes that the proposed inequality is true then the inequality obtained *via* a chain of transformations is true, then the inequality obtained *via* a

chain of transformations is also true. But it is obvious that the final inequality is true as it stands; and we continue to know nothing about the inequality which we set out to prove.

It is logically correct to reason in the reverse order. It is necessary to take some obviously valid inequality and perform manipulations (which of course must be legitimate from the viewpoint of algebra and trigonometry) that will bring us to the inequality to be proved. This is justified reasoning : we started with a valid inequality and via a chain of legitimate transformations arrived at the new inequality, which, hence must also be valid.

Of course there remains the most important question. From what inequality are we to proceed so as to transform it into the required inequality? To answer his question we can perform the transformation of the proceed inequality that leads us to an obviously valid inequality. However, this stage in the solution of the problem must be regarded as an exploratory search for the proof, as an attempt to get the proper approach, but not as proof in itself. If as a result of this exploration (manipulations) we have obtained an obviously true inequality, then we can begin the proof proper : take this obviously correct inequality and manipulate it as we did in the exploratory search, but in reverse order; inverse the manipulations, so to say. If this "work backward procedure" is everywhere legitimate, then the inequality being proved is indeed valid.

Incidentally, a somewhat different procedure is often followed. If, in the process of exploring the proof via a reduction of our inequality to an obvious inequality, we always replaced the given inequality by an equivalent one, then the last inequality will be equivalent to the original one, and therefore its validity implies the validity of the original inequality. Hence, if at each stage in the transformation we specially verified and stressed the equivalence of the inequality, then the " work backward procedure" is not necessary.

We shall follow this reasoning in carrying out the proof of the following inequalities.

Illustration 4. Prove the inequality

$$\frac{a^3 + b^3}{2} \geq \left(\frac{a+b}{2}\right)^3$$

where $a > 0$, $b > 0$

Replace this inequality by the equivalent one

$$\frac{a^3 + b^3}{2} - \left(\frac{a+b}{2}\right)^3 \geq 0$$

Removing brackets and regrouping, we can write it in the equivalent form

$$\frac{3}{8}(a+b)(a-b)^2 \geq 0$$

Since, $a > 0$ and $b > 0$, this inequality is obvious and, thus, the validity of the equivalent original inequality is proved.

Illustration 5. Prove that if $a > 0$, $b > 0$, then for any x and y the following inequality holds true :

$$a.2^x + b.3^y + 1 \leq \sqrt{4^x + 9^y + 1} \cdot \sqrt{a^2 + b^2 + 1}$$

By hypothesis, both sides of this inequality are positive and so it is equivalent to the following :

$$(a \cdot 2^x + b \cdot 3^y + 1)^2 \leq (4^x + 9^y + 1)(a^2 + b^2 + 1)$$

or to

$$a^2.4^x + b^2.9^y + 1 + 2ab\, 2^x\, 3^y + 2^a\, 2^x + 2b3^y$$
$$\leq 4^x a^2 + 4^x b^2 + 4^x + 9^y a^2 + 9^y b^2 + 9^y + a^2 + b^2 + 1$$

Transposing all terms of this inequality to the right side, and then collecting like terms and regrouping, we can write it in the equivalent from

$$(a^2 9^y - 2ab2^x 3^y + 4^x b^2) + (4^x - 2a2^x + a^2) + (9^y - 2b \cdot 3^y + b^2) \geq 0$$

Since, each parenthesis is a perfect square, the original inequality is equivalent to the following obvious inequality :

$$(a3^y - b2^x)^2 + (2^x - a)^2 + (3^y - b)^2 \geq 0$$

Hence, the original inequality is true.

Note that this inequality is also true for any real values of a and b (the proof of this fact is left to the reader).

Illustration 6. Prove that the inequality

$-1 \leq \dfrac{\sqrt{3} \sin x}{2 + \cos x} \leq 1$ is valid for arbitrary x.

Since, both members of this inequality are non- negative, then after squaring and multiplying by the positive expression $(2 + \cos x)^2$, we get an equivalent inequality : $3 \sin^2 x \leq (2 + \cos x)^2$. Replacing $\sin^2 x$ by $1 - \cos^2 x$ and grouping, we finally get $(2 \cos x + 1)^2 \geq 0$. This inequality holds true for all x, and since it is equivalent to the original one, the original inequality is also true, which is what we set out to prove.

The original inequality may be proved differently by making use of inequality (1). Indeed since $2 + \cos x > 0$ for all x, then, after multiplying by $2 + \cos x$, we get the following double inequality which is equivalent to the original one :

$$-2 - \cos x \leq \sqrt{3} \sin x \leq 2 + \cos x$$

The inequality on the left may be written as

$$-2 \leq \sqrt{3} \sin x + 1 \cdot \cos x$$

It is now evident that this is a special case of inequality (1) which is a true inequality. The validity of the inequality on the right is proved similarly.

Illustration 7. Prove that for arbitrary α the inequality $4 \sin 3\alpha + 5 \geq 4 \cos 2\alpha + 5 \sin \alpha$ is valid.

One of the crudest errors made in proving this inequality is the "proof" by substitution of specific values.

At an examination, *a* number of aspirants reasoned something like this : "for $\alpha = 0°$ the inequality holds because 5>4, for $\alpha = 30°$ the inequality is true because

$$4 + 5 > 4\cdot\frac{1}{2} + 5\cdot\frac{1}{2}$$, for $\alpha = 45°, 60°, 90°$ it is also obviously true, which means it holds true for all values of α."

Actually, of course, these aspirants proved the inequality only for several separate values of α and offered no proof what so ever for the remaining values of α. A correct proof is as follows.

We know that $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$, $\cos 2\alpha = 1 - 2\sin^2\alpha$ and so the original inequality may be rewritten as

$$4(3\sin\alpha - 4\sin^3\alpha) + 5 \geq 4(1 - 2\sin^2\alpha) + 5\sin\alpha$$

or, $16\sin^3\alpha - 8\sin^2\alpha - 7\sin\alpha - 1 \leq 0$

The latter inequality should be valid for all values of α. Denoting $\sin\alpha$ by x, we rewrite it as

$$16x^3 - 8x^2 - 7x - 1 \leq 0$$

We now have to prove that this inequality is valid for arbitrary values of x in the interval $-1 \leq x \leq 1$

or $(x - 1)(4x + 1)^2 \leq 0$. This inequality is clearly valid, and so the original inequality is proved.

The proof of certain inequalities requires skill in utilizing the properties of functions that enter into the inequalities.

Illustration 8. Prove that the inequality

$$\cos(\cos x) > 0 \text{ is true for all } x.$$

For all x we have $-1 \leq \cos x \leq 1$. Put $\alpha = \cos x$ to get $-1 \leq \alpha \leq 1$. Since $-\pi/2 < -1$ and $1 < \pi/2$, it follows all the more so that α satisfies the condition $-\pi/2 < \alpha < \pi/2$. The properties of the function $y = \cos x$ imply that $\cos\alpha$ is positive for all these values of α, which actually is what we set out to prove.

Illustration 9. Prove the inequality $\cos(\sin x) > \sin(\cos x)$.

It can be rewritten as

$$\cos(\sin x) - \cos\left(\frac{\pi}{2} - \cos x\right) > 0$$

or, 2 sin

$$\left(\frac{\pi}{4} + \frac{\sin x - \cos x}{2}\sin\left(\frac{\pi}{4} - \frac{\sin x + \cos x}{2}\right) > 0\right)$$

We shall that the factors in the left-hand member are positive since

$$|\sin x - \cos x| = |\sqrt{2}\sin(x - \pi/4) \leq \sqrt{2} < \pi/2$$

it follows that $-\frac{\pi}{4} < \frac{\sin x - \cos x}{2} < \frac{\pi}{4}$

and therefore $0 < \frac{\pi}{4} + \frac{\sin x - \cos x}{2} < \frac{\pi}{2}$

Consequently $\sin\left(\frac{\pi}{4} + \frac{\sin x - \cos x}{2}\right) > 0$

for all x. A similar proof is given that

$$\sin\left(\frac{\pi}{4} + \frac{\sin x - \cos x}{2}\right) > 0$$

A decisive factor in the example which is the use of properties of the exponential function $y = a^x$ if $a > 1$, a larger value of the argument is associated with a larger value of the function and, hence, a greater value of the function is associated with a greater value of the argument; if $a < 1$, to a greater value of the argument corresponds a smaller value of the function and, hence, to a larger value of the function corresponds a smaller value of the argument.

Illustration 10. Prove that possible numbers c and d and arbitrary $\alpha > 0$, the inequalities $c < d$ and $c^\alpha < d^\alpha$ are equivalent.

Let c and d be positive numbers and $\alpha > 0$. Consider the function $y = (c/d)\, x$.

If $c < d$, then $0 < c/d < 1$. By the property of an exponential function with base less than unity, we have

$$\left(\frac{c}{d}\right)^\alpha < \left(\frac{c}{d}\right)^\circ$$

whence follows $c^\alpha/d^\alpha < 1$, or $c^\alpha < d^\alpha$

Conversely, if $c^\alpha < d^\alpha$ then $c^\alpha/d^\alpha < 1$,

or $\left(\frac{c}{d}\right)^\alpha < \left(\frac{c}{d}\right)^\circ$

This means that the larger value of argument ($\alpha > 0$) of our function is associated with a smaller value of the function. But this is true only when the base is less then unity, that is $c/d < 1$, whence $c<d$.

The statement we have just proved is ordinarily formulated as follows : an inequality between positive numbers may be raised to any positive power : in particular, a root of any degree may be extracted.

Illustration 11. Prove the inequality

$(\alpha^\alpha + b^\alpha)^{1/\alpha} \leq (a^\beta + b^\beta)^{1/\beta}$

For $a \geq 0, b \geq 0, \alpha > \beta > 0$

If $a = 0$ or $b = 0$, then the proposition is obvious. Now, let $a > 0$ and $b > 0$. It is clear that one of these numbers does not exceed the other.

Suppose, say,

$0 < a \leq b$. Then $0 < a/b \leq 1$, and since $\alpha > \beta$,

it follows that

$0 < (a/b)^\alpha \leq (a/b)^\beta$ and $1 + (a/b)^\alpha \leq 1 + (a/b)^\beta$

From the latter inequality we get (see example 10)

$$[1 + (a/b)^\alpha]^{1/\beta} \leq [1 + (a/b)^\beta]^{1/\beta}$$

Furthermore, since

$$1 + (a/b)^\alpha \geq 1 \text{ and } 0 < 1/\alpha < 1/\beta$$

It follows that

$$[1 + (a/b)^\alpha]^{1/\alpha} \leq [1 + (a/b)^\alpha]^{1/\beta}$$

Now, we can write

$$[1 + (a/b)^\alpha]^{1/\alpha} \leq [1 + (a/b)^\alpha]^{1/\beta} \leq [1 + (a/b)^\beta]^{1/\beta}$$

whence

$$\left(\frac{a^{\alpha}+b^{\alpha}}{b^{\alpha}}\right)^{1/\alpha} \leq \left(\frac{a^{\beta}+b^{\beta}}{b^{\beta}}\right)^{1/\beta}$$

Since, $b > 0$, the inequality being proved follows from the last inequality.

Illustration 12. Prove the inequality $0 < \sin^8 x + \cos^{14} x \leq 1$.

It is quite obvious that $\sin^8 x + \cos^{14} x \geq 0$. But the equality $\sin^8 x + \cos^{14} x = 0$ is valid only if we simultaneously have $\sin^8 x = 0$ and $\cos^{14} x = 0$, which, of course, is impossible. Therefore, the strict inequality $\sin^8 x + \cos^{14} x > 0$ holds true.

The properties of trigonometric functions imply that $\sin^2 x \leq 1$ and $\cos^2 \leq 1$. for arbitrary real x. But since $8 > 2$ and $14 > 2$, it follows there from that

$\sin^8 x \leq \sin^2 x$ and $\cos^{14} x \leq \cos^2 x$

Combining these inequalities term wise and noting that $\sin^2 x + \cos^2 x = 1$, we obtain

$$\sin^8 x + \cos^{14} x \leq 1.$$

It is obvious here that, say, for $x = \pi/2$ we have equality; in other words, the weak inequality cannot be replaced by the strict inequality $\sin^8 x + \cos^{14} x \leq 1$.

One of the techniques used in proving inequalities consists in the following. For instance, let it be required to prove the inequality A < B, where A and B are certain expressions. If we succeed in finding an expression C such that A < C and at the same time C ≤ B then the required inequality A < B will have thus been proved.

Illustration 13. Prove that for every positive integer n the following inequality holds true :

$$\frac{1}{9}+\frac{1}{25}+\ldots\ldots+\frac{1}{(2n+1)^2}<\frac{1}{4}$$

Noting that $\dfrac{2}{(2k+1)^2} < \dfrac{1}{2k} - \dfrac{1}{2k+2}$

we replace the sum in the left member of the inequality to be proved by the greater expression

$$\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots+\frac{1}{(2n+1)^2}$$

$$<\frac{1}{2}\left[\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\ldots\left(\frac{1}{2n}-\frac{1}{2n+2}\right)\right]$$

However, this latter expression is equal to

$$\frac{1}{2}\left[\frac{1}{2}-\frac{1}{2n+2}\right]=\frac{1}{4}-\frac{1}{4n+4}$$

and, obviously, is less than 1/4. Hence, the sum $\dfrac{1}{9} + \dfrac{1}{25} + \ldots + \dfrac{1}{(2n+1)^2}$ is all the more so less than 1/4.

Illustration 14. Prove that for any positive integer $n > 1$ the inequality

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots\frac{1}{\sqrt{n}}>2\left(\sqrt{n+1}-1\right)$$ holds true.

To prove this, reduce each term of the sum in the left-handed member :

$$\frac{1}{\sqrt{k}}>\frac{2}{\sqrt{k}+\sqrt{k+1}}=2\left(\sqrt{k+1}-\sqrt{k}\right)$$

Therefore, the left side of the inequality we want to prove can be reduced :

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots\frac{1}{\sqrt{n}}>2\left(\sqrt{2}-\sqrt{1}\right)$$

$$+2\left(\sqrt{3}-\sqrt{2}\right)+\ldots\ldots+2\left(\sqrt{n}-\sqrt{n-1}\right)$$

$$+2\left(\sqrt{n+1}-\sqrt{n}\right)$$

Since, the right side of this latter inequality is exactly equal to $2\sqrt{n+1}-2$, in the original inequality is valid.

In the next example, an apt combining of factors leads us to the result we need.

Illustration 15. Prove that $n! < \left(\dfrac{n+1}{2}\right)^n$ where n is an integer exceeding unity.

The validity of this inequality will follows from the validity of the equivalent inequality

$$(n!)^2 < \left(\frac{n+1}{2}\right)^{2n}$$

Let us multiply the number $n! = 1.2\ldots\ldots\ldots\ldots k\ldots\ldots(n-1)n$ by the number $n! = n\,(n-1)\ldots\ldots(n-k+1)\ldots\ldots 2.1$ arranging them one above the other :

1	2	…………k	……….$(n-1)$	n	
n	$(n-1)$	……$(n-k+1)$	……….2	1	

Multiplying the numbers in each column, we get

$(1.n)\,[2(n-1)]\ldots[k\,(n-k+1)]\ldots[(n-1)\cdot 2]\,(n\cdot 1)$

In order to obtain $(n!)^2$ we have to multiply the terms of this row. Applying inequality (2) to each term of this row, we get

$$\sqrt{k\,(n-k+1)} \leq \frac{k+-k+1}{2}$$

$$=\frac{n+1}{2},\, k = 1, 2, \ldots\ldots\ldots, n$$

Equality being achieved only when $k = n - k + 1$, that is to say, for $k = (n+1)/2$. In other words, only for n odd; and only then for one term of our row in this inequality is equality possible. Hence, for all brackets except possibly one, the inequalities

$[k\,(n-k+1)] < \left(\dfrac{n+1}{2}\right)^2$ hold true.

Since, there n terms in the row, we get

$$(n!)^2 < \left[\left(\frac{n+1}{2}\right)^2\right]^n$$

A sufficiently large number of inequalities can be proved by the method of mathematical induction.

Illustration 16. Prove that for any real number $\alpha \geq -1$ and any positive integer n the inequality

$(1 + \alpha)^n \geq 1 + n\alpha$ ………………………. (4)

holds true.

The inequality is clearly true for $n = 1$. Suppose that the inequality $(1 + \alpha)^k \geq 1 + k\alpha$ holds true; we will prove that in that case in inequality $(1 + \alpha)^{k+1} \geq 1 + (k + 1)\alpha$ is valid indeed :

$(1 + \alpha)^{k+1} = (1 + \alpha)^k (1 + \alpha) \geq (1 + k\alpha)(1 + \alpha) = 1 + (k + 1)\alpha + k\alpha^2 \geq 1 + (k + 1)\alpha$.

This means that the original inequality holds true.

Illustration 17. Prove that the inequality $|\sin nx| \leq n|\sin x|$, is valid for any positive integer n.

For $n = 1$ the inequality is obviously true. Assuming that $|\sin kx| \leq k|\sin x|$, we will prove that $|\sin (k + 1)x| \leq (k + 1)|\sin x|$. Indeed, taking advantage of the inequality $|\cos kx| \leq 1$ we have

$$\begin{aligned}|\sin (k+1)x| &= |\sin kx \cdot \cos x + \sin x \cdot \cos kx| \\ &\leq |\sin kx| \cdot |\cos x| + |\sin x| \cdot |\cos kx| \\ &\leq |\sin kx| + |\sin x| \leq k|\sin x| + |\sin x| \\ &= (k+1).|\sin x|\end{aligned}$$

Hence, the original inequality is true.

Illustration 18. Prove the following theorem : if the product of $n \geq$ positive numbers is equal to 1, then the sum of the numbers is greater than or equal to n, that is if,

$x_1 x_2 \ldots\ldots x_n = 1, x_1 > 0, x_2 > 0, \ldots\ldots, x_n > 0$,

then $x_1 + x_2 + \ldots\ldots + x_n \geq n$

If $n = 2$ we have to prove the statement : if $x_1 x_2 = 1$, then $x_1 + x_2 \geq 2$. but this is obvious since the arithmetic mean $(x_1 + x_2)/2$ of two positive numbers is greater than or equal to the geometric mean $\sqrt{x_1 x_2} = 1$ or $x_1 + x_2 \geq 2$. Besides, equality (that is, $x_1 + x_2 = 2$) is attained only when $x_1 = x_2 = 1$.

Using induction, we take any positive numbers $x_1 \ldots\ldots x_k, x_{k+1}$ which satisfy the condition $x_1 \ldots\ldots x_{k-1} . x_k . x_{k+1} = 1$. If each of these number equals 1, then the sum $x_1 + \ldots\ldots x_k + x_{k+1} = k + 1$ so that in this case the original inequality is valid.

If this is not so, there will be a number among them less than 1 and a number greater than 1. Suppose that $x_k > 1$. $x_{k+1} < 1$ we have the equality

$x_1 \ldots\ldots x_{k-1} (x_k x_{k+1}) = 1$

This is a product of k numbers and so the induction hypothesis is applicable and we can assert that

$x_1 + \ldots\ldots + x_{k-1} + x_k x_k + 1 \geq 1$

But then

$$\begin{aligned}x_1 + \ldots\ldots + x_{k-1} + x_k + x_{k+1} &\geq k - x_k x_{k+1} + x_k + x_{k+1} \\ &= k + 1 + (x_k - 1)(1 - x_{k+1}) > k + 1\end{aligned}$$

Since, $x_{k-1} > 0$ and $1 - x_{k+1} > 0$ which completes the proof.

Note that we also established the fact that equally in the relation at hand is only possible if all $x_i = 1$; now if not all x_i are equal to unity, then the strict- inequality sign holds true in this relation.

From this theorem follows the generalized inequality between the arithmetic mean and the geometric mean for $n \geq 2$ positive numbers :

$$\frac{x_1 + \ldots\ldots + x_n}{n} \geq \sqrt[n]{x_1 \ldots\ldots x_n}, x_1 > 0, \ldots\ldots x_n > 0 \qquad \ldots(5)$$

Indeed denote $\sqrt[n]{x_1 \ldots\ldots x_n}$ by c and $x_{i/c}$ by y_i. Then $y_i \ldots\ldots y_n = (x_1 \ldots\ldots x_n)/c^n$. By what has been proved, $y_1 + \ldots\ldots y_n \geq n$, whence $(x_1 + \ldots\ldots + x_n)/n \geq c$ and this completes the proof.

This inequality is widely used in the proof of other inequalities. For example, if we apply it to the numbers $1, 2, \ldots\ldots, n$, then we immediately get the inequality

$$\sqrt[n]{1 \cdot 2 \ldots\ldots n} < \frac{1 + 2 + \ldots\ldots + n}{n}$$

or $\sqrt[n]{n!} < (n + 1)/2$, whence $n! < [(n + 1)/2]^n$. We proved this inequality in problem 15 via a special technique. This new proof is clearly simpler.

The foregoing examples show that the method of mathematical induction can successfully be applied in the proof of a variety of inequalities. At the same time, one should not overestimated the power of the induction method : there are many problems that would seem particularly suited to this method, whereas attempts to employs it encounter insuperable difficulties.

To illustrate, let us try to use in induction on the inequality

$$\frac{1}{9} + \frac{1}{25} + \ldots\ldots + \frac{1}{(2n+1)^2} < \frac{1}{4}$$

For $n = 1$ it has form $1/9 < 1/4$, which is true. Suppose that this inequality is valid for $n = k$:

$$\frac{1}{9} + \frac{1}{25} + \ldots\ldots + \frac{1}{(2k+1)^2} < \frac{1}{4}$$

For $n = k + 1$, the left side is

$$\frac{1}{9} + \frac{1}{25} + \ldots\ldots + \frac{1}{(2k+3)^2}$$

$$= \left[\frac{1}{9} + \frac{1}{25} + \ldots\ldots + \frac{1}{(2k+1)^2}\right] + \frac{1}{(2k+3)^2}$$

By the induction hypothesis, the sum in the square brackets is less than 1/4 and therefore

$$\frac{1}{9} - \frac{1}{25} + \ldots\ldots \frac{1}{(2k+3)^2} < \frac{1}{4} + \frac{1}{(2k+3)^2}$$

Quite obviously this inequality does not in the least imply that the left- hand member is less than 1/4. Thus, proof by induction has come to an impasses, whereas the inequality is very simply proved by an entirely different method. This is done in Illustration 13.

In conclusion we offer two inequality in the proof of which the techniques suggested above and certain others are used; these inequality can be solved by several methods involving algebra, trigonometry and even geometry.

Illustration 19. Prove that if $x^2+y^2 = 1$, then $-\sqrt{2} \le x + y \le \sqrt{2}$.

Algebraic solution. Let us write the obvious inequality $(x - y)^2 \ge 0$ or $x^2 + y^2 \ge 2xy$, whence $2(x^2 + y^2) \ge x^2 + 2xy + y^2$. In so far as $x^2 + y^2 = 1$, from the latter inequality we have $(x + y)^2 \le 2$, whence,

$$|x + y| \le \sqrt{2} \text{ or } -\sqrt{2} \le x + y \le \sqrt{2}$$

Trigonometric solution. If x and y satisfy the condition $x^2 + y^2 = 1$, then we can find an angle α such that $x = \cos\alpha$, $y = \sin\alpha$. Then we have to prove that for any value of α

$$-\sqrt{2} \le \cos\alpha + \sin\alpha \le \sqrt{2}$$

Since $\cos\alpha + \sin\alpha = \sqrt{2}\sin(\alpha + \pi/4)$ and $-1 \le \sin(\alpha + \pi/4) \le 1$, it follows that –

$-\sqrt{2} \le \sqrt{2}\sin(\alpha + \pi/4) \le \sqrt{2}$ for all values of α, which completes the proof of the required inequality.

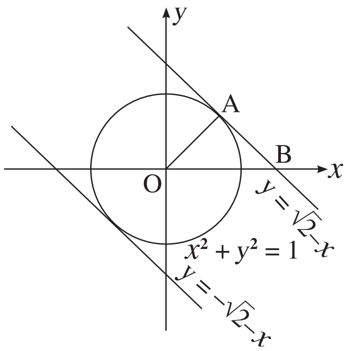


Fig. 1

Geometric solution. We will consider x and y as co-ordinate of point in a plane in a given system of co-ordinate. Then the condition $x^2+y^2 =1$ is satisfied by the points(x, y) lying on a circle of radius 1 centered at the origin **(Fig. 1)**. The points which satisfy the inequality $x + y \le \sqrt{2}$ lie on the straight line $y = \sqrt{2} - x$ and below that line.

Let B be the point of intersection of this straight line with axis of abscissas and OA a perpendicular dropped on this line from the origin. Then $OB = \sqrt{2}$, $\angle ABO = 45°$ and therefore OA =1. Hence, the point A lies on the circle and the straight line $y = \sqrt{2} - x$ is perpendicular to the radius OA at its end point, which is to say it is tangent to the circle.

Similarly, the inequality $-\sqrt{2} \le x + y$ is satisfied by points lying on the straight line $y = -\sqrt{2} - x$ and above it; this line is also tangent to the circle $x^2 + y^2 = 1$.

Thus, the double inequality to be proved is satisfied by points lying in the strip between the straight line $y = \sqrt{2} - x$ and $y = -\sqrt{2} - x$ (these lines included). But the circle $x^2 + y^2 = 1$ lies entirely insides this strip and so the co-ordinates of any points of it satisfy the inequality $-\sqrt{2} \le x + y \le \sqrt{2}$ the proof is complete.

Illustration 20. Let $a + b = 2$, where a and b are real numbers. Prove that $a^4 + b^4 \ge 2$.

Note that if one of the numbers, a or b, is negative the inequality is almost obvious. Suppose, say, $b < 0$. Then $a > 2$ and the inequality $a^4 + b^4 \ge 2$ is true, since $b^4 > 0$ and $a^4 > 16$. We will therefore assume that $a \ge 0$ and $b \ge 0$.

First solution. Since $a + b = 2$, then $(a + b)^2 = 4$. Using the inequality between the arithmetic mean and the geometries mean, $ab \le (a^2 + b^2)/2$, we have $4 = (a + b)^2 = a^2 + b^2 + 2ab \le 2(a^2 + b^2)$, or $2 \le a^2 + b^2$ squaring this inequality (this is legitimate since the numbers on the right and left are positive), we get

$$4 \le (a^2 + b^2)^2$$

On the basis of the inequality between the arithmetic mean and the geometric mean $a^2b^2 \le (a^4 + b^4)/2$. Therefore we have $4 \le (a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2 \le 2(a^4 + b^4)$ whence $2 \le a^4 + b^4$, and the proof is complete.

Second solution. We again assume that $a \ge 0$ and $b \ge 0$. Since $a + b =2$, then $(a + b)^4 = 16$ or

$$\begin{aligned}(a + b)^4 &= (a^2 + 2ab + b^2)(a^2 + 2ab + b^2)\\ &= a^4 + b^4 + 4ab(a^2 + b^2) + 6a^2b^2 = 16\end{aligned}$$

But since

$a^2 + b^2 = 4 - 2ab$, the last equality can be rewritten

$$a^4 + b^4 = 16 - 16ab + 2a^2b^2$$

If we are able to demonstrate that $16 - 16ab - 2a^2b^2 \ge 2$, then our inequality will have been proved.

By hypothesis, $ab \le 1$. Indeed, $\sqrt{ab} \le (a + b)/2$. Since $a + b = 2$, it follows that $\sqrt{ab} \le 1$., whence $ab \le 1$. And so we have to prove the inequality $16 - 16ab + 2a^2b^2 \ge 2$, provided that $ab \le 1$. We set $x = ab$. Then we have to prove of the inequality $x^2 - 8x + 7 \ge 0$ with the provison that $x \ge 1$. The roots of the quadratic trinomial $x^2 - 8x + 7$ are $x_1 = 1$, $x_2 = 7$. Therefore, the last inequality may be written as $(x - 1)(x - 7) \ge 0$.

But the last $x \le 1$ this inequality is obvious. We have thus obtained $16 - 16ab + 2a^2b^2 \ge 2$, which is what we set out to prove.

Third solution. Let $a = 1 + c$, $b = 1 - c$. Since, we earlier assumed that $a \ge 0$ and $b \ge 0$, it follows that $-1 \le c \le 1$ and so we can take advantage of inequality (4) (see problem 16 of this section) :

$$(1 + c)^4 \ge 1 + 4c, \quad (1 - c)^4 \ge 1 - 4c$$

Thus,

$$a^4 + b^4 = (1 + c)^4 + (1 - c)^4 \ge (1 + 4c) + (1 - 4c) = 2$$

In conclusion we note that a more general statement is valid : if $a + b = 2$, then $a^n + b^n \ge 2$ for any positive

integer n. This can easily be proved by, say, the third method given above.

Solving Inequalities

A great deal of mistake are made in the solution of inequalities. The point is that in most cases the solution of inequalities given at examination does not require any particular ingenuity or artificial techniques, and so, as a rule, the student sees at a glance what steps must be taken. However, in carrying out the manipulations, the students makes serious mistakes due to a failure to recognize the fundamental theoretical propositions involving inequalities.

Actually, solving inequalities hardly requires anything more than the ability to reduce an inequality to the solution of elementary inequalities (without either losing a solution or introducing any extraneous ones), and then to solve these elementary inequalities. To carry out the latter part, the student has to know the fundamental properties of the functions studied at school (algebraic, exponential, logarithmic and trigonometric functions); to carry out the former part, the student must be able to handle the basic concepts involving the equivalence of inequalities, the sources of loss of solutions and of the introduction of extraneous solutions.

The basic definitions needed in the solution of inequalities repeat almost word for word those required for equations. Note the following two differences in terminology however : the term "root" is not used when speaking of inequalities; one always uses the term "solution"; also, for the sake of brevity, one speaks of the solution being a certain set of values of x, for instance, the interval $a < x < b$, whereas in actuality every value of x of the set is a solution.

The similarity of equations and inequalities is quite naturally not confined to that of the basic definitions. It is obvious, for example, that everything that has been said about transforming equations which extend or restrict the domain of the variable is just as valid when applied to inequalities.

However, it must be stressed that solving inequalities has its peculiarities in that the same manipulations applied to equations and inequalities lead to different results. For instance, when multiplying both members of an equation by some nonzero factor (which is meaningful in the domain of the variable), an equation is replaced by an equivalent equation, whereas for inequalities we have to deal with the additional restriction that the factor be non-negative in the domain of the variable. In the same way, squaring both sides of an equation does not lead to a loss of roots, while squaring an inequality can lead either to a loss of solutions or to the introduction of solutions. Students often lose sight of these peculiarities and make mistakes in the solution of inequalities that they never would make when solving equations.

It is a matter of wonder that so many mistakes are made by students when solving the simplest kind of inequality. Apparently this is due to a formally understood analogy between equations and inequality. The reasoning goes roughly like this : "Since the solution of the equation $\log_{1/2} x = 1$ is $x = 1/2$, the solution of the inequity $\log_{1/2}$ X > 1 constitutes the values $x > 1/2$." Similarly, solutions to the inequalities $(1/5)^x < 2$ are written as $x < \log_{1/5} 2$, and so on. Yet the actual solutions to the two foregoing inequalities are different : in the first case, $0 < x < 1/2$, in the second, $x > \log_{1/5} 2$. A false analogy between equations and inequalities led to these mistakes.

Actually, when the student tackles an elementary inequality, he should consciously take advantage of the properties of the functions participating in the inequality. Let us now consider examples in solving some elementary inequalities.

We wish to note first of all that the solution of linear (first-degree) I and quadratic (second-degree) algebraic inequalities is usually quite thoroughly explained in textbooks and hardly ever causes any trouble.

Here we wish to dwell on elementary exponential, logarithmic, and trigonometric inequalities.

An elementary exponential inequality is an inequality of the type $a^x > a^b$ ($a^x < a^b$). When handling such inequalities, it must be remembered that the properties of an exponential function differ for bases greater than unity and less than unity.

Illustration 21. Solve the inequality $-1 \le (1/3)^x < 2$.

To solve a double inequality means to find all the values of x which simultaneously satisfy the two inequalities : $(1/3)^x \ge -1$ and $(1/3)^x < 2$. **(Fig. 2)**

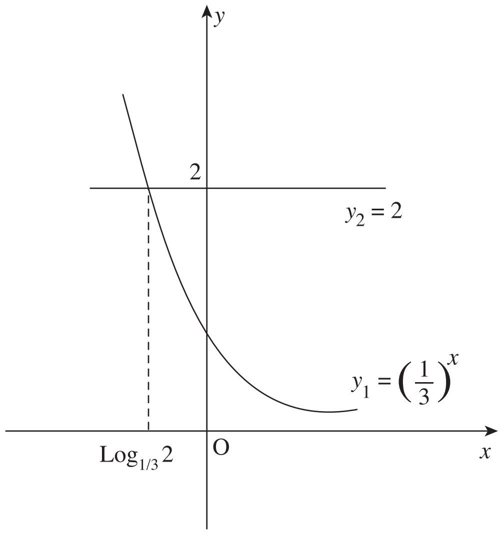


Fig. 2

Since, an exponential function is always positive, the first of these inequalities is valid for all values of x. Rewriting the second inequality as $(1/3)^x < \left(\frac{1}{3}\right)^{\log_{1/3} 2}$.

We take advantage of the property of an exponential function : to a base less than unity, the greater value of the function is associated with the smaller value of the argument and conversely, to the smaller value of the function correspond the greater value of the argument. This inequality is therefore equivalent to the inequality $x > \log_{1/2} 2$.

This solution is well illustrated by the graph shown in **Fig. 2,** namely, the solutions are those values of x for which the graph of the function $y = (1/3)^x$ lies below the horizontal straight line $y = 2$; that is, all x to the right of the abscissa of the point of intersection of these graphs (this abscissa is a solution of the equation $(1/3)^x = 2$). Thus, the solution of our inequality is the interval $x > \log_{1/3} 2$.

When solving inequalities containing the unknown under the sign of the logarithm, one must also bear in mind that the properties of a logarithmic function differ depending on whether the base is less than or greater than unity. However, another essential point in solving these inequalities is that the logarithmic function is not defined for all values of x. This is lost sight of by many students when solving an inequality like $\log_2 x < 1$. They reason this way : "We rewrite the inequality as $\log_2 x < \log_2 2$. The greater number to a base greater than 1 has the larger logarithm, and so the inequality is valid for $x < 2$."

Nothing would seem to be wrong in this argument, but still the answer is faulty because extraneous solutions were introduced. Indeed, any negative number is less than 2, but the original inequality is meaningless for negative values of x (because negative numbers do not have logarithms).

Why were extraneous solutions introduced? When "solving" the inequality, we passed from $\log_2 x < \log_2 2$ to $x < 2$. The latter inequality is meaningful for all values of x while the original inequality has meaning only for those values of x for which $\log_2 x$ is meaningful, that is to say, for $x > 0$. Hence, extraneous solutions were introduced simply because the fact was disregarded that a logarithmic function is defined only for positive values of x.

A correct answer is obtained if we choose from among the solutions of the latter inequality those whose values of $x > 0$; thus, the solution of our inequality is the interval $0 < x < 2$.

This simple example makes it abundantly clear that one should bear in mind, when solving logarithmic inequalities in this manner, that a logarithmic function is only defined for positive values of x. However, these inequalities may be solved in a different way : instead of using the domain of definition of the logarithmic function and its property of monotonicity we can immediately take advantage of Properties VII and VIII of logarithms.

Thus, using Property VII in the above example, we can directly replace the inequality $\log_2 x < \log_2 2$ by the equivalent inequality $0 < x < 2$, which yields the answer.

Taking into account the simplicity of solving logarithmic inequalities by means of Properties VII and VIII, we will henceforth solve such inequalities by using these properties.

Illustration 22. Solve the inequality $\log_{1/2} x > \log_{1/3} x$.

Taking the logarithm of the right member to the base 1/2 (Rule V), we get an equivalent inequality :

$$\log_{1/2} x \left(1 - \log_{1/3} \frac{1}{2}\right) > 0$$

Since, $1/2 > 1/3$, it follows that $\log_{1/2} 1/2 > \log_{1/3} 1/3$ or $1 - \log_{1/3} 1/2 > 0$.

Noting that $0 = \log_{1/2} 1$, we find that the original inequality is Equivalent to $\log_{1/2} x > \log_{1/2} 1$.

Applying Property VIII to this inequality, we get the solution of the original in quality : $0 < x < 1$.

Now, let us examine trigonometric inequalities. Despite the fact that the solutions of the more elementary trigonometric inequalities are thoroughly explained in the standard textbooks, students continue to make serious mistakes even when solving the simplest inequalities. We now examine a few typical mistakes of this nature.

(a) Knowing that the solutions of the equation $\sin x = a$ ($|a| \le 1$) are given by the formula $x = (-1)^k \arcsin a + k\pi$, where $k = 0, \pm 1, \pm 2, \ldots$, many students write that "the solution of the inequality $\sin x < a$ consists of all values of $x < (-1)^k \arcsin a + k\pi$, $k = 0, \pm 1, \pm 2, \ldots\ldots$"

It is quite often difficult to convince the student of the absurdity of such an answer.

(b) Many mistakes are made that are connected with the formal use of the symbols arcsin a, arccos a, etc. These symbols are frequently employed when the student has not yet investigated whether they are **(Fig. 3).**

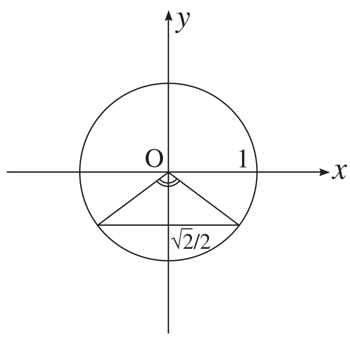


Fig. 3

Meaningful or not. For instance, the solution to the inequality $\sin x \le \log_4 5$ is written as $\arcsin(\log_4 5)$, which is meaningless since $\log_4 5 > 1$. Yet this inequality is valid for all values of x; this is evident from, the very start because $\log_4 5 > 1$.

(c) Mistakes occur due to improper use of the trigonometric circle. For example, when solving an inequality like $\sin x \le -\sqrt{2}/2$, the students correctly

indicate the angles that yield the solutions of the inequality **(Fig. 3)** but err when they give the analytic notation as

$$\frac{5\pi}{4} + 2k\,\pi \le x \le -\frac{\pi}{4} + 2k\,\pi,\ k = 0,\ \pm 1, \pm 2,\ldots\ldots$$

It is clear that this notation is meaningless since the left member of the inequality is greater than the right member for ail values of k.

When solving elementary trigonometric inequalities, it is best to make use of the graphs of trigonometric functions. This is a practical guarantee against mistakes and makes for a pictorial representation of the regions in which the inequality is valid. When giving their analytic notation, it is convenient to take advantage of the following fact : if $f(x)$ is a periodic function, then to solve the inequality $f(x) > a$ it suffices to find the solution in any interval that is equal to the length of the period of the function $f(x)$, then all values of x thus found and also all x that differ from these values by an integral number of the periods of the function $f(x)$ constitute a solution of our inequality.

Illustration 23. Solve the inequality $\sin x > 1/2$.

We construct the graphs of the functions $y_1 = \sin x$ and $y_2 = 1/2$ **(Fig. 4).** This inequality is satisfied for all values of x for which the **(Fig. 4).**

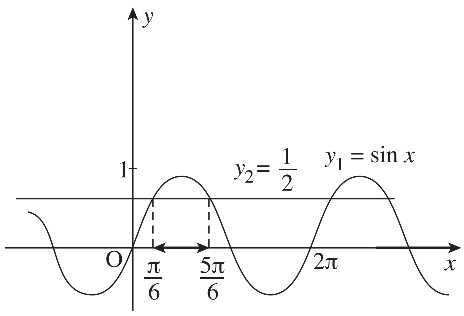


Fig. 4

First graph lies above the second one. Since the period of the function $\sin x$ is 2π, it is sufficient for us to solve the proposed inequality on some interval of length 2π. It is easy to see that the most convenient interval is that from 0 to 2π : the solutions can most simply be written then as $\pi/6 < x < 5\pi/6$.

Thus, the complete solution of the inequality is

$$\frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{3} + 2k\pi,\ k = 0, \pm 1, \pm 2,\ldots\ldots$$

This notation is to be understood as follows : there is a certain interval for each integer k, and the set of all these intervals constitutes the solution of the inequality.

Illustration 24. Solve the inequality $\cos x \ge -1/2$.

We construct the graphs of the functions $y_1 = \cos x$ and $y_2 = -1/2$ **(Fig. 5)**. The period of the function $\cos x$ is also equal to 2π, but

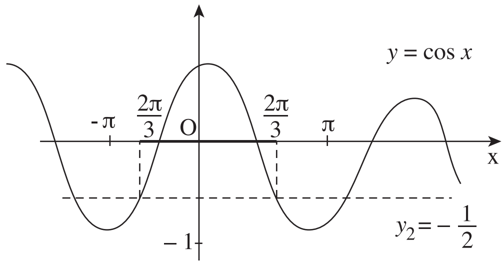


Fig. 5

Fig : 5 the drawing shows us that it is no longer convenient to take the interval from 0 to 2π for the basic interval because the solution of the inequality there will consist of two "pieces". It is therefore more convenient to seek the solution of this inequality on the interval from $-\pi$ to π. This is the interval $-2\pi/3 \le x \le 2\pi/3$. Consequently, the complete solution is

$$-\frac{2\pi}{3} + 2k\,\pi \le x \le \frac{2\pi}{3} + 2k\,\pi, k = 0, \pm 1, \pm 2,\ldots\ldots$$

Illustration 25. Solve the inequality $|\tan x| < 1/7$.

The period of the function $|\tan x|$ is equal to π. We consider the inequality on the interval from $-\pi/2$ to $\pi/2$ and construct the graphs of the functions $y_1 = |\tan x|$ and $y_2 = 1/7$.

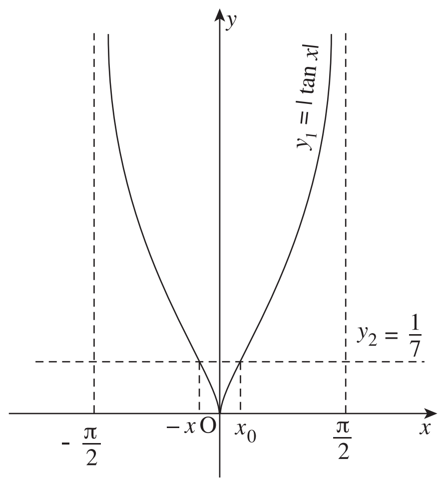


Fig. 6

It is evident that the solution will consist of all x lying in the interval $-x_0 < x < x_0$, where x_0 is the abscissa of the intersection point of the graphs under consideration that lies between 0 and $\pi/2$, that is, the root of the equation $\tan x = 1/7$ located in the interval $0 < x < \pi/2$. Hence, $x_0 = \arctan(1/7)$. Taking into account the period of the function $y = |\tan x|$, we find that the solution of our inequality consists of all values of x located in the intervals

$$-\arctan\frac{1}{7} + k\,\pi < x < \arctan\frac{1}{7} + k\pi, \text{ where } k = 0, \pm 1,$$

$\pm 2,\ldots$

Note that the original inequality can be written as a double inequality $-1/7 < \tan x < 1/7$ and solved by using the graph of the function $y = \tan x$.

Now, let us consider raising to a power. In the sequel we will frequently make use of the following statement.

Theorem—If $f(x) \geq 0$ and $\phi > (x) \geq 0$ on some set of values of x, then the inequalities $f(x) > \phi(x)$ and $[f(X)]^2 > [\phi(X)]^2$ are equivalent on that set.

Proof–Let x_0 be an arbitrary solution of the first inequality taken from the set of values of x under consideration. If $\phi(x_0) > 0$, then from the validity of the inequality $f(x_0) > \phi(x_0)$ follows, on the basis of the theorem on raising numerical inequalities to a power, the validity of the inequality $[f(x_0)]^2 > [\phi(x_0)]^2$. But if $\phi(x_0) = 0$, then it is obvious that the validity of the inequality $f(x_0)>0$ implies $[f(x_0)]^2 > 0$. This proves that every solution of the inequality $f(x) > \phi(x)$ is a solution of the inequality $[f(x)]^2 > [\phi(x)]^2$.

The converse is proved in similar fashion : that every solution of the inequality $[f(x)]^2 > [\varphi > (x)]^2$ is a solution of the inequality $f(x) > \phi(x)$.

The proof of the theorem is complete.

Note that in the statement of the theorem the strict inequalities $f(x) > \phi(x)$ and $[f(x)]^2 > [\phi(x)]^2$ may be replaced by the weak inequalities $f(x) \geq \phi(x)$ and $[f(x)]^2 \geq [\phi(x)]^2$. The proof of this fact is carried out in the same way as the proof of the theorem.

When equations are raised to a power, it is only possible to introduce extraneous solutions, which may occur due to an extension of the domain of the variable or when the signs of the two sides of the equation are disregarded. Similarly, extraneous solutions can be introduced in the solution of inequalities; they too are introduced because of an extension of the domain of the variable and also when the signs of the two members of the inequality are disregarded. Below are some examples which illustrate how extraneous solutions are introduced in both cases.

However, unlike the case of equations, raising an inequality to a power can result in the loss of solutions as well. The reason why student, make mistakes here is that they remember that raising an equation to a power cannot result in the loss of a solution but forget that raising an inequality to a power can result in the loss of solutions. We will show below how it is possible to lose a solution when raising an inequality to a power .

Let us begin with an example that illustrates how extraneous solutions are introduced due to extension of the domain of the variable when raising an inequality to a power.

Illustration 26. Solve the inequality $\sqrt{(x-3)(2-x)} > \sqrt{4x^2 + 12x + 11}$.

Some aspirants gave this solution : "Since the right and left members of this inequality are non-negative (this is because we have principal square roots on the right and left), the inequality may be squared to obtain the equivalent inequality $5x^2 + 7x + 17 > 0$. The quadratic trinomial in the left-hand member of this equation does not have any real roots and therefore this inequality holds true for a1l real values of x. It then follows, because the inequalities are equivalent, that the original inequality too holds true for all values of x". This reasoning appears to be correct, but there is one serious defect. It is true in the domain of the variable of the original inequality.

The proper solution is in the domain of the variable, both member of the original inequality are non-negative; for this reason it is equivalent, in the domain, to the inequality $5x^2 + 7x + 17 > 0$ and hence is true for all Values of x in the domain. It is now easy to find domain of the original inequality and thus to obtain the answer : $2 \leq x \leq 3$.

In the problem that follows, extraneous solutions are introduced not because of an extension of the domain of the variable but because of raising to a power without investigating the signs of both members of the inequality.

Illustration 27. Solve the inequality $x + 1 > \sqrt{x+3}$.

Here is an instance of reasoning that gives rise to extraneous solutions : "The domain of the variable of our inequality is $x \geq -3$. For any x in the domain we have a non-negative number (principal square root) on the right; hence, the number on the left is a positive number. For this reason, squaring yields the equivalent inequality $x^2 + x - 2 > 0$, the solution of which is $x > 1$ and also $x < -2$. Taking into account the domain of the original inequality, we get the answer : the solution of the original inequality consists of all values of $x > 1$ and also of all values of x located in the interval $-3 \leq x < -2$.''

Actually, all values of x in the interval $-3 \leq x < -2$ are not solutions, to the original inequality. The point is that for x in the domain, the right member of the inequality is indeed non-negative, whereas the left member is negative for certain values of x located in the domain and is non-negative for others. It is clear that for those values of x in the domain for which the left member is negative, the inequality is invalid and so there are no solutions of our inequality among them. It is thus necessary to seek solutions of the original inequality among those values of x in the domain for which the left-hand member of the inequality is non-negative, which is to say among $x \geq -1$. For these x, both members of the inequality are indeed non-negative, and it can be squared to obtain the inequality $x^2 + x - 2 > 0$, which is equivalent to the original inequality on the set $x \geq -1$. It is now necessary to choose from among the solutions of the inequality $x^2 + x - 2 > 0$ those which satisfy the condition $x \geq -1$. They will yield the solutions of the original inequality, which are $x > 1$.

The mistake that was made in the earlier reasoning was due to the fact that the student did not notice the shift in concepts. It is true that for any value of x which is a solution of the original inequality there is a non-negative number (principal square root) on the right and a positive

number on the left. However, it is obvious that not all values of x located in the domain will be solutions of the original inequality, and so the number on the left will not be positive for all x of the domain. The student replaced the words "for any x which is a solution" by the phrase "for every value of x in the domain." This was his mistake.

Illustration 28. Solve the inequality

$$\sqrt{4-\sqrt{1-x}}-\sqrt{2-x}>0$$

Difficulties here spring up when we begin to compute the domain of the variable. The domain of this inequality is defined from the conditions : $2-x \geq 0$, $1-x \geq 0$, $4 \geq \sqrt{1-x}$. The first two of these inequalities are true for $x \leq 1$. But both sides of the third inequality are non-negative for these values of x, and so it can be squared to get an equivalent inequality : $x \geq -15$. Thus, the domain of the original inequality is $-15 \leq x \leq$. We rewrite our inequality thus : $\sqrt{4-\sqrt{1-x}}>\sqrt{2-x}$. Within the domain, both members of this inequality are non-negative; therefore squaring yields an inequality are non-negative, therefore squaring yields an inequality, $2+x>\sqrt{1-x}$, that is equivalent in the domain. For values of $x<-2$ and such that enter into the domain, the left member of this inequality is negative, while the right member is non-negative, which means that there are no solutions to the original inequality among these values of x. It remains to consider the values of x in the interval $-2 \leq x \leq 1$. For these x, both members of the inequality $2+x>\sqrt{1-x}$ are non-negative, and so squaring yields the quadratic inequality $x^2+5x+3>0$, which is equivalent to the original inequality on the set $-2 \leq x \leq 1$. This latter inequality holds true for $x>(-5+\sqrt{13})/2$ and for $x<(-5-\sqrt{13})/2$. Now to get the answer we have to choose from among these solutions those which lie in the interval $-2 \leq x \leq 1$. These consist of all values of x in the interval $(-5+\sqrt{13})/2<x \leq 1$. They are the ones which constitute the answer to this problem.

Note that if we had not taken the domain of the variable into account, we would have introduced extraneous solutions, for example, all $x>1$; and if we had not taken into consideration that the inequality $2+x>\sqrt{1-x}$ has solutions only for $-2 \leq x \leq 1$, we would also have introduced extraneous solutions, for example, all the values of $x<(-5-\sqrt{13})/2$.

Let us now examine some problems in which one can lose solutions by raising the inequality to a power.

Illustration 29. Solve the inequality $\sqrt{x+2}>x$.

If we square this inequality at once, we will lose solutions even if we take into account the domain of the variable. Indeed, the domain for this inequality is $x \geq -2$. Squaring, we get the inequality $x+2>>x^2$, whose solution will consist of all the values of x in the interval $-1<x<2$. All these values of x enter into the domain, and so some students wrote that these values constitute the answer to the problem.

Actually, in thus reasoning they lost the solutions $-2 \leq x \leq -1$, because it is easy to see that for any number in this interval the left member of the inequality is non-negative, while the right member is negative.

So as not to lose solutions, the student must keep careful watch of the signs of the left and right members. The proper solution of this inequality is as follows.

The domain of the variable in this inequality consists of all $x \geq -2$. The left member of the given inequality is non-negative in the domain, while the right member may be positive or negative. Clearly, the original inequality will be true for all those values of x in the domain for which the right member is negative. Hence, all the values of x in the interval $0>x \geq -2$ are solutions of the original inequality.

Now let us consider the remaining values of x, that is, $x \geq 0$. Both members of the original inequality are nonnegative for all these x, and so the inequality can be squared to obtain $x+2>x$ 2, which is an equivalent inequality for all $x \geq 0$.

The solution of the last inequality consists of all x in the interval $-1<x<2$. In this case, the solution of the original inequality consists of all values of x in the interval $0 \leq x<2$.

Combining these two cases, we find that the solution to the original inequality will consist of all values of x lying in the interval $-2 \leq x<2$.

In the next problem, it will be possible to lose solutions if one fails to take into account the signs of the right and left members of an intermediate inequality.

Illustration 30. Solve the inequality

$$\sqrt{x^2+3x+2}<1+\sqrt{x^2-x+1}$$

The domain of the variable here consists of two intervals : $x \leq -2$ and $x \leq -1$. In the domain, both members of our inequality are non-negative and so squaring yields the equivalent (in the domain) inequality $2x<\sqrt{x^2-x+1}$.

(a) For $x \leq -2$ and $-1 \leq x<0$, this inequality is true since for each of these values of x there is a negative number on the left and a positive number on the right. Thus, all these values of x are solutions to the original inequality.

(b) For $x \geq 0$, both members of the inequality $2x<\sqrt{x^2-x+1}$ are non-negative and so squaring yields the equivalent (for these x) inequality $3x^2+x-1<0$. The solution of this inequality consists of values of x in the interval $(-1-\sqrt{13})/6<x<(-1+\sqrt{13})/6$.

Taking Condition (b) into account, we find that in the latter case the solution of the original inequality will consist of all values of x in the interval $0 \le x < (-1 + \sqrt{13})/6$.

Combining both cases we get the answer : $x \le -2$ and also $-1 \le x < (1 + \sqrt{13})/6$.

It will be noted that those students who did not consider the cases (a) and (b) and squared the inequality $2x < \sqrt{x^2 - x + 1}$ from the start naturally lost some of the solutions. Most likely what happened was that since at the beginning of the solution of the inequality the signs of the left and right members had already been investigated, there was a kind of loss of "vigilance" in the second squaring.

Exercise

1. Solve the inequality $\sin x - \cos x > 0$.
2. Solve the inequality
$$x(x+1)(-x+\sqrt{2})(x^2 - x + 1)(3x+1)^2(x+\sqrt{17})^3 \times (1-x)(2x - \pi^2)(-x+\pi)(x - \sin^x) < 0$$
3. Solve the inequality
$$9^x - 10 \cdot 3^x + 9 \le 0$$
4. Solve the inequality
$$\text{Log}^2{}_2\, x + 3\log_2 x \ge \frac{5}{2}\log_{4\sqrt{2}} 16$$
5. Solve the inequality
$$\left(\frac{1}{2}\right)^{(x^6 - 2x^3 + 1)^{1/2}} < \left(\frac{1}{2}\right)^{1-x}$$
6. Solve the inequality
$$5 + 2\cos 2x \le 3|2\sin x - 1|$$
7. Solve the inequality
$$\log_5 . \sin x > \log_{125}(3\sin x - 2)$$
8. Solve the inequality,
$$\cos[\pi(x^2 - 10x)] - \sqrt{3}\sin[\pi(x^2 - 10x)] > 1$$
9. Solve the inequality $\sqrt{x} > -1$.
10. Solve the inequality $\sqrt{\log_{10} x > 0}$.
11. Solve the inequality $\log_{2-x}(x-3) \ge -5$.
12. Solve the inequality $\sqrt{x+2} + \sqrt{x-5} \ge \sqrt{5-x}$.
13. Solve the inequality $\sqrt{2 + x - x^2} > x - 4$.
14. Solve the inequality $\sqrt{\sin x + 2\cot x} < -1$.
15. Solve the inequality
$$(x-2)/(x+2) \ge (2x-3)/(4x-1)$$
16. Solve the inequality
$$\log_{1/2}\frac{5x+4}{x-2} > \tan\frac{5\pi}{4}$$
17. Solve the inequality
$$(\log_x 2)(\log_{2x} 2)(\log_2 4x) > 1$$
18. Solve the inequality
$$\sqrt{3 + 2\tan x - \tan^2 x} \ge \frac{1 + 3\tan x}{2}$$
19. Solve the inequality
$$\log_{\frac{25 - x^2}{16}}\left(\frac{24 - 2x - x^2}{14}\right) > 1$$
20. Solve the inequality
$$\log_{\frac{2\cos x}{\sqrt{3}}}\sqrt{1 + 2\cos 2x} < 1$$
21. Solve the inequality
$$x^4 \cdot 7^{\log 7^{1/3} 5} \le 5^{-\log_{1/x} 5}$$
22. Solve the inequality
$$(x^2 + x + 1)^x < 1$$
23. Solve the inequality
$$\text{Log}_x{}^2\left(\frac{4x-5}{|x-2|}\right) \ge \frac{1}{2}$$
24. Solve the inequality
$$\log_x 2x \le \sqrt{\log_x(2x^3)}$$
25. Solve the inequality
$$\sqrt{4\sin^2 x - 1}\,\log_{\sin x}\frac{x-5}{2x-1} \ge 0$$

Solutions

1. Using a consequence of the addition formula and $\pi/4$ as an auxiliary angle (we call this the auxiliary-angle formula), we get the inequality $\sqrt{2}\sin[x - (\pi/4)] > 0$. Of course it can be solved by considering the graph of the function $y = \sin[x - (\pi/4)]$. However, it is best to do otherwise. Denoting $x - (\pi/4)$ by z, let us consider the inequality $\sin z > 0$. Its solution $2\pi k < z < \pi + 2\pi k$, $k = 0, \pm 1, \pm 2, \ldots$ is directly obtained from the graph of the function $y = \sin z$. Now, substituting

$x - (\pi/4)$ in place of z, we find the appropriate intervals of variation of x :

$$\frac{\pi}{4} + 2k\pi < x < \frac{5\pi}{4} + 2k\pi,\ k = 0, \pm 1, \pm 2.$$

This technique-replacing $x - (\pi/4)$ by z-enabled us to dispense with constructing the graph of the function $y = \sin[x - (\pi/4)$. Its convenience is still more evident when solving elementary trigonometric inequalities with a complicated argument. For example, it allows us to get around constructing an extremely involved graph when solving inequalities like $\sin(\sqrt{2x} + 7)\ -1/2$. Here of course it is easier to denote $\sqrt{2x} + 7$ by z and solve the inequality $\sin z > -1/2$ using the graph of the function $y = \sin z$, and then pass to x.

Higher-degree algebraic inequalities can also be classed as elementary inequalities. Students sometimes

solve them by investigating various cases, which is to say, by passing to a solution of several systems of inequalities. Confusion often begins when the student is not able to find the common portion of the solutions and is undecided about whether or not to combine these solutions, yet there is a unified standard method for solving such inequalities. It is the so-called method of intervals that we now give.

Suppose, for example, we have to solve the inequality

$(x - x_1)(x - x_2) \ldots (x - x_{n-1})(x - x_n) < 0$

Where $x_1, x_2, ..., x_n$ are distinct real numbers. We will assume that

$x_1 < x_2 < \ldots < x_{n-1} < x_n$

Plot these points on the real number line

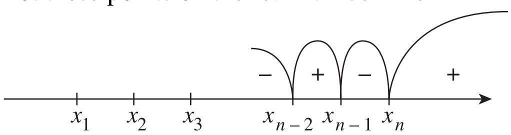


Fig. 7

(Fig. 7) and consider the polynomial

$P(x) = (x - x_1)(x - x_2) \ldots$

$(x - x_{n-1})(x - x_n)$ ....(1)

It is clear that for all $x > x_n$ all the parenthetic expressions in (1) are positive and, hence, for $x > x_n$ we have $P(x) > 0$. Since, for $x_{n-1} < x < x_n$ the last parenthesis in the expression $P(x)$ is negative, and all the other parentheses are positive, it follows that for $X_{n-1} < X < X_n$ we have $P(x) < 0$. Similarly, we obtain $P(x) > 0$ for $X_{n-2} < X < X_{n-1}$ and so on. That is the underlying idea of the method of intervals. On the number line, the numbers X_1 $X_2, \ldots, X_n$ must be arranged in order of increasing magnitude. Then place the plus sign in the interval to the right of the largest number. In the next interval (from right to left) place the minus sign, then the plus sign, then the minus sign, etc. The solution of the inequality $P(x) < 0$ will then consists of intervals having the minus sign.

2. It is quite obvious that if we reduce this inequality to systems of inequalities, then we will have a large number of cases to consider.

Let us solve it by the method of intervals. First, we have to reduce it to the proper form. Note that $x^2 - x + 1 > 0$ for any value of x and for this reason this factor can be cancelled from both members of the inequalities. Further note that $(3x + 1)^2 > 0$ for $x \neq -1/3$ and therefore this factor can likewise be cancelled. Remember however that $x = -1/3$ is not a solution of the inequality. Besides, it is clear that the sign of $(x + \sqrt{17})^3$ coincides with that of $x + \sqrt{17}$ and therefore we can replace $(x + \sqrt{17})^3$ by $x + \sqrt{17}$ without impairing the inequality. Finally, represent each factor as $x - a$, where a is a number.

All these manipulations result in the inequality

$(x - 0)\ [x - (-1)]\ (x - \sqrt{2})\ [x - (-\sqrt{17})] \times (x - 1)$

$(x - \frac{\pi^2}{2})(x - \pi)(x - \sin^2 1) > 0$

which is equivalent to the original one for all x # $-1/3$(since we multiplied three parentheses by -1, the sense, of the inequality is reversed).

Plot the numbers $0, -1, -\sqrt{17}\ 1, \pi^2/2, \sqrt{2}$ and $\sin^2 1$ on the real number line

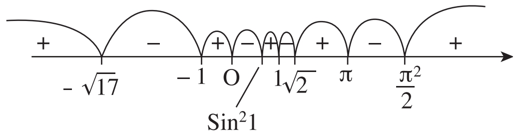


Fig. 8

(Fig. 8). Then the last inequality is true for x located in the intervals

$x < -\sqrt{17}, -1 < x < 0, \sin^2 1 < x < 1,$

$\sqrt{2} < x < \pi, \frac{\pi^2}{2} < x$

The solutions of the original inequality are these values of x, with the exception of $x = -1/3$, that is,

$$x < -\sqrt{17}, -1 < x < -\frac{1}{3}, -\frac{1}{3} < x < 0,$$

$\sin^2 1 < x < 1, \sqrt{2} < x < \pi, \frac{\pi^2}{2} < x$

It is also to be noted that the weak inequality

$$(x - x_1)(x - x_2) \ldots (x - x_n) \leq 0$$

can also be solved by the method of intervals, but the answer is written in the form of the intervals $x_i \leq x \leq x_{i+1}$ with the endpoints included.

Frequently, problems involving inequalities can be reduced to elementary inequalities by means of simple algebraic manipulations and the introduction of a new unknown.

3. Denoting 3 by y, rewrite the inequality thus : $y^2 - 10y + 9 \leq 0$. This quadratic inequality is true for all values of y in the interval $1 \leq y \leq 9$.

Substituting 3^x in place of y, we obtain that the original inequality holds true for all x satisfying the double inequality $1 \leq 3^x \leq 9$.

Solving this elementary exponential inequality, we get the answer $0 \leq x \leq 2$.

4. Denoting $y = \log_2 x$ and noting that $5/2 \log_{4\sqrt{2}} 16 = 4$, we rewrite our inequality thus : $y^2 + 3y - 4 \geq 0$. The solution set of this quadratic inequality is made up of all $y \geq 1$ and also all $y \leq -4$. Hence, the original inequality will hold true for all x for which $\log_2 x \geq 1$ and also for those x for which $\log_2 x \leq -4$. Solving these elementary logarithmic inequalities by means of Property VIII of logarithms, we get the answer : $x \geq 2, 0 < x \leq 2^{-4}$.

5. If we disregard the exponents, we can say that this is an elementary exponential inequality with base less than unity : $(1/2)^a < (1/2)^b$. Solving it, we find that the original inequality is equivalent to the inequality $(x^6 - 2x^3 + 1)^{1/2} > 1 - x$.

Since, $(x^6 - 2x^3 + 1)^{1/2} = \sqrt{(x^3-1)^2} = |x^3 - 1|$, it follows that we have yet to solve the inequality

$$|x^3 - 1| > 1 - x$$

Since, the left member here is non-negative, it is automatically satisfied for $1 - x < 0$, that is, when $x > 1$.

We now consider $x \le 1$. In this case, $x^3 \le 1$, and so $|x^3 - 1| = 1 - x^3$.

and we have the inequality $1 - x^3 > 1 - x$ or $x(x-1)(x+1) < 0$

Solving this inequality by the method of intervals, we find that it is true for $x < -1$ and for x located in the interval $0 < x < 1$. All these values of x lie in the domain $x \le 1$ under consideration and so are solutions of the original inequality.

Thus, the original inequality is valid for $x < -1$, $0<x<1$, $x > 1$.

6. Taking advantage of the formula for the cosine of a double angle and denoting sin x by y, we can rewrite our inequality as $7 - 4y^2 \le 3\,|2y - 1|$. To get rid of the absolute-value sign, consider two cases : $y \ge 1/2$ and $y < 1/2$.

(a) Suppose $y \ge 1/2$, then our inequality is written $7 - 4y^2 \le 3(2y-1)$ or $2y^2 + 3y - 5 \ge 0$. The solution set of the latter inequality is $y \ge 1$ and $y - 5/2$. But taking into account that we only consider $y \ge 1/2$, we find that this condition is satisfied by $y \ge 1$ alone.

(b) Let $y < 1/2$. Then the original inequality is rewritten $7 - 4y^2 \le -3(2y-1)$ or $2y^2 - 3y - 2 \ge 0$. The solution set of this last inequality consists of $y \ge 2$ and $y \le -1/2$. But condition (b) is satisfied solely by $y \le -1/2$.

Thus, the solutions of the inequality in y are $y \le -1/2$ and $y \ge 1$. If in these inequalities we replace y by $\sin x$, we find the solutions of the original inequality to be all x that satisfy the elementary trigonometric inequality $\sin x \le -1/2$ and all x satisfying the inequality $\sin x \ge 1$.

The solution set of the first inequality consists of all x lying in the intervals

$$-\frac{5\pi}{6} + 2k\pi \le x \le -\frac{\pi}{6} + 2k\pi, k = 0, \pm 1, \pm 2, \ldots\ldots$$

The second inequality will be true only for those values of x for which $\sin x = 1$; that is, for

$$x = \frac{\pi}{2} + 2k\pi \qquad k = 0, \pm 1, \pm 2\ldots\ldots$$

Thus, finally, the solution set of the original inequality consists of all

$x = \pi/2 + 2k\pi$ and all x located in the intervals

$$-\frac{5\pi}{6} + 2k\pi \le x \le -\frac{\pi}{6} + 2k\pi, k = 0, \pm 1, \pm 2, \ldots\ldots$$

7. Noting that $\log_5 \sin x = \log_{125} \sin^3 x$, we rewrite our inequality as

$$\text{Log}_{125} \sin^3 x > \log_{125} (3 \sin x - 2)$$

Now, applying Property VIII of logarithms , we see that our inequality is equivalent to the inequality $\sin^3 x > 3 \sin x - 2 > 0$. Denoting $y = \sin x$, we arrive at the system of inequalities

$$y^3 - 3y + 2 > 0$$
$$3y - 2 > 0$$

Regrouping, represent the left member of the first inequality as

$$y^3 - 3y + 2 = y(y^2 - 1) - 2(y - 1)$$
$$= (y - 1)(y^2 + y - 2)$$
$$= (y - 1)^2 (y + 2)$$

It then follows that this inequality is true for all $y > -2$ with the exception of $y = 1$.

The second inequality of this system is valid for $y > 2/3$. Hence, the solution of the system includes all $y > 2/3$, except $y = 1$.

Returning to x, we find that the original inequality is equivalent to the following double inequality : $2/3 < \sin x < 1$.

The solutions of this elementary trigonometric inequality are given by the intervals

$$\arcsin 2/3 + 2k\pi < x < \pi/2 + 2kn,$$
$$n/2 + 2k\pi < x < \pi - \arcsin 2/3 + 2k\pi,$$
$$k = 0, \pm 1, \pm 2, \ldots$$

8. Putting $y = n(x^2 - 10x)$, rewrite the inequality as

$$\frac{1}{2}\cos y - \frac{\sqrt{3}}{2}\sin y > \frac{1}{2}$$

Using the auxiliary-angle formula, we get $\cos[y + (\pi/3)] > 1/2$. The solution of this elementary inequality consists of the intervals

$$-\frac{\pi}{3} + 2k\pi < y + \frac{\pi}{3} < \frac{\pi}{3} + 2k\pi, k = 0, \pm 1, \pm 2\ldots\ldots$$

Reverting to x, we find that for every integer k we have to solve the following system of quadratic inequalities :

$$x^2 - 10x - 2k < 0$$
$$x^2 - 10x - 2k + 2/3 > 0$$

The first inequality has solutions if and only if the discriminate of the quadratic expression $x^2 - 10x - 2k$ is positive, that is, $25 + 2k > 0$ or $k \ge -12$ (k an integer). And so the second inequality of the system will also be considered only for $k \ge -12$.

Note that for these k, the discriminate of the second inequality is also positive. For any fixed $k \ge -12$, the solution of the first quadratic inequality is the interval $5 - \sqrt{25 + 2k} < x < 5 + \sqrt{25 + 2k}$ while the solutions of the second one consist of two infinite intervals : $x > 5 + \sqrt{25 + 2k - (2/3)}$ and $x < 5 - \sqrt{25 + 2k - (2/3)}$. The common portions of the solutions of these two inequalities (in the terminology of set theory we would say "the intersection of the solutions of these two inequalities") yield the solution of the system and, hence, of the original inequality. Clearly, $\sqrt{25 + 2k - (2/3)}$

$< \sqrt{25+2k}$ for all k ≥ -12. Taking this remark into account, it is easy to write out the answer :

$$5 - \sqrt{25+2k} < x < 5 - \sqrt{25+2k-2/3},$$

$$5 + \sqrt{25+2k-2/3} < x < 5 + \sqrt{25+2k}$$

where k is an integer ≥ -12.

Besides inequalities that are combinations of elementary inequalities, the student often has to deal with inequalities in the solution of which he has to apply various transformations and the associated concepts.

9. In this question we will show how the concept of the domain of the variable is used. Since, the left member is a non-negative expression, the inequality is true for all values of x for which it is meaningful, that is to say, in the domain of the variable x. But the domain of this inequality consists of the set $x \geq 0$; this is the solution of the inequality.

10. This time again, the expression on the left-hand side is non-negative and so the inequality holds true for all x in the domain of the variable, with the exception of those for which the left member vanishes. This domain is determined by the condition $\log_{10} x \geq 0$; which is to say it is the set $x \geq 1$. But when $x = 1$, the left member vanishes and so this value of the unknown is not a solution of the inequality; the interval $x > 1$ constitutes the solution of the original inequality.

11. The domain of the variable here is defined by the conditions $x - 3 > 0$, $2 - x > 0$, $2 - x \neq 1$. But the inequalities $x - 3 > 0$ and $2 - x > 0$ do not have common solutions. Hence, the domain of our inequality does not contain a single number and so the inequality does not have a solution.

12. The domain of the variable is defined by the inequalities $x + 2 \geq 0, x - 5 \geq 0, 5 - x \geq 0$. But this system of inequalities has the solution $x = 5$. Hence, the domain of the original inequality consists of the unique solution $x = 5$. Therefore, no transformations are needed to solve this inequality since it is sufficient to verify that it is satisfied for $x = 5$. A direct verification shows that $x = 5$ is the solution.

13. The domain of this inequality is the interval $-1 \leq x \leq 2$. Thus, the left member of the original inequality assumes real and non-negative values for $-1 \leq x \leq 2$. It is meaningless for other values of x. But it is obvious that the right member of the inequality is negative of all $x < 4$ and, in particular, for all x in the interval $-1 \leq x \leq 2$; thus the proposed inequality is valid. Hence, the solution of the inequality is the interval $-1 \leq x \leq 2$.

14. The left member of this inequality is non-negative for all permissible x and, consequently, it cannot be true for any value of x, which means there are no solutions.

The foregoing examples make it clear that we cannot give a general recipe of how to employ the notion of the domain of the variable of an inequality in various specific cases. In the first two examples we simply could not have found the solutions without computing the domain, in the third, fourth and fifth we first found the domain and this immediately gave us our answer. On the contrary, in the sixth example it would have been a complicated job to find the domain; what is more, it would have been senseless since there were no solutions anyway among the permissible values of x.

For this reason, when solving complicated problems, it is sometimes useful to find the domain at the start, but occasionally this is useless since later on it turns out to be superfluous for the given case. A general piece of advice may be given : if computing the domain is not complicated, then it is best to do so (since it will never do any harm), but if it is a complicated affair, then put off computing the domain until it is really needed.

At competitions one often encounters problems that require transformations which can result in a loss of solutions or the introduction of extraneous solutions. Here again, as in the case of equation solving, a principal role is played by the concept of equivalence. Previously we examined the equivalence of equations and demonstrated why the student has to be sure that the newly derived equations and the original equations are equivalent. All this basically holds true for inequalities as well, in fact it is still more important than for equations.

Indeed, for equations it usually suffices to point out that for a certain transformation certain extraneous roots may be introduced and then to check the roots. In the case of inequalities, it is not possible to verify solutions by substitution since ordinarily there are an infinity of solutions. It is therefore necessary to pay special attention to the derived and original inequalities being equivalent.

It is to be noted that the transformations which lead to non-equivalence of equations naturally lead to non-equivalent inequalities.

Certain manipulations only extend or restrict the domain of the variable of the inequalities. A general procedure can be suggested for such transformations : manipulations restricting the domain are forbidden since that might result in a loss of solutions; as for manipulations extending the domain, first carry them out and then choose from the solutions of the final inequality those values which enter into the domain of the original inequality. These will yield the answer.

The most common types of transformations that alter domains are the "identity transformations", which have already been mentioned. Besides these, the solution of inequalities involves other transformations as well : clearing of fractions, taking certain functions of both members. These include powering, taking logarithms, antilogarithms, and the like. We will now take these up in more detail.

We start with the most "harmless" one, that of clearing fractions. Recall equations. There is no loss of solutions when clearing fractions, and extraneous solutions are introduced only due to the extension of the

domain of the variable, which is to say, *via* adding to the domain of the original equation those values of the unknown which make the denominator vanish.

Many think that the same holds true of inequalities, and so they "solve" the inequality $l/x < 1$ this way : "clearing fractions we get $1 < x$; all these values of x yield the solutions of the original inequality since the denominator of the original inequality does not vanish for any value."

But it is easy to see that the original/inequality holds true for all negative values of x as well. All these solutions are thus lost by the student because clearing of fractions in equations is quite different from that operation in inequalities.

Actually, clearing of fractions in an equation (or inequality) consists in multiplying both members of the equation (or inequality) by the expression in the denominator. In this operation, equations remain equivalent if they are multiplied by a non-zero expression, but for inequalities this property is more involved : multiplication of both members of an inequality by a positive expression does not change the sense of the inequality, multiplication by a negative expression reverses the sense of the inequality.

Therefore, when multiplying both members of the inequality at hand by x, one should have taken into account that the x could have assumed negative values as well as positive values, and then he should have reversed the sense of the inequality in the latter case.

Thus, in every case when we wish to multiply both members of an inequality by an expression that is dependent on x and assumes both positive and negative values, the student should examine the two appropriate cases. This rule is often forgotten and is the cause of a lot of trouble.

15. The domain of the variable in this inequality consists of all values of x except $x = -2$ and $x = 1/4$. From now on we will consider only those values of x which lie in the domain. At the examination, many students cleared fractions and wrote that it can be replaced by the following inequality :

$(x - 2)(4x - 1) \geq (2x - 3)(x + 2)$.................. (2)

This is clearly wrong since the manipulation actually amounts to multiplying both members of the original inequality by the expression $(x + 2\}(4x - l)$, which may be negative as well as positive. The original inequality may be replaced by (2) if and only if the expression $(x + 2)(4x - 1)$ is positive, and so also we have to consider the case. Then it is negative. Thus, the solution of the original inequality reduces to solving systems of inequalities.

It is simpler however to do as follows. Transpose all terms of the original inequality to the left side and reduce it to a common denominator :

$$\frac{2(x^2 - 5x + 4)}{(x + 2)(4x - 1)} \geq 0$$

The roots of the quadratic expression $x^2 - 5x + 4$, *i.e.*, $x_1 = 1$ and $x_2 = 4$, are the solutions of our inequality.

We will now assume that $x \neq 4$ and $x \neq 1$. and we will solve the inequality

$$\frac{(x - 1)(x - 4)}{(x + 2)(x - 1/4)} \geq 0 \qquad \ldots(3)$$

At this point, students often reduce the inequality to two systems of inequalities : the numerator and denominator are both greater than zero or are both less than zero. It is simpler however to solve it by the method of intervals.

Multiply both sides of the last inequality by the expression $(x + 2)^2 \times (x - 1/4)^2$, which is positive for the x under consideration. Then for all these values of x our inequality will be equivalent to the following one :

$$(x + 2)(x - 1/4)(x - 1)(x - 4) > 0 \qquad \ldots(4)$$

This inequality is in a form convenient for application of the method of intervals.

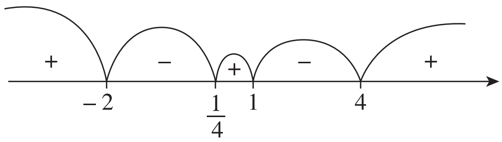


Fig. 9

Fig. 9. shows us that the solutions of the last inequality consist of all x in the intervals $x < -2$, $1/4 < x < 1$, $4 < x$. Since, we have already found that $x = 1$ and $x = 4$ are solutions to the original inequality, we get the answer :

$$x < -2,\ 1/4 < x \leq 1,\ 4 \leq x$$

In the foregoing solution, we replaced the inequality (3) by the Inequality (4) by multiplying, the first one by the square of the denominator. Similarly, we can assure ourselves that, generally, the inequalities

$$\frac{p(x)}{q(x)} > 0 \text{ and } p(x)\,q(x) > 0$$

are equivalent. Therefore, to solve the inequality

$$\frac{p(x)}{q(x)} > 0$$

where $P(x)$ and $Q(x)$ are polynomials, one applies the method of intervals to the inequality $P(x)\,Q(x) > 0$, which need not even be written out explicitly, it being sufficient to locate the roots of the polynomials $P(x)$ and $Q(x)$ on the number line and affix the appropriate sign to each of the resulting intervals.

16. Noting that $\tan(5\pi/4) = 1$ and applying Property VII of logarithms, we see that our inequality is equivalent to the double inequality $0 < (5x + 4)/(x - 2) < 1/2$ or, what is the same thing, to the system of inequalities

$$\frac{5x + 4}{x - 2} > 0$$

$$\frac{5x + 4}{x - 2} < \frac{1}{2}$$

Transposing 1/2 to the left member of the second inequality and carrying out the obvious manipulations, we rewrite it in the form $[x + (10/9)]/(x - 2) < 0$.

We use the method of intervals to solve each of the inequalities of this system and find that the first inequality holds true for $x > 2$ and for $x < -4/5$, while the second one is valid for x located in the interval $-10/9 < x < 2$. We now have to find the common part of these solutions (their intersection, in the terminology of set theory). This is conveniently done on the number line **(Fig. 10).**

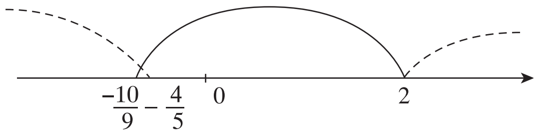


Fig. 10

Plotting the points $-10/9$, $-4/5$ and 2, we denote the solutions of the first inequality by the broken line and the solutions of the second one by the solid line. The overlapping (common) portion of these two ranges is readily found to be $-10/9 < x < -4/5$. This is the solution of the original inequality.

17. Using the properties of logarithms, this inequality may be rewritten

$$\frac{\log_2 4x}{\log_2 x \cdot \log_2 2x} > 1$$

Denoting $\log_2 x$ by y, we rewrite the last inequality as

$$\frac{2 + y}{y\,(1 + y)} > 1$$

Transposing all terms to the right and reducing to a common denominator, we get

$$\frac{y^2 - 2}{y\,(1 + y)} < 0$$

Factoring the numerator, we locate the roots of the numerator and denominator on the number line **(Fig. 11)** and then apply the method of intervals to get the solution of the inequality : all values of y in the intervals

$$-\sqrt{2} < y < -1 \text{ and } 0 < y < \sqrt{2}$$

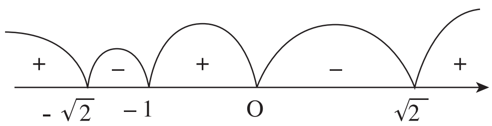


Fig. 11

Recalling that $y = \log_2 x$, we see that the solution of the original inequality includes all values of x that satisfy either the inequality $-\sqrt{2} < \log_2 x < -1$ or the inequality $0 < \log_2 x < \sqrt{2}$. The solution of the first inequality consists of the x in the interval $2^{-\sqrt{2}} < x < 2^{-1}$; the solution of the second inequality consists of the values of x located in the interval $1 < x < 2^{\sqrt{2}}$.

18. Denoting $\tan x$ by y, rewrite the inequality as

$$2\sqrt{3 + 2y - y^2} \geq 1 + 3y \qquad \ldots(5)$$

The domain of the variable in (5) is the interval $-1 \leq y \leq 3$. Our inequality is obvious for those values of y in the domain for which $1 + 3y < 0$; that is, all values of y in the interval $-1 \leq y < -1/3$ are solutions of inequality (5).

It remains to consider Case (b) : $-1/3 \leq y \leq 3$. Here both members of (5), are non-negative and so squaring in the case at hand yields the equivalent inequality $13y^2 - 2y - 11 \leq 0$.

The solution of the last inequality consists of all values of y in the interval $-11/13 \leq y \leq 1$. Taking into consideration Condition (b), we find that in this case the solution of inequality (5) consists of all values of y in the interval $-1/3 \leq y \leq 1$.

Combining both cases we find the solution to inequality (5) to be all values of y in the interval $-1 \leq y \leq 1$.

Returning to x, we find the solution of the original inequality to be all x satisfying the inequality $-1 \leq \tan x \leq 1$. We solve this elementary trigonometric inequality to get the answer :

$$-\frac{\pi}{4} + k\pi \leq x \leq \frac{\pi}{4} + k\pi, \text{ where } k = 0, \pm 1, \pm 2, \ldots\ldots$$

Ordinarily, taking antilogarithms of inequalities is only employed in the solution of inequalities involving the unknown under the logarithmic sign. We have already considered the solution of elementary logarithmic inequalities and have seen that they are very simply solved by taking advantage of Properties VII and VIII of logarithms.

The more complicated logarithmic inequalities should therefore also be solved on the basis of these properties. This will help the student to avoid many mistakes.

One more remark is in order : despite the fact that taking antilogs is always involved in the solution of logarithmic inequalities (either with regard for the domain of definition of the logarithmic function or by Properties VII and VIII), that term is not always used and we find phrases like this : "on the basis of the properties of logarithms (or the logarithmic function) we have.. .".

Taking antilogarithms is investigated in the next few problems.

19. The natural thing to do is to take antilogarithms. Since, the logarithmic base contains x and since the properties of a logarithmic function differ according as the base is greater than or less than unity, we cannot take antilogs straight off and will have to consider two cases.

(a) Let $\dfrac{25 - x^2}{16} > 1$, that is, $x^2 < 9$. In this case the given inequality is equivalent to

$$\frac{24 - 2x - x^2}{14} > \frac{25 - x^2}{16}$$

This inequality may be rewritten as $x^2 + 16x - 17 < 0$. The solution of this inequality consists of all values of x in the interval $-17 < x < 1$, but the condition of this

case (that is, $-3 < x < 3$) is only satisfied by those x located in the interval $-3 < x < 1$. All these values of x constitute the solution of the original inequality in the case at hand.

(b) Now, let $0 < (25 - x^2)/16 < 1$. Here, the original inequality is equivalent to the double inequality

$$0 < \frac{24 - 2x - x^2}{14} < \frac{25 - x^2}{16}$$

Thus, in this case we have to solve the following system of double inequalities :

$$0 < \frac{25 - x^2}{16} < 1$$

$$0 < \frac{24 - 2x - x^2}{14} < \frac{25 - x^2}{16}$$

The first one is readily reduced to $9 < x^2 < 25$ and its solution consists of two intervals : $-5 < x < -3$ and $3 < x < 5$. The second double inequality is equivalent to the system of inequalities

$$x^2 + 2x - 24 < 0$$

$$x^2 + 16x - 17 > 0$$

The first inequality of this system has the solution $-6 < x < 4$, the solution of the second one consists of two infinite intervals $x > 1$ and $x < -17$, so the solution of the latter system is the interval $1 < x < 4$.

We now have to choose from these values of x those which satisfy the first double inequality; they are the values of x in the interval $3 < x < 4$.

Thus, combining the two cases, we have the solution of the original inequality which consists of two intervals : $-3 < x < 1$ and $3 < x < 4$.

20. Denoting $\cos x$ by y and taking advantage of the formula for the cosine a double angle, we rewrite the inequality as

$$\log_{2y/\sqrt{3}} \sqrt{4y^2 - 1} < 1 \qquad \ldots(6)$$

The domain of this inequality is defined by the conditions $y^2 > 1/4$, $y > 0$, $y \neq \sqrt{3}/2$, or $y > 1/2$ and $y \neq \sqrt{3}/2$. Since, the logarithmic base may be greater than 1 or less than 1 for values of y, we consider two cases :

(a) Let $1/2 < y < \sqrt{3}/2$. Then the base is less than 1 and we obtain the equivalent inequality $\sqrt{4y^2 - 1} > 2y/\sqrt{3}$ or, since both members positive, $4y^2 - 1 > 4y^2/3$. This inequality is true for $y^2 > 3/8$, or $y > \sqrt{6}/4$. Taking into account the condition of the case at hand, we find the solution of inequality (6) to be the interval $\sqrt{6}/4 < \sqrt{3}/2$.

(b) Let $y > \sqrt{3}/2$. Then we get the equivalent inequality $\sqrt{4y^2 - 1} < 2y/\sqrt{3}$ whence, after squaring, follows $y^2 < 3/8$. But under the condition of our case $y^2 > 3/4$, that is, we do not obtain any new solutions to inequality (6).

It thus remains to solve the elementary trigonometric inequality $\sqrt{6}/4 < \cos x < \sqrt{3}/2$ which is satisfied for all values of x in the intervals

$$-\text{arc cos}\frac{\sqrt{6}}{4} + 2k\pi < x < -\frac{\pi}{6} + 2k\pi,$$

$$\frac{\pi}{6} + 2k\pi < x < \text{arc cos}\frac{\sqrt{6}}{4} + 2k\pi$$

where k is any integer **(Fig. 12).**

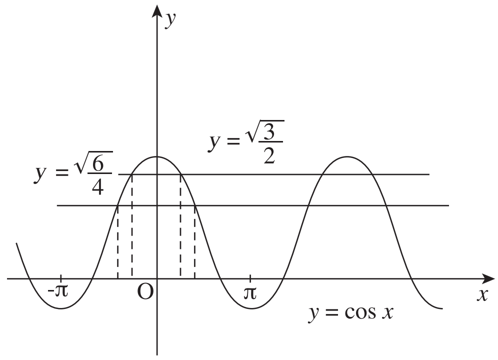


Fig. 12

With respect to taking logarithms of inequalities, it is easy to see in which cases this operation leads to an equivalent inequality. However, it is well to bear in mind that unwise logarithm-taking of inequalities can result in a restriction of the domain of the variable and a loss of solutions. Therefore, prior to taking logarithms always check to see that both members of the inequality are positive. Only then (and naturally with regard for the base of the logarithm) are we able to generate an equivalent inequality.

Earlier we solved the elementary inequality $(1/3)^x < 2$, using the properties of the exponential function let us now solve this inequality by taking logarithms. Since, both members of the inequality $(1/3)^x < 2$ are positive, we can take advantage of property VIII of logarithms and take logarithms of the inequality to the base 1/3 to get $\log_{1/3} (1/3)^x > \log_{1/3}2$ (note that the sense of the inequality has been reversed!) whence $x > \log_{1/3}2$.

Thus, the solution set of the inequality is the set $x > \log_{1/3}2$.

Note that all the elementary exponential inequality discussed above could have been solved *via* logarithms.

Let us solve a few problems by taking logarithms.

21. The domain of the variable here consists of all $x > 0$, except $x = 1$. Since, $7^{\log_{7 1/3} 5} = 5^3$ and $\log_{1/x}5 = -\log_x 5$, our inequality may be rewritten in the form

$$x^4 \cdot 5^3 \leq 5^{\log_x 5}$$

Both members are positive within the domain of the variable and so we can take logarithms of both sides of the inequality to the base 5 (greater than unity) and obtain the equivalent (in the domain) inequality $4\log_5 x + 3 \leq \log_x 5$. Denoting $\log_x 5$ by y and transposing all terms to the left-hand side, we rewrite the inequality thus; $4y + 3 - 1/y \leq 0$ or, reducing to a common denominator, thus : $(y + 1)(y - 1/4)/y \leq 0$.

Now, we apply the method of intervals and find the solution to be $y \leq -1$ and y in the interval $0 < y \leq 1/4$.

Now, reverting to x, we see that the original inequality is true for those values of x for which $\log_5 x \leq -1$ and also for those x for which $0 < \log_5 x \leq 1/4$. Solving these elementary logarithmic inequalities, we get the answer : $0 < x \leq 1/5, 1 < x \leq \sqrt[4]{5}$.

22. For arbitrary real x, the quadratic expression $x^2 + x + 1$ is positive and therefore the domain of the variable consists of all real values of x.

Since, both sides of the original inequality are positive for all x, we take logs to the base 10 to get the equivalent inequality $x \log_{10}(x^2 + x + 1) < 0$. This inequality holds true in two cases : when x satisfies the system of inequalities

$$x > 0$$
$$\log_{10}(x^2 + x + 1) < 0$$

and when x satisfies the system of inequalities

$$x < 0$$
$$\log_{10}(x^2 + x + 1) > 0$$

Let us solve the first system of inequalities. From the properties of logarithms we find that it is equivalent of the system

$$x > 0$$
$$(x_2 + x + 1) < 1$$

Since, the solution of the second inequality of the system is $-1 < x < 0$ and the solution of the first is $x > 0$, this system is inconsistent, which means that in this case original inequality does not have a solution.

The second system is equivalent to

$$x < 0$$
$$(x^2 + x + 1) > 1$$

The solution set of this system consists of all $x < -1$, whence the solution of the original inequality is the set of all values of $x < -1$.

A different solution of this inequality may be suggested. Since the properties of a power depend on whether the base is greater or less than unity, it is natural to consider two cases.

(a) Suppose that $x^2 + x + 1 < 1$, or $-1 < x < 0$. For all these values of x, the quadratic $x^2 + x + 1$ is raised to a negative power x. And since for all these values of x the trinomial $x^2 + x + 1 < 1$, it follows that for them $(x^2 + x + 1)\ x > 1$, which contradicts the condition. Hence, these values of x cannot be solutions of our inequality.

(b) Suppose that $x^2 + x + 1 > 1$. This is clearly valid for $x > 0$ and for $x < -1$. Therefore we have to consider two cases here.

Let $x > 0$. Then $x^2 + x + 1 > 1$ and after raising the expression to a positive power x the sense of the inequality remains unchanged, which means that for these x we have $(x^2 + x + 1)\ x > 1$. Hence, neither can these values of x be solutions of our inequality.

Let $x < -1$. Then $x^2 + x + 1 > 1$. If the quadratic expression $x^2 + x + 1$ is now raised to a negative power x, the result will be less than unity, which means that for all $x < -1$ we have $(x^2 + x + 1)\ x < 1$.

Thus, the solution set of the original inequality consists of all values of $x < -1$.

We have been making considerable use of the concept of domain of the variable of our inequalities. However, with the exception of just a few very elementary cases, we did not stress whether this has been helpful or not in solving inequalities. We will therefore consider two examples involving inequalities to see whether it is necessary to compute the domain of the variable beforehand.

23. The domain of x here is defined from the conditions $(4x - 5)/|x - 2| > 0$, $x^2 > 0$, $x^2 \neq 1$, whence $x > 5/4$ and $x \neq 2$. But for all these values of x we have $x^2 > 1$ and so our inequality, by the property of logarithms to a base exceeding unity, is equivalent (within the domain of x) to

$$\frac{4x - 5}{|x - 2|} \geq x$$

Since, $x \neq 2$, the expression $|x - 2|$ is positive and therefore the original inequality is equivalent, within the domain of x, to

$$4x - 5 \geq x\,|x - 2|$$

we now consider two cases.

(a) Let $x > 2$. Then our inequality will be rewritten as $4x - 5 \geq x^2 - 2x$ or $x^2 - 6x + 5 \leq 0$. The solution set of the last inequality consists of all x in the interval $1 \leq x \leq 5$, and the solution set of the original inequality in this case consists of all values of x in the interval $2 < x \leq 5$.

(b) Now, let $5/4 \leq x < 2$. Then our inequality takes the form $4x - 5 \geq -x^2 + 2x$ or $x^2 + 2x - 5 \geq 0$. Its solution set will consist of all values of x in the intervals $x \geq \sqrt{6} - 1$ and $x \leq -\sqrt{6} - 1$. The solution set of the original inequality will in this case consist of all values of x in the interval $\sqrt{6} - 1 \leq x < 2$.

Combining both cases, we find the solution of the original inequality to consist of all values of x in the intervals $\sqrt{6} - 1 \leq x < 2$ and $2 < x \leq 5$.

24. In this problem, it is not advisable to establish the domain of the variable beforehand since it does not simplify the solution and is a rather complicated matter.

Without finding the domain of x, we note only that in the domain, $x > 0$ and $x \neq 1$.

Denote $\log_x 2$ by y and rewrite the inequality as

$$y + 1 \leq \sqrt{y + 3} \qquad \ldots(7)$$

Here, the domain of y consists of all $y \geq -3$. But the inequality is obvious for y in the interval $-3 \leq y < -1$, which means that all these values of y constitute the solution.

Now, let $y \geq -1$. Then both members of (7) are non-negative; this inequality may be squared to obtain the equivalent (for $y \geq -1$) inequality $(y+1)^2 \leq y+3$, whose solution consists of all y in the interval $-2 \leq y \leq 1$. In this case, the solution of (7) consists of all values of y in the interval $-1 \leq y \leq 1$.

Combining both cases we see that inequality (7) is satisfied for $-3 \leq y \leq 1$.

Now, returning to x we find that the original inequality will have solutions for all x that satisfy the double inequality $-3 \leq \log_x 2 \leq 1$.

This inequality may be solved in two ways.

First solution : Since, the properties of logarithms differ for bases greater than or less than unity, we consider two cases : $x > 1$ and $0 < x < 1$.

(a) Let $x > 1$. Then $\log_x 2 > 0$, and all the more so $\log_x 2 \geq -3$. It remains to solve the inequality $\log_x 2 \leq 1$, whence $2 \leq x$. Thus, here the solution set of the original inequality consists of all values of $x \geq 2$.

(b) Let $0 < x < 1$. Then $\log_x 2 < 0$ and all the more so $\log_x 2 \leq 1$. It remains to solve the inequality $-3 \leq \log_x 2$, whence $x^{-3} \geq 2$, or $x \leq 2^{-1/3}$. Thus in this case the solution set of the original inequality consists of all x in the interval $0 < x \leq \sqrt[3]{1/2}$.

Combining both cases we have the solution of the original inequality : $0 < x < \sqrt[3]{1/2}$ and $x \geq 2$.

Second solution : Since, $\log_x 2 = 1/\log_2 x$, then by denoting $\log_2 x$ by z, we get the system of inequalities

$$\frac{1-z}{z} \leq 0$$

$$\frac{1+3z}{z} \geq 0$$

Solving each of these inequalities by the method of intervals, we find that the system will hold true for $z \geq 1$ and also for $z \leq -1/3$.

To get the answer we have to solve the elementary logarithmic inequalities $\log_2 x \geq 1$ and $\log_2 x \leq -1/3$, whence we obtain $x \geq 2$ and $0 < x \leq \sqrt[3]{1/2}$.

25. At the very start, many students made the mistake of discarding the first factor. Their reasoning probably went like this : since $\sqrt{4\sin^2 x - 1} \geq 0$, it is necessary for the second factor to be non-negative as well. This argument contains two mistakes at once : firstly, the domain of the variable is extended when the radical is discarded; secondly, when the first factor is zero, the inequality is valid even when the second factor is negative. The first mistake introduces extraneous solutions, the second mistake results in a loss of solutions.

A proper solution must take into account both of these items and can be carried out as follows. The given inequality is valid in two cases : when the second factor is greater than or equal to zero and when the first factor is zero. Then, we naturally have to take only those solutions of the resulting inequality and equation which enter into the domain of the variable of the original inequality.

This domain is defined by the system of inequalities

$$4\sin^2 x - 1 \geq 0, 0 < \sin x < 1, \frac{x-5}{2x-1} > 0$$

From the first two inequalities it follows that $1/2 \leq \sin x < 1$; the third is satisfied for $x < 1/2$ and $x > 5$.

Let us begin with the inequality $\log_{\sin x} [(x-5)/(2x-1)] \geq 0$. Since, $\sin x < 1$, it follows that the inequality is equivalent, in the domain, to the inequality $(x-5)/(2x-1) \leq 1$, the solutions of which are $x \leq -4$ and $x \geq 1/2$ of these values of x, only $x \leq -4$ and $x > 5$ lie in the domain of x. It remains to take into account the inequality $1/2 \leq \sin x < 1$.

The graph shown in **Fig. 13** makes it evident that the solution of this double inequality consists of the intervals

$$\frac{\pi}{6} + 2n\pi \leq x \leq \frac{5\pi}{6} + 2n\pi, n = 0, \pm 1, \pm 2, \ldots\ldots$$

with the points $x = \pi/2 + 2n\pi$ eliminated. Since, we only need $x \leq -4$ and $x > 5$, it follows (this too is found from the graph) that the values $n = 0$ and $n = -1$ do not satisfy us, and there remains a portion of the interval corresponding to $n = -1$: $-(11\pi)/6 \leq x \leq -4$ without $x = -3\pi/2$.

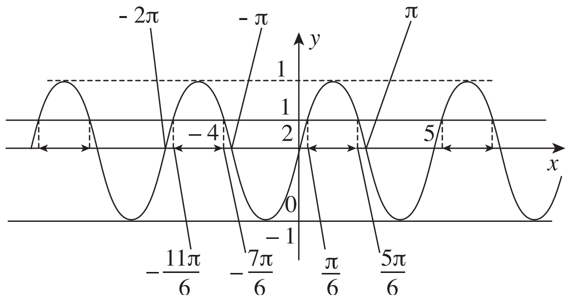


Fig. 13

We now consider the equation $4\sin^2 x - 1 = 0$, from which, taking into account that $\sin x, > 0$ in the domain, we get $\sin x = 1/2$. However, we have just solved an inequality which is clearly satisfied by the solutions of the equation $\sin x = 1/2$. Therefore, all the solutions of the equation at hand have been obtained and there would be no reason to include them if (and this is yet another underwater reef of the given problem) the solutions of the double inequality $1/2 \leq \sin x < 1$ had not partially been discarded because of the conditions $x \leq -4$ and $x > 5$. In this operation, the values $-7\pi/6$, $\pi/6$ and $5\pi/6$ were eliminated; the latter two do not enter into the domain of the original inequality, and the first one, that is, $x = -7\pi/6$, is to be adjoined to the intervals obtained above.

We thus get the answer :

$$\frac{\pi}{6} + 2n\pi \leq x \leq \frac{5\pi}{6} + 2n\pi, x \neq \pi/2 + 2n\pi$$

where n is any integer except 0 and 1, $-11\pi/6 \leq x \leq -4$ and also $x = -7\pi/6$.

6 Permutations and Combinations

Permutation means 'arrangement'; Combination means 'selection'.

There are two fundamental principle of counting :

1. Product Rule : If one process can be performed in m ways, and if corresponding to each of the m ways of performing this process there are n ways of performing a second process, then the number of ways of performing the two operations together is $m \times n$.

Example : If Ram can go from his house to Sita's house in 3 different ways and from Sita's house to the school in 5 different ways.

Then total number of ways of going from his house to the school is $3 \times 5 = 15$ ways.

2. Addition Rule : If a work w consists of two parts w_1 and w_2 of which one part can be done in m ways and the other part in n ways, then the work w can be done in $(m + n)$ ways.

Example : If Ram can go from Meerut to Delhi in 5 different ways and from Meerut to Faridabad in 3 different ways, then total number of different ways he can go to Delhi or Faridabad from Meerut = 5 + 3 = 8

Factorial : A factorial is the name obtained by multiplying all the positive integer less than or equal to a given positive integer.

The factorial of a given integer n is usually written as $n\,!$ or $\lfloor n$

$$\therefore \quad n\,! = n \times (n-1) \times (n-2) \times (n-3) \times \ldots 2 \times 1$$

Example :

$$\begin{aligned} 6\,! &= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= 30 \times 24 \\ &= 720 \end{aligned}$$

Permutation : 'n' number of given things can be arranged in different orders by taking some or all of them at a time each of these different arrangements is called a permutation.

The number of permutations of 'n' different things taking 'r' of them at a time is denoted by ${}^nP_r$ where P stands for permutation.

${}^nP_r$ is defined as

$${}^nP_r = \frac{n\,!}{(n-r)\,!}$$

$${}^nP_r = n\,(n-1)\,(n-2)\,(n-3) \text{ ------------- } (n-r+2)\,(n-r+1)$$

$$\begin{aligned} {}^8P_6 &= \frac{8\,!}{(8-6)\,!} = \frac{8\,!}{2\,!} \\ &= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} \\ &= 20160 \end{aligned}$$

$${}^nP_n = \frac{n\,!}{(n-n)\,!} = \frac{n\,!}{0\,!} = n\,!$$

$$0\,! = 1$$

Or, $\lfloor 0 = 1$

Illustration 1. Find the total number of arrangement possible with 6 letters (A, B, C, D, E, F) taking 4 of them at a time.

Solution : The total number of arrangements possible with 6 letters taking 4 of them at a time is given by

$${}^6P_4 = \frac{6\,!}{(6-4)\,!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$$

$$= 30 \times 12 = 360$$

Illustration 2. If ${}^nP_4 = 56 \times {}^nP_2$. Find the value of n.

Solution : $\dfrac{n\,!}{(n-4)\,!} = 56 \times \dfrac{n\,!}{(n-2)\,!}$

$$\frac{n\,!}{(n-4)\,!} = 56 \times \frac{n\,!}{(n-2)\,(n-3)\,(n-4)\,!}$$

Or, $n^2 - 5n + 6 = 56$

Or, $n^2 - 5n - 50 = 0$

Or, $n^2 - 10n + 5n - 50 = 0$

Or, $n\,(n-10) + 5\,(n-10) = 0$

Or, $(n-10)\,(n+5) = 0$

$n = 10, \; n \neq -5$

Negative value of n is not allowed.

Illustration 3. $\dfrac{{}^nP_3}{{}^{n-1}P_4} = \dfrac{5}{21}$ · Find the value of n.

Solution : $\dfrac{n\,!}{(n-3)\,!} \times \dfrac{(n-5)\,!}{(n-1)\,!} = \dfrac{5}{21}$

Or, $\dfrac{n\,(n-1)\,!}{(n-3)\,(n-4)\,(n-5)\,!} \times \dfrac{(n-5)\,!}{(n-1)\,!} = \dfrac{5}{21}$

Or, $\dfrac{n}{(n-3)\,(n-4)} = \dfrac{5}{21}$

Or, $21n = 5\,(n^2 - 7n + 12)$

Or, $5n^2 - 35n + 60 - 21n = 0$

Or, $5n^2 - 50\,n - 6n + 60 = 0$

Or, $5n(n-10) - 6(n-10) = 0$

Or, $(n-10)(5n-6) = 0$

$$n = 10, \quad n = \frac{6}{5}$$

Since, n represents number of things so it can't be a fraction.

$\therefore$ Answer $n = 10$

Combination : Each of different selections or collections or groups that can be formed by taking some or all of a number of given things at a time irrespective of the order in which the things appear in the group is called a combination.

The number of combinations of n different things r of them at a time is denoted by ${}^nC_r$,

where 'C' stands for combinations.

$$\therefore \quad {}^nC_r = \frac{n!}{r! \times (n-r)!}$$

Relation between ${}^nP_r$ and ${}^nC_r$:

${}^nP_r$ is written as $= \dfrac{n!}{(n-r)!}$

while ${}^nC_r$ is written as $\dfrac{n!}{r! \times (n-r)!}$

$\therefore \quad {}^nP_r = r! \times {}^nC_r$

$\therefore \quad {}^nC_n = {}^nC_0 = 1$

Illustration 4. Find the total number of possible groups of 3 men that can be formed out of 5 men A, B, C, D and E.

Solution : We have five men A, B, C, D and E.

$\therefore$ Combination of 3 men can be formed as

A B C	A C D	B C E
A B D	A C E	B D E
	C D E	A D E
A B E	B C D	

Clearly, number of groups = 10

Alternate Method : The number of groups can be formed by taking 3 men from 5 men at a time is given by

$$ {}^5C_3 = \frac{5!}{3! \times 2!} = \frac{5 \times 4 \times 3!}{3! \times (2 \times 1)}$$

$$= \frac{5 \times 4}{2} = 10$$

Illustration 5 . How many triangles can be formed with in 5 points which are co-planar and no three points are collinear?

Solution : A triangle is formed by joining 3 points which are co-planar but not collinear clearly using property of combination. We have to select three points out of 5 points.

So, number of triangles formed $= {}^5C_3$

$$= \frac{5!}{3! \times 2!}$$

$$= 10$$

Illustration 6. At an annual meet of a company there are 40 males and 25 females. Each male exchange greeting only with all the males and each female exchange greeting only with all females. Find the total number of greeting exchanged at the meet.

Solution : Each exchange of greeting occurs between two persons, either two males or two females. Exchange of greeting among males from one class while the exchange of greeting among females from another class and the two classes are independent of each other.

Number of greeting exchanged among males

$$ {}^{40}C_2 = \frac{40!}{2! \times (40-2)!}$$

$$= \frac{40 \times 39 \times 38!}{2 \times 1 \times 38!}$$

$$= 780$$

No. of greeting exchanged among females

$$ {}^{25}C_2 = \frac{25!}{2! \times 23!} = 300$$

Total number of greeting exchanged

$$= 780 + 300 = 1080$$

Illustration 7. In a class there are 20 boys and 16 girls. They are to be divided into subgroups each containing either 2 boys or 3 girls, what is the maximum number of ways of forming such subgroups ?

Solution : Boys have to be divided into subgroups of 2 each while girls have to be divided into subgroups of 3 each. The two types of subgroups are independent of each other.

Total number of possible subgroups among boys

$$= {}^{20}C_2 = \frac{20!}{2! \times 18!} = \frac{20 \times 19}{2}$$

$$= 190$$

Total number of possible subgroups among girls

$$= {}^{16}C_3 = \frac{16!}{3! \times 13!}$$

$$= \frac{16 \times 15 \times 14}{3 \times 2} = 16 \times 5 \times 7$$

$$= 80 \times 7 = 560$$

Maximum number of ways of forming such subgroups

$$= 190 + 560$$

$$= 750$$

Illustration 8 . A palace has 10 entry gates but and 8 exit gates. Entry gates cannot be used for exit and *vice-versa*. In how many ways can a tourist enter the palace and then come out of it?

Solution : The whole process can be broken into two part—

(i) entry and (ii) exit.

Number of ways in which the tourist can enter the palace $= m = 10$

Number of ways in which the tourist can exit from the palace $= n = 8$

Once he has entered the palace through any of the 10 entry gates. He has 8 ways open to him to exit from the palace. In other words, we can say that for each way of entering the palace he has 8 ways of coming out of it.

$\therefore$ For 10 ways of entering the palace there will be $10 \times 8 = 80$ ways of coming out of it.

Illustration 9. A young lady has 10 sarees, 8 pairs of shoes, 6 sets of jewellery and 4 watches. In how many different ways can she dress up? Using multiplication rule find the number of permutations of n different things taking r of them at a time or derive the relation

$$^{n}P_{r} = \frac{n\,!}{(n-r)\,!} \text{ for } r \leq n$$

Solution : We have all n different things and none of the n things is used more than once.

Let us select a space and divide it into r column and two rows.

I	II	III	-------------------------	$(r-1)^{th}$	r^{th}

Now, from n different things, r of them are to be selected for putting one of each column.

(I) The first column can be filled in n ways as any one of the given n things can be chosen to fill up this column.

(II) When the first column has been filled has been filled up in any one of the n ways. We are left with $(n-1)$ things. Then the second column can be filled up in $(n-1)$ ways with any one of the remaining $(n-1)$ things.

(III) When the first two columns have been filled up we are left with $(n-2)$ things. Therefore, the third column can be filled up in $(n-2)$ ways with any one of $(n-2)$ things that are remaining.

Similarly, $(r-1)^{th}$ column can be filled up in $\{n-(r-1)-1\} = [n-(r-2)]$ ways with any one of the remaining $(n-r+2)$ things.

$\therefore$ So, all the r columns are to be filled in succession, we can use the generalized multiplication rule to find the possible number of ways of filling these r columns.

$\therefore$ Total number of ways filling r columns

$= n\,(n-1)\,(n-2)$ ---------------- $(n-r+2)\,(n-r+1)$

Total number of permutation of n different things taking r of them at a time.

i.e. $^{n}P_{r} = n\,(n-1)\,(n-2)$ ------------- $(n-r+1)$

Illustration 10. How many different numbers of 4 digits can be formed with five digits 2, 5, 7, 6, 9 without repeating any of them in any number ?

Solution : Total possible ways of forming these numbers is equal to the number of permutations of 5 digits taking any 4 of them at a time

$$^{5}P_{4} = \frac{5\,!}{(5-4)\,!}$$

$$^{5}P_{4} = 5 \times 4 \times 3 \times 2 = 120$$

PERMUTATION

(A) General Permutation : When 'n' different objects and taken all at a time.

Place	1st	2nd		nth
Number of ways to fill	N	$(n-1)$		$n-(n-1) = 1$

$\therefore$ To fill all the places total number of ways

$= n \times (n-1) \times (n-2)$ -------------- $\times 1$

$= \lfloor n = {}^{n}P_{n}$

Example : How many number of 4-digits formed from four digits 9, 5, 7 and 6?

Solution : According to above formula—

Required number of numbers $= \lfloor 4$

$= 4 \times 3 \times 2 \times 1$

$= 24$

(B) Permutation without Repetition : We have 'n' different objects and 'r' objects are taken at a time.

Place	Ist	2nd	3rd	--------	rth
Number of ways	n	$(n-1)$	$(n-2)$	-------	$(n-r+1)$

Total number of ways

$= n\,(n-1)\,(n-2)$ ------------ $[\,n-(r-1)]$

$= n\,P_{r}$

$= n\,P_{r} = \dfrac{n\,!}{(n-r)\,!}$

Illustration 11. How many of different numbers of 3 digits can be formed with five digits 1, 2, 3, 4, 5 without repeating any of them in any number?

Solution : 3-digit number can be formed as

Hundred's place	Ten's place	Unit's place
5 ways	4 ways	3 ways

Total number of 3-digit numbers

$= 5 \times 4 \times 3 = 60$

(C) Permutation with Repetition : We have 'n' different objects and r blank spaces. Now,

Place	Ist	2nd	3rd	---------------	rth
Number of ways to fill with n objects	n	n	n	---------------	n

$\therefore$ Total number of ways $= n \times n \times n \times$ ------------ n

$= n^{r}$

Illustration 12. How many four digit numbers can be formed with digit 5, 6, 7, 8, 9 repetitions of digits allowed any number of times?

Solution : The four places in a four digit numbers are

Thousand's place	Hundred's place	Ten's place	Unit's place
It can be filled by one of digits 5, 6, 7, 8, 9 in 5 ways	It can be filled by one of digits 5, 6, 7, 8, 9 in 5 ways	It can be filled by one of digits 5, 6, 7, 8, 9 in 5 ways	It can be filled by one of digits 5, 6, 7, 8, 9 in 5 ways

$\therefore$ Total number of ways of forming four digits numbers with given 5 digits (5, 6, 7, 8, 9) repetition of digits being allowed any number of ways.

$$= 5 \times 5 \times 5 \times 5 = 5^4 = 625$$

(D) Permutation of Certain Things Occur Together : 'n' different objects of which 't' number of objects are to be always together ($t > 1$)

Block 'A'	Block 'B'
No. of objects remaining = $n - t$	Objects always together = t

If objects in block 'B' is considered as 1.

$\therefore$ Total number of objects in block = A + B

Hence, number of arrangements = $(n - t + 1)$

Number of ways 't' objects can be arranged in block 'B' itself = t

Using product rule :

Required number of arrangements = $(n - t + 1) \times \underline{|t}$

Illustration 13. Find the total number of words formed with 6 different alphabets, taking any four of them at a time.

Solution : We consider 4-different alphabets taking as a single alphabet.

Now, Total alphabet = $(6 - 4) + 1 = 2 + 1 = 3$

$\therefore$ 4-different alphabets can be arrangements in $\underline{|4}$ ways.

Block 'A'	Block 'B'
$\underline{|3}$ ways	$\underline{|4}$ ways

Now, Total number of words from 6-different alphabets taking any four of them at a time

$$= \underline{|3} \times \underline{|4}$$
$$= 3 \times 2 \times 4 \times 3 \times 2 \times 1$$
$$= 6 \times 24$$
$$= 144$$

(E) Certain things never together :

When r number of objects among 'n' different objects never together

Required permutation = $\underline{|n} - (\underline{|n} - r + 1) \times \underline{|r}$

(F) Objects of some group occur together : 'n' different groups ($n \geq 2$) where each group consists of number of the objects (≥ 1) such that the objects of the some group are always together.

We have,

Number of different groups = N

Number of objects in group I = n_1

Number of objects in group II = n_2

Number of different objects in group w = n_w

Group	1st	2nd	3rd		wth
No. of objects	n_1	n_2	N_3		n_w
No of ways	$\underline{|n_1}$	$\underline{|n_2}$	$\underline{|N_3}$		$\underline{|n_w}$

Number of ways objects can be arranged in each group = $\underline{|n_1} \times \underline{|n_2} \times \underline{|n_3} \ldots\ldots \times \underline{|\ \ }$

Number of ways each group can be arranged among themselves = n

Required number of ways = $n \times n_1 \times n_2 \ldots\ldots n_w$

Illustration 14. There are two books each of four volumes and two books each of three volumes to how many ways can these books be arranged in a shelf so that the volumes of the some book remain together?

Solution : Formula , Required arrangement

$$= \underline{|4} \times \underline{|4} \times \underline{|3} \times \underline{|3}$$
$$= 24 \times 24 \times 6 \times 6$$
$$= 36 \times 24 \times 24$$

(G) All objects are not different : n objects of which all are not different but 's_1' are alike, 's_2' are alike, 's_3' are alike and rest 'k' are all different

$\therefore$ Required number of permutations

$$= \frac{n\,!}{(s_1\,!) \times (s_2\,!) \times (s_3\,!)}$$

(H) Arrangements in a row (linear permutation) : Two different groups- one group consisting of 'n' distinct objects and other group have r number are arranged in a row such that

(a) All of 'r' are together :

Group I	Group II
No of object = n	No of objects together = r

If objects in Group II is considered as one, then total no of objects = $n + 1$

Required number of arrangement

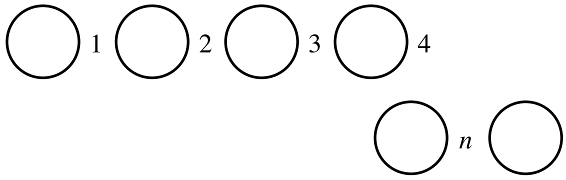
$$= \underline{|n} + 1 \times \underline{|r}$$

(b) All of 'r' are ever together : Required number of arrangements

$$= \lfloor \underline{n + r} - \lfloor \underline{(n+1)} \times \lfloor \underline{r}$$

(c) 'r' objects under restriction :

(1) No two of 'r' are together :



Given circle can be filled by 'r'objects to full fill the restriction .

Required number of arrangements = $(n\,!) \times {}^{n+1}P_r$

(d) Objects of both groups are alternate :

(i) If $n \neq +$

Required arrangements $= \lfloor \underline{n} \times \lfloor \underline{+}$

Or (ii) $n = +$

Required arrangements $= 2\,(\lfloor \underline{n}\,)^2$

Illustration 15. How many different words can be formed with the letters of the word 'illusion' taken all at a time, such that all vowels occur together ?

Solution : In 'illusion'

We have four (i, u, i, o) vowels

In which there are two

Now,

Required number of words in which all 4 vowels occur together

$$= 5 \times \frac{4}{2}$$

$$= 5 \times 4 \times 3 \times 2 \times 1 \times 4 \times 3$$

$$= 120 \times 12$$

$$= 1440$$

Illustration 16. In the above question if no two vowels occur together. Then find the how many different words can be formed?

Solution : From the above formula :

No. of words formed $= 4^{n+1} P_r \times \frac{1}{\lfloor \underline{2}}$

Here, we have $n = 4\,(1, 1, s, n)$

$R = 4\,(\text{I, I, o, u})$

There are two I, 2

$\therefore$ Number of words formed

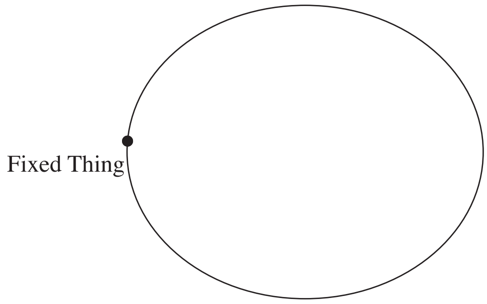
$$= 4 \times {}^5_4P \times \frac{1}{2}$$

$$= \lfloor \underline{4} \times \frac{\lfloor \underline{5}}{\lfloor \underline{1}} \times \frac{1}{\lfloor \underline{2}}$$

$$= 4 \times 3 \times 5 \times 4 \times 3 \times 2$$

$$= 120 \times 12 = 1440$$

(I) Circular Permutation : 'n' different things taken all at a time :

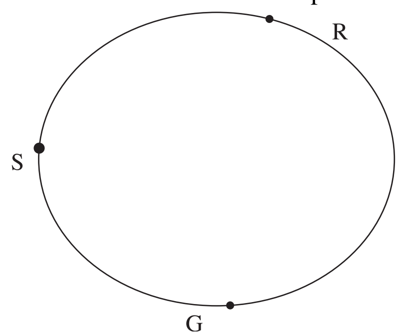


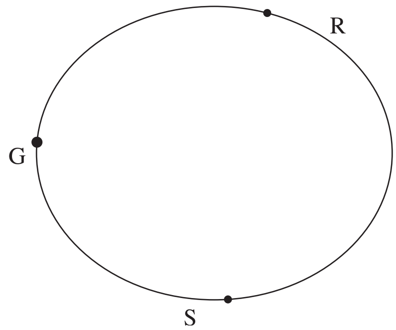
Here, having fixed 'n' things, the remaining $(n - 1)$ things can be arranged round the table **in $(n - 1)$! Ways.**

Particular case : necklace

Number of arrangements of n beads all different to form a necklace or on a circular wire will be $\frac{1}{2}(n - 1)$!

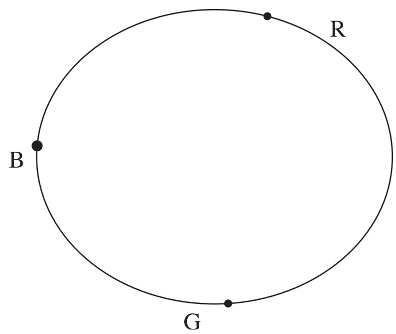
Explanation : R = Ram G = Gopal S = Sita

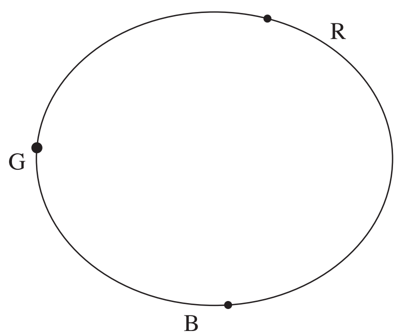




Round Table

The above seating arrangement of Ram, Gopal and Sita on the round table are different as shown in figure and that is why we say $(n - 1)$! Clockwise and anti-clockwise make different arrangements.





R = red flower, G = green flower, B = blue flower

The above arrangements of three flowers to form a necklace is the some because on the necklace we get the arrangement and that is why we say the total number of arrangements of n beads for forming **a necklace is** $\frac{1}{2}(n-1)!$

Here, clockwise or anticlockwise does not change the character of the necklace. It remarks the same.

Illustration 17. In how many ways a wedding garland can be formed out of 15 flowers of different colors?

Solution : Number of wedding garland formed from out of 15 flowers $= \frac{1}{2}(15-1)!$

$$= \frac{1}{2} \times (14)!$$

Illustration 18. Find the number of six-digit telephone numbers in a city if at least one of their digit is repeated and (i) Zero (0) is allowed at the beginning of telephone number (ii) Zero (0) cannot initiate the number.

Solution : Here, all the ten digits 0,1,2,----------9 have equal importance since '0' can also start the telephone number.

Number of six- digit telephone number

$= 10^6$ (when digit may be repeated)

If repetition of digit is not allowed

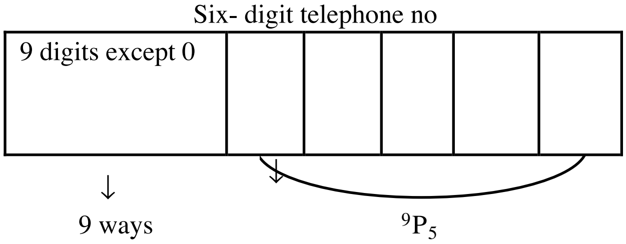
$$= {}^{10}P_6$$

Hence, required number of telephone numbers

$$= 10 - {}^{10}P_6 = 8{,}48{,}800$$

(ii) Here , 0 (zero) cannot come at the start of the telephone number.

Six- digit telephone no



Now,

Number of six- digit telephone number $= 9 \times 10^5$ (if repetition of digits allowed)

Again if repetition of digits not allowed

Number of six-digit telephone number $= 9 \times {}^9P_5$

Required number of telephone number

$$= 9 \times 10^5 - 9 \times {}^9P_5$$

$$= 7{,}63{,}920$$

COMBINATION

(A) Combination of 'n' distinct objects and taken all at a time

Required number of combination $= {}^nC_n = 1$

(B) Combination without repetition : Here we have 'n' distinct objects and taken 'r' objects at a time but no object is repeated.

Required number of combinations $= {}^nC_r$

$$= \frac{n\,(n-1)\ \ldots\ldots t \text{ or terms}}{r}$$

Illustration 19. Find the number of ways 6 identical balls can be distributed among 10 identical boxes if not more than one ball can go into a box.

Solution : Here balls are identical and boxes are also identical. So, it is the case of combination it also has been given that one ball can go into one box only *i.e.*, there is no repetition

Required number of ways $= {}^nC_r$

Here, $n = 10$; $r = 6$

Required number of ways $= {}^{10}C_6 = \frac{10!}{6!\,4!} = 210$

(C) Combination of with repetition : 'n' distinct objects and taking 'r' objects at a time when each objects may be repeated.

Required number of combination $= {}^{n+r-1}C_r$

Illustration 20. A box has 12 balls find the how many ways 8 balls can be selected if any ball may be repeated any number of times.

Solution : Using above formula-Required number of combination $= {}^{n+r-1}C_r$

Here, $n = 12$, $r = 8$

$\therefore$ Required number of combination

$$= C_8^{12+8-1} = C_8^{19} = \frac{19!}{(8!)\,(11!)}$$

$$= \frac{19\cdot18\cdot17\cdot16\cdot15\cdot14\cdot13\cdot12}{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1} = 19.18.17.13$$

(D) Certain things always occur :

'n' distinct objects taken 'r' at a time $r \le n$ such that particular 'P' no of objects always occur.

BlockA	**Block B**
Remaining no. of objects $= n - p$	No. of objects always occur = P

Number of combination in Block B = ${}^{p}C_{p}$

Since, all P objects to occur in each combination, remaining $r - p$ objects are to be taken from $n - p$ objects in block A

Number of combination in Block A = ${}^{n-p}C_{r-p}$

$\therefore = {}^{n-p}C_{r-p} \cdot {}^{p}C_{p} = {}^{n-p}C_{r-p}$

Illustration 21. A basket contain 14 tennis balls, 6 green 4 yellow and 2 blue and 2 white. Find the no. of ways a player can select some or all of tennis balls.

Solution : Total number of balls $n = 14$

Number of green balls, $m = 6$

No. of yellow balls, $p = 4$

No. of blue balls, $q = 2$

No. of white balls, $r = 2$

$\therefore$ Required no. of ways selecting some or all of tennis balls

$= [(m+1)(p+1)(q+1)(r+1)] - 1$
$[(6+1)(4+1)(2+1)(2+1)] - 1$

$= 7.5.3.3 - 1$

$= 314$

(E) Certain things never occur : 'n' distinct object taken 'r' at a time ($r \leq n$) when particular 'p' ($1 \leq p \leq n$) number of objects never occur.

Required number of combination = ${}^{n-p}C_{r}$

Illustration 22. A person has 10 friends and he wants to invite 8 of them to a birthday party. How many times 2, particular friends will never attend the parties ?

Solution : Here, total number of friends, $n = 10$

Particular friends = $p = 2$

We have to select only 8 friends

So, required number of combination ${}^{10-2}C_{8}$

$= {}^{8}C_{8} = 1$

(F) Equal Distribution of given different objects among persons :

Number of different objects $= n$

Number of persons $= r$

Since, distribution is made equally. So, each person gets equal number of objects $= \frac{n}{r}$

Persons	1st	2nd	3rd	-------------	r^{th}
	$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$
	$\frac{n}{r}$	$\frac{n}{r}$	$\frac{n}{r}$		$\frac{n}{r}$

Total number of ways for such distribution

$$= \frac{n}{\left(\frac{n}{r}!\right)^{r}}$$

Illustration 23. Find the number of ways 10 different things can be divided in two equal groups.

Solution : Now here $n = 10$

Number of groups = $r = 2$

$$\text{Required number of ways} = \frac{10!}{2\left(\frac{10}{2}!\right)^{2}}$$

$$= \frac{10}{2\cdot 5!\cdot 5!} \quad \frac{10\cdot 9\cdot 8\cdot 6}{5\cdot 4\cdot 3\cdot 2\cdot 2}$$

$$= 126$$

(G) Distribution of given different objects in several groups/packets :

Number of different objects $= n$

Number of groups $= p$

Distribution of different objects is made in 'p' number of groups

Again each group can be arranged among themselves in $p!$ ways

Group	1st	2nd	3rd	--------------	p^{th}
	$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$
	$\frac{n}{p}$	$\frac{n}{p}$	$\frac{n}{p}$		$\frac{n}{p}$

Total number of ways for such distribution

$$= \frac{n!}{\left(\frac{n}{p}!\right)^{p} \times p!}$$

Illustration 24. Find the number of ways 6- different sarees can be divided equally among 3 ladies.

Solution : Number of sarees = 6 ; Total ladies = 3; they are different.

So, required of ways $= \frac{6!}{(2!)^{3}} = 90$

Illustration 25. How many different numbers can be formed from the digits 1, 3, 5, 7, 9 (without repetition) when taken all at a time and what is their sum ?

Solution : 1, 3, 5, 7, 9 – five digits

Number of numbers = 5! = 120

Suppose 9 is the unit place, then the remaining 4 can be arranged in 4!= 24 ways.

Hence, sum of due to the unit place all the 120 number = 24(9+7+5+3+1) = 600 units.

Again, suppose 9 is in the 2nd place *i.e.,* ten's place and it will be so in 24 numbers. Similarly, each digit will be in ten's place 24 times. Hence, the sum of digits due to ten's place of all the 120 numbers is

= 24 (9 + 7 + 5 + 3 + 1) ten = 6000

Proceeding exactly for hundreds, thousands and ten hundreds. We have the sum of the numbers

$= 600(1+10+100+1000+10000)$

$= 600 \times 11111 = 6666600$

Illustration 26. A number of 4- different digits is formed by using the digits 1,2,3, 4,5,6,7 in all possible ways.

Find :

(a) How many such numbers can be formed ?

(b) How many of them are greater than 3400 ?

Solution : 4 – digit number

thousand	hundred	ten	one
			σ

We have 4-places and 7- digits

So, number of numbers formed

$$= {}^7P_4 = \frac{7\,!}{3\,!}$$

$$= 7 \times 6 \times 5 \times 4 \;\; = 840$$

(b)

thousand	hundred	ten	one
3			
When 3 is fixed At thousand place	Hundred's place can be filled from any of digits 4, 5, 6, 7	can be filled ${}^5P_2$	can be filled ${}^5P_2$

So, number of numbers formed when 3+5 fixed at thousands place $= 4 \times {}^5P_2 = 80$

Again

↓ 4,5,6,7			

Required number

$= 4 \times {}^6P_3 = 4 \times 6 \times 5 \times 4 = 24 \times 20 = 480$

Required number of numbers formed

$= 480 + 80 = 560$

Illustration 27. How many different numbers of 4 digits can be formed with nine digits 1,2,3,4,5,6,7,8,9 without repeating any of them in any number ?

Solution : A four digits numbers has four places

thousand	hundred	ten	one
↓ Thousand's place can filled from one of the nine digits in 9 ways	↓ Hundred's place be filled from one of 8 digits in 8 ways	↓ Similarly ten's place can be filled in 7 ways	↓ One's can be filled in 6 ways

∴ Total number of different numbers formed by 9 – digits

$$= 9 \times 8 \times 7 \times 6$$

$$= 72 \times 42 = 3024$$

Illustration 28. On a shelf there is space for 4 books in how many ways can it be filled 12 different books?

Solution : There is space for 4 books we have to fill 12 different books on that space.

∴ Number of ways the space of the shelf can be filled $= {}^{12}P_4 = 11880$.

Illustration 29. Husband has 12 relatives (7m, 5f) and his wife has 12 relatives (5m, 7f) in how many ways can they invite 12 people (6m, 6f) for a dinner party so that each invites 6 of his / her relations?

Solution :

Husband			**Wife**	
(7m, 5f)		(6m, 6f)	(5m, 7f)	
M -	F		M -	F
6	0		0	6
5	1		1	5
4	2		2	4
3	3		3	3
2	4		4	2
1	5		5	1

∴ Total number of ways $= ({}^7C_6 \cdot {}^5C_0)_H \times ({}^5C_0 \cdot {}^7C_6)_W + ({}^7C_5 \cdot {}^5C_1)_H \times ({}^5C_1 \times {}^7C_5)_W + ({}^7C_4 \cdot {}^5C_2)_H \times ({}^5C_2 \cdot {}^7C_4)_W + ({}^7C_3 \cdot {}^5C_3)_H \times ({}^5C_3 \cdot {}^7C_3)_W + ({}^7C_2 \cdot {}^5C_4)_H \times ({}^5C_4 \cdot {}^7C_2)_W + ({}^7C_1 \times {}^5C_5)_H \times ({}^5C_5 \cdot {}^7C_1)_W$

Illustration 30. In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back?

How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats?

Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seat ?

Solution : 3G, 9 B = 12 person

3,3 front, 4,4 back = 14 seats

(if no condition) number of arrangement $= {}^{14}P_{12}$

Take out 3G are taken as a single number of seating of 3G together on adjacent seats, there are 4 ways of selecting three seats 1,2,3 or 2,3,4 Ist van and 1,2,3 or 2,3,4 of 2nd van in each set the three girls can be arranged in 3! ways each

Total number of ways $= 4 \cdot 3\,! = 24$

After seating the three girls as desired we are left with 9 boys to be seated on remaining 11 seats which can be done in ${}^{11}P_9$ ways

∴ Hence by fundamental the or m total number of seating arrangement is ${}^{11}P_9 \cdot 24$

Required probability $= {}^{11}P_9 \cdot 24 \div {}^{14}P_{12} = \frac{1}{91}$

Illustration 31. A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each sides. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated ?

Solution : Having seated 4 on side A and 2 on side B we are left 10 person.

Out of which we choose 4 for side A in ${}^{10}C_4$ ways and now for side B we are left with 6 persons and 6 have to be seated so that they can be seated

$$\text{In } {}^{6}C_6 = 1 \text{ way}$$

Hence the number of selection for the two sides is

$${}^{10}C_4 \times 1 = \frac{10\,!}{6\,!\,4\,!}$$

Again 8 person on each side can be arranged amongst themselves in 8! ways

Total number of seating arrangement is

$$= \frac{10\,!}{6\,!\,4\,!} \times 8\,! \times 8\,!$$

$$= 341397504000$$

Illustration 32. Find r if ${}^{15}C_{3r} = {}^{15}C_{r+3}$.

Solution : We know that ${}^{n}C_x = {}^{n}C_y$

$$x + y = n$$

$$\therefore \quad {}^{15}C_{3r} = {}^{15}C_{r+3}$$

$$3r + r + 3 = 15$$

$$\Rightarrow \quad 4r = 15 - 3 = 12$$

$$\Rightarrow \quad R = \frac{12}{4} = 3$$

Illustration 33. If ${}^{n}C_8$ find ${}^{n}C_{17}$ and ${}^{22}C_n$.

Solution : We know that ${}^{n}C_x = {}^{n}C_y$

$$x + y = n$$

$$\therefore \quad 12 + 8n = 20$$

$$\text{Now,} \quad {}^{n}C_{17} = {}^{20}C_{17} = \frac{20 \times 19 \times 18}{3 \cdot 2} = 1140$$

$$\text{Now,} \; {}^{22}C_n = {}^{22}C_8 = {}^{22}C_{20} = \frac{22 \times 21}{2}$$

$$= 11 \times 21 = 232$$

Illustration 34. ${}^{9}P_5 + 5 \cdot {}^{9}P_4 = {}^{10}P_r$, find r.

Solution : $$\frac{\angle 9}{\angle 5 \angle 4} + 5 \cdot \frac{\angle 9}{\angle 4 \angle 5} = {}^{10}P_r$$

$$\frac{\angle 9}{\angle 4} + \frac{\angle 9}{\angle 4} = \frac{\angle 10}{\angle 10 - r}$$

$$\left(\frac{5}{\angle 5} = \frac{1}{\angle 4}\right)$$

$$2 \cdot \frac{\angle 9}{\angle 4} = \frac{\angle 10}{\angle 10 - r}$$

$$\frac{2}{\angle 4} = \frac{10}{\angle 10 - r}$$

$$\angle 10 - r = \angle 5$$

Comparing, we get $\quad 10 - r = 5$

$$r = 5$$

Illustration 35. ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$

Solution : ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$

We know that $\quad {}^{n}C_r = {}^{n}C_{n-r}$

$$\therefore \quad {}^{15}C_8 + {}^{15}C_{15-8} = {}^{15}C_7$$

$$\therefore \quad {}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$$

$$= \quad {}^{15}C_7 + {}^{15}C_6 - {}^{15}C_6 - {}^{15}C_7 = 0$$

Illustration 36. If ${}^{n}C_{n-r} + 3\;{}^{n}C_{n-r+1}\;\; 3\;{}^{n}C_{n-r+2} + {}^{n}C_{n-r+3} = {}^{P}C_r$

Find the value of P in terms of n.

Solution : We know that ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$$\therefore \; {}^{n}C_{n-r} + 3\,{}^{n}C_{n-r+1} + 3\,{}^{n}C_{n-r+2} + {}^{n}C_{n-r+3} = {}^{P}C_r$$

We can write

$${}^{n}C_{n-r} + {}^{n}C_{n-(r-1)} + 2\,{}^{n}C_{n-(r-1)} + 2\,{}^{n}C_{n-(r-2)} + {}^{n}C_{n-(r-2)} + {}^{n}C_{n-r+3} = {}^{P}C_r$$

$$\text{Or, } |{}^{n}C_{n-r} + {}^{n}C_{n-(r-1)}| + 2\,|{}^{n}C_{n-(r-1)} + {}^{n}C_{n-(r-2)}| + |{}^{n}C_{n-(r-2)} + {}^{n}C_{n-(r-3)}| = {}^{P}C_r$$

$$\text{Or, } {}^{n+1}C_{n-(r-1)} + 2\,{}^{n+1}C_{n-(r-2)} + {}^{n+1}C_{n-(r-3)} = {}_{P}C_r$$

$$\text{Or, } {}^{n+1}C_{n-(r-1)} + {}^{n+1}C_{n-(r-2)} + {}^{n+1}C_{n-(r-2)} + {}^{n+1}C_{n-(r-3)} = {}^{P}C_r$$

$$\text{Or, } \quad {}^{n+2}C_{n-(r-2)} + {}^{n+2}C_{n-(r-3)} = {}^{P}C_r$$

$$\text{Or, } \quad {}^{n+2}C_{n-(r-3)} = {}_{P}C_r$$

$$\text{Or, } \quad {}^{n+3}C_{n-(r-3)} = {}^{P}C_r$$

$$\text{Or, } \quad {}^{n+3}C_r = {}^{P}C_r \; \therefore \; P = n + 3$$

Illustration 37. If ${}^{n}C_{r-1} = 36$, ${}^{n}C_r = 84$ and ${}^{n}C_{r+1} = 126$

Find the value of n and r.

$$\frac{84}{36} = \frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \frac{n - r + 1}{r}$$

$$\Rightarrow \quad \frac{14}{6} = \frac{n - r + 1}{r}$$

$$\text{Or,} \quad \frac{7}{3} = \frac{n - r + 1}{r} \qquad \ldots(1)$$

$$\text{and} \quad \frac{{}^{n}C_{r+}}{{}^{n}C_r} = \frac{126}{84}$$

$$\text{Or,} \quad \frac{n - r}{r + 1} = \frac{21}{14} = \frac{3}{2} \qquad \ldots(2)$$

Solving eq. (1) & (2), we get

$n = 9 \qquad r = 3$

SOME IMPORTANT FORMULA

(1) $^nC_r = \dfrac{n\,!}{(n-r)\,!\,r\,!} = \dfrac{^nP_r}{r\,!}$

$r\,!\,nC_r = {}^nP_r$

(2) $^nC_n = {}^nC_0 = 1$

(3) $^nc_{r_1} = {}^nc_{r_2}$

$\Rightarrow r_1 + r_2 = n$

$\Rightarrow \quad ^nc_{r_1} = {}^nC_{n-r_2}$

(4) $^nc_r + {}^nc_{r-1} = {}^{n+1}c_r$

(5) $^nc_r \div {}^nc_{r-1} = \dfrac{n-r+1}{r}$

(6) $^{n-1}P_r + r \cdot {}^{n-1}P_{r-1} = {}^nP_r$

Illustration 38. Permutations with all n things taken at a time in which 2 specified things always occur together.

Solution : Let us consider the 2 specified things as a single entity. The number of things in the given sample reduces to $(n-1)$.

Number of possible permutations with these $(n-1)$ things among themselves $= (n-1)$.

For each of these $(n-1)$ permutations the 2 specified things can be rearrangement in two ways by interchanging their places = 2.

$\therefore$ Total number of possible permutation of n things among themselves such that 2 specified things always occur together $(n-1)\,! \times 2$.

Illustration 39. Permutations of n different things taken r at a time in which 2 specified things occur together.

Solution : We consider 2- specified things as a single thing /entity

We have $(n-2)$ things

1	2	3	------	-----	----	r places

But taking two things as a single thing/entity we have

1	2	3				r-2 places

Considering the extremes we have $r-1$ places we have to fill $(r-1)$ places from $(n-2)$ digits

$\therefore$ Required number of permutation

$$= {}^{n-2}P_{r-2} \times (r-1) \times 2\,!$$

Because the single entity /thing can be

Added at $(r-1)$ places

and the 2 specified things can be arranged in 2! = 2 ways by inter changing their places.

Illustration 40. Find the number of ways in which 8-different colored pencil can be arranged among themselves red and black colours are never together.

Solution : Considering red and black colors as a single colors.

So, we have $n-1 = 8-1 = 7$ different colors

$\therefore$ Required number of permutations $= n\,! - n\,! \times 2\,!$

$$= 8\,! - 7\,! \times 2\,!$$
$$= 7\,!\,(8-2)$$
$$= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \times 6$$
$$= 42 \times 120 \times 6 = 30240$$

Illustration 41. How many six digit numbers can be formed with 1 to 8 without repetition of any digit in any number such that digits 2 and 5 always occur together?

Solution : Here, $n = 8$

Here, 2 and 5 always occur together we have to form six digit number

$\therefore \qquad r = 6$

So, Required number of seven digit numbers

$$= 2(r-1)\,{}^{n-1}P_{r-1}$$
$$= 2\,(6-1)\,{}^6P_4$$
$$= 2 \times 5 \times \frac{6 \cdot 5}{2} = 150$$

Illustration 42. How many 4-digit numbers can be formed with digits 1to 8 without repetition any digit in any number such that digit 3 and 4 never occur together ?

Solution: $n = 8$ $\qquad$ r = 4

Required number of 4 digit number

$^nP_r - 2\,(r-1)^{n-2}\,P_{r-2} = {}^8P_4 - 2\,(4-1)^6\,P_2$

= 8.7.6.5-2.3.6.5 = 56.30-6.30

=1680 – 180 = 1500

Illustration 43. How many four digit numbers can be formed with digits 1, 3, 5, 7, 9 such that 3 is always at hundred's place ?

Solution :

Thousand	hundred	ten	one

Rest digit = 4

Now we have to fill 3 place by the digits 4 (1,5,7,9).

So, required number of numbers formed

$= {}^4P_3 = 4! = 4.3.2 = 24$

Illustration 44. How many seven digit number can be formed with digit 1, 2, 3, 4, 5, 6, 7, 8, 9. Such that they start with 2 and end with 6?

Solution :

6							2

Here, 2, and 6 are fixed so we have to fill 5- places with only 7-digits (1, 3, 4, 5, 7, 8, 9).

Required number of 7-digit number

$$= {}^7P_5 = \frac{7\,!}{2\,!} = 7{\cdot}6{\cdot}5{\cdot}4{\cdot}3 = 2520$$

Illustration 45. How many non-zero numbers can be formed with digits 0, 1, 2, 3, 4, 5 and 6 repetition of digits not being allowed in any number ?

Solution : (i) Single digit number. Total ways of forming single digit numbers

$$= 6$$

(ii) Two digits numbers. Number of permutations of 7 digits taken 2 at a time ${}^7P_2$

But this also includes ${}^6P_1$ two digit numbers starting with 0 and hence are not two digit numbers.

Total number of such two digit numbers starting with 0 is equal to the number of permutation of remaining six digits to fill units place this is given by ${}^6P_1$

Total number of two digit numbers

$$= {}^7P_2 - {}^6P_1$$

Similarly, total number of three digit numbers

$$= {}^7P_3 - {}^6P_2$$

Total number of four digit numbers

$$= {}^7P_4 - {}^6P_3 \text{ and so on}$$

$\therefore$ Total number of non- zero numbers

$$= {}^6P_1 + ({}^7P_2 - {}^6P_1) + ({}^7P_3 - {}^6P_2) + ({}^7P_4 + {}^6P_3) + ({}^7P_5 - {}^6P_4) + ({}^7P_6 - {}^6P_5) + ({}^7P_7 - {}^6P_6)$$

$$= ({}^7P_2 + {}^7P_3 + {}^7P_4 + {}^7P_5 + {}^7P_6 + {}^7P_7) - ({}^6P_2 + {}^6P_3 + {}^6P_4 + {}^6P_5 + {}^6P_6)$$

$$= 11742$$

Illustration 46. How many four digits numbers greater than 6000 can be formed with the digits 0, 3, 4, 6, 7 and 9 repetition not being allowed ?

Solution : Four digit numbers greater than 6000 must start with 6,7 or 9.

Suppose, the number starts with 6, the other three places in the number can be filled up by any of the 3-digits from remaining 5 digits (0,3,4,7,9).

The number of permutation of 5 digits taken 3 of them at a time is

$$= {}^5P_3 = 60$$

This hold for in each case of number starting with 7 and 9 too.

Required number of numbers = 3 × 60 = 180

Illustration 47. Shyam has 8 different color suits and 12 ties of different colors he has to take 4 suits and 8 ties on a tour. In how many ways he can select his dresses for the tour ? If a grey tie is not be selected in case he selects a blue suit, find the number of ways he can find the number of ways he can make his select his dresses for the hour.

Solution : (i) Number of ways four suits can be selected out of 8 suits

$$= {}^8C_4 = \frac{8{\cdot}7{\cdot}6{\cdot}5}{4{\cdot}3{\cdot}2} = 70$$

Number of ways 8 ties can be selected out 12 ties

$$= {}^{12}C_8 = \frac{12\,!}{8\,!\,4\,!}$$

$$= \frac{12{\cdot}11{\cdot}10{\cdot}9}{4{\cdot}3{\cdot}2{\cdot}1} = \frac{990}{2} = 445$$

$\therefore$ Total number of ways to select 4 suits and 8 ties

$$= 70 \times 495 = 34650$$

(ii) First we will find the number of selections in which both green tie and blue suit appear together. In such cases, 3 suits more are to be selected from remaining 7 suits and 7 ties are to be selected from remaining 11 ties.

$$\text{Number of ways} = {}^7C_3 \times {}^{11}C$$

$$= \frac{7\,!}{3\,!\,4\,!} \times \frac{11\,!}{7\,!\,4\,!}$$

$$= \frac{11{\cdot}10{\cdot}9{\cdot}8{\cdot}7{\cdot}6{\cdot}5}{3{\cdot}2{\cdot}4{\cdot}3{\cdot}2{\cdot}1}$$

$$= 11.10.7.3.5 = 11550$$

Exercise – A

1. If ${}^{2n+1}C_{n-1} : 2^{n-1}C_n = 7 : 4$, then the possible value of n will be—
   (A) 3 (B) 5
   (C) 4 (D) 2
2. In how many ways can the letters of the word "IGNOU" be arranged so that the vowels are never separated ?
   (A) 36 (B) 72
   (C) 1·8 (D) 18
3. How many different ways are possible to arrange the letters of the word "GRACE" so that the vowels may occupy only the even positions ?
   (A) 10 (B) 12
   (C) 14 (D) 15
4. The total number of ways in which letters of the word "DISCIPLINE" can be arranged so that the three I's never come together will be—
   (A) 4500 (B) 60,000
   (C) 564480 (D) 9500
5. The total number of ways in which 5 books can be distributed amongst 4 students (when repetitions are allowed) are—
   (A) 5^2 (B) 4^5
   (C) 1 (D) 93

6. A man invites 3 men and 3 women to a party. In how many ways can they sit at a round table so that no two men are together ?

(A) 10 (B) 12

(C) 16 (D) 24

7. 4 men and 5 women have to be seated in a straight row so that no two women are together. The number of ways in which this can be done is—

(A) 1440 (B) 2880

(C) 7200 (D) 7505

8. In how many ways can 11 different alphabets (1, 2, 3, …..) be arranged so that the alphabets f and g never come together ?

(A) $\lfloor 11 - \lfloor 10$ (B) $\lfloor 11 - \lfloor 10 \times \frac{1}{2}$

(C) $\lfloor 11 - 2 \times \lfloor 10$ (D) None of these

9. There are 9 points in a plane out of which 6 are collinear. The number of straight lines then can be drawn by joining these points will be—

(A) 22 (B) 24

(C) 36 (D) 38

10. In the above problem the number of triangles that can be drawn will be—

(A) 35 (B) 40

(C) 46 (D) 64

11. How many numbers greater than 6000000 can be formed with 1, 1, 6, 6, 9, 9, 0 ? (repetitions not allowed)

(A) 360 (B) 720

(C) 180 (D) 240

12. Out of 6 consonants and 3 vowels, all distinct, how many words can be formed each having 3 consonants and 1 vowel ?

(A) 2400 (B) 4800

(C) 3600 (D) 7200

13. How many numbers greater than one million can be formed with 3, 4, 5, 0, 2, 3 ? (repetitions not allowed)

(A) 575 (B) 300

(C) 625 (D) 675

14. How many numbers lying between 10 and 1000 can be formed with 2, 3, 4, 0, 8, 9 ? (repetitions not allowed)

(A) 360 (B) 120

(C) 125 (D) 142

15. Each consisting of 4 official and 5 non-official members—

(A) 1260 (B) 1800

(C) 3360 (D) 1600

16. Each containing at least two non-official members—

(A) 368 (B) 456

(C) 1029 (D) 1120

17. The letters T and E respectively occupy the first and the last places ?

(A) 7 ! (B) 6 !

(C) 5 ! (D) 2 × 6 !

18. The letters T, H, O are never together ?

(A) 2560 (B) 4320

(C) 5420 (D) None of these

19. The vowels always occupy even places—

(A) 3 ! × 3 ! (B) 4 ! × 4 !

(C) 3 ! × 4 ! (D) 5 ! × 4 !

20. How many words of 4 letters each can be formed, each containing 2 consonants and 2 vowels ?

(A) 432 (B) 5 ! × 4 !

(C) 4 ! × 4 ! (D) 241

21. How many different words beginning with Y and ending with I can be formed with the letters of the word "INDUSTRY"?

(A) 8 ! (B) $\frac{7\,!}{2\,!}$

(C) 6 ! (D) 2 × 5 !

22. The number of different permutations of the letters of the word 'MISSISIPPI'—

(A) $\frac{10\,!}{5\,!\,4\,!}$ (B) $\frac{10\,!}{4\,!\,3\,!}$

(C) $\frac{10\,!}{5\,!\,3\,!}$ (D) $\frac{10\,!}{4\,!\,3\,!\,2\,!}$

23. How many of them are greater than 3400 ?

(A) 840 (B) 560

(C) 480 (D) 120

24. How many of them are exactly divisible by 25 ?

(A) 20 (B) 35

(C) 40 (D) 50

25. How many of them are exactly divisible by 4 ?

(A) 150 (B) 160

(C) 120 (D) 200

26. From 7 men and 3 ladies, a committee of 4 is to be formed. The number of ways in which this can be done such that at least one lady is included, is—

(A) 70 (B) 72

(C) 75 (D) 92

27. The total number of seats at a particular management college is X, out of which Y seats are reserved for SC/ST candidates. If a total of 100 candidates including 17 SC/ST candidates take the MBA entrance test, then how many ways are there in which the section can be done ? (Assume Y < 17)

(A) $17C_Y \times {}^{11}C_{(X-Y)}$ (B) ${}^{17}C_Y \times {}^{(100-Y)}C_{(X-Y)}$

(C) ${}^{17}C_{(X-Y)} \times {}^{17}C_X$ (D) ${}^{(100-17)}C_X \times {}^{17}C_{(X-Y)}$

28. There are 5 boys and 6 girls in a family. They are photographed in groups of 6 children such that there are not least girl. The number of different photographs will be—

(A) 220 (B) 260
(C) 350 (D) 386

29. An army code languages consists of 4 symbol codes out of which the first two have to be numbers (digits) and the last two have to be alphabets. What is the total number of codes possible under such a scheme?

(A) 84656 (B) 24230
(C) 56346 (D) 67600

30. There are 40 volunteers for earthquake relief who have to be sent to four small villages requiring 12, 3, 11, and 9 men respectively. The number of ways in which this can be done is—

(A) ${}^{40}C_{12} \times {}^{28}C_3 \times {}^{25}C_{11} \times {}^{14}C_9$
(B) ${}^{40}C_{11} \times {}^{28}C_3 \times {}^{25}C_{11} \times {}^{14}C_9$
(C) ${}^{40}C_{12} \times {}^{28}C_3 \times {}^{25}C_{11} \times {}^{14}C_8$
(D) ${}^{40}C_{12} \times {}^{28}C_3 \times {}^{25}C_{10} \times {}^{14}C_9$

Exercise – B

1. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$, then the possible value of n will be—

(A) 3 (B) 5
(C) 4 (D) 2

2. In how many ways can the letters of the word "VALEDICTORY" be arranged so that the vowels are never separated ?

(A) 883490 (B) 967680
(C) 563680 (D) 483840

3. How many different ways are possible to arrange the letters of the word "PINAK" so that the vowels may occupy only the even positions ?

(A) 10 (B) 16
(C) 24 (D) 12

4. The total number of ways in which letters of the word ACCOST can be arranged so that the two C's never come together will be—

(A) 120 (B) 360
(C) 240 (D) 6! – 2!

5. The total number of ways in which 'X' things can be distributed amongst N persons (when repetitions are allowed) are—

(A) X^n (B) N^x
(C) ${}^nP_x \times X\,!$ (D) ${}^XP_N \times N\,!$

6. A man invites 4 men and 4 women to a party. In how many ways can they sit at a round table so that no two men are together ?

(A) 24 (B) 6
(C) 144 (D) 120

7. 5 men and 6 women have to be seated in a straight row so that no two women are together. The number of ways in which this can be done is—

(A) 48400 (B) 39600
(C) 9900 (D) 86400

8. In how many ways can 13 different alphabets (a, b, c, ….m) be arranged so that the alphabets f and g never come together ?

(A) 13! – 12! (B) 13! – 12!/2!
(C) 13! – 2 × 12! (D) None of these

9. There are 10 points in a plane out of which 5 are collinear. The number of straight lines than can be drawn by joining these points will be—

(A) 35 (B) 36
(C) 45 (D) 24

10. In the above problem the number of triangles that can be drawn will be—

(A) 120 (B) 110
(C) 100 (D) 78

11. How many numbers greater than 6000000 can be formed with 1, 1, 6, 6, 9, 9, 0 ? (repetitions not allowed)

(A) 360 (B) 720
(C) 180 (D) 240

12. Out of 5 consonants and 2 vowels, all distinct, how many words can be formed each having 2 consonants and 1 vowel ?

(A) 120 (B) 240
(C) 360 (D) 150

13. How many numbers greater than one million can be formed with 2, 3, 0, 3, 4, 2, 3 ? (repetitions not allowed)

(A) 720 (B) 360
(C) 120 (D) 240

14. How many numbers lying between 10 and 1000 can be formed with 2, 3, 4, 0, 8, 9 ? (repetitions not allowed)

(A) 360 (B) 120
(C) 125 (D) 142

15. Each consisting of 3 official and 2 non-official members—

(A) 360 (B) 180
(C) 336 (D) 160

16. Each containing at least two non-official members—

(A) 368 (B) 456
(C) 366 (D) 360

For Q. 17 – 20 : How many different words can be formed from the letters of the words GANESHPURI when :

17. The letters P and I respectively occupy the first and the last places ?

(A) 9 ! (B) 8 !
(C) 8 !/2 ! (D) 2 × 8 !

18. The letters E, H, P are never together ?
   (A) 8 ! (B) 8 ! × 42
   (C) 8 ! × 28 (D) 8 ! × 84

19. The vowels always occupy even places ?
   (A) 7 ! × 6 ! (B) 5 ! × 6 !
   (C) 6 ! × 5 !/2! (D) 2×6 ! × 7!

20. How many words of 5 letters each can be formed, each containing 3 consonants and 2 vowels ?
   (A) 5 ! × 5 ! (B) 5 ! × 6 !
   (C) 7 ! × 9 ! (D) 2 × 5 !

21. How many different words beginning with O and ending with E can be formed with the letters of the word ORDINATE ?
   (A) 8 ! (B) 6 !
   (C) 7 ! (D) 7 !/2 !

22. The number of different permutations of the letters of the word 'BANANA' are—
   (A) 120 (B) 60
   (C) 180 (D) 100

23. How many of them are greater than 3400 ?
   (A) 840 (B) 560
   (C) 480 (D) 120

24. How many of them are exactly divisible by 25 ?
   (A) 20 (B) 35
   (C) 40 (D) 50

25. How many of them are exactly divisible by 4 ?
   (A) 150 (B) 160
   (C) 120 (D) 200

26. From six men and 4 ladies, a committee of 5 is to be formed. The number of ways in which this can be done such that at least one lady is included, is—
   (A) 382 (B) 246
   (C) 482 (D) 336

27. The total number of seats at a particular management college is X, out of which Y seats are reserved for SC/ST candidates. If a total of 100 candidates including 17 SC/ST candidates take the MBA entrance test, then how many ways are there in which the section can be done ? (Assume Y < 17)
   (A) $17C_Y \times {}^{11}C_{(X-Y)}$
   (B) ${}^{17}C_Y \times {}^{(100-Y)}C_{(X-Y)}$
   (C) ${}^{17}C_{(X-Y)} \times {}^{17}C_X$
   (D) ${}^{(100-17)}C_X \times {}^{17}C_{(X-Y)}$

28. There are 5 boys and 3 girls in a family. They are photographed in groups of 2 boys and one girl. The number of different photographs will be—
   (A) 360 (B) 180
   (C) 30 (D) 60

29. An army code languages consists of 4 symbol codes out of which the first two have to be numbers (digits) and the last two have to be alphabets. What is the total number of codes possible under such a scheme?
   (A) 84656 (B) 24230
   (C) 56346 (D) 67600

30. There are 40 volunteers for earthquake relief who have to be sent to four small villages requiring 12, 3, 11, and 9 men respectively. The number of ways in which this can be done is—
   (A) ${}^{40}C_{12} \times {}^{28}C_3 \times {}^{25}C_{11} \times {}^{14}C_9$
   (B) ${}^{40}C_{11} \times {}^{28}C_3 \times {}^{25}C_{11} \times {}^{14}C_9$
   (C) ${}^{40}C_{12} \times {}^{28}C_3 \times {}^{25}C_{11} \times {}^{14}C_8$
   (D) ${}^{40}C_{12} \times {}^{28}C_3 \times {}^{25}C_{10} \times {}^{14}C_9$

Solutions – A

Solution : 1. (A)

Given $$2^{n+1}C_{n-1} : 2^{n-1}C_n = \frac{\dfrac{\lfloor 2n+1}{\lfloor n+1 \lfloor n}}{\dfrac{\lfloor 2n}{\lfloor n \lfloor n}} = \frac{7}{4}$$

$$\Rightarrow \frac{\lfloor 2n+1 \lfloor n \lfloor n}{\lfloor n+1 \lfloor n \lfloor 2n} = \frac{7}{4}$$

$$\Rightarrow \frac{(2n+1) \lfloor 2n \lfloor n}{(n+1) \lfloor n \lfloor 2n} = \frac{7}{4}$$

$$\Rightarrow \frac{2n+1}{n+1} = \frac{7}{4} \Rightarrow n = 3$$

Solution 2. (A)

Consider the vowels to be one entity (i, o, u), g, n have to be permuted and the 3 vowels can be also permute in the set.

⇒ Total number of arrangements possible

$$= {}^3P_3 \times {}^3P_3$$

$$\Rightarrow \frac{L_3}{L_0} \times \frac{L_3}{L_0} = 6 \times 6 = 36.$$

Solution 3. (B)

'GRACE' has 3 consonants and 2 vowels.

The vowels can be placed in position no. 2 or 4

∴ Total ways possible

= 2! = 2.

The consonants can occupy the position in 3! Ways.

So, the required number of ways

= 2! × 3! = 12.

Solution 4. (C)

Total number of ways to permute 10 alphabets 3 of which are common

$$= \frac{L_{10}}{L_3} = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4$$

= 720 × 840

= 604800.

Assume three I's as one

⇒ Number of possible ways = ${}^8P_8$

= L8 = 40320

So, required number of ways = Total arrangements – Number of arrangements in which they always come together

= 604800 – 40320 = 564480.

Solution 5. (B)

Books 1	books 2	books 3	books 4	books 5
4	4	4	4	4

$\Rightarrow$ Total number of ways $= 4 \times 4 \times 4 \times 4 \times 4 = 4^5$.

Solution 6. (C)

Fix the position of 1 women M

$\Rightarrow$ Remaining women can sit in ${}^2P_2$ ways

$\Rightarrow$ Remaining men can sit in ${}^3P_3$ ways.

$$\Rightarrow \text{Total } {}^3P_3 \times {}^2P_2 = L_3 \times L_2$$
$$= 6 \times 2 = 12$$

Solution 7. (B)

The arrangements will be : W M W M W M W M W

$\Rightarrow$ Total possible arrangements will be

$$= {}^4P_4 \times {}^5P_5$$
$$= L_4 \times L_5 = 2880.$$

Solution 8. (C)

Total possible arrangements $= |11$

Total number in which 6 and 7 are together

$$= 2 \times {}^{10}P_{10} = 2 \times |10$$

So, required arrangement $= |11 - 2 \times |10$

Solution 9. (A)

To draw a straight line, we need two points.

Hence, ${}^9C_2$ lines are possible.

But 6 points are collinear, hence we subtract ${}^6C_2$

But these 6 points give 1 straight line.

$\Rightarrow$ Number of straight lines possible

$$= {}^9C_2 - {}^6C_2 + 1$$
$$= \frac{|9}{|2|7} - \frac{|6}{|2|4} + 1$$
$$= 9 \times 4 - 3 \times 5 + 1$$
$$= 37 - 15 = 22.$$

Solution 10. (B)

A triangle requires 3 non collinear points, ${}^9C_3$ combinations. But 6 points give us straight line .

Hence, number of triangles

$$= {}^9C_3 - {}^6C_3$$
$$= \frac{9 \times 8 \times 7}{6} - \frac{6 \times 5 \times 4}{6}$$
$$= 84 - 20 = 64.$$

Solution 11. (A)

Case 1 : First place is occupied by 6

$\Rightarrow$ Total arrangements $= {}^6P_6/\,(2!\,2!)$

Because 9 and 1 each appear twice

Case 2 : First place is occupied by 9, then total ways are exactly similar to case 1.

$\Rightarrow$ Total numbers $= 2 \times ({}^6P_6/\,2!\,2!) = 360$.

Solution 12. (D)

Total number of ways in which 3 consonants can be selected out of 6 = ${}^6C_3$

Total number of ways in which 1 vowel can be chosen out of 3 = ${}^3C_2$

$\Rightarrow$ Total numbers $= {}^6C_3 \times {}^3C_2 \times {}^5P_5 = 7200$.

(The last term denotes the internal arrangements in each word)

Solution 13. (B)

Required number is greater than 1 million (6 digits).

From given digits, total numbers which can be formed $= |6$

Number starting with zero $= |5$

$\Rightarrow$ Required number $= |6 - |5 = 720 - 120$
$= 600$

Since, repetition not allowed, so required answer

$$= \frac{600}{|2} = 300.$$

Solution 14. (C)

Between 10 and 100,

Possible numbers

$= {}^6P_2 - 5$ (those starting with a zero) = 25.

Between 100 and 1000, possible numbers

$= {}^6P_3 - {}^6P_2$ (starting with a zero) = 100

$\Rightarrow$ Total $= 25 + 100 = 125$.

For Q. 15 & 16 : How many committees of 6 members each can be formed from 9 official and 5 non-official members in the following cases :

Solution 15. (A)

4 official and 2 non-officials

4 official out of 9 officials

$$= {}^9C_4 = \frac{|9}{|4\,|5} = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126$$

2 non-official out of 5 can be selected in ${}^5C_2$

= 10 ways

$\Rightarrow$ The number of ways in which the committee can be formed is

$= 126 \times 10 = 1260$.

Solution 16. (B)

Three non-official and 3 official

$$= {}^5C_3 \times {}^9C_3 = 10 \times 84 = 840$$

Four non-official and 2 officials

$$= {}^5C_4 \times {}^9C_2 = 5 \times \frac{9 \times 8}{2}$$
$$= 36 \times 5 = 180.$$

5 non-official and 1 official

$$= {}^5C_5 \times {}^9C_1 = 1 \times 9 = 9.$$

Total 840 + 180 + 9

$= 1029$.

For Q.17 – 20 : How many different words can be formed from the letters of the words CATHODE when :

Solution 17. (C)

T occupies first and E occupies the last place and so we have to arrange the remaining 5 letters which can be done in 5 ! ways.

Solution 18. (B)

Take T, H, O as one letter and so the number of letters will be 5 ! 3 !

The number of words in which T, H, O are together will be 5 ! 3 !

The total number of arrangements is 7 !

Hence the number of words when T, H, O are never together is

$= 7! - 5!\ 3! = 5!\ [7 \times 69 - 6] = 5!(42 - 6)$

$= 36 \times 120 = 4320$

Solution 19. (B)

We have 7 places, out of which 4 places are odd and 3 places are even *i.e.,* 1st , 3rd , 5th , 7th are odd places and 2nd, 4th , and 6th place are even places. We have with $^3P_3 = 3\,!$ ways.

Total number of ways is $= 3\,! \times 4\,!$

Solution 20. (A)

Each word is to contain 2 consonants out of 4 and 2 vowels out of 3 in $^4C_2$ and $^3C_2$ways. Thus, by fundamental theorem, the total number of combinations (groups) of 5 letters will be $^4C_2 \times {}^3C_2 = 6 \times 3 = 18$ ways. Thus, we have 18 groups each containing 4 letters *i.e.,* 2 consonants and 2 vowels. Now, the 4 letters in each group can be arranged amongst themselves in 4! Ways, *i.e.,* 24 ways. Hence, the total number of different words will be $24 \times 18 = 432$.

Solution 21. (C)

Fix the position of Y and I. Then here are 6! Ways of arranging the words in different letters.

Solution 22. (D)

In the word MISSISIPPI, $3S^{15}$, $2P^{15}$, $4T^5$ and 1m.

Number of words taken all at a time is $\dfrac{10\,!}{3\,!\,2\,!\,4\,!\,1\,!}$.

For Q. 23 – 25 : Refer to the following information to answer the question that follow—

A number of 4 different digits is formed by using the digits 1, 2, 3, 4, 5, 6, 7 in all possible ways without repetition.

Solution 23. (B)

Numbers greater than 3400 will have, 4 or 5 or 6 or 7 in the first place. Having filled the first place say by 4, we have to choose 3 digits out of the remaining 6 and the number will be $^6P_3 = 6!/3! = 6 \times 5 \times 4 = 120$.

Therefore, total of such numbers will be

$4 \times 120 = 480$ … (a)

Numbers greater than 3400 can be those which have 34, 34, 36, 37 in the first two places. Having filled up 34 in the first two places we will have to choose 2 more out of remaining 5 and the number will be

$^5P_2 = 5\,!/3\,! = 5 \times 4 = 20.$

Therefore, total as above will be $20 \times 4 = 80$ …(b)

Hence, all the numbers greater than 3400 will be

$480 + 80 = 560.$

Short-cut : Numbers less than 3400 will have 1 or 2 in 1st place or 31, 32 in the first two positions.

$^6P_3 + {}^6P_3 = 120 + 120 = 240.$

$^5P_2 + {}^5P_2 = 20 + 20 = 40.$

Total numbers which are less than 3400

$= 240 + 40$

$= 280$

Also total number of numbers formed is $^7P_4$

$= 840.$

Hence, numbers greater than 3400 is $840 - 280$

$= 560.$

Solution 24. (C)

A number will be divisible by 25 if the last two digits are divisible by 25 and this can be done in two ways for either 25 or 75 can be there and remaining two places out of 5 digits can be filled in $^5P_2$ ways. Hence, the required number $= 2 \times {}^5P_2 = 2 \times 20 = 40$.

Solution 25. (D)

A number divisible by 4 if the last two digits are divisible by 4 which can be done in 10 ways (12, 16, 24, 32, 36, 52, 56, 64, 72, 76)

Hence, required numbers

$= 10 \times {}^5P_2 = 10 \times 20 = 200.$

Solution 26. (A)

This can be done in 2 ways

(1) There are 2 ladies and 2 men $^3C_2 \times {}^7C_2$

(2) There are 3 ladies and 1 men $^3C_3 \times {}^7C_1$

Total number of ways $= 3 \times 21 + 1 \times 7$

$= 63 + 7 = 70$

Solution 27. (B)

Total numbers of possible ways

$= {}^{17}C_Y \times {}^{(100-Y)}C_{(X-Y)}.$

Solution 28. (D)

Different photographed in following ways :

(1) 3 girls and 3 boys $= {}^6C_3 \times {}^5C_3 = 200$

(2) 4 girls and 2 boys $= {}^6C_4 \times {}^5C_2 = 150$

(3) 5 girls and 1 boy $= {}^6C_5 \times {}^5C_1 = 6 \times 5 = 30$

(4) 6 girls $= {}^6C_6 = 6$

Total number of ways $= 200 + 150 + 30 + 6 = 386$

Solution 29. (D)

Each code is of the form NN' AA' where first two are digits and last two are alphabets.

$\Rightarrow$ Total number of codes possible

$= (10)^2 \times (26)^2 = 67600.$

Solution 30. (A)

For the first village, we can select volunteers in ${}^{40}C_{12}$ ways, for the second village, we have to select from the remaining volunteers 28 and so on.

Total number of ways = ${}^{40}C_{12} \times {}^{28}C_3 \times {}^{25}C_{11} \times {}^{14}C_9$.

Solutions – B

Solution 1. (C)

Given ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3/5$

$$\Rightarrow \frac{(2n+1)\,!}{(2n+1-n+1)\,!} \div \frac{(2n-1)\,!}{(2n-1-n)\,!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)\times 2n \times (2n-1)\,!}{(n+2)\,(n+1)\,n\,(n-1\,!)} \times \frac{(n-1)\,!}{(2n-1)\,!} = \frac{3}{5}$$

$$\Rightarrow \frac{2\,(2n+1)}{(n+2)\,(n+1)} = \frac{3}{5}$$

$$\Rightarrow \quad 5\,(4n+2) = 3\,(n^2+3n+2)$$

$$\Rightarrow \quad 20n+10 = 3n^2+9n+6$$

$$\Rightarrow \quad 3n^2-11n-4 = 0$$

$$\Rightarrow \quad (3n+1)\,(n-4) = 0$$

$$\Rightarrow \quad n = 4.$$

Short-cut : Work from the option.

Solution 2. (B)

Consider the vowels to be one entity (a, e, i, o), v, l, d, c, t, r, y have to be permuted and the 4 vowels can be also permute in the set.

$\Rightarrow$ Total number of arrangements possible

$= {}^8P_8 \times {}^4P_4 = 967680.$

Solution 3. (D)

'PINAK' has 3 consonants and 2 vowels.

The vowels can be placed in position no. 2 or 4

$\therefore$ Total ways possible $= 2! = 2.$

The consonants can occupy the position in 3! ways.

So, the required number of ways

$= 2! \times 3! = 2 \times 6 = 12.$

Solution 4. (C)

Total number of ways to permute 6 alphabets 2 of which are common = 6! / 2! = 360.

Treat the two C's as one

$\Rightarrow$ Number of possible ways = ${}^5P_5 = 120$

So, reqd. number of ways = Total arrangements – Number of arrangements in which they always come together

= 360 – 120 = 240.

Solution 5. (B)

The first can be given in N ways; the second thing can also be given in N ways; the third thing can also be given in N ways etc.

$\Rightarrow$ Total number of ways = N.N.N.... times = N^x.

Solution 6. (C)

Fixed the position of one woman

$\Rightarrow$ Remaining women can sit in ${}^3P_3$ ways

$\Rightarrow$ Total ${}^3P_3 \times {}^4P_4 = 144$ ways.

Solution 7. (D)

Total seats = 5 + 6 = 11

Arrangements will be : W M W M W M W M W M W

$\Rightarrow$ Total possible arrangements will be : ${}^6P_6 \times {}^5P_5$

= 86400.

Solution 8. (C)

Total possible arrangements

$= {}^{13}P_{13} = 13\,!$

Total number in which f and g are together

$= 2 \times {}^{12}P_{12} = 2 \times 12\,!$

Hence Answer is ${}^{13}P_{13} - 2.\,{}^{12}P_{12}$

$= 13\,! - 2 \times 12\,!.$

Solution 9. (B)

To draw a straight line, we need two points. Hence, ${}^{10}C_2$ lines are possible. But 5 points are collinear, hence we subtract ${}^5C_2$. But these 5 points give 1 straight line.

$\Rightarrow$ Number of straight lines possible

$= {}^{10}C_2 - {}^5C_2 + 1 = 45 - 10 + 1 = 36.$

Solution 10. (B)

A triangle requires 3 non-collinear points, ${}^{10}C_3$ combinations. But 5 points give us straight line .

Hence, number of triangles $= {}^{10}C_3 - {}^5C_3$

$= 120 - 10 = 110.$

Solution 11. (A)

Case 1 : First place is occupied by 6

$\Rightarrow$ Total arrangements = ${}^6C_6$ / (2! 2!)

Because 9 and 1 each appear twice

Case 2 : First place is occupied by 9, then total ways are exactly similar to case 1.

$\Rightarrow$ Total numbers = $2 \times ({}^6C_6 / 2!\,2!) = 360.$

Solution 12. (A)

Total number of ways in which 2 consonants can be selected out of $5 = {}^5C_2$

Total number of ways in which 1 vowel can be chosen out of $2 = {}^2C_1$

$\Rightarrow$ Total numbers $= {}^5C_2 \times {}^2C_1 \times {}^3P_3 = 120.$

(The last term denotes the internal arrangements in each word)

Solution 13. (B)

Required number is greater than 1 million (7 digits).

From given digits, total numbers which can be formed

$= 7\,!$

Number starting with zero $= 6\,!$

$\Rightarrow$ Required number $= 7\,! - 6\,!$

$\because$ Repetition not allowed, so required answer

$$= \frac{7\,! - 6\,!}{2\,!\,3\,!} = 360.$$

Solution 14. (C)

Between 10 and 100,

Possible numbers

= $^6P_2$ - 5 (those starting with a zero) = 25.

Between 100 and 1000, possible numbers

= $^6P_3$ - $^6P_2$ (starting with a zero) = 100

⇒ Total = 25 + 100 = 125.

For Q. 15 & 16 : How many committees of 5 members each can be formed from 8 official and 4 non-official members in the following cases :

Solution 15. (C)

3 official and 2 non-officials

3 official out of 8 can be selected $^8C_3 = 56$.

2 non-official out of 4 can be selected in $^4C_2$

= 6 ways

⇒ The number of ways in which the committee can be formed is $56 \times 6 = 336$.

Solution 16. (B)

At least two non-official members

⇒ Two non-official and 3 officials

$= {}^4C_2 \times {}^8C_3 = 6 \times 56 = 336.$

Three non-official and 2 officials

$= {}^4C_3 \times {}^8C_2 = 4 \times 28 = 112.$

Four non-official and 1 official

$= {}^4C_4 \times {}^8C_1 = 1 \times 8 = 8.$

Total 336 + 112 + 8 = 456

Short-cut : At least two non-officials

= Total – One non-official – No non-official.

$= {}^{12}C_5 - ({}^8C_4 \times {}^4C_1) - {}^8C_5 = 792 - 280 - 56 = 456.$

Solution 17. (B)

P occupies 1st and I occupies the last place and so we have to arrange the remaining 8 letters which can be done in 8! ways.

Solution 18. (D)

Take E, H, P as one letter and so the number of letters will be 10 – 3 + 1 = 8.

The number of words in which E, H, P are together will be 8!, 3!

The total number of arrangements is 10!

Hence, the number of words when E, H, P are never together is 10! – 8! 3! = 8! [10 × 9 – 6] = 84! × 8!.

Solution 19. (B)

We have ten places out of which 5 places are odd *i.e.*, 1st , 3rd , 5th , 7th, 9th, and five are even *i.e.*, 2nd, fourth, sixth, eight, and tenth. In the five even places we have to fix up 4 vowels which can be done in $^5P_4$ ways. Having fixed up the vowels in even places, we will be left with six places namely 5 odd and one even after fixing the four vowels. In these six places we have to fix six consonants which can be done in $^6P_6$ *i.e.*, 6! ways. Thus, the total number of ways is $^5P_4 \times {}^6P_6$ or 5! × 6!

Solution 20. (A)

Each word is to contain 3 consonants out of 6 and 2 vowels out of 4. We can select them in $^6C_3$ and $^4C_2$ ways. Thus, by fundamental theorem, the total number of combinations (groups) of 5 letters will be $^6C_3 \times {}^4C_2$. But $^6C_3 \times {}^4C_2 = 120$ ways. Thus, we have 120 groups each containing 5 letters *i.e.,* 3 consonants and 2 vowels. Now, the 5 letters in each group can be arranged amongst themselves in 5! Ways, *i.e.*, 120 ways. Hence, the total number of different words will be 120 × 120 = 14400.

Solution 21. (B)

6 ! ways, O fixed in 1st and E fixed in last.

Solution 22. (B)

BANANA.3 As, 2Ns, 1B, *i.e.,* 6 letters, 3 alike of one type and 2 of another type.

Number of words taken all at a time is $\frac{6\,!}{3\,!\,2\,!}$

$= \frac{6 \times 5 \times 4}{2} = 60.$

For Q. 23–25 : Refer to the following information to answer the question that follow—

A number of 4 different digits is formed by using the digits 1, 2, 3, 4, 5, 6, 7 in all possible ways without repetition.

Solution 23. (B)

Numbers greater than 3400 will have, 4 or 5 or 6 or 7 in the first place. Having filled the first place say by 4, we have to choose 3 digits out of the remaining 6 and the number will be $^6P_3 = 6!/3! = 6 \times 5 \times 4 = 120$.

Therefore, total of such numbers will be 4 × 120

= 480 .... (1)

Numbers greater than 3400 can be those which have 34, 34, 36, 37 in the first two places. Having filled up 34 in the first two places we will have to choose 2 more out of remaining 5 and the number will be $^5P_2 = 5\,!/3\,!$

= 5 × 4 = 20.

Therefore, total as above will be 20 × 4

= 80 ...(2)

Hence, all the numbers greater than 3400 will be 480 + 80 = 560.

Short-cut : Numbers less than 3400 will have 1 or 2 in 1st place or 31, 32 in the first two positions. $^6P_3 + {}^6P_3$

= 120 + 120 = 240.

$^5P_2 + {}^5P_2 = 20 + 20 = 40$. Total numbers which are less than 3400 = 240 + 40 = 280 Also, total number of numbers formed is $^7P_4 = 840$.

Hence, numbers greater than 3400 is 840 – 280

= 560.

Solution 24. (C)

A number will be divisible by 25 if the last two digits are divisible by 25 and this can be done in two ways for either 25 or 75 can be there and remaining two places out of 5 digits can be filled in $^5P_2$ ways. Hence, the required number = $2 \times {}^5P_2 = 2 \times 20 = 40$.

Solution 25. (D)

A number divisible by 4 if the last two digits are divisible by 4 which can be done in 10 ways (12, 16, 24, 32, 36, 52, 56, 64, 72, 76)

Hence, required numbers = $10 \times {}^5P_2 = 10 \times 20 = 200$.

Solution 26. (B)

There are 6 men and 4 ladies, in forming the groups, they may be classified as follows :

1. 1 lady, 4 men $\Rightarrow$ ${}^4C_1 \times {}^6C_4 = 60$
2. 1 lady, 3 men $\Rightarrow$ ${}^4C_2 \times {}^6C_3 = 120$
3. 3 ladies, 2 men $\Rightarrow$ ${}^4C_3 \times {}^6C_2 = 60$
4. 4 ladies, 1 man $\Rightarrow$ ${}^4C_4 \times {}^6C_1 = 6$

Total = 60 + 120 + 60 + 6 = 246.

Solution 27. (B)

Total numbers of possible ways

$= {}^{17}C_Y \times {}^{(100-Y)}C_{(X-Y)}$.

Solution 28. (B)

The number of ways of forming the group

$= {}^5C_2 \times {}^3C_1 = 30.$

Members of each group can be arranged among themselves in 3! ways

= 6 ways

Therefore, the number of photographs 30×6

= 180

Solution 29. (D)

Each code is of the form NN' AA' where first two are digits and last two are alphabets.

$\Rightarrow$ Total number of codes possible = $(10)^2 \times (26)^2$

= 67600.

Solution 30. (A)

For the first village, we can select volunteers in ${}^{40}C_{12}$ ways, for the second village, we have to select from the remaining volunteers 28 and so on.

Total number of ways = ${}^{40}C_{12} \times {}^{28}C_3 \times {}^{25}C_{11} \times {}^{14}C_9$.

7 Probability

Introduction : Human life is full of uncertainties in our day to day life very often we make guess and use statements like : "*possibility* of a particular party to win this election is more…………" Or "Most *probably* it will rain today."

Whenever we use such statement we have intuition which enables us to claim that one event is more likely to happen than the other.

In probability theory, the degree of certainty or uncertainty of such events is measured in terms of numbers lying between 0 and 1.

SOME IMPORTANT TERMS

1.Experiment : An operation which result in some well defined out come is called an experiment.

2.Random Experiment : An experiment whose out come can not be predicted with certainty is called a random experiment.

In other words if an experiment is performed many times under similar condition and the out come each time is not the same, then this experiment is called a random experiment.

Example : Tossing of a fair coin, is a random experiment because if we toss a coin either a head or a tail will come up. But if we toss a coin again and again the out come each time will not be the same.

Sample Space : The set of all possible out comes of a random experiment is called the sample space for that experiment. It is usually denoted by S.

Example : (i) When a coin is tossed either a head or a tail will come up. If H denotes the occurrence of head and T denotes the occurrence of tail.

Sample Space, $S = \{H, T\}$

(ii) When a die is thrown any one of the numbers 1, 2, 3, 4, 5 and 6 will come up.

$\therefore$ Sample Space, $S = \{1, 2, 3, 4, 5, 6\}$

Sample Point or Event Point : Each element of the sample space is called a sample point or an event point.

Example : When a die is thrown sample space.

$$S = \{1, 2, 3, 4, 5, 6\}$$

Here 1, 2, 3, 4, 5, 6 are the sample points.

Discrete Sample Space : A sample space S is called a discrete sample space if S is a finite set.

Event : A subset of the sample space S is called an event.

Example : When two coins are tossed

Sample Space S = {HH, HT, TH, TT }

Here HT denotes the occurrence of head on first coin and tail on second coin.

Simple Event or Elementary Event : An event is called a sample event if it is a singleton subset of the sample space S.

Example : When a coin is tossed,

Sample Space $S = \{H, T\}$

Let $A = \{H\}$ = the event of occurrence of head

$B = \{T\}$ = the event of occurrence of tail

Here A and B are simple events.

Mixed Event or Compound Event or Composite Event : A subset of the sample space S which contains more than one element is called a mixed event.

Example : When a die is thrown, Sample Space

$S = \{1, 2, 3, 4, 5, 6\}$

Let $A = \{1, 3, 5\}$ = the event of occurrence of an odd number and

$B = \{5, 6\}$ = the event of occurrence of a number greater than 4

Trial : When an experiment is repeated under conditions and it does not give the same result each time but may result in any one of the several possible. Out comes, the experiment is called a trial and the out comes are called cases.

The number of times the experiment is repeated is called the number of trials.

Example : One toss of a coin is a trial when the coin is tossed 5 times.

Occurrence of an Event : For a random experiment, Let E be an event.

Let $E = \{a, b, c\}$ if the out come of the experiment is a or b or c then we say that event E has occurred.

Equally Likely Cases (Events) : Cases (out comes) are said to be equally likely when we have no reason to believe that one is more likely to occur than the other. Thus when an unbiased die is thrown all the six faces 1, 2, 3, 4, 5 and 6 are equally likely to come up.

Exhaustive Cases (Events) : For a random experiment A, Set cases (event) is said to be Exhaustive if one of them must necessarily happen every time the experiment is performed.

Example : When a die is thrown cases or events 1, 2, 3, 4, 5, 6 form on exhaustive set of events.

Probability of Occurrence of an Event

Let S be the sample space, then the probability of occurrence of an event E is denoted by P (E) and it is defined as

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of elements in E}}{\text{number of elements in S}}$$

$$P(E) = \frac{\text{number of cases favourable to event E}}{\text{Total Number of Cases}}$$

Illustration 1. When a die is tossed, sample space, S = { 1, 2, 3, 4, 5, 6 }?

Solution : Let A = The event of occurrence of an odd Number = { 1, 3, 5 }

B = The event of occurrence of a number greater than 4 = { 5, 6 }

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Illustration 2. Probability of occurrence of an event is a number lying between 0 and 1?

Solution : Let S be the sample space and E be an event.

Then, $\phi \subseteq E \subseteq S$

$n(\phi) \subseteq n(E) \subseteq n(S)$

Or, $0 \subseteq n(E) \subseteq n(S)$

Or, $\frac{0}{n(S)} \subseteq \frac{n(E)}{n(S)} \subseteq 1$

$\therefore$ $0 \leq P(E) \leq 1$

Now, it is clear that

(i) ϕ is the impossible event.

(ii) $P(S) = \frac{n(S)}{n(S)} = 1$

(iii) $P(E) = 0$

$\Rightarrow$ $E = \phi$

(iv) $P(E) = 1$

$\Rightarrow$ $E = S$

Complement of an event : Let S be the sample space for a random experiment.

Let E be an event complement of event E is denoted by E^1 or E^c or $\overline{\in}$

E^1 means non occurrence of event E

Odds in favour and odds against on event :

Let S be the sample space and E be an event.

Let E^1 denotes the complement of event E, then

(i) Odds in favour of event E

$$= \frac{n(E)}{n(E^1)} = \frac{\text{Number of cases favourable to event E}}{\text{Number of cases against E}}$$

(ii) Odds against an event E

$$= \frac{n(E^1)}{n(E)} = \frac{\text{Number of cases against the event E}}{\text{Number of cases favourable to event E}}$$

Some Important information about playing cards :

(a) A pack of 52 playing cards has 4 suits

(i) Spade (ii) Hearts

(iii) Diamonds (iv) Clubs

(b) Spades and clubs are black faced cards.

(c) Heart and diamonds are red faced cards.

(d) Each suit consists of 13 cards.

(e) The aces, kings, queens, jacks are called face cards or honors cards.

Illustration 3. One ticket is drawn from a bag containing 30 tickets numbered 1 to 30. Represent the sample space and the event of drawing a ticket containing number which is a multiple of 5.

Solution : Since, the bag contains 30 tickets numbered 1 to 30.

Here Random experiment is :- Drawing of one ticket from the bag.

Let S be the sample space and E be event of drawing a ticket containing a number which is a multiple of 5.

Let 1, 2, 3, ----------- 30 denotes the out comes of drawing tickets containing numbers 1, 2, 3, --------- 30 respectively.

Then, S = {1, 2, 3, 4, -------------- 30 }

and E = {5, 10, 15, 20, 25, 30 }

Illustration 4. A coin is tossed successively three times. Find the probability of getting exactly one head or two heads.

Solution : Let S be the sample space and E be the event of getting exactly one head or exactly two heads, then

S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT }

and E = {HHT, HTH, THH, HTT, THT, TTH }

$n(E) = 6$ and $n(S) = 8$

Now, required probability, $P(E) = \frac{n(E)}{n(S)}$

$$= \frac{6}{8} = \frac{3}{4}$$

Illustration 5. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

Solution : A leap year has 366 days *i.e.,* 52 complete weeks and two days more. These two days will be two consecutive days of a week. A leap year will have 53

Tuesdays if out of the two consecutive days of a week selected at random one is a Tuesday.

Now, S = { (Monday, Tuesday) (Tuesday, Wednesday) (Wednesday, Thursday) (Thursday, Friday) (Friday, Saturday) (Saturday, Sunday) (Sunday, Monday)}

n (S) = 7

and E = {(Monday, Tuesday) (Tuesday, Wednesday) }

n (E) = 2

∴ Required probability, P (E)

$$= \frac{n\,(E)}{n\,(S)} = \frac{2}{7}$$

Illustration 6. What are the odds in favour of throwing at least 8 in a single throw with two dice?

Solution : Here random experiment is : Throwing of two dice

S = Sample Space

$= \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$

n (S) = 6×6

Throwing at least 8 with two dice means throwing 8 or 9 or 10 or 11 or 12.

∴ E = {(6, 2) (5, 3) (4, 4) (3, 5) (2, 6) (6, 3) (5, 4) (4, 5) (3, 6) (6, 4) (5, 5) (4, 6) (6, 5) (5, 6) (6, 6)}

n (E) = 15

$n\,(E^1) = 36 - 15 = 21$

Now, odds in favour of E

$$= \frac{n\,(E)}{n\,(E^1)} = \frac{15}{21} = \frac{5}{7}$$

Illustration 7. If a number of two digits is formed with the digits 2, 3, 5, 7, 9 without repetition of digits. What is the probability that the number formed is 35?

Solution : Here random experiment is : Formation of two digit number with the digits 2, 3, 5, 7, 9 without repetition.

Let S = Sample Space

E = The event that the number formed is 35

Now,

n (E) = 1

n (S) = Total number of numbers of two digits formed with the digits 2, 3, 5, 7, 9 without repetition.

$= {}^5P_2 = 5 \times 4 = 20$

∴ Required probability,

$$P\,(E) = \frac{n\,(E)}{n\,(S)} = \frac{1}{20}.$$

Illustration 8. From a group of 8 men and 5 ladies a committee of 8 persons is formed. What is the probability that the committee will consist of exactly 2 ladies?

Solution : n (S) = Sample Space = Total number of selections of 8 persons out of 13 persons

$= {}^{13}C_8$

n (E) = Number of selection of 6 men and 2 ladies out of 8 men and 5 ladies

$= {}^8C_6 \times {}^5C_2$

So Required probability

$$= P\,(E) = \frac{n\,(\in)}{n\,(S)}$$

$$P\,(E) = \frac{{}^8C_6 \times {}^5C_2}{{}^{13}C_8}$$

$$= \frac{8\,! \times 5\,!}{6\,! \times 2\,! \times 2\,! \times 3\,!} \times \frac{8\,! \times 5\,!}{13\,!}$$

$$= \frac{8 \times 7 \times 5 \times 4}{2 \times 2} \times \frac{5 \cdot 3 \cdot 4 \cdot 2 \cdot 1}{13 \times 12 \times 11 \times 10 \times 9}$$

$$= \frac{280}{1287}$$

Illustration 9. If from a pack of 52 playing cards one card is drawn at random, what is the probability that it is either a king or a queen?

Solution : Random experiment : Drawing of one card from a pack of 52 playing cards.

Let n (S)= Number of ways of selecting one card out of 52 playing cards

$= {}^{52}C_1 = 52$

n (E) = Number of selection of a card which is either a king or a queen

$= {}^4C_1 + {}^4C_1 = 4 + 4 = 8$

∴ Required probability, P (E)

$$= \frac{n\,(E)}{n\,(S)} = \frac{8}{52} = \frac{2}{13}$$

Probability in terms of Symbols

Let A and B any two events.

(a) $A \cap B$ or AB denotes the event of simultaneous occurrence of both the events A and B.

Thus, $A \cap B$ occurs if and only if both A and B occurs.

(b) $A \cup B$ or A + B denotes the events of occurrence of at least one of the events A and B.

Thus, $A \cup B$ occurs

⇒ at least one of A and B occurs.

(c) A – B denotes the occurrence of events A but not B.

Thus, A – B occurs

⇒ A occurs and B does not occur.

Clearly, $A - B = A \cap B^1$

$B - A = B \cap A^1$

(A) Mutually Exclusive or Disjoint events :

Two or more events are said to be mutually exclusive if one of them occurs, others cannot occur.

Thus, two or more events are said to be mutually exclusive if no two of them can occur together.

Thus, events A_1, A_2, --------------- A_n are mutually exclusive if and only if

$$A_i \cap A_J = \phi \qquad \text{for } i \neq j$$

Example : When a die is thrown sample S

$$= \{1, 2, 3, 4, 5, 6\}$$

Let A = the event of occurrence of a number greater than 4 $= \{5, 6\}$

B = the event of occurrence of an odd number

$$= \{1, 3, 5\}$$

C = the event of occurrence of an even number

$$= \{2, 4, 6\}$$

Then B and C are mutually exclusive events but A and B are not mutually exclusive because they can occur together.

(B) Independent or Mutually Independent Events :

Two or more events are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of other events.

Example : Let a bag contain 3 red and 2 black balls. Two balls are drawn one by one with replacement.

Let A = The event of occurrence of a red ball in first draw

B = The event of occurrence of a black ball in the second draw

$P(A) = \frac{3}{5}$ when a red ball is drawn in the first raw.

$$P(B) = \frac{2}{5}$$

Here probability of occurrence of event B is not affected by occurrence or non-occurrence of event A. Here A and B are independent events.

But if two balls are drawn one by one without replacement, then probability of occurrence of a black ball in the second draw when a red ball has been drawn in the first draw $= \frac{2}{4}$ probability of occurrence of a black ball in the second draw when a red ball is not drawn in the first draw $= \frac{1}{4}$

Here the events of drawing a red ball in the first draw and the event of drawing a black ball in the second draw are not independent.

So Clearly,

(i) A and B^1 are independent events.

(ii) A^1 and B are independent events.

Illustration 10. To prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

where A and B be any two events in a sample space S, then the probability of occurrence of at least one of the events A and B is given by $P(A \cup B)$.

Solution : From Set-theory we know that–

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Or, $\frac{n(A \cup B)}{n(S)}$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Case-I

If A and B are mutually exclusive events.

Then, $A \cap B = \phi$

Hence, $P(A \cap B) = 0$

$\therefore$ $P(A \cup B) = P(A) + P(B)$

Case-II

Two events are mutually exclusive if and only if

$$P(A \cup B) = P(A) + P(B)$$

Case-III

$$1 = P(S) = P(A \cup A^1) = P(A) + P(A^1)$$

$$P(A^1) = 1 - P(A)$$

Illustration 11. If A, B and C are any three events in a sample space S, then prove that

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

Solution : From Set-theory we know that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Now, $n(A \cup B \cup C)$

$= n[A \cup (B \cup C)]$

$= n(A) + n(B \cup C) - n[A \cap (B \cup C)]$

$= n(A) + n(B \cup C) - n[(A \cap B) \cup (A \cap C)]$

$= n(A) + n(B \cup C) - n(X \cup Y)$

{where $X = A \cap B, Y = A \cap C$}

$= n(A) + n(B \cup C) - [n(X) + n(Y) - n(X \cap Y)]$

$= n(A) + n(B) + n(C) - n(B \cap C) - n(X) - n(Y) + n(X \cap Y)$

$= n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$\therefore \frac{n(A \cup B \cup C)}{n(S)}$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} + \frac{n(C)}{n(S)} - \frac{n(B \cap C)}{n(S)} - \frac{n(A \cap C)}{n(S)} - \frac{n(A \cap B)}{n(S)} + \frac{n(A \cap B \cap C)}{n(S)}$$

$\therefore P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Illustration 12. Two cards are drawn at random from a pack of cards. Find the probability that both the cards are of red colour or they are queen.

Solution : Here random experiment is –

Drawing of two cards from a pack of 52 cards.

Let S be the sample space

A = The event that the two cards drawn are red

B = The event that the two cards drawn are Queen.

Number of cards drawn are both red or both queen

$= A \cup B$

$A \cap B$ = the event that the two cards drawn are queen of red colour.

Total number of cards = 52

Number of red cards = 26

Number of queen = 4

$\therefore \quad n(S) = {}^{52}C_2$

$n(A) = {}^{26}C_2$

$n(B) = {}^{4}C_2$

$\therefore$ Required probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2} - \frac{{}^{2}C_2}{{}^{52}C_2}$$

$$= \frac{26 \times 25}{52 \times 51} + \frac{4 \times 3}{52 \times 51} - \frac{2 \times 1}{52 \times 51}$$

$$= \frac{660}{2652}$$

Illustration 13. For a post three selected is twice that of B and the probability of B being selected is thrice that of C, what are the individual probabilities of A, B and C being selected?

Solution : Let E_1, E_2, E_3 be the events of selection of A, B and C respectively.

Let $\quad P(E_2) = X$

Then, According to question –

$$P(E_2) = 3P(E_3) = 3X$$

$$P(E_1) = 2P(E_2) = 6X$$

Here there are three candidates A, B and C one must be selected and exactly one will be selected.

$\therefore \quad P(E_1 \cup E_2 \cup E_3) = 1$

and E_1, E_2, E_3 are mutually exclusive.

Now, $\quad 1 = P(E_1 \cup E_2 \cup E_3)$

$$= P(E_1) + P(E_2) + P(E_3)$$

$$1 = 6X + 3X + X = 10X$$

$$X = \frac{1}{10}$$

$$\therefore \quad P(E_3) = X = \frac{1}{10}$$

$$P(E_1) = 6X = 6 \times \frac{1}{10} = \frac{3}{5}$$

$$P(E_2) = 3X = \frac{3}{10}$$

Illustration 14. The odds in favour of standing first of three students appearing at an examination are 1 : 2, 2 : 7 and 1 : 9 respectively. What is the probability that either of them will stand first?

Solution : Let the three students be P, Q and R.

Let A, B and C denotes the events of standing first of the three students P, Q and R respectively.

Given, Odds in favour of A = 1 : 2

Odds in favour of B = 2 : 7

and Odds in favour of C = 1 : 9

So, $\quad P(A) = \frac{1}{3}$

$P(B) = \frac{2}{9}$

$P(C) = \frac{1}{10}$

Since, events A, B, C are mutually exclusive.

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$= \frac{1}{3} + \frac{2}{9} + \frac{1}{10}$$

$$= \frac{30 + 10 + 9}{90} = \frac{49}{90}$$

Fundamental Principal of Counting

1. (a) Addition Rule : If a work is done when exactly one of a number of works A_1, A_2, A_3 --------- A_n is done, then number of ways of doing the work.

A = Sum of the number of ways of doing all the works A_1, A_2, ----------- A_n

(b) Multiplication Rule : If a work A is done when all of a number of works A_1, A_2, A_3, --------- A_n are done, then number of ways of doing the work.

A = Product of the number of ways of doing all the works A_1, A_2, A_3 ---------- A_n

2. (a) Number of permutations of n different things taken r at a time is given by

$${}^nP_r = \frac{n!}{(n-r)!}$$

(b) Number of ways of arranging of n different things = $n!$

(c) Number of ways of arranging n things out of which P are like and are of one type, Q are like and are of second type and rest are all different.

$$= \frac{n!}{P!\,Q!}$$

(d) Number of ways of arranging n different things along a circle when clockwise and anticlockwise arrangements are different *i.e.* when observations can be made from one side only. = $(n-1)!$

(e) Number of ways of arranging n different things along a circle when clockwise and anticlockwise arrangements are not different *i.e.*, when observation can be made from both sides $= \frac{(n-1)!}{2}$

3. Number of ways of selecting r different things out of n different things

$$= {}^nC_r$$

Illustration 15. If 12 persons are seated at a round table, what is the probability that three particular persons sit together?

Solution : Let S be the sample space

$$n(S) = (12-1)! = (11)!$$

= Total number of ways of seating 12 persons along around table.

Now, considering the three persons as one person.

Now, We have only 9 + 1 = 10 persons

These 10 persons can be seated at a round table in (9)! ways

But three particular persons can be arranged among themselves in 3! ways

$$\therefore \quad n(E) = (9)!.(3)!$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3!\cdot 9!}{11!} = \frac{3\times 2}{11\times 10} = \frac{3}{35}$$

Illustration 16. If from a pack of 52 playing cards, one card is drawn at random, what is the probability that it is either two kings or two queens?

Solution : S = Sample space

$n(S)$ = Number of ways of selecting two cards out of 52 playing cards

$= {}^{52}C_2$

$n(E)$ = Number of selecting of a card which is either a king or a queen

$$= {}^4C_2 + {}^4C_2 = \frac{4\times 3}{2} + \frac{4\times 3}{2}$$

$$= 6 + 6 = 12$$

Required probability

$$= P(E) = \frac{n(E)}{n(S)} = \frac{12}{{}^{52}C_2}$$

$$= \frac{12\times 2}{52\times 51} = \frac{6}{13\times 51} = \frac{6}{663}$$

$$= \frac{2}{221}$$

Illustration 17. Find the probability of drawing two kings a queen and a jack from a pack of 52 playing cards?

Solution : $n(S)$

= Total number of ways of selecting 4 card. from a pack of 52 cards

$= {}^{52}C_4$

$n(E)$ = Number of selection of 4 cards out of which two is a kings, one is a queen and one is a jack

$$= {}^4C_2 \times {}^4C_1 \times {}^4C_1 = 6\times 4\times 4$$

Now, P (E)

$$= \frac{n(E)}{n(S)} = \frac{6\times 4\times 4}{{}^{52}C_4} = \frac{6\times 4\times 4\times 4\times 3\times 2}{52\times 51\times 50\times 49}$$

$$= \frac{2}{13}\times\frac{1}{17}\times\frac{8}{25}\times\frac{2}{49}$$

BINOMIAL PROBABILITY

If an experiment is repeated n times under similar conditions, we say that n trials of the experiment have been made.

To solve such conditional problem we use Binomial Theorem.

Binomial Theorem on Probability :

Statement : Let E be the event and P be the probability of occurrence of event E in one trial.

$q = 1 - P$ = Probability of non occurrence of event E in one trial.

Again Let A be the number of success or number of times event E occurs in n trials.

Then, the probability of occurrence of event E exactly r times in n trials is given by

$$P(r) = n\,C_r\,P_r\;q^{(n-r)}$$

$$= (r+1)^{\text{th}} \text{ term in the expansion } (q+p)^n$$

Proof : Probability of occurrence of event E in one trial

$$= P$$

Probability of non-occurrence of event E in one trial

$$= q = 1 - p$$

Event E occurs exactly r times in n trials means that the event E occurs r times and it does not occur $(n-r)$ times in n trials.

Now, r trials in which event E occurs can be selected out of n trials in ${}^nC_r$ ways.

Also, the n trials are independent.

Therefore, probability of occurrence of event E exactly r times out of n trials is given by

P $(A = r)$ = ${}^nC_r$ (P.P.P. -------- r times) [q.q. ------------ $(n-r)$ times]

$$P(A = r) = {}^nC_r\,P^r\,q^{(n-r)}$$

Probability Distribution

Random Variable : A random variable is a real valued function defined over the sample space of an experiment. A random variable is usually denoted by the capital letters X, A, B, C, Z etc.

Discrete Random Variable : A random variable which can take finite or countable infinite number of values is called a discrete random variable.

Continuous Random Variable : A random variable which can take any value between two given limits is called a continuous random variable.

Probability distribution of a random variable :

If the values of a random variable together with the corresponding probabilities are given, then this description is called a probability distribution of the random variable.

Probability distribution when two coins to seed :

Let A denote the number of heads occurred then

P (A = 0) = Probability of occurrence of zero head

$$= P(TT) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(A = 1) = \text{Probability of occurrence of one head}$$
$$= P(HT) + P(TH)$$
$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$$

$$P(A = 2) = \text{Probability of occurrence of two head}$$
$$= P(HH) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Thus, the probability distribution when two coins are tossed is as given below :

X	0	1	2
P (A)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Another Form :

$$\begin{pmatrix} A \\ \\ P(A) \end{pmatrix} \begin{matrix} \rightarrow \\ \\ \rightarrow \end{matrix} \begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

Binomial Probability of throwing a die :

When a die is thrown,

Sample Space = {1, 2, 3, 4, 5, 6 }

Let E = the event of occurrence of a number greater than 4

$$E = \{5, 6\}$$
$$n(E) = 2$$

P = probability of occurrence of event E when a die is thrown once

$$= \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

$$q = 1 - P = 1 - \frac{1}{3} = \frac{2}{3}$$

Now, if probability of occurrence of event E three times when a die is thrown 10 times is given by

$$P(A = 3) = {}^{10}C_3 \times P^3 \times q^{(10-3)}$$
$$= {}^{10}C_3 \times \left(\frac{1}{3}\right)^3 \times \left(\frac{2}{3}\right)^7$$

Binomial Distribution :

Let P = probability of occurrence of event E in one trial

and

$q = 1 - P$ = probability of non-occurrence of event E in one trial

Let E = the event = a success

If X denote the number of success in n trials, then P $(X = r)$ = probability of r success

Therefore, the probability distribution of the random variable X in as given below :

X	0	1	2
P(X)	q^n	${}^nC_1 P\, q^{(n-1)}$	${}^nC_2\, P^2 q^{(n-2)}$
--------	r	--------	n
---------	${}^nC_r\, P^r\, q^{(n-r)}$	--------	P^n

Mean and Variance of a Random Variable :

Let X be a random variable which takes values X_1, X_2, ------------ X_n with corresponding probabilities P_1, P_2, ----------- P_n, then

$$\text{Mean, } \mu = \frac{\sum_{X=1}^{n} P_i X_i}{\Sigma P_i} = \Sigma P_i X_i \qquad \{\Sigma P_i = 1\}$$

and Variance,

$$(\sigma)^2 = \frac{\sum_{i=1}^{n} (X_i - \mu)^2 P_i}{\Sigma P_i}$$
$$= \sum_{i=1}^{n} (X_i - \mu)^2 P_i$$

Mean of the Binomial Distribution :

$$\text{Mean, } \mu = \Sigma Pi\, Xi = \sum_{r=0}^{n} r\, {}^nC_r\; P^r\; q^{(n-r)}$$

Here $x_i = r$ and $P_i = {}^nC_r\, P^r\, q^{(n-r)}$

$$= \Sigma n \times {}^{(n-1)}C_{(r-1)} \times P^r \times q^{(n-r)}$$
$$= n\, P \sum_{r=1}^{n} {}^{(n-1)}C_{(r-1)} \times P^{(r-1)} \times q^{n-1-(r-1)}$$
$$= n\, P\, (q + P)^n$$
$$= n\, P \qquad (P + q = 1)$$

$$\text{Variance} = \Sigma(Xi - \mu)^2 \times P_i$$
$$= \sum_{r=0}^{n} (r - n\,P)^2 \times {}^nC_r \times P^r \times q^{(n-r)}$$
$$= \sum_{r=0}^{n} r^2 \times {}^nC_r\, P^r\, q^{(n-r)} - 2n\,P \sum_{r=0}^{n} r\, {}^nC_r\; P^r\; q^{(n-r)} + n^2\, P^2 \sum_{r=0}^{n} {}^nC_r\, P^r\, q^{(n-r)}$$
$$= n\,P \sum_{r=1}^{n} {}^{(n-1)}C_{(r-1)}\, P^{(r-1)}\, q^{(n-r)} - 2\,n\,P\,.\,n\,P + n^2\, p^2 \;(P + q)^n$$
$$= n\,P \sum_{r=1}^{n} (r - 1 + 1) \times {}^{(n-1)}C_{(r-1)}\; P^{(r-1)}\, q^{(n-r)} - 2\,n^2\, P^2 + n^2\, P^2 \;(P{+}q = 1)$$
$$= nP \sum_{r=1}^{n} (r-1) \times {}^{(n-1)}C_{(r-1)}\; P^{(r-1)}\, q^{(n-r)} + nP \sum_{r=1}^{n} {}^{(n-1)}C_{(r-1)}\; P^{(r-1)}\, q^{(n-r)} - n^2\, P^2$$
$$= n\,P\,(n-1)\,P \sum_{r=2}^{n} {}^{(n-2)}C_{(r-2)}\, P^{(r-2)}\, q^{n-2-(r-2)} + n\,P\; (P + q)^{(n-1)} - n^2\, P^2$$
$$= n\,(n-1)\,P^2\,(P + q)^{(n-2)} + n\,P - n^2\,P^2$$
$$= n\,(n-1)\,P^2 + n\,P - n^2\,P^2$$

$= -nP^2 + nP$

$= nP(1-P)$

$= nPq$

Illustration 18. 10 coins are tossed what is the probability that exactly 5 heads appear ? Also, find the probability of getting at least 8 heads.

Solution : Let E = the event of occurrence of head on one coin

$\therefore \quad P = P(E) = \frac{1}{2}$ and $q = 1 - P = \frac{1}{2}$

Here $n = 10$ (Since 10 coins have been tossed)

Now, P (r) = Probability that event 6 will occur exactly r times in 5 trials.

$$= {}^nC_r \times P^r \times q^{(n-r)}$$

Required probability

$$= {}^{10}C_5 \times P^2 \times q^5$$

$$= \frac{10\,!}{5\,!\,5\,!} \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^5$$

$$= \frac{63}{256}$$

Second Part : Required Probability

$$= P(8) + P(9) + P(10)$$

$$= {}^{10}C_8 \times P^8 \times q^2 + {}^{10}C_9 \times P^9 \times q^1 + {}^{10}C_{10} \times P^{10} \times q^0$$

$$= 45 \times \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^2 + 10 \times \left(\frac{1}{2}\right)^9 \times \left(\frac{1}{2}\right) + 1 \times \left(\frac{1}{2}\right)^{10}$$

$$= 56 \times \left(\frac{1}{2}\right)^{10}$$

$$= \frac{7}{2^7} = \frac{7}{128}$$

Illustration 19. A lot of 100 bulbs from a manufacturing process is known to contain 10 defective and 90 non-defective bulbs. If 8 bulbs are selected at random, what is the probability that

(i) There will be 3 defective and 5 non-defective bulbs.

(ii) There will be at least one defective bulb.

Solution : Since, out of 100 bulbs, 10 bulbs are defective, therefore probability of drawing a defective bulb when one bulb is selected is given by

$$P = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \quad q = 1 - P = \frac{9}{10}$$

= probability that a bulb selected is non defective.

Since, 8 bulbs are selected, therefore number of trails

$$n = 8$$

$\therefore$ probability that out of 8 bulbs selected 3 will be defective and 5 non-defective is given by

P (X = 3) or P (3)

$$= {}^8C_3\, P^3\, q^5$$

$$= \frac{8\,!}{3\,!\,5\,!} \times \left(\frac{1}{10}\right)^3 \times \left(\frac{9}{10}\right)^5$$

$$= \frac{3306744}{100000000}$$

Probability that no bulb will be defective.

$$= 1 - P(0) = 1 - \left(\frac{9}{10}\right)^8$$

$$= 1 - \frac{43046721}{100000000} = \frac{56953279}{100000000}$$

Illustration 20. The first of three urns contains 7 white and 10 black balls, the second contains 5 white and 12 black balls and the third contains 17 white balls. A person chooses an urn at random and draws a ball from it and finds it to be white. Find the probability that the ball come from the second urn.

Solution : Urn – I = 7 white, 10 Black

Urn – II = 4 white, 12 Black

Urn – III = 17 white

Let $P\left(\frac{A}{A_1}\right)$ denote the probability of drawing a white ball when the ball is drawn from the second urn.

A denote the event of drawing a white ball. When one ball drawn at random from one of the three urns.

A_1 = the event that one ball drawn came from the second urn.

A_2 = the event that the ball drawn is from the first urn.

A_3 = the event that the ball drawn is from the third urn.

Baye's theorem :

$$P\left(\frac{A_1}{A}\right) = \frac{P(A_1) \times P\left(\frac{A}{A_1}\right)}{P(A_1) \times P\left(\frac{A}{A_1}\right) + P(A_2) \times P\left(\frac{A}{A_2}\right) + P(A_3) \times P\left(\frac{A}{A_3}\right)}$$

Now, $\quad P\left(\frac{A_1}{A}\right) = \frac{5}{17}$

Similarly, $\quad P\left(\frac{A_2}{A}\right) = \frac{7}{17}$

$\quad P\left(\frac{A_3}{A}\right) = \frac{17}{17}$

We assume that the probabilities of choosing first, second and third urn are equal.

$$P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$$

From equation (1), Required probability

$$P\left(\frac{A_1}{A}\right) = \frac{\frac{1}{3}\times\frac{5}{17}}{\frac{1}{3}\times\frac{5}{17}+\frac{1}{3}\times\frac{7}{17}+\frac{1}{3}\times\frac{7}{17}} = \frac{5}{29}$$

Illustration 21. An urn contains 4 red and 6 black balls and another urn contains 3 red and 7 black balls. If one ball is drawn from each urn. Find the probability that

(i) both balls are of same colour.

(ii) both balls are of different colours.

Solution : Let

I- 4R : 6B

$$P(E_1) = \text{probability of drawing red ball} = \frac{4}{10} = \frac{2}{5}$$

II- 3R : 7B

$$P(E_2) = \text{probability of drawing black ball} = \frac{6}{10} = \frac{3}{5}$$

Similarly,

$$P(E_3) = \frac{3}{10};\quad P(E_4) = \frac{7}{10}$$

Now, $P(E_1 \cap E_3) = P(E_1) \times P(E_3)$

E_1 and E_3 are independent.

$$P(E_1 \cap E_3) = \frac{2}{5}\times\frac{3}{10} = \frac{3}{25}$$

$$P(E_2 \cap E_4) = P(E_2)\times P(E_4) = \frac{3}{5}\times\frac{7}{10} = \frac{21}{50}$$

Now, probability of drawing the same colour

$$= \frac{3}{25}+\frac{21}{50} = \frac{6+21}{50} = \frac{27}{50}$$

Now, $P(E_1 \cap E_4)$

= probability of drawing the one red and one black ball

$$= \frac{2}{5}\times\frac{7}{10} = \frac{7}{25}$$

$$P(E_2 \cap E_3) = \frac{3}{5}\times\frac{3}{10} = \frac{9}{50}$$

∴ Probability of drawing the different colour

$$= \frac{7}{25}+\frac{9}{50} = \frac{14+9}{50} = \frac{23}{50}$$

Compound and Conditional Probability

Compound Events : When two or more events occur together their joint occurrence is called a compound event.

Example : Drawing a red and a black ball from a bag containing 5 red and 6 black balls when two balls are drawn from the bag is a compound event.

Compound events are two types :

(i) Independent events

(ii) Dependent events

Compound Probability : Let A and B be any two events $B \neq \phi$, then $P\left(\frac{A}{B}\right)$ denotes the conditional probability of occurrence of event A when B has already occurred.

Example : Let a bag contain 2 red balls and 3 black balls. One ball is drawn from the big and this ball is not replaced in the bag. Then second ball is drawn from the bag.

Let B denote the event of occurrence of a red ball in the first draw and A denote the event of occurrence of a black in the second draw.

When a red ball has been drawn, the number of balls left is 4 and out of these four balls one is red ball and three are black balls.

Illustration 22. If a dice is thrown then find the probability of occurrence of a number greater than 4, when an odd number has occurred.

Solution : When a die is thrown :

Sample Space S = {1, 2, 3, 4, 5, 6}

Let A = the event of occurrence of a number greater than 4.

= {5, 6}

B = the event of occurrence of a number greater than 4.

Then, $P\left(\frac{A}{B}\right)$ = probability of occurrence of a number greater than 4 when an odd number has occurred.

But among odd number only 5 is greater than 4.

So, here when an odd number has occurred total number of case is only 3 and favourable case is 1.

Hence, $P\left(\frac{A}{B}\right) = \frac{1}{3}$

(A) Multiplication Theorem of Probability or Theorem of Compound Probability :

Let S = Sample Space. In case of occurrence of event A when B has already occurred, B works as the sample space and $A \cap B$ works as the event.

Now, $$P\left(\frac{A}{B}\right) = \frac{n(A\cap B)}{n(B)} = \frac{n(A\cap B)/n(S)}{n(B)/n(S)}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$$

Case–I : If A and B are independent events, then probability of occurrence of event A is not affected by occurrence or nor occurrence of event B.

$$\therefore\quad P\left(\frac{A}{B}\right) = P(A)$$

$$\therefore\quad P(A\cap B) = P(A)\times P(B)$$

Case–II : If A, B, C are any three independent events

$$P(A\cap B\cap C) = P(A)\times P(B)\times P(C)$$

(B) Complementation Rule :

Case–I : If A and B are two independent events.

$\therefore \quad P(A \cup B) = 1 - P(A^1) \times P(B^1)$

Similarly, if A_1, A_2, A_3 ....A_n are independent events

$P(A_1 \cup A_2 \cup A_3 \cup A_4 \ldots\ldots\ldots \cup A_n)$

$= 1 - P(A^1{}_1)\, P(A^1{}_2)\, P(A^1{}_3) \ldots\ldots P(A^1{}_n)$

Illustration 23.

A dice is thrown twice. What is the chance of coming up of the number 6 in the first throw and an odd number in the second throw?

Solution :

For first throw of the die

Let S = Sample Space = {1, 2, 3, 4, 5, 6}

A = the event of occurrence of 6

Then, $n(A) = 1$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

For 2nd throw of die :

Its contain sample space

= {1, 2, 3, 4, 5, 6}

B = the event of occurrence of an odd number

= {1, 3, 5}

$n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Clearly, $A \cap B$ = the event of occurrence of 6 in the first throw and occurrence of an odd number in the second throw.

Now, since A, B are independent events.

$$\therefore P(A \cap B) = P(A) \times P(B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

Illustration 24.

A bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. If one ball is drawn from each bag. Find probability that :

(i) both are white

(ii) both are black

(iii) one is white and one is black.

Solution :

Bag 1 = 4 white and 2 black

Bag 2 = 3 white and 5 black

A = the event of drawing a white ball from first bag

B = the event of drawing a black ball from first bag.

A^1 = the event of drawing a white ball from second bag.

B^1 = the event of drawing a black ball from second bag.

Let $E_1 = A \cap A^1$

$E_2 = B \cap B^1$

$$\text{Now,} \quad P(A) = \frac{4}{6} = \frac{2}{3}$$

$$P(B) = \frac{2}{6} = \frac{1}{3}$$

$$P(A^1) = \frac{3}{8}$$

$$P(B^1) = \frac{5}{8}$$

(i) $P(E_1) = P(A \cap A^1) = P(A) \times P(A^1)$

Since, A and A^1 are independent.

$$P(E_1) = \frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$$

(ii) $P(E_2) = P(B \cap B^1)$

$= P(B) \times P(B^1)$

Since B and B^1 are independent.

$$\therefore \quad P(E_2) = \frac{1}{3} \times \frac{5}{8} = \frac{5}{24}$$

(iii) P (E) = probability of occurrence of one is white and one is black.

Probability of occurrence of one is white and one is black

$$= P(A \cap B^1) + P(B \cap A^1)$$

$$= \frac{4}{6} \times \frac{5}{8} + \frac{2}{6} \times \frac{3}{8}$$

$$= \frac{5}{12} + \frac{1}{8}$$

$$= \frac{10 + 3}{24} = \frac{13}{24}$$

$$\therefore \quad P(E) = \frac{13}{24}$$

Illustration 25.

If X and Y are independent events then

(i) X and Y^1 are independent events.

(ii) X^1 and Y are independent events.

(iii) X^1 and Y^1 are independent events.

Solution :

Since, X and Y are independent events.

$\therefore \quad P(A \cap B) = P(A) \times P(B)$

(i) $X = (X \cap Y) \cup (X \cap Y^1)$

$P(X) = P(X \cap Y) + P(X \cap Y^1)$

Since, $X \cap Y$ and $X \cap Y^1$ are mutually exclusive events.

$P(X) = P(X) \times P(Y) + P(X \cap Y^1)$

Or, $P(X \cap Y^1) = P(X) - P(X) \times P(Y)$

$= P(X)\{1 - P(Y)\}$

$\Rightarrow \quad P(X \cap Y^1) = P(X) \times P(Y^1)$

Clearly, X and Y^1 are independent events.

(ii) Similarly, we can show that X^1 and Y are independent events.

$$\Rightarrow \quad P(X^1 \cap Y) = P(X^1) \times P(Y)$$

$$\begin{aligned}
\text{(iii) } P(X^1 \cap Y^1) &= P(X \cup Y)^1 \\
&= 1 - P(X \cup Y) \\
&= 1 - [P(X) + P(Y) - P(X \cap Y)] \\
&= [1 - P(X)] + P(Y)[1 - P(X)] \\
P(X^1 \cap Y^1) &= P(X^1) \times P(Y^1)
\end{aligned}$$

Hence, clearly X_1 and Y_1 are independent events.

Illustration 26.

A speaks the truth in 75% cases and B in 80% of the cases, in what percentage of cases they likely to contradict each other in starting the same fact?

Solution :

Let E_1 = the event that A speaks the truth

E_2 = the event that B speaks the truth

${E^1}_1$ = the event that A tells a lie

${E^1}_2$ = the event B tells a lie

$X = E_1 \cap {E_2}^1$

= the event that A speaks the truth and B tells a lie

$Y = {E_1}^1 \cap E_2$

= the event that A tells a lie and B speaks the truth

Z = the event that A and B contradict each other in starting the same fact then

$Z = X \cup Y$

Clearly, A and B are independent events because A and B are independent with cases.

So, E_1 and E_2 are independent events.

E_1 and ${E_2}^1$ are independent events.

${E_1}^1$ and E_2 are independent events.

$$Z = X \cup Y = \left[\frac{3}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{4}{5}\right] = \frac{7}{20}$$

Required percentage

$$= \frac{7}{20} \times 100 = 35\%$$

Illustration 27.

Two persons Ram and Shyam throw a coin alternately till one of them gets 'head' and wins the game. Find their respective probabilities of winning.

Solution :

Let E = the event of occurrence of head in one throw of a coin

E_1 = the event of occurrence of tail in one throw of a coin

$$\therefore \quad P(E) = \frac{1}{2} \text{ and } P(E^1) = 1 - P(E) = \frac{1}{2}$$

Let E_1 = the event that Ram wins

E_2 = the event that Shyam wins.

Clearly, exactly one of E_1 and E_2 will certainly happen.

$$\therefore \quad E_2 = {E_1}^1$$

For Ram to win, he must throw a head in first, third, fifth ……… throws and Shyam should not throw a head in second, fourth, sixth ……… throws.

$$\therefore \quad P(E_1) = \frac{1}{2} + \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{2}\right) \times \frac{1}{2} + \ldots + \infty$$

$$\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{2}\right) \times \frac{1}{2} + \ldots + \infty$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 \times \frac{1}{2} + \left(\frac{1}{2}\right)^4 \times \frac{1}{2} + \ldots + \infty$$

$$= \frac{1}{2} \times \left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \ldots\ldots\right]$$

$$= \frac{1}{2} \times \left(\frac{1}{1 - \frac{1}{2}}\right) = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$$

$$\text{and } P(E_2) = P({E_1}^1) = 1 - P(E)$$

$$= \frac{1}{3}$$

$$P(E_1) = \frac{2}{3}, P(E_2) = \frac{1}{3}$$

Illustration 28.

If a coin is tossed n times, what is the probability that head will appear an odd number of times ?

Solution :

Here number of trials = n

P = probability of occurrence of head when coin is tossed once = $\frac{1}{2}$

$$\therefore \quad q = 1 - P = \frac{1}{2}$$

Now, probability of occurrence of head exactly r times in n trials is given by

$$P(r) = {}^nC_r \times P^r \times q^{(n-r)}$$

$\therefore$ Probability of occurrence of head odd number of times in n trials

$$\begin{aligned}
&= P(1) + P(3) + P(5) + \ldots\ldots \\
&= {}^nC_1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^{n-1} + {}^nC_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^{n-3} + \ldots\ldots \\
&= \left(\frac{1}{2}\right)^n \times ({}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\ldots) \\
&= \left(\frac{1}{2}\right)^n \times 2^{n-1} \quad \{\because C_1 + C_3 + \ldots = 2^{n-1}\} \\
&= \frac{1}{2}
\end{aligned}$$

BAYES'S THEOREM

Partition of a Set : A family of sets $A_1, A_2, \ldots\ldots A_n$ is said to form a partition of a set A if

(i) $A_1, A_2, \ldots\ldots\ldots A_n$ are non-empty

(ii) $A_i \cap A_j = \phi$ for $i \neq j$

(iii) $A = A_1 \cup A_2 \cup \ldots\ldots\ldots \cup A_n$

Baye's Theorem : If $A_1, A_2, A_3, \ldots\ldots\ldots A_n$ be n mutually exclusive and exhaustive events and A is an event which occurs together (in conjunction) with either of A_i *i.e.* if $A_1, A_2, \ldots\ldots A_n$ form a partition of the sample space S and A be any event, then

$$P\left(\frac{A_k}{A}\right) = \frac{P(A_k) \times P\left(\frac{A}{A_k}\right)}{P(A_1)\, P\left(\frac{A}{A_1}\right) + P(A_2)\, P\left(\frac{A}{A_2}\right) + \ldots\ldots + P(A_n)\, P\left(\frac{A}{A_n}\right)}$$

1. If $P(A_1) = P(A_2) = P(A_3) = \ldots\ldots\ldots = P(A_n)$

$$P\left(\frac{A_k}{A}\right) = \frac{P\left(\frac{A}{A_k}\right)}{P\left(\frac{A}{A_1}\right) + P\left(\frac{A}{A_2}\right) + \ldots\ldots + P\left(\frac{A}{A_n}\right)}$$

2. The probabilities $P(A_1), P(A_2) \ldots\ldots P(A_n)$ which are known before the experiment takes place are called priori probabilities and $P\left(\frac{A_i}{A}\right)$ are called posteriori probabilities.

● If $A_1, A_2, \ldots\ldots\ldots A_n$ form a partition of an event A then prove that

$$P\left(\frac{A_k}{A}\right) = \frac{P(A_k)}{P(A_1) + P(A_2) + \ldots\ldots + P(A_n)}$$

Proof : Since $A_1, A_2, \ldots\ldots A_n$ for a partition of A therefore

(i) $A_1, A_2, \ldots\ldots A_n$ are non-empty.

(ii) They are pair wise disjoint *i.e.* no two of $A_1, A_2, \ldots\ldots A_n$ have any common element.

(iii) $A = A_1 \cup A_2 \cup A_3 \ldots\ldots \cup A_n$

From (i), (ii) and (iii) it is clear that

$(A \cap A_1), (A \cap A_2), \ldots\ldots (A \cap A_n)$ are non-empty pair wise disjoint (they are mutually exclusive)

and By using additional theorem :

$P(A) = P(A \cap A_1) + P(A \cap A_2) + \ldots + P(A \cap A_n)$

$$\text{Now, } P\left(\frac{A_k}{A}\right) = \frac{P(A_k \cap A)}{P(A)}$$

$$= \frac{P(A_k \cap A)}{P(A \cap A_1) + P(A \cap A_2) + \ldots\ldots + P(A \cap A_n)}$$

$$= \frac{P(A_k) \times P\left(\frac{A}{A_k}\right)}{P(A_1) \times P\left(\frac{A}{A_1}\right) + P(A_2) \times P\left(\frac{A}{A_2}\right) + \ldots\ldots}$$

Since, $A_1, A_2, \ldots\ldots$ and A_n are subsets of A.

$$\therefore\ P\left(\frac{A}{A_i}\right) = 1 \text{ for } i = 1, 2, 3, \ldots\ldots n$$

$$\therefore\ P\left(\frac{A_k}{A}\right) = \frac{P(A_k)}{P(A_1) + P(A_2) + \ldots\ldots + P(A_n)}$$

Illustration 29.

A man known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Solution :

Let A = the event that man reports occurrence of E

A_1 = the event of occurrence of 6 when a die is thrown

A_2 = the event of non-occurrence of 6 when a die is thrown

Now, By Baye's Theorem :

$$P\left(\frac{A_1}{A}\right) = \frac{P(A_1) \times P\left(\frac{A}{A_1}\right)}{P(A_1)\, P\left(\frac{A}{A_1}\right) + P(A_2)\, P\left(\frac{A}{A_2}\right)}$$

$$P(A_1) = \frac{1}{6},\ P(A_2) = \frac{5}{6}$$

$P\left(\frac{A}{A_1}\right)$ = Probability that man reports occurrence of 6 when 6 has actually occurred = $\frac{3}{4}$

$P\left(\frac{A}{A_2}\right)$ = Robability that man reports occurrence of 6 when 6 has not actually occurred.

$P\left(\frac{A}{A_2}\right)$ = Probability that the man tells a lie = $\frac{1}{4}$

$$P\left(\frac{A_1}{A}\right) = \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}$$

Illustration 30.

If $P(A) = \frac{3}{5}$, $\quad P(B) = \frac{2}{3}$ then

(a) $P(A \cup B) \geq \frac{2}{3}$

(b) $\frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$

Solution :

Since, $\quad P(A \cup B) \geq P(B)$

$\therefore \quad P(A \cup B) \geq \frac{2}{3} \quad \ldots\ldots (1)$

Or, $\quad P(A \cap B) \leq P(A)$

$\Rightarrow \quad P(A \cap B) \leq \frac{3}{5} \quad \ldots\ldots (2)$

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &\geq P(A) + P(B) - 1 \\ &\geq \frac{3}{5} + \frac{2}{3} - 1 = \frac{4}{15} \quad \ldots(3) \end{aligned}$$

Now, from equation (2) and equation (3), we get

$$\frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$$

Illustration 31.

In a test, an examinee either guesses or copies or known the answer to a multiple choice question with four choice the probability that he makes a guess is $\frac{1}{3}$ · The

probability that he copies the answer is $\frac{1}{6}\cdot$ The probability that the answer is correct, given that he copies, is $\frac{1}{8}\cdot$ Find the probability that he knows the answer to the question, given that he correctly answered it.

Solution :

Let A = the event that the examinee gives the correct answer

A_1 = the event that the examinee knows the answer

A_2 = the event that the examinee guesses the answer

A_3 = the event that the examinee copies the answer

By Baye's theorem,

Required probability = $P\left(\frac{A_1}{A}\right)$

$$= \frac{P(A_1)\times P\left(\frac{A}{A_1}\right)}{P(A_1)\times P\left(\frac{A}{A_1}\right)+P(A_2)\times P\left(\frac{A}{A_2}\right)+P(A_3)\times P\left(\frac{A}{A_3}\right)}$$

According to question :

$$P(A_2) = \frac{1}{3} \qquad P(A_3) = \frac{1}{6}$$

$$\therefore \quad P(A_1) = 1 - P(A_2) - P(A_3)$$

$$= 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$$

Also, $P\left(\frac{A}{A_1}\right)$ = probability that the examinee gives the correct answer.

Since, probability of occurrence of a sure event = 1

$\therefore$ $P\left(\frac{A}{A_2}\right)$ = probability that the examinee gives the correct answer. When he makes a gives = $\frac{1}{4}$

$$P\left(\frac{A}{A_3}\right) = \frac{1}{8}$$

$$P\left(\frac{A_1}{A}\right) = \frac{\frac{1}{2}\times 1}{\frac{1}{2}\times 1+\frac{1}{3}\times\frac{1}{4}+\frac{1}{6}\times\frac{1}{8}}$$

$$= \frac{24}{29}$$

Illustration 32.

X and Y play a series of 6 games in badminton. X^1s chance of winning a single game against Y is $\frac{2}{3}\cdot$ Find the chance that X wins

(i) exactly 4 games

(ii) at least 4 games

Solution :

Here each game is trial and X^1s winning the game is a success. All games are independent. So, using the Binomial Distribution.

$$P(r) = {}^nC_r \times P^r \times (1-P)^{n-r}$$

where r = wins exactly 4 games

$$P = \frac{2}{3} \qquad q = 1 - P = 1 - \frac{2}{3} = \frac{1}{3}$$

$$P(4) = {}^6C_4 \times \left(\frac{2}{3}\right)^4 \times \left(\frac{1}{3}\right)^2$$

$$= \frac{6\times 5}{2}\times\left(\frac{2}{3}\right)^4\times\frac{1}{9}$$

$$= \frac{80}{243}$$

(ii) X wins at least games means $r \geq 4$

i.e. $\quad X = 4, 5, 6$

$$\therefore \quad P(r\geq 4) = P(r=4) \text{ or } P(r=5) \text{ or } P(r=6)$$

$$= {}^6C_4\times\left(\frac{2}{3}\right)^4\times\left(\frac{1}{3}\right)^2 + {}^6C_5\times\left(\frac{2}{3}\right)^5 \times\left(\frac{1}{3}\right) + {}^6C_6\times\left(\frac{2}{3}\right)^6$$

$$= \frac{80}{243}+\frac{16}{243}+\frac{64}{243\times 3}$$

$$= \frac{240+48+64}{243\times 3} = \frac{352}{729}$$

Exercise – A

1. A bag contains 6 black balls and an unknown number (not greater than six) of white balls. Three balls are drawn successively and not replaced and all are found to be white. The chance that a black ball will be drawn next is—

(A) $\frac{677}{909}$ (B) $\frac{609}{677}$

(C) $\frac{480}{909}$ (D) $\frac{280}{909}$

2. It is known that at noon at a certain place the sun is hidden by clouds on an average two days out of every three. The chance that the sun will be shining at noon on at least four out of five specified future days is—

(A) $\frac{21}{4096}$ (B) $\frac{19}{4096}$

(C) $\frac{20}{4096}$ (D) $\frac{17}{4096}$

3. Supposing that it is 4 to 7 against a person A who is now 30 years of age, living till he is 80, and 5 to 6 against a person B now 25 living till he is 75. The chance that at least one of these persons will be alive 30 years hence is—

(A) $\frac{57}{121}$ (B) $\frac{91}{121}$

(C) $\frac{101}{121}$ (D) None of these

4. Three dice are thrown together. The probability that the sum of all faces showing up will be more than 16 is—

(A) $\frac{1}{54}$ (B) $\frac{1}{27}$

(C) $\frac{1}{108}$ (D) $\frac{3}{154}$

5. A bag contains 5 white and 6 black balls, and 6 are successively drawn out and not replaced. What's the chance that they alternately of different colours ?

(A) $\frac{15}{77}$ (B) $\frac{30}{77}$

(C) $\frac{31}{77}$ (D) $\frac{16}{77}$

6. Three balls are drawn successively from box containing 9 white, 10 red, 5 blue balls. If each drawn balls is not replaced, then the probability of these are drawn in order red, white and black is—

(A) $\frac{11}{2024}$ (B) $\frac{12}{2024}$

(C) $\frac{25}{2024}$ (D) $\frac{29}{2024}$

7. X, Y, Z in order, cut a pack of cards, replacing them after each cut, on condition that the first who cuts a heart shall win a prize. Then A's chance of winning is—

(A) 16/37 (B) 12/37

(C) 9/37 (D) 14/37

8. A party of n persons is sitting at a round table. The odds against two specified individuals sitting next to each other are—

(A) $2 : (n-1)$ (B) $(n-3) : 2$

(C) $2 : (n-3)$ (D) $(n-1) : 2$

9. Triangles are formed by joining vertices of a octagon. Any one those triangles is selected at random. What is the probability that the selected triangle has no side common with the octagon ?

(A) $\frac{5}{7}$ (B) $\frac{14}{5}$

(C) $\frac{7}{9}$ (D) $\frac{5}{14}$

10. If on an average 1 bulb out of every 7 is fused, then the chance that out of 5 bulbs expected, at least 3 will arrive lighed, is—

(A) $\frac{16416}{75}$ (B) $\frac{16529}{75}$

(C) $\frac{75}{16416}$ (D) $\frac{910}{75}$

11. In an examination, there are 500 students. 120 passed the first paper and 250 passed the second paper. 150 students passed both the papers. The probability that a student at random has failed in both the papers is—

(A) $\frac{2}{25}$ (B) $\frac{1}{25}$

(C) $\frac{4}{25}$ (D) $\frac{3}{50}$

12. A person is asked to randomly pick two balls which has 20 black & 10 red balls. The probability that the persons pick two balls of the same colour is—

(A) $\frac{45}{87}$ (B) $\frac{46}{87}$

(C) $\frac{47}{87}$ (D) $\frac{87}{45}$

13. From a bag containing 5 white and 13 black balls a man drawn 5 at a random; what are the odds against these being all black ?

(A) 8567 : 1 (B) 1 : 8567

(C) 5423 : 1 (D) 1 : 5423

14. The sincere to casual ratio in a class is 12 : 10. If this trend is expected to continue, the probability that a newcomer is casual, will be—

(A) $\frac{11}{12}$ (B) $\frac{5}{11}$

(C) $\frac{6}{11}$ (D) $\frac{12}{11}$

15. Two fruits are to be picked from a basket having 6 mangoes and 10 oranges. The probability that both of them are mango is—

(A) $\frac{2}{21}$ (B) $\frac{5}{42}$

(C) $\frac{3}{21}$ (D) $\frac{6}{21}$

16. Out of integers from 5 to 40 (both 5 & 40 included), a number is picked at random. The probability that the number is divisible by 5 is—

(A) $\frac{2}{9}$ (B) $\frac{3}{5}$

(C) $\frac{7}{9}$ (D) $\frac{6}{7}$

17. In a plane, 6 lines of lengths 2, 3, 4, 5 & 6 cm are lying. What is the probability that by joining the three randomly chosen lines end to end a triangle can't be formed ?

(A) $\frac{2}{5}$ (B) $\frac{3}{10}$

(C) $\frac{5}{7}$ (D) $\frac{7}{10}$

18. If a rod is marked at random in n points and divided at those points, then the chance that none of the parts shall be greater than $\frac{1}{n}$ th of the rod is—

(A) $\frac{1}{n^{n-1}}$ (B) $\frac{1}{n^n}$

(C) $\frac{n\,!}{n^n}$ (D) ${}^{2n}P_{n-1}$

19. Two drawing, each of 2 balls, are made from a bag containing 6 black and 14 yellow balls, the balls not being replaced before the second trial. The chance that the first drawing will be give 2 yellow and the second drawing will give 2 black balls is—

(A) $\frac{16}{323}$ (B) $\frac{15}{323}$

(C) $\frac{14}{323}$ (D) $\frac{16}{350}$

20. X and Y throw one die for a stake of Rs. 22, which will be won by the player who first throws a 1. If X has the first throw, then what could be their respective expectations ?

(A) Rs. 10, Rs. 12 (B) Rs. 11, Rs. 11

(C) Rs. 13, Rs. 9 (D) Rs. 12, Rs. 10

21. Before a race the chance of three runners, A, B, C, were estimated to be proportional to 5, 3, 2; but during the race A meets with an accident which reduces his chance to one-third . What is the chance of B ?

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$

(C) $\frac{4}{15}$ (D) $\frac{1}{5}$

22. In the above question (Q. 21), what is the chance of C ?

(A) $\frac{4}{15}$ (B) $\frac{2}{15}$

(C) $\frac{3}{5}$ (D) $\frac{2}{5}$

23. 10 persons draw lots for the occupancy of the six seats in a first class railway compartment the chance that two specified persons obtain opposite seats is—

(A) $\frac{9}{10}$ (B) $\frac{7}{10}$

(C) $\frac{1}{10}$ (D) $\frac{3}{10}$

24. Karunesh Verma and Kaloesh Sharma appear for an interview for two vacancies in an organisation for the same post. The probabilities of their selection are $\frac{3}{5}$ and $\frac{2}{3}$ respectively. What is the probability that none of them will be selected ?

(A) $\frac{3}{15}$ (B) $\frac{1}{15}$

(C) $\frac{2}{15}$ (D) $\frac{4}{15}$

25. The odds against a certain events are 3 : 5 and odds in favour of another independent events are 4 : 7. The probability that none of the events will occur is—

(A) $\frac{3}{22}$ (B) $\frac{5}{22}$

(C) $\frac{7}{22}$ (D) $\frac{9}{22}$

26. Rahul and Shahil throw a die alternately till one of them gets '6' and wins the game. What are their respectively probabilities of winning ?

(A) $\left(\frac{6}{11}, \frac{5}{11}\right)$ (B) $\left(\frac{7}{11}, \frac{4}{11}\right)$

(C) $\left(\frac{3}{5}, \frac{2}{7}\right)$ (D) $\left(\frac{7}{11}, \frac{2}{7}\right)$

27. Eight children are standing in a line outside a ticket window at Appu Ghar, New Delhi. Four of these children have a one a rupee coin each and the remaining 4 children have a two rupee coin each. The entry ticket is priced at rupee one. If all the arrangements of the eight children are equally likely, then the probability that no child will have to wait for change is—(assume that the cashier at the ticket window begins with no change)

(A) $\frac{1}{2}$ (B) $\frac{1}{5}$

(C) $\frac{1}{8}$ (D) $\frac{1}{6}$

28. A box contains 13 white balls and 12 black balls. One ball is taken out from the box and not replaced back. If a ball is now taken at random from the box, then what is the probability that it is black ?

(A) $\frac{13}{25}$ (B) $\frac{12}{25}$

(C) $\frac{11}{25}$ (D) $\frac{14}{25}$

29. A closed box contains two balls whose colours are not known. [Each an be either white or black.] A white ball has been put [which is indistinguishable from the other size] into the box. What is the probability of drawing a white ball ?

(A) $\frac{1}{3}$ (B) 1

(C) $\frac{2}{3}$ (D) $\frac{1}{6}$

30. Three dice are thrown. What is the probability that the numbers shown on the dice are not same?

(A) $\frac{103}{108}$ (B) $\frac{105}{108}$

(C) $\frac{21}{54}$ (D) $\frac{7}{36}$

Exercise – B

1. A bag contains 6 black balls and an unknown number (not greater than six) of white balls. Three balls are drawn successively and not replaced and all are found to be white. The chance that a black ball will be drawn next is—

(A) 677/909 (B) 609/677

(C) 480/909 (D) 280/909

2. It is known that at noon at a certain place the sun is hidden by clouds on an average two days out of every three. The chance that the sun will be shining at noon on at least four out of five specified future days is—

(A) 1/81 (B) 16/243

(C) 11/243 (D) 7/20

3. Supposing that it is 9 to 7 against a person A who is now 35 years of age, living till he is 65, and 3 to 2 against a person B now 45 living till he is 75. The chance that at least one of these persons will be alive 30 years hence is—
(A) 1/2 (B) 27/80
(C) 3/4 (D) 53/80

4. Three dice are thrown together. The probability that the sum of all faces showing up will be more than 15 is—
(A) 5/108 (B) 5/6
(C) 1/54 (D) 5/18

5. A bag contains 5 white and 3 black balls, and 4 are successively drawn out and not replaced. What's the chance that they alternately of different colours ?
(A) 1/6 (B) 1/5
(C) 1/4 (D) 1/7

6. Three balls are drawn successively from an urn containing 8 red balls, 5 white balls and 7 black balls. If each drawn balls is not replaced, then the probability of these are drawn in order red, white and black is—
(A) $\frac{7}{19}$ (B) $\frac{1}{171}$
(C) $\frac{7}{171}$ (D) $\frac{19}{7}$

7. A, B, C in order, cut a pack of cards, replacing them after each cut, on condition that the first who cuts a spade shall win a prize. Then A's chance of winning is—
(A) 16/37 (B) 12/37
(C) 9/37 (D) 14/37

8. A party of n persons is sitting at a round table. The odds against two specified individuals sitting next to each other are—
(A) $2 : (n-1)$ (B) $(n-3) : 2$
(C) $2 : (n-3)$ (D) $(n-1) : 2$

9. Triangles are formed by joining vertices of a octagon. Any one those triangles is selected at random. What is the probability that the selected triangle has no side common with the octagon ?
(A) 3/7 (B) 2/7
(C) 5/7 (D) 1/7

10. If on an average 1 vessel out of every 10 is broken, then the chance that out of 5 vessels expected, at least 4 will arrive safely is—
(A) 34712/50000
(B) $(9/10)^5$
(C) 30618/50000
(D) 45927/50000

11. In an examination, there are 500 students. 150 passed the first paper and 350 passed the second paper. 50 students passed both the papers. The probability that a student at random has failed in both the papers is—
(A) 1/5 (B) 1/10
(C) 3/10 (D) 3/5

12. A person is asked to randomly pick two balls from a bag which has 15 yellow & 5 red balls. The probability that the persons pick two balls of the same colour is—
(A) 33/38 (B) 23/38
(C) 38/43 (D) 15/38

13. From a bag containing 4 white and 5 black balls a man drawn 3 at a random; what are the odds against these being all black ?
(A) 37 : 5 (B) 5 : 37
(C) 23 : 19 (D) 19 : 23

14. The sincere to casual ratio in a class is 10 : 27. If this trend is expected to continue, the probability that a newcomer is casual, will be—
(A) 10/37 (B) 27/37
(C) 10/27 (D) 17/27

15. Two balls are to be picked from a bag having 6 white balls and 9 black balls. The probability that both of them are black is—
(A) 17/49 (B) 27/35
(C) 12/35 (D) 23/35

16. Out of integers from 10 to 50 (both 10 & 50 included), a number is divisible by 7 is—
(A) 6/40 (B) 6/41
(C) 6/39 (D) 7/41

17. In a plane, 5 lines of lengths 2, 3, 4, 5 & 6 cm are lying. What is the probability that by joining the three randomly chosen lines end to end a triangle can't be formed ?
(A) 3/10 (B) 7/10
(C) 1/2 (D) 1

18. If a rod is marked at random in n points and divided at those points, then the chance that none of the parts shall be greater than $\frac{1}{n}$ th of the rod is—
(A) $\frac{1}{n^{n-1}}$ (B) $\frac{1}{n^n}$
(C) $\frac{n\,!}{n^n}$ (D) ${}^{2n}P_{n-1}$

19. Two drawings, each of 3 balls are made from a bag containing 5 white and 8 black balls, the balls not being replaced before the second trial. The chance that the first drawing will be give 3 white and the second drawing will give 3 black balls is—
(A) 7/429 (B) 7/15
(C) 5/143 (D) 8/15

20. A and B throw one die for a stake of Rs. 11, which will be won by the player who first throws a 6. If A has the first throw, then what could be their respective expectations ?

(A) Rs. 7, Rs. 4
(B) Rs. 6, Rs. 5
(C) Rs. 4, Rs. 7
(D) Rs. 5, Rs. 6

21. Before a race the chance of three runners, A, B, C, were estimated to be proportional to 5, 3, 2; but during the race A meets with an accident which reduces his chance to one-third . What is the chance of B ?

(A) 2/5 (B) 3/5
(C) 4/15 (D) 1/5

22. In the above question (Q. 21), what is the chance of C ?

(A) 4/15 (B) 2/15
(C) 3/5 (D) 2/5

23. Seven persons draw lots for the occupancy of the six seats in a first class railway compartment the chance that two specified persons obtain opposite seats is—

(A) 6/7 (B) 5/7
(C) 1/7 (D) 3/7

24. Ram Gopal Verma and Shyam Gopal Sharma appear for an interview for two vacancies in an organisation for the same post. The probabilities of their selection are 1/6 and 2/5 respectively. What is the probability that none of them will be selected ?

(A) 5/6 (B) 1/5
(C) 1/2 (D) 3/5

25. The odds against a certain events are 5 : 2 and odds in favour of another independent events are 6 : 5. The probability that none of the events will occur is—

(A) 6/11 (B) 5/11
(C) 52/77 (D) 25/77

26. Two persons A and B throw a coin alternately till one of them gets head and wins the game. What are their respectively probabilities of winning ?

(A) $\frac{2}{3}, \frac{1}{3}$ (B) $\frac{1}{5}, \frac{2}{7}$
(C) $\frac{1}{3}, \frac{2}{7}$ (D) $\frac{1}{5}, \frac{1}{3}$

27. Eight children are standing in a line outside a ticket window at Appu Ghar, New Delhi. Four of these children have a one a rupee coin each and the remaining 4 children have a two rupee coin each. The entry ticket is priced at rupee one. If all the arrangements of the eight children are equally likely, then the probability that no child will have to wait for change is— (assume that the cashier at the ticket window begins with no change)

(A) $\frac{1}{2}$ (B) $\frac{1}{5}$
(C) $\frac{1}{8}$ (D) $\frac{1}{6}$

28. A box contains m white balls and n black balls. One ball is taken out from the box and not replaced back. If a ball is now taken at random from the box, then what is the probability that it is white ?

(A) $\frac{m}{m+n}$ (B) $\frac{m}{m+n-1}$
(C) $\frac{m-1}{m+n-1}$ (D) $\frac{m-1}{m+n}$

29. A closed box contains two balls whose colours are not known. [Each n be either white or black.] A white ball has been put [which is indistinguishable from the other size] into the box. What is the probability of drawing a white ball ?

(A) $\frac{1}{3}$ (B) 1
(C) $\frac{2}{3}$ (D) $\frac{1}{6}$

30. Three dices are thrown. What is the probability that the sum of numbers shown on the dice is neither 3 nor 18 ?

(A) 1/36 (B) 5/36
(C) 1/108 (D) 107/108

Solutions – A

1. (A) The bag contains 6 black balls and there are four way of the following containing white balls :

(a) It may contain three white balls.
(b) It may contain four white balls.
(c) It may contain five white balls.
(d) It may contain six white balls.

(a) The probability that at the three successive draws give white ball is :

drawn $= p_1 = \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7}$ because balls are not replaced.

(b) The probability of drawing white balls at three successive draws is

$$p_2 = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}.$$

Similarly,

$$p_3 = \left(\frac{5}{11}\right) \times \left(\frac{4}{10}\right) \times \left(\frac{3}{9}\right)$$

$$p_4 = \frac{6}{12} \times \frac{2}{11} \times \frac{4}{10}$$

When there are 6 white balls, *i.e.*,

$$p_1 = \frac{1}{84}, p_2 = \frac{1}{30}, p_3 = \frac{2}{33}, p_4 = \frac{1}{11}$$

If Q_1, Q_2, Q_3, Q_4 be denoting posteriori probability of (a), (b), (c) and (d) cases.

$$\therefore \frac{Q_1}{55} = \frac{Q_2}{154} = \frac{Q_3}{280} = \frac{Q_4}{420} = \frac{1}{909}$$

Therefore, the chance of drawing the black ball next

$$Q_1 \times 1 + Q_2 \times \frac{6}{7} + Q_3 \times \frac{3}{4} + Q_4 \times \frac{2}{3}$$

$$= \frac{55}{909} \times 1 + \frac{154}{909} \times \frac{6}{7} + \frac{280}{909} \times \frac{3}{4} + \frac{420}{909} \times \frac{2}{3}$$

$$= \frac{677}{909}.$$

2. (C) The probability that the sun is hidden $= \frac{3}{4}$

$\therefore$ The probability that it is out $= \frac{1}{4}$

Now, at least 5 days shining = 5 days out and 1 day hidden + 6 days out

$$= \left(\frac{1}{4}\right)^5 \times {}^6C_1 \frac{3}{4} + \left(\frac{1}{4}\right)^6$$

$$= \left(\frac{1}{4^6}\right) [6 \times 3 + 1]$$

$$= \frac{19}{46} = \frac{19}{16 \times 16 \times 16}$$

$$= \frac{19}{4096}$$

3. (C) The chance that A will die within 50 years is $\frac{4}{11}$; the chance that B will die within 50 years is $\frac{5}{11}$; therefore the chance that both will die is $\frac{4}{11} \times \frac{5}{11}$ or $\frac{20}{121}$;

Therefore, the chance that both will not be dead, that is at least one will be alive, is $1 - \frac{20}{121}$ or $\frac{101}{121}$.

4. (A) Total possible outcomes

$$= 6 \times 6 \times 6 = 216$$

Favourable cases are (6, 6, 6), (6, 6, 5), (6, 5, 6), (5, 6, 6)

Total number of favourable cases = 4

$$\text{Probability} = \frac{4}{216} = \frac{1}{54}.$$

5. (B) Total number of balls = 11.

Let the first drawn ball is white.

So, required probability

$$= \frac{5}{11} \times \frac{6}{10} \times \frac{4}{9} \times \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} = \frac{15}{77}.$$

When we start with a black ball, the required

$$\text{Probability} = \frac{5}{11} \times \frac{5}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{15}{77}.$$

Since, these two cases are mutually exclusive.

Total probability

$$= \frac{15}{77} + \frac{15}{77} = \frac{2 \times 15}{77} = \frac{30}{77}.$$

6. (C) On first drawing, Probability of drawing red

$$= \frac{{}^{10}C_1}{{}^{24}C_1} = \frac{10}{24} = \frac{5}{12}$$

On second drawing, Probability of drawing white

$$= \frac{{}^{9}C_1}{{}^{23}C_1} = \frac{9}{23}$$

On third drawing, Probability of drawing black

$$= \frac{{}^{5}C_1}{{}^{22}C_1} = \frac{5}{22}$$

$\therefore$ Required probability

$$= \frac{5}{12} \times \frac{9}{23} \times \frac{5}{22} = \frac{25}{2024}.$$

7. (A) There are 13 cards of spade in a pack of 52 cards. So, the chance that any of them will cut's a spade is 13/54 = 1/4 $\Rightarrow$ Probability that it is not a spade = 1 – 1/4 = 3/4. Consider the following mutually exclusive ways in which A may win : A wins in the first cut, OR X, Y, Z have failed and then A wins, OR X, Y, Z, X, Y, Z have failed and then A wins and so on upto infinity $\Rightarrow$ Respective chance of these events are = 1/4, [(3/4) × (3/4) × (3/4) × (1/4)], [(3/4) × (3/4) × (3/4) × (3/4) × (3/4) × (3/4) × (1/4)] .... To infinity $\Rightarrow$ A's chance of succeeding = (1/4) + $(3/4)^3$ × (1/4) + $(3/4)^6$ × (1/4) + ... = 1/4 [1/(1 – 27/64)] = 16/37.

8. (B) Total numbers of ways in which n persons can sit a round table = $(n - 1)!$ As the two specified individuals are to sit together, we may consider them to be a single individuals and then we have $(n - 2)!$ Different arrangements in which the two specified individuals can sit together in the same order. Since, they can permute among themselves, the total possible ways favourable to the events are $2 \times (n - 2)!$ $\Rightarrow$ Probability of two specific individuals sitting together = $[2 \times (n - 2)!]/(n - 1)! = 2/(2 + n - 3)$. Hence, the odds against the events are $(n - 3)$ to 2.

9. (D) Here number of sides are = 9

Total number of triangles are

$${}^9C_3 = \frac{\underline{|9}}{\underline{|3}\ \underline{|6}} = \frac{9 \times 8 \times 7}{6} = 84$$

Total number of triangles with no sides common

$$= {}^nC_3 - (n-4)\,n - n$$

$$= {}^9C_3 - (9-4)\,9 - 9$$

$$= 84 - 45 - 9$$

$$= 84 - 54 = 30$$

So, required probability

$$= \frac{30}{84} = \frac{5}{14}.$$

10. (A) The chance that the bulb is fused $= \frac{1}{7}$

$\Rightarrow$ Chance that the bulb is lighted $= \frac{6}{7}$ at least 3 will be lighted if all 5 are light or 4 are lighted and one is fused or 3 will be lighted and 2 are fused.

Required Probability

$$= \left(\frac{6}{7}\right)^5 + 5\left(\frac{6}{7}\right)^4 \times \frac{1}{7} + {}^5C_2\left(\frac{6}{7}\right)^3 \times \left(\frac{1}{7}\right)^2$$

$$= \left(\frac{6}{7}\right)^5 + \frac{5}{7} \times \left(\frac{6}{7}\right)^4 + \frac{10}{7^2} \times \left(\frac{6}{7}\right)^3$$

$$= \frac{6^3}{7^5}[36 + 30 + 10]$$

$$= \frac{76 \times 216}{7^5} = \frac{16416}{7^5}$$

11. (B) Total number of students = Number passing in I paper only + Number of student passing in II paper only + Number passing in both + Number failed in both

$\Rightarrow$ $500 = (120 - x) + (250 - x) + 150 + x$

$\Rightarrow$ $500 = 520 - x \Rightarrow x = 20$

$\Rightarrow$ Required probability

$$= \frac{20}{500} = \frac{1}{25}.$$

12. (C) Total possible outcomes $= {}^{30}C_2$

The person can pick up two black balls in ${}^{20}C_2$ ways and two red balls in ${}^{10}C_2$ ways

Required probability

$$= \frac{{}^{20}C_2}{{}^{30}C_2} + \frac{{}^{10}C_2}{{}^{30}C_2} = \frac{\frac{19 \times 20}{2} + \frac{10 \times 9}{2}}{\frac{30 \times 29}{2}}$$

$$= \frac{190 + 45}{435} = \frac{235}{435} = \frac{47}{87}$$

13. (A) The total number of ways in which 5 balls can be drawn is ${}^{18}C_5$ and the number of ways of drawing 5 white ball is ${}^5C_3$; therefore the chance of drawing 5 white balls

$$= \frac{{}^5C_5}{{}^{18}C_5} = \frac{1}{|18}$$

$$= \frac{\lfloor 13 \lfloor 5}{\lfloor 18} = \frac{\lfloor 13 \times 5 \times 4 \times 3 \times 2 \times 1}{18 \times 17 \times 16 \times 15 \times 14 \lfloor 13}$$

$$= \frac{1}{8568}$$

Thus, the odds against the event are 8568 to 1.

14. (B) Net cases $= 12 + 10 = 22$.

The required Probability

$$= \frac{\left(\frac{1}{22}\right)}{\left(\frac{1}{10}\right)} = \frac{10}{22} = \frac{5}{11}.$$

15. (C) The total number of ways of selecting two fruits

$= {}^{15}C_2$

Favourable case $= {}^5C_2$

$\Rightarrow$ Probability $= \frac{{}^5C_2}{{}^{15}C_2} = \frac{5 \times 2}{15 \times 7} = \frac{2}{21}.$

16. (A) Possible comes $= 36$.

Favorable case $= 8$

Required probability $= \frac{8}{36} = \frac{2}{9}.$

17. (B) $\because$ The sum of two sides in a triangle is greater than the third side.

$\therefore$ In following cases, triangle cannot be formed.

2	3	5
2	3	6
2	4	7
2	4	6
2	4	7
2	5	7

$\therefore$ Required probability $= \frac{6}{{}^6C_3} = \frac{6 \times 6}{6 \times 5 \times 4}$

$= \frac{3}{10}.$

18. (b) The care, none of the parts shall be greater than $\frac{1}{n}$ th of the rod is possible if the rod is divided in n equal parts. The probability of one part being $\frac{1}{n}$ th of the rod $= \frac{1}{n}$.

$\therefore$ Required probability of n parts being $\frac{1}{n}$ th of the rod each

$$= \frac{1}{n} \times \frac{1}{n} \times \frac{1}{n} \times \ldots\ldots \frac{1}{n} = \left(\frac{1}{n}\right)^n.$$

19. (C) The chance of drawing 2 yellow balls $= \frac{{}^{14}C_2}{{}^{20}C_2}$ and without replacing, the chance that of drawing two black balls $= \frac{{}^6C_2}{{}^{18}C_2}$.

Required Probability $= \frac{{}^{14}C_2}{{}^{20}C_2} \times \frac{{}^6C_2}{{}^{18}C_2}$

$= \frac{14 \times 6}{10 \times 19} \times \frac{3 \times 5}{9 \times 17}$

$= \frac{14}{19 \times 17} = \frac{14}{323}.$

20. (D) In his first throw X's chance is $\frac{1}{6}$; in his second throw it is $\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$, because each player must have failed once before X can have a second throw; in his third throw his chance $= \left(\frac{5}{6}\right)^4 \times \frac{1}{6}$ because each

player must have failed twice; and so on. Thus, X's chance is the sum of the infinite series

$$\frac{1}{6}\left\{1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+\ldots\ldots\right\}.$$

Similarly, Y's chance is the sum of the infinite series

$$\frac{5}{6}, \frac{1}{6}\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+\ldots\right]$$

$\therefore$ X's chance is to Y's as 6 is to 5. Their respective chances are therefore $\frac{6}{11}$ and $\frac{5}{11}$, and their expectations are Rs. $\frac{6}{11} \times 22 = 12$ and Rs. 10 respectively.

21. (A) The chance of A, B, C before the starts are $\frac{5}{10}$, $\frac{3}{10}$, $\frac{2}{10}$ *i.e.*, $\frac{1}{2}$, $\frac{3}{10}$, $\frac{1}{5}$ respectively. A can lose in two ways; either by the winning of B or C.

As A's chance of winning is 1/2; therefore A's chance of losing is 1/2. After the accident his chance of winning is 1/3, and hence his chance of losing becomes 2/3. $\Rightarrow$ his chance of losing is increased in the ratio of 4 to 3, also B's and C's chances of winning are increased in the same ratio. Thus B's chance of winning $= \frac{3}{10} \times \frac{4}{3} = \frac{2}{5}$; and C's chance of winning $= \frac{2}{10} \times \frac{4}{3} = \frac{4}{15}$.

22. (A)

23. (C) Let the X & Y are two specified persons. The chance that X occupies any seat in the first class railway compartment is $\frac{9}{10}$, X has occupied his seat there is only one case favourable to the event that Y occupies is seat opposite to X. Thus, the chance of the compound events that Y gains an opposite seat to X is $\frac{9}{10} \times \frac{1}{9} = \frac{1}{10}$.

24. (C) Required Probability

$$= \left(1-\frac{3}{5}\right) \times \left(1-\frac{2}{3}\right)$$

$$= \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}.$$

25. (B) Let A & B be the event.

$$\Rightarrow \quad P(A) = \frac{5}{8},$$

$$\Rightarrow \quad P(B) = \frac{4}{11}.$$

$\therefore$ Required probability

$$\frac{5}{8} \times \frac{4}{11} = \frac{5}{22}$$

26. (A) Probability of coming six

$$= \frac{1}{6} = P(S)$$

and Probability of coming another then 6

$$= \frac{5}{6} = P(F)$$

Now, Rahul wins if he throws a six in 1st or 3rd or 5th or 7th …..

$\therefore$ Probability of Rahul wins = P[S or (FFS) or (FFFFS) or ……]

$= P(S) + P(FFS) + P(FFFFS) + \ldots\ldots$

$= P(S) + P(F) . P(F) . P(S) + P(F) . P(F) . P(F) . P(F) . P(SH) + \ldots\ldots$

$$= \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \ldots\ldots$$

$$= \frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)^2 + \frac{1}{6}\left(\frac{5}{6}\right)^4 + \ldots\ldots$$

$$= \frac{1}{6}\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+\ldots\right]$$

$$= \frac{1}{6} \times \frac{1}{1-\left(\frac{5}{6}\right)^2} = \frac{36}{6\,(36-25)} = \frac{6}{11}$$

$$P(\text{Rahul}) = \frac{6}{11},$$

$$P(\text{Shahil}) = 1 - \frac{6}{11} = \frac{5}{11}$$

$$\Rightarrow \text{Required Probability} = \left(\frac{6}{11}, \frac{5}{11}\right).$$

27. (b) Total possible combinations of distribution of ticket that no child will have to wait for change are as follows :

1. 1 1 1 1 2 2 2 2	2. 1 1 1 2 1 2 2 2
3. 1 1 1 2 2 1 2 2	4. 1 1 1 2 2 2 1 2
5. 1 1 2 1 1 2 2 2	6. 1 1 2 2 1 1 2 2
7. 1 1 2 1 2 1 2 2	8. 1 1 2 1 2 2 1 2
9. 1 1 2 2 1 2 1 2	10. 1 2 1 2 1 2 1 2
11. 1 2 1 1 1 2 2 2	12. 1 2 1 1 2 1 2 2
13. 1 2 1 1 2 2 1 2	14. 1 2 1 2 1 1 2 2

Now, these 14 combinations can again be arranged in 4 ! × 4 ! Ways.

So, total favourable combinations

$$= 14 \times 4! \times 4!$$

And total possible combinations = 8 !

$$\text{Required Probability} = \frac{14 \times 4! \times 4!}{8!} = \frac{1}{5}.$$

28. (B) **Case I :** Let the first ball removed be black

$\Rightarrow$ Black balls left = 11 and white balls left = 13

Probability of getting a black ball in the first and second draw

$$= \frac{12}{25} \times \frac{1}{24} = \frac{1}{50}$$

Case II : Let the first ball removed be white

$\Rightarrow$ Black balls left $=$ 12

White ball left $= n - 1$

Probability of getting a white ball in the first and black in second draw

$$= \frac{13}{25} \times \frac{12}{24} = \frac{13}{50}$$

Required Probability $=$ Sum of these two

$$= \frac{11}{50} + \frac{13}{50} = \frac{12}{25}.$$

29. (C) **Case I :** Assume that the box has two white balls. When a white ball is added, it now has 3 white balls $\Rightarrow$ Probability of drawing a white ball = Probability of drawing one ball $\times$ Probability of white ball

$$= \frac{1}{3} \times \frac{3}{3} = \frac{1}{3}$$

Case II : Assume that the box has one white and one Black ball.

When a white ball is added the box will then contain 2 white balls and 1 Black ball $\Rightarrow$ Probability of drawing a white ball = Probability of drawing one ball $\times$ Probability of white ball

$$= \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}.$$

Case III : Assume that the box has two Black balls. When a white ball is added, it now has 1 white ball and 2 Black balls

$\Rightarrow$ Probability of drawing a white = Probability of drawing one ball $\times$ Probability of white ball

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \Rightarrow \text{Overall Probability}$$

$$= \frac{1}{3} + \frac{2}{9} + \frac{1}{9} = \frac{2}{3}$$

[because all these are mutually exclusive cases].

30. (B) Possible outcomes $= 6^3 = 6 \times 6 \times 6 = 216$

The outcomes which are not favourable are (1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5) and (6, 6, 6)

$\Rightarrow$ Required probability $= \dfrac{(216 - 6)}{216}$

$$= \frac{210}{216} = \frac{105}{108}.$$

Solutions – B

1. (A) The bag contains 6 black balls and there are four ways of following containing white balls :

(a) It may contain three white balls.

(b) It may contain four white balls.

(c) It may contain five white balls.

(d) It may contain six white balls.

(a) The probability that at the three successive draws give white ball is :

drawn $= p_1 = \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7}$ because balls are not replaced.

(b) The probability of drawing white balls at three successive draws is $p_2 = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}$.

Similarly, $p_3 = (5/11) \times (4/10) \times (3/9)$

$$p_4 = \frac{6}{12} \times \frac{2}{11} \times \frac{4}{10}$$

When there are 6 white balls, *i.e.*,

$$p_1 = \frac{1}{84}, p_2 = \frac{1}{30}, p_3 = \frac{2}{33}, p_4 = \frac{1}{11}$$

If Q_1, Q_2, Q_3, Q_4, be denoting posteriori probability of (a), (b), (c) and (d) cases.

$$\therefore \quad \frac{Q_1}{55} = \frac{Q_2}{154} = \frac{Q_3}{280} = \frac{Q_4}{420} = \frac{1}{909}$$

Therefore, the chance of drawing the black ball next

$$Q_1 \times 1 + Q_2 \times \frac{6}{7} + Q_3 \times \frac{3}{4} + Q_4 \times \frac{2}{3}$$

$$= \frac{55}{909} \times 1 + \frac{154}{909} \times \frac{6}{7} + \frac{280}{909} \times \frac{3}{4} + \frac{420}{909} \times \frac{2}{3}$$

$$= \frac{677}{909}.$$

2. (C) Here the probability that the sun is hidden = 2/3

$\therefore$ The probability that it out = 1/3

Now, at least 4 days shining = 4 days out and 1 day hidden + 5 days out

$$= 5\left(\frac{2}{3}\right) \times \left(\frac{1}{3}\right)^4 + \left(\frac{1}{3}\right)^5$$

$$= \frac{5 \times 2}{243} + \frac{1}{243} = \frac{11}{243}.$$

3. (D) The chance that A will die within 30 years is $\frac{9}{16}$; the chance that B will die within 30 years is $\frac{3}{5}$; therefore the chance that both will die is $\frac{9}{16} \times \frac{3}{5}$ or $\frac{27}{80}$; therefore the chance that both will not be dead, that is at least one will be alive, is $1 - \frac{27}{80}$ or $\frac{53}{80}$.

4. (A) (a) Total possible outcomes = $6 \times 6 \times 6 = 216$

(b) Favourable cases are (6, 6, 6), (6, 6, 5), (6, 5, 6), (6, 4, 6), (5, 6, 6), (4, 6, 6), (6, 5, 5), (5, 5, 6), (5, 6, 5)

$\Rightarrow$ Total number of favourable cases = 10

$\Rightarrow$ Probability = 10 / 216 = 5 / 108.

5. (D) Total number of balls = 8. Let the first drawn ball is white.

So, required probability $= \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5} = \frac{1}{14}$. But here we had started with a white ball. When we start with a black ball, the required

$$\text{Probability} = \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} \times \frac{4}{5} = \frac{1}{14}.$$

Since, these two cases are mutually exclusive.

Total probability $= \frac{1}{14}+\frac{1}{14}=\frac{2}{14}=\frac{1}{7}$.

6. (C) On first drawing, Probability of drawing red

$$= \frac{{}^8C_1}{{}^{20}C_1}=\frac{8}{20}=\frac{2}{5}$$

On second drawing, Probability of drawing white

$$= \frac{{}^5C_1}{{}^{19}C_1}=\frac{5}{19}$$

On third drawing, Probability of drawing black

$$= \frac{{}^7C_1}{{}^{18}C_1}=\frac{7}{18}$$

∴ Required probability

$$= \frac{2}{5}\times\frac{5}{19}\times\frac{7}{18}=\frac{7}{171}.$$

7. (A) There are 13 cards of spade in a pack of 52 cards. So, the chance that any of them will cut's a spade is 13/52 = 1/4 ⇒ Probability that it is not a spade = 1 – 1/4 = 3/4. Consider the following mutually exclusive ways in which A may win : A wins in the first cut, OR A, B, C have failed and then A wins, OR A, B, C, A, B, C have failed and then A wins and so on upto infinity ⇒ Respective chance of these events are = 1/4, [(3/4) × (3/4) × (3/4) × (1/4)], [(3/4) × (3/4) × (3/4) × (3/4) × (3/4) × (3/4) × (1/4)] …. To infinity ⇒ A's chance of succeeding = $(1/4) + (3/4)^3 \times (1/4) + (3/4)^6 \times (1/4) + \ldots = 1/4[\,1/(1 - 27/64)] = 16/37$.

8. (B) Total numbers of ways in which n persons can sit a round table = $(n - 1)!$ As the two specified individuals are to sit together, we may consider them to be a single individuals and then we have $(n - 2)!$ Different arrangements in which the two specified individuals can sit together in the same order. Since, they can permute among themselves, the total possible ways favourable to the events are $2 \times (n - 2)!$ ⇒ Probability of two specific individuals sitting together = $[2 \times (n - 2)!]/(n - 1)! = 2/(2 + n - 3)$. Hence, the odds against the events are $(n - 3)$ to 2.

9. (B) Total number of triangles that can be formed

$$= {}^8C_3 = 56.$$

Total number of triangles with no sides common = ${}^nC_3 - (n - 4)n - n = 16$, where $n = 8$ here. S, required probability = 16/56 = 2/7.

10. (D) The chance that the vessel is broken = 1/10 ⇒ Chance that the vessel is safe = 1 – 1/10 = 9/10. At least 4 will arrive safely if all 5 are safe or if 4 are safe and 1 is broken. Probability that all five are safe $= \left(\frac{9}{10}\right)^3$; Probability that 4 are safe and 1 is broken $= 5 \times \left(\frac{9}{10}\right)^4 \times \frac{1}{10}$ because the 4 may be safe in 5 mutually exclusive ways.

⇒ Required probability

$$= \left(\frac{9}{10}\right)^5 + 5\times\left(\frac{9}{10}\right)^4\times\frac{1}{10}$$

$$= 45927/50000.$$

11. (B) Total number of students = Number passing in I paper only + Number passing in II paper only + Number passing in both + Number failed in both

⇒ 500 = (150 – 50) + (350 – 50) + 50 + X

⇒ X = 50

⇒ Required probability

= 50/500 = 1/10.

12. (B) Total probability

$$= \frac{{}^{15}C_2}{{}^{20}C_2}+\frac{{}^5C_2}{{}^{20}C_2}$$

$$= \frac{105}{190}+\frac{10}{190}=\frac{23}{38}.$$

13. (A) The total number of ways in which 3 balls can be drawn is ${}^9C_3$ and the number of ways of drawing 3 black ball is ${}^5C_3$; therefore the chance of drawing 3 black balls $= \frac{{}^5C_3}{{}^9C_3} = \frac{5\times4\times3}{9\times8\times7} = \frac{5}{42}$. Thus, the odds against the event are 37 to 5.

14. (B) Net cases = 27 + 10 = 37.

Required probability = (1/37) /(1/27) = 27/37

15. (C) Total possible outcomes

$= {}^{15}C_2$; Favourable outcomes

$= {}^9C_2$

⇒ Probability $= {}^9C_2/{}^{15}C_2 = 36/105 = 12/35$.

16. (B) Total possible outcomes = 41. Therefore, the required probability = 6/41.

17. (A) ∵ The sum of two sides in a triangle is greater than the third side.

∴ In following 3 cases, triangle is not formed.

2,	3,	5
2,	3,	6
2,	4,	6

∴ Required probability $= \frac{3}{{}^5C_3} = \frac{3}{10}$.

18. (B) The care, none of the parts shall be greater than $\frac{1}{n}$ th of the rod is possible if the rod is divided in n equal parts. The probability of one part being $\frac{1}{n}$ th of the rod $= \frac{1}{n}$.

∴ Required probability of n parts being $\frac{1}{n}$ th of the rod each $= \frac{1}{n}\times\frac{1}{n}\times\frac{1}{n}\times\ldots\ldots\frac{1}{n} = \left(\frac{1}{n}\right)^n$.

19. (A) At the first trial 3 balls may be drawn in ${}^{13}C_3$ ways and 3 white balls may be drawn in ${}^{5}C_3$ ways. Therefore, the chance of 3 white balls at first trial $= \frac{5 \times 4}{1 \times 2} \div \frac{13 \times 12 \times 11}{1 \times 2 \times 3} = \frac{5}{143}$ · When 3 white balls have been drawn and removed, the bag contains 2 white and 8 black balls; therefore at the second trial 3 balls may be drawn in ${}^{10}C_3$ ways; and 3 black balls may be drawn in ${}^{8}C_3$ ways; therefore the chance of 3 black balls at the second trial

$$= \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \div \frac{10 \times 9 \times 8}{1 \times 2 \times 3} = \frac{7}{15}.$$

So, the chance of the compound event

$$= \frac{5}{143} \times \frac{7}{15} = \frac{7}{429}.$$

20. (B) In his first throw A's chance is $\frac{1}{6}$; in his second throw it is $\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$, because each player must have failed once before A can have a second throw; in his third throw his chance $= \left(\frac{5}{6}\right)^4 \times \frac{1}{6}$ because each player must have failed twice; and so on . Thus, A's chance is the sum of the infinite series

$$\frac{1}{6}\left\{1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots\right\}.$$

Similarly, B's chance is the sum of the infinite series

$$\frac{5}{6}, \frac{1}{6}\left\{1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots\right\}$$

$\therefore$ A's chance is to B's as 6 is to 5. Their respective chances are therefore $\frac{6}{11}$ and $\frac{5}{11}$, and their expectations are Rs. 6 and Rs. 5 respectively.

21. (A) The chance of A, B, C before the starts are $\frac{5}{10}$, $\frac{3}{10}$, $\frac{2}{10}$ *i.e.,* $\frac{1}{2}$, $\frac{3}{10}$, $\frac{1}{5}$ respectively. A can lose in two ways; either by the winning of B or C.

As A's chance of winning is 1/2; therefore A's chance of losing is 1/2. After the accident his chance of winning is 1/3, and hence his chance of losing becomes 2/3. $\Rightarrow$ his chance of losing is increased in the ratio of 4 to 3, also B's and C's chances of winning are increased in the same ratio. Thus B's chance of winning $= \frac{3}{10} \times \frac{4}{3} = \frac{2}{5}$; and C's chance of winning $= \frac{2}{10} \times \frac{4}{3} = \frac{4}{15}$.

22. (A)

23. (C) Let the two specified persons be denoted by A and B.

The chance that A occupies any seat in the first class railway compartment is $\frac{6}{7}$, because there are seven persons and the first class seats in the compartment are 6. Wherever, A has occupied his seat there is only one case favourable to the event that B occupies is seat opposite to A. Thus, the chance of the compound events that B gains an opposite seat to A is $\frac{6}{7} \times \frac{1}{6} = \frac{1}{7}$.

24. (C) Required probability

$$= \left(1 - \frac{1}{6}\right) \times \left(1 - \frac{2}{5}\right)$$

$$= \frac{5}{6} \times \frac{3}{5} = \frac{1}{2}.$$

25. (D) Let the events be A & B. Now, P(A) = 2/7, P(B) = 6/11.

$\therefore$ The probability that none of the events (A & B) will occur

$$= \left(1 - \frac{2}{7}\right) \times \left(1 - \frac{6}{11}\right)$$

$$= \frac{5}{7} \times \frac{5}{11} = \frac{25}{77}.$$

26. (A) Since, Probability of coming head $= \frac{1}{2}$ = probability of coming tail

$$\Rightarrow \quad P(H) = \frac{1}{2} = P(T).$$

Now, A wins if he throws a head in 1st or 3rd or 5th or 7th …..

$\therefore$ Probability of A wins

= P[H or (TTH) or (TTTTH) or (TTTTTTH),……]

= P(H) + P(TTH) + P(TTTTH) + ……..

= P(H) + P(T) . P(T) . P(H) . P(T) . P(T) . P(T) . P(T) . P(H) + ……..

$$= \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \ldots\ldots\ldots$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^5 + \ldots\ldots$$

$$= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \ldots\right]$$

$$= \frac{1}{2} \times \frac{1}{1 - \left(\frac{1}{2}\right)^2} = \frac{1}{2} \times \frac{4}{4-1} = \frac{2}{3}$$

$$P(A) = \frac{2}{3}, P(B) = 1 - \frac{2}{3} = \frac{1}{3}$$

$\Rightarrow$ Required Probability $= \left(\frac{2}{3}, \frac{1}{3}\right)$.

27. (B) Total possible combinations of distribution of ticket that no child will have to wait for change are as follows :

1.	1 1 1 1 2 2 2 2	2.	1 1 1 2 1 2 2 2
3.	1 1 1 2 2 1 2 2	4.	1 1 1 2 2 2 1 2
5.	1 1 2 1 1 2 2 2	6.	1 1 2 2 1 1 2 2
7.	1 1 2 1 2 1 2 2	8.	1 1 2 1 2 2 1 2
9.	1 1 2 2 1 2 1 2	10.	1 2 1 2 1 2 1 2
11.	1 2 1 1 1 2 2 2	12.	1 2 1 1 2 1 2 2
13.	1 2 1 1 2 2 1 2	14.	1 2 1 2 1 1 2 2

Now these 14 combinations can again be arranged in $4! \times 4!$ Ways.

So, total favourable combinations = $14 \times 4! \times 4!$

And total possible combinations = 8!

$$\text{Required Probability} = \frac{14 \times 4! \times 4!}{8!} \frac{1}{5}.$$

28. (A) **Case I :** Let the first ball removed be White

$\Rightarrow$ White balls left = $(m - 1)$ Black balls left = n

Probability of getting a white ball in the first and second draw = $\dfrac{m}{(m+n)} \times \dfrac{(m-1)}{(m-1+n)}$

Case II : Let the first ball removed be Black

$\Rightarrow$ White balls left = m

Black ball left = $n - 1$

Probability of getting a Black ball in the first and White in second draw

$$= \frac{n}{(m+n)} \times \frac{m}{(m+n-1)}$$

$\Rightarrow$ Total Probability

= Sum of these two

$$= \frac{m \times (m-1)}{(m+n)(m-1+n)} + \frac{mn}{(m+n)(m+n-1)}$$

$$= \frac{m^2 - m + mn}{(m+n)(m-1+n)}$$

$$= \frac{m(m-1+n)}{(m+n)(m-1+n)} = \frac{m}{m+n}.$$

29. (C) **Case I :** Assume that the box has two white balls. When a white ball is added, it now has 3 white balls $\Rightarrow$ Probability of drawing a white ball = Probability of drawing one ball $\times$ Probability of white ball

$$= \frac{1}{3} \times \frac{3}{3} = \frac{1}{3}$$

Case II : Assume that the box has one white and one Black ball.

When a white ball is added the box will then contain 2 white balls and 1 Black ball $\Rightarrow$ Probability of drawing a white ball = Probability of drawing one ball $\times$ Probability of white ball

$$= \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}.$$

Case III : Assume that the box has two Black balls. When a white ball is added, it now has 1 white ball and 2 Black balls

$\Rightarrow$ Probability of drawing a white = Probability of drawing one ball $\times$ Probability of white ball

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \Rightarrow \text{Overall Probabiity}$$

$$= \frac{1}{3} + \frac{2}{9} + \frac{1}{9} = \frac{2}{3}$$

[because all these are mutually exclusive cases].

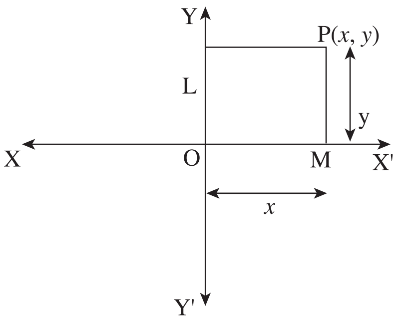
30. (D) There will be $6 \times 6 \times 6 = 216$ outcomes.

The outcomes which are not favourable are (1, 1, 1) and (6, 6, 6)

$\Rightarrow$ Required probability = [214/216] = [107/108].

8 Co-ordinate Geometry
(Lines in two dimensions)

1. Co-ordinate axes : Two perpendicular lines XOX′ and YOY′ are intersecting at the point O. XOX′ and YOY′ are called *co-ordinate axes*. The point O is called *origin*. XOX′ is called *x-axis* and YOY′ is called y axis.



2. Co-ordinate of a point : The perpendicular distance of a point P from x-axis and y-axis is shown by P (x, y) and also known as the co-ordinate of point P.

The length OM or LP is called *x-co-ordinate* or *abscissa* of point P and OL or MP is called *y-co-ordinate* or *ordinate* of point P.

3. Quadrants : The x-axis and y-axis divide the plane into four parts called *quadrants*. Regions XOY, YOX′, X′OY′ and Y′OX are respectively called the first, second, third and fourth quadrants.

4. Signs of the Co-ordinates of Points in Different Quadrants :

Quadrant	*x*-co-ordinate	*y*-co-ordinate	Point
First quadrant	+	+	(+, +)
Second quadrant	–	+	(–, +)
Third quadrant	–	–	(–, –)
Fourth quadrant	+	–	(+, –)

Note : 1. Abscissa is the perpendicular distance of a point from y-axis.

2. Ordinate is the perpendicular distance of a point from x-axis.

3. Abscissa is positive to the right of y-axis and is negative to the left of y-axis.

4. Ordinate is positive above x-axis and negative below x-axis.

5. Abscissa of any point on y-axis is zero.

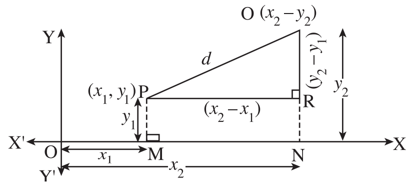
6. Ordinate of any point on x-axis is zero.

7. Co-ordinate of the origin are (0, 0).

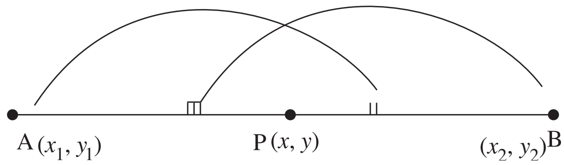
5. Distance formula : The distance between two points P (x_1, y_1) and Q (x_2, y_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Note : Distance of a point P (x, y) from the origin (0, 0) $= \sqrt{x^2 + y^2}$.



6. Section formula : The co-ordinate of point P, dividing the line segment joining A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m : n$ are given by



$$x = \frac{mx_2 + nx_1}{m + n}, y = \frac{my_2 + my_1}{m + n}$$

Note : The mid-point P (x, y) of a line segment joining the points A (x_1, y_2) and B (x_2, y_2) is given by

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

The co-ordinates of a point dividing the line segment joining the points A (x_1, y_1) and B (x_2, y_2) in the ratio of K : 1 is $\left(\frac{kx_2 + x_1}{K + 1}, \frac{ky_2 + y_1}{K + 1}\right)$

7. The co-ordinate of the centroid of a triangle whose vertices are A (x_1, y_1) B (x_2, y_2) and C (x_3, y_3) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

8. Four points will form : (i) a parallelogram, if its opposite sides are equal.

(ii) a rectangle, if its opposite sides are equal and two diagonals are also equal.

(iii) a rhombus, if its all the four sides are equal.

(iv) a square, if its all sides are equal and diagonals are also equal.

9. Three points will form : (i) an equilateral triangle, if its all the three sides are equal.

(ii) an isosceles triangle, if its any two sides are equal.

(iii) a right angled triangle, if the sum of square of its any two sides is equal to the square of third largest side.

10. Three points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are collinear or a lie on a line if any one of the following holds :

(i) AB + BC = AC

(ii) AC + CB = AB

(iii) CA + AB = CB

(iv) $[x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0$

11. Area of a triangle: Let the vertical of a triangle be A (x_1, y_1) B (x_2, y_2) and C (x_3, y_3). Then, area of ΔABC $= \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

If the value of area calculation is negative then we take its absolute value.

12. (a) If we have to show that three given points are collinear using **area of a triangle**, then the area of triangle formed by them is 0.

i.e.,, $[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0$

(b) If we have to find area of a quadrilateral, then we divide the quadrilateral into appropriate number of triangles and after finding **areas of triangles**, add them.

(c) If in question, it is given that the **area of a triangle** is p square units and we have to find same condition, then we must take two cases as $\pm p$ after removing the modulus sign from the formula of triangle.

Exercise – A

1. The area of quadrilateral with vertices (3, 5), (0, 5), (0, – 6), (3, – 6) is equal to (sq. units) :
   (A) 12 (B) 20
   (C) 33 (D) 50
2. The points (0, 0), (1, 2), (2, 0) are vertices of a/an :
   (A) equilateral triangle.
   (B) isosceles triangle.
   (C) right triangle.
   (D) scalene triangle.
3. The co-ordinates of a point which divides the join of (7, – 7) and (4, – 5) in the ratio 5 : 4, externally, are :
   (A) (5, 4) (B) (– 8, 6)
   (C) (– 8, 3) (D) (5, 3)
4. The centroid of the triangle whose vertices are (5, 7), (8, 9), (2, 11), is :
   (A) (5, 4) (B) (3, 9)
   (C) (5, 9) (D) None of these
5. If A (0, 0), B (3, 4), C (5, 8) are three vertices of a parallelogram ABCD, then the fourth vertex D will be :
   (A) (1, 2) (B) (2, 3)
   (C) (2, 4) (D) (4 – 4)
6. Point P is equidistant from A (1, – 8) and B (4, 5) and its ordinate is twice its abscissa, then its co-ordinates are :
   (A) (1, 3) (B) (3, 9)
   (C) (2, 6) (D) (5, 15)
7. The vertices of triangle ABC are A (2, 2), B (3, 4) and C (6, 6), then angle ABC is equal to :
   (A) 45° (B) 60°
   (C) 120° (D) $\tan^{-1} \left(\frac{1}{3}\right)$
8. The equation of the line passing through the points (1, –1) and (2, 4) is :
   (A) $5x - y + 6 = 0$ (B) $5x - y + 6 = 0$
   (C) $5x - y - 6 = 0$ (D) $5x + y - 6 = 0$
9. The slope of the line $9x - 5y + 16 = 0$, where a is constant, is :
   (A) $-\frac{9}{5}$ (B) $-\frac{5}{9}$
   (C) $\frac{9}{5}$ (D) $\frac{5}{9}$
10. The equation of the line passing through (1, – 3) and parallel to X + 2Y + 5 = 0, is :
    (A) 2X – Y – 1 = 0 (B) 2X + Y + 1 = 0
    (C) 2X + 2Y – 1 = 0 (D) 2X – Y + 1 = 0
11. If $(a, -2)$ lies on the line which passes through (2, 7) and which is parallel to $3x + 5y - 6 = 0$, then the value of P is :
    (A) $\frac{1}{16}$ (B) 16
    (C) 8 (D) $\frac{2}{9}$
12. The vertices of a ΔABC are A (0, 2), B (3, 0) and C (4, 0). The orthocenter H of the triangle is :
    (A) $\left(\frac{7}{3}, \frac{7}{3}\right)$ (B) $\left(1, \frac{4}{3}\right)$
    (C) (0, – 6) (D) None of the above
13. If A (a, b) lies on $3x + 2y - 12 = 0$ and B(b, a) lies on $2x - 5y + 4 = 0$, then the equation of the line AB is :
    (A) $y + x = 5$ (B) $y - a = 10$
    (C) $x + y + 5 = 0$ (D) $x + y - 10 = 0$
14. The equation of perpendicular bisector of the line joining A (3, 6) and B (5, 2) is :
    (A) $x + 2y + 4 = 0$ (B) $x - 2y + 4 = 0$
    (C) $x + 2y - 4 = 0$ (D) $x - 2y - 4 = 0$
15. A triangle ABC is given by A (5, 4), B (2, -7), C (4, 5). The equation of median drawn on BC from A, is :
    (A) $y + 3x = 19$ (B) $5x - 2y = 17$
    (C) $5x + 3y = 17$ (D) $3x - 5y = 20$

16. The equation of the line perpendicular to the line L : $3X + 5Y + 5 = 0$ and having Y intercept same as L is :

(A) $5X + 3Y = 17$ (B) $5X + 3Y = 3$

(C) $5X - 3Y = 3$ (D) $5X - 3Y = 13$

17. The line with X intercept = – 2 and passing through the intersection of $x + 5y - 5 = 0$ and $x - y + 2 = 0$, will be :

(A) $8x - 5y = 3$ (B) $3x + 9y = 7$

(C) $7x - 5y = 17$ (D) $7x + 17y = 14$

18. The distance between the lines $2x + 3y - 6 = 0$ and $24x + 36y + 84 = 0$, will be :

(A) $\frac{14}{\sqrt{13}}$ (B) $\sqrt{13}$

(C) $\frac{49}{\sqrt{13}}$ (D) None of these

19. The point whose abscissa is equal to its ordinate and which is equidistant from the point (3, 5) and (1, 3) is :

(A) (2, 1) (B) (4, 2)

(C) (6, 3) (D) (8, 4)

20. The co-ordinates of the points A, B, C, D, are (1, 3), (2, 4), (6, 8) and (3, b) respectively. If the line AC and BD are perpendicular, then a = ?

(A) 1 (B) – 2

(C) 3 (D) $\frac{4}{5}$

21. If the middle points of the sides of a triangle be (3, – 2), (– 3, 4) and (7, 9), then the centroid of the triangle is :

(A) $\left(\frac{5}{3}, 4\right)$ (B) $\left(\frac{7}{5}, \frac{6}{5}\right)$

(C) $\left(\frac{7}{3}, 4\right)$ (D) $\left(\frac{7}{3}, \frac{11}{3}\right)$

22. The mid-points of sides of a triangle are (0, – 4), (1, 4) and (3, 2). Then the co-ordinates of its vertices are :

(A) (3, 6), (– 3, – 14), (5, 6)

(B) (2, 3), (0, 1), (4, 5)

(C) (2, 3), (– 2, 4), (5, 9)

(D) (2, -3) (– 2, – 5) (4, 7)

23. If the points (2, 0), (0, 5) and (a, b) are collinear, then :

(A) $a = \frac{5}{2}, b = \frac{9}{5}$ (B) $a = \frac{7}{3}, b = \frac{15}{2}$

(C) for $b = \frac{15}{2}$ (D) none of the above

24. Area of a triangle whose vertices are $(a \sin \theta, - b \cos \theta)$, $(- a \sin \theta, b \cos \theta)$, $(a \sin \theta, - b \cos \theta)$ is :

(A) $4ab \sin \theta \cos \theta$ (B) $ab \sin 2\theta \cos \theta$

(C) $3ab \sin \theta \cos \theta$ (D) $4ab \sin 2\theta$

25. The points (2, 4) and (6, – 2) are the opposite vertices of a rectangle. The other two vertices lie on the line $2y = 3x + k$, then the value of k will be :

(A) 10 (B) – 10

(C) 5 (D) – 8

26. The extremities of a diagonal of a parallelogram are the points (– 1, 3) and (3, 2) and (1, 0). If the vertex is (– 2, 1), then fourth vertex is :

(A) (– 1, 5) (B) (2, 4)

(C) (1, 5) (D) (– 1, – 5)

27. The equation of the straight line passing through the points (– 1, 2) and perpendicular to the line $y = - x$ is :

(A) $y - x + 5$ (B) $y = x + 3$

(C) $2y = x + 7$ (D) $y = 5x + 5$

28. The angle between the lines $y = \sqrt{3} + 9$ and $\sqrt{3} + x = 3$ is :

(A) 45° (B) 60°

(C) 90° (D) 120°

29. A line passes through the point (4, 6) and cuts off intercepts from the co-ordinates axes such that their sum is 20. The equation of the line is :

(A) $3x - 2y = 24$ (B) $x - y = 20$

(C) $x + y = 10$ (D) $2x + 3y = 15$

30. Point of intersection of the diagonals of square is at origin and co-ordinate axes are drawn along the diagonals. If the side is of length a, then which one is not the vertex of square ?

(A) $(a\sqrt{2}, 0)$ (B) $(0, a/\sqrt{2})$

(C) $(a/\sqrt{2}, 0)$ (D) $(- a/\sqrt{2}, 0)$

Exercise – B

1. The area of quadrilateral with vertices (2, 4), (0, 4), (0, – 4), (2, – 4) is equal to (sq. units) :

(A) 8 (B) 12

(C) 16 (D) 32

2. The points (3, 0), (– 3, 0), $(0, - 3\sqrt{3})$ are vertices of a/an :

(A) equilateral triangle (B) isosceles triangle

(C) right triangle (D) scalene triangle

3. The co-ordinates of a point which divides the join of (5, – 5) and (2, – 3) in the ratio 4 : 3, externally, are :

(A) (3, 4) (B) (– 7, 3)

(C) (– 7, 9) (D) (8, 3)

4. The centroid of the triangle whose vertices are (3, 10), (7, 7), (– 2, 1), is :

(A) (8/3, 6) (B) (6, 8/3)

(C) (– 4, – 7/3) (D) None of these

5. If A (0, 2), B (– 2, – 2), C (1, – 1) are three vertices of a parallelogram ABCD, then the fourth vertex D will be :

(A) (1, 2) (B) (2, 3)

(C) (3, 3) (D) (2, -2)

6. A point is equidistant from A (3, 1) and B(5, 3) and its abscissa is twice its ordinate, then its co-ordinates are :

(A) (2, 1) (B) (1, 2)

(C) (4, 2) (D) (2, 4)

7. The vertices of triangle ABC are A (4, 4), B (6, 3) and C (2, -1) ; then angle ABC is equal to :

(A) 45° (B) 90°

(C) 60° (D) None of these

8. The equation of the line passing through the points (5, 3) and (3, 5) is :

(A) X – Y + 8 = 0 (B) X – Y + 8 = 0

(C) X – Y – 8 = 0 (D) X + Y – 8 = 0

9. The slope of the line $a^2X - aY + 1 = 0$, where a is constant, is :

(A) $-a^2$ (B) $-a$

(C) a (D) None of these

10. The equation of the line passing through (2, – 4) and parallel to X – 2Y – 5 = 0, is :

(A) 2X + Y + 3 = 0 (B) X – 2Y – 10 = 0

(C) X – 2Y + 8 = 0 (D) X – 2Y + 13 = 0

11. If (4, P) lies on the line which passes through (2, 3) and which is parallel to 4X + 3Y – 6 = 0, then the value of P is :

(A) 3 (B) 1

(C) 1/3 (D) 1/2

12. The vertices of a ΔABC are A $(0, a)$, B $(b, 0)$ and C $(c, 0)$. The orthocenter H of the triangle is :

(A) $(b + c)/3, a/3$ (B) $(b + c)/c, (a^2 + bc)/2a$

(C) $(0, -bc/a)$ (D) None of these

13. If R (r, s) lies on 6X – Y = 1 and S (s, r) lies on 2X – 5Y = 5, then the equation of the line RS is :

(A) Y + X = 6 (B) X + Y + 2 = 0

(C) X + Y + 1 = 0 (D) None of these

14. The equation of perpendicular bisector of the line joining A (1, 2) and B (7, 4) is :

(A) 3X + Y = 0 (B) – 13X + 13Y – 11 = 0

(C) 3X + Y – 15 = 0 (D) None of these

15. A triangle ABC is given by A (2, 5), B(– 1, – 1), C (3, 1). The equation of median drawn on BC from A, is :

(A) 2X + Y – 9 = 0 (B) 5X – Y – 5 = 0

(C) 3X + Y = – 9 (D) None of these

16. The equation of the line perpendicular to the line L_1 : 2X – 5Y = 0 and having X intercept same as L_1 is :

(A) 5X – 2Y + 3 = 0 (B) 2X – 5Y + 9 = 0

(C) 5X – 2Y + 9 = 0 (D) 10X + 4Y - 5 = 0

17. The line with Y intercept = – 1 and passing through the intersection of 7X + 2Y = 20 and 2X – 3Y + 12 = 0, will be :

(A) 149X – 137Y + 9 = 0

(B) 153Y – 137X + 9 = 0

(C) 4X – 5Y + 8 = 0

(D) 149X – 36Y – 36 = 0

18. The distance between the lines X + 2Y – 3 = 0 and 11X + 22Y + 88 = 0, will be :

(A) 7/5 (B) $49/\sqrt{5}$

(C) $7/\sqrt{5}$ (D) None of these

19. The point whose abscissa is equal to its ordinate and which is equidistant from the point (1, 0) and (0, 3) is :

(A) (1, 1) (B) (2, 2)

(C) (3, 3) (D) (4, 4)

20. The co-ordinates of the points A, B, C, D, are $(2, a)$, (3, 5), (3, 4) and (0, 6) respectively. If the line AC and BD are perpendicular, then a = ?

(A) 7 (B) 1

(C) – 1 (D) – 7

21. If the middle points of the sides of a triangle be (– 2, 3), (4, – 3) and (4, 5), then the centroid of the triangle is :

(A) (5/3, 2) (B) (5/6, 1)

(C) (2, 5/3) (D) (1, 5/6)

22. The mid-points of sides of a triangle are (2, 1), (– 1, – 3) and (4, 5). Then the co-ordinates of its vertices are :

(A) (7, 9), (– 3, – 7), (1, 1)

(B) (– 3, – 7), (1, 1), (2, 3)

(C) (1, 1), (2, 3), (– 5, 8)

(D) None of the above

23. If the points $(a, 0)$, (0, b) and (1, 1) are collinear, then :

(A) $1/a^2 + 1/b^2 = 1$ (B) $1/a^2 - 1/b^2 = 1$

(C) $1/a + 1/b = 1$ (D) $1/a - 1/b = 1$

24. Area of a triangle whose vertices are $(a \cos\theta, -b \sin\theta)$, $(-a \sin\theta, -b \cos\theta)$, $(-a \cos\theta, -b \sin\theta)$ is :

(A) $a \cos\theta \sin\theta$ (B) $ab \sin\theta \cos\theta$

(C) 1/2 (D) ab

25. The points (1, 3) and (5, 1) are the opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$, then the value of c will be :

(A) 4 (B) – 4

(C) 2 (D) – 2

26. The extremities of a diagonal of a parallelogram are the points (3, – 4) and (3, – 4) and (– 6, 5). If the vertex is (– 2, 1), then fourth vertex is :

(A) (1, 0) (B) (– 1, 0)
(C) (1, 1) (D) None of these

27. The equation of the straight line passing through the points (3, 2) and perpendicular to the line y = x is :

(A) $x - y = 5$ (B) $x + y = 5$
(C) $x + y = 5$ (D) None of these

28. The angle between the lines $y = (2 - \sqrt{3})X + 5$ and $y = (2 + \sqrt{3})X - 7$ is :

(A) 30° (B) 60°
(C) 45° (D) $\tan^{-1} 3$

29. A line passes through the point (3, 4) and cuts off intercepts from the co-ordinates axes such that their sum is 14. The equation of the line is :

(A) $4x - 3y = 24$ (B) $4x + 3y = 24$
(C) $3x - 4y = 24$ (D) $3x + 4y = 24$

30. Point of intersection of the diagonals of square is at origin and co-ordinate axes are drawn along the diagonals. If the side is of length a, then which one is not the vertex of square ?

(A) $(a\sqrt{2}, 0)$ (B) $(0, a/\sqrt{2})$
(C) $(a/\sqrt{2}, 0)$ (D) $(-a/\sqrt{2}, 0)$

Solutions – A

1. (C) Area of quadrilateral ABCD = Area of ΔABD + ΔBCD

A (3, 5), B (0, 5), D (3, – 6), C (0, – 6)

$$\text{Area } \Delta ABD = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[3(5 + 6) + 0(\ ) + 3(5 - 5)]$$

$$= \frac{1}{2} \times 3 \times 11 = \frac{33}{2} \text{ sq. unit.}$$

$$\text{Area of } \Delta BCD = \frac{1}{2}[0(\ ) + 0(\ ) + 3(5 + 6)]$$

$$= \frac{11 \times 3}{2} = \frac{33}{2} \text{ sq. unit.}$$

$$\text{Area of ABCD} = \left(\frac{33}{2} + \frac{33}{2}\right) \text{ sq. units.} = 33.$$

2. (A) Here A (0, 0), B (1, 2), C (2, 0)

$$AB = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{1+4} = \sqrt{5}$$

$$BC = \sqrt{(2-1)^2 + (0-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$CA = \sqrt{2^2 + 0^2} = 2$$

AB = BC = CA

So, ABC is isosceles triangle.

3. (C) (x_1, y_1) (x_2, y_2) in m : n (7, – 7), (4, – 5), 5 : 4

$$x = \frac{m \times x_2 - m \times x_1}{m - n}$$

$$= \frac{5 \times 4 - 4 \times 7}{5 - 4} = \frac{20 - 28}{1}$$

$$= -8$$

$$y = \frac{m \times y_2 - m \times y_1}{m - n}$$

$$= \frac{5 \times -5 - 4 \times 7}{5 - 4}$$

$$= \frac{28 - 25}{1} = 3$$

$$(x, y) = (-8, 3)$$

4. (A) Centroid (x, y), where x & y are given by

$$x = \frac{5 + 8 + 2}{3} = 5$$

$$y = \frac{7 + 9 + 11}{3} = 9$$

Centroid = (5, 9)

5. (C) As diagonals bisect in a parallelogram, the mid point of AC will be the same as mid point of BD.

Let D = (X, Y)

Then we have mid point of AC

$$= \left(\frac{0+5}{2}, \frac{0+8}{2}\right)$$

$$= \left(\frac{5}{2}, 4\right)$$

$$\Rightarrow \frac{x+3}{2} = \frac{5}{2} \Rightarrow x = 2$$

$$\Rightarrow \frac{y+4}{2} = 4 \Rightarrow y = 4$$

So, D = (2, 4)

6. (C) Let co-ordinate of P is (x, y).

As given y = $3x$.

$$\sqrt{(x-1)^2 + (3x+8)^2} = \sqrt{(x-4)^2 + (3x-5)^2}$$

$$\Rightarrow x^2 - 2x + 1 + 9x^2 + 64 - 48x = x^2 + 16 - 8x + 9x^2 + 25 - 30x$$

$$\Rightarrow -50x + 65 = -39x + 41$$

$$\Rightarrow 65 - 41 = (50 - 38)x$$

$$\Rightarrow 24 = 12x$$

$$\Rightarrow x = 2, y = 6$$

7. (D) ∠ABC is the angle between the lines AB & BC.

$$\text{Now slope of line AB}(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-2}$$

$$= 2$$

$$\text{\& slope of line BC }(m_2) = \frac{6-4}{5-3} = \frac{2}{2} = 1$$

$$\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{2-1}{1 + 2 \times 1} = \frac{1}{3}.$$

8. (D) $$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $(x_1, y_1) = (1, -1)$

$(x_2, y_2) = (2, 4)$

$$\Rightarrow \quad y - (-1) = \frac{4-(-1)}{2-1}(x-1)$$

$$\Rightarrow \quad y + 1 = \frac{5}{1}(x-1)$$

$$\Rightarrow \quad y + 1 = 5x - 5$$

$$\Rightarrow \quad 5x - y = 6$$

9. (C) $$\text{Slope} = -\frac{\text{coefficient of X}}{\text{coefficient of Y}}$$

$$= -\frac{9}{-5} = \frac{9}{5}.$$

10. (B) Slope of given line(m_1)

$$= \frac{\text{coefficient of X}}{\text{coefficient of Y}}$$

$$m_1 = -\frac{1}{2}$$

Let slope of perpendicular line = m_2

$$\Rightarrow \quad m_1 \times m_2 = -1$$

$$\Rightarrow \quad -\frac{1}{2} \times m_2 = -1$$

$$\Rightarrow \quad m_2 = 2$$

So, equation of line that passes through (1, 3) and having slope 2 is

$$y - y_1 = \text{slope}\ (x - x_1)$$

$$\Rightarrow \quad y - 3 = 2(x-1)$$

$$\Rightarrow \quad 2x - 2 = y - 3$$

$$\Rightarrow \quad 2x - y + 1 = 0$$

11. (C) Equation of line parallel to $3x + 54 - 6 = 0$ is

$$3x + 5y + \lambda = 0 \quad \text{.....(i)}$$

it is given it passes through (1, 7)

$$\Rightarrow 3 \times 1 + 5 \times 7 + \lambda = 0$$

$$\Rightarrow \quad 3 + 35 + \lambda = 0$$

$$\Rightarrow \quad \lambda = -38$$

So, equation of line is $3x + 5y - 38 = 0$

Since $(a, -2)$ lines on this line.

$$\Rightarrow \quad 3a - 10 - 38$$

$$\Rightarrow \quad 3a = 0 \Rightarrow a = 16$$

12. (C) $$\text{Slope of AB} = \frac{0-2}{3-0} = -\frac{2}{3}$$

$$\text{Slope of AB's altitude} = \frac{1}{(2/3)} = \frac{3}{2}$$

Equation of AB's altitude

$$= (y - 0) = \frac{3}{2}(x - 4)$$

$$= 2y = 3x - 12 \quad \text{...(i)}$$

$$\text{Slope of BC} = \frac{0-0}{4-3} = \frac{0}{1} = 0$$

Slope of BC's altitude

$$= \frac{1}{\infty} = 0]$$

Equation of BC's altitude

$$= y - 2 = \frac{1}{0}(x - 0)$$

$$\Rightarrow \quad x = 0 \quad \text{...(ii)}$$

From equations (i) & (ii)

$$x = 0$$

$$y = -6$$

Hence orthocenter $(0, -6)$

13. (A) (a, b) lies on $3x + 2y = 12$

$$\Rightarrow \quad 3a + 2b = 12 \quad \text{...(i)}$$

$$(b, a) \text{ lies on } 2x - 5x + 4 = 0$$

$$\Rightarrow \quad 2b - 5a + 4 = 0 \quad \text{...(ii)}$$

From equations (i) & (ii)

$$3a + 2b = 12$$

$$-5a + 2b = -4$$

$$8a = 16$$

$$a = 2$$

$$\Rightarrow \quad b = 3$$

So, A(2, 3), B(3, 2)

Equation of line AB

$$y - 3 = \frac{2-3}{3-2}(x - 2)$$

$$\Rightarrow \quad y - 3 = -(x-2)$$

$$\Rightarrow \quad x + y - 5 = 0$$

$$\Rightarrow \quad x + y = 5$$

14. (C) $$\text{The slope AB} = \frac{2-6}{5-3} = \frac{-4}{2} = -2$$

Slope of line perpendicular to AB

$$= \frac{-1}{-2} = \frac{1}{2}$$

and this passes through mid point of AB *i.e.*,

$$= \left(\frac{3+5}{2}, \frac{6+2}{2}\right)$$

$$= (4, 4)$$

$\Rightarrow$ Equation of perpendicular bisector

$$\Rightarrow \quad y - 4 = \frac{1}{2}(x - 4)$$

$$\Rightarrow \quad 2y - 8 = x - 4$$

$$\Rightarrow \quad x - 2y + 4 = 0$$

15. (B)

Median will be the line through A & mid-point of BC $\left(\frac{2+4}{2}, \frac{-7+5}{2}\right)$

i.e, (3, –1). Hence equation through (5, 4) & (3, – 1) will be

$$y - 4 = \frac{-1-4}{3-5}(x - 5)$$

$\Rightarrow \quad 2(y-4)+5(x-5)$

$\Rightarrow \quad 2y-8 = 5x-25$

$\Rightarrow \quad 5x-2y = 17$

16. (C) $\quad 5Y = -3X-5$

$Y = -\frac{3}{5}X - \frac{5}{5}$

So slope of L_1 is $= \frac{1}{(-3/5)} = \frac{5}{3}$

Put $x = 0$ in L. $\quad Y = -1$

Now equation of L_1 is $= Y-(-1)$

$= \frac{5}{3}(X-0)$

$= 3(Y+1) = 5X$

$= 5X-3Y = 3$

17. (D) Equation of line which passes through the inter section of $x+5y-5=0$ and $x-y+2=0$ is

$x+5y-5+\lambda(x-y+2) = 0$

$(a+\lambda)x+(5-\lambda)y+(-5+2\lambda) = 0$

Put $\quad y = 0$

$\Rightarrow \quad (a+\lambda)x = 2\lambda - 5$

$\Rightarrow \quad x = \frac{2\lambda-5}{1+\lambda}$

$= -2$ $\quad$ given

$\Rightarrow \quad 2\lambda-5 = -2-2\lambda$

$\Rightarrow \quad 4\lambda = -2+5$

$\Rightarrow \quad 4\lambda = 3$

$\Rightarrow \quad \lambda = \frac{3}{4}$

$\Rightarrow$ Equation is

$(a+5y-5)+\frac{3}{4}(x-y+2) = 0$

$4x+20y-20+3x-3y+6 = 0$

$7x+17x-14 = 0$

$\Rightarrow \quad 7x+17y = 14$

18. (B) $\quad$ Distance $= \frac{C_1-C_2}{\sqrt{(a^2+b^2)}}$

Here $\quad C_1 = 6, C_2 = -7$

$= \frac{6-(-7)}{\sqrt{(4+9)}} = \frac{13}{\sqrt{13}}$

$= \sqrt{13}$

19. (B) Let the point is (x, y)

Given that $x = 2y \Longrightarrow$ points $(2y, y)$

and $\quad \sqrt{(2y-3)^2+(y-5)^2} = \sqrt{(2y-1)^2+(y-3)^2}$

$4y^2+9-12y+y^2+25-10y = 4y^2-4y+1+y^2 + 9-6y$

$\Rightarrow \quad -22y+34 = -10y+10$

$\Rightarrow \quad 34-10 = 12y$

$\Rightarrow \quad y = 2.$

So, $\quad$ abscise $= 4$

So, point is (4, 2).

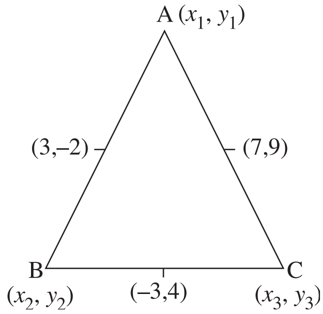
20. (B) $\quad$ Slope AC $= \frac{8-3}{6-1} = 1$

Slope BD $= \frac{b-4}{3-2} = b-4$

$1 \times (b-4) = -1$

$\Rightarrow \quad b = -1+4 = 3$

21. (C)



$\frac{x_1+x_2}{2} = 3,$

$x_1+x_2 = 6$

$\Rightarrow \quad \frac{y_1+y_2}{2} = -2$

$y_1+y_2 = -4 \quad$ …(i)

$\Rightarrow \quad \frac{x_2+x_3}{2} = -3$

$x_2+x_3 = -6 \quad$ …(ii)

$\Rightarrow \quad \frac{y_2+y_3}{2} = 4$

$y_2+y_3 = 8$

and $\quad \frac{x_1+x_3}{2} = 7$

$\Rightarrow \quad x_1+x_3 = 14$

and $\quad y_2+y_3 = 18 \quad$ …(iii)

From (i) + (ii) + (iii)

$x_1+x_2+x_2+x_3+x_3+x_1$

$= 6-6+14$

$\Rightarrow \quad 2(x_1+x_2+x_3) = 14$

$\Rightarrow \quad x_1+x_2+x_3 = 7$

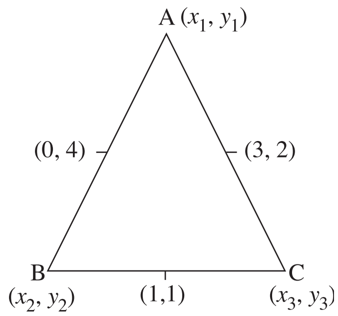
and $\quad 2(y_1+y_2+y_3) = -4+8+18$

$y_1+y_2+y_3 = \frac{22}{2} = 11$

So, $\quad$ Centroid is $= \frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}$

$= \left(\frac{7}{3}, \frac{11}{3}\right)$

22. (D)



$$x_1 + x_2 = 0 \quad \ldots(i)$$
$$x_2 + x_3 = 2 \quad \ldots(ii)$$
$$x_3 + x_1 = 6 \quad \ldots(iii)$$
$$y_1 + y_2 = -8 \quad \ldots(iv)$$
$$y_2 + y_3 = 2 \quad \ldots(v)$$
$$y_3 + y_1 = 4 \quad \ldots(vi)$$

From Eqns. (i) & (ii)

$$x_1 + x_3 = -2$$
$$x_1 + x_3 = 6$$

$$2x_1 = 4$$
$$x_1 = 2$$
$$\Rightarrow \quad x_2 = -2$$

From Eqns. (iv) & (v)

$$y_1 - y_3 = -10$$
$$y_1 + y_3 = 4$$

$$2y_1 = -6$$
$$y_1 = -3$$
$$\Rightarrow \quad y_3 = 7 \;\&\; y_2 = -5$$

$\Rightarrow$ Co-ordinate of vertices are $(2, -3)$, $(-2, -5)$ and $(4, 7)$

23. (C) For three points to be collinear

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ 0 & 5 & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & -5 & 0 \\ 0 & 5-b & 0 \\ a & b & 1 \end{vmatrix} \; R_1 \to R_1 \to R_2 \to R_2 \to R_2 \to R_3$$

$\Rightarrow$ Expand along C_3

$$\Rightarrow \quad +1\,(2 \times 5 - b) - (-5) = 0$$
$$\Rightarrow \quad 10 - 2b + 5 = 0$$
$$\Rightarrow \quad b = \frac{15}{2}$$

24. (B) Area Δ whose co-ordinates are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are

$$\Rightarrow \quad = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$$
$$\Rightarrow \quad = \frac{1}{2}[a \sin\theta\,(2b\cos\theta) - a\sin\theta\,(-2b\cos\theta) + a\sin\theta(0)]$$
$$\Rightarrow \quad = \frac{1}{2}[4ab\sin\theta\cos\theta]$$
$$\Rightarrow \quad = 2ab\sin\theta\cos\theta$$
$$\Rightarrow \quad = ab\sin\theta.$$

25. (B) We know that the mid-point of diagonals lies on line $2y = 3x + k$.

Here mid-point is $(4, 1)$ hence $2 \times 1 = 3 \times 4 + k$

$$\Rightarrow \quad k = -10.$$

26. (C) Let A $(-1, 3)$ and C $(3, 2)$ be the ends of diagonals of parallelogram ABCD. Let B $(1, 0)$ and D be (x, y). Then mid-points of diagonals AC and BD coincide.

So,

$$\Rightarrow \quad \frac{x+1}{2} = \frac{-1+3}{2}$$
$$\Rightarrow \quad x + 1 = 2 \Rightarrow x = 1$$
$$\Rightarrow \quad \frac{y+0}{2} = \frac{3+2}{2}$$
$$\Rightarrow \quad y = 5$$
$$\Rightarrow \quad = (1, 5)$$

27. (B) Slope of $y = -x$ is

$$m = -1$$

Equation of line with slope 1 and passing through point $(-1, 2)$ is

$$y - 2 = 1\,(x + 1)$$
$$\Rightarrow \quad y - 2 = x + 1$$
$$\Rightarrow \quad x - y + 3 = 0$$

28. (C) $m_1 = \sqrt{3}, m_2 = -\frac{1}{\sqrt{3}}$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2} = \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)}{1 - 1} = \frac{2}{\sqrt{3} \times 0} = \infty$$

Short cut : here $m_1 \times m_2$

$$= \sqrt{3} \times -\frac{1}{\sqrt{3}} = -1.$$

29. (B) Here $a + b = 20$

$$\Rightarrow \quad a = 20 - b$$

Equation of line in intercept's form is

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 1$$
$$\Rightarrow \quad \frac{x}{20-b} + \frac{y}{b} = 1$$

It passes through $(4, 6)$

$$\Rightarrow \quad \frac{4}{20-b} + \frac{6}{b} = 1$$
$$\Rightarrow \quad 4b + 120 - 6b = (20 - b)b$$
$$\Rightarrow \quad -2b + 120 - 20b + b^2 = 0$$
$$\Rightarrow \quad b^2 - 22b + 120 = 0$$
$$\Rightarrow \quad b^2 - 12b - 10b + 120 = 0$$
$$\Rightarrow \quad b(b - 12) - 10(b - 12) = 0$$
$$\Rightarrow \quad (b - 12)(b - 10) = 0$$
$$\Rightarrow \quad b = 12 \text{ or } 10$$

So, $a = 8$ or 10

Equation of line is

$$\Rightarrow \quad \frac{x}{8} + \frac{y}{12} = 1$$
$$\Rightarrow \quad 12x + 8y = 96$$

or $3x + 2y = 24$

or it can be $\frac{x}{10} + \frac{y}{10} = 1$

$$\Rightarrow \quad x + y = 10$$

30. (A) Obviously from right angled triangle BOA

$$OA = OB = a/\sqrt{2}$$

Hence the vertex $(a/\sqrt{2}, 0)$ is not the vertex of square.

Solutions – B

1. (C) Let A, B, C, D be the four vertices. Then Area of quadrilateral = Area of two triangles ABD and BCD. Now Area of triangle ABD with A $(2, 4)$, B $(0, 4)$, D $(0, -4)$

$= \frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$= \frac{1}{2}[2 (4 + 4) + 0 (- 4 - 4) + 2 (4 - 4)]$

$= \frac{1}{2} \times 16 = 8$ square units.

Area of Triangle BCD with B (0, 4), C (0, – 4), D(2, – 4) will be

$= \frac{1}{2}[0 (- 4 + 4) + 0 (- 4 - 4) + 2 (4 + 4)]$

$= \frac{1}{2} \times 16 = 8$ square units.

⇒ Total Area

$= 8 + 8 = 16$ square units.

2. (A) Find the three length separately

$$AB = 6, BC = \sqrt{3^2 + (3\sqrt{3})^2} = 6$$

$$AC = \sqrt{3^2 + (3\sqrt{3})^2} = 6.$$

Hence the point are the vertices of equilateral triangle.

3. (B) Let the ratio be 4 : 3 or 4/3 : 1.

Now X = (4/3 × 2 – 5)/(4/3 – 1)
= (8/3 – 5)/(1/3)
= (– 7/3)/(1/3) = – 7
Y = [(4/3X – 3]/(4/3 – 1)
= 1/(1/3) = 3.

Hence (– 7, 3).

4. (A) Centroid = (3 + 7 – 2), (10 + 7 + 1)/3
= (8/3, 6).

5. (C) As diagonals bisect in a parallelogram, the mid point of AC will be the same as mid point of BD. Let D = (X, Y) Then we have mid point of AC = (1/2, 1/2) & mid point of BD = [(– 2 + X)/2, (– 2 + Y)/2] Equating (– 2 + X)/2 = 1/2 or -2 + X = 1 or X = 3 Similarly Y = 3.
Hence (3, 3).

6. (C) Let the point be P (2X, X).

$$AP = \sqrt{(3 - 2X)^2 + (1 + X)^2},$$

$$BP = \sqrt{(5 - 2X)^2 + (3 - X)^2}$$

$$AP = BP\ (3 - 2X)^2 + (1 + X)^2$$

$$= (5 - 2X)^2 + (3 - X)^2$$

$$10 - 14X + 5X^2 = 34 - 26X + 5X^2$$

$$2X = 24$$

$$\Rightarrow x = 2, 2x = 4$$

$$P(2X, X) = P(4, 2).$$

7. (D) Angle ABC is the angle between the lines AB & BC.

Now slope of line AB = $m_1 = (3 - 4)/(6 - 4) = - 1/2$
and slope of line BC = $m_2 = (- 1 - 3)/(2 - 6)$
$= - 4/- 4 = 1$

Now
$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-\frac{1}{2} - 1}{1 + \left(-\frac{1}{2}\right)(1)}$$

$$= \frac{-3/2}{1 - \frac{1}{2}} = -3.$$

8. (D) Direct formula. The equation will be

$$Y - 3 = \frac{(5 - 3)}{(3 - 5)} \times (X - 5)$$

$$\Rightarrow Y - 3 = (X - 5)$$

or $X + Y - 8 = 0$

9. (C) Slope = – coefficient of X/coefficient of Y
$= - a^2/- a = a.$

10. (B) Line parallel to X – 2Y – 5 = 0 will be X – 2Y + K = 0 Put the point (2, -4) in this equation we have 2 + 8 + K = 0 or K = -10 or line is X – 2Y – 10 = 0.
Short-cut : Substitute (2, -4) in the options. Only (b) satisfies.

11. (C) Line parallel to 4X + 3Y – 6 = 0 will be 4X + 3Y + K = 0
Put (2, 3)
⇒ 8 + 9 + K = 0 or K = – 17.
Hence the required line is 4X + 3Y – 17 = 0
Now (4, P) lies on this line so, 4 × 4 + 3 × P – 17 = 0
Or 3P = 1 or P = 1/3.

12. (C) We have the triangle as in the figure.
We have slope of BC = 0
Hence slope of BC's altitude = ∞
Equation of BC's altitude = $(Y - a) = m(X)$
⇒ $(Y - a) = 1/0 \times X$
X = 0 … (a)
Now slope of AB = $(a - 0)/(0 - b)$
$= - a/b$ and
Slope of AB's altitude = b/a
⇒ Equation of AB's altitude = (Y – 0)
$= b/a(X - c)$
$aY - bX + bc = 0$ …(b)
Solve (a) & (b) to get X = 0
$Y = - bc/a.$

13. (A) We have 6r – s = 1 … (1)
2s – 5r = 5 … (2)
Solving $r = 1$, $s = 5$. Hence equation of line through (1, 5), (5, 1) will be Y – 5 = (1 – 5)/(5 – 1) × (X – 1)
⇒ Y – 5 = 1 – X
or Y + X = 6.

14. (C) The slope of given line = (4 – 2) /(7 – 1) = 2/6 = 1/3, hence slope of line perpendicular to this will be – 3 & it passes through mid point of AB (1 + 7)/2, (2 + 4)/2, = (4, 3).
Hence equation is Y – 3 = – 3(X – 4)
or Y – 3 = – 3X + 12
or Y + 3X = 15.

15. (B) Median will be the line through A & mid-point of BC $\left(\frac{-1+3}{2}, \frac{-1+1}{2}\right)$
i.e, (1, 0) . Hence equation through (2, 5) & (1, 0) will be Y – 5 = (0 – 5)/(1 – 2) × (X – 2) or 5X – Y – 5 = 0.

16. (D) Let the required line be L.

$$\text{Now } 2X - 5Y - 1 = 0 \text{ or } Y = 2/5\,X - 1/5.$$
$$\text{Slope of } L_1 = 2/5,$$
$$\text{Slope of L} = -5/2, \text{ Put } Y = 0 \text{ into } L_1.$$

X = 1/2 (X intercept of L = X intercept of L_1 = 1/2)

Now equation of L is $Y - 0$
$$= -5/2\,(X - 1/2)$$
or $\quad 10X + 4Y - 5 = 0$

17. (D) The required line
$$= 7X + 2Y - 20 + K\,(2X - 3Y + 12)$$
$$= 0 \qquad \ldots(1)$$

Or $(7 + 2K)\,X + (2 - 3K)Y - 20 + 12\,K = 0$

Now Y intercept is obtained by putting X = 0 in the above equation.

or $\quad Y = (20 - 12K) / (2 - 3K) = -1$

or $\; 20 - 12\,K = 3K - 2$ or $15K = 22$ or $K = 22/15$.

Putting K into (1) we have the line as $149X - 36Y - 36 = 0$.

18. (D) We have the lines as
$$X + 2Y - 3 = 0 \;\&\; X + 2Y + 8 = 0$$
$$\text{Now the distance} = \frac{C_1 - C_2}{\sqrt{(a^2 + b^2)}} + \frac{8 + 3}{\sqrt{1^2 + 2^2}}$$
$$= \frac{11}{\sqrt{5}}.$$

19. (B) Let the point be (X, X), so according to the condition
$$(X - 1)^2 + (X - 0)^2 = (X - 0)^2 + (X - 3)^2$$
$$\Rightarrow \quad -2X + 1 = -6X + 9$$
$$\Rightarrow \quad X = 2.$$

Hence the point is (2, 2).

20. (B) $\quad$ Slope AC $= (4 - a)/(3 - 2) = 4 - a$

Slope BD $= (6 - 5)/\,(0 - 3) = -1/3$

For perpendicular $(4 - a)(-1/3) = -1 \Rightarrow a = 1$.

21. (C) Let the vertices of the triangle are A (X_1, Y_1), B (X_2, Y_2) and C (X_3, Y_3), then
$$X_1 + X_2 = 8 \qquad \ldots(i)$$
$$Y_1 + Y_2 = 10 \qquad \ldots(ii)$$
$$X_2 + X_1 = -4 \qquad \ldots(iii)$$
$$Y_2 + Y_3 = 6 \qquad \ldots(iv)$$
$$X_3 + X_1 = 8 \qquad \ldots(v)$$
$$Y_3 + Y_1 = -6 \qquad \ldots(vi)$$

On solving these equations, we get $X_1 = 10, X_2 = -2, X_3 = -2Y_1 = -1, Y_2 = 11, Y_3 = -5$.

Hence the centroid is (2, 5/3).

Shortcut : As we know that the centroid of the triangle ABC and that of the triangle formed by joining the middle points of the sides of triangle ABC is same. $[(4 + 4 - 2)/3, (5 - 3 + 3)] \equiv (2, 5/3)$.

22. (A) $\quad (X_1 + X_2)/2 = 2\,(X_2 + X_3)/2 = -1,$
$$(X_3 + X_1)/2 = 4$$
$$\Rightarrow \quad X_1 = 7, X_2 = -3, X_3 = 1.$$

Similarly y_1, y_2, y_3 can be found
$$Y_1 + Y_2 = 2, Y_2 + Y_3 = -6, Y_3 + 4_1 = 10$$
$$\therefore \quad Y_1 = 9, Y_2 = -7, Y_3 = 1.$$

23. (C)
$$1/2 \begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$
$$\Rightarrow a\,(b - 1) + 0 + 1\,(-b) = 0$$
$$\Rightarrow \quad ab - a - b = 0$$
$$\Rightarrow \quad 1/a + 1/b = 1$$

24. (D) Area
$$= 1/2 \begin{vmatrix} a\cos\theta & -b\sin\theta & 1 \\ -a\sin\theta & b\cos\theta & 1 \\ -a\cos\theta & -b\sin\theta & 1 \end{vmatrix}$$
$$= 1/2\,(a \times b) \begin{vmatrix} \cos\theta & \sin\theta & 1 \\ -\sin\theta & \cos\theta & 1 \\ -\cos\theta & -\sin\theta & 1 \end{vmatrix}$$
$$= ab/2\,[\cos\theta\,(\cos\theta + \sin\theta) - \sin\theta\,(-\sin\theta + \cos\theta) + 1\,(\sin^2\theta + \cos^2\theta)$$
$$= ab/2\,(1 + 1) = ab.$$

25. (B) We know that the mid-point of diagonals lies on line $\quad y = 2x + c$.

Here mid-point is (3, 2), hence $c = -4$.

26. (B) Let A (3, – 4) and C (– 6, 5) be the ends of diagonals of parallelogram ABCD. Let B (– 2, 1) and D be (X, Y). Then mid-points of diagonals AC and BD coincide.

$$\text{So } (X - 2)/2 = (-6 + 3)/2 \text{ and } (Y + 1)/2 = \frac{5 - 4}{2}$$
$$\Rightarrow \quad X = -1, Y = 0$$

$\Rightarrow$ Co-ordinates of D are (–1, 0).

27. (B) Let the required equation be $y = -x + c$ which is perpendicular to $y = x$ and passes through (3, 2). So
$$2 = -3 + c \Rightarrow c = 5$$

Hence required equation is x + y = 5.

28. (B)
$$\theta = \tan^{-1}\left(\frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + 4 - 3}\right)$$
$$= \tan^{-1}(-\sqrt{3}) = 120°$$

Considering smaller angle $\theta = 60°$.

29. (B) Given $\quad a + b = 14 \Rightarrow a = 14 - b$.

Hence the equation of straight line is
$$X/(14 - b) + y/b = 1.$$

Also it passes through $(3, 4) \Rightarrow 3/(14 - b) + 4/b = 1$
$\Rightarrow b = 8$ or 7

Therefore equations are
$$4x + 3y = 24 \text{ and } x + y = 7.$$

Shortcut : This equation can be checked with the options as the line $4x + 3y = 24$ passes through (3, 4) and also cuts the intercepts from the axes whose sum is 14.

30. (a) Obviously from right angled triangle
$$\text{BOA, OA} = \text{OB} = a/\sqrt{2}$$

Hence the vertex $(a/\sqrt{2}, 0)$ is not the vertex of square.

9 Puzzles, Verbal Problems and Equations

We use the term Puzzle, verbal, or story, problems for those which traditionally are called problems leading to equations. Examination problems nowadays frequently require the use not only of equations but of inequalities as well, and sometimes of other conditions which are not written in the form of equations and inequalities. The most common feature of problems of this type is that the condition is given in verbal form without any formulas or even without any literal designations of the unknowns. The habit of most aspirants to regard any verbal problem as a problem which involves the setting up of equations is sometimes a drawback in that they are psychologically unprepared when it turns out that the equations alone are not sufficient to obtain a solution.

The ordinary type of problem in which all the conditions can be written down in the form of equations do not, as a rule, cause any particular difficulty, though even in these problems there are certain items that, occasionally are troublesome. Usually the real difficulty of more complicated problems lies in their unusual aspect, the necessity to reason and not merely to solve certain systems of equations or inequalities.

It frequently happens that simple arguments without any setting up of equations and inequalities, even if that is possible, proves to be faster and simpler. What is more, an occasional problem is more amenable to simple, everyday reasoning than to ordinary mathematical techniques. Incidentally, the "commonsense" solution is not always rigorous and so one has to supplement it with rigorous mathematical arguments.

Let us begin with some mixture problems. The aspirant frequently finds difficulty in setting up the equations in these problems.

Illustration 1.

Given three mixtures consisting of three components A, B and C. The first mixture only contains components A and B in the weight 3 : 5, the second mixture contains components B and C in the weight ratio of 1 : 2, and the third mixture contains only the components A and C in the weight ratio 2 : 3. In what ratio must we take these mixtures so that resulting mixture of components A, B and C stand in the weight ratio 3 : 5 : 2 ?

Some aspirants have found the phrase "weight ratio" difficult to grasp others fear the word "mixture". Actually, the problem is not at all difficult.

Since components A and B are in the ratio 3: 5 by weight in the first mixture, each gram of the first mixture contains 3/8 gram of component A and 5/8 gram of component B. Similarly, 1 gram of the second mixture contains 1/3 gram of B and 2/3 gram of C, and 1 gram of the third mixture contains 2/5 gram of A and 3/5 gram of C.

If we take x grams of the first mixture, y grams of the second, and z grams of the third and mix them, we get $(x + y + z)$ grams of the new mixture, which will contain $(3/8x + 2/5z)$ grams of A, $\left(\frac{5}{8}x + \frac{1}{3}y\right)$ of B and $\left(\frac{2}{3}y + \frac{3}{5}z\right)$ grams of C. We have to take the first, second and third mixtures in quantities such that in the new mixture the components A, Band C are in the weight ratio 3: 5 : 2, that is, in one gram of the new mixture there must be 3/10 gram of A, 5/10 gram of B and 2/10 gram of C. But then $x + y + z$ grams of the new mixture will contain $\frac{3(x+y+z)}{10}$ grams of A, $\frac{5(x+y+z)}{10}$ grams of B and $\frac{2(x+y+z)}{10}$ grams of C. Equating the different expressions for one and the same quantity of components A, B, C, we get the following system of equations:

$$\begin{aligned} \frac{3}{8}x + \frac{2}{5}z &= \frac{3}{10}(x+y+z) \\ \frac{5}{8}x + \frac{1}{3}y &= \frac{5}{10}(x+y+z) \qquad \ldots(1) \\ \frac{2}{3}y + \frac{3}{5}z &= \frac{2}{10}(x+y+z) \end{aligned}$$

Note from the start that although we have three equations in three variables, there are only two independent equations in this system.

This can easily be shown, for example, by subtracting from the equation $x + y + z = x + y + z$ the sum of the first two equations; we get the third equation. For this reason, from system (1) we find only the ratio $x: y:$ z, and not x, y and z themselves. For instance, eliminating from the first two equations of (1), we find that $y = 2$z. Substituting this value of y into any equation of the system, we get $x = (20/3)z$.

Consequently, $x: y: z = 20 : 6 : 3$, which means the mixtures, have to be taken in the weight ratio 20 : 6 : 3.

Percentage problems represent another difficult type of problem for aspirants. Yet there is nothing hard about the notion of a per cent. We can get rid of percentages simply by considering hundredths of a number. The

following problem is one involving both mixtures and percentage.

Illustration 2.

The percentages (by weight) of alcohol in three solutions form a geometric progression. If we mix the first, second and third solutions in the weight ratio of 2: 3: 4, we obtain a solution containing 32% alcohol. If we mix them in the weight ratio 3 : 2 : I, we obtain a solution containing 22 % alcohol. What is the percentage of alcohol in each solution ?

Let there be x% alcohol in the first solution, y% in the second, and z%in the third. This means that 1 gram of the first solution contains x/100gram of alcohol, 1 gram of the second solution, y/100gram of alcohol, and 1 gram of the third solution, z/100gram of alcohol. If we take 2 grams of the first solution, 3 grams of the second, and 4 grams of the third, we get 9 grams of a mixture containing $\left(2\cdot\frac{x}{100}+3\cdot\frac{y}{100}+4\cdot\frac{z}{100}\right)$ grams of alcohol. By the statement of the problem, the resulting mixture contains 32% alcohol, which means that 9 grams of the mixture contains $9.\frac{32}{100}$ grams of alcohol. From this condition we get equation

$$\frac{2x+3y+4z}{100} = \frac{9\cdot 32}{100}$$

In similar fashion, we get yet another equation :

$$\frac{3x+2y+z}{100} = \frac{6\cdot 22}{100}$$

Finally, by hypothesis, the numbers x, y, z form a geometric progression and so $y^2 = xz$.

It now remains to solve the system of equations

$$\begin{aligned} 2x+3y+4z &= 288 \\ 3x+2y+z &= 132 \\ y^2 &= xz \end{aligned}$$

Solving the first two equations for y and z and substituting the resulting expressions into the third equation, we get the equation $x^2 - 76x + 768 = 0$ with roots $X_1 = 64$ and $X_2 = 12$.

But the value $X_1 = 64$ does not satisfy the conditions of the problem since the corresponding value of $Y = 48 - 2x$ is negative. Hence, there remains only $x = 12$, from which we easily find $y = 24$ and $z = 48$.Thus, the first solution contains 12% alcohol, the second 24%, and the third 48% .

In many cases difficulties arise in the solution of the systems, particularly when finding the unknown requires a certain amount of guesswork or an artificial device. Such techniques often simplify computations or even suggest the only way to solve the problem. .

Illustration 3.

A tributary flows into a river. On the tributary, at a certain distance from the mouth of the tributary, lies point A. On the river, point B lies at the same distance from the mouth of the tributary. The time required for a motorboat to cover the distance from A to the mouth of the tributary and back stands in the ratio of 32 to 35 to the time required to go from B to the mouth of the tributary and back. If the rate of the motorboat were 2 km/hr more, then this ratio would be 15: 16, and if the rate of the motorboat were 2 km/hr less, then the ratio would be 7: 8. Find the rate of flow of the river (distances are measured along the tributary and river, respectively).

Let the rate of river flow be u km/hr, the rate of the motorboat in still water v km/hr and the rate of flow of the tributary, w km/hr. Furthermore, let the distance from A to the mouth of the tributary be equal to s km. Then the time the boat takes to cover the distance from A to the mouth of the tributary and back is $t_1 = s/(v+w) + s/(v-w) = 2sv/(v^2 - w^2)$ (hrs.).

Since the distance from B to the mouth of the tributary is also s km, the time required for the boat to go from B to the mouth of the tributary and back is $t_2 = s/(v+u) + s/(v-u) = 2sv/(v^2 - u^2)$ (hrs.). From the statement of the problem, $t_1 : t_2 = 32 : 35$ and we get the first equation

$$\frac{v^2-u^2}{v^2-w^2} = \frac{32}{35}$$

Two more equations are set up in similar fashion:

$$\frac{(v+2)^2-u^2}{(v+2)^2-w^2} = \frac{15}{16}$$

$$\frac{(v+2)^2-u^2}{(v+2)^2-w^2} = \frac{7}{8}$$

Simplifying we get the following system of equations :

$$\begin{aligned} 3v^2 &= 35u^2 - 32w^2 \\ (v+2)^2 &= 16u^2 - 15w^2 \\ (v-2)^2 &= 8u^2 - 7w^2 \end{aligned}$$

We have to find u. This is best done by first eliminating u, which is just the unknown we are seeking. Eliminating u, we obtain the system

$$\begin{aligned} 2(v-2)^2 - (v+2)^2 &= w^2 \\ 35(v-2)^2 - 24v^2 &= 11w^2 \end{aligned}$$

Now eliminating w, we get the equation $13(v-2)^2 + 11(v+2)^2 - 24v^2 = 0$, whence $v = 12$. Now it is easy to find $w = 2$ and, finally.

$u = 4$. We thus have the answer: the rate of flow of the river is equal to 4 km/hr.

The next problem involves a system of three linear equations in three unknowns. This would appear to be easy to solve, but at the examinations many aspirants failed because they got entangled in the manipulations with the *literal coefficients*. Problems involving literal data come up rather often at competitions and the aspirant should be able to handle them.

Illustration 4.

Two rivers flow into a lake. A steamship leaves port M on the first river, steams downstream to the lake then across the lake (still water) and up the second river

(upstream) to port N, and then makes the return trip. The ship has a speed of v in still water, the rate of flow of the first river is v_1 that of the second river v_2, the transit time between M and N is t, the distance from M to N is S. The return-trip time from N to M along the same route is equal to t. What distance does the steamship cover across the lake in one direction?

Denote by S_1 and S_2 respectively the distances from ports M and N to the lake, and by s the distance over the lake. By the statement of the problem, we have $S_1 + s + S_2 = 8$. It is then easy to see that the time required for the steamship to cover the distance from M to N is

$$\frac{s_1}{v+v_1}+\frac{s}{v}+\frac{s_2}{v-v_2}=t$$

The time required for the return trip is figured similarly. We thus obtain a system of three equations in three unknowns S_1, S_2, s :

$$\begin{aligned} s_1 + s + s_2 &= S \\ \frac{s_1}{v+v_1}+\frac{s}{v}+\frac{s_2}{v-v_2} &= t \qquad \ldots(2) \\ \frac{s_1}{v-v_1}+\frac{s}{v}+\frac{s_2}{v+v_2} &= t \end{aligned}$$

We are interested in s.

This system looks rather imposing, though actually there is nothing really complicated in it, particularly if we recall that v, v_1, v_2, S, t are given constants; it is quite obvious that the system (2) is a system of three equations in three unknowns, which of course can always be solved, say, by successive elimination of the unknowns.

However, it often happens that what is simple in theory turns out to be extremely awkward in practice. In this problem, such an approach would be unwieldy in the extreme, involving cumbersome manipulations merely because the coefficients of system (2) are rather complicated.

We will solve (2) in a somewhat artificial manner, but one which is shorter. The second equation of the system can be rewritten as

$$v^2s_1 - vv_2s_1 + v^2s + (v_1 - v_2)vs - v_1v_2s + v^2s_2 + vv_1s_2 = tv(v^2 + vv_1 - vv_2 - v_1v_2)$$

Replacing the sum $v^2s_1 + v^2s + v^2s_2$ in the left member by v^2S on the basis of the first equation, and collecting terms, we get the equation

$$v^2s + v[v_1s_2 - v_2s_1 + (v_1 - v_2)s] - v_1v_2s = tv(v^2 + vv_1 - vv_2 - v_1v_2) \qquad \ldots(3)$$

In the same way we can transform the third equation of our system. But we can save on manipulations if we notice that the third equation is very much like the second: simply replace s_1 by s_2 and v_1 by v_2 and *vice-versa*, and we have the second equation. In (3) replace s_1 by s_2 and v_1 by v_2 and vice versa to get the third transformed equation :

$$v^2S + v\,[v_2s_1 - v_1s_2 + (v_2 - v_1)s] - v_2v_1s = tv\,(v^2 + vv_2 - vv_1 - v_2v_1) \qquad \ldots(4)$$

Now adding (3) and (4) we have $2v^2S - 2v_1v_2s = tv(2v^2 - 2v_1v_2)$, whence follows the distance over the lake:

$$S = v\,\frac{vS - v^2t + v_1v_2t}{v_1v_2}\,vt + v^2\,\frac{S - vt}{v_1v_2} \qquad \ldots(5)$$

The solution of the problem is complete. Some aspirants, however, considered it necessary, after obtaining the answer with literal data [formula (5), say], to determine under what relationships of the data the answer has "real meaning" (such requirements are imposed as positivity of rates, distances, etc.; conditions are introduced that guarantee nonvanishing denominators, and the like). Quite naturally, a properly conducted investigation does not detract from the solution of a problem, but this investigation is not logically necessary, since it is usually taken for granted that the events actually took place and, hence, the literal data already satisfy the necessary relations. Quite naturally, such an investigation must be carried out if it is explicitly required by the conditions of the problem.

It rather often happens that in problems demanding the setting up of equations, the resulting system has second-degree homogeneous equations in two variables (equations of the type $ax^2 + bxy + cy^2 = 0$, where a, b and c are certain numbers). Unfortunately, however, few aspirants realize that the presence of homogeneous equations helps to solve a system of equations. A homogeneous equation of second degree in two unknowns directly defines the relationship of the unknowns, and this naturally simplifies subsequent computations. Let us examine a problem whose solution makes essential use of this fact.

Illustration 5.

An automobile leaves A for B and a motorcycle leaves B for A at the same time and with a smaller speed. After a time they meet, and at that instant another motorcycle leaves B for A and encounters the automobile at a point which is distant from the meeting point of the automobile and first motorcycle 2/9 of the distance from A to B. If the speed of the automobile were 20 km/hr less, the distance between the meeting points would be 72 km and the first meeting would have occurred 3 hours after the automobile left A. Find the distance between A and B (the motorcycles have the same speeds).

Let the speed of the automobile, be u km/hr, that of the motorcycle v km/hr, the distance AB, s km, and let t hours elapse before the automobile and first motorcycle meet. We readily set up the following system of equations :

$$\begin{aligned} tu + tv &= s \\ 3(u - 20) + 3v &= s \\ 3(u - 20) + 3v &= s \\ \frac{\frac{2}{9}s}{u} &= \frac{vt - \frac{2}{9}s}{v} \end{aligned}$$

$$\frac{72}{u-20} = \frac{3v-73}{v}$$

Eliminating the auxiliary unknown t and simplifying, we get the following system :

$$s = 3(u+v-20)$$
$$9uv = 2(u+v)^2$$
$$v(u-20) = 24(u+v-20)$$

To find s we have to find u and v from the last two equations. Noting that the second equation is a homogeneous second-degree equation in two variables, we can easily find the ratio $u : v$.

Since we are interested in u and v different from zero, then dividing the second equation by v^2, we get a quadratic equation in the new variable $z = u/v$:

$$2z^2 - 5z + 2 = 0$$

The roots of this equations $z_1 = 2$ and $z_2 = 1/2$, and so either $u = 2v$ or $u = v/2$.

But from the statement of the problem $u > v$, and so we take $u = 2v$.

Substituting this value of u into the third equation, we get either $v = 40$ or $v = 6$. But if $v = 6$, then $u = 12$, but it is given that $u > 20$. The problem is satisfied only by $v = 40$. But then $u = 80$ and $s = 300$. Therefore, the distance AB is found to be 300 km.

Almost insuperable difficulties stem from problems in which the aspirant finds, after correctly setting up the system of equations, that the number of unknowns is greater than the number of equations. The following problem illustrates this point.

Illustration 6.

Two boys with one bicycle between them set out from A in the direction at B, one by bicycle and the other on foot. At a certain distance from A the one riding the bicycle left it by the road and continued towards B on foot. The one who had started out on foot reached the bicycle and rode the rest of the distance. Both reached B at the same time. On the return trip from B to A, they did as before, but this time the cyclist rode one kilometer more than the first time and so his comrade arrived in A 21 minutes after he did. Find the rate of each of the boys on foot if they both did 20 km/hr cycling, and, on foot, the first takes 3 minutes less to cover each kilometer than the second.

Let us introduce the following notation :

s km for the distance between A and B;

v km/hr for the rate on the foot of the first boy;

w km/hr for the rate on the foot of the second boy;

a km for the distance that the first boy cycled from A to B (he thus first his bicycle a km from A and covered the rest of the distance to B on foot).

It is quite clear that the whole trip from A to B was made by the first boy $a/20 + (s-a)/v$ hours, by the second boy in $a/w + (s-a)/20$ hours. The fact that they set out at same time and arrived at the same time yield the first equation :

$$\frac{a}{20} + \frac{s-a}{v} = \frac{a}{w} + \frac{s-a}{20}$$

The information on the B to A trip enables us to set up the second equation in similar fashion :

$$\frac{a+1}{20} + \frac{s-a-1}{v} = \frac{a+1}{w} + \frac{s-a-1}{20} - \frac{7}{20}$$

(21 minutes = 7/20 hr).

Since the first boy spent $1/v$ hours per km and the second $1/w$ hours, we immediately get (from the statement of the problem) the third equation :

$$\frac{1}{w} - \frac{1}{v} = \frac{1}{20}$$

We now have a system of three equations in four unknowns. It is impossible to determine all the unknowns s, a, v, w from this system. In this sense, the system is indeterminate. But does this mean we are not able to solve the problems ? Of course not. This is because we only have to find two unknown quantities, the rates v and w, and those can be found uniquely from the system of equations.

To do this, subtract the first equation from the second to get $\frac{1}{w} + \frac{1}{v} = \frac{9}{20}$ and consider the result together with the third equation. Obvious computations yield $v = 5$ km/hr and $w = 4$ km/hr.

In this problem we were able to find the unknowns we needed despite the fact that there were fewer equations than unknowns. In the next problem we will obtain a system of equations without being able to determine any of the unknowns, though we will be able to determine the greater one, which is what is required in the problem.

Illustration 7.

A aspirant spends a certain sum of money on a booking, a fountain pen and a book. If the cost of the bookbag were less by a factor of 5, the pen by a factor of 2 and the book by a factor of 2.5, the overall cost would be 8 dinar. Now if, compared to the original cost the prices were reduced- twofold for the bookbag, fourfold for the pen, and threefold for the book-then the total outlay would be 12 dinar. How much money was spent and what item cost more, the book bag or the fountain pen ?

Suppose the bookbag cost x dinar, the pen y dinar and the book z dinar. Together they cost $x + y + z$, and that is what we wish to find out.

The first equation is set up on the condition that under the original assumption the outlay was 8 dinar :

$$\frac{x}{5} + \frac{y}{2} + \frac{z}{2 \cdot 5} = 8$$

Similarly we set up the second equation :

$$\frac{x}{2}+\frac{y}{4}+\frac{z}{3} = 12$$

We have a system of two equations in three unknowns and cannot of course determine all the unknowns, but we can find the total sum, which is what we need. To do this, rewrite the equations thus:

$$2x + 5y + 4z = 80$$
$$6x + 3y + 4z = 144 \quad \ldots(6)$$

Adding these two equations, we get the sum of the unknowns : $x + y + z = 28$. The total outlay comes 28 dinar, thus answering the first question.

Now let us try to determine which item, bookbag or fountain pen, is more expensive: in other words we want to know which of the inequalities $x > y$ or $y > x$ is valid.

If we subtract the first equation of (6) from the second, we obtain

$$2x - y = 36 \quad \ldots(7)$$

From this it is clear that $x > y/2$, because otherwise we would have $32 = 2x - y < 0$. But the inequality $x > y/2$ does not yet answer the question given in the problem. This is because we have not yet made full use of equation (7). Namely, we merely noticed that the difference $2x - y$ is positive. Now let us try to make use of the fact that it is equal to 32 and also take into account that $x + y + z = 28$ and that all the unknowns, x, y, z must, realistically, be positive numbers.

Rewrite (7) as $x + (x - y) = 32$. Since the total outlay is 28 dinar, then certainly $x < 28$ and from the latter equations $x - y > 0$ which states that the bookbag is more expensive than the fountain pen.

Nearly all the foregoing problems implicitly involve inequalities. In problem 5 for instance there were even two : $u > v$ and $u > 20$. The inequalities that appear in such problems do not ordinarily upset the aspirant; what does cause a lot of trouble is when the condition of the problem have to be written out explicitly as inequalities. Many aspirants get as far as writing down the system of equations and inequalities, but no farther. Apparently they are not ready psychologically to solve systems of this kind to illustrate, take the following problem.

Illustration 8.

A first train leaves A for C at 9 A.M. At same time two passenger trains leave B (located between A and C), the first with destination A, the second with destination C. The two passenger trains have the same speed. The fast train meets the first passenger train not later then 3 hours after departure, then passes B not earlier than 14:00 of the same day and, finally, arrives in C together with the second passenger train exactly 12 hours after meeting the first passenger train. Find the time of arrival at A of the first passenger train.

Let the speed of the fast train be v_1 km/hr, that of the passenger train, v_2 km/hr, and let the distance AB be s km. From the statement that the fast train meets the first passenger train not later than three hours after departure, we get

$$\frac{s}{v_1 + v_2} \leq 3$$

From the condition that the fast train arrived in B not earlier than 5 hours after departure, we obtain

$$\frac{s}{v_1} \geq 5$$

Since the time elapse prior to the first meeting is $s/(v_1 + v_2)$ hours, it follows that the fast train will overtake the second passenger train in 12+ $[s/(v_1 + v_2)]$ hours, and so

$$\left(12 + \frac{s}{v_1 + v_2}\right)(v_1 - v_2) = s$$

We have to find $x = s/v_2$, whence $s = xv_2$. Substituting this expression for s in the preceding equation and inequalities and denoting v_1/v_2 α we get the system

$$x \leq 3(\alpha + 1)$$
$$x \geq 5\alpha$$
$$x = 6(\alpha^2 - 1)$$

It was precisely this system that stumped many aspirants.

Actually it is not so complicated. It is necessary to eliminate either x or α and go over to a system of two inequalities in one unknown. Since it appears easier at first glance to eliminate x, let us do so. Putting $x = 6(\alpha^2 - 1)$ into the first two inequalities, we obtain the system of inequalities

$$2\alpha^2 - \alpha - 3 \leq 0$$
$$6\alpha^2 - 5\alpha - 6 \geq 0$$

The solutions of the first inequality are $-1 \leq \alpha \leq 3/2$, the solutions of the second are $\alpha \geq 3/2$, and $\alpha \leq -2/3$. Hence, the solution of the system is $\alpha = 3/2$, and also all α lying in the interval $-1 \leq \alpha \leq -2/3$. Since we are only interested in positive values of α, the condition of the problem is satisfied by the sole value α, the condition of the problem is satisfied by the sole value $\alpha = 3/2$. From this it is easy to find $x = 15/2$ and we get the answer : the first passenger train arrives at A at 16:30.

This problem allows for a solution in which all the conditions of the problem are written down in the form of equations. This is achieved by introducing supplementary unknowns and obtaining a system of equations in which the number of unknowns is greater than the number of equations. However, it is more difficult to solve that type of system of equations than it is to solve a system of inequalities.

Let us solve the problem the second way, retaining all the earlier notation. Let the fast train meet the first passenger train after an elapse of $(3 - t_1)$ hours $(t_1 \geq 0)$, let it pass B in $(5 + t_2)$ hours $(t_2 \geq 0)$ and let it catch up with the second passenger train in $[(3 - t_1) + 12]$ hours. Then we can easily set up the equations

$$(v_1 + v_2)(3 - t_1) = s$$
$$v_1(5 + t_2) = s$$
$$(15 - t_1)(v_1 - v_2) = s$$
$$xv_2 = s$$

Eliminating s and denoting v_1/v_2 by α, we get the system of equations

$$(\alpha + 1)(3 - t_1) = x$$
$$\alpha(5 + t_2) = s \quad \ldots(8)$$
$$(\alpha + 1)(15 - t_1) = x$$

This is a system of three equations in four unknowns. We only want x. proceeding as before, eliminate x to get

$$\alpha t_2 + (\alpha + 1) t_1 = 3 - 2\alpha$$
$$(1 - \alpha)t_1 - \alpha t_2 = 15 - 10\alpha \quad \ldots(9)$$

Noting that the right member of the second equation is five times the right member of the first, multiply the first by 5 and subtract the second to get

$$6\alpha t_2 + (6\alpha + 4) t_1 = 0 \quad \ldots(10)$$

Since $\alpha > 0$, $t_1 \geq 0$, $t_2 \geq 0$, it follows that this equation can only be valid for $t_1 = 0$ and $t_2 = 0$. But then from (9) it is easy to find $\alpha = 3/2$ and from (8), $x = 15/2$. We get the same answer. Many aspirants failed to notice that (10) can be derived from (9) and therefore they could not conclude, from (9), that $t_1 = t_2 = 0$ and so could not solve the problem.

The foregoing shows that the first method of solution is much easier than the second method.

Illustration 9.

A cyclist starts out from city A at 9 A. M. and proceeds at a constant rate of 12 km/hr. Two hours later, a motorcyclist starts out with an initial speed of 22 km/hr and proceeds with uniformly decelerated motion so that in one hour the speed diminishes by 2 km/hr. A man in a car driving to A at 50 km/hr meets first the motorcyclist and then the cyclist. Will the car driver be able to reach A by 19: 00 that same day ?

This problem can likewise be solved by setting up a system of equations and inequalities, but it would require extended reasoning. It is best to approach the problem by simple reasoning instead of attempting to solve it by formally setting up of a system of equations and inequalities.

If follows from the statement of the problem that first the motorcyclist catches up with the cyclist and then the cyclist catches up with the motorcyclist. Let the cyclist spend t hours prior to a meeting (first or second, no matter which). Then the motorcyclist will spend $(t - 2)$ hours over the same distance. Since they cover the same distance before meeting, we can equate their paths to obtain.

$$12t = 22(t - 2) - 2\frac{(t - 2)^2}{2}$$

Solving this equation, we see that up to the first meeting the cyclist rode 6 hours and thus covered 72 km; prior to the second meeting he rode 8 hours and hence covered 96 km. It is given that the car driver met the cyclist before the latter had covered 96 km, which means the motorist has less than 96 km left to drive to A. He will spend less than 96/50 hours. Since the cyclist will spend less than 8 hours before meeting the car, their meeting will occur earlier than 17:00. Thus, after meeting the cyclist there remain over two hours for the car driver to reach *A* before 19:00 hours. But he requires less than 96/50 hours, that is less than two hours, to cover this distance. And so the motorist will reach A before 19:00.

At competitions, problems appear in which the aspirant is asked to find an optimal solution, say, to buy the largest possible quantity of goods for a given sum of money or to choose the best possible (cheapest) variant in transportation of goods, and the like.

Solutions of this type of problem may consist of setting up systems of equations and inequalities and their solution. However, the most necessary element in such cases is the reasoning that helps to choose the best possible variant.

Illustration 10.

The task is to construct a number of identical dwelling houses with a total floor-space of 40,000 sq. metres. The cost of one house of N sq. metres (m^2) of dwelling floor-space consists of the cost of the above ground portion (superstruction), which is proportional to $N\sqrt{N}$ and the cost of the foundation, which is proportional to $\sqrt{N}$. The cost of a house of 1600 m^2 is set at 176,800 dinar, in which case the cost of the above ground portion is 36 % of that of the foundation. Determine how many houses should be put up so that the overall cost is the smallest possible and find that sum.

Suppose we decide to construct n identical structures each of which y m^2 of dwelling floor-space. Then the equation $yn = 40{,}000$ is valid. Let the cost of one structure of y square metres of floor-space be z thousand dinar; then the cost x of the construction job as a whole is computed from the equation $x = zn$.

The cost of a house consists of the cost, v, of the above-ground portion of the structure and the cost, w, of the foundation: thus, $z = v + w$.

It is given that the cost of the above-ground portion of a house of y squares metres is proportional to $y\sqrt{y}$, that is, $v = \alpha y\sqrt{y}$, where α is a certain coefficient. Similarly, $w = \beta\sqrt{y}$ where β is a certain coefficient.

In the particular case of the construction of a house of 1600 square metres, taking into account that the cost of the superstructure is 36% of the cost of the foundation, we get $\alpha \cdot 1600 \cdot \sqrt{1600} = \frac{36}{100} \cdot \left(\beta \cdot \sqrt{1600}\right)$ and taking into consideration that the construction of a house of 1600 square metres costs 176,800 dinar, we have

$$176{,}800 = \alpha \cdot 1600\sqrt{1600} + \beta\sqrt{1600}$$

All he conditions of the problem have been written down, we now have to determine x as a function of n and then determine for which n, will be a minimum.

It is easy to find α and β from the last two equations: $\alpha = 117/160000$, $\beta = 13/4$. Substituting v and w into the expression for z, we get $z = (117/160000)\, y\sqrt{y} + (13/4)\sqrt{y}$. Now putting this value of z and the value y=40,000/n from the first equation into the second,

we get

$$X = 650\left(\frac{9}{\sqrt{n}} + \sqrt{n}\right)$$

We, thus conclude that x, the cost of construction, is the above function of n, the number of houses. We now have to determine the smallest value of x. Applying to the right member of this equation between the arithmetic mean and the geometric mean, we find $X \geq 2 \cdot 650\sqrt{9} = 3900$ equality being attained only when $8/\sqrt{n} = \sqrt{n}$, that is, for $n = 9$. In other words, the cost of construction will always be at least 3.9 million dinar and will be exactly equal to this figure only when $n = 9$.

Thus, in this housing construction project, the smallest expenditure will be in building 9 houses and the total cost in that case will be 3·9 million dinar.

Illustration 11.

It is agreed to spend 100 dinar on Christmas tree decorations. Such decorations are sold in sets consisting of 20 items per set costing 4 dinar a set, sets of 35 items at 6 dinar, and sets of 50 items at 9 dinar. How many of what kinds of sets should be bought in order to have the largest number of items ?

Let x, y, z be the number of sets, respectively, of the first, second and third kind which must be bought to ensure a maximum quantity of items (this solution is termed the optimal solution of the problem) Then

$$4x + 6y + 9z = 100$$

This is the only equation that can be set up on the basis of the statement of the problem. But we also know that x, y and z are nonnegative integers and that the number of items of decoration in this purchase is greater than any other. These conditions, it turns out, are quite is greater sufficient to ensure an unambiguous determination of all unknowns. The first idea is to solve the equation by running through all possible values of the unknowns but this is rather hopeless due to the enormous number of cases.

However, this number may be appreciably reduced with the aid of arguments of an economic nature. Indeed, 12 dinar can buy 3 sets of the first type or 2 sets of the second; in the former case we get 60 items, in the latter 70. It is then clear that in an optimal solution the number of sets of the first type should not exceed 2. Comparing sets of the second and third type in similar fashion, we find that in the optimal solution there should not be more than one set of the third kind. We have thus obtained the inequalities $x \leq 2$, $z \leq 1$.

Now there are fewer cases and we can run through them. For $x = 0$ we get the equation $6y + 9z = 100$ to determine y and z. It clearly does not have a solution since the left member is divisible by 3 but the Right is not. Furthermore, when $x = 1$ we get the equation $2y + 3z = 32$, which [taking into account the inequality $z \leq 1$) has the unique solution $y = 16$. $z = 0$. Finally, when $x = 2$, there is no solution, as in the case of $x = 0$.

Thus, to ensure the largest number of decorations, we have to buy 1 set of 20 items and 16 sets of 35 items.

It is possible in this solution to dispense with examining all cases if a more detailed use is made of divisibility, as witness: from the given equation it follows that the number x yields a remainder of 1 upon division by 3, and the number z is even. It therefore follows immediately from the inequalities $x \leq 2$ and $z \leq 1$ that $x = 1$ and $z = 0$; from the equation we get $y = 16$.

Note in conclusion that in actuality the foregoing reasoning signifies that the optimality condition of the solution may be written in the form of the following system of equations and inequalities :

$$4x + 6y + 9z = 100,$$

$$0 \leq x \leq 2, 0 \leq y, 0 \leq z \leq 1 \qquad \ldots(11)$$

With the supplementary provison that x, y and z are integers. Now the condition of x, y and z being integers means that $z = 2n$ and $x = 1 + 3k$, where n and k are also integers. Substituting these values of z and x into the appropriate equations, we get $n = k = 0$, or $x = 1$ and $z = 0$. It is now easy to find $y = 16$ from equation (11).

Illustration 12.

A forestry has to deliver 1590 trees. The vehicles assigned to this job are $1\frac{1}{2}$ – ton, three-ton and five-ton trucks. A$1\frac{1}{2}$ – ton truck carries 26 trees at a time, a three-ton truck carries 45 trees, and a five-ton truck, 75 trees. The cost of one run of a $1\frac{1}{2}$ – ton truck is 9 dinar, that of a three-ton truck, 15 dinar, and of a five-ton truck, 24 dinar. The forestry wishes to minimize the overall cost of the deliveries. How is this to be done (it is assumed all trucks are fully loaded) ?

Let x, y, z be the number of $1\frac{1}{2}$-ton, three-ton, and five-ton trucks, respectively, in the case of optimal distribution. Since all vehicles are fully loaded, the number of transported trees in this setup will be $26x + 45y + 75z$ and thus we get the equation $26x + 45y + 75z = 1590$.

We are now in the same position as in Problem 11, but attempts to reduce the number of cases that succeeded then do not yield any substantial simplification. For example, x is divisible by 15. That's about all. We might

add that 45 $1\frac{1}{2}$-ton trucks would cost 405 dinar a trip, 26 three-ton trucks carrying the same number of trees would cost 390 dinar so that in an optimal solution the number of $1\frac{1}{2}$-ton trucks should not exceed 44. And so for x we get three possibilities : $x = 0$, $x = 15$, $x = 30$. For each of these values we would have to solve the equation for y and z, which also have a good many solutions.

This is a very tedious approach, though in the absence of any other, it is acceptable.

Here's an attractive idea, which, unfortunately, doesn't do the job. From the statement of the problem it is easy to figure out that for 45 dinar using 5 $1\frac{1}{2}$-ton trucks we can deliver 130 trees and using 3 three-ton trucks, 135 trees. It would therefore appear that the number of $1\frac{1}{2}$-ton trucks should not exceed 4, otherwise those same trees could be delivered more cheaply. From this and from earlier considerations it follows that $x = 0$ and the number of cases left to be examined is much less.

Actually, however, this argument only implies that for a given sum of money we can deliver a larger number of trees, but our aim is to deliver a given number of trees at minimum cost. Nevertheless, it is still possible to get around brute-force tactics by just a little commonsense reasoning.

Any reasonable person would first estimate which of the given types of trucks is the most efficient by determining the cost of delivering a single tree. We find the following : for a $1\frac{1}{2}$-ton truck, a three-tonner and a five-tonner the cost is 9/26, 1/3 and 8/25 dinar respectively. Since $9/26 > 1/3 > 8/25$, it is clear that it is more profitable to use five-ton trucks first, then, if necessary, three-tonners and, finally, $1\frac{1}{2}$-ton trucks.

It is easy to see that the greatest number of trees that can be delivered by five-ton trucks comes out to 1575. However, seeing that all vehicles must be fully loaded, we get 1500 trees for delivery by five-ton trucks. Then 90 trees can be delivered by three-tonners and so it is natural to suppose that the optimal distribution will be 20 five-ton and 2 three-ton trucks.

It is easy to demonstrate that this plan is indeed an optimal plan : if we reduce the number of five-tonners, then the "undelivered" trees apportioned to these vehicles would have to be delivered by $1\frac{1}{2}$-ton and three-ton trucks, but delivery costs per tree on these trucks are higher, and so the total cost or the assignment would increase.

We thus have the optimal distribution of 20 five-ton and 2 three-ton trucks, while all the unknowns and the single equation that was set up remain unused! To summarize, then, we set out in the ordinary way, but in the process of solution we found an approach which made all the earlier arguments unnecessary. Quite obviously the method given here would be quite sufficient at any competition.

SOLVING EQUATION

At competitions a rather strange situation would appear to develop around equation. These problems are ordinarily not considered difficult and most aspirants handle them fairly well. Yet many serious mistakes are made.

This situation is strange only at first glance. The point is that very often there is a big gap between computational techniques and a conscious grasp of the logical foundations that underlie them.

Most aspirants can of course simplify an equation by means of clear-cut manipulations, but by far not every aspirant is capable of realizing that a solution has been lost or acquired, and many don't even give thought to such things still others may know certain parts of the theory pertaining to these matters but the knowledge is only formal and such aspirants are often completely helpless in a slightly altered situation.

Let us say that the aspirant is quite familiar with fact that squaring both sides of an irrational equation can give rise to extraneous roots. Yet time and again aspirants square trigonometric equations and then fail to discard extraneous roots! This mistake would not be made if the aspirant realized **why** squaring leads to the introduction of extraneous roots.

Or take checking. There seem to be two opposing opinions among aspirants here. Some regard checking as a whim of the teacher, some thing that simply has to be done in order to pass while others regard checking as necessary in all cases without exception. They even go on to check the roots of quadratic equations. Both views are based on a total misconception of what checking really is and what place it occupies in problem solving.

In short, the aspirant should have a firm grasp of the fundamentals of the theory that is needed in the solution of equations. Let us examine this minimum of theoretical knowledge.

First some definitions.

1. The domain of the variable of an equation is the set of values of the variable (unknown) for which its left and right members are meaningful (defined); it is thus the set of all eligible replacement for a variable in an equation.

2. A number is a solution (root) of an equation if when substituted for the unknown (variable) makes the equation a true statement (converts it into a true numerical equation).

According to this definition, the solution set (all the solutions) of an equation is a subset (a part) of the domain

of the variable, otherwise substitution in place of the unknown would not yield a true statement and would be meaningless.

3. To solve an equation means to find all the roots or prove that there are no roots.

4. If all the roots (solution set) of one equation are the roots of another equation, then the latter equation is a consequences of the former.

5. Two equations are termed equivalent if each is a consequence of the other. From this definition it follows immediately that equivalent equations have the same solution sets.

6. Two equations are equivalent on some set of values of the variable (unknown) if they have exactly the same solution belonging to this set.

Let us illustrate these concepts with two examples.

This domain of x in the equation $x - 3 = \sqrt{3}$ consists, according to the definition, of all x for which the left member $x - 3$ and the right member $\sqrt{x}$ are meaningful. Clearly, the left member is defined for any x and the right member for $x \geq 0$. Therefore the domain of x in this equation consists of $x \geq 0$.

Yet many aspirants erroneously state that the domain of the variable is $x \geq 3$, since "for $x < 3$ the left member is negative and the right member cannot be negative." The quoted part of the statement is true and it is used in the solution of the given equation. It shows that the roots of the equation are not less than 3. But it does not follow there from that all permissible values are less then 3, because not all permissible (eligible) values are roots ! Consider the two equations.

$\log_6 (x - 2) + \log_6 (x + 3) = 2$ and $\log_6 (x - 2)(x + 3) = 2$

Obviously every root of the former equation is a root of the latter one, so that the latter equation is consequence of the former the latter equation can readily be solved to yield the solution set (roots) $x_1 = 6$ and $x_2 = -7$. The root x_2 does not satisfy the first equation, it is not even in the domain of the variable. Thus, the two given equations are not equivalent, but they are equivalent in the domain of x of the first equation (in this domain they have the one root $x = 6$).

It is easy to see why this is so. The domain of x in the first equation consist of $x > 2$,while the domain of x in the second equation is broader, including these x and also $x < -3$. It is therefore natural that in passing from the first equation to the second an extraneous root $x = -7$ appeared that does not belong in the domain of the variable of the first equation.

How do these newly introduced concepts operate in the solution of equations? The point is that in most cases a solution is obtained after a long chain of manipulations and transformations from one equation to the next. Thus, in the solution process, each equation is replaced by a new one, and quite naturally the new equation can have a new solution set (new roots). The prime task in a correct solution of any equation is to follow the variations in the solution set and not to allow for any loss of roots or any failure to discard extraneous roots.

The best method is clearly , each time to replace the given equation with an equivalent equation. Then the roots (solution set) of the last equation will be the roots (solution set) of the original equation. In practice however this ideal version is rare. As a rule, an equation is replaced by a consequence that is not equivalent ; then, by the definition of consequence, all the roots of the first equation are the roots of the second, that is to say there is no loss of roots, but extraneous roots may appear (on the other hand, they may not). And when in the process of manipulations the equation is replaced by a nonequivalent consequence, the roots have to be investigated. This is a check and it is necessary. Note here and now that, as we shall see later on, this investigation does not at all mean that we have to substitute the roots obtained into the original equation.

To summarize then, if a solution is carried through without an investigation of equivalence and source of extraneous roots, then verification is an necessary part of the solution without which it cannot be regarded as complete even if no extraneous roots appeared in actual fact. Of course if they did appear and were not discarded, the solution is simply incorrect. On the other hand, if each time the equation was replaced by an equivalent equation (which, incidentally, is an extremely rare case), and this fact is stipulated at each stage in the solution, then no verification is required. We thus see that the notion of checking plays a very definite and extremely essential role in the solution of equation and does not by any means merely reduce to a simple checking through of computations.

As for checking computations, that is up to the aspirant. He may do it or he may not according to how carefully he feels the computation have been carried through. It is of course always best to check oneself at an examination, but this should be done on a separate sheet of paper and there is no need to include it in the solution.

It must be stressed that it is not permissible to replace a given equation by one which is not a consequence of the first, because then there is a root of the first equation which is not a root of the second, and so solving the second equation does not yield all the roots of the first. A root will be lost for good. That is the essential difference between loss of roots and the introduction of extraneous roots.

Such is the theory. In practical situation, one has to know the specific sources of introducing or losing roots. In the main, these sources are of two types: the so-called "identity transformations" and the performance of operations such as raising to a power, taking logarithms, antilogarithms, etc. in both members of an equation.

At first glance," identity transformations" are quite harmless, but actually they often lead to nonequivalent equations since they change the domain of the variable (unknown). Say, if in the solution of an irrational equation we replace $\left(\sqrt{2x+1}\right)^2$ by $2x + 1$, we immediately extend the domain of the variable since $2x + 1$ is meaningful for all x, while $\left(\sqrt{2x+1}\right)^2$ is valid only for $x \geq -1/2$. The same goes for the example that we analyzed earlier; the use of the formula for the logarithm of a product led to an extension of the domain and, as a result, to the introduction of an extraneous root.

There is nothing strange in this : it is simply that most formulas used in transformations are such that their left and right members are meaningful for different values of the letters used. Such, for example, are the formulas

$$\begin{aligned}\sqrt{ab} &= \sqrt{a}\sqrt{b}, \left(\sqrt{x}\right)^2 \\ &= x, \log_a xy = \log_a x + \log_a y \\ \log_a x_n &= n \log_a x, \\ a^{\log ab} &= b, \cot x = \frac{1}{\tan x}, \sin 2x \\ &= \frac{2 \tan x}{1 + \tan^2 x}, \\ \tan(x+y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y}\end{aligned}$$

Therefore, replacing one part of a formula by another leads to an extension or narrowing of the domain of the variable. Extending the domain opens the way to acquiring extraneous roots, while narrowing (restricting) the domain makes possible the loss of roots, so that narrowing is never permissible. As for extraneous roots, if they are acquired through the extension of the domain, then it is not necessary to substitute them directly in the original equation in order to separate them from the roots of the original equation; it suffices to check to see whether they are in the domain of the variable. If they are not, discard them, if they are, leave them.

This fact is of exceptional importance in the solution of equations and so we will dwell on it in more detail.

A. If in the process of transforming an equation, extraneous roots appear only due to extending the domain of the variable, then those roots (and only those) which appear in the domain will be the roots of the original equation.

This rule relieves us of the necessity to substitute the roots found into the equation the purely mechanical checking of numerical equations, which at times is exceedingly difficult or even sometimes completely impossible due to the fact that there are an infinite of numbers to be verified.

Thus, in place of direct substitution we can employ a test for membership in the domain, but only when the source of extraneous roots is extension of the domain of the variable, this case and no other. As to talking function of both members of an equation, we consider only two of the more important cases: squaring and the taking of antilogarithms.

It often happens (especially in the solution of irrational equations) that one has to pass from a certain equation $f(x) = g(x)$ to the equation $\{f(x)\}^2 = [g(x)]^2$. What happens to the roots in this transition ? First of all it is clear that the second equation is a consequence of the first: if the number a is a root of the first equation, that is $f(a) = g(a)$, then $\{f(a)\}^2 = [g(a)]^2$ and a is a root of the second equation. But, generally speaking, the converse is not true: the second equation is satisfied also by the roots are not lost but extraneous roots may appear.

A very important practical consequence follows from this statement.

B. If both members of an equation are non-negative on some set of values of the argument, then, upon squaring, we obtain an equation that is equivalent to the original one on that set.

Indeed, in this case the "extraneous" equation clearly has no roots, except for those for which both sides vanish, but such are not extraneous for our equation. How this is done practically will be demonstrated in concrete examples below.

Similarly we consider taking antilogarithms in equations, that is, passing from the equation $\text{Log}_c f(x) = \log_c g(x)$ to the equation $f(x) = g(x)$. Let a be a root of the original equation, that is $\text{Log}_c f(a) = \log_c g(a)$ then $c^{\text{Log}_0 f(a)} = c^{\text{Log}_0 g(a)}$, or $f(a) = g(a)$. Hence, any root of the original equating is a root of the second equation. On the other hand the domain of the variable of the second equation is greater then that of the first and so it is natural to expect extraneous roots, but they will be due precisely to the extension of the domain. Hence, it suffices to find the roots of the second equation and test them for membership in the domain of the first equation.

Such is the theoretical foundation that he aspirant needs. It is also worth stressing that it is not always advisable to employ the whole theory; moderation is the best advice and one should strive for the simplest solution. If, say, in the solving process it becomes apparent that a simple verification of the resulting roots is not difficult, then there is no reason to seek out the sources of introducing roots or take an interest in the variation of the domain in the solution process, or even to find the domain at all. But if this verification is complicated, then theoretical reasoning can save the day: at the proper time (and in the final version) the aspirant should investigate the transformation that might lead to extraneous roots.

At the same time, in any solution we must make sure that no loss of roots occurs. It is useful to state this explicitly, particularly if the transformation employed is sufficiently complicated.

Below will illustrate the more typical cases and also the most insidious sources (though not all of them, naturally) that give rise to extraneous roots. They include the formulas for transforming radicals the fundamental logarithmic identity, and the formulas for taking logarithms of a product and a power, clearing of fraction, the cancellation of similar terms, the replacement of an equation by a collection of simpler equations, and certain "verbal" arguments. We will then consider some sources of loss of roots. Some of the last and more involved problems will be analyzed with the aim of indicating certain difficulties of a different nature, not connected with the acquisition or loss of roots

Illustration 13.

Solve the equation $\sqrt{2x-6}+\sqrt{x+4}=5$.

Squaring both sides and using the formulas for transforming radicals, we get the equation

$$2x-6+2\sqrt{(2x-6)(x+4)}+x+4=25$$

Or $2\sqrt{2x^2+2x-24}=27-3x$ ...(1)

Again squaring and getting rid of the radicals we arrive at the equation $x^2-170x+825=0$ whose roots are $x_1=5$ and $x_2=165$. Direct substitution of these values in the original equation shows that x_1 is a root and x_2 is not.

Illustration 14.

Solve the equation :

$$\sqrt{5x+7}-\sqrt{3x+1}=\sqrt{x+3}$$

Squaring both members and manipulating we get

$$2\sqrt{(5x+7)(3x+1)}=7x+5$$

Whence, again squaring we obtain the quadratic equation $11x^2+34x+3=0$, whose roots are $x_1=-1/11$ and $x_2=-3$ direct verification shows that $x=-1/11$ is the root of the original equation. Checking the value $x=-3$ many aspirant at the examination got the equation $\sqrt{-8}-\sqrt{-8}=0$. They considered this statement to be true on the ground that the left member is a case of "equals subtracted from equals". Thus the value $x=-3$ proved to be a root of the original equation. But this argument is baseless since the expression $\sqrt{-8}$ is devoid of meaning: as we know, irrational equations are only considered in the domain of real numbers, and the symbol $\sqrt{a}$ is used for real a only to denote the principal square root of a non-negative number a. Therefore, the value $x=-3$ does not lie in the domain of the variable and hence, is not a root of the original equation.

The situation is quite different in the problem which now follows. Here, checking the "bad" roots is very involved and the simplest approach to a solution is by applying the theory we have developed when the sources of extraneous roots are taken into account in the very process of solution.

Illustration 15.

Solve the equation $\sqrt{x+3}+\sqrt{2x-1}=4$.

Both members of this equation are non-negative in the domain of the variable, and so after squaring we obtain an equation which, according to statement B is equivalent to the original one in the domain of the variable :

$$\left(\sqrt{x+3}\right)^2+2\sqrt{x+3}\sqrt{2x-1}+\left(\sqrt{2x-1}\right)^2=16$$

Using the formulas for transforming radicals, which clearly extend the domain, we get the equation

$2\sqrt{2x^2+5x-3}=14-3x$ ...(2)

In these transforming, extraneous roots could appear only due to the extension of the domain of the variable.

Then we reason as follows. The left member of (2) is non-negative for every (permissible) value of x; but the right member is negative for $x>14/3$. It is quite obvious that these values of x cannot be solutions of the equation, and so from now on we will consider equation (2) only in the domain $x\le 14/3$. But in this domain both member of (2) are non-negative (for the permissible values of x with respect to (2), naturally) and according to the statement B, squaring yields an equation that is equivalent to (2) on the set $x\le 14/3$:

$$\left(2\sqrt{2x^2+5x-3}\right)^2-(14-3x)^2$$

From this, once more extending the domain of the variable, we get the quadratic equation $x^2-104x+208=0$ whose roots are $x_{1,2}=52\pm 8\sqrt{39}$. Both these roots, as will readily be seen, lie in the domain of the variable of the original equation and for this reason we have only to check that they satisfy the condition $x\le 14/3$: It is easy to compute that $x_1>14/3$ and $x_2\le 14/3$ so that x_2 is the only root of equation (2) and consequently, of the original equation.

We once again stress that one should resort to this kind of detailed, "theoretical", solution only in case of necessity, only when working through the rough draft as the aspirant sees that the roots are "bad" which is to say that a direct substitution into the equation leads to a rather complicated problem: the proof or disproof of the equations

$$\sqrt{55+8\sqrt{39}}+\sqrt{103+16\sqrt{39}}=4,$$

$$\sqrt{55+8\sqrt{39}}+\sqrt{103-16\sqrt{39}}=4,$$

Incidentally, the first of these equations is clearly not true. The second one can easily be proved if one knows the formula for transforming expressions of the form $\sqrt{A\pm\sqrt{B}}$. The alternative approach of squaring involves considerable computational difficulties. It is quite clear that both these methods are more complicated than the one we gave, where all we had to do was to test the roots x_1 and x_2 for their validity under the condition $x\le 14/3$. Nevertheless, in this problem it is still possible to overcome the difficulties of direct verification and avoid application of the theory.

However, in equations containing a parameter, direct verification is appreciably more difficult and practically the only way out is to employ the theory.

Illustration 16.

Solve the equation $x - 1 = \sqrt{a - x^2}$.

The right side is non-negative for all (permissible) x, and the left side is non-negative for $x \geq 1$. Therefore, the given equation is, in the domain $x \geq 1$, equivalent to the equation $(x - 1)^2 = \left(\sqrt{a - x^2}\right)$ which can be reduced to

$$2x^2 - 2x + 1 - a = 0 \quad \ldots(3)$$

(in the process, the domain of the variable was extended and we will finally have to check the resulting roots to see if they lie in the domain). And so we have to solve equation (3) and choose the roots for which $x \geq 1$ and $a - x^2 \geq 0$. The discriminate of this equation is equal to $2a - 1$, so that for $a < 1/2$ it does not have any real roots; all the more so, the original equation has no roots for these values of a.

Now we assume that $a \geq 1/2$; the roots of (3) are $x_{1,2} = \left(1 \pm \sqrt{2a - 1}\right)/2$. The root x_2 clearly does not satisfy the condition $x \geq 1$ and so is not a root of the original equation. In order to find out about x_1 we have to solve the inequality $\left(1 \pm \sqrt{2a - 1}\right)/2. \geq 1$ or $\sqrt{2a - 1} \geq 1$; It clearly holds true $a \geq 1$. And so for a<1the original equation does not have any roots, but for $a \geq 1$ we still have to verify the validity of the inequality $a - x_1^2 \geq 0$ which is equivalent to the inequality $a \geq\geq \sqrt{2a - 1}$. Both member of this inequality are nonnegative (we are considering $a \geq 1$) and they can be squared, yielding $a^2 \geq 2a - 1$ or $a^2 - 2a + 1 \geq 0$, which is valid for all values of a.

And so for a < 1 the original equation has no roots, but for $a \geq 1$ it has the root $x = \left(1 \pm \sqrt{2a - 1}\right)/2$.

Note that the verification of the last condition $a - x^2 \geq 0$, which logically speaking is obligatory, can be conducted without any computations at all. Indeed, x_1 and x_2 have been obtained as roots of the equation $(x - 1)^2 = a - x^2$ and hence for $x = x_1$ and $x = x^2$ the right side is non-negative.

We once again stress the fact that a direct substitution as a check of the roots would have reduced to equations in a :

$$\frac{a \pm \sqrt{2a - 1}}{2} - \sqrt{\frac{a \mp \sqrt{2a - 1}}{2}} = 1$$

the outward aspect alone of which is quite saddening. Thus, without a conscious mastering of the approach given here to the solution of equations, such problems can cause great difficulties.

One of the most common sources of extraneous roots is the use of various logarithmic formulas, in particular, the formula for taking logarithms of a product. Indeed, replacing $\log_a f(x) + \log_a g(x)$ by $\log_a f(x)\, g(x)$, we extend the domain of the variable, permitting values of the unknown x for which we simultaneously have $f(x) < 0$ and $g(x) < 0$. And so extraneous roots can appear, but only due to extension of the domain of the variable, so that to discard them, on the basis of statement A, it suffices to verify their membership in the domain. Also note that the converse replacement- the logarithm of a product by the sum of the logarithms- can lead to a narrowing of the domain of the variable and so is not permissible.

Illustration 17.

Solve the equation $\log_2 (x + 2) + \log_2 (3x - 4) = 4$.

Passing to the logarithms of a product we get $\log_2 (x + 2)(3x - 4) = 4$ whence $(x + 2)(3x - 4) = 16$ the roots of this equation are $x_{1,2} = (-1 \pm \sqrt{73})/3$. It is easy to see that only x_1 lies in the required domain of the original equation and, on the basis of statement A, is its root.

A direct substitution of the "bad" root x_1 would not have required very cumbersome computations but there would unpleasant enough "irrational-logarithmic" manipulations, whereas the employment of statement A yielded the answer at once.

The appearance of extraneous roots as a result of applying the fundamental logarithmic identity ordinarily surprises the aspirant, although there is actually nothing strange here. It is due to the extension of the domain when replacing the expression $a^{\log ab}$ by b if a or b contain the unknown.

Illustration 18.

Solve the equation $x \log_{\sqrt{x}} 2x = 4$.

Replacing $\log_{\sqrt{x}} 2x$ by $\log_x (2x)^2$, we get ${}_x\log x^{(2x)^2} = 4$.

Now employing the fundamental identity, we get $(2x)^2 = 4$, which means $x_1 = -1$, $x_2 = 1$. But neither x_1 nor x_2 lie in the domain of the variable of the original equation: $x_1 < 0$, and $\sqrt{x_2} = 1$ cannot be a logarithmic base. Hence, the given equation does not have any roots.

The appearance of extraneous roots may not be so noticeable as in the problems given above. As a rule, this is due to the fact that the reasoning and computations employed lead to an extension of the domain of the variable. In the next problem, extraneous roots appear in a mutual canceling of like terms. Again there is no cause for surprise: in canceling, we remove the restriction that the eliminated terms must be meaningful and thus extend the domain of the variable.

Illustration 19.

Solve the equation

$\log_{10} \sqrt{1 + x} + 3 \log_{10} \sqrt{1 - x} = \log_{10} \sqrt{1 - x^2} + 2$. we transform $\log_{10} \sqrt{1 - x^2}$

$\log_{10} \sqrt{1 - x^2} = \log_{10} \sqrt{1 + x}\,\sqrt{1 - x} = \log_{10} \sqrt{1 - x} + \log_{10} \sqrt{1 + x}$

It is easy to see that this manipulation does not change the domain of the variable, and the transformed equation $\log_{10}\sqrt{1+x} + 3\log_{10}\sqrt{1-x} = \log_{10}\sqrt{1-x} + \log_{10}\sqrt{1+x} + 2$ is equivalent to the given one. Eliminating $\log_{10}\sqrt{1+x}$ in both members, we obtain the equation $2\log_{10}\sqrt{1-x} = 2$ whose domain consists of the number $x < 1$, which, as is evident, is greater than that of the original equation. We should thus expect extraneous roots. Solving the last equation we get the root $x = -99$, which does not lie in the domain of the original equation and therefore is not its root. Thus, the given equation does not have any roots.

One of the sources of mistakes is the (explicit or implicit) clearing of fractions. But this causes an extension of the domain of the variable-those values of x are included for which the denominator is equal to 0.

Illustration 20.

Solve the equation,

$$\frac{1}{\log_6(3+x)} + \frac{2\log_{0\cdot 25}(4-x)}{\log_2(3+x)} = 1$$

Taking all logarithms to the base 2 and manipulating, we get the equivalent equation

$$\frac{\log_2 6 - \log_2(4-x)}{\log_2(3+x)} = 1 \qquad \ldots(4)$$

Whence $\log_2 6 - \log_2(4-x) = \log_2(3+x)$ and then

$$\frac{6}{4-x} = 3+x$$

This last equation is reduced to a quadratic equation and its roots are found to be : $x_1 = 3, x_2 = -2$.

During the solution process, extraneous roots could have appeared only due to an extension of the domain of the variable because of clearing of fractions in equation (4) and (5). It is therefore sufficient to test the resulting roots for membership in the domain of the variable of the original equation. We thus find that x_2 does not lie in the domain, but x_1 does and hence, it is a root of the original equation.

Mistake that occur in solving equations in which the left member is a fraction and the right member is zero are due to this disregard for the domain of the variable. Frequently, the aspirant simply discards the denominator in such cases and equates the numerator to zero. For a correct solution, one should equate the numerator to zero, find the roots of the resulting equation and discard those for which the denominator vanishes.

Illustration 21.

Solve the equation $\tan 3x = \tan 5x$.

Rewrite the equation as

$$\frac{\sin 3x}{\cos 3x} - \frac{\sin 5x}{\cos 5x} = 0$$

Whence, after a few elementary manipulations, we get

$$\frac{\sin 2x}{\cos 3x \cos 5x} = 0$$

Now, solving the equation $\sin 2x = 0$, we get $x = k\pi/2$, $k = 0, \pm 1, \ldots\ldots$ Now discard extraneous solutions, which is to say, those for which the denominator $\cos 3x \cos 5x$ vanishes, which obviously happens when the values of k are odd. The solution of the original equation will then be the angles $x = k\pi/2$, where k is even: $k = 2n$, $n = 0, \pm 1, \pm 2\ldots\ldots$

That is, $x = n\pi, \quad n = 0, \ \pm 1, \ \pm 2, \ldots\ldots$

Quite obviously it is a grave mistake to take the set of values $x = k\pi/2$ for the answer.

It is the same disregard for the domain of the variable that accounts for mistakes in equation solving in which the left-hand member has been factored and the right-hand member is zero. In solving such an equation, the aspirant ordinarily equates each factor to zero in succession and combines the solutions obtained, completely discarding the fact that for certain values of x which make one of the factors vanish the other factor may be meaningless, and in that case such values of x will not be roots of the proposed equation. Therefore a proper solution requires a check to see that all the values of x obtained do indeed lie within the domain of the variable. This can occasionally give rise to considerable difficulties.

Illustration 22.

Solve the equation

Sin $2x \cos^2 2x \sin^2 6x \tan x \cot 3x = 0$

Equating each factor to zero in succession, we finally get five groups of roots :

$$x = \frac{k\pi}{2}, x = (2k+1)\frac{\pi}{4}, x = \frac{k\pi}{6}, x = k\pi, x = (2k+1)\frac{\pi}{6}$$

Where, throughout, k is any integer.

But this is not yet the answer, the point being that $\tan x$ and $\cos 3x$ are not defined for all values of x and therefore many values of x in these groups may prove to be extraneous. Let us consider each group separately.

(1) $x = k\pi/2$. If k is even, $k = 2l$, then $x = l\pi$ and $\cot 3x$ is meaningless ; if k is odd, $k = 2l + 1$, then $x = l\pi + \pi/2$ and $\tan x$ is meaningless.

Thus, not a single angle x of the first group is actually a solution of the equation

(2) $x = (2k + 1)\pi/4$. Then, as is evident, $\tan x$ is meaningless. Besides, $3x(6k + 3)\ \pi/4$ so that $\cot 3x$ is likewise meaningful.

Thus, all angles x of the second group are solutions of the equation.

(3) $x = k\pi/6$. It is easy to see that, in the trigonometric circle, the terminal side of the angle x coincide with the vertical diameter for $x = 6l + 3$ and hence $\tan x$ is meaningful for $k \neq 6l + 3$. Furthermore, $3x = k\pi/2$ and $\cot 3x$ has meaning only for odd values of k. Hence, the only suitable values are odd numbers k not equal to $6l + 3$, or numbers k of the form $k = 6l + 1$, $k = 6l + 5$.

Thus, of the angles of the third group, the following angles are solutions :

$x = l\pi + \pi/6, \quad x = l\pi + 5\pi/6$

Where l is any integer.

(4) $x = k\pi$. In this case cot $3x$ is meaningless and so there are no solutions.

(5) $x = (2k + 1)\ \pi/6$. By the trigonometric circle, it is evident that the terminal side of the angle x coincides with the vertical diameter for $k = 3l + 1$, and hence, tan x will be meaningless for $k = 3l$, $k = 3l + 2$.

Thus, in the fifth group we have the angles

$x = 1\pi + \pi/6, \quad x = l\pi + 5\pi/6$

where l is any integer; these are the same angles as in the third group. The final answer can be written as follows :

$x = (2n + 1)\pi/4$, $x = n\pi + \pi/6$, $x = n\pi + 5\pi/6$, n an arbitrary integer, or more compactly,

$x = (2n + 1)\ \pi/4$, $x = n\pi \pm \pi/6$, n an arbitrary integer.

Acquiring extraneous roots is not always so explicit as occurs in the last two examples. Sometimes the cause is what would appear at first glance to be quite harmless reasoning.

For example the equation tan $3x$ = tan $5x$ that was analyzed earlier is frequently solved as follows: "The tangents of two angles are equal if and only if the difference of the angles is equal to an integral multiple of π; hence $2x = k\pi$, $x = k\pi/2$, $k = 0, \pm 1, \pm 2, \ldots$ But we know that this answer is not correct.

Where does the mistake lie ?

The explanation is rather simple: the assertion on which the solution is based is correct although it is quite common among aspirants. Indeed, if tan α = tan β, then $\alpha - \beta = k\pi$, where k is an integer, but the converse is not true : if $\alpha - \beta = k\pi$, then the equation tan α = tan β may simply be meaningless (say if $\alpha = \pi/2$, $\beta = -\pi/2$). Therefore, in the replacement of equation tan $3x$ = tan $5x$ by $2x = k\pi$, there was no loss of roots, but certain extraneous roots appeared.

Let us now consider some sources of the loss of roots and appropriate measures for avoiding such loss. Aspirants most often lose roots when replacing a given equation by a new one having a more restricted domain of the variable. Such a restriction (narrowing) of the domain results, as we have already seen, from the use of logarithmic formulas, trigonometric formulas and certain "verbal" reasoning.

As we have already noted, replacing the logarithms of a product by a sum of the logarithmic leads to a narrowing of the domain, just as does rule III which has to do with taking the logarithm of a power. To avoid such restrictions, one should employ Rule I and III instead of Rules I and III. The use of the former can at worst extend the domain, that is, lead to extraneous roots. And we already know what to do with extraneous roots.

That is how we will solve the following problem.

Illustration 23.

Solve the equation

$$\frac{3}{2}\log_{\frac{1}{4}}(x+2)^2 - 3 = \log_{\frac{1}{4}}(4-x)^3 + \log_{\frac{1}{4}}(x+6)^3$$

Since,
$$\log_{\frac{1}{4}}(x+2)^2 = 2\log_{\frac{1}{4}}|x+2|$$
$$\log_{\frac{1}{4}}(4-x)^3 = 3\log_{\frac{1}{4}}(4-x),$$
$$\log_{\frac{1}{4}}(x+6)^3 = 3\log_{\frac{1}{4}}(x+6)$$

The equation takes the form

$$\log_{\frac{1}{4}}|x+2| - 1 = \log_{\frac{1}{4}}(4-x) + \log_{\frac{1}{4}}(x+6)$$

Whence, $\log_{\frac{1}{4}}|x+2| = \log_{\frac{1}{4}}(4-x)\cdot(x+6)$

and consequently,

$$4|x+2| = 4(4-x)\,(x+6)$$

(there is an extension of the domain in the last two transformations and so we can expect the appearance of extraneous roots). The roots of this equation are $x_1 = 2$, $x_2 = 1 - \sqrt{33}$

Extraneous roots could appear during the process of solution only because of an extension of the domain of the variable and so, on the basis of statement A, all we need to do is test the values x_1 and x_2 for membership in the domain. It is readily seen that all the expression under the sign of the logarithms in the given equation for $x = x_1$ and are roots of the equation.

The restriction of the domain and, hence, the loss of roots can also occur when passing to a new logarithm base.

Illustration 24.

Solve the equation

$$\log_{0.5x} x^2 - 14\log_{16x} x^3 + 40\log_{4x}\sqrt{x} = 0$$

Here is a aspirant's solution. Talking of the change-of-base rule and taking x as new logarithmic base, we have

$$\frac{\log_x x^2}{\log_x 0{\cdot}5x} - \frac{14\log_x x^3}{\log_x 16x} + \frac{40\log_x \sqrt{x}}{\log_x 4x} = 0$$

But it is quite evident that the new equation is devoid of meaning for $x = 1$, whereas the original equation not only is meaningful for $x = 1$ but has unity as its root. This is precisely where most aspirants lose the root $x = 1$.

We must therefore reason as follows : we want to pass to the base x; to do this we must be sure that $x > 0$ and $x \neq 0$. Since all the x of our domain are positive the first condition $x > 0$ is satisfied; on the other hand, unity lies in the domain and substitution shows that $x = 1$ is a root. Thus, one root of the original equation has been found: $x = 1$. Now let us seek roots that differ from unity. Then we can pass to the base x without losing roots.

From now on the solution is not difficult using the properties of logarithms and denoting $\log_x 2$ by y we have

$$\frac{2}{1-y} - \frac{42}{1+4y} + \frac{20}{1+2y} = 0$$

This equation is reduced to the quadratic $2y^2 + 3y - 2 = 0$, whose roots are $y_1 = 1/2$, $y_2 = -2$. Then we get $\log_x 2 = 1/2$, whence $x = 4$ and $\log_x 2 = -2$, whence $x = 1/\sqrt{2}$. Both of these values, 4 and $1/\sqrt{2}$, are roots of the original equation. Hence, the original equation has the roots.

A common and very grave mistake that results in a loss of roots is the canceling of a common factor from sides of an equation. It is clear that in the process, roots may be lost which make the common factor vanish.

In such cases it is best to transpose all terms to the left side, take out the common factor and consider two cases: (1) the common factor is equal to zero; (2) the common factor is not equal to zero; then of course the expression in the brackets is zero. It is also possible to consider first the case when the common factor is equal to zero and then cancel the common factor.

Illustration 25.

Find all the solutions of the equation

$$x^2 2^{x+1} + 2^{|x-3|+2} = x^2 2^{|x-3|+4} + 2^{x-1}$$

We consider two cases.

(a) Let $x \geq 3$. Here we have the equation

$$x^2 2^{x+1} + 2^{x-1} = x^2 2^{x+1} + 2^{x-1}$$

Which is evidently satisfied for every x, and so in this case the solutions of the given equation will be all values of $x \geq 3$.

(b) Let $x < 3$. Then the equation takes the form

$$x^2 2^{x+1} + 2^{5-x} = x^2 2^{7-x} + 2^{x-1}$$

whence

$$2^{x-1}(4x^2 - 1) = 2^{5-x}(4x^2 - 1)$$

It was precisely at this point where many of the aspirants at the examination were taken in by the exponential expressions and disregarded the "insignificant" power expression and simply cancelled them obtaining the equation $2^{x-1} = 2^{5-x}$. They then obtained the root $x = 3$ and, what is more, failed to noticed that it does not satisfy the condition $x < 3$.

It is abundantly clear that before canceling out $4x^2 - 1$ the aspirants should have considered the case of $4x^2 - 1 = 0$ then they would have found the roots $x_{1,2} = \pm\, 1/2$ which also satisfy the condition $x < 3$.

Thus, the solutions of the given equation are: any

$$x \geq 3, x_1 = 1/2, x_2 = -1/2.$$

A common mistake made by aspirants is the incorrect use of the following statement: "If two powers are equal and if their bases are equal and different from 0 and 1, then their exponent are equal as well:" What is usually forgotten is the phrase "differently from 0 and 1". The result is a loss of roots, namely those of which the base is equal to 0 or 1.

Illustration 26.

Solve the equation $x^{\sqrt{x}} = \sqrt{x^x}$.

This equation may be rewritten as

$$x^{\sqrt{x}} = x^{x/2}$$

Thus, the powers are equal and the bases are equal. So as not to lose any roots let us see whether the base can be 0 or 1. Since the expression 0° is meaningless, the number 0 is not an element in the domain set and therefore $x = 0$ is not a root of the equation. Contrariwise, $x = 1$ is obviously a root. Now let us seek roots that are different from 0 and 1. Using the indicated rule, we obtain $\sqrt{x} = x/2$, whence we find the second rot of the equation, $x = 4$.

One sometimes hears the erroneous assertion: "If the power of a number is 1, then the exponent is equal to zero." This is only true if the base is different from 1, but if the base is 1, then for any exponent the power will be 1.

Problem involving logical difficulties

Very considerable difficulties of a logical nature are ordinarily caused by equations, inequalities or systems containing ***parameters***, in which it is required to find the values of the parameters for which certain supplementary requirements are fulfilled (say, the equation has a unique solution or, contrariwise, is satisfied by all admissible values of x, or every solution of one system of equations is a solution of another system, or every solution of one inequality is a solution of another, and the like)

This type of problem is probably the most difficult, for it requires a high degree of logical culture. The student must at every step clearly realize what has been done and what still remains to be done, and what the results obtained signify.

Illustration 27.

For what values of a does the equation $1 + \sin^2 ax = \cos x$ have a unique solution ?

It is clear that $\sin^2 ax$ cannot, for arbitrary values of a, be expressed in terms of $\sin x$ and $\cos x$. For this reason, the equation at hand cannot be solved by ordinary methods; a new idea for the solution is needed.

Due to the fact that we have the inequality $\cos x \leq 1 \leq 1 + \sin^2 ax$, the original equation is valid if and only if one of the following systems of equations is fulfilled :

$$\begin{cases} 1 + \sin^2 ax = 1 \\ \cos x = 1 \end{cases} \quad \text{or} \quad \begin{cases} \sin ax = 0 \\ \cos x = 1 \end{cases}$$

We thus have to solve the last system and investigate for which values of $\boldsymbol{a}$ it has a unique solution. Since the original equation is equivalent to this system, the values of a thus found will be the required values.

Here is where the most serious logical complications begin. It is precisely at this point that we see which students understands the problem and which merely performs the manipulations without realizing what he is doing and why it is necessary.

Here is an instance of one student's solution of the system :

$$“ax = \pi k,\ x = 2\pi n,\ 2a\pi n = \pi k,\ a = \frac{k}{2\pi}”$$

That and nothing else! Not a single word, merely the equation $a = k/(2n)$ was underlined and this was apparently taken to mean the answer. This is no solution of course.

We now give a real solution, which repeats the manipulations of the preceding "solution" but is supported by arguments that were lacking there.

The solutions of the first equation of the latter system are

A$x = k\pi, k = 0, \quad \pm 1, \pm 2......$

The solutions of the second equation are also obvious:

X $= 2n\pi, \quad n = 0, \pm 1, \pm 2...$

We need x such that satisfy simultaneously both equations that is, we have to find the numbers k and n for which we obtain one and the same value of x for both sets. Thus, we have to solve one equation in two unknown's n and k (and also with the parameters a) :

$2an\pi = k\pi$ ...(1)

It is obvious that for any a the number pair $n = 0, k = 0$, is a solution of this equation. To it corresponds the root $x = 0$. Thus, for arbitrary a the original equation has the solution $x = 0$. If $n \neq 0$, then equation

(1) can be rewritten

A $= k/(2n)$ ...(2)

Now let us recall our basic problem : not to solve the equation but merely to determine for which values of a it has a unique solution. Now any pair of numbers k and n which satisfies (2) will yields a solution of the original equation $x = 2\pi n = \pi k/a$. Since for arbitrary a we have already found one root of the original equation ($x = 0$), we must now seek values of a for which no integers k and n exist such that relation(2) is valid. Clearly, if a is irrational, then no such k and n relation (2) is valid. Clearly if a is irrational then no such k and n exist the first result is obtained: if a is an irrational number, then the given equation has a unique solution.

Is the problem solved ? Of course not, since we have not yet investigated the rational values of a. However, if a is rational, that is $a = p/q$, then it can be written in the form a = (2p)/(2q) and in equation (2) we get the solution $k = 2p,\ n = q$. Hence, in this case, with the exception of $x = 0$, there will at least be one solution (actually there will even be infinitely many). To summarize then: for a rational the original equation has more than one solution. The problem is solved.

All these steps are needed so as to solve the problem for ourselves and obtain the answer. This might be called the rough solution. We now show what the final version might look like.

Obviously, $x = 0$. is a root of the equation for any a. we will demonstrate that for a irrational there are no other solutions and for a rational, there are. Indeed, first suppose that a is irrational. From the inequality $\cos x \leq 1 \leq 1 + \sin^2 ax$ it follows that x is a solution if and only of the following system is satisfied :

$1 + \sin^2 ax = 1$ $\quad\quad$ $\sin ax = 0$

$\cos x = 1$ $\quad$ that is $\quad$ $\cos x = 1$

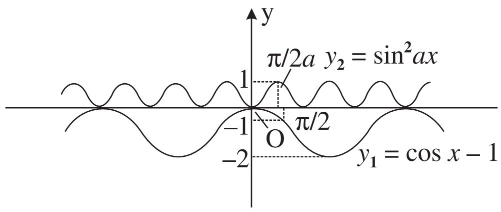
If $x \neq 0$ is a solution of the last system, then, firstly, $ax = \pi k$, k an integer and secondly $x = 2\pi\, n$, n an integer, $n \neq 0$. But then $2a\pi n = \pi k$, whence $a = k/(2n)$, that is to say, a is a rational number, which runs counter to the assumption. Now let a be rational, $a = p/q$. Then $x = 2\pi q$ will clearly be a solution and, moreover, one different from zero. Thus, the given equation has a unique solution if and only if the number a is irrational.

This problem can also be solved graphically. We assume that $a \neq 0$, since for $a = 0$ the equation clearly has infinite of solutions let us rewritten our equation thus :

$$\sin^2 ax = \cos x - 1$$

And denote

$$Y_1 = \cos x - 1,\ y^2 = \sin^2 ax = \frac{1 - \cos^2 ax}{2}$$



We draw both graphs on one drawing. The original equation clearly has a solution if and only if the graphs of the functions y_1 and y_2 have a point in common. It is evident from the drawing that for arbitrary a there is a point of intersection of the graphs: $x = 0$. The drawing also shows that subsequent intersection of graphs is only possible at points where both curves touch the x-axis, that is, at points where, simultaneously, $\sin^2 ax = 0$ and $\cos x = 1$. But $\sin^2 ax = 0$ for $ax = n\pi$, where $n = 0, \pm 1, \pm 2$ and $\cos x = 1$. for $x = 2k\pi$, where $k = 0, \pm 1, \pm 2, ...$ therefore the point $x = 0$ will be the sole meeting point of the graphs only when $2ak\pi \neq n\pi$ for no non zero integers n and k. In other words, we have shown that there will be a unique solution only when $a \neq n/(2k)$, where n and k are nonzero integers. Then, as before, it is demonstrate that this occurs only when a is an irrational number.

In solving problems involving parameters, one often reasons as follows. Let a parameter a be some fixed number that satisfies the condition of the problem; such values of a will be called suitable. We then derive consequences from the statement of the problem and assumptions concerning a. We thus obtain certain conditions which the suitable values of the parameter must satisfy. The values of the parameter which do not satisfy these consequences are automatically classed as unsuitable, and we have only to consider the values of the

parameter that satisfy the consequence obtained. In particular, if these consequences are satisfied solely by certain concrete values, then the problem reduces to a verification of these values.

Illustration 28.

Find all the values of ***a*** for which the system

$$2^{|x|} + |x| = y + x^2 + a$$
$$x^2 + y^2 = 1$$

has only one solution (*a, x, y* are real numbers).

In accordance with the above, we first assume that a is some suitable number, that is, a number that satisfies the statement of the problem. In other words, for this values of a the given system of equation has exactly one solution; denote it by (x_0, y_0). But it is easy to notice that both equations of the system remain unchanged upon replacing x by $-x$, which means that the pair $(-x_0, y_0)$ is also a solution of the system for the value under consideration. The original assumption, however, was that the system has a unique solution with respect to a. There is only one way out, (x_0, y_0) and $(-x_0, y_0)$ are one and the same pair. This then simply means that $x_0 = -x_0$, that is $x_0 = 0$. So far this reasoning does not yields any information about y_0. But if we put the solution $(0, y_0)$ into the original system, we get the equations

$$1 = y_0 + a, \qquad y^2_0 = 1$$

Whence it follows that y_0 is equal either to 1 or – 1; accordingly, a is equal either to 0 or to 2.

We have thus demonstrated that if a suitable number, then either $a = 0$ or $a = 2$. It must be stressed that the foregoing reasoning has in no way proved that the numbers 0 and 2 are suitable. Quite the contrary, that is precisely what we must now find out.

We first consider the value $a = 0$ in this case we have the system

$$2^{|x|} + |x| = y + x^2$$
$$x^2 + y^2 = 1 \qquad \ldots(3)$$

If we can prove that this system has a unique solution, then this will signify that the value $a = 0$ satisfies the condition of the problem. Note that the value $a = 0$ was obtained above when substituted the pair (0,1) into the original system. It is easy to verify that this pair does indeed satisfy the system (3) and thus for $a = 0$ the original system already has one solution. Now let us find out whether (3) have any other solutions.

This system is not solvable by ordinary procedures. We will have to reason in a special way. From the second equation of the system it follows that $|x| \le 1$, $|y| \le 1$, whence $x^2 \le |x|$ and $y \le 1$. Beside $2^{|x|} \ge 1$, since $|x| \ge 0$. From all these inequalties we get,

$$2^{|x|} + |x| \ge 1 + x^2 \ge y + x^2$$

And, hence, the first equation is satisfied only when equality occurs in both weak inequalities; that is, when

$$2^{|x|} = 1, |x| = x^2, y = 1$$

And this is true only for $x = 0$, $y = 1$. Thus, for $a = 0$ the given system has the unique solution of (0, 1).

We now consider the value $a = 2$. In this case we have the system

$$2^{|x|} + |x| = y + x^2 + 2$$
$$x^2 + y^2 = 1$$

As before, we note that the pair (0, – 1) is a solution and again we have to find out whether there are any other solutions. But by substituting $x = 1$, $y = 0$, we are assured that the pair (1, 0) is also a solution of the system and, hence, for $a = 2$ the system has more than one solution.

To summarize then, the given system has a unique solution for $a = 0$ alone.

The foregoing solution requires a remark or two, not of a mathematical but rather of a psychological nature. As so often happens, the solution is easy to understand, but how is it found ? There is of course no cut and-dry answer to that.

In our solution there are three possible guesses.

Firstly, we noticed that the system does not change upon replacement of x by $-x$. This was an essential hint. Anyone with some idea about the evenness and oddness of a function and with some experience in handling functions would realize this.

Secondly, we started out by working system (3) by nonstandard procedures using inequalities. This conjecture is somewhat more complicated, but the examples of earlier sections showed us that it is often necessary to employ inequalities in equation solving.

Finally, we realized that for $a = 2$ the original system has yet another solution: $x = 1$, $y = 0$. We therefore tried simply to pick a solution, and it worked out. This approach proved successful merely due to the existence of "good" integral solutions. In certain cases, such a choice is the only possible route for solving a problem.

Illustration 29.

Find all the values of a and b for which the system

$$xyz + z = a$$
$$xyz^2 + z = b$$
$$x^2 + y^2 + z^2 = 4$$

Has only one solution (*a,b,x,y,z* are real numbers)

Let (a, b) be a suitable pair of values of the parameters and (x_0, y_0, z_0) the corresponding unique solution. It is readily seen that the system remain unchanged if, simultaneously, we replace x by $-x$ and y by $-y$. This implies that the triple $(-x_0. -y_0, z_0)$ is also a solution of the system and, as in the preceding problem, we conclude that $x_0 = y_0 = 0$. Substituting the triple $(0, 0, z_0)$ into the system, we get $z_0 = a$, $z_0 = b$, ${z_0}^2 = 4$, whence $z_0 = \pm 2$ and $a = b = \pm 2$.

Thus, if the pair (a, b) is suitable then either $a = b = 2$ or $a = b = -2$

Again, as in the preceding problem, we have to establish whether these pairs of values of the parameters are suitable or not. For $a = b = 2$ we have the system

$$\begin{aligned} xyz + z &= 2 \\ xyz^2 + z &= 2 \\ x^2 + y^2 + z^2 &= 4 \end{aligned}$$

One of the solution of which, as can readily be verified, is $x = 0, y = 0, z = 2$. From the second and first equation it follows that xy $(z^2 - z) = 0$. If $x = 0$, then from the second and third equations we get $z = 2$ and $y = 0$. We already know this solution. The same solution is obtained if $y = 0$.

We will now assume that $z^2 - z = 0$ *i.e,* $z = 0$ or $z = 1$. However, when $z = 0$ we see that the first two equations are contradictory, and for $z = 1$ we get the system

$$\begin{aligned} xy &= 1 \\ x^2 + y^2 &= 3 \end{aligned}$$

Which, as it easy to see, have four real solutions. Thus, for $a = b = 2$ the original system has five solutions and therefore the pair $a = b = 2$ is not a suitable one.

Now let $a = b = -2$. We have the system

$$\begin{aligned} xyz + z &= -2 \\ xyz^2 + z &= -2 \\ x^2 + y^2 + z^2 &= 4 \end{aligned}$$

One of the solutions of which, as we can readily see, is $x = 0, y = 0, z = -2$. Reasoning as before, we see that the system does not have any other real solutions and so for $a = b = -2$ the original system has a unique solution, which means this pair of values of the parameters is suitable.

Hence the condition of the problem is satisfied only by the values $a = b = -2$.

Illustration 30.

Find all the values of ***a*** for which the system

$$\begin{aligned} (x^2 + 1)^a + (b^2 + 1)^y &= 2 \\ a + bxy + x^2y &= 1 \end{aligned}$$

has at least one solution for any value of b (a, b, x, y are real numbers).

Let a be a suitable value of the parameter, that is, a value for which the given system has at least one solution for any value of b. We choose some value of b; this can be done in arbitrary fashion, but we will choose b so that the system takes on the simplest possible aspect. Clearly the best to choose is $b = 0$. Then the system looks like this

$$\begin{aligned} (x^2 + 1)^a &= 1 \\ a + x^2y &= 1 \end{aligned}$$

and since a is a suitable value, the system has at least one solution, which we denote by (x_0, y_0).

In this solution, x_0 is either zero or non-zero. if $x_0 = 0$ then from the second equation we get $a = 1$ and $x_0 \neq 0$ then $x_0^2 + 1 \neq 1$ and from the first equation we get $a = 0$.

Thus, if a is a suitable number then either $a = 0$ or $a = 1$. Now we have to determine whether these value are indeed suitable or not. When $a = 0$ the system is of the form

$$\begin{aligned} (b^2 + 1)^y &= 1 \\ bxy + x^2y &= 1 \end{aligned}$$

We now have to find out whether this system has any solution for arbitrary values of b. For $b \neq 0$ it follows from the first equation that $y = 0$, and then the second equation is inconsistent. Hence, the value $a = 0$ is not a suitable value.

Let a =1, then the system is

$$\begin{aligned} x^2 + (b^2 + 1)^y &= 1 \\ bxy + x^2y &= 0 \end{aligned}$$

clearly, $x = y = 0$ for any b is a solution and so $a = 1$ is a suitable value.

Thus, the condition of the problem is satisfied by the unique value $a = 1$.

Illustration 31.

Find all the numbers a for each of which any root of the equation

$$\sin 3x = a \sin x + (4 - 2)\,|a|\,\sin^2 x \qquad \ldots(4)$$

is a root of the equation

$$\sin 3x + \cos 2x = 1 + 2\sin x \cos 2x \qquad \ldots(5)$$

and, contrariwise, any root of the latter equation is a root of the former.

The problem can more briefly be stated thus : for which values of a are the equation (4) and (5) equivalent? There are fundamentally two ways of determining the equivalence of two equations : the first is to obtain each equation from the other by means of certain manipulation, the second, in accord with the definition of equivalence, is to prove that every root of one equation is a root of the other, and vice-versa.

In our example, the first approach is apparently inapplicable and we have to take advantage of the second approach. Here, too, however things are not so simple. It is hard to reason about the coincidence of the roots of two equations which are so unlike. The only thing that can save us is knowledge of all these roots or the roots of at least one of the equations.

In our case, equation (5) has a simple solution and so the problem readily reduces to the following one: for which values of a does equation (4) have exactly the same roots as (5) ?

To simplify computations, denote sin x by y. then (5) becomes

$$2y^2 - y = 0 \qquad \ldots(6)$$

This equation has the roots $y_1 = 0$, $y_2 = 1/2$. Similarly, upon replacing $\sin 3x = 3y - 4y^2$, equation (4) becomes

$$[4y^2 + (4 - 2|a|)y + a - 3]\, y = 0 \qquad \ldots(7)$$

Many aspirants replaced sin x by y and this "helped" them to make two serious mistakes. Thus, many decided immediately that the required values of a do not exist

since equation (6) is quadratic and equation (7) is cubic and, hence, they are not equivalent because they have different numbers of roots. This argument contains two mistakes at once. Firstly, a quadratic and a cubic equation can be equivalent (for instance, the equations $x^2 = 0$ and $x^3 = 0$ both have the unique root $x = 0$) and, secondly, we will see for ourselves below, (4) and (5) can be equivalent even if (6) and (7) are not.

There lies the second mistake. At first glance it would appear quit obvious that our problem was reduced to the following : for which values of a does not equation (7) have only the root 0 and 1/2. But actually, if we recall that $y = \sin x$, we can indicate yet another possibility for the value of a to be suitable: if (7) has the roots 0, 1/2, and its third root y_3 is greater than unity in absolute value, then (4) and (5) are equivalent because the corresponding value $\sin x = y_3$ will not given equation (4) any additional solutions. Naturally, the equation (4) and (5) are equivalent when the third root of (7) is equal to 0 or to 1/2.

Our problem is now clear: we have to find values of a such that (7) has the roots 0, 1/2 and its third root is either 0 or 1/2 or exceeds 1 in absolute value.

It is evident at once that 0 is a root of equation (7) so that we will henceforth consider the equation

$$4y^2 + (4 - 2|a|)y + a - 3 = 0 \qquad \ldots(8)$$

One of the roots of this equation must be 1/2. Substituting $y = 1/2$ into it, we find that 1/2 is a root when $a = |a|$, or $a \geq 0$. By Viete's theorem, the second root is equal to $(a - 3)/2$ and according to what has just been said, the value of a will be suitable in the following three cases :

$$(1)\ \frac{a-3}{2} = 0, \qquad (2)\ \frac{a-3}{2} = \frac{1}{2}, \qquad (3)\ \left|\frac{a-3}{2}\right| > 1$$

(also bear in mind that $a \geq 0$).

We than have the answer :

$a = 3, a = 4, 0 \leq a < 1, a > 5$

Illustration 32.

Find all the numbers a such that for every root of the equation

$$2 \sin^7 x - (1 - a) \sin^3 x + (2a^3 - 2a - 1) \sin x = 0 \ \ldots(9)$$

is a root of the equation

$$2 \sin^6 x + \cos 2x = 1 + a - 2a^3 + a \cos^2 x \qquad \ldots(10)$$

and, contrariwise, every root of the second equation is a root of the first equation.

Here, both the given equations are complicated and so we cannot proceed as in the preceding problem. But we can note that equation (9) has solution of the form $x = k\pi$, where k is any integer, and perhaps some other solutions. This remark will enable us to solve the problem.

Let a be a suitable value of the parameter. Then the values $x = k\pi$ – roots of (9) – are roots of (10), and this immediately yields the equation $a^3 = a$ (since $\sin^6 k\pi = 0$, $\cos^2 k\pi = 1$). Therefore, the suitable values are to be chosen from among only three numbers : 0, 1 and – 1 the task now is to verify all three values.

Let $a = 0$, then the equation will assume the form

$$\sin x\,(\sin^2 x - 1)\,(2\sin^4 x + 2\sin^2 x + 1) = 0$$

$$\sin^2 x\,(\sin^2 x - 1)\,(\sin^2 x + 1) = 0$$

Since $1 + \sin^2 x > 0$ and $2 \sin^4 x + 2 \sin^2 x + 1 > 0$, these equations are equivalent.

Let $a = 1$, then the equations can be rewritten as

$\sin x\,(2 \sin^6 x - 1) = 0$ and $\sin^2 x\,(2 \sin^4 x - 1) = 0$

Since the first equation has the solution $\sin x = \sqrt[6]{1/2}$, which does not satisfy the second equation, these equations are not equivalent,

Let $a = -1$. We then have the equations

$$\sin x(2 \sin^6 x - 2 \sin^2 x - 1) = 0$$

and $$\sin^2 x\,(2\sin^4 x - 3) = 0$$

Since $2 \sin^4 x - 3 < 0$ and $2\sin^6 x - 2 \sin^2 x - 1 = 2 \sin^2 x(\sin^4 x - 1) - 1 < 0$, it is clear that the equations are equivalent. Thus, the condition of the problem is satisfied only by $a = 0$ and $a = -1$. In this problem too, many aspirants replaced $\sin^2 x$ and y and could not figure out what to do with the value $a = -1$, since in the inequalities that have to be proved in this case, essential use is made of the fact that $0 \leq y \leq 1$.

7. Find all number pairs a, b for which every number pair x, y ($x \neq \pi/2 + k\pi, y \neq \pi/2; k, n = 0, \pm 1, \pm 2, \ldots\ldots\ldots$) that satisfies the equation $x + y = a$ also satisfies the equation,

$$\tan x + \tan y + \tan x \tan y = b \qquad \ldots(11)$$

Let a and b be a suitable pair of values of the parameters. We take a number pair $x = 0, y = a$ which clearly satisfies the equation $x + y = a$. If $a \neq \pi/2 + n\pi$, then this pair satisfies the restrictions imposed on x and y in the statement of the problem and, for this reason, by virtue of (11), the equation $\tan a = b$ must be valid. Let us now take the number pair $x = \pi/4, y = a - \pi/4$, must also satisfies the equation $x + y = a$. If $a \neq 3\pi/4 + k\pi$, then this pair likewise satisfies the restrictions on x and y and therefore (since a and b are assumed to be suitable) the following equation must hold :

$$1 + 2 \tan\left(a - \frac{\pi}{4}\right) = b \qquad \ldots(12)$$

Since $b = \tan a$, then a thus satisfies the equation

$$1 + 2 \tan\left(a - \frac{\pi}{4}\right) = \tan a$$

Which can readily be reduced to the quadratic equation $\tan^2 a - 2 \times \tan a + 1 = 0$. Hence, $\tan a = 1$ and the suitable pairs a, b must be sought among the infinitude of pairs of the form

$$a = \frac{\pi}{4} + m\pi, b = 1, m = 0, \pm 1, \pm 2, \ldots\ldots$$

Let us determine which of these pairs are actually suitable. Let $x + y = \frac{\pi}{4} + m\pi$ for some integer m, and $x \neq \pi/2 + k\pi, y \neq \pi/2 + n\pi$, $\tan x + \tan y + \tan x \tan y$

$$= \tan x + \tan\left(\frac{\pi}{4} + m\pi - x\right) + \tan x \tan\left(\frac{\pi}{4} + m\pi - x\right)$$

$$= \tan x + \tan\left(\frac{\pi}{4} - x\right) + \tan x \tan\left(\frac{\pi}{4} - x\right)$$

$$= \tan x + \frac{1 - \tan x}{1 + \tan x} + \tan x \frac{1 - \tan x}{1 + \tan x}$$

The last expression is equal to unity. Thus all pairs $a = \pi/4 + m\pi$, $m = 0, \pm 1, \pm 2, \ldots, b = 1$ are suitable.

However, the solution is not yet complete since in the course of our discussion we excluded the values $a = \pi/2 + n\pi$ and $a = 3\pi/4 + k\pi$, $k, n = 0, \pm 1, \pm 2, \ldots$ it remain to consider these values as well.

Let $a = \pi/2 + n\pi$, n an integer. In this case, obviously, $a \neq 3\pi/4 + k\pi$ and therefore (12) must be valid, from it follows, for the values of a under consideration, that $b = 3$. We will attempt to find suitable pairs among

$$a = \frac{\pi}{2} + n\pi, b = 3, n = 0, \pm 1, \pm 2, \ldots\ldots$$

The pairs $x = -\frac{\pi}{4}, y = 3\pi/4 + n\pi$ satisfies the equation $x + y = a$; on the other hand,

$$\tan\left(-\frac{\pi}{4}\right) + \tan\left(\frac{3\pi}{4} + n\pi\right) + \tan\left(-\frac{\pi}{4}\right) \tan\left(\frac{3\pi}{4} + n\pi\right)$$

$= -1 \neq 3$ and therefore there are no suitable pairs among the number pairs a, b under consideration.

Now let $a = 3\pi/4 + k\pi$, k an integer. Since in this case $a \neq \pi/2 + n\pi$, the equation $\tan a = b$ must be true, *i.e.*, $b = -1$. We wish to find suitable pairs among the pairs

$$a = \frac{3\pi}{4} + k\pi, b = -1, k = 0, \pm 1, \pm 2, \ldots\ldots$$

The pair $x = 3\pi/8$, $y = 3\pi/8 + k\pi$ satisfies the equation $x + y = a$ on the other hand,

$$\tan\frac{3\pi}{8} + \tan\left(\frac{3\pi}{8} + k\pi\right) + \tan\frac{3\pi}{8} \tan\left(\frac{3\pi}{8} + k\pi\right)$$

$$= 2\tan\frac{3\pi}{8} + \tan^2\frac{3\pi}{8} > 0$$

(because the angle $3\pi/8$ lies in the first quadrant) and so the left member of (11) is different from -1 and thus there are no suitable pairs among the pairs a, b under consideration.

The final answer is this: the condition of the problem is satisfied by infinity of pairs

$$a = \frac{\pi}{4} + m\pi, b = 1, m = 0, \pm 1, \pm 2, \ldots\ldots$$

Illustration 33.

Find all the values of a for which any values of x that satisfies the inequality $ax^2 + (1 - a^2)x - a > 0$ does not exceed two in absolute value.

In its logical form, the statement of this problem is quite analogous to that of the preceding problem. Namely, it is required to find the values of the parameters a for which from the inequalities $ax^2 + (1 - a^2)x - a > 0$ follows the inequalities $-2 \leq x \leq 2$. However, the method of solution applied in the preceding problem is not suitable here mainly because we have to do with inequalities and not with equations.

In this problem we have to determine for which values of a all the solutions of given inequality lie in the interval $-2 \leq x \leq 2$. If $a \neq 0$, then the given inequality is quadratic, and we first consider this general case. Thus, let $a \neq 0$, we know that the solutions of a quadratic inequality, if they exist, form on the number line either a finite interval, two infinite intervals, or the entire set of real numbers; and this depends on the signs of the discriminate and the leading coefficient. We therefore compute the discriminate of the quadratic trinomial in the left member of the inequality :

$$D = (1 - a^2)^2 + 4a^2 = a^4 + 2a^2 + 1 = (a^2 + 1)^2$$

Thus, $D > 0$ for any a, so that the roots of the trinomial are real and distinct and easily found: $x_1 = a$, $x_2 = -1/a$.

Now, depending on the sign of the number a, the solutions of the given inequalities form an interval between roots (for $a < 0$ when the parabola is concave downwards) or two infinite intervals (for $a > 0$).

By hypothesis, we need values of a for which all solutions of the inequalities lay in the interval $-2 \leq x \leq 2$. Therefore the values $a > 0$ are not suitable: two infinite intervals cannot fit into a finite interval. It remains only to consider the values $a < 0$. In this case, $x_1 < 0 < x_2$ and the solution of the given inequality is the interval $a < x < -1/a$.

We want the entire interval $a < x < -1/a$ to lie in the interval $-2 \leq x \leq 2$ and this occurs obviously if and only if the endpoints of the first interval lie on the interval $-2 \leq x \leq 2$. (coincidence of endpoints is admissible), that is, if the inequalities

$$-2 \leq a < -\frac{1}{a} \leq 2$$

Hold true. From the inequality $-1/a \leq 2$, taking into account that $a < 0$, we get $a \leq -1/2$ and, hence $-2 \leq a \leq -1/2$.

Thus, any solution of the complete the solution we have not exceed two in absolute value when $-2 \leq a \leq -1/2$, but we obtained this on the assumption that $a \neq 0$. To complete the solution we have yet to consider this special case. For $a = 0$, the original inequality assumes the form $x > 0$ and not all its solutions fail to exceed two in absolute value so that the value $a = 0$ is not a suitable value. To summarize, then, the inequality obtained above $-2 \leq a \leq -1/2$ is the final answer.

Illustration 34.

Find all the values of a for which, for all x not exceeding unity in absolute value, we find the inequality $\frac{ax - a(1 - a)}{a^2 - ax - 1} > 0$ to be valid.

First replace the given inequality by the equivalent but more customary quadratic inequality $(ax + 1 - a^2)$ $[ax - a(1 - a)] < 0$

We have been a bit hasty in calling this inequality quadratic, since we have not yet checked to see if, when the brackets are removed, the coefficient of x^2 is different from zero. This coefficient is equal to a^2 and is zero when $a = 0$, but for $a = 0$ the given inequality takes the form $0 < 0$, which is to say that it does not hold for any x. Therefore, $a = 0$ is not a suitable value and we discard it, considering henceforth $a \neq 0$ everywhere.

We will solve the resulting quadratic inequality by following the same ideas as in the preceding problem. We see at once that the roots of the trinomial are real and so the discriminate need not be computed. Beside, the leading coefficient a^2 is positive and, hence, the solutions of the quadratic inequality form an interval between its roots $x_1 = (a^2 - 1)/a$, $x_2 = 1 - a$, if these roots are distinct. But if the roots coincide, the quadratic inequality is not satisfied for a single value of x and therefore the corresponding values do not interest us.

We thus need the values of a for which the entire interval $-1 \leq a \leq 1$ lies between the number $a - 1/a$ and $1 - a$. But in order to wire this geometric condition in the language of inequality, we have to know which of the two numbers is greater. This clearly depends on the number a, and so we consider two cases.

(a) $(a^2 - 1)/a < 1 - a$. As in the preceding problem, for the interval $-1 \leq a \leq 1$ to lie entirely within the interval $(a^2 - 1)/a < x < 1 - a$, it is necessary that the endpoints (-1 and 1) lie in the interval; thus, the following inequalities must be valid :

$$\frac{a^2 - 1}{a} < -1 < 1 < 1 - a$$

(Coincidences of extreme points, that is, the equation $a - 1/a = -1$ and $1 - a = 1$ are not admissible, since, for instance, when $a - 1/a = -1$, the number -1 in the interval $-1 \leq x \leq 1$ is exterior to the interval $-1 \leq x \leq 1 - a$.)

From the inequalities $1 < 1 - a$ it follows that $a < 0$, and then from the inequality $(a^2 - 1)/a < -1$ we get $a^2 - 1 > -a$ or $a^2 + a - 1 > 0$. The solution of this inequality are the values of $a < (-1 - \sqrt{5})/2$ and $a > (-1 - \sqrt{5})/2$, Since $a < 0$, we only leave the values $a < (-1 - \sqrt{5})/2$. Now, from the resulting values of a we have to choose which satisfy condition (a), *i.e.,* the inequality $a - 1/a < 1 - a$. But this condition is automatically satisfied for $a < (-1 - \sqrt{5})/2$. Indeed the indicated values are obtained as solutions of the inequalities $a - 1/a < -1 < 1 - a$.

(b) $1 - a < (a^2 - 1)/a$. in this case we have to solve the inequalities

$$1 - a < -1 < 1 < \frac{a^2 - 1}{a}$$

From $1 - a < 1$ we have $a > 2$ and then from $(a^2 - 1)/a > 1$ follows $(a^2 - 1) > a$, or $a^2 - a - 1 > 0$. The solutions of this inequalities are the values $a < (-1 - \sqrt{5})/2$ and $a > (1 + \sqrt{5})/2$. Since $a > 2$, we only leave the values $a > 2$. As in Case (a), these values automatically satisfy condition (b). Thus, the condition of the problem is satisfied by the values $a < (-1 - \sqrt{5})/2$ and $a > 2$.

Exercises

1. Three cyclists start out simultaneously from the same place in one direction around a circular track 1 km in length. The rates of the cyclists form, in a certain order, an arithmetic progression with common difference 5 km/hr. After some time, the second one catches up with the first, having made one extra circuit; 4 minutes later the third arrives at that point, having covered the same distance that the first did at the time he caught up with the second cyclist. Find their rates.

2. Three brothers, whose ages form a geometric progression, divide among themselves a certain sum of money in direct proportion to the age of each. If this were done in three years time, when the youngest becomes one-half the age of the oldest, then the youngest would receive 105 dinar, and the middle one, 15 dinar more than at the present time. Find the ages of the brothers.

3. Two groups of tourists start out from A in the direction of B at the same time. The first group goes by bus (at 20 km/hr) and reaches C, midway between A and B, and then continues on foot. The second group starts out on foot, but in one hour boards a car which proceeds at 30 km/hr and reaches B. The first group passes C 35 minutes before the second group, but arrives at B 1 hr and 25 minutes later than the second group. What is the distance from A to B if the rate of the first group (on foot) is 1 km/hr greater than that of the second group ?

4. Two identical vessels are filled with alcohol. We draw off a litres of alcohol from the first vessel and add that amount of water. Then we draw off a litres of the resulting mixture of water and alcohol and add that amount of water. In the case of the second vessel, we draw off $2a$ litres of alcohol and add that amount of water, and then draw off $2a$ litres of the resulting mixture and add that amount of water. Determine what part of the volume of the vessel is taken up by a litres if the strength of final mixture in the first vessel is 25/16 times the strength of the final mixture in the second vessel. (By strength is meant the ratio of volume of pure alcohol in the mixture to the total volume of the mixture. It is assumed that the volume of the mixture is equal to the sum of the volumes of its components.)

5. Two bodies are in uniform motion around a circle in the same direction, and one of them catches up with the other every 46 seconds. If these bodies were in motion in opposite directions, they would meet every 8 seconds. Determine the rates of the bodies if we know that the radius of the circle is equal to 184 cm.

6. Towns A and B are located on a river, B downstream from A. At 9 A.M. a raft starts floating downstream from A in the direction of B (the rate of the relative to the bank of the river is the same as the rate of the current). At that time, a boat starts out from B for A and meets the raft in 5 hours. Upon reaching A, the boat round and returns to B arriving at the same time as the raft. Did the boat and raft at B by 9 P.M. (of the same day) ?

7. Three workers receive an assignment, which each separately completes in a specified time, the third completing the job one hour faster than the first. Working together they can complete the job in one hour. But if the first worker does one hour and then the second 4 hours, together they can complete the job. How long does it take each worker separately to complete the full assignment ?

8. We have two solutions of a salt in water. To obtain a mixture containing 10 grams of salt and 90 grams of water, one takes twice as much (by weight) of the first solution as the second. One week elapses and 200 grams of water has evaporated from each kilogram of the first and second solution. Now to obtain the same mixture as before, we require four times more (by weight) of the first solution than of the second. How many grams of salt did 100 grams of each solution originally contain ?

9. A freight train going from A to B arrives at station C together with a passenger train going from B to A with a speed m times that of the freight train. The two trains stop at C for of hours and then continue on their ways, each train increasing its 25% over the original speed it had prior to arrival at C. Then the freight train arrives at B later by t_1 hours and the passenger train arrives at A later by t_2 hours than if they had gone nonstop at their original speeds. How much earlier did the freight train start out from A than the passenger train from B ?

10. Three points A, B and C are connected by straight roads. Adjoining section AB of the road is a square field with a side of 1/2 AB; adjoining BC is a square field with a side equal to BC, and adjoining CA is a rectangular section of the woods of length equal to AC and of width 4 km. The wooded area is 20 square kilometres greater than the sum of the areas of the square fields. Find the area of the woods.

11. Thirty aspirants received marks of 2, 3, 4, 5 at a competition. The sum of the marks is 93, there are more 3's than 5's and fewer 3's than 4's. Besides, the number of 4's is divisible by 10 and the number of 5's is even. Determine the number of the various marks received by the thirty aspirants.

12. A motorcycle and a car (Volga) leave A for B at the same time; and at the Same time another car (Moskvich) leaves B for A. the Moskvich arrives in A in 5 hours and 50 minutes. The cars meet 2 hours 30 minutes later, and the motorcycle and Moskvich meet 140 km from A. If the rate of the motorcycle were twice what it was, it would have met the Moskvich car 200 km from A. Find the speeds of the motorcycle and the two cars.

13. An empty tank is being filled through two pipelines simultaneously with pure water and a constant concentration of an acid solution. When filled, the tank has a 5% solution of the acid. If when the tanks were half full, the water supply were cut off, the full tank would have a 10% solution of the acid. Determine which pipeline delivers liquid faster and how much faster ?

14. A car leaves point A for B. At the same time, a cyclist starts out in the opposite direction (from B). Three minutes after they meet, the car turns around and follows the cyclist; after catching up with the cyclist, it turns around and goes to B. If the car had turned around 1 minute after the meeting, and the cyclist (after the meeting) had increased his speed 15/7 times, the car would have spent the same amount of time for the entire trip. Find the ratio of the speeds of the bicycle and the car.

15. Two men start out at the same time from A in the direction of B, which is 100 km from A, one on a bicycle, the other on foot. Also at the same time, a car starts out from B and goes in the direction of A. An hour later the car meets the cyclist and, continuing another $14\frac{2}{17}$ km, it meets the man on foot, who boards the car, and the car overtakes the cyclist. Compute the speeds of the bicycle and the car if we know that the man on foot was going 5 km/hr. The time required to get into the car and turn the car around is disregarded.

16. A laboratory has to order a certain number of identical spherical flasks with a total capacity of 100 litres. The cost of one flask consists of the cost of labour, which is proportional to the square of the surface area of the flask, and the cost of the material, which is proportional to the surface area. A flask of capacity 1 litre costs 1 rouble 25 kopeks, and in this case the labour cost is 20% of the cost of the flask (the wall thickness of the flask is negligible). Will 100 rubal (100 kopeks to a rubal) be enough to cover the cost ?

17. Bus No. 1, which a aspirant uses to get to his place of study (Institute of Perfection, Hardwar) without changing buses, covers the distance in 2 hours and 1

minute. He can also get to the institute by anyone of the buses No.2, 3, ... , No. K, but the only way to make a change to Bus No. P is from Bus No. (P – 1). The routes of these buses are such that if the aspirant gets to the institute on one of them, he will spend en route (disregarding transfers) a time inversely proportional to the number of buses used. Moreover, he will have to spend 4 minutes at each transfer. Is there a route he can take such that the total time is less than 40.1 minutes ?

18. Between town A and city E is a petrol pump 0 and a water supply station B, which divide the distance between A and E into three equal parts (AO=OB=BE). A motorist and cyclist start out simultaneously from A in the direction of E and, at the same time, a truck starts out from E in the direction of A and at the water supply station passes the car and at the petrol pump passes the cyclist. At the petrol pump, the cyclist increases his speed 5 km/hr. The motorist reaches E and then sets out on the return trip at 8 km/hr slower than before. As a result, when the truck arrives in A, the cyclist still has 7.5 km to go, to B, while the motorist is in between 0 and A, 14 km from O. Find the distance from the town to the city and also the speeds of the car, truck and bicycle.

19. A rectangular plot of area 900 square metres is to be fenced in; two adjoining sides to be brick and the two others to be wooden. One metre of the wooden fence casts 10 dinar and one metre of the brick fence costs 25 dinar. A total of 2000 dinar has been allotted for the job. Will this sum be sufficient?

20. The tank at a water supply station is filled with water by several pumps. At first, three pumps of the same capacity are turned on; 2.5 hours later, two more pumps (both the same) of a different capacity are set into operation. One hour after the additional pumps were set into operation the tank was almost filled to capacity (15 cubic metres were still lacking), in another hour the tank was full. One of the two additional pumps could have filled the tank in 40 hours. Find the volume .of the tank.

21. At a 10,000-metre ski race, the first skier starts out and is followed shortly by a second one, the rate of the second skier being 1m/sec more than that of the first one. When the second skier catches up with the first, the latter increases his rate by 2 m/sec, while the rate of the second skier remains unchanged. As a result, the second skier finishes 7 minutes and 8 seconds after the first one. If the distance had been 500 metres longer, then the second skier would have finished 7 minutes and 33 seconds after the first one. Find the time lapse between the start of the first and second participants.

22. Three skaters whose rates, taken in same order, form a geometric progression, start out at the same time on a skating circuit. After a time, the second one overtakes the first, covering 400 metres more. The third skater covers the same distance that the first did when the latter was overtaken by the second during a time that is 2/3 minutes more than the first. Find the rate of the first skater.

23. A farm has tractors of four models, A, B, C, D. Four tractors (2 of model B and one of models C and D) plough a field in two days. Two model A tractors and one model C tractor take three days to do this job, and three tractors of models A, B and C take four days. How long will it take to do the job if a team is made up of four tractors of different models?

24. Grass was mowed on three fields in the course of three days. On the first day, all the grass of the first field was moved in 16 hours. On the second day, all the grass of the second field was mowed in 11 hours. On the third day, all the grass was cut on the third field in 5 hours, four hours of which the work was done with scythes and one hour by a moving machine. During the second and third days, together, four times mare grass was cut than on the first day. Haw many hours was the mowing machine in operation if in one hour it mows five times as much grass as is cut by hand. It is assumed that the hand and machine operations were separate in time (did not overlap) and there were no breaks in the work.

25. A factory has to deliver 1100 parts to a client. The parts are packed in boxes of three types. One box of type one holds 70 parts, one of type Two holds 40, and one of type Three holds 25 parts. The cast of delivery in a type one box is 20 dinar, in a type two box, 10 dinar, in a type three box, 7 dinar. What kind of boxes should be used in order to minimize the cost of delivery? All boxes must be used to full capacity.

26. A stamp collector decides to put all his stamps in a new album. If he puts 20 stamps on one sheet, there will be some left over; if he puts 23 stamps on a sheet, there will be at least one empty sheet left in the album. If the stamp collector is presented with an album of the same kind, each sheet holding 21 stamps, he will have a total of 500 stamps How many sheets are there in the album?

27. Two pipelines operating together fill a pool 3/4 full of water If one pipeline fills one-fourth of the pool first, and then the second (the first is then switched off) brings the volume of water up to 3/4 the capacity of tile pool, then this will require 2.5 hours. Now if the first pipeline is in operation for one hour and the second for half an hour, they will bring the water level up to more than one-half the pool. How long will it take each pipeline separately to fill the pool?

28. Points A and B are located on a river so that a raft floating downstream from A to B with the rate of the current covers the distance in 24 hours. A motorboat

goes from A to B and returns in less than 10 hours. If the rate of the motorboat in Still water were 40% greater, the same distance (from A to B and back) could be covered in not more than 7 hours. Find the time it takes the motorboat to go from A to B at the original rate (not increased).

29. At 8 A.M. a fast train leaves A for B. At the same time a passenger train and an express train leave B for A, the speed of the former being one half that of the latter. The fast train meets the express train not earlier than 10:30 A.M. and arrives at B at 13:50 the same day. Find the time of arrival of the passenger train at A if we know that not less than an hour elapsed between the meetings of the fast train and express train and the fast train and the passenger train.

30. At 9 A.M. a cyclist starts out from A in the direction of B. Two hours later a motorist sets out and overtakes the cyclist not later than 12:00 noon. The motorist continues on and reaches B, then immediately turns round and heads back for A. On the way, the motorist meets the cyclist and arrives in A at 17:00 hours that same day. When does the cyclist arrive in B if we know that no more than 3 hours elapsed between the two encounters of the cyclist and motorist?

Answer

1. 20 km/hr; 25 km/hr; 15 km/hr
2. 27, 18 and 12 years old
3. 30 km
4. 1/6
5. 19π cm/sec and 27π cm/sec
6. No
7. 3 hours, 6 hours, 2 hours
8. 5 grams and 20 grams
9. $5m\,(t - t_2) - 5m^{-1}\,(t - t_1)$
10. The area of the forest is 40 km^2. Obtain the equation $AC = 5 + 1/4BC^2 + 1/16\ AB^2$ from the statement of the problem; besides for any three points A, B and C the inequality $AC \le AB + BC$ is valid, whence $5 + 1/4\ BC2 + 1/16\ AB^2 \le AB + BC$ or $(1/2BC - 1)^2 + (1/4AB - 2)^2 \le 0$, which is possible only for AB = 8 and BC = 2.
11. The number of marks 2, 3, 4 and 5 are equal, respectively, to 11, 7, 10 and 2.
12. Velocities are : motorcycle, 40 km/hr, Moskvich car, 60 km/hr and Volga car, 80 km/hr
13. The water is delivered twice as fast
14. 1 : 3
15. 20 km/hr and 80km/hr
16. No
17. No
18. The rate of cyclist is 20 km/hr, that of the truck, 40 km/ hr, of the Volga car 80 km/hr. The distance from A to D is 60 km.
19. No
20. 60 m^3
21. 2 minutes
22. 0.6 km/min.
23. 12/7 days.
24. 12 hours
25. Four Boxes of the third type and 25 boxes of the second type.
26. 12 sheets
27. The first pipe will fill the pool in 2 hours, the second in 4 hours
28. 4 hours
29. 16 hours and 45 minutes
30. 18 hours

PART–II : VERBAL

Part A : English Verbal

1 Structure of Sentence

A. Sentences – Types and Interchange

1. The art of arranging words, phrases and clauses in correct order so that they make complete sense is called a sentence. The arrangement of such words, phrases and clauses in a systematic and proper manner is called the structure of sentence.

2. Every sentence consists of two parts—

(a) Subject

(b) Predicate

The word or words about which we say something, is called the subject.

What is said about the subject is called the Predicate of the sentence.

For Example—

Raj	killed a tiger.
↓	↓
Subject	*Predicate*

Raj is the person the sentence is saying something about; therefore, 'Raj' is the subject. What is said about the subject (Raj) 'killed a tiger'. This then is called the Predicate of the sentence.

3. There are Four types of sentences—

(a) Assertive

(b) Interrogative

(c) Imperative/ Optative

(d) Exclamatory

4. Assertive/Statement (Affirmative or Negative) Declarative Sentences—

When a statement that gives sense information or description whether in the negative or positive it is called an Assertive Sentence.

As,

He is a very handsome man. (*Positive*)

Harry can never be depended upon. (*Negative*)

Honesty is the best policy. (*Positive*)

She is not working properly. (*Negative*)

A positive sentence does not contain negative words such as, 'not', 'never', 'hardly', 'seldom', 'rarely' etc.

Whereas, a negative sentence utilizes such words.

5. Interrogative (Question) Sentences—Such sentences inquire about something. They ask, what ? Where ? When ? Why ? Who ? Whose ? How ? Etc.

As,

Where are you working ?

What are you doing ?

Do you know me ?

When are you returning ?

Why do you sleep so much ?

Who are you ?

Whose book is this ?

How are you ?

How far is Jaipur ?

6. Imperative Sentences—A sentence that expresses a command, request or wish is called an imperative sentence. They are also called optative sentences when they express a wish, prayer or curse.

As,

Close the door please. *(Request)*

Don't walk on the grass. *(Probation)*

Stand to attention. *(Command / Order)*

May you live long? *(Wish / Optative)*

Wish you a happy birthday. *(Wish)*

Note—In sentences 1, 2, 3 the subject is understood. Close the door please = You close the door.

7. Exclamatory Sentences—Sentences which express strong emotion or feeling or reaction in connection with a statement are called exclamatory sentences.

As,

What a goal ! (*Surprise*)

Bravo ! What spirit. (*Joy*)

How strong he is ! (*Surprise*)

How cute ! (*Joy*)

Alas ! We are ruined. (*Sorrow*)

What a pity! (*Regret*)

How dare he! (*Anger*)

Tut! Tut! (*Disapproval*)

Such sentences or words are followed by exclamation marks (!).

8. Structure Sentences—To structure a sentences one must be familiar with Auxiliary Verbs, Subject, Main Verbs, Object (Direct and Indirect) and other words.

8. (a) Auxiliary Verbs—Words (Auxiliary) of the Present, Past and Future Tense.

As,

Sub	Aux	V^3	O			
Mother	*will have*	cooked	the *food*	before	father	*arrives*.
				Timeword		$V^1 + S$

Present	Subject	Main Verb
(*Continuous*)		
Am	I	
Are	You, we, they (Plural)	V^1 + ing
Is	Name, he, she, it (Singular)	
(*Indefinite*)		
Do	I, you, we, they (Plural)	
Does	Name, he, she, it (Singular)	V^1
(*Perfect continuous*)		
Has been	Name, he, she, it (Singular)	
Have been	I, you, we, they (Plural)	V^1 + ing + for / since
(*Perfect*)		
Has	Name, he, she, it (Singular)	
Have	I, you, we, they (Plural)	V^3
8(b)		

Past	Subject	Main Verb
Was	You, we, they (Plural)	V^1 + ing
Were	Name, he, she, it, I (Singular)	
(*Indefinite*)		
Did	All subjects	$*^1V^1$
(*Perfect continuous*)		
Had been	All subjects	V^1 + ing + for / since
(*Perfect*)		
Had	All subjects	$V^3 + (V^2)$
8(c)		

Future	Subject	Main Verb
Will	You, we, they, Name, he, she, it	V^1
Shall	I, we	
(*Continuous*)		
Will be	You, we, they, Name, he, she, it	V^1
Shall be	I, we	
(*Perfect continuous*)		
Will + have + been	You, we, they, name, he, she, it,	V^1 + ing + for $*^2$
Shall + have + been	I, we	
(*Perfect*)		
Will have	You, we, they, name, he, she, it,	$V^3 + *^3$ (V^1, or + s, es, ies)
Shall have	I, we	

Note—

*1 In case of the affirmative the second form of the verb is utilized.

*2 The Perfect Continuous Future utilizes only 'for' (period of time) and not 'since' as it is in the future and hence since (point of time) is not applicable.

*3 The Future Perfect tense indicates an action that will have finished before another starts and is followed by the Present Indefinite with affirmative statements add s, es or ies to the first form of the verb.

Remember,

The Present Indefinite in affirmative statements or answers add s, es or ies to the first form of the verb according to the nature of the word.

As,

He runs. (V^1 + s)

She goes to school. (V^1 + es)

Tom replies to all my letters. (V^1 + ies)

* However, in Interrogative and negative sentences Do, Does + (not) or simply V^1 Not is negative.

9. Structure of Interrogative (Question) Sentence—Helping Verb (Aux.) + Subject + Verb + others ?

9(a) Do You read books?

Formula—

It becomes simpler if you follow and understand it in an easy mathematical formula.

Think of the subject as 1, the Auxiliary verb as 2 and the main verb as 3. Got it.

Now consider this.

To form a question use the formula. 2 + 1 + 3, and you have created a question. 2 stands for the Auxiliary Verb, 1 stands for its subject and 3 stands for the main verb.

The above formula changes its auxiliary verb and main verb according to the time frame required. Viz. Present, Past and Future.

As,

(a)	2	+	1	+	3 ?	
	Do		you		exercise?	*(Present)*
(b)	2	+	1	+	3 ?	
	Did		he		play ?	*(Past)*
(c)	2	+	1	+	3 ?	
	Will		they		talk ?	*(Future)*

There isn't it easy just remember 1, 2, 3 and you can structure an Interrogative sentence. Not to mention the assertive (positive and negative) which follows.

How to make a question? Easy, 2 + 1 + 3? And you're got a question. Simple, isn't it? Lets move on to the assertive (positive answer / statement / Declaration) formula. It's easy as 1, 2, 3.

9(b) The formula for the assertive is as shown below:

Positive Answer = Yes + 1 + 2 + 3

(Are you going ?) = Yes, I am going

There, by remembering 1, 2, 3 and placing yes before it you have a positive answer. To make a positive statement, simply remove the 'yes'.

As,

1	+	2	+	3
I		am		going

9(c) To make a negative sentence (answer)

Use,

No	+	1	+	2	+	not	+	3
No		I		am		not		going

Just remember, No, 1, 2 not 3, and you have a negative answer.

If you desire to create a negative statement or declaration simply remove the 'no' from the formula.

As,

1	+	2	+	not	+	3
I		am		not		going

Recall—

Question	=	2	+	1	+	3 ?
Positive Answer	=	Yes	+	1	+	2 + 3
Negative Answer	=	No	+	1	+	2 + not + 3
Positive Statement	=	1	+	2	+	3
Negative Statement	=	1	+	2	+	not + 3

10. Question Words—The interrogative sentence may begin with (a) an auxiliary verb (b) a main verb or (c) a question word.

As,

(a) Is he coming ? (*Auxiliary Verb*)

(b) Has he a car ? (*Main Verb*)

(c) Where are you going ? (*Question word*)

There are basically nine question words.

They are—

1. What
2. Where
3. When
4. Why
5. Who
6. Whom (with)
7. Which
8. Whose
9. How

11. The question word structure is simple—You now know that the formula for a question is 2 + 1 + 3. However, while making an interrogative sentence with a question word place the question word/s before 2.

Example—

QW	+	2	+	1	+	3 ?
Where		do		you		live ?
Q.word		*Aux.*		*Sub.*		*M.Verb*

12. (a) What (*Pronoun / Adjective*)—Inquires about persons / things etc.

(b) Where (*Adverb*)—Inquires about place.

(c) When (*Adverb*)—Inquires about time.

(d) Why (*Adverb*)—Inquires about reason.

(e) Who (*Pronoun*)—Inquires about persons.

(f) Whom (*Pronoun*)—Inquires about persons (*Object*).

(g) Which (*Pronoun/Adjective*)—Inquires about persons or things when options are limited.

(h) Whose (*Pronoun/Adjective*)—Inquires about possession (*Subject or object*)

(i) How (*Adjective/ Adverb*)—To find out methodology, health, distance, route, size etc.

13. How + Adjective / Adverb—

1. How many—(*Countable*) number.

2. How much—Quantity (*Quantitative–Uncountable*)

3. How far— Distance

4. How long— Duration

5. How often— Frequency

6. How quickly—Time span

Also

7. At what point—(*exact place*)

8. At what time—(*exact time*)

9. What brand of / sort of / etc. (*variety*)

14. (a) Examples of the Question word **'What'**—

(a) What do you play ?

(b) What does your father do ?

(c) What is the time ?

(d) What is the matter ?

(e) What is he doing ?

(f) What can you see ?

(g) What did she ask ?

(h) What are you doing this evening ?

14. (b) Examples of the Question word **'Where'**—

(a) Where is my pen ?

(b) Where are your brothers staying ?

(c) Where is Iceland ?

(d) Where can I find a petrol pump ?

(e) Where is the railway station ?

(f) Where do you put your shoes ?

14. (c) Examples of the Question word **'When'**—

(a) When does Aunty visit you ?

(b) When will the train arrive ?

(c) When shall I see you again ?

(d) When did the police catch the thief ?

(e) When do these flowers bloom ?

(f) When does Hari exercise ?

14. (d) Examples of the question word **'Why'**—

(a) Why are you leaving ?

(b) Why is she sad ?

(c) Why are they rejoicing ?

(d) Why do bird fly and not humans ?

(e) Why did she resign ?

(f) Why are they quarrelling ?

14. (e) Examples of the question word **'Who'**—

(a) Who is calling ?

(b) Who is your father ?

(c) Who are you ?

(d) Who saw me cheating ?

(e) Who told a lie ?

(f) Who stole the cream ?

14. (f) Examples of the question word **'Whom / With whom'**—

(a) Whom do you consider the best ?

(b) With whom is she going ?

(c) Whom do you know that speaks Greek ?

(d) Whom are you talking to ?

14. (g) Examples of the question word **'Which'**—

(a) Which book are you reading?

(b) Which of these mangoes do you want?

(c) Which movie is being shown?

(d) Which coat do you like?

(e) Which actor is your favourite?

(f) Which day is auspicious?

* Which is generally followed by a noun or a noun phrase.

14. (h) Examples of the question word **'Whose'**—

(a) Whose watch are you wearing ?

(b) Whose books are those ?

(c) Whose house are you living in ?

(d) Whose orders are you following ?

(e) Whose cell phone is that ?

(f) Whose money are you spending ?

* 'Whose' like 'Which' is followed by a noun.

14. (i) Examples of the question word **'How'** :

The question word 'How' covers a wide range of areas when attached to an adjective or adverb.

Single use of 'How'—

(a) How are you ? (*Health / well being*)

(b) How did you fix it ? (*Methodology / way*)

(c) How do you reach the station ? (*Route*)

How + Adjective

(a) How far is Delhi ? (*Distance*)

(b) How handsome is your brother ? (*Degree of Beauty*)

(c) How tall is Ravi ? (*Height*)

(d) How much oil do you use ? (*Quantity uncountable*)

(e) How many spoonfuls of sugar do you take ? (*Number countable*)

(f) How long have you been in Haridwar ? (*Duration*)

(g) How often do you visit Raj ? (*Frequency*)

(h) How many times have I warned you ? (*Frequency*)

Now, that you have an idea and a certain grasp of question words test your skill by doing the following exercises.

Exercise—1

Fill in the blanks with the appropriate **Question words**—

1. .... are you doing ?
2. .... is she going ?
3. .... school do you study in ?
4. .... far is the railway station ?
5. .... pen is this ?
6. .... are you talking to ?
7. .... beautiful the mountain look ?
8. .... close are we to Delhi ?
9. .... is the doctor coming ?
10. .... is the matter ?

Exercise—2

Fill in the blanks with appropriate **Adjectives or Adverbs**—

1. How .... boys are in the class ?
2. How .... money do you have ?
3. How .... can you count ?
4. How .... is America ?
5. How .... your brother is !
6. How .... the crow is !
7. How .... can you come ?
8. How .... he has become !
9. How .... to your house does he live ?
10. How .... from your house, is he ?

Note—The answer page will give you an idea of the kinds of words you can use. But, it should suit the sense of your question or exclamation.

Exercise—3

Fill in suitable words :

1. How .... the rises. (*Adverb of time*)
2. Whose .... have they stolen ? (*Common Noun*)
3. .... have you done ? (*Question Word*)
4. Which .... has been selected ? (*Common Noun*)
5. .... is the bus arriving ? (*Question Word*)
6. .... is my bag ? (*Question Word*)
7. How .... is the river ? (*Distance*)
8. How .... petrol is there ? (*Quantity*)
9. How .... books do you have ? (*Number*)
10. How .... your sister is! (*Beauty*)

Note—All words except question words and words of quality and number a wide range of options.

Exercise—4

Fillers—

1. .... will you buy me a present ? (*Question Word*)
2. .... watch are you wearing ? (*Question Word*)
3. .... are they doing ? (*Question Word*)
4. .... is that boy ? (*Question Word*)
5. With .... are you going ? (*Question Word*)
6. .... hotel are you staying in ? (*Question Word*)
7. .... old are you ? (*Question Word*)
8. .... are you watching ? (*Question Word*)
9. .... are you laughing ? (*Question Word*)
10. .... is the child crying ? (*Question Word*)

Exercise—5

1. .... fast can you run ?
2. .... do you think you are going ?
3. .... is the computer not working ?
4. .... does the shop close ?
5. .... house is being sold ?
6. .... is the matter ?
7. .... stupid she is !
8. How .... knowledge do you have ?
9. How .... litres of milk did you buy ?
10. .... met your father in the market ?

Answers

Exercise—1

1. What 2. Where 3. Which 4. How 5. Whose 6. Who 7. How 8. How 9. When 10. What

Exercise—2

1. Many 2. Much 3. Much/many 4. Far 5. Handsome 6. Clever 7. quickly 8. thin 9. close 10. far

Exercise—3

1. early 2. watch 3. what 4. girl 5. when 6. where 7. deep 8. much 9. many 10. beautiful

Exercise—4

1. When 2. Whose 3. What 4. Who 5. Whom 6. Which 7. How 8. What 9. Why 10. Why

Exercise—5

1. How 2. Where 3. Why 4. When 5. Which 6. What 7. How 8. much 9. many 10. Who

B. ARTICLES - *Types of Articles*

A, AN and THE are the three articles of two types in the English Language.

(i) Indefinite Article – 'A and An'

(ii) Definite Article- 'The'

Which one is right, wrong, preferable and acceptable

1. Horse is a faithful animal.
2. A horse is a faithful animal (A horse = Any horse)
3. An horse is a faithful animal
4. The horse is a faithful animal (The horse = All horse)

Sentence

1. is right and acceptable
2. is right and preferable
3. is wrong and unacceptable here 'H' sounded as consonants
4. is right and preferable

In 1st sentence Horse is used in its widest sense. In 2nd sentence Horse used as a common noun and it is work of representative of class mean 'Any Horse' Hence omission of article before horse is writing and the sense of 4th sentence described the quality of whole class. Mean each horse is a faithful animal.

So, it is clear that in Standard English the use of Articles play a vital role to describe the meaning of sentences. For better understanding we present rules of the articles.

Position of Articles

Case A—Articles are used before **noun** like – a **book**, the **day**, an **apple**; but if noun is use with **adjective** then article should placed before adjective. Like—a **good** library.

Case B—Always use article after **many/such/what** not before these specific words. Like—**many** a man; **such** an event; **what** a song.

Case C—If **as/how** is used before adjective then articles should be used after adjective. Like—How **good** a book; How **nice** a girl you are; As **good** a show as that.

Case D—If **so/too** used before adjective, then articles can be used before or after to adjective. Like—So **serious** an attempt or a so **serious** attempt.

Rules of Articles

Rule 1—The Article 'An' is used in the following cases:-

Rule 1(a)—Before words beginning with an open vowel which is sounded as open vowel.

Illustrations1—An **a**ss; An **a**pple; An **u**mbrella; An **o**wl; An **e**ar;

Exception—But if an open vowel is not sounded as open vowel we used 'a' before such vowel- A **u**seful book; a **u**niversity; a **o**ne rupee note.

Rule 1(b)—Before a word beginning with consonants which sounded as open vowels.

Illustrations—An **M**BA, An **S**P, An **F**IR, An **L**LB.

Rule1(c)—Before word beginning with silent 'H'

Illustrations—An **h**our; An **h**eir; An **h**onour; An **h**onest; An **h**onorary

NB:- Incorrect An Humble; Correct: A Humble

*(Due to its present pronounce as **Hotel** etc.)*

Rule 2—The Article 'A' is used in the following cases—

Rule 2(a)—Before a verb which is used as a noun

Illustrations—You should have a **rest** now; He has gone for a **walk**

Rule 2(b)—Before a common noun in the singular to suggest the sense of one.

Illustrations—Wheat sells twelve rupees a **kilo**; Wait for a **minute**.

Rule 2(c)—Before abstract noun when we use abstract noun as a kind of quality.

Illustrations—He has a **working knowledge** of Thai grammar.

Rule 2(d)—Before the common noun in the singular to suggest the sense of any.

Illustrations—A **bird** has wings (a bird = any bird)

Rule 2(e)—Before a common noun in singular to suggest the sense certain.

Illustrations—While going to church, I saw a **thief** being beaten by the police.

Rule 2(f)—Before a proper noun when it is used as common noun.

Illustrations—He thinks. He is a **Hercules**. He is a **Milkha Singh**.

Rule 2(g)—Before a uncountable noun when it is used as a countable

Illustrations—I have a **good news** for you. I have a **milk bar.**

Rule 2(h)—Some more sense are define by 'a' are as follows—

Sense of each—we observe **holiday** once a **week.**

Sense of same—**Elephant** and **mouse** are of an **age**.

Some idiomatic phrases—To make a five, a piece of advice, I had a headache, a dozens eggs.

Rule 3—The definite Article 'The' is used.

Rule 3(a)—Before a particular person or things, or one already referred to or known as the speaker.

Bring a ticket from the **station**. (only station of city)

Let us go to the **institute**. (both or all are the students of same institute)

Rule 3(b)—When a singular noun is meant to represent a whole class.

The **horse** is a faithful animal.

The **cow** is a useful animal.

Rule 3(c)—Before a common noun to give it the force of superlative.

This is a **thing** to do

He is the **leader** (he is the best leader)

Rule 3(d)—Before a common noun to give it the meaning of an abstract noun.

The **mother** (parented affection) in her pitied the child.

He is the lover of the **good**, the **pure** and the **beautiful**.

Rule 3(e)—Before a common noun instead of possessive adjectives—

He caught her by the **arm**. (instead of by her arm)

Rule 3(f)—When the thing or noun is understood

We should help the poor.

Kindly return the book.

Rule 3(g)—With superlative

She is the **wiser** of the two

He is the **greatest** philosopher of the world.

Rule 3(h)—As an adverb with a comparative

The **sooner**, the **better.**

The **more** you eat, the **fatter** you become.

The **higher** you go the **colder** is it.

Rule 3(i)—I Some more uses of 'the' Before

(i) Circus/ picture/ theater/ cinema/ office/ station/ bus stop—station

he is at the **office**

(ii) Before the name of Musical instruments.

he plays the **Violin**; the **sitar**; the **table**

(iii) Before the name of clubs or foundations.

Dr. Ramesh is a member of the **JRD Young Club**

(iv) Before the nationality, community or political party

The **Indian**, the **Hindus**

(v) Before the unique things

The **sun**, the **moon**, the **air**, the **sea**,

(vi) before the name of river sea, ocean, mountain ranges, group of islands, famous buildings, ships, famous boots and news papers

The **Taj**, The **Times of India**, The **Ramayana**, The **West**

(vii) Before the name of some profession

The **author**, the **poet**, the **press**.

(viii) Before the name of invented things

The **radio**, the **barometer**.

(ix) Before the name of post

The **director** will decide the matter.

He is the president and CEO of the ganga gloves maker. (mean one persone holding two offices)

The president and the CEO of the company are twins. (mean two different person with different posts)

(x) In certain fixed idiomatic phrases.

In the town; Off the mark; In the worry; At the time

Rule 4—Omission of articles

Rule 4(a)—Before name of disease, names of regular meals seasons, games, language, colour and name of something single kind.

Fever, consumption. Like—she is suffering from fever.

But when the diseases are plural in their form, the article is not omitted. Like—The **measles** is a contagious disease.

—Breakfast, lunch, dinner

—This summer is very hot in comparison to last summer

—I play chess

—He does not know Thai that but he knows French

—I like pink and yellow

— Hell, heaven, god, parliament (but the pope, the devil are exceptions)

Rule 4(b)—Before such as are plural in their sense though singular in form.

Furniture, scenery, cattle, gentry, business, society, folk, people, mankind

Rule 4(c)—Before a noun used in its widest sense

Birds' fly, Dogs have very keen noise

Rule 4(d)—'The' is not used before market, library, institute, church, hospital, school, prison, table, bed,

temple, parliament, and university etc, when these places are visited or used for their primary purpose.

—He went to library (it means that he went to library to study or for books)

—They went to the library and from there they joined the rally. (it means their purpose was thought of rather than actual building)

Rule 4(e)—Before relatives (like brother, sister, father, nurse, or cook) days, months and material

—Father has returned from the office

—Sunday is a holiday

—Gold is a precious metal

Rule 4(f)—Before abstract noun that express quality, feelings, action or processes of thought

Honesty is the best policy

Rule 4(g)—Before a noun following the expression 'kind of'

What kind of suit do you like?

But if kind of/ sort of indicate any quality/ capacity then use of a / an is must.

—What kind of an artist is she?

Rule 4(h)—Some more places where articles are prohibited

(i) When noun in pairs

—Door to door, neck and neck, brother & sister, face to face

(ii) When the post is only one

—he has been appointed headmaster

—he was appointed as a teacher (because teacher can be more than one)

(iii) When noun follows by preposition

—on demand, by hand, by train, by courier, on foot, on earth

(iv) When objects are followed by transitive verb

—To send word, to cast anchor, to lose heart, to leave home

(v) Before some title or name

—Emperor Ashoka, President APJ Kalam, Captain George.

Exercise—1

In each of the following passage there are Articles, each of which has been bold and italic. Identify the rule of article by which you can justify its position.

VEERING to ***the*** left of ***the*** Hero Honda roundabout in Manesar, Gurgoan is ***a*** nondescript road. ***A*** 100 meters down this path stands ***a*** building, which symbolizes ***the*** rising global acceptance of India's fashion design industry –***the*** 3,50,000–square feet designing and manufacturing unit of Orient Craft, one of India's largest export houses. On ***the*** ground floor of this Rs.750 crore company, in ***a*** glass cabin overlooking ***the*** work stations of 100 associates, 42 year old Anoop Thataj, Joint managing Director and CEO of ***the*** company, is busy discussing ***the*** new spring collection for ***a*** US customer, Finally, after hours of discussions, ***a*** few cuts, silhouettes and fabrics are short–listed. Then ***the*** design team of around 100, along with ***a*** support staff of 700, begins work on rolling out ***the*** products. Say Anoop Thataj. "***The*** team has to complete ***the*** projects in ***the*** next 14 days. Then we begin work for ***a*** major European retail brand. I am running at full capacity. Besides manufacturing prototypes, we are developing our own design lines".

For Orient Craft, it has been ***an*** eventful journey, for, just 10 years back it was manufacturing apparel for international clients with little value addition. But ***the*** company has climbed up ***the*** value chain. Says Sudhir Dhingra, Chairman and Managing Director, Orient Craft: " Out of ***the*** 65% woman's wear produced by us, almost 40% have our own design input and we produced 2,000 design samples a day.This differentiates from competition and certain clients get back to us for particular designs."

The design element in ***the*** apparels and accessories industry – apparel alone is ***a*** Rs.30,000 crore market–has risen by almost 80%. Graduating from assembly line operations for Western labels, Indian design firms are now creating there own lines based on strong in–house R and D capabilities. Says Devangshu Dutta, Chief Executive, Third Eyesight, ***a*** Delhi based fashion consulting firm. "***The*** days of cut, copy, paste are coming to ***an*** end as every exporter looks for ***a*** distinct image. This is possible only if you innovate in design."

While big export houses like Orient Craft are enhancing their business by emphasizing on design, international firms are looking at India as ***an*** outsourcing hub. This is spawning many start–ups, such as Bangalore–based Munch Design and Delhi–based Bricolage, which are developing lines of apparel and accessories for global brands like Nike, Reebok, esprit, Adidas, Zara, Guess, Macy's and Gucci. Says Narinder Mahajan, Founder, Bricolage: "Clients depend on us for forecasts and trends. Right from deciding on ***the*** theme–based collection names to ***the*** final sampling, every thing is done by us." Bricolage is now developing ***a*** casual clothing division for Reebok and ***a*** range of shirts and Tees for Benton.

A Cut above the Rest

Design in apparel as ***a*** key differentiates comes at a premium. According to industry sources, ***a*** prototype consignment of 10,000 shirts to ***the*** US would cost $ 10 per shirt. But with elements of design like embroidery, embellishments and cuts, ***the*** same shirt would cost $20 or more. Says Vijay Agarwal, President of Apparel Export Promotion Council–India's strength is design; where is mass producer. We need to balance ***the*** two–numbers and design innovations–for enhancing exports."

However, not just exports, ***the*** design elements are slowly creeping into ***the*** lives of domestic consumers too. No more ***the*** plain shirts for ***the*** Indian male. ***The*** choice has widened to embroidered, pleated, crushed, crystal–laden and metallic shirts. For women though, there's practically no end to ***the*** need for choice.

Homegrown companies like Pantaloon and Madura garments are busy satiating ***the*** design needs of Indian consumers. Says Hemchandra Jaweri, Senior Executive President, Madura Garments lifestyle Brands and retail: "***The*** importance of design will be further heightened in future as Indian consumers get more in sync with global trends. Indian companies will have to compete in design, branding and retail. I see this as the key differentiate of ***the*** future." Madura Garments owns brands like Allen Solly, Allen Solly Womens, Peter England, Van Heusen, SF Jeans, Louis Philippe, Byford, Elements and San Frisco. "We try to balance fashion, Innovations and Commercial logic," he adds.

Pantaloon Retail too offers ***a*** variety of apparel and accessories targeted at men, women and kids. Says 43 year old Kishor Biyani, MD of pantaloon.

Discussion—Rules of Articles

STANZA–1

Sentence–1	VEERING to ***the*** left of ***the*** Hero Honda roundabout in Manesar, Gurgoan is ***a*** nondescript road.	**Rule 3(i)-x; Rule 3 (i)-vi; Rule 2(b)**
Sentence–2	***A*** 100 meters down this path stands ***a*** building, which symbolizes ***the*** rising global acceptance of India's fashion design industry – ***the*** 3,50,000 – square feet designing and manufacturing unit of Orient Craft, one of India's largest export houses.	**Rule 2(b); Rule 2(e); Rule 3(c); Rule 3(i)-x**
Sentence–3	On ***the*** ground floor of this Rs.750 crore company, in ***a*** glass cabin overlooking ***the*** work stations of 100 associates, 42 year old Anoop Thataj, Joint managing Director and CEO of ***the*** company, is busy discussing ***the*** new spring collection for ***a*** US customer, Finally, after hours of discussions, ***a*** few cuts, silhouettes and fabrics are short - listed.	**Rule 3(i)-x;2(e); Rule 3(a); Rule 3(a); Rule 3(c); Rule 2(e); Rule 2(d)**
Sentence–4	Then ***the*** design team of around 100, along with ***a*** support staff of 700, begins work on rolling out ***the*** products.	**Rule 3(i)-x; Rule 2(b); Rule 3(a)**
Sentence–5	Say Anoop Thataj. "***The*** team has to complete ***the*** projects in ***the*** next 14 days.	**Rule 3(a); Rule 3(a); Rule 3(i)-x**
Sentence–6	Then we begin work for ***a*** major European retail brand.	**Rule 2(e)**
Sentence–7	I am running at full capacity	**NA**
Sentence–8	Besides manufacturing prototypes, we are developing our own design lines".	**NA**

STANZA–2

Sentence–1	For Orient Craft, it has been ***an*** eventful journey, for just 10 years back it was manufacturing apparel for international clients with little value addition chain.	**Rule 1(a)**
Sentence–2	But ***the*** company has climbed up ***the*** value chain.	**Rule 3(a); Rule 3(i)-x**
Sentence–3	Says Sudhir Dhingra, Chairman and Managing Director, Orient Craft: "Out of ***the*** 65% women's wear produced by us, almost 40% have our own design input and we produced 2,000 design samples ***a*** day.	**Rule 3(a); Rule 2(d)**
Sentence–4	This differentiates from competition and certain clients get back to us for particular designs."	**NA**

STANZA–3

Sentence–1	***The*** design element in ***the*** apparels and accessories industry – appel alone is ***a*** Rs.30,000 crore market – has risen by almost 80%.	**Rule 3(a); Rule 3(i)-x; Rule 2(b)**
Sentence–2	Graduating from assembly line operations for Western labels, Indian design firms are now creating their own lines based on strong in – house R & D capabilities.	**NA**
Sentence–3	Says Devangshu Dutta , Chief Executive, Third Eyesight, ***a*** Delhi based fashion consulting firm.	**Rule 2(b)**
Sentence–4	"***The*** days of cut, copy, paste are coming to ***an*** end as every exporter looks for ***a*** distinct image.	**Rule 3(i)-x; Rule 1(a); Rule 2(e)**
Sentence–5	This is possible only if you innovate in design."	**NA**

	STANZA–4	
Sentence–1	While big export houses like Orient Craft are enhancing their business by emphasizing on design, international firms are looking at India as ***an*** outsourcing hub.	**Rule 1(a)**
Sentence–2	This is spawning many start - ups, such as Bangalore – based Munch Design and Delhi – based Bricolage, which are developing lines of apparel and accessories for global brands like Nike, Reebok, esprit, Adidas, Zara, Guess, Macy's and Gucci.	**NA**
Sentence–3	Says Narinder Mahajan, Founder, Bricolage: "Clients depend on us for forecasts and trends.	**NA**
Sentence–4	Right from deciding on ***the*** theme – based collection names to ***the*** final sampling, every thing is done by us."	**Rule 3(i)-x; Rule 3(i)-x**
Sentence–5	Bricolage is now developing ***a*** casual clothing division for Reebok and ***a*** range of shirts and Tees for Benton.	**Rule 2(b); Rule 2(b)**
	STANZA–5	
Sentence–1	Design in apparel as ***a*** key differentiates comes at ***a*** premium.	**Rule 2(b); Rule 2(b)**
Sentence–2	According to industry sources, ***a*** prototype consignment of 10,000 shirts to ***the*** US would cost $ 10 per shirt.	**Rule 2(b); Rule 3(i) –iv**
Sentence–3	But with elements of design like embroidery, embellishments and cuts, ***the*** same shirt would cost $20 or more.	**Rule 3(a)**
Sentence–4	Says Vijay Agarwal, President of Apparel Export Promotion Council – India's strength is design; where is ***a*** mass producer.	**Rule 2(b)**
Sentence–5	We need to balance ***the*** two – numbers and design innovation – for enhancing exports."	**Rule 3(a)**
	STANZA–6	
Sentence–1	However, not just exports, ***the*** design elements are slowly creeping into ***the*** lives of domestic consumers too.	**Rule 3(a); Rule 3(a)**
Sentence–2	No more ***the*** plain shirts for ***the*** Indian male.	**Rule 3(a); Rule 3(a)**
Sentence–3	***The*** choice has widened to embroidered, pleated, crushed, crystal – laden and metallic shirts.	**Rule 3(a)**
Sentence–4	For women though, there's practically no end to ***the*** need for choice.	**Rule 3(a)**
	STANZA–7	
Sentence–1	Homegrown companies like Pantaloon and Madura garments are busy satiating ***the*** design needs of Indian consumers.	**Rule 3(a)**
Sentence–2	Says Hemchandra Jaweri, Senior Executive President, Madura Garments lifestyle Brands and retail: "***The*** importance of design will be further heightened in future as Indian consumers get more in sync with global trends.	**Rule 3(a)**
Sentence–3	Indian companies will have to compete in design, branding and retail.	**NA**
Sentence–4	I see this as the key differentiate of ***the*** future."	**Rule 3(a)**
Sentence–5	Madura Garments owns brands like Allen Solly, Allen Solly Womens, Peter England, Van Heusen, SF Jeans, Louis Philippe, Byford, Elements and San Frisco.	**NA**
Sentence–6	"We try to balance fashion, Innovations and Commercial logic," he adds.	**NA**
	STANZA–8	
Sentence–1	Pantaloon Retail too offers ***a*** variety of apparel and accessories targeted at men, women and kids. Says 43 year old Kishor Biyani, MD of pantaloon.	**Rule 2(e)**

Exercise –2

In each of the following passage there are Articles, each of which has been bold and italic. Identify the rule of article by which you can justify its position.

Lakshmi mittal is keen to invest over $ 15 billion in India.

EVEN AS KASHMI MITTAL goes about ***the*** challenging task of making synergies work for Arcelor Mittal, ***the*** president and CEO of ***the*** world's largest steel company hasn't lost track of his plans for manufacturing in India. Mittal is looking to set up two Greenfield projects in India in ***the*** states of Jharkhand and Orissa. ***The*** total investment across these projects is expected to be in excess of $ 15 billion (Rs.61, 500 crore).

Announcing ***the*** first quarter results of Arcelor Mittal, Mittal in ***a*** conference call with ***the*** media said that discussions and negotiations were in progress with ***the*** two state governments in India. Responding to ***a*** query from BT on ***the*** states of these too much talked about projects, Mittal said: "***The*** negotiations are with respect to ***the*** allotment of land and also for ***the*** allotment of iron ore. Once these issues are resolved, we will start work on ***the*** construction site". He added that there was work in progress on ***a*** detailed project reports. "We expect that this to be ready in 18 months."

Mittal, of course, knows he isn't ***the*** only one in expansion mode. Large players like Tata Steel, Essar Steel, Jindal Steel & Power and SAIL have already announced expansion plans. "We expect ***the*** total capacity in ***the*** steel sector to be at around 90 – 100 million tonnes by 2015(from ***the*** current 40 million tonnes)," says Mittal.

Meanwhile, Mittal also gave an update on ***the*** Arcelor–Mittal merger and said ***the*** objective was to complete it as soon as possible during ***the*** course of 2007. "***The*** integration process has been in line with our plans and there have been savings from synergies to ***the*** extent of $ 573 million (Rs.2, 349.3 crore) during ***the*** first quarter of 2007. This is against our expectation of $ 500 million (Rs.2, 050 crore)," he told ***the*** media. Mittal isn't called ***a*** steel magnate for nothing.

	STANZA–1	
Sentence–1	EVEN AS KASHMI MITTAL goes about ***the*** challenging task of making synergies work for Arcelor Mittal, ***the*** president & CEO of ***the*** world's largest steel company hasn't lost track of his plans for manufacturing in India.	**Rule-3(i)-x Rule-3(i)-x Rule3(g)**
Sentence–2	Mittal is looking to set up two Greenfield projects in India in ***the*** states of Jharkhand and Orissa.	**Rule – 3(i) –iv**
Sentence–3	***The*** total investment across these projects is expected to be in excess of $ 15 billion (Rs.61, 500 crore).	**Rule- 3(i)-x**
	STANZA–2	
Sentence–1	Announcing ***the*** first quarter results of Arcelor Mittal, Mittal in ***a*** conference call with ***the*** media said that discussions and negotiations were in progress with ***the*** two state governments in India.	**Rule-3(i)- x Rule -2 (e) Rule -3(b) Rule- 3 (i)- iv**
Sentence–2	Responding to ***a*** query from BT on ***the*** states of these too much talked about projects, Mittal said: "***The*** negotiations are with respect to ***the*** allotment of land and also for ***the*** allotment of iron ore.	**Rule- 2(e) Rule- 3(i)-x Rule 3(a) Rule-3(a) Rule-3(a)**
Sentence–3	Once these issues are resolved, we will start work on ***the*** construction site".	**Rule- 3(a)**
Sentence–4	He added that there was work in progress on ***a*** detailed project reports.	**Rule- 2(e)**
Sentence–5	"We expect that this to be ready in 18 months."	
	STANZA–3	
Sentence–1	Mittal, of course, knows he isn't ***the*** only one in expansion mode.	**Rule- 3(i)-iv**
Sentence–2	Large players like Tata Steel, Essar Steel, Jindal Steel & Power and SAIL have already announced expansion plans.	
Sentence–3	"We expect ***the*** total capacity in ***the*** steel sector to be at around 90 – 100 million tonnes by 2015(from ***the*** current 40 million tonnes)," says Mittal.	**Rule- 3(b) Rule- 3(b) Rule- 3(i)-iv**
	STANZA–4	
Sentence–1	Meanwhile, Mittal also gave an update on ***the*** Arcelor – Mittal merger and said ***the*** objective was to complete it as soon as possible during ***the*** course of 2007.	**Rule-3(a) Rule-3(a) Rule-3(i)-x**
Sentence–2	"***The*** integration process has been in line with our plans and there have been savings from synergies to ***the*** extent of $ 573 million (Rs.2, 349.3 crore) during ***the*** first quarter of 2007.	**Rule- 3(i)-x Rule-3(i)-x Rule-3(i)-x**
Sentence–3	This is against our expectation of $ 500 million (Rs.2, 050 crore)," he told ***the*** media.	**Rule- 3(i)-iv**
Sentence–4	Mittal isn't called ***a*** steel magnate for nothing.	**Rule- 2(b)**

2 Parts of Speech

1. In order to become adept in Grammar one must develop a strong grounding in the basic rules and application of English grammar.

First and foremost, the student must be well versed in the Eight Parts of Speech.

Words are slotted into different kinds or classes in accordance to the purpose for which they are utilized. These are called parts of speech and they consist of 8 parts in all.

1. Noun
2. Pronoun
3. Adjective
4. Verb
5. Adverb
6. Preposition
7. Conjunction
8. Interjection

As,

Horse	Noun
The Horse	Article + Noun
The Horse stands	Article + Noun + *Verb*
The Horse stands firmly.	Article + Noun + *Verb* + *Adverb*
The Horse stands firmly on the Hill.	Article + Noun + *Verb* + *Adverb* +*Preposition* + *Article* + *Noun*
The black Horse stands firmly on the Hill.	Article + Adjective + *Noun* + *Verb* + *Adverb* + *Preposition* + *Article* + *Noun*
It stands on the Hill.	Pronoun + *Verb* + *Preposition* + *Article* + *Noun*
Since it stands on the hill it overlooks the plain.	*Conjunction* + *Pronoun* + *Verb* + *Preposition* + *Article* + *Noun* + *Pronoun* + *Verb* + *Article* + *Noun*

It is quite often very difficult to say what part of speech a word belongs to. Unless and until we see the placement of the word in a sentence as it can assume different roles based on its usage in a sentence.

Consider these sentences—

(a) Plant the tulips *(Here 'plant' is used as a verb)*

(b) Fetch some plants. *(Here 'plants' are used as a Noun)*

(c) He spotted a plant bug. *(Here 'plant' is used as an adjective)*

NOUN

A **Noun** is a naming word (derived from Latin *nomen* which means name.)

Such as, tree, house, car, woman, Raj, Ravi, Sita, Gold, Silver, Hatred, Anger, Team, bunch etc.

PRONOUN

A **Pronoun** (the second part of speech) is used in the place of a noun.

It refers to a noun, an individual/s or thing/s whose identity has been determined earlier in a sentence/the text.

Consider these examples—

(a) I met a boy on the way. He was pleased to meet me.

(Here 'he' refers to 'a boy' and works as a pronoun)

(b) My uncle and aunt are avid readers. They love to read whenever they can.

(Here 'they' refer to 'uncle and aunt' and works as a pronoun)

In the first sentence, the pronoun 'he' is used instead of repeating 'boy'. In the second example, the pronoun 'they' is used in the place of 'uncle and aunt' instead of repeating uncle and aunt over and over again.

ADJECTIVE

An **Adjective** is a qualifying word. That is, it adds something to the meaning of a noun. It is also called a describing word.

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Adjectives usually precede a noun.

As,

(a) The tall man with black hair.

(b) A beautiful flower has many hues.

It is also used as the object of a noun or pronoun.

As,

(a) Ram is tall.

(b) That flower is beautiful. Its hues are many.

VERB

A **Verb** is an action word. It does some-thing. Nouns names things; persons things etc. Verbs predicate or say something about the Noun or Pronoun.

As,

(a) A group of boys played football. *(Here it tells us what the group did)*

(b) He walked into the room. *(Here it tells us what 'he' (pronoun) did)*

(c) She dances well. *(Here the action of the pronoun 'she' is expressed)*

ADVERB

An **Adverb** modifies the verb; it indicates how the action of a verb is carried out.

As,

(a) The horse stands firmly.

(b) She speaks well.

(c) He dresses beautifully .

It can also modify an adjective or another adverb.

The house is very firm.

She answered most considerately

PREPOSITION

A **Preposition** connects a noun (with or without an article) or a pronoun to some other word.

As,

(a) It stands on a hill.

(b) Australia is over the sea.

(c) She told the good news to him.

CONJUCTION

A **Conjunction** is a joining word. It is used for no other purpose.

A conjunction is never connected with an object as a Preposition.

A conjunction doesn't qualify a word as an Adverb does. Its function is to merely join words or sentences.

Therefore, the same word can be an Adverb in one place, a Preposition in another, or a conjunction in yet another place.

As,

(a) I have never seen him before. (as *Adverb*)

(b) They worked here before I joined. (as *Preposition*)

(c) The sun rose before we reached the park. (as *Conjunction*)

Conjunctions are sub. divided into two main clauses.

(a) Co-ordinating conjunctions : These are so named as they join words, phrases or clauses of equal rank.

(b) Subordinating conjunctions : These are called so because they join a dependant clause to a principle clause. (*That is to a clause of higher rank*).

INTERJECTION

An **Interjection**, so to speak, is not a part of speech as it has no grammatical connection with any word or words in a sentence.

It is simply an exclamatory sound used in a sentence to indicate strong emotion or feeling.

An Interjection is involuntary and unplanned and expresses strong emotion that overtakes one suddenly. It can be of joy, sorrow, admiration, disgust or disapproval.

Some examples,

Joy	–	Hurrah !	(We won)
Grief	–	Alas ! Oh !	(We lost)
Amusement	–	Ha ! Ha !	(Good joke)
Approval	–	Bravo !	(Well done)
Contempt	–	Bah !	(How stupid)
Ridicule	–	Bosh !	(How stupid)
Disapproval	–	Tut ! Tut !	(Bad boy)
To call	–	Hi ! Hello !	

A Closer look at Noun

1. A noun is a naming word. It is employed to name a person or thing.

A noun consists of 5 kinds or parts.

They are as follows—

1. Proper Noun
2. Common Noun
3 Collective Noun
4. Material Noun
5. Abstract Noun

2. Proper Noun—A Proper Noun stands for one particular person, place or thing as distinct from any other; as Raman (a person) Jamuna (a river), Haridwar (a city) America (a country).

A Point to note—The writing of a Proper Noun should always begin with a capital letter.

As,

1. Shane and Sita went to bathe in the Ganges.

2. Tom met Harry in Haridwar.

3. Betty went to America where she met Blake in Beverly Hills.

4. The Prime Minister of Britain lives at 10, Downing Street.

3. Common Noun—A common noun indicates no one person or thing in particulars, instead it is common to any and every person or thing of the same kind, As 'woman' 'table'.

Thus, woman does not indicate any one particulars woman, such as Julia but can be employed for any and every woman. Similarly, 'river' does not point out to any particular river such as Jamuna and can be used for any

and every river. Like 'country' does not point out to any one country, such as America but can be used for any and every country anywhere in the world.

Some common nouns,

1. He is a boy.
2. That table is made of wood.
3. Woman makes the man.
4. Roads are wide in London.
5. Shirts and pants are western attire.

4. Collective Noun—A collective Noun stands for a group or collection of individuals, animals, places or things as one complete unit.

For example, There may be many buffaloes in the posture, but only one herd. Here herd is a collective noun because it stands for all the buffaloes together.

A point to note—

A distinction is made between a collective noun and a noun of multitude. A collective Noun denotes one united whole, and thus the verb following it is singular.

As,

The class (*Collective Noun*) consists (*Singular Verb)* of 25 students.

The team (*Collective Noun*) has (*Singular Verb*) 11 members.

A noun of multitude indicates the individuals of the group. Thus, the verb that follows it is plural even though the noun is singular.

As,

The class (*Collective Noun*) are (Plural Verb) divided in their hobbies.

The jury (*Collective Noun*) are (Plural Verb) divided in their verdict

5. Material Noun—A material noun denotes the matter or substance of which things are made.

Hence, 'chicken' is a common noun; but chicken (or the flesh of chicken) is a Material Noun.

A wood may be a Material Noun or a common noun according to the context it is used in.

As,

1. Chickens (*Common Noun*) live in coops.
2. Chicken (*Material Noun*) is good for cancer patients.

In sentence 1 the noun indicates individual chicken or chickens, and is therefore a common noun.

In sentence 2 it denotes the matter of which the bodies of chickens are made, and is thus a Material Noun.

Some common Material Nouns—

Gold	Mutton	Wood	Fur
Silver	Chicken	Clay	Wool
Iron	Fruit	Leather	Steel
Lead	Cotton	Jute etc.	

6. Abstract Noun—An Abstract Noun denotes some quality state or action apart from anything possessing such quality, state or action, and can be perceived through the five senses.

Such as,

Quality	=	Kindness, Bravery, Honesty
State	=	Childhood, Poverty, Love
Action	=	Hatred, Laughter, Love.

Note—The Abstract Noun takes singular verbs.

6.(a) Abstract Nouns can be formed from Adjectives, Common Nouns or from verbs.

Take a look at the following—

Adjective	**Abstract Noun**
Deep	Depth
Brave	Bravery
False	Falsehood
True	Truth
Proud	Pride
Wise	Wisdom
Good	Goodness
Mad	Madness etc.

6.(b) From verb to Abstract Noun

Verb	**Abstract Noun**
Order	order
Love	Love
Regret	Regret
Sing	Sing
Taste	Taste
Sleep	Sleep
Talk	Talk

6. (c) From Common Noun to Abstract Noun

Common Noun	**Abstract Noun**
Boy	Boyhood
Child	Childhood
Man	Manhood
Mother	Motherhood

7. Some Abstract Nouns are considered as male, like, sun, thunder, death.

As,

The sun showers his light on the brave war places his cold barrel at the head of Emperors.

On the flip side, states or qualities expressed by Abstract Noun and that which is assumed to possess beauty, fertility, grace and inferiority etc, are regarded as females, such as, the earth (Mother Earth) fertility

(bearing), mercy (forgiveness), charity (giving) virtue (pure) etc.

Consider the following:

A ship is always spoken of as she even though the noun does not begin with a capital letter. The same is also said of trains, motorcycles and other machines.

As,

1. The ship sailed along merrily on her maiden journey.

2. The Betsy will get her first test drive tomorrow.

Noun Gender—The difference of sex in grammar is called the difference of gender.

There are four different types of gender—

1. Masculine gender – indicates the male form.
2. Feminine gender – indicates the female form.
3. Common gender – denotes persons or animals of either sex.
4. Neuter gender – denotes things of either sex, that is, inanimate things.

There are three different ways by which the Masculine Noun can be identified from the Feminine Noun.

(a) By changing the word.

As,

Masculine	Feminine
Boy	Girl
Father	Mother
Brother	Sister
Dog	Bitch
Horse	Mare
King	Queen
Sir	Madam
Drake	Duck
Boar	Sow
Husband	Wife

(b) By Adding a Prefix—

As,

Masculine	Feminine
Cock – sparrow	Hen – sparrow
He – goat	She – goat
Jack – ass	She – ass
Man – servant	Maid – servant
Buck – rabbit	Doe – rabbit

(c) By changing the Suffix—

As,

Masculine	Feminine
Washer man	Washer woman
Pea-cock	Pea-hen
Milk-man	Milk-maid
Grand-uncle	Grand-aunt
Grand-father	Grand-mother

Other ways to identify the masculine from the feminine.

(d) By adding–ess to the Masculine and not alternating its form.

As,

Masculine	Feminine
Shepherd	Shepherdess
Host	Hostess
God	Goddess
Peer	Peeress
Lion	Lioness

(e) By adding ess and dropping the vowel of the last syllable of the Masculine.

As,

Masculine	Feminine
Tiger	Tigress
Director	Directress
Waiter	Waitress
Founder	Foundress
Inspector	Inspectress
Hunter	Huntress
Actor	Actress
Instructor	Instructress
Poster	Postress
Mister	Mistress
Sorcerer	Sorceress

(f) Exceptional Cases—

Masculine	Feminine
Bridegroom	Bride
Widower	Widow

(g) Foreign Feminines—

Masculine	Feminine
Czar	Czarina
Beau	Belle
Signer	Signora
Hero	Heroine
Administrator	Administrators

Noun Number—There are singular nouns and plural nouns. In order to recognize the single number from the plural number.

Read and retain the following rules and points—

Rule 1—Nouns that add – s to the singular.

Singular	Plural
Boy	Boys
Girl	Girls
Cat	Cats
Dog	Dogs
Fan	Fans
Goat	Goats
Hen	Hens
Jackal	Jackals
King	Kings
Queen	Queens
Ship	Ships

Rule 2—Nouns which are used in the plural.

Spectacles	Credentials
Scissors	Alms
Measles	Gymnastics etc.
Wages	
Riches	
Gallows	

However, some forms may be plural in form but are employed in the singular.

Like, Mathematics, Economics, Physics, News etc.

As,

1. Mathematics is a good subject (*not 'are'*)
2. The news was good. (*not 'were'*)
3. His innings was sparkling. (*not 'were'*)

Rule 3—Some nouns are used in the singular only and are not led by 'a' or 'an' neither are they pluralized.

As,

1. The information is correct. (*not informations'*)
2. Susan's hair is red. (*not 'hairs'*)
3. Tom is up to mischief again. (*not 'mischiefs'*)
4. I took his advice. (*not 'advices'*)

Rule 4—When a noun does the job of a compound word it is not pluralized though it may be preceded by a plural number.

As,

1. A three year old.
2. A ten rupee note.
3. A weekend holiday.
4. A six mile track.

Rule 5—Use of apostrophe

Don't put the apostrophe (') sign or put an 's' after plurals that end in 's'/

It may be used after plurals that do not end with 's'.

As,

1. Girls' hostel; Dogs' ear.
2. Men's toilet; Children's toys.

Rule 6—The possessive does not carry the possessive (apostrophe) sign (') words like—yours, ours, its, his, hers, theirs, mine, omit the possessive sign.

As,

1. This is my pen and not your's / yours.
Incorrect / correct

2. Either it is his or her's / her pen. *Incorrect / correct*

Rule 7—With a noun or title of several words. The apostrophe (possessive) sign is used only with the last word.

As,

1. His daughter-in-law's mother.
2. The Governor-General's house.
3. Ram and Deshmukh's company.

In case two nouns in the possessive case are joined by 'and' the apostrophe is added to both to indicate individual possession. But the use of the apostrophe in the last word indicates joint ownership.

As,

1. Sarita and Rohan's collection. *(Joint Possession)*
2. Sarita's and Rohan's collection. *(Separate Possession)*

Rule 8—Some nouns have the same form in the singular as well as the plural. Like Fish, Deer, Sheep.

Ex.

1. Fishes live in water. (x)
2. Fish live in the water. ()
3. Sheeps give us wool. (x)
4. Sheep give us wool. ()

Rule 9—Nouns consisting of several words take the plural in the first word or last word.

As,

Singular	Plural
Sister-in-law	Sisters-in-law
Commander-in-chief	Commanders-in-chief
B.A.	B.As.
Lord Justice	Lord Justices
Major General	Major Generals

Rule 10—Noun words that end in 'f' get pluralized by adding 'ves' and dropping the 'f'.

As,

Knife	Knives

Other old 'ren', en; es; ies or change the vowels to form the plural number.

Singular	Plural
Child	Children
Ox	Oxen
Potato	Potatoes
Foot	Feet
Goose	Geese
Exceptions	
Chief	Chiefs
Gulf	Gulfs
Chef	Chefs
Zoo	Zoos

Determiners

1. Definite Determiners (Demonstrative)

When a person or thing is pointed out precisely it is called a Definite Demonstrative.

The most common being this, that take singular nouns. These, those take plural nouns.

2. When the adjective points out in an inexact fashion it is called an Indefinite Demonstrative.

Common Indefinite Demonstrative are—any, a certain, some, other, any other etc.

3. Take a look at the following to get a clearer understanding of Definite Demonstratives—

1. This boy met me today. These books are for him.
2. That house is on fire. Those people will be burnt alive.
3. This is not the shirt I wanted. Bring the other shirt.
4. Such people (*dishonest people*) can not be trusted.

4. Take a look at the following examples of Indefinite Demonstratives

1. Harry met some people at the party.
2. I do not know if there are any houses vacant in Haridwar.
3. There is a certain amount of oil I can not find.
4. Do you know of any other camps?

5. Adjective : Numerals

Adjective Numerals show how many persons or things or in which order they stand—They identify the number or the position.

Such adjectives are divided into two main classes.

(a) Definite (b) Indefinite

(a) Definite Numerals indicate an exact number, which show how many persons or things. Viz. one, two, three etc. These are called Cardinals.

(b) Those which demonstrate serial order such as, first, second, third etc. are called Ordinals.

Cardinals	Ordinals
One	First
Two	Second
Three	Third
Four	Fourth
Five	Fifth
Six	Sixth
Seven	Seventh
Eight	Eighth
Nine	Ninth
Ten	Tenth

Note—Cardinals can be used at random or as one desires whereas, ordinals have to follow a serial order.

As,

He bought 5 apples but I bought 2. *(The shaded numbers are cardinals)*

He stood first in line whereas I stood last. *(The shaded words are ordinals)*

6. Indefinite Numerals indicate number of some kind without specifically saying what the number is.

The main Indefinite Numerals are; all, some, enough, no or none, many, few, several etc.

Examples—

All men are equal.

No body attended the seminar.

Many women are biased.

Several horses trotted by.

Some of the teachers protested.

Enough bread was available.

Few people are born rich.

Note—The words some, enough, all, no or none are all adjectives of number or Adjectives of Quantity according to its structure.

If the noun is material or abstract the adjective is Quantitative. However, if the noun is common, the Adjective then is Numeral.

Some more examples:

1. She ate some *(a certain quantity)* cake.
2. She did not eat any *(any quantity of)* cake.
3. She ate enough or sufficient cake.
4. She ate all *(the whole quantity of)* cake.
5. Half a loaf of bread is better than none *(no quantity)*.

Note—'No' is used when the noun that it qualifies is expressed. 'None' is used when the noun is understood *(as in example 'e')*

Such adjectives must be followed by a singular noun which is either a material noun or an abstract noun.

7. The Adjectives of number can be classed into two parts. Quantitative (Uncountable) and Numerals (Countable)

Lets consider the following—

1. She has much wealth. *(Quantitative)*
2. He has many 500 rupee notes. *(Numerals)*

The first example (1) has an uncountable number and thus is quantitative. Whereas, the second example (2) a countable number and therefore is a Numeral.

To make things a little more clear. Consider this.

Quantitative *(Uncountable)*	**Numerals *(Countable)***
1. She has little wealth.	She has a few rupees.
2. She has enough wealth.	She has enough rupees.
3. She has some wealth.	She has some rupees.
4. She has no wealth.	She has no rupees.
5. She has all the wealth.	She has all the rupees.
6. Has she any wealth. (*Abstract Noun*).	Has she any 100 rupees notes. (*Common noun*)

To make sure you have understood the difference between quantitative and Numeral Adjectives. Test your skills by filling in the blanks of the following exercises. Stating whether they are quantitative or numeral (Q or N).

1. John has many cars. (................)
2. He has no gold. (................)
3. She has some money. (................)
4. Has she any 10 rupee notes. (................)
5. Susan has some wealth. (................)
6. Ray has enough knowledge. (................)
7. Harry and Tom have all the food. (................)
8. Betty has a few dollars. (................)
9. He has little taste. (................)
10. We have no regret. (................)

Ans. 1. N, 2. Q, 3. Q, 4. N, 5. Q, 6. Q, 7. Q, 8. N, 9. Q, 10.Q.

Mention whether the following adjectives according to the sentence are quantitative (Q) or Numeral (N) Adjectives.

1. He has no money. (Q / N)
2. They have all the coins. (Q / N)
3. She has enough wealth. (Q / N)
4. I have no cars. (Q / N)
5. Ram has a few rupees. (Q / N)

Ans. 1. Q, 2. N, 3. Q, 4. N, 5. N.

A Closer look at Pronoun

1. A Pronoun must agree in person, number and gender with the antecedent or noun it stands for; but its case relies solely upon its own sentence.

Example—

1. After Rajiv Gandhi was declared Prime Minister (*Nominative*), the LTTE assassinated him (*Accusative*).

2. He must return the cycle (*Accusative*) which (*Nominative*) you lent him.

2. The objective form of the Pronoun (*me, him, her, us, them*) is used when; (i) it is the object of a Preposition (ii) It is placed after the Verb 'to be' and this infinitive is preceded by a transitive verb with its object (iii) It is placed after the adjectives; like, unlike or near.

Example—

1. Between you and me (*not I*), he is quite stupid.
2. This is ample food for you and me (*not I*).
3. She invited my friend and me (*not I*) to lunch.
4. No one can dance like him (*not he*).
5. Let you and me (*not I*) play chess.

3. A Relative Pronoun, when it has two Antecedents which are not of the same person, it then agrees with the antecedent nearest to it—

Example—

1. You are the girl who *has been* selected.
2. He is the man who *wishes* to help.
3. I, who *am* speaking said these words.
4. I am the person who *is* most affected.
5. This is one of the most tastiest dishes that *have* (*not has*) ever been cooked. (The Antecedent of that is dishes not one.)
6. He is one of those who *know* (*not knows*) nothing.

Rules of Pronouns

Rule 1—*Relative and Demonstrative pronouns must be of the same number, person and gender as their antecedents.*

As,

One must not waste his time.

(*Change 'his' to one's*)

I am not one of those who can not practise what I preach.

(*change 'I' to they*)

Rule 2—*When two singular nouns joined by 'and' are preceded by 'each' or 'every' the pronoun must to singular.*

As,

Each boy and each man was in his uniform.

Every winter and every summer has its charm.

Rule 3—*'Neither.........nor'; 'Either.....or'; 'or' when they join singular nouns the pronoun is singular.*

As,

Tom or Harry will enjoy his holiday.

Either Greg or Robert lost his wallet.

Neither Sheila nor Hema bought her bag.

Rule 4—*But if a plural noun and a singular noun are joined by 'or', or 'nor', the Pronoun agrees with the closest noun.*

As,

Ram or his friends must finish their work.

Either the boys or mother will do her chores.

Neither he nor we have done our job well.

Rule 5—*It is considered that well mannered and proper to place the first personal pronoun in the singular last. The second should come prior to the third. Whereas in the plural, 'we' come before 'you' and 'you' before 'they'. Such is the preferred order.*

You and I must go quickly. (*Not 'I' and 'You'*)

You and She should join the club. (*not 'she' and 'you'*)

You, he and I must play poker. (*not 'you', 'I' and 'he'*)

We and you should work together. (*not 'you' and 'we'*)

You and they have failed. (*not 'they' and 'you'*)

Rule 6—*When a Pronoun refers to more than one noun or pronoun of different person, the first person plural in preference to the second, and the second in preference to the third.*

As,

You and I have completed our task. (*our is the first person plural*)

You and Tom played your cards. (*your is the second person plural*)

She and I did our jobs. (*our is the first person plural*)

Rule 7—*Who / Whom*

'Who' is used when it is the subject of a verb. 'Whom' is used when it is the object of a verb.

He is the boy who stole your watch.

(Here 'who' is the subject of the verb 'stole')

When I talked to him whom do you reckon interrupted ?

(Here 'whom' is the object 'talked')

Rule 8—*When the complement of the verb 'to be' is expressed by the pronoun it should be in the Nominative form.*

As,

It was she (*not 'her'*).

It must be he (*not 'him'*).

I am he you desire (*not 'him'*)

It was I (*not 'me'*)

Rule 9—*When the object of a verb or a preposition is a pronoun, it should be in the objective form.*

As,

He gave it to you and me. (*not I*)

(object of the verb 'gave')

Between you and me. (*not I*)

(object of the preposition between)

Let(*mean allow*) you and me do it together.

(object of the verb 'let' allow)

Rule 10—*When a pronoun follows 'than' and 'as' the verb is mentally placed.*

As,

Tom is taller than I. (*I am not 'me'*)

He runs faster than I. (*I run, not me*)

I love you more than she. (*loves you*)

I run as fast as he. (*he runs*)

I love you as much as (*I love*) her.

Rule 11—*Try to avoid the usage of 'the same' in place of a personal pronoun.*

As,

After you have examined the contents kindly return the same to the office.

(*use 'them'*)

Rule 12—*'This' when used as a pronoun usually follows the sentence or idea it refers to. It is also used in the place of 'it' for emphasis.*

As,

His son stole the purse and this *(Refers to the act)* hurt him deeply.

This *(For Emphasis in place of 'it')* is definite *(I will go to Goa)*

Rule 13—*A Reflexive Pronoun does not stand alone a noun or pronoun must precede it.*

As,

You and myself will go there. *(Use I)*

Both Tom and myself won the prize. *(Use I)*

Both himself and Greg completed the project. *(Place 'he' before himself)*

Myself cooked the food.*(Place 'I' before myself)*

Rule 14—*Don't omit the Reflexive Pronoun when the following verbs are used reflexively 'exert'; 'resign'; 'apply'; 'revenge'; 'oversleep'; 'over-reach'; 'acquit'; 'absent'; 'enjoy'; 'drink'; 'avail'.*

As

She enjoyed *(Insert 'herself')* at the party.

I applied *(Insert 'myself')* to the job.

He revenged *(Insert 'himself')* by killing his enemy.

Rule 15—*The Reflexive Pronoun is omitted after, such verbs as 'steel', 'stop', 'lengthen', 'make', 'gather', 'bathe', 'move', 'open', 'spread', 'feed', 'draw', 'rest', 'role', 'burst', 'keep', 'turn', 'set', 'break', etc. The verb is then considered intransitive.*

As,

He stopped himself eating meat. *(omit)*

She kept herself far from him. *(omit)*

He made himself away from office. *(omit)*

Rule 16—*Each other / One anothe—While talking about two persons or things use 'each other' and while speaking of more than two persons or things use 'one another'. However, both forms are now commonly used nowadays. But, the above, is considered grammatically correct.*

As,

Tom and Susan are not speaking to each other. (*Between two*)

The guests at the seminar spoke to one another. (*More than two*)

Rule 17—*Either / Anyone—In reference to two persons or things either is used while 'anyone' is used in reference to more than two.*

As,

Either of the two girls must be selected.

Has anyone in the class seen my mobile?

Rita was smarter than anyone in her group.

Rule 18—*What— 'What' refers to things solely, and is used without an antecedent. It is equivalent to 'that which' or the 'thing which'.*

As,

What the eye can not see, the heart does not feel.

('What' is equal to that which)

He can do anything what you like.

(Wrong, as 'what' is used with an antecedent 'anything'. Change 'what' to 'that')

Rule 19—*Whose / Which—As a rule 'whose' is used for persons and 'which' for non living things.*

As,

Meet the man **whose** exploits are legendary.

This is the watch **which** he gave me.

Drill—Use the correct form of the relative pronoun in each of the following sentences.

1. The answer .... you gave is correct.
2. I saw the man .... was hurt.
3. I know the woman .... child was lost.
4. Listen to .... I say.
5. This is the magician .... we saw last night.
6. Is this the book for .... you were asking?
7. This is the girl .... ring you bought.
8. I do not believe .... you say.
9. My son .... you have not met, is here.
10. I need such a man .... he is.
11. .... wants to meet me ?
12. .... umbrella is this ?
13. .... are you waiting for ?
14. The is the Taj .... Shah Jahan built.
15. I gave a coin to a man .... leg was broken.

Answer : 1. which, 2. who, 3. whose, 4. what, 5. who, 6. who, 7. whose, 8. what, 9. whom, 10. that, 11. who, 12. whose, 13. what, 14. that, 15. whose

A Closer look at Adjective

1. The primary use of an adjective is to modify a noun or pronoun. It is a part of speech which describes, qualifies and identifies a noun or pronoun.

In modern day grammar, grammarians are doing away with the separation of Articles from its big brother the Adjective.

The new concept or creation now includes Determiners, as a class a part or distinct from what was formerly clubbed under 'Adjectives'.

Let us start with determiners. They can be categorized as shown below.

(a)	Articles	:	A, An, The
(b)	Quantifiers	:	All, Few, Many, Several
(c)	Genitive or Possessives	:	His, Her, Its, My, Our, Their, Your
(d)	Demonstratives	:	This, That, These, Those.
(e)	Numbers	:	One, Two, Ten, Hundred.
(f)	Negative	:	No, None

Note—They indicate the number and gender of the Noun. Thus, they possess several forms.

Exercises for Determiners are on pages .... To .... after further detailed study and examples.

2. Adjectives are divided into eight parts or categories.

(a) Proper : In which it describes a thing by referring it to a Proper Noun.

(b) Descriptive : Shows of what quality or state a thing is.

(c) *Quantitatives : Show "how much" ?

(d) *Numerals : Tells us "how many" or in which order.

(e) *Demonstratives : Show which or what thing is intend or meant.

(f) Distributives : Tell us whether things are taken separately or in separate lots.

(g) Interrogative : Inquires which or what thing is meant.

(h) *Possessives : Show relationship or ownership.

* As mentioned earlier these adjectives are now classed as Determiners.

Degrees of Comparison

Rule 1—*Most adjectives of one syllable form the comparative by adding 'er' to the positive. Whereas 'est' is added to the superlative.*

Positive	Comparative	Superlative
Bold	Bolder	Boldest
Plain	Plainer	Plainest
Deep	Deeper	Deepest
Cool	Cooler	Coolest
Strong	Stronger	Strongest

Rule 2—*When the positive end 'e' only 'r' and 'st' are added to form the comparative and superlative.*

Positive	Comparative	Superlative
Able	Abler	Ablest
Brave	Braver	Bravest
True	Truer	Truest
Wise	Wiser	Wisest
Rude	Ruder	Rudest

Rule 3—*Where the positive ends in one consonant and the consonant is preceded by a short vowel, the final consonant is doubled.*

Positive	Comparative	Superlative
Big	Bigger	Biggest
Thin	Thinner	Thinnest
Wet	Wetter	Wettest
Red	Redder	Reddest
Fit	Fitter	Fittest

Rule 4—*When a positive ends in 'y' and the 'y' is preceded by a consonant the 'y' is changed into 'i' and 'er' and 'est' are added to form the comparative and superlative.*

Positive	Comparative	Superlative
Dry	Drier	Driest
Pretty	Prettier	Prettiest
Lovely	Lovelier	Loveliest
Healthy	Healthier	Healthiest
Merry	Merrier	Merriest

Rule 5—*All adjectives of more than two syllables and adjectives of two syllables are preceded by 'more' to the positive to form the comparative and 'most' to form the superlative.*

Positive	Comparative	Superlative
Beautiful	More Beautiful	Most Beautiful
Intelligent	More Intelligent	Most Intelligent
Courageous	More Courageous	Most Courageous
Pleasant	More Pleasant	Most Pleasant
Magnificent	More Magnificent	Most Magnificent

Rule 6—*Some adjectives form the comparatives and superlatives in an irregular manner.*

Positive	Comparative	Superlative
Far	Farther	Farthest
Good	Better	Best
Late	Later, Latter	Latest, Last
Many	More	Most
Much	More	Most
Old	Older, Elder	Oldest, Eldest
Little	Less	Least

It is to be remembered that 'than' follows the comparative while 'the' precedes the superlative.

As,

Comparative

1. He is taller than me.
2. Ram is stronger than Shyam.
3. Sita is more beautiful than Rita.
4. Rishikesh is farther than Haridwar.

Superlative

1. He is the tallest boy in the class.
2. Ram is the strongest of all.
3. Sita is the most beautiful girl in Delhi.
4. Rishikesh is the farthest town in our area.

Adjectives of Degree

Adjectives of Degree are divided into three classes. The Positive Degree, The Comparative Degree and The Superlative Degree.

1. Positive Degree—The Positive Degree is used when two persons or things are said to be equal in regard to some quality. The Positive Degree is then used with as.....as, or the comparative Degree can be applied with 'not'.

Consider these sentences—

(a) She is as clever as Sita.

(b) She is no less clever than Sita.

(c) She is not more clever than Sita.

Again,

(a) She is as beautiful as Sita.

(b) She is no less beautiful than Sita.

(c) She not more beautiful than Sita.

These sentences depict the same idea or message that Sita and 'she' are equal in regard to beauty.

2. Comparative Degree—The Comparative Degree is usually used to compare two unequal qualities of persons or things, therein showing a difference between two persons or things in regard to some quality.

Examples—

1. Shyam is heavier than Lakshman.

2. This sword is sharper than that sword.

3. Lakshman is not as heavy as Shyam. (*Positive Degree with 'not'*)

4. Shyam is the heaviest of all. (*Superlative Degree*)

You can now see that the Positive Degree and the Superlative Degree can be used as a Comparative while comparing two unequal qualities of persons or things.

3. Superlative Degree—The Superlative Degree is used when one person or thing is greater than all other persons or things of a similar kind the Superlative is used with 'the', 'of'.

As,

1.She is the most beautiful girl of her class.

2.Paras is the cleverest of all.

3.Rome is one of the most beautiful countries in the world.

4.Raj is the fastest runner of the team.

Note—The word inferior, superior, senior, junior, prior etc. are followed by 'to' instead of 'than'.

Examples—

1. This paper is inferior to that.

2. Mittal is senior to Manoj.

3. Gold is superior to Iron.

4. She is junior to me.

5. Prior to this job I worked in Wipro.

4. Use of the Comparative Degree—The Comparative Degree is preferred to the Superlative Degree when comparing two things of the same quality.

Consider this,

1. She is the cleverest girl in the class. *(It would be better to say, 'the cleverer')*

But,

2. This boy is the fastest of the three. *(correct)*

As, there is no comparison between two persons or things.

Thus, when the number exceeds two it is correct to use the superlative Degree.

3. Who was the most famous king Ashoka or Akbar ?

(The preferred use is 'more famous')

The Comparative Degree is generally followed by 'than' and 'the' goes before the superlative.

As,

1. He is better than I. *(Comparative)*

2. He the best student in the class. *(Superlative)*

5. Some Important Adjectives—At times, some important adjectives are misused or misunder-stood. This leads to confusion and incorrect usage.

Lets take a look at the following adjectives. Elder, Eldest, Older, Oldest.

1. She is my elder sister.

2. She is older than her sister.

3. Her elder daughter is a doctor.

Older and oldest are employed for things as well as persons and indicate age.

But, we can not say,

1. Mumbai is the eldest city in Maharastra. *(Wrong) Instead use, 'oldest'*

2. That is the eldest building in the city. *(wrong) Use 'oldest'*

By this we can by now guess that older, oldest, elder, eldest can be used for persons and older, oldest for things.

6. Further / Furthest / Farther / Farthest—The word 'further' 'denotes something additional 'er' extra while 'farther' denotes a greater distance between two points. However, nowadays the two forms 'further' and 'farther' can be used to indicate distance.

As,

1. Kashmir is further / farther than Manali or Vashist.

2. Kashmir is the furthest / farthest of the three.

3. Kashmir is the furthest / farthest town.

7. Later / Latest / Latter / Last / Former—'Later' and 'Latest' indicate time whereas 'Latter' and 'Last', 'former' and 'Latter' denote position.

As,

1. The student walked in later than his teacher.

2. This dress is the latest fashion.

3. Rohan and Sohan took an entrance examination. The former (Rohan) passed while the latter (Sohan) failed.

4. Ravi walked into the room last.

8. Nearest / Next—'Nearest' denotes space or distance. 'Next' indicates order or position.

As,

1. The cinema hall is nearest to my house. *(distance)*

2. Harry's house is next to mine. *(Position)*

9. Fewer / Less—'Less' usually denotes quantity while 'fewer' denotes number.

But, we can say, 'No less than a thousand guest attended their marriage' when we are thinking not of

individual guests but of the number as a mathematical quantity.

'Fewer' and 'Less' are usually followed by 'than'.

Consider the following—

1. No fewer than 50 girls failed in a class of 70 students. *(Number)*

2. Mr. Gupta does not purchase less than 50 kgs. of sugar. *(Quantity)*

3. There are fewer boys than girls in my school. *(Number)*

4. Factories in Noida do not purchase less than a ton of raw material. *(Quantity)*

10. Countable and Uncountable Adjectives—Many / Much—'Many' is used before countable nouns.'Much' is placed before uncountable nouns.

As,

1. He has read many books.

2. Sheetal has many friends.

Both 1 and 2 have countable nouns.

3. I have not much time to spare.

4. How much oil do you need?

Both 3 and 4 denote quantity and are uncountable.

11. Many / Much / More / Most—These words can be used as Pronouns when the Noun is understood.

As,

1. He buys a lot of books but I do not buy many.

2. He spends a lot of money but I do not spend much.

3. He bought many books but I bought more.

4. He bought a large amount of sugar but I bought most.

12. 'Good' and 'Well'—At times we get into a sticky issue over the application of Good and Well, the Adjective and the Adverb respectively. Clearly, when modifying a verb it is not a problem. Use the Adverb!.

Consider this,

He runs well.

She knows well enough not to disturb her father at this time.

While modifying the verbs 'runs' and 'knows' simply use the Adverb 'well'.

But, when using a linking verb that involves the five senses, the sensible thing to do is to employ the adjective 'Good'.

As,

1 He is not feeling good this morning.

2. The boy's vision is pretty good.

3. He feels quite good after exercising.

A Closer look at Verb

1. The Infinitive is regularly used with 'to' except in the following cases—

(a) The bare infinite that is,

The infinitive without 'to' is applied after auxiliary verbs like, 'shall', 'will', 'can', 'may', 'did', 'should'. However, 'ought' is the exception.

(b) The bare infinitive is employed in the Active voice but not in the Passive Voice after words of perception, feeling etc. such as, see, laugh, feel, watch.

As,

I saw her dance. *(not 'to dance')*

She made him laugh *(not 'to laugh')*

We felt her touch our feet. *(not 'to touch')*

However, the bare infinitive is used after 'let' in both the active and passive voice.

I let her loose. *(not 'to loose)*

She was let loose. *(not 'to loose')*

(c) The bare infinitive is used after 'need' and 'dare' in the negative and interrogative forms.

As,

She need not speak *(not 'to speak')*

I dare not ask *(not 'to ask')*

Need you go ? *(not 'to go')*

Dare for argue ? *(not 'to argue')*

(d) After phrases like, 'as soon as', 'had better' and 'had rather' the bare infinitive is used.

As,

He would as soon sleep as work *(not 'to work')*

She had better talk soon *(not 'to talk)*.

(e) After 'but' and 'than' use the bare infinitive.

They boys did everything but study. *(not 'to study')*

I did nothing more than sing *(not 'to sing')*

2. **The split Infinitive**—When the 'to' is separated from its verb by inserting between them an adverb or adverbial phrase, it is not acceptable to grammarians. (However, in modern times such insertions are now commonly used and accepted).

(a) It is necessary to swiftly act. *(use – to act swiftly)*

He is to quickly leave. *(use – to leave quickly)*

(b) But, if the insertion makes the meaning more clearly expressed it can be done.

As,

To really appreciate the play one must sit in the front row.

(c) The 'to' of one infinitive can serve as 'to' for another infinitive if they are synonymous.

But, if separate ideas are expressed by the two infinitives then, the 'to' of the first infinitive can not serve the second infinitive.

As,

He went on to succeed and prosper.

It is not my intention either to kill or to cure.

(d) An infinitive should be in the present tense unless it denotes an action prior to an action of the Principal verb.

As,

She appears to have enjoyed herself at the party.

3. **The Verbal Noun and The Gerund**—

(a) The Verbal Noun is preceded by the Definite Article 'the' and followed by 'of', whereas, a gerund does not attach them.

As,

The humming of bees frightened him. *(Verbal Noun)*

Humming bees frightened him. *(Gerund)*

(b) A Verbal Noun is qualified by an adjective and a gerund by an Adverb.

The reading of a holy book quickly is not advisable. *(wrong)*

The quick reading of a holy book is not advisable. *(Right)*

Quick reading a holy book is not advisable. *(wrong)*

Reading a holy book quickly is not advisable. *(Right)*

(c) A Gerund and a Verbal Noun have distinct positions in English Grammar no error should be made in their composition.

The driving a car gives one a feeling of control. *(Here driving is not a verbal noun as it does not have of after it. So, cancel the before driving and it changes into a gerund).*

(d) Don't mix a Gerund with a Verbal Noun in a sentence. Consider the following.

She joined me in the applying of my desires and in attaining my goals.

('the applying of' is a verbal noun and 'attaining' a gerund. Either cancel the 'of' to make applying a gerund or place 'the' and 'of' to the gerund to make Verbal Noun.)

(e) Do not confuse or substitute a gerund for an infinitive.

It is easy recognizing him from her. *(wrong)*

It is easy to recognize him from her. *(Right)*

(f) When a Gerund is preceded by a preposition and has reference to the subject of the sentence instead of any other word, the gerund should not be used with a preposition.

As,

They served me ice-cream after having taken a bath. *(change 'after having taken' to after 'I had taken')* If not 'after having' taken will refer to the subject 'they' not 'I'.

(g) A gerund governed by a noun or pronoun should be in the possessive case.

As,

I disapprove my brother coming late. *(wrong)*

I disapprove my brother's coming late. *(correct)*

Please excuse me being untidy. *(wrong)*

Please excuse my being untidy. *(correct)*

4. **Participle**—

(a) A participle should not be alone without any proper agreement. It must be joined with a noun or pronoun to which it refers.

As,

Walking on the footpath, a man tripped him.

(Rewrite; Walking on the footpath he was tripped by a man.)

Being my day off, I overslept.

(Rewrite; It being my day off, I overslept.)

(b) But, words like, 'considering' 'concerning', 'taking', 'speaking', 'touching', 'owing to' are not used in agreement to any noun or pronoun.

Taking all possibilities into consideration.

Broadly speaking it is difficult to assess.

Here the unexpressed pronoun *(subject)* is indefinite e.g. It means if one takes, 'speaks' etc.

(c) A present participle is not used to express an action which is not in accordance with the Principal verb.

He walked home this evening and arriving there at night. *(wrong)*

He walked home this evening and arrived there at night. *(correct)*

A Closer look at Preposition

A Preposition is a word placed before a Noun or its equivalent to indicate what relation the person or thing denoted therein stands to something else. The Noun or Noun equivalent is called the object of the Preposition.

As,

I put the book under my pillow.

He sat on the chair.

They dived into the river.

If the words 'under', 'on' and 'into' are removed, the sentences make no proper sense.

The book can be placed on or under the pillow. Thus, until a Preposition is used, the relationship between the book and the pillow is not known.

Participle Prepositions : These were originally present or past participles used sometimes with the Noun expressed and sometimes with the Noun understood.

As,

Noun Expressed

During the winter, the winter during or still lasting.

Noun Understood

Owing to heavy rains, the crops were destroyed.

Compound Prepositions

Two or more words frequently thrown together and ending with a single Preposition may be termed compound Preposition.

Examples—

In spite of ; Because of; By means of; On account of; With reference to; On behalf of ; In the event of; In place of

Use of 'But' as a Preposition

Example—

(a) All but *(except)* Hari were present.

(b) They were all but *(everything except)* destroyed.

Kinds of Preposition

At / In / On

(a) 'At' is used for an exact time.

(b) 'In' is used for months, years, seasons.

(c) 'On' is used for days and dates.

As,

AT—at 10'clock ; at noon; at bedtime; at sunset; at the moment;

IN—In June; In winter; In 2010; In the next century; In the past; In the future;

ON—On Sunday; On Monday; On 7th January; On Ram's birthday; On Diwali;

The use of Preposition in reference to place.

(a) 'At' is used for a point.

(b) 'In' is used for enclosed places.

(c) 'On' is used for surface.

Examples—

(a) At the corner; At the bus stop; At the end of the lane; At my house; At the door

(b) In Haidwar; In the Box; In my coat pocket; in America

(c) On the shelf; On the table; On the bed; On the desk; On the floor

Suitable Prepositions

A preposition shows relationship—what one person or thing has to do with another person or thing. It expresses some relation between them. But, one must be aware that preposition employed expresses the relation required by the sense. Prepositions must conform to the sense intended by the author as well as the idiom laid down by custom. A case in point, when we speak of a death caused by a disease, we say, 'she died of T.B.' but if a death was caused by something else, we say, 'she died from a kick'. Here both prepositions indicate cause, but one fits in one sense while the other is suitable to the other. Likewise, we 'agree with' him but 'agree to' disagree. The lists of such types are endless.

However, if one uses the following appropriate prepositions containing such words as take dissimilar prepositions after them, then one can retain a fair amount of suitable prepositions that are appropriate.

Suitable Prepositions—

1. Abide with (a person)
2. Abide by (a thing)
3. Abide at (a place)
4. Accused of (a crime)
5. Accused by (a person)
6. Apply for (a thing)
7. Apply to (a person)
8. Appeal to (a person)
9. Appeal against (a thing)
10. Angry at (a thing)
11. Angry for (refers to action)
12. Angry with (a person)
13. Arrive at (a person)
14. Arrive in (a country)
15. Admit of (a thing)
16. Admit to / into (a place)
17. Alight on (a ground, a thing)
18. Alight at (a place)
19. Alight from (a vehicle, animal (car, house etc.)
20. Amused at (a thing)
21. Annoyed with (an action)
22. Annoyed at (a thing)
23. Annoyed with (a person)
24. Answer to (a person)
25. Answer for (a thing)
26. Antipathy to (a thing)
27. Antipathy against (a person)
28. Anxious about (result of something)
29. Anxious for (safety, health)
30. Ask from (a person)
31. Ask for (a thing)
32. Affiliated to (a board, university)
33. Affiliated with (a part of)
34. Arm against (a danger)

35. Arm with (weapons)
36. Atone to (a person)
37. Atone for (a thing)
38. Attend to (a thing)
39. Attend upon (serve a person)
40. Award for (a thing)
41. Award to (a person)
42. Account for (a thing)
43. Account to (a person)
44. Argue against / about (a thing)
45. Argue with (a person)
46. Apologize for (a thing)
47. Apologize to (a person)
48. Appoint (a person)
49. Appoint to (a position)
50. Arbitrate between (two parties)
51. Arbitrate in (a dispute)
52. Accomplice with (a person)
53. Accomplice in (a crime)
54. Affinity with (something)
55. Antidote to (medicine for poison)
56. Antidote against (infection)
57. Authority over (a person)
58. Authority on (a subject)
59. Aspire after (fame)
60. Agree with (a person)
61. Agree to (a proposal)
62. Agree on (a subject)
63. Beg for (a thing)
64. Beloved of (noun) (a person)
65. Blind in (one eye)
66. Blind to (faults / defects)
67. Born in (a country)
68. Born of (parents)
69. Busy at (a thing)
70. Busy with (refers to an action)
71. Cause for (anxiety)
72. Cause of (problems)
73. Claim to (a thing)
74. Claim on (a person)
75. Compete with (a person)
76. Conform to (a rule)
77. Contribute to (a fund)
78. Controversy with (a person)
79. Controversy on / about (a thing)
80. Complain against (a person)
81. Complain of (a thing)
82. Consult with (a person)
83. Consult on (a thing)
84. Clothed in (garment)
85. Clothed with (some quality)
86. Care for (to like)
87. Charge (a person) with (a crime)
88. Condemn to (punishment)
89. Condemn for (a certain crime)
90. Confer with (consult with a person)
91. Confer about (consult about a thing)
92. Converse with (a person)
93. Correspond with (a person)
94. Correspond about (a thing)
95. Concerned at / about (affected by)
96. Convenient for (means suitable)
97. Convenient to (means easy)
98. Clash with (means to strike against)
99. Capable of (a thing)
100. Capacity for (doing something)

Preposition Rules

Rule 1—*As a preposition is not powerful enough to stand in for much emphasis and importance, it is not positioned at the end of a sentence*. But, there are the following exceptions—

(a) When if combines a preceding intransitive verb to form a compound transitive verb.

As,

Francis loves being talked to.

She hates being ogled at.

(b) When the sentence object is a relative pronoun 'that'.

There are the keys that he was looking for.

Such was the end that he came to.

(c) When the object is an interrogative pronoun that is understood.

As,

What is he searching for ?

What are you getting at ?

Where are you going to ?

Rule 2—*'In' and 'at' / 'to' and 'into'*

When speaking of things that are fixed or at rest. 'In' and 'at' are employed.

As,

The lawyer is in his chamber.

He is at the peak of his carrier.

'To' and 'into' are used while speaking things in motion.

As,

The boys dived into the river.

He traveled to China.

Rule 3—*Similar types of prepositions should not be employed with two words. Until and unless it is suited to each word.*

As,

Cotton is similar to and superior to jute.

(This is inappropriate as the words 'similar' and superior take different prepositions. As one preposition can not exercise two, functions)

Instead say, 'Cotton is better then jute and superior to it'.

Rule 4—*Between / Among*

'Between' is used when referring to two persons or things and 'among' when referring to more than two.

As,

Share the ice cream between John and Mary.

Distribute the books among members of the library.

Rule 5—*On / To*

'On' is also used of time / day / event and 'to' indicates place.

As,

Tim arrived on my birthday.

She visited us on Saturday.

He came on Christmas day.

He is going to the market.

Don is walking to China.

She is coming to the theatre.

Rule 6—*With / By*

'With' frequently indicates instrument and 'by' the doer.

As,

He shot Tom with an AK 47.

She was injured by a car.

Harry was stabbed with a dagger by his friend, Joe.

Rule 7—*In / At*

'In' is used with names of countries, big towns while 'at' is utilized for small villages and towns.

As,

Greg lives at Ranipur More in Haridwar.

Susan lives in America.

Rule 8—*Since / For / From*

'Since' denotes some point of time like 'from' and is employed before a noun or phrase. But 'since' is preceded by a verb in a perfect tense, 'from' is used with all tenses except the perfect tense 'for' refers to a period of time unlike 'since' which denotes some point of time. Therefore, 'for' should not be substituted by 'since' or from.

As,

Robert has not eaten since morning.

She has been waiting in line since 8 a.m.

We were closed from Friday.

I will resume working from January.

She will start work from today.

He has been talking to him for 2 hours.

Rule 9—*Ago / Before*

'Ago' refers to past time and 'before' shows precedence between two events.

As,

He left town five years ago.

She joined us before she got married.

Rule 10—*It is incorrect to place prepositions after the following words; 'accompany', 'attack', 'assist', 'combat', 'order', 'reach', 'resist', 'violate', 'pick', 'afford', 'precede', 'request', 'inform', 'succeed' etc.*

When used in the active voice.

As,

1. We wanted for(x) 4 pies.
2. She reached to(x) office late.
3. He stabbed on(x) her mercilessly.

A Closer look at Adverbs

Rule 1—*Adverbs should be placed as close as possible to the word or words they qualify.*

As,

They walked slowly.

We screamed loudly.

The couple danced beautifully.

Rule 2—*With an intransitive verb the adverb or adverbial phrase is positioned after it and not before.*

She heartily laughed. (×)

She laughed heartily. (√)

The boys happily played. (×)

The boys played happily. (√)

They excitedly screamed. (×)

They screamed excitedly. (√)

Rule 3—*With a transitive verb the adverb should be placed either before or after it, never between the verb and object.*

He sadly mourned his father's death. (√)

He mourned his father's death sadly. (√)

He mourned sadly his father's death. (×)

Rule 4—*Adverbs which denote time 'like', 'never', 'ever', 'always', 'sometime', 'often', 'frequently', 'seldom', 'during', 'rarely', etc. are positioned before the verbs they modify.*

She never talks nonsense.

He rarely tells a lie.

They frequently visit the Gupta's.

When an auxiliary verb functions as a main verb the adverb of time is positioned after it.

She is seldom late.

He is never happy.

Rule 5—*The Adverb enough is positioned after the word it qualifies.*

He was strong enough to compete. (√)

He was enough strong to compete. (×)

Rule 6—*'Only' and 'even' are placed immediately before the word it means to qualify.*

I played only one round. (√)

I only played one round. (×)

Rule 7—*When an adverb intends to qualify not any word in particular but a sentence as a whole it is positioned at the beginning of the sentences.*

Sadly, their plan did not work.

Surprisingly, they gave up all hope.

Rule 8—*The Adverbs, 'enough', 'not', 'how' and 'more' should not be left out when their presence is required.*

As,

She is not smart enough to pass.

They know how to fix it.

We are expecting more grants from them after our sterling performances.

He knows not what to do.

Rule 9—*Adjectives should not replace adverbs.*

He moved slower than I expected. (*Used more slowly)*

The birds flew away swifter when the cat arrived. *(Swiftly)*

Rule 10—*An Adverb does not qualify a Noun.*

Quite a bunch of flowers were given to the chief guest.*(Change 'quite' to 'many')*

Explain the above paragraph.

(Change to above mentioned)

Rule 11—*In place of 'firstly' use first as 'first', itself is an adverb.*

Firstly, we must plan and secondly activate the plan. *(Use first)*

Rule 12—*'Else' is followed by the adverb 'but'. Do not follow 'else' with 'than'.*

He speaks nothing else than Spanish. *(Use 'but')*

She does nothing else than sleep. *(Use 'but')*

Rule 13—*The use of 'never' in place of 'not' is wrong as 'never' means 'not ever' whereas* 'not' means negative.

During our conversation he never mentioned her name. *(Use 'did not')*

(Never indicates 'for all time' while 'not' indicates during the conversation there was no mention of her)

I never remember talking to you. *(Use 'do not')*

Rule 14—*'Very' / 'Much' / 'So'*

As an adverb of degree 'so' should not be used without a correlative.

Tigers are so strong. *(Use 'very')*

Tigers are so strong that they can kill man easily. *(Correct as it is followed by its correlative 'that')*

'Very' modifies adjectives or adverbs in the positive degree and 'much' in the comparative degree.

She sang very softly. *(Not 'much')*

She is much faster than I. *(Not 'very')*

'Very' modifies present participles while 'much' modifies past participles.

The game was very interesting. *(Present Participle)*

He was much disturbed. *(Past Participle)*

'Very' is frequently employed to modify the adverb.

This dish is very much tastier than that.

(adverb) (adverb)

Rule 15—*While employing the adverb 'too' and 'very' understand the implication of the words to avoid getting confused.*

'Too' means excess of some kind denoting more than sufficient something that goes beyond what is meant to be expected. 'Very' on the hand means to 'a great extent' or 'really'.

As,

Sharon is too generous to feed the poor children.

(Change to 'very')

The news is very good to be true. *(Change to 'too')*

Enough / Too

'Enough' means sufficient or indicate a proper limit has been reached whereas 'too' means more than enough.

He is enough weak to walk. *(Change to 'too')*

He is too old to get married.

(Change to 'old' enough)

He is too dark to be an African.

(Change to 'dark enough')

Exercises

1. They were intelligent than the others.
2. Sarah seldom finds something in the fridge.
3. They are so exhausted to exercise.
4. He does not know to play tennis.
5. They did not fortunately get injured.
6. The boys are happily playing.
7. She reached quick than I.
8. Rarely he visits the church.
9. I never spoke to him about my salary.
10. Tom only came to my house this morning.
11. He has faithfully obeyed my orders.
12. No one hardly talks to me.
13. I am very surprised at your behaviour.
14. He always is happy.
15. The officer was much angry with us.

Answers

1. place 'more' before 'intelligent', 2. Change 'something' to 'anything', 3. Change 'so' to much, 4. Insert 'how' before 'know', 5. Place 'fortunately' at the beginning of the sentence, 6. Place 'happily' before 'playing', 7. Place 'much' before 'quicker', 8. Place 'rarely' before 'visits', 9. Change 'never spoke' to 'did not speak', 10. Place 'only' before 'this morning', 11. Place 'faithfully' before 'has', 12. Change 'no one hardly' to 'Hardly anyone', 13. Change 'very' to 'much', 14. Place 'always' after 'is', 15. Change 'much' to 'very.

Exercises

Tick the most suitable word.

1. Harry won the toss .... Tom lost.
   (a) not (b) whereas (c) nor (d) where
2. She studies hard .... she may pass.
   (a) so (b) because (c) so that (d) but
3. He waited .... I returned.
   (a) when (b) till (c) unless (d) so
4. He was not informed .... He was terminated.
   (a) so (b) so that (c) that (d) but
5. He has never eaten meat .... hopes to do so.
   (a) or (b) never (c) always (d) neither
6. I shall join you .... you want me to.
   (a) whether (b) if (c) only (d) lest

Answers

1. (b) 2. (c) 3. (b) 4. (c) 5. (a) 6. (b)

A Closer look at Conjunction

A conjunctions function is solely to join words, phrases, clauses and sentences together.

Types of conjunctions—There are two types of conjunctions, coordinating conjunctions and subordinating conjunctions.

1. Coordinating conjunctions—Coordinating conjunctions knit words, phrases, clauses and sentences of equal status or grammatical unit of a similar kind.

Consider this.

(a) John and Mary are dancing. *(Two subjects)*
Noun Noun
Subject Subject

The above conjunction joins two sentences.

E.g. 'John is dancing, Mary is dancing'.

(b) You and I are intelligent. *(Two subjects)*
Pronoun Pronoun
Subject Subject

Here 'and' joins 'you are intelligent' and 'I am intelligent'.

(c) Harry and I took a walk. *(Two subjects)*
Noun Pronoun
Subject Subject

Here the conjunction and joins a noun and a pronoun to form a single sentence.

(d) He is rich but generous.
Adjective Adjective

Here the conjunction 'but' joins two adjectives to form a single sentence. 'He is rich', 'He is generous' to express a contrast.

(e) He swims fast and perfectly.
Adverb Adverb

Here, the conjunction 'and' joins two adverbs and sentences into a single sentence, 'He swims fast', 'He swims perfectly' to express manner. *(How did he swim ?)*

(f) We study during the day and play in the evening.
Phrase Phrase

Here, 'and' joins adverbial phrases of time.

(g) She is the girl who works hard and whose father is employed.
Two Clauses—Clause Clause

The above sentence shows that the conjunctions 'and' and 'but join two nouns, two pronouns, a noun and a pronoun, two clauses and two sentences. These conjunctions join these parts of speech, phrases, clause and sentences of equal rank. And they are called coordinating conjunctions.

Important coordinating conjunctions are as follows:

And, but, nor, too, yet, so, as well as, only, then, therefore, no less than, otherwise, else, still, while, whereas, nevertheless, either……or, neither……nor, both…..and, not only……but also, nonetheless etc.

Coordinating conjunctions are divided into four classes.

(a) Cumulative or Copulative

(b) Alternative or Disjunctive

(c) Adversative

(d) Illative

(a) Cumulative or Copulative—Such conjunctions simply join two statements or facts.

And, also, too, not less than, as well as, not only……but also, both…and.

Consider the following—

He is a king and I am a pauper.

Sam is both ugly and stupid.

You as well as your father is a thief.

The police no less than the politicians are guilty.

My father went to Delhi too.

He is not only dishonest but also mean.

Note—1. 'Also' and 'as well as' are never used after 'both'.

2. When two subjects are joined by 'as well as' the verb ought to agree with the first subject.

(b) Alternative or Disjunctive—Conjunctions like, 'either……or', 'neither…..nor', 'or', 'else', 'otherwise', that give an alternative choice or offer are called Alternative or Disjunctive conjunctions.

As,

(a) Either you or I am lying.

(b) Neither England nor America is my country.

(c) Dress quickly or (else) you will miss your bus.

(d) Confess, otherwise you will be punished.

(c) Adversative—Conjunctions like, 'but', 'still', 'yet', 'nevertheless', 'while', 'however', 'only', 'whereas' show contrast or opposition and are called Adversative Conjunctions.

Consider the following sentences—

(a) Meera is gentle while / whereas her sister is coarse.

(b) Tell the truth only don't involve me.

(c) We had no money, nevertheless we survived.

(d) She was unhappy still / yet she smiled.

(e) Bill Gates is very wealthy but he is humble.

(d) Illative—Conjunctions such as 'hence', 'therefore', 'thus', 'so', 'consequently', 'then', 'so', 'for', prove or infer statement or fact from another. Such conjunctions are called Illative Conjunctions.

Consider the following sentences—

(a) He stole the money therefore / so he was beaten.

(b) Raymond did not pass the entrance examination; hence he could not join the college.

(c) Time will make you forget for time is a great healer.

2. Subordinating Conjunctions—Subordinating conjunctions are used when words, phrases, clauses and sentences depend on a Principal Clause. Conjunctions such as 'since', 'that', 'lest', 'until', 'even if', 'as', 'since', 'as', 'when', are a case in point.

As,

(a) He knows that the grass is greener in the other side.

(b) Greg can not enjoy life since his wife has died.

(c) Study well lest you should fail.

(d) Wait until dark.

(e) Do not be afraid even if I'm not here.

(f) She is not so kind as her mother.

(g) I talked to him when he was angry.

Points to note—

(a) Subordinating conjunctions assist in joining one independed clause to another dependent clause *(subordinate clause)*

(b) Important Dependent Clauses—

'If', 'unless', 'until', 'in case', 'that', 'so that', 'till', 'until', 'before', 'after', 'so long as', 'because', 'why', 'when', 'where', 'while', 'whether', 'how', 'whence', 'who', 'which', 'what', 'as if', 'as soon as', 'than', 'as', 'since', etc.

1. Principal Clause—A Principal Clause expresses a complete meaning. It is also called independent clause.

Subordinate / Dependent Clause—A subordinate or Dependent clause unlike a Principal Clause does not express complete meaning and is dependent on the Principal Clause to make sense.

2. Time—Conjunctions 'like', 'when', 'as', 'while', 'before', 'since', 'after', 'until', 'till', 'as soon as', 'as long as' etc. indicate the time of an action, state etc.

Consider these sentences—

(a) <u>I witnessed as accident</u> (*Principal clause*) <u>while</u> (*Conjunction*) <u>I was driving</u> (*Dependent Clause*).

(b) <u>He phoned me</u> *(Principal Clause)* <u>when</u> (*Conjunction*) <u>I was dressing.</u> *(Dependent Clause)*

(c) <u>I shall be faithful to him</u> *(Principal Clause)* <u>as long as</u> *(Conjunction)* <u>he is alive.</u> *(Dependent Clause)*

(d) <u>Stay there</u> *(Independent Clause)* <u>till</u> *(Conjunction)* <u>I call.</u> *(Dependent Clause)*

(e) <u>Mother had cooked the food</u> *(Independent clause)* <u>before</u> *(conjunction)* <u>father arrived.</u> *(Dependent Clause)*

3. Place—Conjunctions like, 'where', 'wherever' indicate place.

As,

(a) He knows *(Independent clause)* where *(Conjunction)* I went. *(Dependent clause)*

(b) She runs (*independent clause)* wherever *(conjunction)* she wants. *(Dependent clause)*

4. Cause or Reason—Conjunctions like, 'because', 'since', 'as', etc. express cause or reason.

Consider the following—

(a) She came late *(Principal Clause)* because *(Conjunction)* she missed the bus. *(Subordinate clause)*

(b) Sam is heard broken *(Principal Clause)* since *(Conjunction)* his wife has died. *(Subordinate clause)*

(c) They retired early *(Principal Clause)* as *(Conjunction)* they were tired. *(Subordinate clause)*

5. Condition—Conjunctions like, 'if', 'as if', 'unless', 'provided' etc. express condition.

As,

(a) I will work for you *(Principal Clause)* if *(Conjunction)* you pay me well. (*Subordinate clause)*

(b) She walked *(Principal Clause)* as if *(Conjunction)* she were a queen. (*Subordinate clause)*

(c) He will marry me *(Principal Clause)* whether *(Conjunction)* his parents agree or not. (*Subordinate clause)*

(d) Harry won't get selected *(Principal Clause)* unless *(Conjunction)* he improves his acting. (*Subordinate clause)*

(e) I will help you *(Principal Clause)* provided *(Conjunction)* you are sincere. (*Subordinate clause)*

6. Purpose—Conjunctions like, 'that', 'so that', in order that 'lest' etc. indicate purpose.

As,

(a) I breathe *(Principal Clause)* that *(Conjunction)* I may live. (*Subordinate clause)*

(b) She studied hard *(Principal Clause)* lest *(Conjunction)* she should fail. *(Subordinate clause)*

(c) They whispered *(Principal Clause)* so that *(Conjunction)* they wouldn't be heard. (*Subordinate clause)*

(d) 'Lest' is, followed by 'should' and 'not' is never employed in the 'lest' clause.

7. Result or Effect—Conjunctions like, 'so…… that', 'such…..that', 'that' etc. express result or effect.

As,

(a) He was so lazy *(Principal Clause)* that *(Conjunction)* he failed all the tests. (*Subordinate clause)*

(b) She ate so much *(Principal Clause)* that *(Conjunction)* she vomited. (*Subordinate clause)*

(c) They are such fools *(Principal Clause)* that *(Conjunction)* everyone fools them. (*Subordinate clause)*

8. Comparison—Conjunctions like, 'as much as', 'as…..as', 'than', etc. express comparison.

(a) Ray is taller *(Principal Clause)* than *(Conjunction)* his brother (is). (*Subordinate clause)*

(b) Our college is as good *(Principal Clause)* as *(Conjunction)* your college (is). (*Subordinate clause)*

(c) I like pepsi as much *(Principal Clause)* as *(Conjunction)* I like tea. (*Subordinate clause)*

9. Concession or Contrast—Conjunctions like, 'though' / 'although', 'however', 'even if', 'notwithstanding that', 'even though', etc. express contrast or concession.

Consider the following sentences—

(d) He is a miser *(Principal Clause)* even though *(Conjunction)* he is very wealthy. (*Subordinate clause)*

(e) You won't get selected *(Principal Clause)* however *(Conjunction)* good you may be. (*Subordinate clause)*

(f) He will give his consent *(Principal Clause)* even if *Cconjunction)* it hurt him. (*Subordinate clause)*

(g) She is not mean *(Principal Clause)* though / although *(Conjunction)* she is very poor. (*Subordinate clause)*

10. Extent or Manner—Conjunctions like, 'so….as', 'as if', 'according to', 'as far as' etc. indicate manner or extent.

As,

(h) He behaves *(Principal Clause)* as if *(Conjunction)* he were a king. (*subordinate clause)*

(i) They will be placed (*Principal Clause*) according to (*Conjunction*) their abilities. (*Subordinate clause*)

(j) He talks (*Principal Clause*) as (*Conjunction*) the French do. (*Subordinate clause*)

Note—Subordinate clauses can come before a Principal Clause sometimes.

As,

Subordinate Clause	**Principal Clause**
(a) Since he is sincere.	I must help him.
(b) Unless you study.	You can not clear the test.
(c) As far as I know.	He is a gentleman.
(d) When he entered the bedroom.	He found her asleep.

11. Correlatives—Conjunctions that come in pairs are called correlatives. These are also coordinating conjunctions some important correlatives—

'neither….nor', 'either…...or', 'though……yet', 'both…….and', 'such…….as', 'such…..that', 'as…..as', 'as…..so', 'so……..that', 'scarcely……when', 'not only……but also', 'rather…..than', 'no sooner……… than', 'whether…….or'.

12. The functions of correlatives—

To join—

I. A noun / pronoun to another noun / pronouns.

II. An Adjective to another Adjective.

III. An Adverb to another Adverb.

IV. A Phrase to another Phrase.

V. A Clause to another Clause.

VI. A Sentence to another Sentence.

VII. A Subject to another Subject.

VIII. An Object to another Object.

IX. A Complement to another Complement.

As,

(a) Either he (*Pronoun*) or she (*Pronoun*) is innocent.

(b) He can neither read (*Verb*) nor write. (*Verb*)

(c) He is not only rich (*Adjective*) but also modest. (*Adjective*)

(d) She is both a nurse (*Complement*) and a compounder. (*Complement*)

(e) Though he is poor (*Sentence*) yet he is honest. (*Sentence*)

13. Usage of Correlative Conjunctions—

Either……or, Neither…….nor

(a) Harry got both a car and a cheque. (√)

Harry both got a car and a cheque. (×)

(b) He is both handsome and rich. (√)

He both is handsome and rich. (×)

(c) They are not only rich but also famous. (√)

Not only they are rich but also famous. (×)

(d) He is not only a doctor but also a surgeon. (√)

He not only is a doctor but also a surgeon. (×)

14. Whether ………….or / not—

(a) They don't know whether she is alive or not.

(b) Whether he sings or not is uncertain.

15. Although / though ……….yet—

(a) Although (though) he was talented yet he is unemployed.

(b) Although (though) she studied hard yet she failed.

16. Rather / other …… than—

(a) He has no other friend than Ravi.

(b) I would rather die than sweep floors.

17. Such………that / Such……….as—

(a) There was such a noise that everyone evacuated the area.

(b) He is such an idiot that no one ever invites him.

(c) She is not such a fool as I thought.

18. No sooner……..than—

(a) No sooner did it thunder than it started to rain.

(b) No sooner does she come than she starts working.

19. Scarcely / Hardly……… when / before—

(a) Scarcely had she lay down when someone knocked at the door.

(b) Scarcely had we reached home before the rain started.

Note—If a sentence contains NO / NOT / NEVER and followed by a clause, use 'or' and not 'nor'.

As,

(a) He has no cycle or scooter. (*not 'nor'*)

(b) John did not talk or look at me. (*not 'nor'*)

(c) They never read or write. (*not 'nor'*)

3 Sentence of Connectors–Linkers

1. A word or words that join two sentences together is called a sentence connector. There are some conjunctions which do the work of connectors.

As,

(a) This restaurant is both popular and inexpensive.

(b) These pills are not only effective but also reasonably priced.

(c) He will either phone or write to you.

(d) Go by taxi or (otherwise) you will be late.

(e) He knows that his father is dead.

(f) She was too late to attend the meeting.

2. **Some more important sentence connectors**—

(a) He has eaten his lunch so has she.

(b) He is exhausted yet he will work.

(c) I can't go to Srinagar, moreover it is not safe.

(d) He had lost a fortune; nevertheless he kept hoping.

(e) I can't lift a 100 kgs. nor can you.

(f) I know their plans; you also know it.

(g) I hate gambling, besides it's very risky.

3. **Words and phrases also serve as connectors**—Like, generally, Particularly, Similarly, Fortunately, Sadly, For example, likewise, at last, Surely, In general etc.

However, it must be noted that conjunctions alone do not serve as sentence connectors, Adverb/ Adverbial phrases also serve as connectors sentence connectors join two sentences and not two phrases.

As,

Attend the party or you will be sorry.

Here 'or you will by sorry' is not a separate sentence. It is a co-ordinate clause.

Raj is my friend. He is great fun.

In the sentence he connects with Raj is my friend. He is a sentence connector (linkers), and not a conjunction. But words like, 'because', 'as', 'since', 'while', 'than', 'just then', 'just', 'until', in the form of conjunctions work as sentence connectors.

4. **As and its functions**—

(a) As–to an equal degree.

Station was as popular as Mark.

(b) Reason

I had to join as they needed me.

(c) 'As' used as 'while'

As he was playing, they cheered him.

(d) In the way / same way or manner

They act as you do.

(e) Used as though / although

Stupid as she is, she's careful with her money.

5. Usage and functions of **'since'**

(a) After a past time

He has become a drunkard since his wife left him.

(b) From that time hence

We have been meeting every weekend ever since our school days.

(c) Because / Seeing (that)

Since you are injured, you should take rest.

(d) From a certain time till now

I have not heard of him since the 1982 Asiad.

(e) From which time

Since when have you been working here?

6. Usage and functions of **'while'**

(a) During a time / period (that)

While he was reading he listened to music.

They joined me while. I was playing golf.

(b) To show contrast

I enjoying playing chess while my brother enjoys football.

(c) In place of although

While she tries fervently, she does not succeed.

7. Usage and function of **'than'**.

'Than' is used as a conjunction and also as a comparative Adjective or Adverb. It always follows the comparative degree.

Consider the following sentences—

(a) Jim is taller (comparative Adjective) than Harry.

(b) He is much smarter (comparative Adjective) than I (am).

(c) I love you more than she (more than she loves you).

(Here, in sentence C, than takes the form of a Preposition and is placed before a noun/pronoun to show comparison)

8. **Just then, then, Just**—

As,

(a) I was about to leave, just then is started to snow.

(b) I was backing into the car park, just then a cat jumped in the way.

(c) Let me finish my breakfast, then I will leave.

(d) They have just left (a moment ago).

9. **Until**—

Until–upto the time when

(a) Wait until she leaves.

(b) Don't talk until the lecture is over.

(c) He will keep trying until he succeeds.

10. Some common errors (conjunctions)—

Unless you are recommended, you will be selected. (x)

If you are recommended, you will be selected. (v)

Until you are recommended, you will not be selected. (x)

Unless you are recommended, you will not be selected. (√)

He will be selected if that he is recommended. (x)

He will be selected provided that he is recommended. (√)

Because he is recommended, he will be selected. (x)

Provided that he is recommended, he will be selected. (√)

Because its' a holiday, I will relax. (x)

Since/as its' a holiday, I will relax. (√)

When he played the spectators cheered. (x)

While he played the spectators cheered. (√)

She studied hard because she may pass. (x)

She studied hard in order that she may pass. (√)

A good woman if she is, must be respected. (x)

A good woman that she is must be respected. (√)

It was rather noon while I spoke to him. (x)

It was hardly/scarcely noon when I spoke to him. (√)

However, be the problem, I will support you. (x)

Whatever be the problem, I will support you. (√)

No sooner I reached the office when he called. (x)

No sooner I reached the office than he called. (√)

Until you are injured, you must rest. (x)

As long as, you are injured, you must rest. (√)

Exercise

1. The food is ……… tasty ……… nutrition.
2. The mall is ……… inexpensive ……… decorative.
3. He will ……… walk ……… commute by train.
4. Harry up ……… you will be late.
5. She was ……… full ……… anymore.
6. She is tried ……… is he.
7. Jake can't buy the book ……… it is too expensive
8. He lost his empire; ……… he still hoped.
9. I dislike bungee jumping ……… it's very dangerous.
10. I can't fly ……… can you.
11. Harold has lost hope ……… he was sacked.
12. They phoned me ……… I was eating lunch.
13. Don't speak ……… the end of the lecture.
14. You will be selected ……… you are recommended.
15. Tom studied hard ……… he may pass.
16. ……… the issue I am not interested.
17. ……… I reached the station ……… the train arrived.
18. you must rest ……… you are ill.
19. It was ……… sundown ……… he rang me.
20. Let me complete my work ……… I will relax.

Answers

1. both ;and 2. not only; but also 3. either; or 4. otherwise/ or 5. too; to eat 6. so 7. as 8. never the less 9. nor 10. besides 11. since 12. while 13. until 14. provided that 15. in order that 16. Whatever 17. No sooner; than 18. as long as 19, hardly/scarcely; when 20. then

4 Subject Verb Agreement

1. *The Subject verb Agreement is vital to both the spoken and the written form. If mismatched it will sound and look wrong.*

The Main thing to remember is that, singular subjects take singular verbs whereas, Plural subjects take Plural Verbs.

Noun + s / es / ies = Plural

As,

Noun Plural	Verb (Plural)
Fans	are / do / have/ were / ……
Dishes	
Ponies	

Verb + s / es / ies = Singular

As,

Singular Verb	Plural Verb
Runs	Run
Goes	Go
Replies	Reply

Singular Subject + Singular Verb

Plural Subject + Plural Verb

As,

Singular	Plural
He runs	They run
It goes	We go
She replies	You reply

By these examples, you must have understood, by now, that Singular Noun + Singular Verb and Plural Noun + Plural Verb. Keep this important rule in mind. It will help you avoid making subject – Verb Agreement mistakes.

To makes a quick and correct choice of the subject-Verb combination. Remember this easy point.

2. *The Auxiliaries (Helping Verb) that end in 's' take singular subjects.*

As,

Present

1. *Is* Ram going home ? (*Present Continuous*)
2. *Does* she play chess ? (*Present Indefinite*)
3. *Has* he come ? (*Present Perfect*)
4. *Has* Rchana been studying ? (*Present Perfect Continuous*)

And auxiliaries that end is 'e' take plural subjects.

As,

Present

1. *Are* you reading ? (*Present Continuous*)
2. *Do* you watch movies ? (*Present Indefinite*)
3. *Have* they enjoyed the play ? (*Present Perfect*)
4. *Have* the boys been playing ? (*Present Perfect Continuous*)

Do the auxiliary which ends in 'o' is an exception like 'am' which takes only 'I' as its sole subject.

3. *Similarly the Past Tense auxiliaries like was and has take singular subjects and 'were' takes plural subjects.*

As,

Singular	Plural
Was he ?	Were they ?
Has she ?	Were we ?

Note—that other auxiliaries that end in 'd' like 'did', 'had' and 'l' like, 'will' / 'shall' take all subjects singular and plural alike.

Past Indefinite

Did he play ?

Did they play ?

Did she play ?

Did we play ?

Past Perfect Continuous

Had I been reading ?

Had she been reading ?

Had it been reading ?

Had we been reading ?

Past Perfect

Had I slept ?

Had you slept ?

Had she slept ?

Had he slept ?

Future Indefinite

Shall I go ?

Shall we go ?

Will he go ?

Will they go ?

Will it go ?

Note that in Modern grammar 'will' takes all subjects and 'shall' is no longer used for I and We subjects. However, in the written form shall is employed with I and We. But, the discretion lies solely with the writer (or speaker).

4. *When two singular nouns talk about the sense person or thing and possess only one article or other qualifying word before them the verb is singular.*

As,

The king and philanthropist is here.

When he was born, his father and predecessor was famous.

5. *When two nouns express the same idea and one is added to the other for the sake of emphasis the verb is Singular.*

As,

Their success and fame is notable.

Etiquette and manners is the mark of a gentleman.

6. *When two singular nouns are not similar but express together a single idea, the verb is singular.*

As,

Bread and butter is our staple food.

Slow and steady wins the race.

7. *When a collective noun is used as a common noun (but shows some division) the verb is plural.*

As,

The jury are divided in their verdict.

The Ministry are divided in their opinion.

8. *But when a collective noun is thought of as a unit the verb is singular.*

As,

The team gathers together.

The mob moves forward.

9. *When the plural noun is a proper name for a collective unit or single object, the verb is singular.*

The U.S.A. has failed in its attempt to curb terrorism.

The Andaman and Nicobar Islands is a wonderful sight.

10. *At times the subject is joined by the conjunction 'and' but refers to one and the same person then the singular verb is used.*

As,

The king and benefactor has (singular verb) arrived.

The king and the benefactor have (Plural verb) arrived.

The first sentence refers to one person as king and benefactor. Thus, singular verb is used. Whereas, the second sentence treats the king and the benefactor as separate entities. Therefore, Plural verb is used.

11. *In case there is a problem in identifying a singular or plural subject. Look out for the Articles 'a', 'an', 'the' to recognize Plural Forms. Also, keep an eye for possessives like his, her, our, your, its etc.*

As,

Mr. Smith, her uncle has (singular verb) arrived.

Mr. Smith and her uncle have (Plural verb) arrived.

Note—Here 'and' join two subjects and does not refer to the same person. Unlike 7.

So, when two subjects are joined by a connector (in this case 'and') look out for the article or possessive to identify plural forms.

12. *When two objects give the idea of being one unit then, the singular verb is used.*

As,

Curry and Chawal is a good dish.

(*Curry and Chawal, which means a 'dish' containing curry and chawal*)

Similarly,

Slow and steady wins the race.

Winning and losing is part of the game.

However, there are exceptions to the rule.

As,

Tears and laugher go (*Plural verb*) hand in hand.

Time and Tide wait (*Plural verb*) for no man.

13. *Adjectives when followed, as it mostly does, by a noun and describes a single noun takes the singular verb.*

As,

The white and blue check shirt is (*Singular verb*) mine.

The black and blue mark on you face shows (*Singular verb*) you have been beaten.

The tall fat man is (*Singular verb*) insane.

14. Each / Every / Neither / Either

Words such as each, every, neither, either one; take singular verbs.

As,

Each of us is going to protest.

Every one of the protesters was arrested.

Either of the two is lying.

Neither of the two brothers is guilty.

One of them is at fault.

15. *But, when the there is a singular subject and a plural subject the subject closest to the verb agrees with the verb.*

As,

Tom or his friends are guilty.

Either she or the girls are lying.

As,

You or he is guilty.

Either he or I am guilty.

Neither they nor he is guilty.

16. *After 'or' and 'nor' the verb agrees with the subject that follows. (Nearest to the verb)*

As,

That boy or the girls are (*Plural verb*) wrong.

Neither the teacher nor the students are (*Plural verb*) present.

He or I am (*Singular verb*) strong.

Neither you nor he is (*Singular verb*) capable.

17. *In case, two or more are joined by 'and' the plural verb is utilized.*

As,

You, he and I are (*Plural verb*) fit.

You and he are (*Plural verb*) going to Delhi.

She and I are (*Plural verb*) winning.

18. *When a subject express distance, time, mass, weight, amount as a single unit. It takes the singular verb.*

As,

Twenty kilometers is not a short walk.

Fifty quintals is enough.

Twenty years is a long time.

Five hundred dollars is a good reward.

19. *But, when taken as separate units. It is followed by a plural verb.*

As,

Ten thousand dollars were (plural verb) expended.

Twenty miles are (plural verb) to be trekked.

20. *Subjects that precede words like–besides, as well as, except alongwith, with together with, not, in addition to etc are followed by verbs which agree with it, according to its number Singular or Plural.*

As,

Ram, besides his assistants has failed the test.

I, with my children am going to Delhi.

Roymond together with his uncles plays cricket.

I, no less than he am guilty of the crime.

Knowledge, in addition to wisdom was Solomon's strength.

He not you is clever.

You as well as the officer is ignored of the fact.

21. *Where different numbers and persons are the subject of words like, 'but' not only….. but also the verb agrees with the second subject.*

As,

Not father but I am (*second subject*) the culprit.

Not only your from but also your friend (*second subject*) is lying.

Not only your friend but also you (*second subject*) are speaking the truth.

22. *Noun words like the following are only in the singular and have no plural form.*

Furniture, Advice, Information, Equipment, Knowledge, Work, Weather.

Diseases : Mumps, Measles etc.

Sport : Darts, Billiards, Caroms etc.

Proper Nouns : The United States, Algiers etc.

Others : Physics, Athletics, Politics, News, Innings etc.

All the above take singular verbs.

As,

Mumps is common among children.

Mathematics is not a difficult subject.

No news is good news.

Furniture is mostly made of wood.

Knowledge is invaluable.

The United States is a wealthy nation.

Politics is a dirty game.

23. *Some, some of, enough, enough of, half, half of, most, most of, a lot, a lot of, lots of, plenty, plenty of, not enough of are words, phrases that are followed by plural countable that are followed by plural and those that are followed by uncountable nouns that are followed by uncountable nouns take singular verbs.*

As,

Plural Verb

Some boys are clever.

Not enough of books are read by the modern generation.

Lots of apples were sold.

Singular Verb

Some food was distributed.

Half of the land was barren.

Not enough rice was stored.

24. *Collective Nouns when placed between a …….. of, followed by countable nouns take singular verbs.*

As,

A bunch (*collective noun*) of flowers (*Countable noun*) was (*Singular verb*) eaten by cow.

A gang of thieves is in the area.

A troupe of dancers is performing.

A group of boys was present.

25. *Though singular in form words like–sheep, poultry, people, cattle etc. take plural verbs always.*

As,

The poultry (*chicken / ducks*) are missing.

The police (*members of the police force*) were informed.

The cattle (*cows and buffalos*) are straying.

26. *The 'to infinitive and Gerund take a singular verb after it.*

As,

Dancing keeps one fit. (*Gerund + singular verb*)

To dance keeps one fit. (*to infinitive + singular verb*)

27. In optative / conditional and imagination sentences the plural verb is utilized.

As,

Long live the king ! (*optative*)

May God smile upon you ! (*Optative*)

If, I were a butterfly. (*Condition*)

I wish he were alive. (*Imagination*)

28. *When a noun is in apposition, the first subject agrees with the verb.*

As,

I, you brother, am happy.

You, teacher, are wise.

29. *In mathematics both the singular and plural verb may be used.*

As,

One and one is two. (*Singular verb*)

One and one are two. (*Plural verb*)

Exercise

Fill in the appropriate words that are in subject - verb agreement.

1. Tommy ……… (go/goes) to school daily.
2. ……… (has/have)it been working perfectly ?
3. The children ……… (be/are) playing hopscotch.
4. I ……… (will/shall) be going home in an hour.
5. Jon ……… (travel/travels) to work by bus.
6. We ……… (were/ was) in a great hurry.
7. His father and mentor ……… (was/were) present.
8. Our fame and power ……… (is/am/are) remarkable.
9. Charm and gallantry ……… (is/am/are) his forte.
10. Bread and butter ……… (am/are/is) our favorite shack.
11. Slow and steady ……… (win/wins) the race.
12. The jury ……… (is/are) divided in their decision.
13. The ministry ……… (is/are) confused about their posting.
14. The unruly gang ……… (move/moves) on.
15. The U.S.A ……… (have/has)failed to gain our confidence.
16. The king and the benefactor ……… (has/have) arrived.
17. Mr.gupta, her father ……… (has/have) spoken.
18. Winning and losing ……… (is/are) part of life.
19. Time and tide ……… (wait/waits) for no one.
20. The black and white striped shirt ……… (is/are) yours.
21. Each of you ……… (is/are) clever.
22. Either of the two ……… (is/ are) lying.
23. His friends or I ……… (am/are) innocent.
24. Neither you nor he ……… (is/are) guilt.
25. Twenty kilometers ……… (is/are) a long walk.
26. Twenty miles ……… (is/are) to be covered.
27. Hari besides his friends ……… (has/have)failed.
28. No news ……… (is/are) good news.
29. Some food ……… (was/were) distributed.
30. A gang of thieves ……… (is/are) in our locality.

Answers

1. goes 2. has 3. are 4. shall 5. travels 6. were 7. was 8. is 9. is 10. is 11.wins 12. are 13. are 14. moves 15. has 16. have 17. has 18. is 19. waits 20. is 21. is 22. is 23. am 24. is 25. is 26. are 27. have 28.is 29. was 30. is

5 Simple, Compound and Complex Sentences

1. Introduction

The conversion of a sentence is to change it from one grammatical form to another without altering its sense. A simple sentence contains a finite verb. If we say, 'Raj is there' we form an idea of Raj, of a place and Raj being in that place. When we say, 'Raj bit his friends' ear' we picture Raj and a bite passing from Raj to his friend. The verb in the first statement is intransitive and in the second the verb is transitive. In each we make more than one assertion. This is called a Simple Sentence.

But, if we say, 'Raj and Hari talked to Sam'. We make two statements James met Sam, James and Hari were together. Here we have a compound Sentence.

Often we make statements modified by some qualification expressed by a clause. A clause is said to be co-ordinate, when it is possible to separate one from another. Such that each makes an independent sentence and independent sense e.g. 'They left their work and returned to their homes. A subordinate clause on the other hand is dependent on the Principal clause or leading assertion. E.g. She studied hard that she might pass.

A sentence of this type containing a subordinate or secondary clause is called a complex sentence.

Given below are sentences which show how the same sense or meaning can be retained and yet conveyed in a Simple, a Compound or a Complex sentence.

Simple—The problems facing him from all sides did not deter him.

Compound—He was faced by problems from all sides but he was not deterred by them.

Complex—He was not deterred by the problems that faced him from all sides.

Note that the above changes are in the form and structure but the meaning remains unchanged. The close study of the following will be of immense value in learning a variety of expression and it will definitely add colour and flavour in preparation of competitive English.

2. Means of transforming conditional sentences

(A) A sentence containing the word 'too' can be transformed in the following manner.

As,

1. He is too honest to tell a lie.

He is so honest that he can not tell a lie.

2. Jack is too impatient to wait.

Jack is so impatient that he can not wait.

3. The weather is too cold to bathe.

The weather is so cold that we can not bathe.

(B) There are many ways in which one can transform a conditional sentence. By using 'if'.

1. Allow me to leave and I will not return.

I will not return if you allow me to leave.

2. Let the band be superb and I will perform.

I will perform if the band is superb.

(C) By employing a conjunctional phrase—

1. Pay me Rs. 10,000.00 and I will work for you.

In case you pay me Rs. 10,000.00 I will work for you.

(D) Where the 'if' is understood.

1. If I had met him, he would have recognized me.

Had I met him, he would have recognized me.

2. If you had been to America. You would have understood.

Had you been to America, you would have understood.

3. If the Government had collapsed, there would have been chaos.

Had the Government collapsed, there would have been chaos.

(E) By employing a participle phrase:

1. You can come to the party, if you dress well.

You can come to the party provided that you dress well.

2. If that were the case, what should we do ?

Supposing that were the case, what should we do ?

Provided that were the case. What should we do ?

(F) By employing 'but' followed by a phrase:

1. Had they not been innocent, they would have been jailed.

But for their innocence, the would have been jailed.

2. If he does not help you, you would be destroyed.

But that he helped you, you were not destroyed.

But for his help, you would have been ruined.

(G) By employing the phrases 'one more' and 'were to'.

1. If we have one more such failure, we are lost.

One more such failure and we are lost.

2. If he questioned me, he would recognize me.

If he were to question me, he would recognize me.

(H) By employing the Imperative state:

1. If you labour, you will achieve.

Labours and you will achieve.

2. If you shut the door, strangers will not enter.

Shut the door, and strangers will not enter.

(I) By using the interrogatives sentence.

1. If you dance well you will be selected.

Do you dance well? Then they you will be selected.

2. You may leave, if you have seen the film.

Have you seen the film? Then you may leave.

3. To change a sentence that denotes contrast or concession

(A) Use the conjunction 'though' or 'although'.

1. He is rich but just.

He is just though rich.

2. Sarah is beautiful but she is still single.

Although Sarah is beautiful, she is still single.

(B) Use 'as'—

1. Though he was healthy, he died.

Healthy as he was, he died.

2. Though he is powerful, he is a coward.

Powerful as he is, he's a coward.

(C) Use 'however' and follow it by some adjective or adverb—

1. Though he was healthy, he died.

However healthy he might have been, he died.

2. Though he is rich, he is a miser.

However rich he may be, he is a miser.

(D) Use the phrase 'all the same'.

He is very weak but he will work.

He is very weak, all the same he will work.

(E) Use the phrase 'for all that' and follow it with a Noun clause—

1. A father will protect his son, though he may be a criminal.

A father will protect his son, for all that he may be a criminal.

2. Though you may be innocent, the jury will not believe you.

You may be innocent; for all that the jury will not believe you.

(F) Use the absolute participle—

1. Though he was very honest at time he would tell a lie.

Admitting that he was very honest, he at times, tells a lie.

2. Though he was not a bright student, they promoted him.

Knowing he was not a bright student, they promoted him.

(G) Use the preposition 'notwithstanding' followed by a Noun Clause.

1. Raj is still working although he not rested for 12 hours.

Raj is still working, notwithstanding that he has not rested for 12 hours.

2. He still borrows money although he earns more than enough now.

He still borrows money, not with standing that he earns more than enough now.

(H) Use 'if' or 'even' or 'even if'—

1. He was a poor man, but he was generous.

Even if he was a poor, he was generous.

2. Though he drank in excess, he always paid his bills.

If he drank in excess, he always paid his bills.

3. Although he changed his job, yet he is not happy.

He changed his job indeed, but he is not happy.

(I) Use 'nonetheless' or 'nevertheless'—

1. Though they are very wealthy, yet they are stingy.

They are very wealthy, nevertheless they are stingy.

2. Though he was selected instead of me. I am not envious.

He was selected instead of me, nonetheless I am not envious.

(J) Use the pronoun 'whatever'—

They will not attend, though I may offer them anything.

They will not attend, whatever I may offer them.

4. You can transform a sentence by simply changing the degree of comparision.

(A) 1. Positive – Ram is as smart as Ravi.

Comparative – Ravi is not smarter than Ram.

2. Comparative – The weather is cooler in Bangalore than Chennai.

Positive – The weather in Chennai in not so cool as Bangalore.

3. Positive – No author writes as well as Sidney Sheldon.

Superlative – Sidney Sheldon writes the best of all.

4. Superlative – She is the most beautiful girl in the class.

Positive – No girl is as beautiful as her in the class.

(B) Positive – No other gem is as expensive as diamond.

Comparative – Diamond is more expensive than any other gem.

Superlative – Diamond is the most expensive of all gems.

(C) Superlative – U.P. is the largest state in India.

Comparative – U.P. is larger than any other state in India.

Positive – No state in India is as large as U.P.

5. Another way is to transform the Active form into Passive form and vice-versa.

(A) Raymond shot a leopard. (*Active*)

(B) A leopard was shot by Raymond. (*Passive*)

(C) The food was prepared by Aunt Jane. (*Passive*)

(D) Aunt Jane prepared the food. (*Active*)

(E) Who killed your cat ? (*Active*)

(F) By whom was your cat killed. (*Passive*)

(G) Free the prisoner. (*Active*)

(H) Let the prisoner be freed. (*Passive*)

6. You can change a sentence from the affirmative to the Negative and also the other way round.

(A) Mary loves Tim. (*Affirmative*)

(B) Mary is not without love for Tim. (*Negative*)

(C) None but the pure hearts will see heaven. (*Negative*)

(D) The pure hearts alone will see heaven. (*Affirmative*)

(E) No other member of the club was as honest as Paul. (*Negative*)

(F) Paul was the most honest member of the club. (*Affirmative*)

7. An Interrogatives sentence can be transformed into the Assertive form and vice versa.

(A) When will they lose a match ? (*Interrogative*)

(B) They will never lose a match. (*Assertive*)

(C) Everyone would hate a heartless father. (*Assertive*)

(D) Who would not hate a heartless father ? (*Interrogative*)

8. The exclamatory sentence can be transformed into the assertive form.

(A) Alas! She is undone. (*Exclamatory*)

(B) It is sad that she is undone. (*Assertive*)

(C) It is a very beautiful flower. (*Assertive*)

(D) How beautiful the flower is ! (*Exclamatory*)

(E) If I could win her hand ! (*Exclamatory*)

(F) I strongly desire to win her hand. (*Assertive*)

9. You may transform a sentence, such that by changing the Part of Speech of a leading word.

(A) Verb – He failed in his Endeavour.

Noun – His Endeavour was shrouded in failure.

Adjective – His Endeavour was a failure.

(B) Verb – I can not permit you to go.

Noun – I can not give you permission to go.

(C) Noun – They accepted our proposal.

Verb – They accepted what we proposed.

(D) Adverb – She acts intelligently.

Noun – She acts with intelligence.

(E) Use 'not only .... but also' in place of 'besides'.

Simple – Besides being handsome, he was intelligent too.

Compound – He was not only handsome, but he was intelligent too.

Simple – Besides scoring a century, he took five wickets.

Compound – He not only scored a century, but he also took five wickets.

(F) By using the affirmative conjunction 'or'.

Simple – He must surrender, to avoid being shot.

Compound – He must surrender or he will be shot.

Simple – You must marry him to avoid a scandal.

Compound – You must marry him or face a scandal.

(G) By using the conjunction 'and so'.

Simple – He was arrested for killing his wife.

Compound – He killed his wife and so they arrested him.

Simple – Owing to an illness he failed the test.

Compound – He was unwell and so he failed the test.

(H) Change a compound sentence into a simple sentence by changing a Participle for a finite verb.

Compound – The sun set and the cows went home.

Simple – The sun having set, the cows went home.

Compound – He completed his work and took a nap.

Simple – Having completed his work he took a nap.

(I) By using a gerund or infinitive.

Compound – He must not be foolish or he will regret it.

Simple – In the event of his being foolish, he will regret it.

Compound – We must fight or we can not win.

Simple – We must fight to win.

10. Transforming a Simple Sentence into a Complex sentence.

(A) Use a noun clause.

Simple – I am sure to win.

Complex – I am sure that I will win.

Simple – He admitted his mistake.

Complex – He admitted that he had made a mistake.

(B) Use an adjective clause.

Simple – He gifted her diamond chain.

Complex – He gifted her a chain which was made of gold.

Simple – Serious students are successful.

Complex – Students who are serious are successful.

(C) Use an adverb clause.

Simple – On his command, the troops will march.

Complex – The troops will march as soon as he commands them.

Simple – She owned her fame to her agent.

Complex – It was owing to her agent that she got famous.

11. To Transform a Complex Sentence into a Simple sentence.

Substitute a noun for the Noun Clause preceded by 'that'.

(A) Complex – She said that she was guilty.

Simple – She declared her guilt.

Complex – She prayed that he lived long.

Simple – She wished a long life for him.

(B) Substitute a noun for a noun clause preceded by 'what'—

Complex – They must reveal what he intends to do.

Simple – They must reveal his plans.

(C) Use a Compound Noun.

Complex – We stopped on a hill where the farmer lived.

Simple – We stopped on the farmer's residence.

12. Transform a compound sentence into a complex one.

(A) Using a conjunction which adds one statement to another.

Compound – Be brave and you will win.

Complex – If you are brave, you will win.

Compound – Study well or you will fail.

Complex – Unless you study well, you will fail.

(B) Use a conjunction which expresses contrast or opposition.

Compound – He is a poor but he is always generous.

Complex – He is always generous although he is poor.

Compound – He had lost but he was not without hope.

Complex – Although he lost, he was not without hope.

13. Transform a Complex Sentence into a Compound Sentence as illustrated below.

(A) Complex – He is sure you have failed the test.

Compound – You have failed the test and of this he is sure.

(B) Complex – As soon as he ate breakfast. He left.

Compound – He ate his breakfast and immediately left.

(C) Complex – If you do not earn, you will starve.

Compound – You must work or you will starve.

(D) Complex – We eat that we may live.

Compound – We desire to live, therefore we eat.

6 Modals

1. Introduction

The words shall, should, will, would, can, could, must, used, need and dare are called Modal or Modal Auxiliaries.

Marginalized Modal Auxiliaries—Marginal Modal Auxiliaries are Need, Dare, Ought to, Used to. They are not a part of a main or important group and marginalize themselves.

2. Shall / Will

These modals are used to express the future tense.

'Shall' agrees with the First person subject, I, We whereas, 'will' agrees with Second and third person subjects.

The modals shall and will express more than just the future. They also express Promise, Intention, Warning, Determination etc.

(A) Future Tense

I shall go to Delhi.

We shall stay at the Oberoi's.

He will pay the bill.

You will go by train.

(B) Intention / Determination / Promise/ Warning are expressed when the subjects I, We

joins with 'will'.

As,

I will not tell anyone (*promise*)

We will try until we succeed. (*Determination*)

We will report you to the Principal. (*Warning*)

I will buy some more shares. (*Intention*)

(C) When 'shall' is used with the second and third person it can imply order, Promise, Determination, Threat or Warning.

As,

Rajesh shall be jailed if he breaks the law. (*Warning / Threat*)

They shall find the trip exciting. (*Promise*)

You shall finish it. (*Order*)

Tom insists he shall become a Judge. (*Intention / Determination*)

Note—According to Modern grammar it is now accepted and alright to use will with all subjects. So, when in doubt just use will.

(D) Shortened form of Shall not, Will not

Shall not = shan't

Will not = won't

Shortened Form (Positive)

I Shall	=	I'll
We shall	=	We'll
You will	=	You'll
They will	=	They'll
He will	=	He'll
She will	=	She'll
It will	=	It'll

3. Uses of Should / Would

Should, is the past form of shall and would, is the past form of will. The indirect speech employs the use of should, would in place of shall, will in the Direct speech pattern.

(A) Use of Should

Should apart from being used in Indirect speech also expresses Advice and suggestion.

As,

He should attend school regularly.

One should not tell lies.

You should see a doctor.

(B) To express Moral Obligation.

As,

You should respect elders.

We should help the disabled.

We should obey our parents.

We should obey traffic rules.

(C) Lest ….. should to express 'Purpose'

As,

Be careful lest you should fall.

Work hard lest you fail.

Be alert lest you fall asleep.

(D) To express a need

As,

There should garbage cans on the sheets.

There should be schools in every village.

There should be a doctor in every school.

There should be transparency to remove corruption.

(E) To express 'Probability'

As,

He should be in the library.

They should arrive by 7 p.m.

Mother should be in the kitchen.

(F) To express 'condition'

Here it takes the role of if.

As,

Should it snow, I shall stay at home.

Should he not arrive, we shall phone him.

Should you repent, you will be saved.

(G) To express a polite wish

I should like to speak to the Director.

We should like your assent on this issue.

(H) To express Moral Duty / Obligation in the Past

As,

We should have visited uncle when he was in hospital.

You should have asked your parents.

You should not have lied to your wife.

The Congress should not have passed such a bill.

4. Use of Might

As, should, would, could, might is also, used in Indirect speech. Apart from expressing Possibility, Request and Permission, Purpose in the Past and such.

(A) Possibility (Might / be + V^1) Present / Future

As,

It might be a holiday.

He might pass the exam.

Sita might visit today.

(B) Request / Permission

Might I come in ?

Might he close the door ?

He might come in ?

(C) It also expresses Purpose in the past. (So + Might + V^1)

He ate so quickly that he might finish first.

They worked so hard that they might finish in time.

Jeef studied all night so that he might pass the test.

(D) Unfilled Possibility in the Past (Might + have + V^3)

They might have seen the movie.

The children might have eaten.

He might have got the job.

5. Use of Must

Must express Necessity, Compulsion, Prohibition, Emphatic, Advice or determination, duty or possibility in the Present.

As,

(A) I must see a doctor. (*Need*)

We must leave now. (*Immediately due to …*)

I must talk to him now. (*Urgency*)

(B) Necessity / Compulsion (*Future*)

I must pass before lest I be expelled.

He must reach Delhi by tomorrow.

He must have at least 70% to enter the college next month.

(C) Emphatic Advice / Prohibition

You must stop drinking alcohol.

We must fight corruption.

We must obey the law.

6. Use of May

(A) May like most modals has many uses. It expresses; Possibility, Permission, Request,

Wish, Prayer, Purpose, Assumption etc.

Permission (May + V^1)

May I use your phone ? (*Am I permitted to use your phone ?*)

May I come in ? (*Am I allowed to enter ?*)

He may join us for lunch. (*I permit him to join us*)

(B) Possibility (May + V^1)

Thomas may come here. (*It is possible that he will come here*)

It may rain today. (*It is possible that it will rain today*)

Sharon may be at home. (*It is possible that Sharon will be at home*)

(C) Request (May + V^1)

May I shut the window, please ?

May I go out to play, please ?

May I borrow your lawn mower ?

(D) Offer (May + V^1)

May I help you ?

May I carry your bag ?

May I fix your motor ?

(E) Purpose (May + V^1)

He works so hard that he may qualify.

She is singing so beautifully that she may win the contest.

(F) Assumption (May + have + V^3)

She may have stolen the watch.

He may not have forgotten the file.

(G) Wish / Prayer / Curse (Optative Sentence) (May + V^1)

May God curse you !

May God bless you !

May you live long.

(H) Uncertainty (May + be)

Who may be lurking around ?

Who may be it at this hour ?

Who may be stealing our hens ?

The above usages and examples of Modals are meant to give the reader a clearer picture and view into the many uses of Modals. However, these can confuse and puzzle. To understand Modal usage. Keep it simple.

For example—

Use Shall / Will simply as the future tense.

Use Can for present ability, informal permission and possibility.

Use Could for past ability, Polite request and possibility.

Use May, for permission and possibility.

Use might for possibility.

Use should for advice / suggestion.

Use would for past habit / routine.

Use Dare for challenge.

Use need / to for necessity.

Use ought to for advice / suggestion.

7. Use of Would

As mentioned earlier 'would' is the past equivalent of will and is used in Indirect

speech besides, other functions.

(A) Direct Speech

They said, "We will pay the bill".

Indirect

They said that they would pay the bill.

Direct

She said to me, "You will regret tomorrow".

Indirect

She told me that I would regret the next day.

(B) To express Past Habits

As,

When I was younger I would walk 10 mile everyday.

Ten years ago I would smoke and packs of cigarettes. Now, I don't smoke at all.

(C) To express Hypothetical Conditions

If I were a butterfly, I would kiss all the flowers.

If you had entered the contest, you would have won.

If I were the President of America, I would not support Pakistan.

8. Use of Can

Can expresses present ability, Can = able to.

Positive statement = Sub. + can + V^1 + other words

= I can eat ten cakes.

Interrogative Sentence = Can + Sub. + V^1 + OW ?

= Can you lift a 100 kgs. ?

Negative Sentence = Sub. + can + not + V^1 + OW

= He can not drink 5 litres of milk.

Examples—

I can pass this test.

He can speak French and Russia.

Can you break the lock?

He can not fix the motor.

(A) Can is also used to express the future tense = Can = Shall / will be able to.

As,

He can meet one tomorrow.

I can give you a loan next week.

(B) Can also operates as–know how to.

He can solve this equation. (*Knows how to*)

I can train the cadets. (*Know how to*)

(C) Can also expresses probability.

As,

Mr. Gupta can arrive anytime.

The rumor can be true.

(D) To express or ask permission (*Informal*).

As,

Can I go out to play ?

Can I borrow your car ?

You can stay as long as you like.

(E) It also expresses offer / request.

As,

Can I help you ?

Can you close the window ?

9. Use of Could

Could is in the Indirect Speech.

As, Direct

She said, "I can solve this issue".

Indirect

She said that she could solve that issue.

Direct

The boss said, "I can employ you".

Indirect

The boss said that he could employ him.

(A) 'Could' is also utilized for Past Ability.

As,

I could lift a 100 kgs. When I was younger.

He could speak Bengali when he loved in West Bengal.

He could run 10 miles when he was fit.

(B) Could used as expression / possibility

As,

They could help the victims of the earthquake.

Ram could have got the job.

(C) 'Could' as a request.

Could I help you ?

Could I come in ?

(D) To express Condition

If I were the P.M., I could pass the bill.

If I were the Principal, I could pass you.

(E) For Suggestion

You could discuss it with your father.

You could join a health club.

(G) Possibility in the past

I could have entered the contest.

You could have joined our team.

10. Use of Need / Dare (Necessity)

Ravi needs a pair of shoes. His old ones are torn. (*Present*)

Tom needed Harry's help. (*Past*)

They will need monetary support. (*Future*)

Thus, the above examples show that 'need' can be utilized in all three tenses.

(A) Need + not denote absence of necessity.

As,

You need not take a bath.

They need not visit us.

She need not enter the contest.

(B) Dare (Challenge)

You dare to say so.

How dare you!

(*I*) dare you ask her.

Exercise

Place the most suitable modal in the blanks—

1. I .... go to Agra this month.
2. He .... answer the phone.
3. I .... not tell a soul.
4. we .... continue fighting until we win.
5. Raymond .... be punished if he cheats.
6. You .... brush your teeth daily.
7. .... it rain, I shall not go ....
8. It .... rain today.
9. I .... meet him now.
10. .... I use your computer ?
11. He .... go for long walks in the woods.
12. I .... lift 50 kgs of weights
13. He .... work for 8 hours non-stop.
14. He said that he .... return the next day. (indirect speech).
15. Harry .... a pair of shoes.
16. How .... you!
17. I .... smoke 6 packs of cigarettes a day.
18. I .... meet you tomorrow.
19. .... I play outside?
20. You .... obey your parents.

Answers

1. shall 2. will 3. will (promise) 4. will (determination) 5. shall (warning) 6. should (advice) 7. should 8. might (possibility) 9. must (urgency) 10. may (permission) 11. would (used to) 12. ability (present) 13. ability (past) 14. could / would 15. needs(necessity) 16. dare (challenge) 17. would (past habit) 18. shall (simple future) 19. can (informal request) 20. suggestion

7 Tenses

A. Verb Forms

1. Before we go into the tenses it is imperative to understand the importance of verb forms which are the pillars on which the tenses can stand.

There are two types of verb forms—

(a) Regular (b) Irregular

Regular verbs form their past tense and past participle by adding 'ed' to its first form. As, play–played (past tense) played (Past Participle).

Irregular verbs are different as they form their past tense and past participle in a different manner as opposed to regular verbs.

They change their form in the following ways.

(a) In which two forms are similar.

(b) In which all three forms are the some.

(c) In which all three form are dissimilar.

2. Examples of Irregular Forms

(a) In which two of the form are similar.

Present	Past	Past Participle
Awake	Awoke	Awoke
Bleed	Bled	Bled
Catch	Caught	Caught
Stand	Stood	Stood
Win	Won	Won

(b) In which all three forms are similar.

Present	Past	Past Participle
Bet	Bet	Bet
Cut	Cut	Cut
Let	Let	Let
Put	Put	Put
Split	Split	Split

(c) In which all three forms are dissimilar.

Present	Past	Past Participle
Be	was / were	Been
Do	Did	Done
Eat	Ate	Eaten
Rise	Rose	Resin
Write	Wrote	Written

Also,

(d) In which the vowels are changed.

Present	Past	Past Participle
Swim	Swam	Swum
Drink	Drank	Drunk
Wring	Wrung	Wrung

B. Tenses

PRESENT TENSE

1. Formation of Sentences

	Positive	Interrogate	Negative
A	I go to school. He / she / it / Ram goes to school. I, You, We, They take the plural form of the verb (do)	Do I go to school ? Does he / she / it / Ram go to school ?	I do not (don't) go to school. He / she / it / Ram does not (doesn't) go to school.
	Subject + V1 + o / ow	A.Verb^1 + Subject + V^1	Subject + Do/Does + not + V1
B	I am going to school. You / We / They are going to school. He / She / It / Ram is going to school.	Am I going to school ? (You / We / They + are) Are we going to school ? Is he / she / it / Ram going to school ?	I am not going to school. You / We / They are not (aren't) going to school. He / She / It / Ram is not (isn't) going to school.
	Subject + Aux. + V1 + ing + ow	Aux. + Subject + V^1 + ing + ow	Subject + Aux. + not + V^1 + ing + ow

C	I / You / We / They have been to school. He / She / It / Ram has been to school.	Have I / You / We / They been to school ? Has he / she / it / Ram been to school ?	I / You / We / They have not (haven't) been to school. He / She / It / Ram has not (hasn't) been to school.
	Subject + Aux. + V^3 + ow	Aux. + Subject + V^3 + ow	Subject + Aux. + not + V^3 + ow
D	I / You / We / They have been going to school + (for / since) He / She / It / Ram has been going to school + (for / since)	Have I / You / We / They been going to school + (for / since) ? Has He / She / It / Ram been going to school + (for / since) ?	I / You / We / They have not been going to school + (for / since) He / She / It / Ram has not been going to school + (for / since)
	Subject + Aux. + V^3 (been) + V^1 + ing + ow	Aux. + Subject + been (V^3) + V^1 + ing + (For / Since)	Subject + Aux. + not + been + V^1 + ing + ow + (For / Since)

Past Tense

2. Formation of Sentences

	Positive	Interrogate	Negative
A	I / You / We / They went to school. He / she / it / Ram went to school.	Did I / You / We / They go to school ? Did he / she / it / Ram go to school ?	I / You / We / They / He / She / It / Ram did not (didn't) go to school.
	Subject + V^2 + o / ow	Aux. (Did) + Subject + V^1 + ow	Subject + Aux. (did) + not + V^1 + ow
B	You / We / They were going to school. I / He / She / It / Ram was going to school.	Were You / We / They going to school ? Was I / he / she / it / Ram going to school ?	You / We / They were not (weren't) going to school. I / He / She / It / Ram was not (wasn't) going to school.
	Subject + Aux. + V^1 + ing + ow	Aux. (was / were) + Subject + V^1 + ing + ow	Subject + Aux. (was / were) + not + V^1 + ing + ow
C	I / You / We / They / He / She / It / Ram had been to school + *(before + V^2) *when one action was completed before another starts.	Had I / You / We / They / He / She / It / Ram been to school ?	I / You / We / They / He / She / It / Ram had not (hadn't) been to school.
	Subject + had + V^3 + ow + (before + V^2)	Had + Subject + V^3 + ow + (before + subject + V^2)	Subject + Aux. (Had) + not + V^3 + ow + (before + subject + V^2)
D	I / You / We / They / He / She / It / Ram had been going to school + (for / since)	Had I / You / We / They / He / She / It / Ram been going to school + (for / since)?	I / You / We / They / He / She / It / Ram had not been going to school.
	Subject + had + (been) + V^1 + ing + (for / Since)	Aux. (had) + Subject + been + V^1 + ing + (For / Since)	Subject + Aux. (had) + not + been + V^1 + ing + ow

Future Tense

3. Formation of Sentences

	Positive	Interrogate	Negative
A	I / We shall go to school. You / They / He / she / it / Ram will go to school.	Shall I / We go to school ? Will You / They / He / She / It / Ram go to school ?	I / We shall not (shan't) go to school. You / They / He / She / It / Ram will not (won't) go to school.
	Subject + Aux. (shall / will) + V^1 + ow	Aux. (shall / will) + Subject + V^1 + ow	Subject + Aux. + not + V^1 + ow
B	I / We shall be going to school. You / They / He / She / It / Ram will be going to school.	Shall I / We be going to school ? Will You / They / He / She / It / Ram be going to school ?	I / We shall not (shan't) be going to school. You / They / He / She / It / Ram will not (won't) be going to school.
	Subject + Aux. (shall / will) + be + V^1 + ing + ow	Aux. (shall / will) + Subject + V^1 + + ow	Subject + Aux. + not + be + V^1 + ing + ow

C	I / We shall have been going to school + (*for) You / They / He / She / It / Ram will have been going to school. (*for) *only 'for' is used in this tense, since is not used.	Shall I / We have been going to school (for) ? Will You / They / He / She / It / Ram have been going to school (for) ?	I / We shall not (shan't) have been going to school. You / They / He / She / It / Ram will not have been going to school.
	Subject + Aux. (shall / will) + be + V^1 + ing + ow	Aux. (shall / will) + Subject + have + been + V^1 + ing + ow	Subject + Aux. + not + Have + been+ V^1 + ing + ow
D	I / We shall have been to school. You / They / He / She / It / Ram will have been to school.	Shall I / We have been to school ? You / They / He / She / It / Ram have been to school ?	I / We shall not have been to school. You / They / He / She / It / Ram will not have been to school.
	Subject + Aux. (shall / will) + have + V^3 + ow	Aux. (will / shall) + Subject + have + V3 + ow	Subject + Aux. + not + been + V^3 + ow

Fill in the appropriate Present Perfect Continuous Tense—

1. The boy .... (cry) .... 2 hours.
2. Students .... (study) .... Morning.
3. A car .... (honk) .... 30 minute.
4. Dogs .... (bark) at people .... Last night.
5. Beggars .... (beg) .... Time immemorial.
6. You .... (exercise) .... 4 years.
7. The grass .... (grow) .... Last winter.
8. Cats .... (mew) all night.
9. It .... (work) .... Last week.
10. Tom and Jill .... (argue) .... Days.
11. Shane .... (writer) .... Years.
12. They .... (swim) non-stop .... Yesterday.
13. King David .... (rule) .... 20 years.
14. He .... (walk) .... 2008.
15. People .... (vote) .... Ages.

Answers

1. has been crying for 2. have been studying since 3. has been honking, for (the past) 4. have been barking, since 5. have been begging, since 6. have been exercising, for 7. has been growing, since 8. have been mewing 9. has been working, since 10. have been arguing, for 11. has been writing, for 12. have been swimming, since 13. has been ruling, for 14. has been walking, since 15. have been voting, for

Simple Present Tense (Exercises)

Fill in the blanks with the appropriate word. Remember subject verb agreement—

1. Sometimes she also .... (sing / sings) songs.
2. Hermits .... (lead / leads) simple lives.
3. No sooner than the music .... (start / starts) he .... (begin / begins) laughing.
4. Normally, 2nd class travelers .... (do / does) not stand in line.
5. Those who .... (work / works) in temples .... (get / gets) free meals.
6. The baby .... (laugh / laughs) like an angel.
7. A loud blast .... (sound / sounds) and a train .... (roll / rolls) in.
8. The wind .... (blow / blows) and dust .... (rise / rises).
9. .... (do / does) you play chess ?
10. .... (do / does) Sita dance?
11. Birds .... (fly / flies) and cattle .... (walk / walks).
12. We .... (love / loves) our country as much as we .... (respect / respects) it.
13. Hari always .... (speak / speaks) the truth.
14. Those flowers .... (look / looks) beautiful.
15. He .... (count / counts) his pennies and .... (waste / wastes) his pounds.
16. God .... (help / helps) those who .... (help / helps) themselves.
17. The office .... (open / opens) at 9 a.m.
18. The teacher .... (give / gives) us a test every month.
19. The sun .... (rise / rises) in the east and .... (set / sets) in the west.
20. She .... (go / goes) to school daily.
21. A baby .... (cry / cries) when its hungry.
22. Raj Sir, our Maths teacher .... (teach / teaches) us but when he is absent Ravi sir .... (take / takes) our class.
23. Mother .... (cook / cooks) our food.
24. I .... (do / does) my work well.

25. The neighbors dog .... (bark / barks) for no rhyme or reason.

26. The cattle .... (eat / eats) all the paddy in our field.

27. The furniture .... (do / does) not match.

28. We .... (drive / drives) very carefully.

29. The cock .... (crew / crows) first before dawn.

30. You .... (talk / talks) to your parents very rudely.

Answer

1. sings 2. lead 3. starts–begins 4. do 5.work 6. laughs 7. sounds 8. blows–rises 9. do 10. does 11. fly 12. walk 13. speaks 14. look 15. counts–wastes 16. helps 17. opens 18. gives 19. rises–sets 20. goes 21. cries 22. teaches–takes 23. cooks 24. do 25. barkes 26. eats 27. does 28. drive 29. crows 30. talks.

Simple Present

Structure—To make a question (Interrogative) all you have to do is follow these simple steps.

Aux. + subject + V^1

The auxiliary verbs (Helping) in the simple tense are Do and Does.

Thus,

Aux. +	Subject +	V^1
Do	you	play ?
Does	he	sing ?

'Do' takes plural subjects while 'Does' takes singular ones.

Positive Answer—

Yes +	Subject +	Aux. +	V^1
Yes,	I	do	play.

In the simple Present Tense one usually omits. Do while making a positive reply or Positive Assertive statement.

Examples—

Do you play ?

Yes, I play.

In a positive statement a simple 'I play' is sufficient.

But, when one omits the Auxiliary 'Does' in a positive answer or statement remember to add, s, es, or ies, as applicable.

Examples—

1. Does he sing ?

Yes, he sings (Add 's' to the V^1)

2. Does she go ?

Yes, she goes (By adding 'es')

3. Does she reply ?

Yes, she replies (By adding 'ies')

However, in the case of the Interrogative and negative the auxiliary Do and Does can not be omitted.

As,

Do you play ?

No, I do not (don't) play.

Does she sing?

No, she does not (doesn't) sing.

Looks simple, doesn't it ? Well, it is. But you need to practice in order to become perfect. Try, the exercises given hereafter to familiarize yourself with the simple Present Tense.

Fill in the blanks (Interrogative)—

1. .... she watch T.V. ? (Do / Does)
2. .... You like movies ? (Do / Does)
3. .... it work ? (Do / Does)
4. .... I teach well ? (Do / Does)
5. .... they dance ? (Do / Does)

* Examples of Interrogative, Positive, Negative will be added.

Present Continuous Tense (Exercises)

Fill in the blanks with the correct Present Continuous Tense—

1. Jamie .... (exercise) at the moment.
2. She .... (sing) a song.
3. The sun .... (rise) now.
4. They .... (play) our song.
5. It .... (work) perfectly.
6. He .... (talk) on the phone.
7. The crowd .... (gather) around him.
8. Mother .... (cook) the breakfast.
9. The baby .... (cry) as it is hungry.
10. You .... (watch) this movie and I .... (watch) that one.
11. His children .... (tear) their books.
12. The troupe .... (entertain) the guests.
13. The plot .... (unfold) after all.
14. The jury .... (decide) the case.
15. The class .... (take) an exam.

Answer

1. is exercising 2. is singing 3. is rising 4. are playing 5. is working 6. is talking 7. is gathering 8. is cooking 9. is crying 10. are watching, am watching 11. are tearing 12. is entertaining 13. is unfolding 14. is deciding 15. is taking.

Present Perfect (Exercises)

Fill in the blanks with the appropriate Present Perfect Tense—

1. The sun .... (rise).
2. I .... (eat) lunch just now.
3. .... You .... (wear) jeans ever ?
4. .... She .... (post) the letter?
5. They .... (see) the TajMahal.
6. Rohit .... (be) to France.
7. The sweeper .... (sweep) the floor.
8. Father's oxen .... (destroy) the crop.
9. I .... (know) him for 5 years.
10. The gardener .... (cut) the grass.
11. The boys .... (play) football.
12. The temperature .... (rise) sharply.
13. The food .... (be) laid out.
14. The fish .... (catch) the baits.
15. The furniture .... (crack).

Answer

1. Has risen 2. Have eaten 3. Have you worn 4. Has she posted 5. Have seen 6. Has been 7. Has swept 8. have destroyed 9. Have known 10. Has cut 11. Have played 12. Has risen 13. Has been 14. Have caught 15. Has cracked.

Present Perfect Continuous Tense (Exercises)

Fill in the blanks with the appropriate Present Perfect Continuous Tense—

1. He .... (work) 2 hours.
2. They .... (teach) .... Morning.
3. She .... (exercise) .... Last week.
4. The doctor .... (operate) on the patient .... 3 hours.
5. The labourers .... (dig) the fields .... 2005.
6. Shanti .... (talk) non stop .... 30 minutes.
7. Robi .... (visit) us .... Last year.
8. They .... (close) friends .... Childhood.
9. The Ganga .... (flow) .... Time immemorial.
10. That man .... (shout) .... Yesterday.

Answer

1. has been working for 2. Have been teaching since 3. has been exercising since 4. has been operating; for 5. have been digging; since 6. has been talking; for 7. Has been visiting; since 8. have been close ; since 9.has been flowing; since 10. has been shouting; since

Change the following sentences into Present Perfect Continuous Tense. Period of time and point of time is given in brackets.

1. I am walking. (2 hours)
2. He is playing. (morning)
3. She is cooking. (childhood)
4. The students are studying. (February)
5. They are constructing. (2003)
6. The baby is crying. (last night)
7. The dogs are barking. (evening)
8. They are listening to music. (5 hours)
9. The soldiers are fighting. (winter)
10. The boys are jogging. (a long time)

Answer

1. I have been walking for two hours 2. He has been playing since morning 3.she has been cooking since childhood. 4. The students have been studying since February . 5. They have been constructing since 2003. 6. The baby has been crying since last month. 7. The dogs have been barking since evening. 8. They have been listening to music for 5 hours. 9. The soldiers have been fighting since winter. 10. The boys have been jogging for a long time.

Present Perfect Tense (Exercises)

Fill in the blanks with the appropriate Present Perfect Tense—

1. He .... (eat) his lunch.
2. The food .... (serve).
3. The Washerwoman .... (wash) the clothes.
4. Jamie .... (see) the Taj Mahal.
5. I .... (be) to America.
6. The children .... (finish) playing.
7. She .... (watch) that clip.
8. You .... (hear) this song.
9. .... You .... (wear) jeans.
10. .... She .... (visit) you ?

Answer

1. has eaten 2. has been served 3. has washed 4. has seen 5. have been 6. have finished 7. has watched 8. have heard 9. have; worn 10. has; visited

Change the following sentences into Present Perfect Tense.

1. I read the Bhagvad Gita.
2. They are walking.
3. She is singing a sing.
4. Doctors examine patients.

5. The baby is drinking milk.
6. The Gardener is watering the plants.
7. Mother stitches our clothes.
8. Mike sells fruits.
9. He phones his mother.
10. Sister cuts vegetables.

Answer

1. I have read the Bhagvad Gita.
2. They have walked.
3. She has sung.
4. Doctors have examined patients.
5. The baby has drunk milk.
6. The Gardener has watered the plants.
7. Mother has stitched our clothes.
8. Mike has sold fruits.
9. He has phoned his mother.
10. Sister has cut vegetables.

Simple Past Tense (Exercises)

Fill in the blanks with the appropriate Simple Present Tense—

1. Tom .... (play) cricket.
2. He .... (walk) to school.
3. The peon .... (post) the letters.
4. The boy .... (brush) his shoes.
5. She .... (sing) sweetly.
6. The noise .... (awake) him.
7. The man .... (eat) a pizza.
8. Michael .... (swim) across the river.
9. Tom .... (drink) a glass of milk.
10. Jessica .... (beat) her brother.

Answer

1. Plays 2. walks 3. posts 4. brushes 5. sings 6. awakes 7. eats 8. swims 9. drinks 10. beats

Now, change the following into Simple Past Tense—

1. He was weeping.
2. They jump on the bed.
3. She rides a motorcycle.
4. They take a bath daily.
5. The boys were running.
6. Jack screams at Jane.
7. He cuts her hair.
8. Trees grow everywhere.
9. Jacklyn is digging a hole.
10. Mother sweeps the temple.

Answer

1. wept 2. jumped 3. rode 4. took 5. ran 6. screamed 7. cut 8. grew 9. dug 10. swept

Simple Past Tense (Structure)

The following timer (Auxiliary verb) is used in the simple past tense—

Timer	Subject	Verb Form
Did	I, You, We, They (Plural)	V^1
	Name, He, She, It (Singular)	(V^2 is applied for positive statement or answer)

To form the Interrogative, Positive statement or answer follow these simple guidelines—

Interrogative

Timer + Subject + V^1

Did you go to school ?

Positive Answer

(a) Yes, Subject + Timer + V^1

(b) Yes, I did go (to school)

(c) Yes, subject + V^2

(d) Yes, I went (to school)

In case of a positive statement or answer the second form of the verb is usually used. However, while forming the interrogative and the negative 'Did' can not be omitted.

Positive Statement

(a) Subject + Timer + V^1 = I did go.

(b) Subject + V^2 = I went.

Negative Statement

Subject + Timer + not + V^1

I did not go.

Tenses (Past Tense) (Exercises)

Exercises–1

1. Tick the appropriate (Simple) Past Indefinite Tense.

1. He .... (walk) down the road when I met him.
   (A) were walking (B) had been walking
   (C) was walking (D) walked
2. They .... (wait) for the train to arrive.
   (A) had been waiting (B) were waiting
   (C) had waited (D) waited
3. Mother and I .... (speak) to them.
   (A) were speaking (B) spoke
   (C) had spoken (D) am speaking
4. The newly wed couple .... (dance) all night.
   (A) danced (B) had been dancing
   (C) are dancing (D) dance

5. The guards ………… (keep) a watch 24 hours for a week.
(A) are keeping (B) kept
(C) had been keeping (D) keep

Answers

1. (D) 2. (D) 3. (B) 4. (A) 5. (B)

Exercises–2

Employing the appropriate simple Past Tense. Choose the correct option.

1. He ………… (run) very fast.
(A) runs (B) is running
(C) has run (D) ran
2. The gardener ………… (cut) the grass.
(A) has cut (B) is cutting
(C) had cut (D) cut
3. Ricky ………… (swim) across the river in an hour.
(A) swam (B) swum
(C) is swimming (D) has swum
4. Toe ………… (win) the 100 metre sprint race.
(A) win (B) wins
(C) won (D) has won
5. Jack ………… (carry) his son on his shoulders.
(A) carries (B) is carrying
(C) carried (D) has carried

Answers

1. (D) 2. (D) 3. (A) 4. (C) 5. (C)

Exercises–3

Fill in the blanks using the correct 'simple past' form—

1. Till ………… (fetch) a pail of water.
2. Hari ………… (sleep) soundly.
3. The cat ………… (creep) into the room.
4. The horse ………… (jump) over the fence.
5. She ………… (talk) to me all night.
6. The doctor ………… (examine) the patient.
7. Snow ………… (fall) heavily in Shimla last night.
8. Jack ………… (repair) his neighbours car.
9. I ………… (write) a letter to my sister.
10. The motorcade ………… (speed) by us.
11. The woodcutter ………… (chop) all the wood quickly.
12. She ………… (dance) all day.
13. He ………… (slap) his younger brother.
14. Father ………… (scold) us for arriving late.
15. Tom ………… (lift) 100 kgs. of wheat.
16. The cat ………… (drink) all the milk.
17. We ………… (sit) on the carpet.
18. Mother ………… (fry) the potatoes.
19. The cobbler ………… (stitch) our shoes.
20. The Hostess ………… (pour) out the tea.

Answers

1. Fetched 2. Slept 3. Crept 4. Jumped 5. Talked 6. Examined 7. Fell 8. Repaired 9. Wrote 10. Sped 11. Chopped 12. Danced 13. slapped 14. Scolded 15. Lifted 16. Drank 17. Sat 18. Fired 19. stitched 20. Poured.

Exercises–4

Change the following sentences to 'simple present tense'. (Tenses have been underlined to assist you.)

1. They <u>are running</u> away from the police.
2. He <u>eats</u> two dozen apples in 10 minutes.
3. Children <u>have burst</u> all the balloons.
4. He <u>was walking</u> towards the river.
5. The dogs <u>were barking</u> loudly.
6. He <u>has been sleeping</u> all day.
7. She <u>races</u> to buy chocolates.
8. The gatekeeper <u>is opening</u> the door.
9. The man <u>has been shouting</u> at me.
10. The man <u>had pleaded</u> guilty.
11. The officer <u>is shooting</u> the dacoit.
12. The mice <u>will nibble</u> the cheese.
13. The girl <u>will be plaiting</u> her hair.
14. Father <u>is snoring</u> so loudly that everyone <u>is waking</u> up.
15. The teacher <u>has torn</u> our homework books.

Answers

1. Ran 2. Ate 3. Burst 4. Walked 5. Barked 6. Slept 7. Raced 8. Opened 9. Shouted 10. Pleaded 11. Shot 12. Nibbled 13. Plaited 14. Shored, Woke 15. Tore.

Exercises – 5

Simple Past Continuous Tense (Exercises)

Fill in the blanks. Assertive (Positive)—

1. It ………… (rain) heavily today.
2. They ………… (play) football..

3. I ………… writing a letter.
4. He ………… laughing loudly.
5. She ………… (sing) a song.
6. We ………… (call) our friends.
7. Uncle and Aunty ………… (visit) their relativies the evening.
8. You ………… (clean) your room.
9. The boy ………… (ride) a cycle.
10. The gardener ………… (dig) a hole.

Answer

1. was raining 2. were playing 3. was 4. was 5. was sing 6. were calling 7. were visiting 8. were cleaning 9.was riding 10. was digging

Exercises

Interrogative

1. ………… she ………… (buy) books ?
2. ………… you ………… (wash) the dishes ?
3. ………… I ………… (leave) today ?
4. ………… we ………… (learn) computers ?
5. ………… they ………… (teach) English ?
6. ………… he ………… (cook) the food ?
7. ………… it ………… (work) ?
8. ………… father ………… (visit) the kapoors' ?
9. ………… Jill and Jane ……….. (brush) their teeth ?
10. …………… John …………… (eat) a burger ?

Answer

1. was; buying 2. were ; washing 3. am; leaving 4. are ; learning 5. are; teaching 6. is; cooking 7. was; working 8. is; visiting 9. were; brushing 10. was ; eating

Simple Continuous Tenses (Exercise)

Assertive Negative

1. He ………… (go) to the market.
2. They ………… (peel) the potatoes.
3. The Gardener ………… (water) the plants.
4. The Doctor ………… (examine) the patient.
5. The girls ………… (dance) on time.
6. The boy ………… (trick) the whole day.
7. Mother ………… (stitch) the clothes.
8. It ………… (move) fast.
9. You ………… (listen) to me.
10. The driver ………… (drive) very slowly.

Answer

1. was going 2. were peeling 3. was watering 4. was examing 5. were dancing 6. was trekking 7. was stitching 8. was moving 9. were listening 10. was driving

Exercises Mixed

1. ……… you ……… (go) to school ? (Interrogative)
2. ……… she ……….. (mix) the flour ? (Interrogative)
3. That man ………… (talk) to father. (Positive)
4. The boys ………… (kick) the ball. (Negative)
5. The dogs ………… (bark) at him. (Positive)
6. Teacher ………… (teach) us. (Negative)
7. ………… I ………… (work) hard ? (Interrogative, Negative)
8. The hen ………… (lay) an egg ? (Positive)
9. That boy ………… (study). (Negative)
10. I ………… (read) these books. (Negative)

Answer

1. were; going 2. was; mixing 3. was talking 4. weren't kicking 5. were barking 6. wasn't teaching 7. wasn't ; working 8. was laying 9. wasn't studying 10. wasn't reading

Past Continuous Tense

Exercises–1

Tick the correct Past Continuous Tense option.

1. Jake ………… (wait) for Jill by the river.
   (A) waited (B) was waiting
   (C) had waited (D) waits
2. He ………… (dig) a hole in the ground.
   (A) was digging (B) has dug
   (C) will dog (D) has been digging
3. The wealthy man ………… (spend) a lot of money to help the poor.
   (A) spent (B) will spend
   (C) does spend (D) was spending
4. She ………… (pluck) flowers to make a garland.
   (A) had plucked
   (B) was plucking
   (C) had been plucking
   (D) will have been plucking
5. They ………… (attack) us from all sides.
   (A) have been attacking (B) will attack
   (C) were attacking (D) attacked
6. She ………… (comb) her hair when the phone rang.
   (A) was combing (B) combed
   (C) combs (D) has combed
7. The pilot ………… (fly) at 500 miles an hour.
   (A) flew (B) was flying
   (C) flown (D) has flown

8. We .... (drive) through a dark and dangerous forest.
(A) drove (B) had driven
(C) were driving (D) drive

9. Sally .... (sit) on the road.
(A) sits (B) was sitting
(C) sat (D) has sat

10. The children .... (swallow) their food lest they be late for school.
(A) was swallowing (B) were swallowing
(C) have been swallowing (D) swallows

11. Cars .... (zoom) by very fast.
(A) is zooming (B) were zooming
(C) was zooming (D) zoom

12. Waiters .... (serve) the guests very efficiently.
(A) serve (B) serves
(C) have been serving (D) were serving

Answers

1. (B) 2. (A) 3. (D) 4. (B) 5. (C) 6. (A)
7. (B) 8. (C) 9. (B) 10. (B) 11. (B) 12. (D)

Exercises–2

Fillers (use Past Continuous Tense)—

1. Mother .... (spread) the sheets on the beds.
2. Sisters .... (peel) the potatoes.
3. Sam .... (read) a new novel.
4. The washerwoman .... (wash) clothes by the river.
5. The boy .... (tie) his shoelaces.
6. I .... (button) my shirt.
7. She .... (iron) her brother's shirt.
8. The South wind .... (blow) ferociously.
9. Sita .... (drape) herself in a saree.
10. The soldiers .... (stand) to attention.
11. The security guards .... (frisk) the visitors.
12. The machine .... (punch) holes in the sheets automatically.
13. He .... (guess) as he did not know the correct answers.
14. The professor and his assistants .... (guide) the students.
15. We .... (view) the famous Eiffel Tower in Paris.
16. It .... (pour) cats and dogs.
17. The patient .... (twist) and .... (turn) in his sleep.
18. Someone .... (bang) on the door.
19. Shane .... (pull) up the chain.
20. The bells .... (ring) in the temples.

Answers

1. was spreading 2. were peeling
3. was reading 4. were washing
5. was trying 6. was buttoning
7. was ironing 8. was blowing
9. was draping 10. were standing
11. were frisking 12. was punching
13. was guessing 14. were guiding
15. were viewing 16. was pouring
17. was twisting, turning 18. was banging
19. was pulling 20. were ringing

Exercises–3

Change the following sentences using the 'Past Continuous Tense'. Tense indicators have been underlined to assist you.

1. Harry and Sue have eaten their lunch.
2. Tom has been working with us.
3. She scrubbed the floor.
4. Father had slapped him for the second time.
5. The police tracked down the thief.
6. The policeman has been issuing tickets to traffic offenders.
7. The officials stamped visas on our passports.
8. The rascals have sprinkled dirty water on the girls.
9. The vandals destroyed all the furniture in the office.
10. A spider will scale the wall.
11. The Principal and founder will have distributed the sweets among the students.
12. Sonam stirred the soup expertly.
13. Sam and Ray had been discussing the project.
14. He jogs around the park.
15. Simi is buying a new dress for her daughter.

Answer

1. were eating 2. was working
3. was scrubbing 4. was slapping
5. were tracking 6. was issuing
7. were stamping 8. were sprinkling
9. were destroying 10. was scaling

11. was distributing
12. was stirring
13. were discussing
14. was jogging
15. was buying

Simple Past Continuous Tense (Exercises)

Fill in the blanks—

1. The girls ………… (hope) on one leg.
2. The teacher ………… (read) from the lesson.
3. The crowds ………… (protest) at the police station.
4. We ………… (watch) the test match.
5. Sita ………… (wrap) a saree.
6. Mother ………… (fold) the sheets.
7. The carpenters ………… (hammer) nails in the planks.
8. The girls ………… (sweep) the rooms.
9. The waiter ………… (serve) tea to the guests.
10. Harry ………… (rub) oil on his hair.

Answer

1. were hopping 2. was reading 3. were protesting 4. were watching 5. was wrapping 6.was folding 7. were hammering 8. were sweeping 9. was serving 10. was rubbing.

Change the following sentences into Simple Past Continuous Tense—

1. He played cricket.
2. The girls cut their nails.
3. Silvia plaited her hair.
4. Mother peeled the bananas.
5. Kalidas wrote many books.
6. Sidney wore a tuxedo with a hat.
7. The shoeshine boy shone my shoes.
8. The cat drank all the milk.
9. The thief stole all the jewellery.
10. The boys crept into the candy store.

Answer

1. was playing 2. were cutting 3. was plaiting 4. was peeling 5. was writing 6. was wearing 7. was shining 8. was drinking 9. was stealing 10. were creeping.

Simple Past Continuous Tense (Structure)

The timers (Auxiliary Verbs) in the Simple Past Continuous Tense are 'was' and 'were'. The subjects which agree with them are as follows.

Timer	Subject	Verb Form
Were	You, We, They (Plural)	V^1 + ing
Was	Name, He, She, It (Singular)	V^1 + ing

However, in a conditional clause—All singular subjects take 'were'.

Example—

If I were a butterfly.

Here, 'I' takes were instead of 'was'.

The structure for the Interrogative, Positive and Negative are as follows.

Interrogative—To form a question follow this simple rule.

Timer + Subject + V^1 + ing

Were you going (home) ?

Positive Answer

Yes, Subject + Timer + V^1 + ing

Yes, I was going (home).

Negative Answer

No, Subject + Timer + not + V^1 + ing

No, I was not going (home) ?

Past Perfect Continuous Tense (Exercises)

Fill in the blanks using Past Perfect Continuous Tense—

1. They ……….. (working) ……….. 2 days.
2. The driver ……….. (drive) ……….. last week.
3. The boys ……….. (play) ……….. a week.
4. Mother …….. (sew) clothes …….. her school days.
5. Rick ……….. (write) ……….. 2002.
6. The grass ……….. (grow) ……….. last summer.
7. He ……….. (pour) tea ……….. morning.
8. The police …….. (whip) him …….. he was caught.
9. You ……….. (read) this book ……….. 30 minutes.
10. The birds ……….. (chirp) ……….. birth.

Answer

1. had been working; for 2. had been driving; since 3. had been playing ; for 4. had been sewing ; since 5. had been writing ; since 6. had been growing;since 7. had been pouring ; since 8. had been whipping ; since 9. had been reading; for 10. had been chirping ; since.

Change the following into Past Perfect Continuous Tense—

1. She has been exercising.
2. They have been praying.
3. John has been listening to music.
4. He has been brushing shoes.
5. The dentists have been drilling teeth.
6. The men have been tying all day.
7. The music has been blaring all night.
8. The masseur has been massaging people all morning.
9. They have been playing cards ever since.
10. The rag picker has been picking garbage since.

Answer 1 to 10 change the shaded words into had.

Past Perfect Continuous Tense (Structure)

Go form the Interrogative, Assertive (Positive and Negative) Positive or Negative answer. Follow the structure given below.

Interrogative

Had + Subject + been + V^1 + ing + for / since

Had he been working for 2 hours?

Positive Answer

Yes, Subject + had + been + V^1 + ing + for

Yes, he had been working for 2 hours.

Negative Answer

No + Subject + had + not + been + V^1 + ing + for

No, he had not been working for 2 hours.

Positive Statement

Subject + had + been + V^1 + ing + for

He had been working for 2 hours.

Negative Statement

Subject + had + not + been + V^1 + ing + for

He had not been working for 2 hours.

Past Perfect Continuous Tense tells us the period of time or point of time an action lasted in the past.

If, for example you say, 'I was exercising', It tells of a past action but it does not mention the period or point of time elapsed. Whereas, I had been exercising for 1 hour or since morning pinpoints the period or point of time the action lasted.

Past Perfect Continuous Tense

Exercises–1

Tick the most suitable option by. Changing the underlined tenses into the Past Perfect Continuous Tense.

1. He eats apples for 1 week.
   (A) has eaten (B) had been eating
   (C) will eat (D) ate
2. They work hard since morning.
   (A) had been work (B) had worked
   (C) had been working (D) works
3. She will peel potatoes for 5 hours.
   (A) will have been peeling
   (B) peels
   (C) will peeling
   (D) had been peeling
4. Tom and Dick fought since 2007.
   (A) had been fighting (B) is fighting
   (C) was fighting (D) are fighting
5. The teacher will have been teaching for 2 hours.
   (A) had been teach (B) had
   (C) is (D) will had
6. They are exercising for days before they left.
   (A) is (B) were
   (C) had been (D) had been exercise
7. Jack has drilled holes in the angles.
   (A) drilling (B) is drilling
   (C) was drilling (D) had been drilling
8. Jennifer was combing her mother's hair when the comb broke.
   (A) combed (B) has combing
   (C) had been combing (D) will comb
9. He has played cricket for years.
   (A) has been playing (B) is playing
   (C) played (D) had been playing
10. Sean had not talk to sue since (the year) 2000.
   (A) has not talking (B) have not talked
   (C) had not been talking (D) had not talking

Answers

1. (B) 2. (C) 3. (D) 4. (A) 5. (B) 6. (C)
7. (D) 8. (C) 9. (D) 10. (C)

Exercises–2

Fillers : Fill in the blanks using the appropriate Past Perfect Continuous Tense—

1. Mother .... (cook) since morning.
2. The crowds .... (arrive) by the hundreds for 3 days.
3. She .... (read) the Bible for 1 month.
4. The tourists .... (drive) for weeks around India.
5. Mother .... (argue) with father over a petty issue for weeks.
6. The Judge .... (issue) summons to all and Sunday.
7. The group members .... (practice) two songs daily.
8. We .... (contemplate) buying a new house.
9. I .... (plead) with quit his job for years.
10. He .... (hit) his wife continually for years.

Answer

1. had been cooking 2. had been arriving
3. had been reading 4. had been driving
5. had been arguing 6. had been issuing

7. had been practicing
8. had been contemplating
9. had been pleading
10. had been hitting

Exercises–3

Change the underlined words into the 'Past Perfect Continuous Tense'.

1. She was dancing since childhood.
2. They are not talking to each other for 2 days.
3. The Doctor prescribes wrong medicines since May.
4. The poacher will kill tigers for a decade.
5. Harold taught for 5 years before he was fired.
6. They train dogs for ages.
7. We win when suddenly our luck changed.
8. The preacher corrupts young minds before he was caught.
9. Sally lies to us since God knows when.
10. The gas attendants stole gas for months.
11. The thieves are removing bolts from the rails.
12. The deer graze when they were shot.
13. I am protesting but to no avail.
14. They will call us regularly.
15. She combs her hair.

Answer

1. had been dancing
2. had not been talking
3. had been prescribing
4. had been killing
5. had been teaching
6. had been training
7. had been winning
8. had been corrupting
9. had been lying
10. had been stealing
11. had been removing
12. had been grazing
13. had been protesting
14. had been calling
15. had been combing

Past Perfect Tense (Exercises)

Exercises–1

Tick the most suitable option to change the following tenses (underlined) in the sentences into the 'Past Perfect Tense'.

1. He spoke to me before he die.
2. The bus arrives before we reach.
3. The cat already drank the milk when we were catching it.
4. They finished playing before the Sun was set.
5. The man died before the doctor arrives.
6. She is writing her will before she leaves.
7. The pirates kill the crew before they sink the ship.
8. Mother was cooking lunch when the bell wings.
9. He will reach Mumbai we speak to him.
10. Roy sang the song before we were stopping him.

Answer

1. had spoken, died
2. had arrived, reached
3. had already drunk, caught
4. had finished, set
5. had died, arrived
6. had written, left
7. had killed, sank
8. had cooked, rang
9. had reached, spoke
10. had sung, stopped

Exercises–2

Fillers : Fill in the blanks with the 'Past Perfect Tense'—

1. He ………. (abuse) me before I ……… (slap) him.
2. The train …….. (arrive) before we …….. (reach) the station.
3. Tom …….. (eat) an apple before he …….. (sleep).
4. The terrorists …….. (shoot) 5 people before they were …….. (kill).
5. Tom and Jerry …….. (break) the vase before we …….. (know) it.
6. The flower …….. (blossom) before the sun …….. (set).
7. Mary …………. (inform) me when she …………. (leave).
8. The author …….. (complete) the book before he ………. (discover) the mistake.
9. The couple …….. (dine) before I …….. (meet) them.
10. The old woman ……..… (trip) before ……… (catch) her.

Answer

1. had abused; slapped
2. had arrived; reached
3. had eaten; slept
4. had shot; killed
5. had broken; knew
6. had blossomed; set
7. had informed; left
8. had completed; discovered
9. had dined; met
10. had tripped; caught

Exercises–3

Change the following sentences into the Past Perfect.

1. He is coming to meet us before he leaves.
2. The train arrived before she has come.

3. Mother will cook the food before we reach.
4. The teacher taught the students before he ate dinner.
5. Mary is killing her husband before he recovers.
6. Mary will kiss her mother before she elopes.
7. He was killed before he confesses.
8. She wipes the furniture before she sweeps the room.
9. He explains the formula before giving us a task.
10. He spoke to her before phoning me.

Answer

1. had come, left	2. had arrived
3. had cooked, reached	4. had taught, ate
5. had killed, recovered	6. had kissed, eloped
7. had killed, confessed	8. had wiped, swept
9. had explained, he gave	10. had spoken, he phoned

Exercises–4

Change the following into the Past Perfect Tense—

1. They are playing before they sleep.
2. She was leaving before he arrives.
3. The workers will build a wall before they start working.
4. He has eaten a light snack before he eats lunch.
5. We turned on the music before dancing.
6. Charlie completed his work before relaxing.
7. He left the office before the bomb exploded.
8. She met him before meeting her.
9. The police surrounded the area before firing.
10. He gave her the job before consulting his senior.

Answer

1. Change 'are playing', and 'sleep' to 'had played' and 'slept'.
2. Change 'was leaving' and 'arrives' to 'had left' and 'arrived'.
3. Change 'will build' and 'start' to 'had built' and 'started'.
4. Change 'has eaten' and 'eats' to 'had eaten' and ate'.
5. Change 'turned' and 'dancing' to 'had turned' and 'they danced'.
6. Place 'had' before completed and use 'he relaxed' in place of 'relaxing'.
7. Insert 'had' before 'left'.
8. Place 'had' before 'met' and 'she met' in place of 'meeting'.
9. Place of had before 'surrounded' and 'they fired' in place of 'firing'.
10. Use had given in place of 'give' and 'he consulted' in place of 'consulting'.

Past Perfect Tense (Exercises)

Fill in the blanks with the appropriate Simple Past Perfect Tense—

1. Mother ………..… (cook) the food before father ……….. (arrive).
2. The boy ……….. (finish) playing before it ……….. (rain).
3. The man ……….. (die) before help ……….. (come).
4. Jerry ……….. (give) the massage before he ……….. (leave).
5. Susan ……….……. (cut) the cake before the roof ……….. (fall).
6. Father ……….. (shut) the door before the wind ……….. (blow).
7. Mother ……….. (lay) the table before we ……….. (eat) the food.
8. The Muncipality ……….. (clean) the road before the Prime Minister's convoy ……….. (pass).
9. The soldiers ……….. (dig) a tunnel before the firing ……….. (start).
10. The host ……….. (serve) the drinks before the movie ……….. (start).

Answer

1. had cooked; arrived 2. had finished; rained 3. had died; came 4. had given; left 5. had cut; fell 6. had shut; blew 7. had laid; ate 8. had cleaned; passed 9. had dug; started 10. had served.

Change the following into Past Perfect Tense—

1. We took shelter. The storm arrives.
2. Mother cooked the food. The guests come.
3. The crowd started clapping. The Hero reach the stage.
4. Sister dried the clothes. The rain come down.
5. The thief ran 100 metres. The police catch him.
6. The car crashed. The driver realize.
7. The roof fell. Anyone escape.
8. The fruit ripened. We pluck them.
9. The train left the station. We arrive.
10. We drank water. We eat food.

Answer

1. We had taken shelter before the storm arrived. 2. Mother had cooked the food before the guest arrived. 3. The crowd had started clapping before the hero reached the stage. 4. Sister had dried the clothes before the rain came down. 5. The thief had run a100 metres before the

police caught him. 6. The car had crashed before the driver realized 7. The roof had fallen before anyone escaped. 8. The fruit had ripened before we plucked them. 9. The train had left the station before we arrived 10. We had drunk water before we ate food.

Past Perfect Tense (Structure)

Structure

Timer (Aux. Verb)	Subject	Verb Form
Had	I, You, We, They Name, He, She, It Singular and Pluarl	III Form

* Had agrees with all subjects.

Interrogative

Had + Subject + V^3

Had she eaten (lunch) ?

Positive Answer

Yes, Subject + had + V^3

Yes, She had eaten (lunch).

Negative Answer

No + Subject + had + not + V^3

No she had not eaten (lunch).

Positive Statement

Subject + had + V^3

She had eaten (lunch).

Negative Statement

Subject + had + not + V^3

She had not eaten (lunch).

Simple Future Tense (Exercise)

From the two given options choose the most suitable 'simple future tense' option to fill in the blanks. Options—Shall and Will

1. Terry ……….. run all the way to Dehradun.
2. I ……….. tell you what is to be done.
3. Reid and Douglas ……….. play the doubles match.
4. We ……….. discuss the matter later.
5. When he meets you he ………… inform us.
6. If the thief escapes, the police ……….. be held responsible.
7. My house ……….. not be sold.
8. Trespassers ……….. be prosecuted.
9. ……….. We break the news?
10. ……….. I meet her at home?

Answer

1. Will	2. Shall	3. Will	4. Shall	5.Will
6. Will	7. Will	8. Will	9. Shall	10. Shall

Simple Future Continuous Tense

Choose the most appropriate 'simple continuous tense' from the 4 options given below the following sentences.

1. This man ……….. (accompany) you to the market.
   (A) will accompany
   (B) is accompanying
   (C) will be accompanying
   (D) accompanies
2. We ……….. (go) to Delhi tonight.
   (A) will have been going (B) will be going
   (C) shall be going (D) will go
3. The farmers ……….. (grow) wheat next month.
   (A) will grow (B) would grow
   (C) had grown (D) will be growing
4. If I'm not wrong, we ……….. (get) married in two months time.
   (A) will get (B) should be get
   (C) shall be getting (D) will have got
5. Jake ……….. (ring) me in an hour.
   (A) shall be ringing (B) would ring
   (C) will be ringing (D) should ring
6. The plane ……….. (land) at Heathrow Airport is five minutes.
   (A) will be landing (B) should be landing
   (C) will land (D) shall have landed
7. I ……….. (play) the next match.
   (A) should play (B) shall be playing
   (C) would play (D) will be playing
8. We ……….. (expect) your call soon.
   (A) shall be expecting
   (B) would be expecting
   (C) should be expecting
   (D) will be expecting
9. The cook ……….. (not) (work) for us any longer.
   (A) shall not work
   (B) will not be working
   (C) shall not be working
   (D) will not work
10. Samanta ……….. (give) us a treat today.
   (A) shall give (B) would give
   (C) should be giving (D) will be giving

Answer

1. (C)	2. (C)	3. (D)	4. (C)	5. (C)	6. (A)
7. (B)	8. (A)	9. (B)	10. (D)		

Exercises–3

Change the following sentences into 'simple future continuous tense' by substituting the underlined words.

1. They <u>are going</u> home in an hour.
2. I <u>am buying</u> a new car.
3. He <u>gave</u> a lecture.
4. We <u>grow</u> tulips in our garden.
5. They <u>thanked</u> me for this.
6. The team <u>has been playing</u> all over the world.
7. The computer <u>does not work.</u>
8. Same and Fred <u>joined</u> us from today.
9. The children are <u>playing</u> on the lawn.
10. The helicopter <u>has landed</u> now.

Answer

1. will be going	2. shall be buying
3. will be giving	4. shall be growing
5. will be thanking	6. will be playing
7. will not be working	8. will be joining
9. will be playing	10. will be landing

Future Perfect Tense

Exercises–1

Out of the form given options which follow each sentence choose the most appropriate future perfect tense.

1. We .... (clinch) the deal before they .... (do).
   (A) shall clinch; did
   (B) will clinch; do
   (C) shall have clinched; do
   (D) shall clinch; do
2. Mother .... (cook) the food before father .... (arrive).
   (A) will have cooked; arrives
   (B) shall be cooking; arrived
   (C) shall have cooked; arrives
   (D) would have cook; arriving
3. The plane .... (land) before we .... (reach).
   (A) shall be landed; reached
   (B) will have landed; arrive
   (C) would be landing; arrives
   (D) should have landed; arrived
4. Rain .... (fall) before the match .... (finish).
   (A) will fall; finishes
   (B) shall have fallen; finishes
   (C) will have fallen; finishes
   (D) will be falling; finishes
5. Thomas .... (marry) before the year .... (end).
   (A) will have married; ends
   (B) shall be marrying; ends
   (C) will marry; ends
   (D) would marry; ends
6. We .... (see) the movies before anyone .... (find) out.
   (A) will have been seeing; find
   (B) shall be seeing; finds
   (C) shall have seen; finds
   (D) should have seen; found
7. He .... (escape) the police dragnet.
   (A) shall be escaping
   (B) will have escaped
   (C) shall escape
   (D) will have escape
8. Harry .... (write) the book before he .... (return) home.
   (A) will have written; returns
   (B) shall be writing; return
   (C) should write; was returning
   (D) would write; returns
9. They .... (read) the book before he .... (question) them.
   (A) will read; questions
   (B) shall have read; questions
   (C) will have read; questions
   (D) will be reading; questioned
10. John and Joe .... (apply) for the post before he .... (apply).
    (A) will have applied; applies
    (B) shall have applied; applied
    (C) will be applying; applied
    (D) will apply; applying

Answers

1. (C)	2. (A)	3. (B)	4. (C)	5. (A)	6. (C)
7. (B)	8. (A)	9. (C)	10. (A)		

Exercises–2

Change the following sentences into 'future perfect tense' by changing the underlined words.

1. We shall be recruiting this year ends.
2. I am employing my relatives before my boss returns.
3. Mother knits our sweaters before winter begins.
4. They police is arresting all the protesters.
5. Ricky gobbles his food very quickly.
6. The train departed the station before we reached.
7. Sammy eats his tiffin before the bell goes off.
8. Our General Manager closed the deal before consulted us.
9. The ships sinks before it starts.
10. I posted the letter then.

Answers

1. Shall have recruited
2. Shall have employed
3. Will have knitted
4. Will have arrested
5. Will have gobbled
6. Will have departed; reach
7. Will have eaten
8. Will have closed; he consults
9. Will have sunk
10. Shall have posted; by then

Future Perfect Continuous (Solved)

1. She .... (type) for 6 hours.
2. They .... (dance) for 2 hours.
3. I .... (teach) for 3 hours.
4. We .... (play) many games by now.
5. He .... (check) out of the hotel at that time.
6. Shane .... (call) me all night.
7. They .... (eat) by now.
8. The gardeners .... (water) the plants for 8 hours.
9. The men .... (cut) the trees for 1 week.
10. The surgeon .... (operate) on the patient for 3 hours.

Answer

1 will have been typing
2. will have been dancing
3. shall have been teaching
4. will have been playing
5. will have been checking
6. will have been calling
7. will have been eating
8. will have been watering
9. will have been cutting
10. will have been operating

8 Para Jumbles

1. Para jumbles are quite difficult to arrange. As, they depend on the writers train of thought. So, it is not an easy task. However, a few indications and rules will help consider this paragraph.
   (a) Like this is called the adaptive unconscious.
   (b) Is one of the most important new fields in technology.
   (c) The part of our brain that leads to conclusions.
   (d) And the study of this kind of decision making.
   (a) c, a, d, b (√) (b) c, d, b, a
   (c) b, a, d, c (d) b, a, d, c

In such cases one has to consider, whether a continuation is or it's a lead sentence and how does it fall into a series of ideas, in accordance with the flow of the central idea.

Take option (b). It starts with as one of ……… and answers the question 'what' therefore eliminating its capacity as a lead sentence.

Option (d) is similar in its opening and this eliminated.

Option (a) makes sense but answers the question 'what' and so fails to connect whereas.

Option (c) follows a definite path of cohesive sentences 'c' is followed by 'a' like this which shows a connection with 'c' and 'd' is joined by the conjunction 'and' which is apt and relative to 'd'. Logically, 'b' will follow as it the last and correct sentence which has to fall into place.

Now, try your hand at the given para jumbles to hone your skills.

2. (a) They didn't weigh
   (b) The second strategy was the path taken by Evelyn and Having.
   (c) Every conceivable stand of evidence and it ended in failure.
   (d) and the Greek scholars.

   Note—Look out for commas and full stops they indicates continuation or the end of a sentence.
   (a) c, d, b, a (b) a, b, c, d
   (c) b, d, a, c (√) (d) d, c, a, b

3. (a) Have the same impression of Tom Hanks.
   (b) You would say he is decent and trustworthy and down to earth and funny.
   (c) If I asked you what he was like.
   (d) My guest is that many of you.
   (a) d, c, a, b (b) b, c, d, a
   (c) c, d, a, b (d) d, a, c, b (√)

4. (a) But Bargh and his colleagues were wrong.
   (b) The people primed to be rude eventually interrupted.
   (c) on average about five minutes.
   (d) But of the people primed to be polite–82 percent never interrupted at all.

   Options—
   (a) d, b, a, c (b) a, b, c, d (√)
   (c) a, b, d, c (d) b, a, d, c

5. (a) But the Emergency Department seemed to cry out for special attention.
   (b) most of the patients entered the hospital through the Emergency Department.
   (c) The list of problems Reilly faced was endless.
   (d) Because so few cook county patients had health insurance.
   (a) c, b, d, a (b) c, a, d, b (√)
   (c) c, a, b, d (d) c, d, b, a

6. (a) Cook County's big experiment began.
   (b) named Brendan Reilly came to Chicago to become Chairman of the Institution.
   (c) The hospital that Reilly inherited was a mess.
   (d) In 1996, a year after a remarkable man.
   (a) a, c, b, d (b) d, b, a, c
   (c) d, a, b, c (d) a, d, b, c (√)

7. (a) Braden found the same problem with the basketball.
   (b) A man revered for his knowledge and insight into the art of hitting.
   (c) {layer Ted Williams was perhaps the greatest hitter of all time.
   (d) He always said that the ball to the point where he made contact.
   (a) a, c, b, d (b) a, c, b, d (√)
   (c) d, a, b, c (d) a, b, d, c

8. (a) First impressions don't work because
   (b) The blind sip test is the wrong contact for thin slicing coke.
   (c) In the case of a blind sip test,
   (d) Colas are not supposed to be sipped blind.
   (a) c, a, b, d (√) (b) b, a, d, c
   (c) c, b, a, d (d) b, a, c, d

9. (a) What Ekman is describing.
   (b) We can all mind read effortlessly and automatically.
   (c) In a very real sense, is the physiological basis of how we thin slice other people.
   (d) Because the clues we need to make sense of someone on the faces of those in front of us.
   (a) c, b, a, d (b) a, b, d, c
   (c) a, c, b, d (√) (d) c, a, d, b

10. (a) If you are like most people.
    (b) I imagine that you find goslings conclusion quite incredible.
    (c) But the truth is that they shouldn't be.
    (d) Not after the lessons of John Gothman.
    (a) a, c, d, b (b) a, b, c, d (√)
    (c) c, a, d, b (d) b, a, c, d

11. (a) Can be very confusing, for the simple reason.
    (b) That most of us aren't very objective.
    (c) What people say about themselves.
    (d) About ourselves.
    (a) b, a, c, d (b) c, d, a, b
    (c) b, a, d, c (d) c, a, b, d (√)

12. (a) What we reveal about ourselves can confuse.
    (b) That's why, when we measure personality, we don't just ask people point blank.
    (c) What they think about themselves.
    (d) We give them a questioner, like the Big Five Inventory, carefully designed to elicit responses.
    (a) a, b, c, d (√) (b) b, c, d, a
    (c) a, d, c, b (d) b, c, a, d

13. (a) These kinds of unconscious – or, as they like to call them.
    (b) Over the past few years.
    (c) Implicit associations play in our beliefs and behaviour based on the IAT test.
    (d) A number of psychologists have begun to look more closely at the role.
    (a) d, b, a, c (b) b, a, c, d
    (c) d, b, c, a (d) b, d, a, c (√)

Directions for questions 1 to 31 : The sentences given in each question, when properly sequenced, form a coherent paragraph. Each sentence is labeled with a letter. Choose the most logical order of sentences from among the given choices to coherent paragraph.

1. (A) These are early days yet, but these are signs that the appreciation of the rupee and the Reserve Bank of India's monetary policy

   (B) The economy seems to be showing sign of wear and tear; the latest index of Industrial Production (IP) data for September are disappointing, IP growth is down to 6.4 percent compared to 12 percent last September. This is the lowest recorded since October last year.

   (C) Which is resulting in higher interest rates, are hurting overall economic growth

   (D) Which the index rose 4.5 percent. The main culprit is the manufacturing sector, which decelerated to 6.6% compared to about 13% in September last year. The consumer durables segments, in particular, has been hit hard, registering a 7.6% fall in output. There's more bad news for the first half of their financial year, IP growth fell to 9.2%, against 11 percent last year.

   (1) ACBD (2) ADCB
   (3) ACBD (4) DACB
   (5) DCBA

2. (A) Typically, real estate grows 2-3 times the GDP growth of a country. So, it is no exaggeration when J P Morgan forecasts the industry size to grow from \$50 billion (Rs 2,00,000 crore) per annum at present to \$90 billion (Rs 3,60,000 crore) by 2010-11, showing a growth rate of 13 per cent over the period.

   (B)What is driving this enormous investor appetite? Economic growth lapping 9 per cent has spurred demand for office space, retail outlets and hotels.

   (C) Kaushal Sampat, Chief Operating Officer of financial data services firm, Dun & Bradstreet, says: "The demand drivers for real estate are on a firm footing. To satiate this demand, the sector will require continuous capital infusion." And with demand showing strength, Sampat does not see any let- up in investor interest in the next 3-5 years.

   (D) And even as the Special Economic Zones get planned and executed, it is the coming together of rising demand in diverse segments that is driving.

   (1) ABCD (2) BDAC
   (3) BADC (4) ACDB
   (5) ACBD

3. (A) Khanna's gamble paid off for a while as soaring stock prices saw his returns increase several-fold. Emboldened, he tripled his stock purchases and likewise extended his margin credit. But , Khanna's,

luck ran out: the stock he bought plunged over 50 percent in a day, his broker quickly squared.

(B) Off his shares without intimation to avoid further losses, and Khanna ended up with dues that wiped the entire gains he made since he started investing through margin funds.

(C) Khanna is one of many who have tried margin credit and have not been successful. Margin funding is not always a win-win situation as it might appear for most investors.

(D) Six months ago, small–time metal trader sunil khanna began investing in the stock market through margin funding with assistance from his broker. This credit facility paid for a part of Khanna's stock purchases, while his broker funded the rest to be settled at a future date.

(1) DCBA (2) DCAB
(3) ACDB (4) ADCB
(5) BACD

4. (A) This quant fund will allocate and re-allocate assets, if necessary on a monthly basis, after identifying new trends in the market. Earlier too, Lotus India introduced an Agile Saver fund that closed for subscription only recently.

(B) World over, quantitative fund management models that identify trends, and use them to make investment decisions have been gaining popularity; and India is no exception. Already, investors have found a few quant-funds, as they are popularly called, coming their way. A latest offering from Lotus India Mutual Fund, the Agile Tax Saver fund, however, is an ELSS fund where the investments will be eligible for a tax deduction under Section 80C of up to a maximum of Rs 1 lakh.

(C) The AGILE Tax Saver Fund will invest in the same stocks as the AGILE Saver Fund with no difference in the asset allocation mix, but the target audience for the tax saver funds are long term investors looking for tax saving. "We are just targeting different segments of investors," says Rajiv Shastri, Head, Business Development and Strategic Initiatives. Lotus India AMC.

(D) In both the newly introduced quant funds, AGILE is an acronym for Alpha Generated from Industry Leader's Fund. Fund managers try to generate Alpha, which essentially means out-performance, by investing in stocks that beat the market or rise faster than the market.

(1) DABC (2) BCDA
(3) DCBA (4) BADC
(5) ABCD

5. (A) That includes inviting private capital-and even foreign investors like Indonesia's Salim Group-to invest in West Bengal. He has even declared that the state's economic salvation lies, not in socialism but, in enlightened capitalism.

(B) Already the doomsayers are predicting the withering away of the Front. But that's not why Editor is writing this editorial. Bhattacharjee has been at the forefront of efforts to reinvent the Marxist movement in India. He has gone on record saying that he doesn't have time for "dogmatic Marxism" and would rather practice a more "pragmatic" version of the ideology.

(C) His initiative is bearing fruit. The prestigious Tata small car (nano) project is coming up in Singur; the Jindals and the Jai Balaji Group are setting up multi-thousand crore steel plants in the state and IT majors of all hues have set up large campuses in Kolkata. In doing all this, he has had to fight stiff opposition from hardliners in his own party who have still not read the writing on the wall.

(D) These are not good times to be in WEST BENGAL Chief Minister Buddhadeb Bhattacharjee's shoes. The flare-up in Nandigram, and before that, in Singur, the ration riots across the state, the allegations of police complicity in graphic designer Rizwanur Rehman's death and reports of rifts within the till now monolithic Left Front have all combined to sully his squeaky clean and suave image.

(1) DCBA (2) BDAC
(3) DBAC (4) BADC
(5) ABCD

6. (A) Some countries, which were lazier than others, merely reused expressions already in vogue. In Swahili, *Afya* ! (to health) is used while drinking and also when someone sneezes and so it is for the term *na zdrowie,* which is used across many parts of Central Europe.

(B) As only clinking glasses seemed too solemn an act, especially considering that you were drinking alcohol, people soon began to say something along with the act, with "to your good health" being one of the more popular early sayings, As races intermingled and travelled, this dradition soon spread from one part of the world to another, with each country soon acquiring their own distinctive nationalistic way of saying cheers.

(C) Did you know that it's considered bad luck to toast with an empty glass and, worse still, to toast with water, as the person so honoured will be doomed to a watery2. Toasts have been around since time immemorial. The more popular legend regarding their origin is that the practice of toasting originated in ancient Greece, when the host and the guest would clink their glasses together, so that a part of each's drink would go in to the other, and thus provide assurance that the drink was not poisoned.

(D) Do remember, however, that if you're in a foreign country, always try and stay true to the country's traditions by using its local toast and not using a generic expression, say like the British *chin-chin,* especially not in Japan, where ,chin-chin sounds remarkably like a word describing male genitalia. Also verify that what you've been told is actually the correct word and not someone stringing you along.

(1) ADCB (2) CABD
(3) ACBD (4) CBAD
(5) ABCD

7. (A) The original Bhullam-buthur (means 'making a gurgling sound') later Sanskritised into Brahmaputra, was once a very busy inland waterway, but asphalt roads snatched all its cargo. Its only travellers, till fairly recently, were paddling fishermen. Now, the river is on course for revival; and it is possible to cruise almost the entire length of the river in Assam on cruise ships.

(B) The BRAHMAPUTRA, one of the most spectacular rivers in India, runs right through Assam. On its way through the state, it goes past Majuli-the largest riverine island in the world and a major cultural centre, winding past the dense forests of the Kaziranga National Park and idyllic Country villages and paddy fields. At some places the river resembles an ocean-nearly 10 km wide.

(C) On the second day of the cruise, they take you on a road trip to Sibsagar, the ancient capital of the Ahom kings of Assam. These kings ruled Assam for nearly 700 years and their culture and architecture are a delightful blend of the Indian and the South-east Asian. The temples in town have stupa-like profiles and palaces like the Rang Mahal are decorated with beautiful carvings of river creatures like crocodiles. Beyond Sibsagar, the river widens and a few hours down bifurcates into two channels, forming Majuli.

(D) The 10 night Assam Despatch Cruise run by the Assam Bengal Navigation Company takes you on its aire-conditioned luxury river boat, from Dibrugarh in the Upper Assam down to Guwahati, covering some of the most exciting parts of the state. Dibrugarh is a typical colonial town and was a major American base during World War II. The cruise starts from either Dikhou Mukh, about 2.5 hours from Dibrugarh or from Neamati, which is closer to the town of Jorhat.

(1) DABC (2) BADC
(3) DBAC (4) BDAC
(5) BACD

8. (A) At the heart of this down-turn lies the massive credit abuse that resulted from the innovative packaging (Collateralised Debt Obligation) of sub-quality home mortgages into high quality paper and selling them to banks and financial institutions. This has led to a grave credibility crisis in the US corporate bond market and eroded the liquidity from the system

(B) Already, the top US banks have reported aggregate losses and write-downs of around $50 billion for the quarter, with worldwide, losses amounting to ground $300 billion-and still rising.

(C) The word is finally out. The sub-prime impact has been far deeper and incisive than what was earlier estimated. The seriousness of the problem can be gauged from the fact that two consecutive rate cuts by the US Fed, totalling 75 bps, have failed to deliver the, desired outcome of boosting consumer expenditure, especially in the realty market. Consequently, the decline of the housing sector, coupled with the rising banking sector woes, pose a recession threat to the US.

(D) As stated earlier, the underlying market for CDOs has gone completely illiquid, making it difficult to 'mark to market' the securities in possession of the large banks. So in order to reactivate the market segment, major US banks, under the behest of the US Treasury Secretary, aim to float a Special Investment Vehicle Fund with a corpus of around $80-100 billion. This fund will buy securities from the market and flush liquidity into the CDO market segment. However, in my opinion, the aim seems more to get these securities offloaded from the balance sheets of the large banks and cover up their potential losses - a case of creative accounting.

(1) ABCD (2) CDAB
(3) CABD (4) CBAD
(5) DCBA

9. (A) If the reviews are overwhelmingly positive, it is advisable to put a cap on publicity and advertising spend-instead of the carpet bombing being practiced. However, if the review is overwhelmingly bad, the producer, with money to spend, may not have much option but to spend it on publicity and advertising.

(B) so, what decides the publicity and advertising approach of a film? If you have the money, should you always use it to publicise the movie? This depends partially on the perception of the likely success of the movies by the producer- this leads to the question as to how a movie-maker can arrive at an accurate estimate of the likely success of the film ex-ante.

(C) But what happens, if the reviews are in the middle range-which might be the case for a large number of films? The ideal way would be to spend, which is why we see many filmmakers engaging in a spate of pre-release publicity and marketing efforts. One sometimes gets the impression that money is being spent because it is available. The need to rise above

the increasing clutter of course, another reason for the increased spending .

(D) Films with mega stars have often bombed at the box office and small budget movies have become big hits. The challenge is to get the "right" type of viewer to view the first cut of the edited movie that is reasonably close to the final version. Many previews tend to be for "people like us" i.e., people who are like the movie-makers -close friends and family of the movie-makers -and this would need to change to obtain a more accurate forecast. Getting the right "pre-viewer" is not as simple as it seems.

(1) ACBD (2) BCDA
(3) BADC (4) BDAC
(5 ABCD

10. (A) After the ceremonies Ellsworth Toohey took keating away to the retreat of a pale-orchid booth in a quite, expensive restaurant. Many brilliant parties were being given in honor of the opening, but Keating grasped Toohey’s offer and declined all the other invitations.

(B) He had to admit that he was bored. But he smiled and shook hands and let himself be photographed. The Cosmo- Slotnick building rose ponderously over the street. Like a big white bromide.

(C) I should be happy, peter keating told himself--- and wasn’t. He watched from a window the solid spread of faces filling broadway from curb to crub. He tried to talk himself into joy. He felt nothing.

(D) In December the Cosmo- Slotnick Building was opened with great ceremony. There were celebrities, flower horseshoes, newsreel cameras, revolving searchlights and three hours of speeches, all alike.

(1) CBAD (2) DCBA
(3) CADB (4) CDAB
(5) BACD

11. (A) The Banner was first to get the newest typographical equipment. The Banner was last to get the newspapermen—Last, because it kept them. Wynand raided his competitions’ city rooms; nobody could meet the salaries he offered. His produced evolved into a simple formula.

(B) He spent money faster than it came in—and he spent it all on the Banner. The paper was like a luxurious mistress whose every need was satisfied without inquiry about the price.

(C) He paid them well; he got nothing but his rent and meals. He lived in a furnished room at the time when his best reporters lived in sites at expensive hotels.

(D) In the first years of the Banner’s existence Gail Wynand spent more nights on his office couch than in his bedroom. The effort he demanded of himself was hard to believe. He drove them like an army; he drove himself like a slave.

(1) DBAC (2) DCBA
(3) BADC (4) CABD
(5) DABC

12. (A) As more people buy higher-end washing machines, consumption of superior quality detergents will rise and that should increase enzyme usage. Similarly, as Indians eat more packaged foods, the food-processing industry will buy enzymes in larger quantities.

(B) But if you consider the money spent and the advantage you get on product performance, enzyme solutions offer an advantage," says GS Krishnan, MD of Novozymes South Asia. But this mind block is now beginning to change; "In our other markets, we have experienced that economic growth changes enzyme consumption patterns. India will be no exception," he adds. Take detergents, for example.

(C) Despite being around for a long time, the Indian market for enzymes is relatively nascent. "It's all about the mindset. Our biggest challenge is to convince some of the potential users in certain industries to start using enzymes-businesses are very reluctant to change. They feel enzymes are expensive compared to chemicals.

(D) Says an industry source: "We, in India, consume almost all the enzymes that are available globally, but in very small quantities. That should change-but it won't be overnight. "

(1) ABCD (2) CABD
(3) ACBD (4) CBAD
(5) DABC

13. (A) Getting into that list is the equivalent of getting a calling card to start meeting clients looking for supercomputing services. At the conference, the Tata supercomputer christened EKA ('one' in Sanskrit) was ranked the fourth fastest in the world.

(B) There are two reasons why many would call Ratan Tata's decision to build a $30-million supercomputer outrageously bold. (Yes, he personally steered the project.) First, none of the top ten supercomputers in the world has been fully funded by the companies that built it. They were built only after users like the US Nuclear Security Administration and New Mexico Applications Center placed an order, and put the money on the table. Bluntly put, no one has built a supercomputer in order to build a business around it.

(C) Second, none of them has had the audacity to try and build such a machine in six weeks. So why did Tata take such a gamble ? The answer is simple: Tata and Computational Research Labs (CRL), the Tata Sons subsidiary that built the machine, wanted to make it to the Nevada SC07 conference last month. The bi-annual event ranks the world's 100 fastest supercomputers.

(D) It can perform 120 trillion calculations per second-or 120 teraflops) "If we had missed the conference, we would have had to wait for the next edition, which is a matter of several months," says N Seetha Rama Krishna, Project Manager at CRL. The team, led by Chief Executive Officer of Tata Consultancy Services S Ramadorai, didn't want that delay.

(1) BCAD (2) ABCD
(3) BCDA (4) ABDC
(5) DCAB

14. (A) Sometimes, acquisitions can provide control of a global brand. Welspun India, a private label supplier of terry towels to big retailers like Wal-Mart and JC Penney, was eying the branded market (where margins are at least 2-3 % higher.) That is where the acquisition of 150-year old towel brand Christy's , with a annual turnover of $65 million, helped.

(B) Similar is the case with Rain Calcining. It was struggling to gain access into large customers like Alcan and Alcoa. That was when it acquired CII Holding, a US-based company. "After this, access to Alcan and Alcoa became far easier," says Kalpesh Kikani, Head of Global Investment at ICICI Bank, who advised Rain Calcining in the deal. The acquisition has helped Rain Calcining grow from Rs 365 cror (2005) to over Rs 4,000 crore (2008 forecast) in revenues.

(C) In 2002, Amtek Auto was struggling to sell its castings and forgings to customers in the UK and us. The two countries account for two-thirds of global automotive volumes. "The biggies want supply security and the vendor to be close by. There was no way we could supply from India," recalls Santosh Singhi, Director of Finance at Amtek Auto.

(D) Its export earning that year was zilch. "That's why we decided to acquire globally. Our first acquisition gave us a Backdoor entry into General Motors, Ford and Toyota," he adds. Today 80% of Amtek Auto's sales or roughly $ 770 Million comes from acquisitions.

(1) CDBA (2) CADB
(3) CDAB (4) CBDA
(5) CABD

15. (A) Already, ministry mandarins say they have incorporated a special chapter in the new aviation policy that would make metro connectivity to major airports mandatory. BIAL and GMR Group are now even willing to part-finance a metro rail hook-up from the airport to their respective cities. But it will still take three years to run the course. "Only if traffic growth exceeds projections would we look at the option to keep the existing airport open.

(B) "Both the airport operators would be within their right to claim compensation from the government if the existing airport remains operational," says CAPA's Kaul. But to look for ways to resolve the issue, "an Oversight Committee involving all the stake-holders has to begin negotiations with the operators," he says.

(C) Although industry analysts aver that the airport operators are on a sound wicket legally, developers may have to seize the initiative to ensure that the long-term interests of their airport are not jeopardized.

(D) Senior officials, however, point out that they cannot be perceived to be sending negative signals to investors by going back on contractual obligations, "at a time when we are inviting private and foreign capital to invest in our airport infrastructure.

(1) CADB (2) DCAB
(3) ADCB (4) ABCD
(5) CBDA

16. (A)Though the nascent FM radio sector has no data on the demand-supply gap, human resources managers admit difficulty in finding suitable staff. Industry sources say common referral programmes and direct hiring are there but RJs are normally spotted during talent hunts.

(B) radio gaga, a jarring saga? Talent crunch has hit the airwaves. FM radios, just when picking up listener signals across the cities, are finding it hard to get radio jockeys (RJs). FM channels are mushrooming; by this December, there will be 245 FM stations across 87 cities. And the Rs 500-crore industry is poised for a growth of 28 % till 2011. But at the moment, it looks like it will most probably be an anchor-less sail.

(C) Estimates of the attrition level in FM radio vary between 25 and 30 %. Radio companies attribute this to the fast addition of newer stations. In the absence of non-poaching agreements, the industry is also seeing rampant raid on talent, pushing up salaries further. On an average a RJ joins at a monthly pay of Rs 5,000 to Rs 12,000, depending on talent, and this can go up to lakhs in a short time, depending on the public appeal.

(D) For format-specific channels like, Meow-a chat-based women's channel-finding right anchors is harder. Says Anil Srivatsa, Chief Operating Officer of Meow: "For a music-oriented channel, it is easier to hire talent. But our case is different. Though everyone loves talking, on radio, it is not quite the same."

(1) ABCD (2) BADC
(3) DBAC (4) ADBC
(5) BACD

17. (A) One of his current research topics is exploring connections between biological cell mechanics and human disease states." Prof Suresh is optimistic the

new responsibilities wouldn't shut out his time for research.

(B) Indian institutions take notice, for Prof Suresh is a proponent of international collaborations. Says he: "There are potentially many possibilities to explore interactions with Asia, including Indian institutions. But they must be very carefully planned in the sense that both sides should derive significant benefits from such partnerships. "

(C) JULY 23 will be marked in golden letters at the Massachusetts Institute of Technology (MIT). For on that day, an Indian, Subra Suresh, would become the dean of its prestigious college of engineering. An IIT Madras alumnus, Prof Suresh's bonds with MIT are deep; he completed his doctorate there, and years later, came back to teach, and did post -doctoral research.

(D) Between IIT and MIT, Prof Suresh had a stop-over at Iowa State University for his masters. Prof Suresh is now the Ford Professor of Engineering at the Department of Materials Science and Engineering, where hundreds of projects on nanotechnology to biotechnology are taken up at an average annual budget of $18 million.

(1) DCAB (2) CADB
(3) BCAD (4) CDBA
(5) CDAB

18. (A) Companies are also getting good realisalions for old ships. According to Clarksons Research, prices. of 5-10-year-old ships are 16% higher compared to new, building ships. "This will continue as long as there is delay in ship acquisition," says Atul Kulkarni, Senior Manager (Consulting) with Deloitte. Tanker prices are already up 17 % compared to last year and shipping firms believe freights will continue to be high for at least two to three years more. Shipping companies can enjoy this delightful dilemma while the good times last.

(B) It’s that time of the shipping cycle once again when companies have to decide whether they want to sail or sell. Last financial year, four companies took the sell decision. At least ten ships were sold and Rs 271.3 crore booked as profits, making up 13 % of the combined FYO7 profits of these companies.

(C) at the peak of the current economic cycle, capacities of ship building firms are fully booked and there is a three to four years' wait for new ships. Freight rates are also higher. That leaves shipping lines with two happy options-sell old vessels and book good profits or sail them and improve earnings.

(D) Surbhi Chawla, an analyst with Angel Broking, says in FYO7, GE Shipping sold its single hull ships, and Shipping Corporation of India offloaded its old fleet as it is expecting delivery of 52 new vessels. "Selling ships are the functions of acquisition price, what EBIDTA is earned by the company and how much capital expenditure is required for maintenance to secure reasonable profitability in future," says Rajat Datta, General Manager of GE Shipping.

(1) ABCD (2) CBDA
(3) ACBD (4) DBCA
(5) BCDA

19. (A) The feeling that industry, and also the government, is not doing enough is growing. The promises made by the industry about creating and mentoring dalit entrepreneurs haven't impressed many. The Confederation of Indian Industry (CII) has promised to mentor 100 SC/ST entrepreneurs this year.

(B) However, these sporadic moves by the Indian industry are few and far between to address the larger issues. "Why shouldn't SC/ST graduates be taken aboard and given on-the-job or in-house training," asks Chandrabhan Prasad, author and dalit ideologue. "These so-called initiatives only send a message that these guys are not employable till they are trained. What they may lack is articulation or soft skills, not core skills".

(C) Affirmative action is once again under the scanner. The recent initiative by Infosys, in which a batch of scheduled caste/scheduled tribe graduates were trained and subsequently absorbed by IT majors, has provoked the industry to take notice. It is seen as yet another validation of the argument that 'employability' is at the core of the issue and that it can be addressed with training and skill programmes.

(D) “These students were graded using the same stringent norms used by Infosys internally for assessing our trainees," says T V Mohandas Pai, Director Human Resources, Infosys. The company may institutionalise this mechanism and take it to other cities. A best-practice document is being prepared. In the same vein, Assocham has announced a programme with Rai Foundation to put 300 girls from below poverty line families through graduate or MBA courses and help those secure jobs.

(1) ABDC (2) CDBA
(3) CBDA (4) ACDB
(5) ACBD

20. (A) these theme-based malls have varying positioning structures that could be adopted depending upon the development potential, catchments characteristics and product penetration.

(B) similarly, developers are also coming up with different mall formats to attract consumers. A notable emerging format is the specialty malls that are planned as unique centers.

(C) It is critical for each retail development to position itself clearly in reference to the neighbourhood, trade and tenant mix. Specialty malls take a step further and create a niche and water-tight positioning for themselves. Specialty malls are generally developed around a product category

(automobile, electronics, furniture, jewellery, health, etc) or a target segment (women, kids, western, ethnic, discount, etc).

(D) The evolution of modern retailing, from a nascent industry to an organised sector, has been a subject of discussion in recent times. Indian retailers are still experimenting with various modern retail formats; we have seen the emergence of various new retail formats such as stationery, laundry, health and beauty, medical, etc in almost all parts of the country.

(1) ADCB (2) DBAC
(3) ABCD (4) CABD
(5) DBCA

21. (A) Everybody in the courtroom was touched–except Steven Mallory. Steven Mallory listened and looked as if he were enduring some special process of cruelty.

(B) At his trial for the assault on Ellsworth Toohey, Steven Mallory refused to disclose his motive. He made no statement. He seemed indifferent to any possible sentence.

(C) The judge gave him two years and suspended the sentence

(D) But Ellsworth Toohey created a minor sensation when he appeared, unsolicited, in Mallory's defense. He pleaded with the judge for leniency; he explained that he had no desire to see Mallory's future and career destroyed.

(1) CADB (2) BACD
(3) CADB (4) BDAC
(5) BDCA

22. (A) The young men talked a great deal about injustices, unfairness, the cruelty of society towards youth, and suggested that everyone should have his future commissions guarantied when he left college.

(B) The woman architect shrieked briefly something about the iniquity of the rich. the contractor barked that it was a hard world and that "fellows gotta help one another." The boy with the innocent eyes pleaded that "we could do so much good…….."

(C) His voice had a note of desperate sincerity which seemed embarrassing and out of place.Goldon L.Prescott declared that the A.G.A was a bunch of old fogies with no conception of social responsibility and not a drop of virile blood in the lot of them, and that it was time to kick them in the plants anyway.

(D) The woman of indefinite occupation spoke about ideals and causes, though nobody could gather just what these were.

(1) ACDB (2) ABDC
(3) DBAC (4) DACB
(5) ABCD

23. (A) Chairman NR Narayana Murthy's answer- unlike founders like Nilekani, Pai was not getting much as dividend. That was clear admission that dividends were a substantial part of promoter earnings. This link between executive pay and promoter dividend may not be entirely appropriate, though.

(B) for many years now, HCL Technologies has lagged behind Wipro in the IT sector sweepstakes. But last financial year, HCL group founder Shiv Nadar got the better of Wipro Chairman Azim Premji on one count. Nadar, through a holding company, took home a dividend cheque of Rs 717.26 crore, a tad higher than Azim Premji's dividend earnings of Rs 696.96 crore. Remember, dividends are taxfree in the hands of the recipients- promoters, in this case.

(C) HCL Technologies gives-away almost two-thirds of its profits as dividends, the highest among leading IT companies. Promoters, who hold over 68 % of the company's equity, are the biggest beneficiaries.

(D) Even as questions are being raised about CEO salaries in India, attention is slowly shifting to dividends, another big source of promoter earnings. At the recent annual general meeting of Infosys, a shareholder wanted to know why director Mohandas Pai was getting more salary than others like co-chairman Nandan Nilekani.

(1) ACBD (2) BACD
(3) ABDC (4) BCDA
(5) BDCA

24. (A) Since paying off a loan takes the debt burden off our shoulders, in the consequent sense of relief, we often forget to tie up all the loose ends. It is also because the lender, after recovering the money, shows little of the interest it did when it was trying to get you to borrow.

(B) most of us worry whether our paperr work is right when we are applying for a bank loan for a house, car, or an MBA programme. What we should be equally careful about are the procedures' we follow and the paperwork we do when we have finished paying off a loan.

(C) Apart from the conventional ones, we can look at credit cards as loans too. Most cards today are 'life time free'. But just in case it has to be returned, there are certain steps that need to be followed to ensure that ending the relationship with the issuing bank is hassle-free.

(D) So, most of the time, the onus of sorting out the papers and recovering documents such as house ownership deeds will be on the borrower, Unless these things are sorted out, you may be in for a surprise later when you try to sell or mortgage your house or car.

(1) ADCB (2) BCDA
(3) BCAD (4) DCBA
(5) BADC

25. (A) It appeared in the Banner and had started as a department of art criticism, but grown into an

informal tribune from which Ellsworth M. Toohey pronounced verdicts on art, Literature, New York restaurants, international cries and sociology- mainly sociology.

(B) He had heard the latest story about Ellsworth Toohey, but he did not want to think of it because it brought him back to the annoying subject of the strike. Six months ago, on the wave of his success with sermons in stone, Ellsworth Toohey had been signed to wire "one small voice," a daily syndicated column for the Wynand papers.

(C) But he had said nothing in his column, for no one could say what he pleased on the papers owned by Gail Wyned save Gail Wyned. However, a mass meeting of strike sympathizers had been called for this evening. Many famous men were to speak, Ellsworth Toohey among them . At least, Toohey's name had been announced.

(D) It had been a great success. But the building strike had placed Ellsworth M. Toohey in a difficult position. He made no secret of his sympathy with he strikers.

(1) BCDA (2) ACBD
(3) BACD (4) BADC
(5) ABCD

26. (A) It was Roark's building on the sketch, very neatly drawn. It was his building, but it had a simplified Doric portico in front a cornice on top, and his ornament was replaced by a stylized Greek ornament.

(B) A great deal more was said by the three men. Roark barely heard it. He was thinking of the first bite of machine into earth that begins an excavation. Then he heard the chairmen saying "…and so it's yours, on one minor condition." He heard that and looked at the chairman.

(C) "It's a small compromise, and when you agree to it we can sign the contract. It's only an inconsequential matter of the building's appearance. I understand that you modernists attach no great importance to a mere façade, it's the plan that counts with you, quite rightly, and we wouldn't think of altering your plan in any way, it's the logic of the plan that sold on the building. So I'm sure you won't mind?"

(D) "What do you want?" "It's only a matter of a slight alteration in the façade. I'll show you. Our Mr. Parker's son is studying architecture and we had him draw us up a sketch, just a rough sketch to illustrate what we had in mind and to show the members of the board, because they couldn't have visualized the compromise we offered. Here it is." He pulled a sketch from under the drawings on the table and handed it to Roark.

(1) BCDA (2) DABC
(3) CADB (4) CDAB
(5) BACD

27. (A) We knew nothing about money, and so we had this sort of gang mentality toward anybody who worked for us," he recalls. "It sounds funny now, but that's all we had to rely on." But he didn't know the difference between a stock and a bond and lost money in real estate. So at the height of his career, he gave up partying and went back to school in 2000 to study business

(B) As Torrent's of money streamed into his wallet from multiplatinum albums in the '80s and '90s, Duff McKagan, then the bass player for the hard rock band Guns N' Roses, had little interest in tracking his cash. Instead, he relied on intimidation and his group's reputation as the "most dangerous band in the world" to prevent managers from ripping him off.

(C) Today, McKagan, 43, tightly monitors the finances of his current band, Velvet Revolver. Like other rockers easting into middle age or seniorhood, McKagan is also experimenting with new partnership, in response to a music business in flux. Amid plunging record sales and internet file Sharing, rockers are eagerly putting their names everywhere.

(D) Their "brands" are now found in television commercials, tour sponsorships and merchandise as diverse as cars, private-label wines and celebrity cruises. The rock band Kiss has been among the most prolific merchandisers, selling products ranging from condoms to the "Kiss Kasket"' a limited-edition coffin. The band's latest offerings include musical toothbrushes, pool cues, window blinds and baby booties. "It's a different ballgame now," compared with rock's baby boomer heyday, says, Joseph Bongiovi, who handles merchandise and partnerships for the rock group Bon Jovi.

(1) CBAD (2) BCAD
(3) BACD (4) BADC
(5) ABCD

28. (A) The Shipping Corporation of India has applied to the government for permission to enter shipbuilding with an investment of at least Rs. 1,000 crore. Others like Jindal, Essar, Mercator Lines and Garware Offshore are in various stages of planning an entry. "About 15 new players have shown interest. At least 10 are likely to enter." Says Ramesh Agarwal, CEO of i-maritime, a ship and port-building consultancy.

(B) On the face of it, this seems to be a simple enough business decision. The industry has seen frenetic growth in recent times. Outstanding orders have grown to 269 million gross tonnage (GT) in june 2007 from 75 million GT in 2002. That could have a big bearing on the current 721 million GT capacity of commercial ships plying the seas.

(C) Ship- building, a $ 500-billion business globally, has many takers in India Inc. Larsen & Toubro is pumping in Rs. 2,000 crore in the hope of earning a billion dollars in revenue by 2015. It has started

building its first ship in Hazira and has orders for ten ship s, two of them big, worth about $400 million.

(D) Pipavav Shipyard has already spent $250 million in its shipbuilding project and plans to invest another $400 million. It has orders for 22 ships worth $900 million.

(1) CBDA (2) ACDB
(3) ACDB (4) CDAB
(5) DCBA

29. (A) "It's all nonsense. It's all a lot of childish nonsense. I can't say that I feel much sympathy for Mr.Hopton Stoddard. He should have known better. It is a scientific fact that the architectural style of the Renaissance is the only one appropriate to our age.

(B) None of the witness looked at Roark. He looked at them. He listened to the testimony. He said: "No question," to each one. Ralston Holcombe on the stand, with flowing tie and gold headed cane, had the appearance of a Grand Duke or a beer garden composer. His testimony was long scholarly, but it came down to:

(C) The attorney gave them leads like an expert press agent. Austen Heller remarked that architects must have fought for the privilege of being called to the witness stand, since it was the grandest spree of publicity in a usually silent profession.

(D) In the next two days a succession of witnesses testified for the plaintiff. Every examination began questions that brought out the professional achievements of the witness.

(1) ABDC (2) CABD
(3) CADB (4) DCBA
(5) ADCB

30. (A) M and As were the flavour of the season with Genpact, WNS, Firstsource, Transworks, eFunds and Techbooks, all acquiring companies. In February, Pune-based HOV Services, in a large deal, snapped up US-based Lason for $148 million. Among other big deals, Infosys acquired Citibank's 23 % stake in BPO arm Progeon last April for $115.13 million.

(B) The sector clocked 28 % growth-including domestic and export segments-in FY07. Nasscom sees exports in the BPO segment at $8.3 billion in FY07, a rise of 32% year-on-year. The IT-BPO sector, Nasscom says, is well on track to hit the target of $60 billion in export revenues by 2010. It contributes roughly 5.4% to India's GDP. A quick snapshot reveals more interesting trends. Domestic BPO operations grew 53 %, faster than the rate of exports.

(C) Globalisation and favourable demographics have made India a sought after ITES-BPO destination. The ITES-BPO industry has seen a flurry of domestic and cross-border deals, IPO listings and private equity syndications in the last two years, indicating that the segment is building up momentum.

(D) The segment has over 400 players and employs more than 410,000 people. Industry body Nasscom estimates the IT-BPO sector to rake in more than $47.8 billion in revenue in FY07-almost a 10-fold increase-against $4.8 billion in FY98.

(1) CDBA (2) ABDC
(3) ABCD (4) CDAB
(5) BDCA

31. (A) Two major areas of interest in IDM are: enabling user access; and user lifecycle management. While users focus on efficiency of the experience (with one ID you can sign-in and use many applications) and apparent security, system administrators look at the efficiency of management (the user to administrators ratio), service level user administration turnaround time and actual security.

(B) So, organisations have to ensure that only the intended recipients can access information. This has changed the scope and importance of security technologies and identity management.

(C) With reports of security breaches, data thefts and electronic privacy violations happening daily, business managers have scaled up their IT security measures and adopted newer methods of encryption. Earlier, IT security was designed to restrict access to outsiders, but today, more internal applications and resources need to be shared with partners, customers and field staff in remote locations.

(D) Organisations in India have begun to realise the importance of identity management (IDM), especially in ITES, retail, telecom, manufacturing and financial services. IDM solutions can raise productivity, lower costs and meet today's stringent compliance need.

(1) BCDA (2) DCAB
(3) ACBD (4) BDAC
(5) CBDA

Answers

1. (3) ACBD	2. (2) BDAC	3. (1) DCBA
4. (4)BADC	5. (3) DBAC	6. (4) CBAD
7. (2) BADC	8. (3) CABD	9. (4) BDAC
10. (2) DCBA	11. (2) DCBA	12. (4) CBAD
13. (1) BCAD	14. (1) CDBA	15. (5) CBDA
16. (2) BADC	17. (5) CDAB	18. (5) BCDA
19. (2) CDBA	20. (2) DBAC	21. (4) BDAC
22. (5) ABCD	23. (4) BCDA	24. (3) BCAD
25. (4) BADC	26. (1) BCDA	27. (3) BACD
28. (4) CDAB	29. (4) DCBA	30. (3) CDBA
31. (5) CBDA		

Part B : English Reading Comprehensive

1 Reading Comprehensive

What is in test of Reading Comprehension ?

CAT Question Format

Eye Span Map (ESM)

39 Example Reading Comprehension

15 Lengthy Type Reading Comprehension

10 Revision Test of Reading Comprehension

What is in test of Reading Comprehension ?

CAT Reading Comprehension is a test of your ability to read and understand unfamiliar materials and to answer questions about them. Reading Comprehension passages and questions appear in the Reading Comprehension Section.

The Reading Comprehension passages are approximately 150 to 2500 words long. Each one is followed by three or more question about its content and some valuable tips from me 'There's no need to worry about what you know or don't know about the topic in a reading comprehension. The answers are all based on information in the Passage, and you won't be required to draw on outside knowledge.

Question Format

Reading Comprehension question follow the standard multiple—choice format with four-five answer choice each. All of the question fall into on of the following types :

- The main idea of the passage
- Specific details mentioned in the passage
- The author's attitude or tone or aim
- The logical and informational structure of the passage
- Future inferences that might be drawn from the text
- Application of the ideas in the text to new situations

For Example

1. The author cites xyz/line # in order to a, b, c, d
2. The author mention all the following ways by which ………… EXCEPT
3. With which of the following statements about xyz would the author most likely agree?
4. The author makes which of the following criticism of the xyz,
5. It can be inferred that xyz.
6. The primary purpose of the passage is to
7. The author raises the questions at the beginning of the x para in order to/ to highlight etc.
8. It can be inferred that the most important difference between x and y mentioned in line z is that …………
9. Which of the following would be the most logical continuation of the passage a, b, c, d, e ……………
10. The author most likely places the word/s in quotations in line x in order to remind/ highlight the reader that/ xyz issue, point
11. The slayer of monsters introduced in line 'x' functions primarily /………./……….as..
12. According to the passage the main / primary difference between x and y is that …
13. In the study in lines x,y the ……… because………
14. The author attitude towards xyz can best be described as……

    Concern, resignation, anxiety, disinterest, approval, pessimism, optimism, anger, bigoted, egoistical etc………
15. In the passage expression "……………" attends to
16. The ………… leads ………… to
17. According to the passage, the term "………" refers to
18. What is the thematic highlight of this passage?
19. In the study* described in lines # prove that:
20. Which of the following would have been true. If ……… ?
21. According to the author which of the following will correspond with (or vice versa)
22. The reason behind ………… are.
23. The primary reason that ………… was.
24. Consider / peruse the following statements and choose the correctly stated one from the options:
25. The stages connected to ……… of ………… is/are A,BC,D ………… (1) (2) (3) (4).
26. The basic symptom of ………… is reflected when …………
27. The idea that ………… is directly linked will.

28. What is the basic idea behind xyz in xyz?
29. Which of the following cannot be inferred from the following
30. '………' mentioned in line # para ……… comes close in meaning to / is synonymous with
31. According to the passage ……… came before / after etc.
32. Which of the following has not caused ………… ?
33. The attitude of the author towards ………… is

    —energetic, nonchalant, sardonic, penetrative, disbelieving, nostalgic, complaisant- acquiescent
34. Which of the following is true as depicted in para/ lines—
35. According to the author the main cause for ……… as ……… was.
36. Which of the following can't be described as "………" ?
37. Which of the following is not 'Z's idea ?
38. The difference in concept xyz between 'x' and 'y' is best characterized by.
39. Going by the author's exposition of 'xyz' which of the following is true/ untrue?
40. Which of the following views of 'xyz' can/ cannot be attributed to the author ?
41. The author holds that.
42. It is the author's contention that.
43. The author makes a distinction between perception* and creation in terms of.
44. A writer* as an artist*.
45. The writer argues that ……… because they feet* that.
46. The author acknowledges xyz because—
47. The main difference between 'x and z' in the matter of 'X' and 'Y' according to the author is:
48. In the statements "…………", it refers to.
49. The importance of xyz is that it:
50. ………… can be exemplified by :
51. Which of the following statements best / strongly conveys the overall idea of the passage ?
52. Which one of the following best describes he approach of the author ?
53. Which of the following parallels between ……… x ………, is not clamed by the author ?
54. In developing ………… to ………… these are based on:
55. The areas in which ……… has / have excelled over ……… societies are/is
56. Pick/choose out the correctly stated statements from the following—
57. The idea / illusion / presumption etc that is (not) susceptible to ……… is directly linked to:
58. What is the basic idea behind ……… in xyz ?
59. Why is 'x' in compatible to 'y' and is its significance
60. An appropriate title for the passage would be—
61. '………' mentioned in para x/line * [come close in meaning to] [signifies] [portrays] [depicts]
62. The example of ……… goes to prove.
63. Which of the following is true of the xyz system?
64. What does the phrase/sentence ……… imply.
65. In which context is the phrase/ sentence used:
66. The author employs the metaphor ( ) to suggest.
67. What is the contradiction in para 'x' the author uses it to: pick out the correct pair—
68. The author implies that xyz in not necessarily profitable for/ of because—
69. The writer recommended which of the following changes*/ solutions etc be made in ' '
70. The passage supplies information to all the questions with the exception of.

Eye Span Map (ESM)—A slow reader reads a line as follows (show by ESM)

A fast reader reads a line as follows (show by ESM)

Very Important—Vocabulary is a vital part of reading comprehension.

Examples of Reading Comprehension.

Passage–1

Word Count–600

As a writer or editor today, you have a lot more at your disposal than pencils and erasers.

- Word-processing software simplifies writing and revising, from note-taking and drafts to final copy.
- The Internet is a reference library at your fingertips.
- E-mail makes keeping in touch with colleagues, friends and family at the press of a button besides easing the interaction among authors, editors, peers, reviewers, project managers, and production staff.

These powerful tools *help you do it,* but a solid foundation in the principal of revising tells you *what needs to be done*.

Rewrite Right ! Is for people who must write at work or in school and for freelance writers struggling to make sales. Editing is improving something written-making it easier to follow, snappier, more interesting. Knowing how to edit means knowing what good writing is in the first place. And good writing comes from knowing how to revise, how to tug on words and adjust them until they say what you want them to say. In other words, writing

and editing are facets of the some subjects : doing a good job of putting ideas into words.

Have advances in telecommunications reduced the need for good writing ? Not at all. The medium may change but the language still needs to be well crafted. People who used to step down the hall to brainstrom with a colleague now sit down at a keyboard and whip out a message on e- mail.

E-mail has many advantages it's convenient, fast, cheap, and less intrusive than a phone call; it provides a paper trail and is easily distributed to a large number of people to work place from their homes.

But the very ease of e-mail can be a hazard. Many people have developed an e-mail style that's breezy and informal; they pay less attention to spelling and grammar than they would in a standard letter. Should they care about good writing? You bet! E- mail recipients shouldn't have to puzzle their way through a message riddled with misspelled or omitted words and confusing reference. Don't let the informally of e-mail fool you into thinking sloppiness is okay. Spontaneity is fine; Sloppiness can be dangerous in any communication.

Muddled instructions create confusion. Costly research is repeated because results are buried in an obscure two pound report. Boring writing is tossed aside unread. Slipshod writing breeds distrust, prompting readers to wonder if language is the writer's only area of incompetence.

At the other end of the spectrum good writing gets things done, its crisp, clear style requires less of the reader's time. Good writing lowers administrative expenses, lightens work-loads, and suggests the writer is competent in other areas as well.

Yet when it comes to writing, many capable people falter. They may be experts at marketing or high-energy physics, but ask them to write it up, and they rely on worn-out expressions and stilted prose.

As a consulting editor, I've learned where most people need help. My first two books, Write Right ! and better letters, provide simple advice.

Rewrite Right ! is for those who want more. It describes two levels of editing; (1) to improve style and content and (2) to correct language.

Rewrite Right ! includes a variety of reference materials; lists of accepted abbreviations, hackneyed expressions, common redundancies, and irregular plural. It suggests ways to make a document look better and explores sophisticated tools available to today's writers and editors. A glossary provides definitions of unfamiliar terms.

I hope Rewrite Right ! helps you learn not only to write and rewrite well, but to enjoy doing it.

1. Which of the following reason/s does the author state to suggest that modern day technological inventions are helpful but still require editing and good writing skills—
   1. Through E-mail is handy, quick and cheap and people's e-mail style is breezy and informal. However, scant attention is paid to spelling, grammar & omitted words such sloppiness can be hazardous in any communication.
   2. People wonder if you have any idea of what you are saying as they don't.
   3. Advance in telecommunications have not reduced the need for good skilful writing. As, they still need to be well crafted.
   4. The informality of e-mail can lead one in thinking sloppiness is right on spontaneity is fine sloppiness is a wet squib in any communication.

   (A) 1,2,4 (B) 2,3,4
   (C) 3,4 (D) 1,3,4
   (E) 1,2,3,4

2. According to the author muddled instructions create confusion as (All of the following are true Except)—
   (A) Thousands of pages of costly research is repeated due to nebulous and ambiguous reporting style
   (B) Writing that cause ennui is put down unread
   (C) Slipshod writing creates doubts and distrust in the readers' minds
   (D) Though well edited and crafted skilfully lack of personal brainstorming instead of e-mail make the work insipid.
   (E) The readers ponder over whether the writer's sole skill is language incompetence.

3. The author's tone in the passage shows that it is—
   (A) Instructive (B) Critical
   (C) Panegyric (D) Persuasive
   (E) Hard sell

4. The Passage is intended for those who want—
   1. Simple advice on how write well
   2. To improve style and content
   3. To learn how to write better letters
   4. To use a proper and correct language
   5. Methods to make documents look better with a glossary of definitions of terms not familiar

   (A) 1, 2, 5 (B) 2, 4, 5
   (C) 1 and 2 (D) 1, 2, 3, 4
   (E) All of the above

Answers

1. (E) 1, 2, 3 and 4 2. (D) 3. (A) 4. (D) 1, 2, 3 and 4

Passage–2

Words–541

More than five billion dollars in US aid to Pakistan often never reached the military units it was intended for to fight al-Qaida and the Taliban - instead it went into weapon systems aimed at India. Much of the money meant, to reimburse frontline Pakistani units was channeled to weapons systems aimed at India and to pay inflated Pakistani reimbursement claims for fuel, ammunition and other costs, unnamed US government and military officials to daily.

Pakistanis critical of president Pervez Musharraf said he used the reimbursements to prop up his government and one European diplomat said the United States should have been more careful with its money. "I wonder if the Americans have been taken for a ride," said the diplomat, who spoke on condition of anonymity.

Money intended to repay Pakistan. for maintaining 100,000 troops in the restive tribal areas apparently does not reach the troops who need it, officials said.

"It is not making its way, for certain, we know, to the broader part of the armed forces which is carrying out the brunt of the operations on the border" with Afghanistan, a senior for US military official said ?

Despite the vast funds flowing to Pakistan, a US official visiting the border recounted finding members of the country's frontier corps "standing there in the snow in sandals".

Several soldiers were wearing world war-I era pith helmets and had battered Kalashnikov rifles with only 10 rounds of ammunition each, the official said. The two countries have forgotten clear strategic goals as to how the US military aid should be spent or how Pakistan could show it was meeting Washington's expectations, according to US and Pakistani officials US aid to Pakistan has come under scrutiny recently in the United States given the strength of the al-qaida and Taliban cells in Pakistan's northwestern tribal areas as well as the failure to secure the

Capture of Osama bin Laden.

Musharraf has also been forced by US pressure to ease back on repressive measures, lift emergency rule, shed his military uniform and move the country toward greater democracy. The US provides the five billion in aid to reimburse Pakistan for carrying out military operations against terrorist threats. A separate US programme delivers 300 million every year to pay for equipment and training for the Pakistan military.

The US Congress on Thurs day slapped restrictions on the 300 million in traditional military aid, 50 million of which will be withheld until Pakistan shows it is restoring democratic rights.

US funds are vital for Pakistan's military, with American aid accounting for about a quarter of the military's entire budget, the paper said.

Pakistani officials interviewed by the New York Times denied their government had overcharged the United States for the "War-on-terror" military aid it gets. But US officials cited helicopter maintenance as an example of the funding programme's failure.

While Pakistan received $ 55 million for helicopter maintenance for an eight-month period in 2007, the officials said they found out that only 25 million had been received by the Pakistani army for helicopter maintenance for 2007.

Allegations that generous military aid to Pakistan has been squandered represent another setback for President George Bush's administration, which has viewed Pakistan as an important ally in the "war on terror".

1. What was the strongest factor in which Pakistan was accused of misuse of funds a U.S. write up—
   (A) Pak used the funds for other reasons than intended
   (B) To strengthen his (Prevez Musharraf) government
   (C) To create weapons systems armed at India
   (D) To reinforce its frontline military forces
   (E) To create disturbance between India and Pakistan
2. Which one of the following statements is **not** True—
   (A) The money was not used for maintaining troops in restive tribal area.
   (B) Pak's frontier troops have inadequate gear and adequate weapon to conn terrorist attacks.
   (C) US Funds were not used to reimburse Pakistan for carrying out military operations against terrorism.
   (D) To build equipment to counter attack any Indian assault
   (E) All of the above
3. In the last paragraph the writer implies—
   (A) Pakistan has lost favour with the USA
   (B) Pakistan will face severe restrictions in military aid
   (C) The USA wants Pakistan to restore democracy
   (D) The President of America who portrayed Pakistan as an ally will not face political and country wide flak
   (E) The country (Pak) will get further aid from USA.
4. The areas in which the US found Pakistan's misuse of billions of dollars in chronological order are
   (A) To provide gear and state of art weapons to combat terrorism.
   (B) Maintenance of troops in restive area terrorism

(C) Reimbursement claims for fuel and ammunation

(D) Helicopter maintenance.

(E) All of the above

Answers

1. (C) 2. (D) 3. (A) 4. (D)

Passage–3

Words–159

Patriotism is a very complex feeling built up out of primitive instincts and highly intellectual convictions. There is love of home and family and friends, making us peculiarly anxious to preserve our own country from invasion. There is the mild instinctive liking for compatriots as against foreigners. There is pride, which is bound up with the success of the community to which we feel that we belong. There is belief, suggested by pride but reinforced by history, that one's own nation represents a great tradition and stands for ideals that are important to the human race. But beside all these, there is another element, at once nobler and more open to attack, an element of worship, of willing sacrifice, of joyful merging of the individual of life of the nation. This religious element in patriotism is essential to the strength of the state, since it enlists the best that is in most men on the side of national sacrifice.

1. A suitable title for the passage could be —

(A) Elements of patriotism

(B) Historical development of a nation

(C) The role of religion and history in patriotism

(D) Religion and patriotism

2. Describing the element of worship "open to attack," the author implies that it—

(A) Is unnecessary

(B) Leads to national sacrifice

(C) Has no historical basis

(D) Can not be justified on rational ground

3. The tone of the passage can be best described as—

(A) Critical (B) Descriptive

(C) Persuasive (D) Analytical

4. Which of the following can clearly be grouped under "intellectual convictions" that the author mentions in the opening sentence ?

(A) Love of family

(B) Love of compatriots

(C) The element of worship

(D) None of these

5. Which one of the following statements is false ?

(A) We tend to like or own countrymen better than we like foreigners

(B) Nations always stand for ideals that are important to the human race.

(C) It is the religious element in patriotism that motivates us for sacrificing ourselves for our nation.

(D) Our pride of the community is bound with the community's success.

Answers

1. (A) 2. (B) 3. (D) 4. (D) 5. (B)

Passage–4

Words–389

The public distribution system,. Which provides food at low prices, is a subject of vital concern. There is a growing realization that though India has enough food to feed its masses two square meals a day, the monster of starvation and food insecurity continues to haunt the poor in our country.

Increasing the purchasing power of the poor through providing productive employment leading to rising income, and thus good standard of living is the ultimate objective of public policy. However, till then, there is a need to provide assured supply of food through a restructured, more efficient and decentralized public distribution system (PDS).

Although the PDS is extensive-it is one of the largest system in the world it has yet to reach the rural poor and the far-off places. It remains an urban phenomenon, with the majority of the rural poor still out of its reach due to lack of economic and physical access. The poorest in the cities and the migrants are left out, for they generally do not possess ration cards. The allocation of PDS supplies in big cities is larger than in rural areas.

In view of such deficiencies in the system, the PDS urgently need to be streamlined. Also considering the large food grains production combined with food subsidy on one hand and the continuing slow starvation and dismal poverty of the rural population on the other, there is a strong case of making PDS target group oriented.

The growing salaried class is provided job security, regular income, and social security. It enjoys almost hundred percent insulation against inflation. The gains of development have not percolated down to the vast majority of our working population if one compares only dearness allowance to the employees in public and private sector and looks at its growth in the past few years, the rising food subsidy is insignificant to the point of inequity. The food subsidy is a kind of D.A. to the poor the self-employed and those in the unorganized sector of the economy. However, what is most unfortunate is that out of the large budget of the so called food subsidy, the major part of it is administrative cost and wastages. A small portion of the above budget goes to the real consumer and an even lesser portion to the poor who are in real need.

1. Which of the following, according to the passage, is true of public distribution system ?
   (A) It is unique in the world because of its effectiveness
   (B) It has remained effective only in the cities
   (C) It has reached the remotest corners of the country
   (D) It has improved its effectiveness over the years
   (E) It develops self-confidence among the people
2. Which of the following, according to the passage, is the main reason for insufficient supply of enough food to the poorest ?
   (A) Production of food is less than the demand
   (B) Government's apathy towards the poor
   (C) Absence of proper public distribution system
   (D) Mismanagement of food stocks
   (E) None of these
3. What, according to the passage, is the main purpose of public policy in the long run ?
   (A) Good standard of living through productive employment
   (B) Providing enough food to all the citizens
   (C) Reducing the cost of living index by increasing supplies
   (D) Equalizing per capita income across different starts of society
   (E) None of these
4. Which of the following in the passage, is compared with dearness allowance ?
   (A) Food for work programme
   (B) Unemployment allowance
   (C) Procurement price of food grains
   (D) Food subsidy
   (E) None of these
5. What, according to the passage, should be an appropriate step to make the PDS effective ?
   (A) To increase the amount of food grains available for distribution
   (B) To increase the amount of food grains per ration card
   (C) To reduce administrative cost
   (D) To make it target group oriented
   (E) To decrease the allotment of food grains to urban sector
6. Which of the following words is the same in meaning as the word 'power' as used in the passage ?
   (A) Energy (B) Vigor
   (C) Authority (D) Influence
   (E) Capacity
7. The author's writing style is
   (A) Simplistic (B) Argumentative
   (C) Verbose (D) Descriptive
   (E) Analytic
8. What, according to the passage, would be the outcome of making the PDS target group oriented ?
   (A) It will remove poverty
   (B) It will give food to the poorest without additional cost
   (C) It will abolish the imbalance of urban and rural sector
   (D) It will motivate the target group population to work
   (E) None of these
9. Which of the following words ids the same in meaning as the word 'system' as used in the passage ?
   (A) Mechanism (B) Routine
   (C) Machine (D) Procedure
   (E) Collection
10. According to the passage, food subsidy leads to which of the following ?
   (A) Shortage of food grains
   (B) Decrease in food grains production
   (C) Increased dependence
   (D) Sense of insecurity
   (E) None of these

Answers

1. (B) 2. (C) 3. (A) 4. (D) 5. (D) 6. (E)
7. (E) 8. (D) 9. (D) 10. (E)

Passage–5

Words–409

Of the many monarchs that ruled in ancient India, it is Ashoka that we remember most. And this for at least two reasons : He is the first on whom reliable historical records exist to this day. Secondly, he was emperor and hero, conquering and noble, powerful and pious. No matter what he did in his earlier years , in the end he proved to be uncommonly virtuous and wise, remarkably gentle and compassionate. His very name has become a matter of pride for the people of India. Yes it has been stated in Buddhist lore that Ashoka was once a bloodthirsty prince who rose to his throne by the slaughter of his ninetyeight brothers; This may be a slight exaggeration, but it is well established that his emperor of the third century B.C., which extended the boundries of his Magadha kingdom far and wide, engaged in a bloody battle to subjugate Kalinga on the eastern coast of India.

But when it was all over, Ashoka saw the cruely of war, the inhumanity of humanity of conquest. He was

deeply touched by the sufferings he had inflicated this was no crafty strategy to win over the people he had subdued. Ashok's feeling were as genuine as they were profound. He adopted the Buddhist religion which insisted on compassion and non injury, became a serious student of the faith. Years later he even relinquished pomp and power, and became and ascetic. Never in human history has a victorious potentate given up pride to become a meek monk, as did Ashoka.

And while he was still in authority, he set up channels for the systematic propagation of the civilized principles to which he had been converted . He sent out missionaries to distance lands, to Syria and Egypt, to Macedonia and Cyrene, as well as to Sri Lanka and beyond. He assumed tremendous responsibilities towards his people. He worked for their material and moral welfare, and built many roads and hospitals. Today we see billboards and public announcements to sell commercial products. Ashoka adopted a very similar mode over two thousand years ago. For within his own realm he carved his messages in rocks and pillars, in public places and in caves, here and there and everywhere, so that people would see them and reflect over them. The idea worked , and many joined the Buddhist faith. Even more importantly, those inscriptions have left for posterity indelible records of Ashok's thoughts and deeds.

1. Which one of the two reasons does the author give for remembering Ashoka ?
   - (A) He was an emperor and hero, powerful and pious
   - (B) He slaughtered ninety eight of his brothers
   - (C) He set up channels for the systematic propagation of civilized principles
   - (D) He adopted the Buddhist religion
   - (E) He was the youngest emperor
2. How did Ashoka attract people to the Buddhist faith ?
   - (A) He worked for the welfare of the people
   - (B) He gave up his throne
   - (C) He carved messages in rocks, pillars and caves
   - (D) He coerced his subjects to adopt the religion
   - (E) None of these
3. Ashoka adopted the Buddhist religion because ....
   - (A) It was his strategy to win over people
   - (B) He wanted to become a serious student of faith
   - (C) He wanted to preach Buddhism
   - (D) He was moved by the suffering he had inflicted on people
   - (E) He had become a powerful king
4. Which of the following statements is not true in the context of the passage ?
   - (A) Ashoka sent out missionaries to distance places for the propagation of his massage
   - (B) Ashoka built roads and hospitals
   - (C) Ashoka became a Monk
   - (D) Ashoka followed the faith of compassion and non-injury
   - (E) Ashoka introduced public announcements and billboards to expand his kingdom
5. What did Ashoka do which had never happened in human history ?
   - (A) He had slaughtered a hundred thousand people
   - (B) A powerful Emperor had become a quiet and gentle Buddhist
   - (C) He had engraved messages in rocks, caves and pillars
   - (D) He had built roads and hospitals
   - (E) None of these
6. Why did Ashoka's name become a matter of pride for Indians ?
   - (A) He was an Emperor
   - (B) He became, in the end, a gentle, compassionate and wise human being
   - (C) He did a lot of charity work then he became , an Emperor
   - (D) He expended his kingdom
   - (E) He had become a Monk
7. Which of the following lands did Ashoka not send his missionaries to ?
   - (A) SriLanka (B) Macedonia
   - (C) Persia (D) Egypt
   - (E) Syria

Answers

1. (A) 2. (C) 3. (D) 4. (E) 5. (B) 6. (B)
7. (C)

Passage–6

Words–662

Richard Dawkins's international bestseller The God Delusion has got it all wrong. Far from being a delusion, at least in India God is alive and kicking butt daily. Week before last the butt He was kicking was mine, in Bangalore. In my case, God had manifested Himself in the avatar of a sub-registrar. A sub-registrar is a divine manifestation who sub-registrars things. As I needed to have a thing (a small property) sub-registrared in Bangalore I'd had for months sought a *darshan* with this exalted being. Finally, my prayers and entreaties prevailed and I was informed that my long sought *darshan* had been granted (at the BDA office, a place of transcendental

malodorous ness, through which more wealth daily passes than through Tirupati) for a date falling six weeks later. I duly booked my flight to Bangalore and back, well ahead of the *darshan* date. Two days before the appointed day, I was informed that the *darshan* had been postponed by three days. Why ? Because the sub-registrar had decided to go on C/L. (C/L is a divine right of sub-registrars. No, it's *more* than a divine right. Tirupati, for. example, can't suddenly decide to go on C/L and shut up shop for three days. There'd be questions in Parliament, riots, army called out. But sub-registrars and other Dominions and Powers of his ilk go on C/L as a matter of scriptural., routine.) Anyway, i now faced the prospect of canceling two non-refundable air tickets and buying two fresh ones. So i decided to cancel only my return ticket (and buy a new one for a later date) while retaining my ticket for the outward journey to Bangalore. This meant that I'd have to spend almost a whole week in Bangalore for the sake of a 30-minute *darshan*, max. Hey, who said meeting God is easy?

Now I like Bangalore. So much so, that i once wanted to shift there permanently from Delhi, and might still do so one of these years. Despite having been turned from Garden City to Concrete Bungle by greedy land 'developers' and builders, Bangalore still enjoys a pleasant climate and a cosmopolitan charm of manner. I like Bangalore. But an enforced stay can make even paradise seem a prison. As I served my time in Bangalore I thought about how the gods rule us in India. Not the 33 million and still counting gods up in Heaven. But the somewhat less than 33 million (but not much less) even more powerful gods down here on earth whom we daily pay homage to and who are generically known as *babus*. Forget politicians, MPs, the judiciary, the cops, whatever figures of supposed authority you care to name. The entities who really rule us are our *babus*, cousin-brothers and cousin-sisters, one and all, to my absconding C/L-taking sub-registrar. Leave alone an insignificant professional clown like me; captains of industry and barons of business needs must genuflect at the altar of *babudom*. And when you think of the time and the money and the effort we all of us expend in thrall to *babudom*, Tirupati looks like a flop show in comparison.

Dawkins and fellow atheists like Sam Harris have argued that God doesn't exist because there's no evidence of His existence. What the dumb clucks don't realise is that the Guy's just gone on C/L for a couple of millennia or so. Then there's the ontological paradox questioning God's existence: Can God build a wall so high that He can't jump over it? In other words: Can God, universal butt-kicker, kick His own butt? i don't know about Dawkins's God. But i do know about my sub-registrar. Who (back from C/L) will doubtless one day have to go see his sub-registrar to get some sub-registration done only to discover that his sub-registrar has gone on C/L. *Babudom* has just kicked it's own butt. Ergo, God-as-*babu* exists. Tough lumps, Dawkins. But *you're* the delusion. And I have my sub-registrar to prove it.

1. The Author is primarily concerned with—
   (A) God
   (B) Bureaucracy
   (C) Mr.Richard Dawkins
   (D) Atheism
   (E) Bangalore
2. The most appropriate title for the passage would be—
   (A) God is red tape
   (B) God, its' a babu
   (C) Bureaucracy and God
   (D) Man is greater than God
   (E) Babus in God's shoes
3. The author justifies his viewpoint with—
   (A) Facts (B) Statistics
   (C) Assumption (D) Personal Experience
   (E) Comparison
4. The author is most likely to agree with which one of the following:
   (A) God is on C/L
   (B) Mr. Dawkins has got it all wrong
   (C) God is manifested in the form of b*abus*
   (D) God is alive and kicking
   (E) None of these
5. The tone of the article is—
   (A) Witty (B) Critical
   (C) Light (D) Admonishing
   (E) Dominating

Answers

1. (E) 2. (B) 3. (D) 4. (C) 5. (A)

Passage–7

Words–189

Of the many aspects of public administration, the ethical aspects are perhaps the most important but the least codified. While administrative rules and procedures have been codified in various public documents and manuals, there is no manual for the ethics of public servants.

While organizational behaviour analyses the factors, which influence the behaviour of individuals in an organization, ethics refers to those norms and standards which behaviour of individuals in an organization, ethics refers to those norms and standards which behaviour of the people in an organization must conform to. While behaviour analysis deals with factual aspects, ethics

relates to the normative aspects of administration. The normative aspects are of the greatest significance. Just as for an individual if character is lost., everything is lost, so also for an administration if the ethics is lost, everything is lost. Neither efficiency nor loyalty could be substitute for high ethical standards. In India, through there is no ethical code for public administrators, there are what are called, the government servant' conduct RULES. These rules lay down what constitutes misconduct for the public servants. It is not permitted, is also unethical conduct.

1. As per the passage, organizations
   (A) Differ in ethics
   (B) Human behaviour in organizations include ethics
   (C) Ethics do not relate to normative aspects of administration
   (D) None of these
2. Ethics is to an administration, what character is for
   (A) An administrator (B) An official
   (C) An individual (D) None of these
3. Government Servants Conduct Rules are meant for
   (A) Guiding the ethical conduct of government servants
   (B) Guiding what constitutes misconduct for public
   (C) Guiding what constitutes misconduct for government servants
   (D) None of these
4. The underlined word 'manual' in the context of the given passage means
   (A) Hand operated (B) Physical
   (C) Guide book (D) None of these

Answers

1. (C) 2. (C) 3. (C) 4. (C)

Passage – 8

Words–1206

The story may be apocryphal but has never been denied. During the 1950s, the Yemeni administration discovered that some denominations of its currency, the Rial, were disappearing from the market. The administration traced the shortage to Aden, a port Yemen, and found to its surprise that a young man in his 20s had placed an unlimited buy order for the rial. The rial was a solid silver coin. The young man simply bought the rials, melted them into silver ingots and sold them to bullion dealers in London at a much higher price because of the exchange rate arbitrage. The name of the young man: Dhirajlal Hirachand Ambani, or Dhirubhai Ambani. Here were the early signs of the making of an iconic entrepreneur who was always looking for a business opportunity and to make a quick buck on the side. Years later, Dhirubhai told an interviewer: "The margins were small but it was money for jam. After three months, it was stopped. But I made a few lakh of rupees. I don't believe in not taking opportunities."

Even in a world overflowing with rags-to-riches stories, there is only one Dhirubhai. A man who rose from the position of a petrol pump attendant to set up the largest grass root oil refinery in the world; from borrowing Rs 100 to buy clothes in order to go to Aden for a job, to set up the most modern textile mill in the country; from being the one who was asked to wait for his paltry bills outside the cabins of purchase officers and cashiers in mills to become the greatest industrialist in the country and from being a member of a lower middle class Indian family to become a member of the Forbes' list of richest men in the world.

And in this long eventful journey spanning four-odd decades, not only did he convert his fledging business he started with about Rs 13,000 as seed capital for his first textile plant in Naroda, near Ahmedabad into a Rs 60,000 crore powerhouse but also enlisted the support of four million Indians weaned on socialism, in an adventure in can-do capitalism, convincing them to ride the Reliance story. Not for nothing that Dhirubhai is dubbed the high priest *of* the equity cult, that began in 1977 when Reliance Industries first went public with 58,000 shareholders, and continues even today under his two sons, Mukesh and Anil, with over four million shareholders. A *fact* that has been acknowledged even by the Wharton School of Business in 1998 while describing "his path-breaking contribution towards the concept of equity investing in India by creating wealth and value for millions *of* shareholders."

The financial acumen of a poor school teacher's son can also be gauged from the *fact* that Reliance Industries did not pay a paisa in taxes on its corporate earnings till 1996, when Finance Minister P Chidambaram introduced the Minimum Alternate Tax. His other innovations ranged from raising funds through the global depository route, introducing convertible debentures and issuing l00-year bonds in the US market.

Beginnings of A Business Genius

But for a true analysis *of* his genius, we have to travel back a few decades and trace Dhirubhai's journey from Aden, where he worked as an attendant in a Shell outlet and then returned to India in 1956 to begin his new journey. With the money earned from the silver content *of* Aden's coinage, he started a trading house called Reliance Commercial Corporation in Mumbai, importing polyester fiber, Dhirubhai opened his first textile mill in Naroda near Ahmedabad, in 1966 and concentrated on building up his business, giving birth to the Vimal brand.

In 1982, India saw another first from Dhirubhai Ambani, the beginning of the concept of backward integration with Reliance Industries setting up a 10,00 million tonne poly ester plant in Patalganga, some 80 km from Mumbai for polyester yarn and fiber intermediates and

finally to the basic raw material, oil, by setting up a 27 million tonne refinery in Jamnagar, Gujarat, in 1998. He subsequently diversified into chemicals, gas, petro-chemicals, plastics, power and telecommunications. Yet his success always had a whiff of controversy to it. Rivals alleged that his success had as much to do with his business acumen as his ability to get official rules and regulations tweaked to undercut his rivals and push his own business interests. His fight-to-the-finish battle with the fiery proprietor of Indian Express, Ramnath Goenka, the war with industrialist Nusli Wadia, the allegations against some Ambani staffers over a plot to murder Wadia and his travails during the VP Singh government, controversies over licensed capacities, export manipulation and share switching are part of the Ambani plot.

As Gita Piramal wrote in her *book Business Maharajas* : "The corporate world is sharply divided between those who feel he is a visionary and those who consider him a manipulator." Mukesh Ambani, Chairman of Reliance Industries, qualifies it : "What galled Dhirubhai's critics was his success in outwitting them at every turn. Not only did he dream bigger but he also found novel methods to realise them. One such way was to ensure that even ordinary citizen shared in the wealth he created."

Thinking Big

So what was it that makes Dhirubhai a respected as well as reviled figure? For R Ravi mohan, Managing Director of Standard and Poor's South Asia operations, his success could be attributed to his penchant for global benchmarking, using state-of-the-art technology while cutting costs, pursuing ambitious goals, flawless implementation and a willingness to take calculated risks. "But his biggest plus point was his ability to align the goal of his enterprise to the needs of the market and pass on that vision to every member of the Reliance family," he says.

His ambitious goals could leave others shell shocked. For instance, in the early 1980s, when Dhirubhai told

Ravimohan, then a project officer with ICICI, that he planned to sell polyester yarn at less than the price of groundnut, his jaw nearly fell. But within just three to four years, Dhirubhai brought down its prices from Rs. 180 to Rs 17 per Kg. Ravimohan says Dhirubhai was responsible for the telecom revolution in India. When the call rates were Rs10 per minute and consultants told him to keep it at RS 6 a minute, Dhirubhai talked of bringing it down to 40 paisa a minute. "It was Reliance Infocomm which slashed prices, and the whole industry followed," he adds.

Soft Spots

But there are certain softer aspects of Dhirubhai's life, too. For instance, his ability to strike up conversation with any body, his willingness to reward talented people and give them a free hand, his open-handed generosity, the skillful blend of head and heart and his legendary management techniques, which A.G. Krishnamurthy, founder Chairman and former MD of Mudra Communications, the advertising arm of Reliance, calls "Dhirubhaism."

So, how will history judge Dhirubhai Ambani ? Will he be talked in the same breath as Henry Ford or Bill Gates when the economic history of India is written. Possibly yes, because unlike others in the West he had to contend with red tapism, serious credit crunch, an abysmal infrastructure and a poorly developed capital market to build a formidable, globally competitive industrial empire. And that's quite an achievement.

1. Which of the following would be the most suitable heading for the passage—
   (A) Entrepreneur – Apart
   (B) Vision of a Visionary
   (C) Visionary of a different kind
   (D) Visionary Extraordinary
   (E) Leading light

2. The following attributes made Dhirubhai Ambani a multibillionaire and iconic entrepreneur 'EXCEPT'—
   (A) Strong visionary
   (B) Sound business acumen
   (C) Astute manipulator skills
   (D) Willingness to take risks and flawless implementation
   (E) Ensuring others did not share in his creation of wealth

3. The author writes that Dhirubhai Ambanis success always had a whiff of controversy to it . By this, he implies that Dhirubhai—
   (A) Was ruthless and would use any means to achieve his goals
   (B) He was an unscrupulous person
   (C) Dhirubhai had criminal tendencies
   (D) He was petty mined and spiteful
   (E) All of the above

4. Dhirubhai can be best summed up by which of the following positive points attributed to Dhirubhai Ambani—
   (A) His ability to converse with all levels of people
   (B) The adroit mix of heart and mind
   (C) His munificent nature
   (D) His legendary management skills and techniques
   (E) All of the above

Answers

1. (C) 2. (E) 3. (A) 4. (E)

Passage - 9

Words–511

Globalisation, liberalization and free market are some of the most significant modern trends in economy. Most economists in our country seem *captivated* by the spell of the free market. Consequently, nothing seems good or normal that does not accord with the requirements of the free market. A price that is determined by the seller or, of for that matter, established by anyone other than the aggregate of consumers seems pernicious.

Accordingly, it requires a major act of will to think of price-fixing as both normal and having a valuable economic function. In fact price fixing is normal in all industrialized societies because the industrial system itself provides, as an effortless consequence of its own development, the price fixing that it requires. Modern industrial planning requires and rewards great size. Hence a comparatively small number of large firms will be competing for the same group of consumers. That each large firm will act with consideration of its own needs and thus avoid selling its product for more than its competitors charge is commonly recognized by advocates of free-market economic theories. But each firm will also act with full consideration of the needs that it has in common with the other large firms competing for the same customer. Each large firm will thus avoid significant stable price cutting, because price cutting will be prejudicial to the common interest in a stable demand for products. Most economists do not see price fixing when it occurs because they expect it to be brought about by a number of explicit agreements among large firms, Moreover, those economists who argue that allowing the free-market to operate without interference is the most efficient method of establishing prices have not considered the economics of non-socialist countries. Most of these economies employ intentional price-fixing, usually in an overt fashion. Formal price-fixing by cartel and informal price-fixing by agreements covering the members of an industry are common place. Was there something peculiarly efficient about the free market and inefficient about price-fixing, the countries that have avoided the first and used the second would have suffered drastically in their economic development. There is no indication that they have.

Socialist industry also works within a framework of controlled prices. In the early 1970's the Soviet Union began to give firms and industries some flexibility in adjusting prices that a more informal evolution has accorded the capitalist system. Economists in the USA have hailed the change as a return to the free-market. But the Soviet firms were not in favour of the price established by a free market over which they exercised little influence; rather, Soviet firm acquired some power to fix price.

Questions

The author's primary objective of writing the passage is to belie the popular belief that the free market helps enhance development of industrial societies advocate that price fixing is unavoidable and it is beneficial to the economy of any industrialized society explain the methodology of fixing price to stabilize free market prove that price fixing and free market are compatible and mutually beneficial to industrialized societies create awareness among the general public regarding combating price by large firms

1. Considering the literal meaning and connotations of the words used in the passage the author's attitude towards "most economists" can best be described as
   (A) Derogatory and antagonistic
   (B) Impartial and unbiased
   (C) Spiteful and envious
   (D) Critical and condescending
   (E) Indifferent

2. The author feels that price fixed by seller seems pernicious because
   (A) People don't have faith in large firms
   (B) People don't want the Government to fix prices
   (C) Most economists believe that price fixing should be in accord with free market
   (D) Most economists believe that no one group should determine price
   (E) People do not want to decide prices

3. Which of the following statements is definitely TRUE in the context of the passage? Price fixing is—
   (A) A profitable result of economic development
   (B) An inevitable result of the industrial system
   (C) The joint result of a number of carefully organized decisions
   (D) A phenomenon uncommon to industrialized societies
   (E) A result of joint venture of the Government and industry

4. According to the passage, price fixing in non-socialistic countries is generally—
   (A) Intentional and widespread
   (B) Illegitimate but beneficial
   (C) Conservative and inflexible
   (D) Legitimate and innovative
   (E) Conservative and scarce

5. What was the result of the then Soviet Union's change in economic policy in the 1970's?
   (A) They showed greater profits
   (B) They had less control over the free market

(C) They were able to adjust to techno advancement

(D) They acquired some authority to fix prices

(E) They became more responsive to free market

6. The author's primary concern seems to
   (A) Summaries conflicting viewpoints
   (B) Make people aware of recent discoveries
   (C) Criticize a point of view
   (D) Predict the probable result of a practice
   (E) Prepare a research proposal

7. Which of the following statements about the socialist industry is/are false?
   1. It has works under certain price restrictions
   2. It has no authority to determine price
   3. It hails the strategy of price fixing, as a major deviation

   (A) Only 1 is false (B) Only 2 is false
   (C) Only 3 is false (D) 1 and 2 are false
   (E) 2 and 3 are false

Answers

1. (B) 2. (C) 3. (B) 4. (A) 5. (E) 6. (A) 7. (C)

Passage - 10

Words–550

The public sector is at the cross roads ever since the launch of economic reforms programme in India. The pendulum has been swinging between survival and surrender. It is the result of a confluence of several factors : a shift in global economic environment, the emergence of the market economy and myths surrounding the performance of the public sector. So virulent has been onslaught that it is becoming axiomatic that by the very concept, the public sector is inefficient and resource waster whereas private enterprise is resource efficient.

The reform programme in India commenced with the policy of restricting of the public participation with the passage of time, the process of liberalization has shifted to privatization in a disguised form couched as strategic role. In the wake of the recent hot pursuit of the wholesale privatization programme a lively and poignant debate has emerged. It provides a golden opportunity to introspect and revisit the issue.

At the very outset, it must be made clear that in the worldwide-liberalized economic environment and very high stake of the state in most public undertakings disinvestments policy seeks to differentiate closed or bankrupt enterprise from the private sector–a fact deliberately overlooked by the champions of privatization. These undertakings need immediate attention. They are an unnecessary drain on the public exchequer. A high priority area for the disinvestments programme ought to be these enterprises but under one or the other argument these remain unattended, may be it involves a tough task. If these cannot be sold lock, stock and barrel asset stripping is the only option. Obviously the government cannot realize good price from these assets but their disposal will help stop the drain. If the assets are depreciated or become obsolete, then there is no point in holding on to them indefinitely and take to softer option of selling the vibrant and highly profit making organizations to reduce the budgetary deficit. Non-performers exist in both the sectors. Why condemn the public sector as whole? Better option will be closure or privatization of loss-making and non-viable units supporting PSU's which could be turned around and become healthy and viable and providing autonomy to the board of PSU's which are performing well and have potential to be globally competitive be welcome. With public participation in the PCU's there will be a good does of accountability in the system. What need to be reviewed are some basic issues; the priorities allocated to the enterprises. Selected for disinvestments, a comprehensive road map delineating the route, the modes and modalities timing and its consequences. These basic issues require greater discussion and participative decision making. In any event, the disinvestments programme in respect of the closed and non-revivable units is a must if the drain of further resources is to be prevented.

Let it be understood that PCU's are a big repository of value and it will take quite some time for privatisation programme to materialize despite the desire to expedite the process. Until then if a vacuums emerges attended by uncertainty it will do a great harm to the investments, which were made with such great dedication although desired now.

The government has withdrawn a budgetary support over the last decade. If some support is extended it is largely directed to closed or losing enterprise, which have no fortune.

1. The basic issue(s) requiring greater discussion and participative decision making regarding the disinvestments programme is / are—
   (A) The priorities allocated to the enterprises selected for disinvestments
   (B) A comprehensive road map delineating the route
   (C) The modes are modalities, timings and its consequence
   (D) All of these

2. "The public sector is inefficient and resources waster whereas private enterprise is resource-efficient". This opinion is due to—
   (A) A shift in global economic environment
   (B) The emergence of market economy
   (C) The myths surrounding the performance of the public sector.
   (D) All of these

3. The reform programme in India started with the policy of restructuring of PSUs has got shifted to
   (A) Liberalisation (B) Privatisation
   (C) Globalization (D) None of these
4. What was made with great dedication earlier, but now derided ?
   (A) Disinvestments
   (B) Investments in PSUs
   (C) Wholesale privatization
   (D) Strategic plans
5. According to the author, non performers exist in
   (A) Government
   (B) Public sector
   (C) Private sector
   (D) Public and private sectors
6. An appropriate title to the passage will be
   (A) "Disinvestments of PSUs"
   (B) "Economic Reforms Programmes in India"
   (C) "Liberalised Economic Environment"
   (D) "Non –performing Assets"

Answers

1. (D) 2. (D) 3. (B) 4. (B) 5. (D) 6. (A)

Passage - 11

Words–115

A book is written, 'not to multiply the voice merely, not to carry it merely but to perpetuate it. The author has something to say which he perceives to be true and useful or helpfully beautiful. So far he knows no one has said it, so far as he knows no one else can say it. He is bound to say it clearly and melodiously if he may; clearly at all events. In the sum of his life, he finds this to be the thing or group of things, manifest him; this, the piece of true knowledge, or sight, which his share of sunshine and earth has permitted him to seize. That is a book.

1. The opening sentence of the passage implies that the aim of writing a book is to—
   (A) repeat the message it contains
   (B) enable the author to express his ideas in writing
   (C) preserve from extinction the message it contains
   (D) propagate the ideology of the author
2. Which of the following would be the most suitable title for the passage ?
   (A) Contribution of an author
   (B) Aim of writing a book
   (C) Book–the source of true knowledge
   (D) Writers and their books
3. According to the writer, a person is impelled to write a book, because—
   (A) he wishes to satisfy his ego
   (B) he has something nice and pleasing to say
   (C) he is capable of expressing whatever he wants to say.
   (D) he has discovered something unique, true and good which he must convey distinctly and musically
4. Which of the following is not implied in the passage ?
   (A) A writer is motivated to write a book if he discerns a great truth
   (B) An author of a book generally gathers some common truths and gives them a popular and pleasing expression
   (C) A great writer is convinced that whatever he says is not an echo or imitation of what others have said
   (D) An eminent writer's message is conveyed through plain unambiguous language
5. Which of the following is opposite in meaning to the word 'manifest' given in the passage ?
   (A) Unclear (B) Dark
   (C) Pure (D) Hard

Answers

1. (D) 2. (C) 3. (D) 4. (B) 5. (A)

Passage - 12

Words–185

The second thing we must do is to observe the caution which John Mill has given to all who are interested in the maintenance of democracy, namely not to lay their liberties at the feet of even a great man, or to trust him with powers which enable him to subvert their institutions. There is nothing wrong in being grateful to great men who have rendered lifelong services to the country. But there are limits to gratefulness. As has been well said by the Irish patriot Daniel O'Connell, no man can be grateful at the cost of his honor, no women can be grateful at the cost of her `chastity, and no nation can be grateful at the cost of her liberty. This caution is far more necessary in the case of India than in the case of any other country. For in India, hero worshipping plays a part in our politics unequalled in magnitude by the part it plays in the politics of any other country of the world. In politics this hero worshipping is a sure road to degradation and to eventual dictatorship.

1. John Stuart Mill cautioned the lovers of democracy against
   (A) Subversion of democracy
   (B) Entrusting powers to even great men
   (C) To sacrifice their liberty
   (D) None of these

2. One should be grateful
   (A) To great men
   (B) To these who render service
   (C) To those who have long life
   (D) All the above
3. Gratefulness cannot be
   (A) Unlimited (B) Limited
   (C) Confined (D) None of these
4. One should be grateful without losing
   (A) Hon our (B) Chastity
   (C) Liberty (D) None of these
5. People pf India are
   (A) Honorable (B) Political
   (C) Hero-worshippers (D) None of these
6. Hero worshipping and politics are linked in
   (A) India (B) In day to day life
   (C) Daily routine (D) None of these
7. 'Magnitude' means
   (A) Degree (B) Intensity
   (C) Power (D) None of these
8. Hero worshipping always
   (A) Degree (B) Exalts
   (C) Degenerates (D) None of these
9. Hero worshipping often leads to
   (A) Dictatorship (B) Democracy
   (C) End of freedom (D) None of these
10. The whole passage refers to
   (A) Conditions for the success of democracy
   (B) Democracy
   (C) Democratic ideal
   (D) None of these

Answers

1. (C) 2. (A) 3. (A) 4. (D) 5. (C) 6. (A)
7. (A) 8. (B) 9. (A) 10. (A)

Passage 13

Words–722

Hyper markets take the up market route

Makrand Desh Panday (35) likes variety when he goes to a hyper market. "I'm a bit of an impulsive shopper and it puts me off if there's not enough variety. Good ambience even for grocery shopping is essential since I don't like being jostled around", says the associate VP at an MNC bank.

Desh Panday's pick in hypermarkets are Mumbai's HyperCity and Bangalore's Spar. "From exotic veggies in the fresh section to good deals in grocery and consumer durables to variety in clothes, these are places to go with my wife(Meenakshi) and sons on a weekend to stock up for the week ahead and treat ourselves," he says.

There's a growing up market population like Desh Panday that likes a hypermarket but without the chaos and milling crowds. And retailers are increasingly tailoring hypermarkets to suit this audience. The K. Raheja group launched HyperCity at Mumbai, Landmark group's Max Hypermarkets launched Spar in Bangalore, Tata's Trent now has three Star Bazaars in Ahmedabad, Mumbai and Bangalore and the RPG group is turning their large stores into the Spencer's Hyper models at Gurgaon and Mumbai.

"There can be only so many players in the hard core value space. There's no end to price slashing and it's not such a sustainable model in the long term," says Samar Sheikhawat, VP (marketing), RPG Retail. "So it makes more sense for hypermarkets to start segmenting themselves."

These hypermarkets offer value and deals, but not the jaw- dropping too- good- to- be- true variety. "When you offer potatoes or onions at ridiculous prices, you get a lot of customers only for those products. The shopfloor turns chaotic and you've got the customer but only till your competitor offers a better price. So we purposely don 't over-advertise our deals," reasons Vine Singh, MD of Max Hypermarkets "We choose to harbour customer loyalty by offering reasonable value and back it up with rich experience, variety, freshness and service."

There's a marked difference in the ambience at these hypermarkets targeting the SEC B to A audience when, compared to the price-driven ones like Big Bazaar, Vishal Megamart and Reliance Hyper Mart. Live bakeries, delis and cafes, wide variety in cheeses, cold cuts and meats and well-stocked wine sections are common features. Aisle spaces are wider, shelves are well lit, there's more focus on visual merchandising, staff tends to be better trained and the merchandising not just the

Branded goods, but even the private labels–has an upmarket look and feel and better variety.

There's greater attention to detail and these stores are better designed. For instance, there more cash tills to reduce waiting time-Spar has 25 cash tills on the foods floor alone.

"By spacing out aisles, it may seem like we're compromising on our per square foot productivity.

But at stores like ours, the per bill value tends to be much higher than price-driven hypermarkets," Points out Neeti Chopra, head of marketing, Trent.

The average value of the bill at premium hypermarkets is around Rs. 600 to Rs. 700 a figure, Which experts say; is at least 30% higher than the average per bill, value of price driven hypermarket, excluding special offer days.

These stores also ensure that they keep other sections well stocked since they offer higher margins compared to

grocery - at least 15%-25% on garments, 10%-20% on electronics, 10% on home decor and 30% on books and music. Even in the foods section traditionally considered a low margin business at 5%-10% profits are boosted by offering imported, organic and exotic ranges.

"These players will create niche micro markets among each other. But with good ambience becoming common, it's service that will become the next key differentiator, " says R. Kannan, president director of retail consultancy firm Ramms. "Wal Mart, for instance, has post offices, salons, movie ticket booking and pharmacies attached to their stores."

In India, services are picking up. RPG Retail has a bill-paying facility and tie-up With laundrettes in some of their stores. A couple of other retailers are planning to train shopfloor staff to take goods from the billing counter to the customer's vehicle. Some are planning to tie-up with multiplexes to offer booking counters and discounts on a certain amount of purchase. Most are also looking to offer co-branded credit cards with special benefits.

1. The most appropriate title for the passage would most likely be—
   (A) Hyper markets are booming
   (B) Hyper markets take the up market route
   (C) Hyper models are the upmarket
   (D) Hyper markets segmenting
   (E) Hyper models emerge in the retail scenario

2. Pick the correct options from the following statements as to what will eventually make the ultimate difference—
   1. Good ambience.
   2. Post office, saloons, movie ticketing bookings and pharmacies attached to their stores.
   3. Taking goods from the billing counter to the customers vehicle.
   4. Offering co-branded credit cards with special benefits.
   5. Efficient service will be the primary differentiator.

   (A) 1 only (B) 1 and 5
   (C) 3 and 4 (D) 1 and 5
   (E) 5 only

3. The author's attitude towards the trend can best be described as—
   (A) Optimism (B) Pessimism
   (C) Bigoted (D) Concern
   (E) Resignation

4. Accounting to the passage the primary difference between upmarket route and Hyper market routes are—
   1. Up markets don't have enough variety whereas Hyper markets do.
   2. Hypermarkets don't have chaos and milling crowds.
   3. Hyper markets offer too good to be true deals.
   4. Aisle spaces wider well lit shelves.

   (A) 1 and 3 (B) 2 and 4
   (C) 3 and 4 (D) 1 and 2
   (E) 1 and 4

Answers

1. (B) 2. (E) 3. (A) 4. (D)

Passage–14

Words–680

Old age, they say, is the most dreaded period of a person's lifetime. Illness and ailments become a part of daily routine. Depletion in the quality of vision and power of eyes is one of the first thing that comes with growing age. And cataract is the most widespread eye ailment which is common among all elderly people, but is not just restricted to them-youngsters too suffer from it.

Many technological breakthroughs have been made in the way cataract surgery is performed and it's getting better by the day. Anyone who suffers from this ailment can chose what kind of surgery one wants to undergo and the amount of money that they want to shell out.

The most recent breakthrough in the cataract surgery is the 1.8mm micro-incision surgery known as Stellaris-MICS. This high-end technology requires only a 1.8mm hole (or even smaller) in the cornea of the eye to pull out cataract and insert a new lens.

This technology has been launched in India at the same time as it has been launched internationally, which is generally not the case. Being a Third-World country and despite having some very talented scientists, India is still on the backburner of scientific inventions and development. But Stellaris-MICS was the brainchild of one of our eye surgeons from Chennia.

Dr. Amar Aggarwal, ophthalmic surgeon, thought the bi-manual way of surgery was a little complex which prevented it from being popular and suggested that there should be a machine that can perform suction and cleaning with the help of one tool and at the same time. This is how the idea for this outstanding surgical machine came into being, which was noticed by international companies like Bosch and Lomb who went on to manufacture it and train the ophthalmologists in its usage.

"The process of cataract surgery started with 12mm incision in the cornea, which is 12mm to 13mm wide. So, the cut went right across the eye because of which there were more chances of infection and a lot of post-surgery precaution were needed "explains Dr Samir Sud, eye surgeon.

Phaco, the technology to minimize the length of the hole in the cornea, then came as a welcome change. It is

used to break the cataract inside the eye and suck it out with the help of the needle. With this it became possible to bring down the size of the cut to 3mm, but still the fresh lens that is 5mm wide could not be inserted through that hole. So the incision had to be 5mm big

Then came the idea of making foldable lens which could be implanted through the smaller hole and then fitted in the eye. The 3mm hole became possible with the help of the intraocular micro incision lens. The surgery became a boon for the patients. Post-surgery precautions became a thing of the past. They could get back to their normal lifestyle within a day. But better was in the offing.

1·8 mm incision technique made it all the more patient-friendly, wherein the patient can even drive back to his destination only 15 minutes after the surgery has been performed. "I did not feel any irritation or that my eye had been tampered with. It's only much better. I can see everything properly with bright colours," says 75 years old Mohan Raj, who now wants to get his second eye operated too, so that he has a perfect vision.

"The reason for this quick recovery is that with the help of a smaller incision, the corneal tissue is being disrupted at all," says Dr Sud. Also a large part of the reasons for an improved vision depends on the Micro-Incision lens which is designed in a way the profession camera lens is. It has a single point focus and is aberration free, aspheric lens. The lens enables good colours, contrasts and hues improving the quality of vision, after the cataract almost blinds the patient.

Technology always leaves you wondering whether any thing better can ever come up to make the lives easier. And it always has a surprise in store.

1. The most appropriate title for the passage would be—
   (A) The cut gets tinier
   (B) Perfect vision is only a small a cut away
   (C) Cut and drive home brighter
   (D) A cut above the rest
2. The authors theme song is—
   1. Technological breakthrough have helped make a tedious and often painful operations, painless while cutting down the time involved to a great extent.
   2. The invention of the suck and clean surgical machine
   3. Technology and scientific inventions will improve still further and make our lives easier.
   4. The invention of Micro incision lens

   Chose the correct answer or combination—
   (A) 3 only (B) 3 and 4
   (C) 2 and 3 (D) 1 and 2
   (E) 1 and 4
3. The author introduces the following lines "With this it became possible to bring down the size of the cut to 3mm, but still the fresh lens that is 5mm wide could not be inserted through that hole" to—
   (A) a logical build up to show case astounding benefits of the latest cataract operation machine
   (B) to show the progress of science break through inventions in cataract surgery
   (C) to authenticate the invention of Micro incision lens
   (D) applaud Phaco, the technology to minimise the length of the hole in the cornea
4. The author in the concluding lines infers that—
   1. India has some very talented scientists
   2. Although India has talent but fails to excel on an international level
   3. India still lags behind in scientific inventions and developments
   4. The 12 mm incision technique was time consuming and often painful
   5. Technology will continue to leave us spell bound

   Chose the correct answer or combination :
   (A) 1 and 4 (B) 3 and 5
   (C) 2 and 4 (D) 5
   (E) 1 and 5

Answers

1. (D) 2. (D) 3. (B) 4. (D)

Passage–15

Words–150

American use archaisms such as the preservation of 'gotten' as the past participle of "get" 'fall' for autumn, 'aim to' for the English 'aim at' and faucet for tap. The word, politician, is used in a disparaging sense in America. Solicitor in America means a canvasser or visiting agent or beggar and the word clerk indicates a shop assistant, usually a female.

The use of the word cut meant for 'education' was originally American but became acceptable in good English largely because of the 1931 financial slump in Britain.

The Americans 'visit with' friends, in Britain one visits them. In Britain the word, welcome, finds use in this way 'anyone who cares to come will be welcome' or 'the guests were welcome by her. In the United States, anyone begging another person's pardon may receive the reply, "you're welcome" which in Britain would be considered a sign of ill breeding.

1. The American use ……… for autumn.
   (A) Past participle of get
   (B) Gotten

(C) Fall

(D) None of these

2. Americans use faucet for—

(A) Tap (B) Aim

(C) Gotten (D) None of these

3. The word 'politician' is used in—

(A) Bad sense in America

(B) Hateful sense

(C) Criticising sense

(D) None of these

4. The shop assistant in America is—

(A) Usually female (B) Clerk

(C) Good person (D) None of these

5. 'Visit with' the friends, in America, mean

(A) Visit them (B) Go with them

(C) See them (D) None of these

6. In America "You'are welcome" is said when a person is—

(A) Begging another person's pardon

(B) When a person is received

(C) When a person is insulted

(D) None of these

7. The meaning of 'ill breeding' is—

(A) Bad manners (b) Uncultured

(C) Uncivilized (d) None of these

8. Which of the following sentences use the word 'sign' correctly—

(A) Please sign this paper

(B) This is a sure sign of success

(C) He called him for sign

(D) None of these

Answers

1. (C) 2. (A) 3. (A) 4. (A) 5. (A) 6. (A)
7. (B) 8. (A), (B)

Passage–16

Words–354

The surge witnessed in mergers, amalgamations and take-over of companies during the past few years is indicative of the shape of thing to come. While these concepts are not new and were recognised even in the Companies Act of 1913 compulsion have undergone a dramatic change. In the past mergers and acquisition were used largely as an instrument for revival of sick units or for obtaining tax benefits. It was not uncommon for a business house to merge a sick company with a profit making one claim tax benefits.

The objective was not necessarily to achieve faster growth. The liberalization process witnessed during the late seventies and the eighties and particularly the relaxation of some of the restrictive provision of MRTP Act. and FERA, brought about a qualitative change in the mergers and amalgamations of companies. Even so, the incentive to grow was almost nonexistent and in fact some companies preferred to "demerge" by splitting one company into two or more so as to escape form the harsh provision of the MRTP Act.

The fast pace of liberalization since July 1991 and the time-bound programme of structural reforms under form the IMF and the World Bank have shaken the Indian industry form a slumber by exposing it to internal as well as international competition. Not surprisingly, the pressure is building up on every enterprise to mordernise and expand to cut costs. Gone are the days of the license and permit raj, high import duties and the prosperity guaranteed by a " sellers' market" with the rising threat of competitions and the "seller market" giving way to a "buyer market" in a large number of industries, the compulsion to look for economies of scale in production and cutting down the selling cost is increasing. Simultaneously, the virtual scrapping of the MRTP provision and relaxation in FER have removed the disincentive to grow. Hence mergers, amalgamations and take-over have assumed greater importance, mergers and acquisitions have now come to represent a short cut for companies to achieve accelerated growth. This is the trend world over and India cannot remain an exception as it moves towards globalization.

1. What was the motive of some companies to resort to demerger ?

(A) To boost their productivity and profitability

(B) To bypass the unfavorable legal provision

(C) To bring about qualitative changes

(D) To increase the number of companies under their

(E) None of these

2. The phrase "sellers' market giving way to Buyers' market" means—

(A) Increase in production is proportionate to the demand

(B) Increase in demand is disproportionate higher than the supply

(C) Market is financially in favor of consumers as compared to in the past

(D) Industry's profit margin is enhanced

(E) None of these

3. Which of the following is true about Indian Industry's scenario prior to July 1991 ?

(A) There had been pressures form the World Bank and the IMF

(B) It was exposed to severe competition on national and international fronts

(C) Liberalization process was at its peak

(D) Structural reform programme were planned and implemented

(E) None of these

4. Which of the following is not an outcome of Indian Industry's exposure to competition ?

(A) Switching over to expansion

(B) Reduction in selling cost

(C) Adopting new technologies

(D) Guarantee for Profits and Prosperity

(E) Need for obtaining licenses and permits

5. For which of the following were the amalgamations largely used in the past ?

(A) Saving on Taxes payable to the Government

(B) Forcing the Government to adopt liberalization process

(C) To achieve accelerated growth

(D) Overcoming the provision of revival of sick units

(E) None of these

6. The mergers of companies in the past and present doffer in respect of which of the following ?

(A) Tax benefits (B) Pace of growth

(C) Objectives (D) Modalities

(E) Profit percentage

7. Which of the following is true about Government's on import duties ?

(A) Import duty is lowered in order encourage imports

(B) Import duty is raised in order to discourage buying of foreign good

(C) Import duty is now lowered to encourage healthy competition

(D) Import duty is raised in order to earn substantial revenue

(E) Import duty is lowered in order to enable Indian industrialists to adopt foreign technology

8. The changing scenario as described in the passage is most likely to result into—

(A) Exorbitant profit margin to industry despite financial respite to consumers

(B) Reasonable profit margin to industry and marginally higher cost to buyers

(C) Marginal losses to industry and considerable benefits to buyers

(D) Adequate profit margin to industry despite lower prices

Shift of Govt. taxes form industry to buyers

9. The term "demerge" as used in the passage means

(A) Formulation of two or more companies out of an existing one

(B) Re-union of companies which had split up of one company

(C) Separation of two or more companies which had merged into one

(D) Renaming a company to claim tax benefits

(E) None of these

10. Which of the following inferences can be drawn form the passage?

(A) In the eighties, the change in FERA and MRTP provision the necessary impetus for growth

(B) FERA provisions were counter-productive to industrial growth earlier

(C) It is only the external financing agencies' pressure that has compelled Indian industry to adopt the present structure

(D) Most of the business house were not inclined to using the merger-tactics for revival of sick units

(E) None of these

11. Which of the following groups of statements is true in the context of the passage ?

Statement (i) : FERA and MRTP provisions were not conductive to industrial growth earlier.

Statement (ii) : Unlike in the rest of the world, in India merge of companies is a way to achieve accelerated growth.

Statement (iii) : The Indian industry shall have to find out profit sources other than form customers pocket.

(A) Only (i) and (iii) are correct

(B) All the three statements are correct

(C) Only (i) and (ii) are correct

(D) Only (ii) and (iii) are correct

(E) None of these three statements is correct

Answers

1. (B) 2. (C) 3. (E) 4. (E) 5. (A) 6. (B)
7. (C) 8. (D) 9. (A) 10. (C) 11. (A)

Passage–17

Words–1199

A lot of boom, a little gloom, but no doom. That perhaps sums up the trajectory of India's growth story so far, and which is also a fair commentary on what's in store in the coming days. For, nothing-not even higher interest rates, inflationary pressures and an appreciating rupee-appear potent enough to arrest the economy's growth, particularly investment growth of the corporate sector. Hence, by all reckoning, economic expansion should touch 8 % in the current financial year.

The growth momentum could be maintained beyond 2007-08. In any case, a lower than 7.5 % growth is unlikely, even if the agriculture sector's performance disappoints and corporate investments slacken.

While the Reserve Bank's monetary tightening, aimed at dousing inflation, and currency appreciation, triggered fears of an economic slowdown, confidence is intact. While export earnings are taking a hit, exports account for just about 12 % of the GDP. So while earnings growth of some companies may come under pressure, there is little concern about a slowdown in demand. In contrast the appreciating rupee may benefit companies that intend to import equipment for capacity expansion.

Belt Tightening-Just that

The monetary tightening over the last one year has not hampered demand for investment credit. Non-food credit grew by 28% in 2006-07 against 31.8% in 2005-06. The Reserve Bank of India hopes to moderate that growth to 24-25 % in the current financial year, even as it targets about 8.5% GDP.

"Interest rate hike has not impacted investment growth as yet," points out Manesh Vyas, Managing Director and Chief Executive Officer of Centre for Monitoring Indian Economy. He reckons that another round of increase in the interest rate may not have a negative impact on investment growth, as the consumer demand is very strong. "The recent hike in the cash reserve ratio is a lot more severe than all the measures taken by RBI earlier, and even these don't seem to be hurting credit growth rate," Vyas explains. So what are the chances of interest rates rising further? "Interest rate are close to peaking," declares DSP Andrew Holland, Merrill Lynch Managing Director.

But he cautions that inflation remains a risk and thus another increase of perhaps 25 bps in interest rates seems probable. A Deutsch Bank report on India published mid April too suggests that policy rates of RBI were near peaking. Holland believes globally, inflation would begin to cool off leading to a fall in interest rates. "We should see interest rate start to come down in the second half of the year in the US and Europe." In contrast, Standard & Poor's Chief Economist David Wyss is less optimistic about an easing of interest rates. "I don't expect any significant rate cuts this year. The US may cut by year-end, but Europe and Japan are still tightening." Wyss reckons the rate cuts may begin in 2010.

The monetary stance of the US Federal Reserve normally serves as a cue for other central banks. The Fed he held interest rates steady since Jun 2006 amid concerns of a slowing economy. In contrast, many central banks, including the Bank of England the European Central Bank and the Bank of Japan, have hiked rates over the last 10 months.

Opening The Flood Gates

The higher interest rate in India has attracted large inflows over the last few months. With RBI ceasing to buy dollars, given its attack on inflation, the rupee appreciated about 6 % in April. While this has given the jitters to exporters and companies with high overseas income, the appreciation is unlikely to hurt growth. "Recent history indicates that exports have grown even in the years the rupee has appreciated. There s not enough evidence to suggest the strengthening rupee would hurt growth, "point out vyas.

Holland adds that income growth of exporters and software companies may remain subdued, given the translation effect on earnings. According to Ashima Goyal, Professor at Indira Gandhi Institute of Development Research, "There has been no real slump in exports and most IT companies have shown better-than-expected results. An appreciation of the rupee means that there will be greater focus on the domestic market." Also, a stronger rupee would aid the country's infrastructure investment needs, Holland notes. Companies and project developers can access equipment and technology at a lower cost, at the same time capacity building would not stoke inflation.

There are other benefits too at the same time. The oil import bill would be lower, as a stronger currency would partly offset the increase in oil prices. Consequentially, the government would have to spend less on subsidizing diesel, Kerosene and liquefied petroleum gas.

However, there is no saying how oil prices will move in the months ahead. Much would depends on the geopolitical scenario. Holland does not expect any sharp movement in either direction, and estimates crude prices to be in the range of \$ 55 to \$ 65 a barrel. S & P's wyss puts it in a narrower band of \$ 60-65 a barrel. Nymex futures market forecasts oil price would rise to \$ 65.7 a barrel by July 2007.

Another area of concern is a possible slowdown in housing, caused by the rise in the cost of funds coupled with an escalation of property prices. but the housing story remains strong, given the employment growth in the services sector. Moreover, with speculation in the sector declining, buyers may find prices more realistic. Demand for housing loans remains high despite recent rate hikes, points out Vyas. A slowdown in housing could have dampened demand for cement and steel, but given the rising incomes, the fear may be far fetched.

The Party Poopers

The growth story is, however, not without any risk. For one, political indecisiveness has the potential to hamper growth. As Kristin Lindow, Vice President and senior credit officer of Moody's investor service, puts it : "India's continuing inflationary pressures and its widening external trade resemblances are emblematic of two of the economy's capacity constraints remain unresolved amidst

an obstructive political climate. The second issue relates to the insufficiency of fiscal adjustment, exacerbated recently by an overly accommodative fiscal stance that puts the onus of inflation control squarely on the Reserve Bank." Infrastructure deficit and slow progress on addressing the problem would be another issue. " The rate of growth was running way ahead of its potential," states Yes bank's Chief Economist Shubhada Rao. Agriculture output would remain a concern, given that the sector supports nearly 70% of the population but contributes less than 20% to GDP. However, companies are going ahead with their capital expenditure plans, a reflection that investment growth is strong. "with the investment to GDP ratio hovering at 33%, sustaining a 9% growth is not an issue," explain Goyal.

Supplementing the domestic sources of investment is the strong inflow of foreign direct investments. "we should get about $15 billion in foreign direct investment this year, and about $20 billion next year," notes Holland. Further, the retail boom would give a thrust to growth. Also, the government's expenditure is unlikely to decelerate as elections are less than two years away and the popularity of the PA- led government is waning. That would provide some impetus to growth.

1. The most appropriate title for the passage would most likely be:
   (A) Economy surge
   (B) Rising higher and swifter
   (C) Boom ,boom on the rise
   (D) The sky's the limit
   (E) No slowdown in sight

2. According to the author which of the following reasons are no cause for alarm to the ever rising boom—
   (A) High interest rates
   (B) The appreciating rupee
   (C) Pressures of inflation
   (D) Shortfall in FDI and FII's
   (E) All of the above

3. According to the author what could cause the fall in interest rates in the near future as started by Holland—
   (A) Depreciation of the rupee
   (B) A surge in FDI's
   (C) US federal interest rates have been steady which would signal global inflation rates will cool off
   (D) Bank of Japan will raise interest rates further
   (E) None of the above

4. The tone of the author in the passage can best be described as—
   (A) Canteous (B) Pessinistic
   (C) Encouraging (D) Didactic
   (E) Pragnatic

Answers

1. (E) 2. (E) 3. (C) 4. (C)

Passage–18

Words–925

Our parliamentary system has created a unique breed of legislator, largely unqualified to legislate, who has sought election in order to wield (Or influence) executive power. It has produced governments more skilled at politics than at policy or performance. It has distorted the voting preferences of an electorate that knows which individuals it wants but not necessarily which policies.

It has permitted parties that are shifting alliances of individuals rather than vehicles of coherent sets of ideas. It has forced governments to concentrate not on governing but on staying in office, and obliged them to cater to the lowest common denominator of their coalitions. It is time for a change.

The fact that the principal reason for entering parliament is to attain governmental office poses two specific problems. First, it limits executive posts to those who are elect able rather than to those who are able. The prime minister cannot appoint a cabinet of his choice; he has to cater to the wishes of the political leaders of 20 parties. Second, it puts a premium on defections and horse-trading. The Anti-Defection Law of 1984 was necessary because in many states (and, after 1979, at the Centre) parliamentary floor crossing had become a popular pastime, with lakhs of rupees, and many ministerial posts, changing hands. Now, musical chairs is an organized sport, with "party splits" instead of defections, and for much the same motives. I shudder to think of what will happen after the next elections produce a parliament of 40 odd parties jostling to see which permutation of their numbers will get them the best rewards.

The case for a presidential system of either the French or the American style has, in my view, never been clearer. The French version, by combining presidential rule with a parliamentary government headed by a prime minister, is superficially more attractive, sine it resembles our own system, except for reversing the balance of power between the president and the council of ministers. This is what the Sri Lankans opted for when they jettisoned the British model.

But, given India's fragmented party system, the prospects for parliamentary chaos distracting the elected president are considerable. An American or Latin American model, with a president serving both as head of state and head of government, might better evade the problems we have experienced with political factionalism. A directly-elected chief executive in New Delhi, instead of being vulnerable to the shifting sands of coalition, support politics, would have stability of tenure free from legislative whim, be able to appoint a cabinet of talents, and above all, be able to devote his or her energies to

governance, and not just to government. The Indian voter will be able to vote directly for the individual he or she wants to be ruled by; and the president will truly be able to claim to speak for a majority of Indians rather than a majority of MPs. At the end of a fixed period of time -let us say five years- the public would be able to judge the individual on performance in improving the lives of Indians, rather than on political skill at keeping a government in office. It is a compelling case.

Why; then, do the arguments for a presidential system get such short shrift from our political class ? At the most basic level, our parliamentarians' fondness for the parliamentary system rests on familiarity: this is the system they know. They are comfortable with it, they know how to make it work for themselves, they have polished the skills required to triumph in it. Most non politicians in India would see this as a disqualification, rather than as a recommendation for a decaying status quo.

The more serious argument advanced by liberal democrats is that the presidential system carries with it the risk of dictatorship. They conjure up the image of an imperious president, immune to parliamentary defeat and impervious to public opinion, ruling the country by fiat. Of course, it does not help that Mrs Gandhi, during the Emergency, contemplated abandoning the parliamentary system *for* a modified form of Gaullism, thereby, discrediting the idea of presidential government in

many democratic Indian eye: Mrs Gandhi is herself the best answer to such fears: she demonstrated with her Emergency rule that even a parliamentary system can be distorted to permit autocratic rule Dictatorship is not the result of a particular type of governmental system.

In any case, to offset the temptation for a national president to become all-powerful, and to give real substance to the decentralisation essential for a country of India's size, an executive chief minister or governor should also be directly elected in each of the states. The case for such a system in the states is even stronger than in the Centre. Those who reject a presidential system on the grounds that it might lead to dictatorship may be assured that the powers of the president would thus, be balanced by those of the directly elected chief executives in the states.

Democracy, as i have argued elsewhere, is vital for India's survival: our chronic pluralism is a basic element of what we are. Yes, democracy is an end in itself, and we are right to be proud of it. But few Indians are proud of the kind of politics our democracy has inflicted upon us. With the needs and challenges of one-sixth of humanity before our leaders, we must have a democracy that,

delivers progress to our people. Changing to a presidential system is the best way of ensuring, a democracy that works.

1. According to the author which of the followings statements can be deemed to be true ?
   (A) India's parliamentary system require no change
   (B) India's parliamentary system needs to switch over to a different system of legislature
   (C) The Parliamentary system is perfect. However its implantation lacks either honest or unqualified. (d) The French or American Presidential system is clearer for India.
   (E) All of the above
2. The author suggests that—
   (A) The French version of governance is suitable for India.
   (B) India counter part legislature in England is a perfect model to ape.
   (C) An American or Latin American model might serve us better
   (D) French or American system suits India best
   (E) None of the above
3. The attitude of the passage is—
   (A) Critical of India's legislation
   (B) Appreciative of India's legislation
   (C) Cynical of India's legislation
   (D) Dissatisfied with India's legislation
   (E) Confused with India's legislation
4. The loopholes and shortcoming of India's parliamentary system is due to—
   (A) The ruling party having to cater to the demands of its coalition partners
   (B) The P.M. cannot appoint a cabinet of his preference
   (C) Floor crossing and horse trading have immense value
   (D) Most part of constitution builts by British
   (E) All of the above

Answers

1. (B) 2. (E) 3. (D) 4. (A and C)

Passage–19

Words–924

Happiness is a Warm Sun

I watched the rerun of a BBC documentary, called The Happiness Formula, over the week end. It delved into scientific research on the subjects and noted how close we were actually to quantifying happiness in individuals I froze in anxiety as the show ended.

Would doctors be telling us soon how happy we were? According to the documentary quite possibly. And poof would go a chunk of our individual right to examine

and decide how exactly we felt on a particular day. Our moods would fit into a formula devised by scientists, who would blithely define what it meant to be happy, sad or otherwise.

Mind you, the scientists are doing astounding work. The documentary showed a series of baby faces in various stages of emotion some were radiating delightfully toothless grains, others were puckered in distress. Each of those moods had been closely scanned by electrodes to produce changing colours in various parts of their brains. The happy babies showed distinctly different, and apparently quantifiable, blobs of blue in the area of the brain where happy moods reside you can are or so the scientists how happy you are or so the scientist claimed adding they were on the job of precisely measuring happiness.

Well, good luck, men in white. You are not going to crowd my skull with electrodes any day soon to tell my mood. And you are not going to define for me what it mean to be happy at a particular point of time, on a particular day, in a particular situation and in a particular place, I will decide for myself thank you; or may be my mercurial temperament will. And, will you measure me after stuffing me full of chocolates or will you deprive me of the one human invention that can make me hit instant high? Besides, do you know we sometimes smile when we're actually fuming inside? I do often it's stressful but controlling one's temper with a smile is a wise forward deal on long-term happiness.

I am happy at new year's eve parties, positively jolly, as such occasions demand but to raise another problem the degree of happiness can very from year to year. And, if you measure my mood in the middle of the festive hours, you might find the happiness area colourful, but with a marked decrease in intensity as evening wears into the late night. The next morning could show a brainful of remorse at having indulged too much for too long. I crawl out of bed swearing never again until the next party that is. So, what do you measure, and exactly when and where, if you want to declare me happy or unhappy ?

But common sense arguments never stopped scientists, Neurologists, Psychiatrists and geneticists will carry on turning common sense on its head as indeed they must if they are true scientists. And now political scientist and social psychologist have joined the band they have gone one gone beyond to classify whole nations in a global happiness index

Researchers at Britain's university of Leicester used a battery of statistical data and surveyed 80,000 people worldwide in 2006 to chart the state of happiness in 178 countries. The happiest nations ? Denmark and Switzerland, which tied for first place. The most unhappy ? Zimbawe and Burundi. The united states came in 23rd in the list, even though it was the first republican democracy in the world to incorporate the pursuit of happiness in its constitution as a worthy national goal.

You might assume we in India are a pretty happy bunch going by the delirious, musical acrobatics that inundate our TV screens day in and night out or by the sheer number of supposedly happy festivals we celebrate year in and year out. Well, India fingers a lowly 125th on the index. If that makes you sad, you can feel happy by looking at Pakistan, which is 166th ,just step above Russia. But then again, China is higher at 82nd.

Interestingly, the countries that fare well on the index are almost all well–off. Being healthy and wealthy boosts national moods. But does it make people wise? If being healthy and wealthy made nations wise, the Iraq war may not have happened capitalism however is a winner all the nations listed high on the index are market economics.

Democracy, curiously does not seem to feature in the global happiness equation, much to the chagrin of us liberals who value freedom - of expression, in particular as a key ingredient of any national happiness formula. Although most national in the top 50 in the list are democracies countries like Saudi Arabia, Brunei and kuwati are also up there, underscoring the wealth factor but raising doubt about popular yearning for freedom.

The one intriguing entry in the list, at number 8, is the happy state of Bhutan. It is neither wealthy nor is it a market economy. But it is the only country which officially measures its well being not by GDP alone but also by a national happiness index. Bhutan's culture minister explained on the BBC show people were kept happy by strict government supervision of what they could see on TV and what outside influence would be allowed into the landlocked kingdom. Clearly, if you don't let people know what excitement lies out there in the wide world, ignorance can indeed be bliss.

I, on the other hand, would rather be sad knowing thing out there under the warm sun then be happy in plato's cave.

1. The author makes which of the following critical of scientists research into qualifying happiness ?
    1. That a huge amount of one's individual also right to decide on how one feels would be usurped.
    2. Neither wealth nor economy play an important role in the happiness quotient.
    3. Controlling one's anger by masking it with an appearance of approval is wise on the term happiness scale.
    4. The degree of happiness various from time to time. So, when, where and what does one measure to slot one as happy or unhappy
    5. Happiness cannot be put to a scale.

    (A) 1 and 2 (B) 3 and 4

    (C) 1 and 4 (D) 1 and 5

    (D) 5 and 3

2. The author state all of the following argument against scientific research to quality happiness in individuals EXCEPT—
   (A) An individual's right to decide one's feelings can be dented
   (B) One cannot enjoy festivals as such occasions demands
   (C) The degree of happiness varies from year to year
   (D) Democracy does not feature in the global equation of happiness
   (E) Happiness can be defined by scientists
3. The author attitude can be best described as one of:
   (A) Approval (B) Cynicism
   (C) Anger (D) Denial
   (E) Criticism
4. The most appropriate title for the passage would undoubtedly be—
   (A) Transferred happiness
   (B) Cloned happiness pines
   (C) Programmed happiness
   (D) Happiness on call
   (E) Happiness a warm sun.

Answers

1. (D) 2. (B) 3. (E) 4. (B)

Passage–20

Words–385

A recent report titled 'Women and Children in India' and a nearly simultaneous report on 'Indian Women-their health and economic productivity' highlight the international importance being given to the problems of the progress of Indian women.

Unfortunately, both the report fail to either convey a fresh understanding of the issue or to suggest innovative and workable approaches for women's development. The former is because the reports restrict themselves to old statistics. We are told for example, that the ratio of women to men remains below its natural level ant that it is related to other parameters, such as the level of literacy and the availability of primary health care.

The failure to suggest a fresh approach is less understandable given that a variety of approaches to women's development has been tried out all over the world. In India, the state has made a strong political and economic commitment to women. In 1990 the National Commission of Women was established. In 1989 two major policy documents were released. Ever since the Sixth-Five plan, there has been a special section on socio-economic programmes for women. Poverty alleviation programmes have a 30 percent target for women. Special programmes like Development of Women and Children in Rural Areas have also been started.

These governmental and other approaches have had mixed results. For example SEWA's (Self Employed Women's Association) and WWF's (Working Women's Forum) approach of using bank credit to organize self-employed women has worked well. So also women's literacy programmes in Kerala. On the other hand, some of the legislated reforms like dowry prohibition have not worked well. The issue of whether women should be identified as separate participants in the development process is also controversial.

In this context, it is disappointing to read in one of the reports that one of the government's targets for 1991-95 ought to be foster "an ethos of caring in the community, not to let a child go to bed hungry, be subjected to a preventable disease or remain without learning opportunity this could mark the beginning of a social process towards a more humane order." Is the report saying then the Indians do not love their children as a matter of policy makers. It is very unfortunate that these reports have become superficial and does not delve sufficiently deep into the problems.

1. Which of the following is correct in the context of the passage in regard to development schemes ?
   (A) All people are the opinion that women should be given special status
   (B) Some people hold the opinion that women should not be given special status
   (C) No one feels that women should be given special status
   (D) There is no controversy regarding women being given special status
   (E) None of these
2. Which of the following is specifically put forward as a reason for the unequal sex ratio in the population ?
   (A) Natural causes
   (B) Economic development
   (C) Literacy level
   (D) Socio economic programmes
   (E) Poverty alleviation programme
3. Which of the following is not put forward by the author of the passage as a defect of the reports ?
   (A) They highlight the problems of Indian women
   (B) They contain outdated statistical data
   (C) The reports have not suggested any new measures
   (D) Some of them suggested new measures
   (E) The report are not analytical
4. Which of the following is true in the context of the passage ?
   (A) Culturally, Indians do not love their children
   (B) All developmental programmes and schemes are generally a success

(C) Indian women do not manage their finances well

(D) Women literacy programme have failed in India

(E) Some developmental programme have mixed result all over India

5. Which of the following has not been mentioned as necessary, in one of the reports , to start new social order ?

(A) Prevent hunger of children

(B) Prevent disease in children

(C) Increase educational opportunities

(D) Community care of children

(E) All of these have been mentioned in the report

6. Why does the author say that the reports are unable to convey a 'fresh understanding of the issues' ?

(A) The problem are dealt with superficially

(B) Old data are used in the reports

(C) Some of the development schemes are not dealt with

(D) Indian problems cannot be understood by foreigners

(E) None of these

7. Which of the following is conveying the same meaning as the 'target' as is used in the passage ?

(A) Aim (B) Margin

(C) Attempt (D) Schemes

(E) Quota

8. Which of the following is conveying the same meaning as the word 'mark' as is used in the passage ?

(A) Symbol (B) Sign

(C) Signal (D) Notice

(E) Start

9. Which of the following is false according to the passage ?

(A) The problem of women in general is attention the world over

(B) National Commission of Women is an attempt for women's development

(C) Indian people love their children and try to take care of them

(D) Right from independence women have been receiving special attention for development

(E) Care of children is an essential starting point for social development

10. Why has, according to the passage the legislation prohibiting dowry failed in India ?

(A) Due to the cultural ethos in the country

(B) Because of the fact that it was passed as law

(C) No social organization were involved in this

(D) Reason not mentioned in the passage

(E) Literacy rate among women is poor

11. Which of the following would indicate the same meaning as the phrase "go to bed hungry" as it has been used in the passage ?

(A) Malnutrition (B) Insomnia

(C) Famine (D) Undernourishment

(E) Unsatisfied

12. Which of the following would correctly reflect the position of the author of the passage to the two report mentioned in the first paragraph ?

(A) Evaluative (B) Critical

(C) Neutral (D) Praiseful

(E) Appreciative

Answers

1. (B)	2. (C)	3. (A)	4. (E)	5. (C)	6. (B)
7. (A)	8. (C)	9. (D)	10. (D)	11. (E)	12. (B)

Passage–21

Words–249

It is strange that, according to his position in life, an extravagant man is admired or despised. A successful business man does nothing to increase his popularity by being careful with his money. He is expected to display his success, to have a smart car, an expensive life, and to be lavish with his hospitality. If he is not so, he is considered mean, and his reputation in business may even suffer in consequence. The paradox remains that if he had not been careful with his money in the first place, he would never have achieved his present wealth.

Among the low-income group, a different set of values exists. The young clerk, who makes his wife a present of a new dress when he hasn't paid his house rent, is condemned as extravagant. Carefulness with money to the point of meanness is applauded as a virtue. Nothing in his life is considered more worthy than paying his bills. The ideal wife for such a man separates her housekeeping money into joyless little piles so much for rent, for food, for the children's shoes; she is able to face the milkman with equanimity every month, satisfied with her economising ways, and never knows the guilt of buying something she can't really afford.

As for myself, I fall into neither of these categories. If I have money to spare, I can be extravagant, but when, as is usually the case, I am hard up, then I am the meanest man imaginable.

1. Which of the following would be the most suitable title for the passage?

(A) Extravagance is always condemnable

(B) Extravagance leads to poverty

(C) Extravagance in the life of the rich and the poor

(D) Miserly habits of the poor

2. In the opinion of the writer, a successful businessman
   (A) should not bother about popularity,
   (B) is expected to have expensive tastes
   (C) is more popular if he appears to be doing nothing
   (D) must be extravagant before achieving success
3. The phrase ‘lavish with his hospitality' in the third sentence of the first paragraph, signifies
   (A) considerateness in spending on guests and strangers
   (B) indifference in treating his friends and relatives
   (C) miserliness in dealing with his friends
   (D) extravagance in entertaining guests
4. The word 'paradox' in the last sentence of the first paragraph means
   (A) statement based on the popular opinion
   (B) that which is contrary to received opinion
   (C) statement based on facts
   (D) that which brings out the inner meaning
5. It seems that low paid people should
   (A) feel guilty if they overspend
   (B) borrow money to meet their essential needs
   (C) not keep their creditors waiting
   (D) not pay their bills promptly
6. How does the housewife, described by the writer, feel when she saves money ? She.....
   (A) wishes she could sometimes be extravagant
   (B) is still troubled by a sense of guilt
   (C) wishes life were less burdensome
   (D) is content to be so thrifty
7. The statement "she is able to face the milk man with equanimity" implies that
   (A) she is not upset as she has been paying the milkman his dues regularly
   (B) she loses her nerve at the sight of the milk man who always demands his dues
   (C) she manages to keep cool as she has to pay the milkman only a month's dues
   (D) she remains composed and confident as she knows that she can handle the milkman tactfully
8. Which of the following is opposite in meaning to the word 'applauded' in the passage ?
   (A) suppressed (B) cherished
   (C) decried (D) humiliated
9. We understand from the passage that—
   (A) thrift may lead to success
   (B) wealthy people are invariably successful
   (C) all mean people are wealthy
   (D) carefulness generally leads to failure
10. As far as money is concerned, we get the impression that the writer—
    (A) doesn't often have any money to save
    (B) would like to be considered extravagant
    (C) is never inclined to be extravagant
    (D) is incapable of saving anything

Answers

1. (C)	2. (B)	3. (D)	4. (B)	5. (A)	6. (D)
7. (A)	8. (C)	9. (A)	10. (A)		

Passage–22

Words–66

The recent rapid growth of industry has, in some cases, been so excessive that too much manufacturing capacity has been developed in some fields of production, which forces companies to sell their surplus products in world markets at prices lower than normal. This will make it almost impossible to develop local industries producing the same items because consumers will prefer to buy the cheaper' imported product.

1. Why is it necessary for companies to sell products at cheaper prices ?
   (A) The cost of production has been considerably low.
   (B) The local industries also manufacture the same product.
   (C) There is a heavy demand for these products.
   (D) The demand has been lowered significantly.
   (E) None of these
2. According to the passage, the situation resulting from the rapid industrial growth is
   (A) favourable to the manufacturers
   (B) disastrous to the exporters
   (C) conducive to the growth of local industries
   (D) unfavourable to the consumers
   (E) None of these
3. 'This will make' in this sentence, 'This' refers most closely and directly to—
   (A) Development of local industries
   (B) The recent rapid growth of industry
   (C) Selling products of excessively higher prices
   (D) Companies manufacturing surplus products
   (E) Companies selling their production at cheaper prices

4. Which of the following is/are most likely to hamper the development of local industries ?
   1. Availability of imported product at cheaper rates
   2. Consumer's tendency to refrain from using imported products
   3. Excessive production capacity and low production cost

   (A) Only A (B) Only B
   (C) Only C (D) A and B
   (E) A and C

5. 'Imported product' as used in the last product refers to'—
   (A) product manufactured locally but of export quality
   (B) product sold to such, other country which can't locally manufacture it,
   (C) product of a foreign country available at a below normal price
   (D) surplus product manufactured by foreign country and sold at a normal price

Answers

1. (E) 2. (E) 3. (B) 4. (E) 5. (C)

Passage–23

Words–169

Liberty is not a personal affair only, but a social contract. It is an accommodation of interests. In matters which do not touch anybody else's liberty of course, I may be as free as I like. If I choose to go down the Strand in a dressing-gown, with long hair and bare feet, who shall say me nay, You have liberty to laugh at me, but I have to be indifferent to you. And if I have a fancy for dyeing my hair. Or waxing my moustache (which heaven forbid), or wearing a tall hat, a frock coat and sandals, or going to bed late or getting up early, I shall follow my fancy and ask no man's permission. I shall not inquire of you whether I may eat mustard with my mutton. And you will not ask me whether you may follow this religion or that, whether you may marry the dark lady or the fair lady, whether you prefer Longfellow to Wordsworth, or champagne to coca cola.

1. 'Not a personal affair' means—
   (A) Not enjoyed in person
   (B) Not having individual affair
   (C) Not concerned with individuals
   (D) None of these

2. 'Social contract' implies—
   (A) A contract for the sake of society
   (B) A contract among the members of society
   (C) A contract of social interests
   (D) None of these

3. "Accomation of interest" means—
   (A) A place for living of interests
   (B) Adjustment of individual interests
   (C) Adjustment of personal ends
   (D) None of these

4. One may be as free as one likes in—
   (A) Matters which do not encroach on 'another's liberty
   (B) Laughing at others
   (C) Laughing at oneself
   (D) None of these

5. "Indifferent to" can be replaced by—
   (A) Having no interest in
   (B) Unconcerned about
   (C) Not worried about
   (D) None of these

6. The word 'fancy' has been used twice in the sentence number 4. It means—
   (A) Liking; liking
   (B) Imagination; Imaginative
   (C) Liking; whim
   (D) None of these

7. 'Inquire of you' means—
   (A) Inquire form you
   (B) Inquire about you
   (C) Inquire into your views
   (D) None of these

8. 'Dark' is contrasted with the word ………. in the last sentence
   (A) Prefer (B) Whether
   (C) Fair (D) None of these

9. 'Prefer' takes the proposition ……… as is clear form the passage.
   (A) To (B) On
   (C) With (D) None of these

10. 'Champagne' is a type of—
    (A) Cold drink (B) Soft drink
    (C) Wine (D) None of these

Answers

1. (C) 2. (B) 3. (B) 4. (A) 5. (B) 6. (C)
7. (A) 8. (C) 9. (A) 10. (C)

Passage 24

Words–577

Naseema Begagum, a government school teacher in Karachi's middle income Garden Road locality, is not a happy woman. She has been assigned election duties in Karachi South, one of the city's more troubled areas.

Here the fight will be between Altaf Hussain's Muttahida Qaumi Movement (MQM) and Benazir Bhutto's Pakistan Peoples Party. Naseema Begum and other teachers at the Government Girls Secondary School on Garden Road say that it is the polling agents who have to bear the brunt of election violence. "Hooligans from one party came in when I was on duty and pointed a gun at me and told me to get lost. It was so humiliating," she says, recalling a previous election.

Naseema believes that the MQM holds sway in most parts of Karachi, but points out that trouble arises in those constituencies where there is opposition. On previous occasions, the government has deputed the army and paramilitary Rangers force to protect polling stations but this time when polling takes place, the government has instead deputed civilians like firemen, civil defence volunteers and private security guards.

But the teachers are not impressed. "*Yeh aik chamaat kay hain*" (They are worth one slap) comments Muhibba Bano, a teacher also doing poll duty in January.

Bano says that they feel insecure when local police take charge as they run away at the slightest hint of trouble. "In fact they are also part of the rigging when it takes place," she alleges. Government employees complain that they are forcibly drafted to election duties without any proper arrangements. "We get very little allowance, no transport and no security," comments Bano.

This time round, they say, there is another danger because of which many are being advised to stay away despite strict government instructions to be present at their stations on polling day. The danger comes from religious militants who have threatened to blow up polling stations, particularly in the troubled tribal areas.

President Musharraf has alluded to this threat in his speeches and vowed that the democratic process "will not be hijacked by those who believe in extremism." However, the rise in suicide attacks over the past week have again raised fears of election related attacks.

The attack on the welcome rally of Ms. Bhutto in October 07 in which hundreds died is fresh in the minds of people. "we are afraid of what can happen at rallies," says Nasheed Ahmad,

a resident of Rawalpindi Who said he did not go to the airport to receive Ms Bhutto as he feared a possible suicide attack. "I cant help it," he says, adding "my family members do not let me go." That is one reason why election rallies are poorly attended, say observers.

The government has also issued a code of conduct under which political rallies can only beheld in the daytime. "Our priority in the elections is to ensure law and order," comments Brigadier Akhtar Zamin, the home minister for Sindh province where the most largely attended rallies are those in which Ms Bhutto is the main speaker.

However, a recent rally in Hyderabad was a disappointment. In such uncertain times, there are expectations that voter turnout will be much lower than expected. The boycott by main religious parties will lead to lower voter turnout. Despite all the hype created by major politicians that this may be the election that will decide the future of President Musharraf, the chances are most Pakistanis will stay away from the polls.

1. Which of the following headings can aptly sums up the passage ?

   (A) Terror stalks Pak polls

   (B) Pak polls in the eye of terror

   (C) Bullet not ballot will decide Pak poll fate

   (D) Pak polls under terror shadow

   (E) Pakistanis are disillusioned by past and present political scenario

2. According to the author low voter turnout will be due to—

   (A) Booth rigging

   (B) Fear of terror attacks and boycott by main religious parties

   (C) Lack of voters faith in political parties

   (D) Fear of religions ostracise and repercussion

   (E) because Government employees not get proper allowances and security

3. Polling agents fear for their safety because of the following reason—

   (A) Inadequate security

   (B) Opposing political cadre clashes

   (C) Intimidation by hooligans

   (D) Terror attacks

   (E) none of these

4. The writer introduces which of the following to support the argument of polling agents ?

   (A) Examples

   (B) past history of Pak

   (C) News clips

   (D) premise

   (E) Interviews and number of terrorist attacks

5. The author is most likely to sympathizes with which statement—

   (A) A few Pakistani will turn up for voting

   (B) The majority of Pakistanis will not exercise the right to vote

   (C) A fifty percent turnout is a fair assumption

   (D) Pakistanis will in masse boycott the polls

   (E) A huge voter turnout, say about 70%

Answers

1. (D) 2. (B) 3. (A) 4. (E) 5. (B)

Passage–25

Words–627

Year of Experiments and 'Neo-Wave' Cinema

The year goes down in Bollywood history as the 'year of living Dangerously'. Forget the regular hits like Om Shanti Om, Namastey London, Partner and Heyy Babby. These were the usual blockbuster that one find in Bollywood annual roster, year after year. What made 2007 special was the spirit of Bollywood. Like a new –age Columbus, it embarked boldly on a voyage into the unknown, caring little about the common concern of cash inflows and revenue curves.

The idea was the lodestar which guided this doughty ship into unchartered terrain, choppy waters and swirling whirpools. And the end result of all this experimentation was a neo- wave cinema that boasted of films like Bheja fry, Black Friday, Parzanta, Johny gaddar, Manorama six feet under and perhaps, the biggest venture of them all, Taare zameen par.

Think about it and you will realize that even the two biggies that charmed your heart chak de India and Jab we met were formula busting films. Chak de struck and instant chord and became the national slogan with its underplayed celebration of patriotism, only because this brand of desh bhakti was so different from the chest beating, flag waving nationalism of films like Border, Gaddar and even Lagaan. Similarity jab we met turned the Dilwale Dulhaniya le jayenge formula on its head, merely because its lead characters streered clear of the stereotype, as did the dhamakedar dialogue which the daily banter of today's youth

Films like Dharma,Black Friday and Parzania displaed the power of docufeatures and showed how cinema could still sell soul stirring messages about peace, humanity, oneness and tolerance. Bheja fry lead the brat pack with its attempts to redefine comedy completely and make the laugh act brain teasing rather than rib tickling. Of course, India still laughed on the banana-peel skids (partner, Heyy Babby, Dhamaal and Dhol)But the Rajat Kapoor Vinay Pathak banter gave the comic a cutting edge satire and black humour.

But the real teasers of the maverick bunch were two films which made the criminal king. Johny gaddar and shootout at lokhandwala stood out for their ekdum human insights into evil, making the anti hero alluring, yet wicked.

Surely, Vivek Oberoi's crook act in shootout at Lokhandwala and Neil Mukesh's Johny gaddar are the two most mesmerising character of the year in the 'ignoble' class, even as Shah Rukh khan's coach Kabir Khan in Chak De India and Aamir Khan's Nikumb sir in Taare zameen par are the hottest characters in the 'noble' category.

The two managed to leave a lasting impact, despite doing away usual romantic track that Bollywood heroes can't seem to do without. In the girlie category, our vote for hot and happening would go to the chak De girls who showed what woman power was all about; and then for the two debutants, Deepika Padukone and Sonam Kapoor who matched dew fresh charisma with Ranbir Kapoor and Neil Mukesh, the male debutants of the year.

Watch out for them in the next two years, they will be heading the power list soon. But only next to the brand new hero of Bollywood: Innovation, experimentation, ideation.

And how do we close account without gleefully anointing the duhs! Of the year. Leading the sad pack is indeed Ram Gopal Verma ki Aag which enunciated how not to remake a classic, followed by Nikhil Advani's Salaame-e-Ishaq and Shaad Ali's Jhoom Baraber Jhoom ,two over hyped films that bit the dust, despite the stars and the reputed directors.

Remains of the day? Year 2008 is going to be tough formula won't work taking the audience for granted will be suicidal, adventurism will be the must have attitude. Truly a year to find out: Bollywood *tussi* great ho or nahin.

1. The main theme of the passage can be summed up by which of the following—
   (A) block buster are an annual feature
   (B) Hits like 'Om Shanti Om' 'Namaste London' and Partner tell a big story
   (C) New age entrepreneurs step into unknown frontiers
   (D) Popular actors like Amitabh Bachan Hrithik Roshan Shah Rukh Khan are respectively for hits
   (E) Living on conservative lines accounts for blockbusters
2. By these lines in the 2nd paragraph, "this doughty ship into uncharted terrain and swirling whirlpool" the authors implies—
   (A) There are fluke films that made it big
   (B) These brand of films are bold and neo wave cinema which will sparkle and pioneer a new trend of cinema that are different and ground breaking.
   (C) Though innovative cannot set a precedent
   (D) Wishful thinking to cash in on the fickle publics mind
   (E) Are not worth the while
3. The author infers that the two films Chak de India and Tare Zameen par the hottest characters are coach Kabir Khan and Nikumb sir because—
   (A) They are the hottest character in two noble category

(B) The left an indelible impression without having to depend on the emotional remarks quotient

(C) Stood out for their insights into evil

(D) They are the two most mesmerising character of the year

(E) Two films which made the criminal king

4. Taking the public for granted in the closing lines of the passage implies—

(A) Formula will work

(B) Taking the public for granted is suicidal. Adventurism will tip the scales.

(C) Over hyped films have hit the dust

(D) Only time will tell whether Bollywood age old formulas will work

(E) Stars will make all the difference

Answers

1. (C) 2. (B) 3. (B) 4. (B)

Passage–26

Words–138

It is firm resolve that we should strive together to build a new world—a world where the differences of rich and poor, colour and castes of brotherhood, where every religion will be respected, where the wealth of the nations would be employed for the developmental works and for the improvement of education, health and nutrition of the children, instead of building up atomic piles for waging wars, where nations would have friendly relations with one another even though they might have subscribed to different ideologies, where the structure of divine power in everyman would be converted into the refulgent light of spiritualism. The road is difficult now like the razor's edge, but if you want to preserve the human race, we will perforce have to walk on this path – with courage, with patience and with self confidence.

1. 'Firm resolve' in the first sentence means—

(A) Definite resolution (B) Accepted view

(C) Unflinching (D) None of these

2. What is 'our firm resolve' ?

(A) To work together

(B) To make a collection effort to build a new world

(C) To build a new world

(D) None of these

3. One of the features of the new world will be—

(A) No distinction of colour and caste will exist

(B) No person belonging to castes will exist

(C) Poor and rich will live together

(D) None of these

4. 'Humanity will be sole test of brotherhood' means—

(A) Human being will judge brotherhood

(B) Human feeling will develop brotherhood

(C) Humanitarian considerations will be the only test of brotherly feeling

(D) None of these

5. The writer wants of say that nations should—

(A) Not spend on education

(B) Not manufacture atoms

(C) Not wage wars

(D) None of these

6. Nations in new world would have—

(A) Different ideologies

(B) Friendly relation

(C) Different ideals

(D) None of these

7. In the new society everyone will have—

(A) Divine power

(B) Ideologies

(C) Light of spiritualism

(D) None of these

8. 'Refulgent' means—

(A) Dazzle (B) Glare

(C) Radiant (D) None of these

9. 'Road' in the last sentence refers to—

(A) Build new world

(B) Achieving spiritualism

(C) Getting divine power

(D) None of these

10. 'Perforce' means—

(A) Under compulsion (B) By force

(C) Forcefully (D) None of these

11. If we do not follow this path—

(A) Human race will come to an end

(B) We will be forced to follow it

(C) Will have to take courage

(D) None of these

Answers

1. (C) 2. (B) 3. (A) 4. (C) 5. (C) 6. (B)
7. (C) 8. (C) 9. (A) 10. (A) 11. (A)

Passage–27

Words–230

When we are suddenly confronted with any terrible danger, the change of nature we undergo is equally great. In some cases fear paralyses us. Like animals, we stand still, powerless to move step in fright or to lift a hand in defence of our lives, and' sometimes we are seized with panic, and again, act more like the inferior animals than rational beings. On the other hand, frequently in cases of

sudden extreme peril, which cannot be escaped by flight, and must be instantly faced, even the most timid' men at once as if by miracle, become possessed of the necessary courage, sharp quick apprehension, and swift decision. This is a miracle very common in nature. Man and the inferior animals alike, when confronted with almost certain death 'gather resolution from despair' but there can really be no trace of so debilitating a feeling in the person fighting, or prepared to fight for dear life. At such times the mind is clearer than it has ever been; the nerves are steel, there is nothing felt but a wonderful strength and daring. Looking back at certain perilous moments in my own life, I remember them with a kind of joy, not that there was any joyful excitement then; but because they brought me a new experience - a new nature, as it were and lifted me for a time above myself.

1. An appropriate title for the above passage would be—
   (A) The Will to Fight
   (B) The Miracle of Confronting Danger
   (C) The Change of Nature
   (D) Courage and Panic
2. The author names three different ways in which a man may react to sudden danger. What are they ?
   (A) He may flee in panic, or fight back or stand still.
   (B) He may be paralyzed with fear, seized with panic or act like an inferior animal.
   (C) He may be paralyzed with fear, or seized with panic, or as if by miracle, become possessed of the necessary courage, and face the danger.
   (D) He may be paralyzed with fear, run away or fight.
3. The distinction between inferior animal and 'rational beings' is that—
   (A) the former are incapable of fighting.
   (B) the latter are clever.
   (C) the latter are stronger.
   (D) the latter are capable of reasoning things out whereas the former cannot do so.
4. Explain the phrase 'gather resolution from danger'—
   (A) Find hope and courage
   (B) A state of utter hopelessness steels one to fight out the danger
   (C) Not to lose hope, but fight
   (D) Find courage to face the danger
5. The author feels happy in the recollection of dangers faced and overcome because—
   (A) they brought him a new experience
   (B) they brought him a new experience, and lifted him above himself for a time
   (C) he survived his ordeal
   (D) he was lucky to be alive

Answers

1. (B) 2. (C) 3. (D) 4. (B) 5. (B)

Passage–28

Words–93

The artificial ways of inducing sleep are legion, and are only alike in their ineffectuality. In Lavengro there is an impossible character, a victim of insomnia, who finds that a volume of Wordsworth's poems is the only sure soporific, but that was Borrow's Malice. The famous old plan of counting sheep jumping over a stile has never served a turn. I have herded imaginary sheep until they insisted on turning themselves into white bears or blue pigs, and I defy any reasonable man to fall asleep while mustering a herd of stupid swine.

1. The author points out that—
   (A) sleep can-easily be induced
   (B) the artificial means of inducing sleep are not good
   (C) artificial ways of inducing sleep are ineffective
   (D) artificial ways of inducing sleep are expensive
2. According to the author the character in Lavengro
   (A) resorts to external aids to get some sleep
   (B) is an admirer of Wordsworth
   (C) spends sleepless nights reading Wordsworth
   (D) is an avid reader of poetry
3. The author uses "impossible" for the character of Lavengro in the sense of—
   (A) funny (B) unrealistic
   (C) queer (D) imaginary
4. Borrow's malice is most probably directed at—
   (A) Sleeplessness
   (B) The artificial ways of inducing sleep
   (C) Wordsworth's poetry
   (D) Poetry in general
5. In order to cure his insomnia, the writer—
   (A) does a lot of reading
   (B) vainly tries to concentrate on imaginary situations
   (C) keeps a flock of sheep
   (D) counts sheep jumping over a stile

Answers

1. (C) 2. (D) 3. (B) 4. (B) 5. (D)

Passage–29

Words–135

Experiments with the Sulfonamides have made clear a fact about germs which is gaining increasing importance in fighting them. Germs, it seems, have the same ability

as all the other living things gradually to change themselves to suit new conditions. But, as the generation of germs lasts only twenty, twenty-five or thirty minutes, before all the germs divide to form new ones, changes, that would take many years in animals can be achieved by germs in a few hours. Perhaps, then you give the attacking germ a dose of Sulfonamides which upsets them somewhat but is not strong enough to prevent them from multiplying; if so, they very rapidly develop new powers which enable them to resist the effects of the drug. After this has happened, even the strongest dose will fail to disturb them.

1. Experiments with Sulfonamides have led to the important discovery that—
   (A) germs are living things, and can change themselves to suit new conditions.
   (B) one generation of germs lasts only twenty, twenty five or thirty minutes.
   (C) germs can adjust themselves to live and multiply in new conditions.
   (D) germs are not disturbed even by the strongest possible dose of Sulfonamides.
2. Like all other living things, germs can change themselves to suit new conditions. This adjustment is possible because the germs have—
   (A) the power of fluctuation
   (B) the power of compliance
   (C) the power of adaptability
   (D) the power of adaptability
3. Since germs can change themselves to suit new conditions, the task of fighting them has become—
   (A) absolutely impossible
   (B) much easier
   (C) much more difficult
   (D) increasingly important
4. Germs which are not disturbed even by the strongest possible dose of the Sulfonamides are said to have become—
   (A) immortal (B) immune
   (C) improvised (D) immobile
5. One generation of germs expires, bringing into existence the next generation—
   (A) in twenty minutes
   (B) in twenty five minutes
   (C) in not more than half an hour
   (D) in a few hours

Answers

1. (A) 2. (D) 3. (C) 4. (B) 5. (C)

Passage–30

Words–220

Though supposed to be the beginning of woman's liberation decade, recent months have been far form being kind to top ladies. All the three really prominent once only a short while ago firmly in power, are now out it. In Argentina, the widow of the former dictator, Juan Peron, was displaced by a military coup; but both Mrs. Indra Ghandhi early this and Mrs. Bandaranaike now, have been defeated in a democratic election. Speculation will be immediate about the extent to which the Indian example influenced even farther south. Mrs. Gandhi had governed by Emergency and extensively rewritten Constitution. Mrs. Bandaranaike used a similar steamroller majority in the last Parliament to give Sri Lanka an entirely new one. Both extended the natural life of Parliament; both imposed severe curbs on the judiciary and the press, both were eventually embarrassed by the prominence given to their sons. The main difference seems to be that Mrs. Bandaranaike at least won her own seat, whereas Mrs. Gandhi did not. It will be a matter of wide satisfaction at home and abroad, for the subcontinent to have once again demonstrated, that special power are not necessarily eternal that public patience is not inexhaustible, and that it is a pure and interested myth to assert that Asia is not a favourable ground for effective democracy.

1. 'Liberation decade' means—
   (A) Decay of liberation movement
   (B) Liberation has come to an end
   (C) A period of 10 year during which liberation movement goes on
   (D) None of these
2. 'Far form being kind' means—
   (A) Has not been kind (B) Not kind
   (C) Kind enough (D) None of these
3. Juan Peron was the—
   (A) Prime Minister of Argentina
   (B) Wife of the former dictator of Argentina
   (C) Lady who brought military coup
   (D) None of these
4. Speculation will be immediate. About what ?
   (A) About democratic elections
   (B) About Mrs. Bandaranaike's defeat
   (C) About the extent to which Indian elections effected Sri Lanka
   (D) None of these
5. Mrs. Gandhi ruled—
   (A) By changing the Constitution
   (B) Through emergency

(C) Through emergency and by rewriting constitution
(D) None of these

6. 'Steamroller majority' means—
(A) Overwhelming majority
(B) Overriding majority
(C) Happy majority
(D) None of these

7. Mrs. Gandhi and Mrs. Bandaranaike resembled—
(A) In curbing judiciary and press
(B) In being ousted
(C) In resigning
(D) None of these

8. Public patience is—
(A) Boundless (B) Not boundless
(C) Pure (D) None of these

9. Myth means—
(A) Fiction (B) Legend
(C) Mythology (D) None of these

Answers

1. (C) 2. (A) 3. (B) 4. (C) 5. (C) 6. (A)
7. (A) 8. (A) 9. (A)

Passage–31

Words–144

I would say with those who say God is Love. But deep down in me I used to say that through God be Love, God is Truth-above all. If it is possible for the human tongue to give the fullest description of God, I have come to the conclusion that for myself, God is Truth. But two years ago I went a step further and said that .Truth is God. I never found a double meaning in connection with Truth, and even atheists had not demurred to the necessity or power of Truth but in their passion for discovering Truth the atheists have not hesitated to deny the very existence of God form there own point of view rightly. And it was because of this reasoning that I saw that rather than say that God is Truth, I should say that Truth is God.

1. The writer believed—
(A) What others said about God
(B) That God is Love
(C) That Love is God
(D) None of these

2. "Deep down in me" means—
(A) In the hearts of my heart
(B) When I was down
(C) In the depth of down

3. "Fullest description of God" means—
(A) Describing God completely
(B) Giving proper description of God
(C) Complete description of God
(D) None of these

4. Why does he use the phrase "a step further" ?
(A) In order to show that his earlier view less comprehensive
(B) In order to prove the second view is larger one
(C) In order to say that Truth makes God higher
(D) None of these

5. 'Atheists' are the persons—
(A) Who do not believe in the existence of God
(B) Who worship many gods
(C) Who are irreligious
(D) None of these

6. Power of Truth—
(A) Cannot be denied even by the atheists
(B) Can be understood
(C) Can be realized
(D) None of these

7. 'Atheists' have not hesitated—
(A) To disbelieve the existence of God
(B) Deny Gods' power
(C) Deny life
(D) None of these

8. In the last sentence the word 'this' refers to—
(A) Argument given in the previous sentence
(B) Existence of God
(C) Hesitate
(D) None of these

9. The main idea of the passage is—
(A) Truth is God (B) God is Truth
(C) God is Love (D) None of these

Answers

1. (D) 2. (A) 3. (C) 4. (A) 5. (A) 6. (A)
7. (A) 8. (A) 9. (A)

Passage–32

Words–188

It will be a mistake to think that he was given only 'bouquets', he also received many 'brickbats'. The Christian missionaries took alarm at his popularity. They used to raise funds 'by preaching that India was a land of heathens waiting to be saved by Christianity. The American press now began to say that it was a shame that any body should try to teach India religion, rather the

world should sit at her feet to learn it. Vivekananda also said that India did not need religion but material support. The missionaries found that the subscriptions they had so long been receiving from the people were steadily declining. They blamed it on Swamiji. They now started denigrating him in all manner of ways. They even began to spread scandals against his personal character. Strangely enough, even some of his own countrymen joined them in this for reasons of their own. But 'Truth alone prevails', as Swamiji always preached. He did not try to defend himself, but others stood up for him and vehemently protested. Finally, all such mean attempts failed and his reputation only rose higher and higher.

1. The passage teaches us
   (A) not to believe in religions other than our own
   (B) not to get involved in scandals
   (C) not to visit foreign lands
   (D) not to deviate from the path of truth
2. Vivekananda was criticised by missionaries in America because—
   (A) he was a bad student of Western theology.
   (B) he opposed the tenets of Christianity
   (C) Americans had become very fond of him.
   (D) he did not allow them to raise funds in India.
3. Swami Vivckananda told the American people that India—
   (A) did not approve of the Catholic Church
   (B) would teach religion to those who sit at her feet
   (C) required religious and material help
   (D) was self sufficient in religion though poor
4. Vivekananda's rapport with the American people—
   (A) helped India get substantial aid
   (B) made his friends desert him
   (C) annoyed the American Government
   (D) caused a drop in Church's collections
5. Vivekananda did not defend himself because—
   (A) he believed in the ultimate triumph of truth.
   (B) he was in a foreign land.
   (C) some of his countrymen were opposing him.
   (D) he had brought many friends along to fight for him.

Answers

1. (D) 2. (C) 3. (D) 4. (D) 5. (A)

Passage–33

Words–109

Religion is the greatest instrument for so raising us. It is amazing that a person not intellectually bright, perhaps not even educated, is capable of grasping and living by something so 'advanced as the principles of Christianity. Yet, there is a common phenomenon. It is not, however, in my province to talk about religion, but rather to stress the power which great literature and the great personal and the great personalities whom we meet in it and in it and in history have to open and enlarge over minds, and to show us what is first rate in human personality and human character by showing us goodness and greatness.

1. In the passage, the author's ultimate intention is to talk about—
   (A) religion (B) history
   (C) education (D) character
2. The phrase "so raising us" means—
   (A) giving us a sense of spiritual superiority
   (B) making us feel that we are more important than we really are
   (C) improving our mental abilities
   (D) making us realise that we all are children of God
3. What surprises the author is that—
   (A) Even uneducated people are attracted towards Christianity.
   (B) Christianity is practiced by a large number of people
   (C) Despite being difficult and complex, the, principles of Christianity are practiced by so many people
   (D) Even very intelligent people cannot understand the principles of Christianity
4. The author hesitates to talk about religion be cause
   (A) he does not feel himself competent to talk about it.
   (B) nobody around him likes to talk about it.
   (C) he does not believe in any religion.
   (D) he does not fully understand its importance.
5. According to the author, we come across examples of greatness and nobility in—
   (A) great works of literature
   (B) literary and historical works
   (C) historical records
   (D) books on Christianity

Answers

1. (C) 2. (D) 3. (C) 4. (A) 5. (B)

Passage–34

Words–691

Judiciary has enjoyed years of activism. Public adulation, emanating from the instant judicial solution to

their problems, has oxygenated filling of PILs in the apex and high court. A fact-over 90% of the PILs are dismissed by the courts. The exercise of its prime role as protector of fundamental rights has made the judiciary wander deep into executive's and sometimes, into legislature's domains. As long as the executive was at the receiving end, there was hardly any protest.

The first ever clear-cut legislative action by the SC happened in Vishakha case. It gave a new guideline, to deal with perverted Romeos at work-place. Though the judgment came more than 10 years ago, it still holds good. For, the legislature has not bothered to enact a law to make sexual harassment at workplace a statutory offence

Protests from legislature became loud after the judgment on President's rule in Bihar castigating the governor and the central government for preventing a coalition from staking claim. This was followed by SC's intervention in Jharkhand trust motion, an order that made legislators hoist the red flag for judicial encroachment into their domain. Buoying the legislative and executive protests, a two judge bench of Justices AK Mathur & Markandey Katju recently penned a judgment warning of impending political anger over judicial activism that could lead to curtailment of judiciary's independence. It also urged judges to exercise restraint so as not to breach the *lakshman rekha* demarcating boundaries.

The judgmenrt, delivered on December 6, did not specify which of the judges-justice Mathur or Justice Katju-authored it. It was a unanimous one- both agreeing to it whole heartedly. The Justices Mathur-Katju bench has in the last four months, given similar judgments. In Union of India vs MS Mohd Rawther the bench on August 16 said, "The court has only judicial power to review the executive order on Wendnesbury principle, but it cannot arrogate itself the power of the executive ………. The court must exercise judicial restraint in such matters. There is a board separation of powers under the Constitution, and one organ of the state should not the ordinarily encroach into the domain of another".

So, the judicial non-encroachment into executive domain is held to be a cardinal principle, by which both Justices Mathur & Katju swear.

If the restraint dice is rolled further, one comes across the Ghazipur abattoir case pending before the court for the last three years. The abattoir is being constructed by the MCD. A three- judge bench of the SC, including Justice Mathur, has been passing orders for speeding up the construction, the money to be paid to the contractor and the release of machinery imported from abroad.

Whose function is it to monitor construction of an abattoir- judiciary or executive? The cost of the abattoir was fixed by an expert at Rs113crore, but the bench has already ensured payment of Rs123crore to the contractor with the direction that it would look into the final estimates of the project cost. Who fixes the project cost-judiciary or executive? Justice Katju has consistently advocated the principle of judicial restraint. This is evident from his judgment right in the beginning of the year.

In Veam china Koteshwar Rao case judgment pronounced by him on February 15, he had said, " it is well settled that there must be judicial restraint regarding administrative decision. " He is not a judge known for keeping any restraint in his utterances during hearings. The famous "hang them from the nearest lamp post" mooted by him for corrupt officials had hit headlines. Well, we are sure that he said it out of anger against rampant corruption in the country and did not actually mean it to be a law. For, it had striking resemblances with Taliban laws. Few days back, he again observed that if a girl was of marriageable age, there was nothing wrong in her eloping with the person she wanted to marry.

Judicial restraint should also from part of their speech at the time of hearing cases. If not, then incongruities-like the two-judge bench terming previous larger bench judgment as judicial aberrations-are bound to creep in .

1. The author states which of the following examples prove that the legislature is of logger heads with the judiciary ?
   1. New policies and ruler to deal with workplace Romes who are perverted advised by the judiciary go Central unheeded a decade on
   2. Berating a government and governor for disbarring a coalition from staking a claim to rule
   3. Anger over judicial activism
   4. Judges do not restrain in breaching demarcating boundaries
   5. All of the above

   Chose the correct answer or combination :

   (A) 1 and 2 (B) 1, 2 and
   (C) 1, 3 and 4 (D) only 3
   (E) only 5

2. In which areap according to the passage the court has no power to execute ?
   (A) To peruse the executive order on Wednesbury principles
   (B) Beston upon itself the power of the executive
   (C) One domain of the state should not trespass into the area of another
   (D) Encroachment into executive domain
   (E) None of these

3. What would be the most suitable and appropriate title for the passage ?
   (A) Should judger look the other way fearing political wreath ?
   (B) Judiciary fearing judiciary & executive back lash
   (C) Executive reigns, judiciary bows
   (D) Executive and judiciary: heads on
   (E) Toe the line, executive to the judiciary
4. What are the problems which may crop up, according to the author if judicial restrained does not became a part of judgment during or at the time of hearing cases ?
   (A) Differ cultists are likely to arise
   (B) A smaller bench accusing larger benches of making wrong decisions
   (C) Executives will take the advantage
   (D) Judge will lose face and executives will stand to gain
   (E) None of the above

Answers

1. (E) 2. (B) 3. (B) 4. (B)

Passage–35

Words–808

The Dust over the Gujarat election has begun to settle and the post-mortems have begun. The pundits have zeroed in on three or four reasons for the Congress' defeat. One of them is the stalling of the Forest Rights Act, which apparently cost the congress a large number of potential tribal supporters and which-belatedly-will be notified today. Is this diagnosis correct? Why should what seems to be an arcane welfare scheme make such a difference to a state election?

The Scheduled Tribes and Other Traditional Forest Dwellers (Recognition of Forest Rights) Act, 2006 is not a welfare scheme. Rather, it aims at correcting the century-long State takeover of resources belonging to the country's tribal and forest communities. Contrary to allegations, this Act does not 'hand out' or distribute land to anyone . It addresses one basic problem : what are called 'forests ' in India law presumed by many to be ' pristine wilderness' are nothing of the kind; crores of people live inside these forests.

Why do many of them not have rights to the land they live on ? The reason is India's forest laws. The forest Acts aimed to bring all forests under a centralized Forest Department's control and to take over the lands and rights of forest-dwellers. At first this was justified in the name of easier timber extraction; then post-Independence., for the nation's industrial requirements; and finally, for conservation. Whatever the justification, the policy has remained the same.

Thus according to the law, at the time of declaring a 'forest' a settlement officer surveys and settles the rights of people. Unsurprisingly, these settlement officers either did nothing or only recorded the rights of those who were powerful. Million of people, mostly tribal, lost their rights and were deemed 'encroachers' in their own homes. To this day, 82 per cent of Madhya Pradesh's forest blocks have not been surveyed, while 40 per cent of Orissa's forests and 60 per cent of our national parks have not completed settlement of rights. In Gujarat, a survey found that even when the Government claimed to recognize land title, only 5 per cent of the eligible claimants actually got *pattas*.

As a result of this, neither forests nor the people gained. Those deemed 'illegal' are subjected to extortion, assault, jail or arbitrary eviction. It is no accident that tribals are the poorest communities in the country today.

Forests, in turn, came under the highly centralized control of the bureaucracy, which treats them as property to be sold to the highest bidder.

This happens through corruption and legal diversion of forests for industrial, mining and development purpose, which has already destroyed five lakh hectares of forest between 2001 and 2006. Forest dwellers who oppose destruction of their homelands find themselves facing criminal cases, arrest and eviction.

The Forest Rights Act aims to address this through two steps. First, it recognizes the trilbals' rights to land and forest resources that they were using as of 2005 (non-tribals have to prove 75 years of residence) No land is 'given'; no one receives title to land that they are not already cultivating. Second, it gives communities the right to protect forests, empowering them to be partners in conversations rather than its victims.

So why has the government delayed the Act? Such change is not liked by the forest bureaucracy by the industrial groups that currently enjoy easy access to forest land or by a few conservationists who now equate the forest bureaucracy with conversation the firestorm of opposition to this Act has taken place in the name of tiger conservation, but its essential theme has been that all power should remain with the bureaucracy. Anything else, we are told will lead to catastrophe for tigers, forests, or both this is despite the fact that the act itself provides safeguards against land-grabbing and a procedure for how people can be resettled where necessary for wildlife, a procedure supported by environmental organizations.

Given such powerful opposition, even when passing the Act, the government modified some key clauses severely weakening the law's provisions. For a full year after its passage, the government stalled the Law's implementation. Today, according to the minister, the Act will be notified into force. But even this may not end the charade, for the rules to the Act will likely again be heavily diluted.

Meanwhile, 10 days ago, more than 100 families were evicted in Madhya Pradesh and their homes demolished; forest guards opened fire twice, seriously injuring six people. Forest destruction also continues, with the ministry hastily clearing new industrial and mining projects. The notification today will be a step forward. But if the rules are diluted further, it will only show that mere electoral defeat is insufficient to shake the government's commitment to vested interests. It is a commitment that has already extracted a heavy price.

1. Which of the following would be the most suitable title for the passage ?
   (A) Don't dilute the Forest Act
   (B) Out of the wilderness
   (C) Arcane scheme rock the congress boat
   (D) Protect tribunal interests or lose
   (E) Rights of tribals must not be denied or watered down
2. Which of the following the author does not mention at the time of declaring a 'forest' ?
   (A) The settlement officers surveys and settler the rights of people
   (B) The settlement officer did nothing
   (C) The settlement officers only recorded the rights of those who were powerful
   (D) Tens of thousands, mostly tribals lost their right
   (E) 82 percent of MP's forest blocks have been surveyed
3. In the starting statement of paragraph six the author implies—
   (A) That corruption and legal methods of distribution deprive tribals of their rights and make bureaucrats rich
   (B) Tribals are charged with criminal cases, arrested and convicted
   (C) It recognizer the tribals rights to land and forest resources
   (D) Communities are given the right to protect forest
   (E) It empowers them to be co-owners in conservation and not destroyers of forests.
4. The firestorm of opposition to the proposed Forest Rights Act is summarized by which of the following—
   (A) Power shall remain with the bureaucracy under the name of tiger conservation
   (B) That it will lead to the destruction and exploitation of tigers instead of conservation of tigers
   (C) The act provides provisions against land snatching
   (D) As to how the tribals can be resettled smoothly and efficiently
   (E) None of the above

Answers

1. (E) 2. (E) 3. (A) 4. (B)

Passage–36

Words–875

Catch Them Young

Education system in India, all the way from adult literacy programmes and rural schools to vocational institutes and research universities, need a comprehensive makeover. This is uppermost amongst the country's development priorities.

Literacy and basic education are required for people to manage their daily lives and participate in democratic processes; vocational skills enable participation in the economy; and higher education enable Indians to play a more effective role in the global knowledge economy and international affairs. However there is another need which runs through all forms of education: it is the need to inculcate the right values to develop responsible citizens of society.

Changing societal values is a recurring theme across the world the erosion of 'family values' has been concerning leader in the USA where both republican and Democratic political leaders regularly call for their restoration. The decay of 'Community life' also bothers Americans. Social studies reveal that family and community values contribute greatly to happiness, and their depletion in economically advanced countries is a significant reason why people in these countries are not happier even when they are richer.

Though children brought up in the education systems of these countries learn to stand on their feet, and be politically and economically independent, often they end up "Bowling Alone" (in sociologist Robert Putnam's memorable phrase) and unhappy. Therefore as, we strive to improve India's education systems to enable our children to become economically independent, they should also imbibe better family and community values in them if we want a more harmonious, happy society.

At the heart of family and community values is the value of inclusion of considering not only one's own needs, but also the needs of others. Some suggest that good values are built at home, in the family and the community rather than in school. However, when good values are disappearing from families and communities themselves, schools must become the place where they are strengthened, and from which required values will be reinserted into society. In India, citizens must think and act more inclusively than many in any other country of the world perhaps, because we are very diverse and a democracy with many inequities.

Values cannot be taught like texts, nor tested in written examinations. They are learned by living. Two Indian schools that are good example of how children imbibe values of inclusion are the Loreto Day school

besides Sealdah station in Kolkata, and the Kathalaya schools in Delhi's enormous Govindpuri slum. The Loreto Day school has about 1'500 children, half from fee paying middle class families, and the rest from poor families.

Amongst the poor are about 350 'rainbow' children, who are taken off the streets, many of whom are orphans. The children come from very different circumstance but no distinction is made amongst them when they are in school. Learning is in teams. The rainbow mix with the rest. The school enters inter-school debates and other competitions and does well.

However it, has an unusual system. If a child wins a debate, that child does not compete in the next one. This may affect the school's chances at winning. But it gives another child a chance to win and shine. Imagine Implementing this system in Indian cricket. If a player scores a centaury, he must step down from the next match. There would be fewer superstars but there may be many more stars!

The Kathalaya school is in the middle of the huge, congested, and trash- strewn slum. It has several hundred children, all poor. Here too, children learn in teams. They begin every semester exploring a new theme, and weave all their lessons, in language, history, geography, science and Vocational skills, around topics that they discover and want to learn about as they explore this theme.

The school is buzzing with activity computer screens flashing, painting sketched on boards, colleges of words and pictures, and voice of children. No bored children sitting in row in this school! The unusual mode of learning in the school not only enables the kids to easily pass the certified central school exams, it also gives them a great ability and desire to learn.

The themes the kids explore in school lead them to the concerns of their communities. While looking into these, they are guided into structured projects. For example, teams are working on improving sanitation and availability of water in the slums of Delhi.They have surveyed families needs and the physical realities of infrastructure. They have discussed solutions with the municipal authorities, some of which are being implemented. They are also persuading their community to change behaviours, regarding sanitary practices for example. The confidence of these 12-14 years old children in the face of staggering odds- that they will make a difference to the lives of their own communalities is most inspiring.

Both these schools are showing how, along with all the utilization knowledge and skills children need to pass exams and get jobs, the children are also learning values of caring for others and for their surrounding, so that they will become not only responsible citizens, but even leaders. These children are some of the millions of fireflies, carrying their own light for others. They can transform India into a more inclusive and better society.

1. The need of the hour according to the author is their India needs to stress on above all—
   (A) To change its societal values
   (B) Literacy and basic education
   (C) Vocational training
   (D) Adult literacy programmes
   (E) All of the above
2. By the phrase decaying of community life the author means—
   (A) Erosion of family values
   (B) Community value play as vital role in contributing to happiness
   (C) That depletion of community values have an adverse effect on the overall happiness quotient
   (D) Restoration of such values are the need of the hour
   (E) People are not happier even when they are richer
3. The main concern of the author is which of the following—
   (A) Literacy and basic education
   (B) To inculcate correct values to make happier and better citizen
   (C) Vocational training; so that our can earn a livelihood
   (D) Improve our educational system to make our children economically independent
   (E) India is diverse and a country with many inequities.
4. The most appropriate title for the passage would be—
   (A) Values are invaluable
   (B) Children our future leaders
   (C) Community values our strength
   (D) Catch them young
   (E) Child, the father of man

Answers

1. (A) 2. (C) 3. (D) 4. (D)

Passage–37

Words–878

A bunch of no-hopers from Iraq-spurred on by their platitude-spilling Brazilian coach and some crazy idea of deliverance - provided the most poignant moment in all sport in 2007, and not just football.

When a hastily-whipped up, scarred-by-the war, no-place-to-call-home team won the Asia Cup this summer, it helped a sport that has become an industry, remember that you cannot manufacture romance. Football become a sport again, albeit briefly...

Because soon, the wheels that turn football today were back in business. And the Iraq story being the story of the year, 2007 actually belonged to two characters - so different and yet so similar with their unique sub-plots in the bigger picture. David Beckham and Juan Roman Riquelme may have crossed paths a few times in the Spanish League turning out for Real Madrid and Villarreal respectively, and the odd footwear commercial, but you couldn't have more diverse personalities living the same life. And that irony was the story of the year in football-pretty -face, money-spinning, most-time model, part-time footballer who can't hide meets out of work misfit who actually happens to be the best footballer in the world at the moment, but there's no one to confirm the fact.

Much before he finally won his only trophy in his high-profile, much-sought-after four years with Real Madrid, Beckham had already signed up to take soccer in the US to its natural plane, i.e. help it break the bank. And so, even as sanity -read, football - was returning to Rea1, Beckham was sulking over not being allowed to attend newest bestest pal Tom Cruise's wedding. That was last year, and, the man who clamped-the curfew back then on1y Cause of the minor detail of Real playing over that weekend–now happens to be Beckham's boss in the England set-up. After making public that he had nothing to do with him, tough talking disciplinarian Fabio Capello did a most undiplomatically turnaround when he was forced to re-induct Beckham into the squad as ,his team floundered; now he faces the dilemma again. But for a while, the onerous task, was his predecessor's, the out-of-depth, Steve Mc-Claren.

As Beckham brought out his best suits to sit on the LA Galaxy bench- ostensibly honouring his $250 million (£128m) five year deal - Mc Claren was doing a Capello, dropping Beckham from the England line-up as they went about the task of qualifying for the European championships next year Futilely it was to prove on both fronts as an inept England superstar/ celebrity side failed to come to terms with the changed standard in Europe, Beck ham returned to the England right wing – hilariously much to the mild consternation of Galaxy who had failed to make the MLS playoffs. He was to be a bit player in the scheme, but apart from cornering all the attention, did little as the most over-hyped football team in the world tumbled out of qualifying.

So, Beckham with millions in the bank over the next five years $ 50m as salary, $50m from club profits, $ 50 from shirt sales merchandising and $ 100m from his various sponsorship deals which include Adidas, Pepsi, Police, Diesel and Rage software, putting his worth for 2007 at 11m pounds - was without a country and one goal for his weekend football club (which incidentally brought out Ruud Gullit from the wood work to help spruce up their *shexy shoccer*)

At the same time, south of the equator from the sunny climes of California, an eerily similar story was unfolding. Having quit the National team after the World Cup last year, since his mother couldn't take the pressure, the moody, publicity-shy Argentine playmaker Riquelme was having Problems at his club, Villarreal which had, for three seasons, revolved around him. Written off, out of favour and still on loan from FC Barcelona, he was farmed out to Boca Juniors in his homeland Argentina where he recovered his joy for the sport leading the side to the South American championships, the Copa Liberator’s.

But then that is the Americas and only Beckham must shine there. In Europe with his club future still uncertain, he returned to the Argentine National fold, donning the No 10 once again, and dishing out some scintillating football as Argentina marched in fear some fashion to the Copa America finals only to self-destruct before a clinical Brazil side in a manner on1y they can. Typically Requiem faded in the big game, but then seemed to regain his groove as the mammoth South American qualifiers for the 2010 World Cup got under-way, majestically dictating play and scoring spectacular goals–off regular play and free-kicks.

So like Beckham and unlike him, here was Riquelme. A man whose bank balance is nothing home to write about-he, lived in the social apartment block in downtown Villarreal without a club, but so important for his country that they wouldn't dream of taking, the field without him.

As the year faded, Boca had offered him a contract which Riquelme had gladly accepted. On the other side, the Italian Capello has admitted he's taking English lessons for his new job. Whether he wants to tell off Beckham in the Queen's best English, or, welcome him back, some how *Ciao* sounds so much better. Either way.

1. The author here in the passage implies that David Beckham with millions in the banks—
   (A) Earns an enormous sum by his sheer scintillating football skills
   (B) Performed superbly job for LA galaxy in the qualifiers for the European championships
   (C) Is a better footballer than Roman Riquelme
   (D) Earns more in sponsorship deals than by his football genius
   (E) None of the above

2. The author points out to the ironical situation between sheer talent and celebrity status through which of the following—
   (A) Pure raw talent besides, being the best footballer in the world is no match for money churning, semi footballer and pretty model celebrity
   (B) An out of work, talented footballer is no match for the likes of Beck ham who has earned fame in the football fraternity
   (C) The diverse personalities of the out of form back ham and inform Riquelme

(D) Requilme was offered sevrals big money contracts while Back ham was paid enormous sum to endorse big products like Adidas, Pepsi, Diesel and Rage

(E) All of the above

3. The tone of the author in the passage can be best described by which of the following word/s ?

1. Encouraging	2. Ironical
3. Disgusted	4. Didactic
5. Cynical	
(A) 1 and 2	(B) 3 and 4
(C) 2 and 3	(D) 2 and 5
(E) 3 and 5	

4. Which of the following headings according to the passage would be most apt ?

(A) Win-some lose some

(B) How the west was won

(C) Glamour vs Raw talent

(D) Pretty face -ugly football

(E) Moderate talent- Model success

Answers

1. (D) 2. (A) 3. (E) 4. (B)

Passage–38

Words–931

The hero is a slayer of monsters. That is not just his corer competence, but his only competence. According to Roberta Calasso, deconstructor of the cross-cultural mythological hero, the destruction of the monstrous is not an attribute of heroism but its constitution; the very stuff heroism is made of. It is Hercules slaying the Nemean lion or the many-headed Hydra; Bal Krishna crushing the giant serpent; These us killing the minotaur; Rama vanquishing Ravana.

It is not incumbent on the hero qua hero to be particularly wise, or intelligent, or indeed decisive. In fact, the hero as hero is the least decisive of persons : his killing of monsters being a form of monomania or an obsessive-compulsive disorder. The hero *must* destroy monsters, even when, like a post-modern Beowulf, he might intuit that the monster he destroys may in fact be somewhat less monstrous than he himself is.

At the moment of his triumph-when he not only bestrides Gujarat like a colossus but casts a prophetic shadow over Delhi-Narendra Modi recalls the great heroes of mythology as defined by Calasso: slayers of monsters and demons. The many, interlocking reasons behind Modi's victory-which totally confounded most supposedly expert predictions-will be discussed and analysed till kingdom come, or at least till the next government comes.

The rebels within the BJP ranks proved toothless when it came to the crunch. The Patel vote did not, as expected, go against him. The development platform he initially campaigned on has a firm foundation. Sonia Gandhi's 'merchant of death' accusation backfired, wounding 'Gujarati pride' which was reinvigorated by Modi's Soharabuddin speech. All these, and many more, reasons will be adduced to explain the spectacular electoral success of 'Moditva', an addition to India's political lexicon which seems to have come to stay, to the great glee of Narendrabhai's supporters and the dismay of his equally vehement opponents, not all of whom are outside the BJP.

But in this welter of instant analysis and even faster comment, when cause and effect often switch place, one factor can perhaps be identified as the underlying theme which links these many disparate notes to compose Modi's victory march: the yearning for a 'strong leader'; a hero, a slayer of monsters. India and Indians-and not just Gujaratis, but all of us–have, ever since the British decamped, longed for strong men (or women) to lead us to our manifest destiny, whatever that might be. The goal (prosperity, literacy, better public health) is not as important as the emotional and psychological reassurance that we are being *led,* inexorably, unswervingly, without any of the diversionary debate which robbed us of our innate robustness of single-minded energy and enterprise thanks to the legacy of Nehruvian social liberalism.

Nehru, despite his patrician mien was after all an end product of the European Enlightenment, as such he was the antithesis of the mythic hero, a description more likely to fit Sardar Patel, the 'strongman' that many even today wish had become India's first prime minister and rewritten our post-independence history by doing so. Subhas Bose also belonged to the mythic-heroic mould, and even now commands a near idolatrous veneration and not just in Bengal.

Indira Gandhi succeed, but only too well, in giving us a 'strong' leader : Like a true hero, she slew the demons of privilege and pelf by cutting the princely privy purses and nationalizing banks, and, as Ma Durga, 'liberating' Bangladesh from Pakistani tyranny. In the end, she met the fate of most heroes who must forever created monster to destroy in order to perpetuate their heroic role : she created one monster too many, which turned around and devoured her.

At first glance, the 'monster' that Modi created, post-Godhra, so as to cast himself as protective hero is all too easy to see : the demoniacal 'other' the lurking terrorist in our midst who threatens every single member of the majority community. Naipaul described the rise of Hindutva as a response to the '700-year-old sense of revenge' that Hindus harboured against Muslim 'invaders', particularly such marauders like Mahmud Ghazni who destroyed Gujarat's Somnath temple. The meat-eating Muslim was seen as a carnivorous predator;

the 'grass-eating' (vegetarian) Hind as effete, herbivorous prey. By 'semitising' Hinduism (one people, one book, one faith) Hindutva developed an aggressive muscularity. But it had yet to find its implacable, invincible avenging hero (Vajpayee ? Too sweet and too found of jalebis. Advani ? All rath and no wrath; too soft on jinnah) .In Modi and Moditva, the true Saffron Avenger has finally been found.

Modi's secret lies in that the demons he has so successfully harnessed to this heroic cause are not out there but inside us, as all true demons are. It is not the Islamic terrorist or fundamentalist which is the real bogeyman that Modi has exploited in his voters' minds. It is their fear of the fear of the Muslim other that he has addressed: You are afraid to be afraid of Them; don't worry I'll always be there to protect you from that subtle demon inside you. That's the real monster that Modi has created : the fear of the fear of the monster. Which phoenix like will keep arising from its ashes, because deliverance from it ashes, because deliverance from it can be achieved not through the intercessionary office of a strongman hero-who actually must preserve the monster in order repeatedly to kill it-but through a process of self-exorcism. But that's difficult to do. Not least because we like to believe in heroes who, in the end, protect us from no other monsters but those within ourselves.

1. The author is this passage is concerned with—
   1. Megalomania at it diplomat best.
   2. Heroes who are concerned with slaying monsters within.
   3. A hero who is exemplary.
   4. A hero hell bent on slaying all those who are intent on destroying our national structure.

   (A) 2 and 4 (B) only 1
   (C) 3 and 1 (D) a and 4
   (E) 3 and 1

2. The enigma of Modi's stratagem and incredulous victory will be according to the author ?
   1. Talked about decades henceforth in glowing termed.
   2. A blunder India will remember.
   3. Something that will confound experts and predictions.
   4. Analysed at least till power changes hands.

   (A) Only 1 (B) 3 and 1
   (C) Only 4 (D) 3 and 4
   (E) Only 3

3. According to the passage Mr. Modi is slaying the monster within which implies—
   (A) Fear of the muslim other
   (B) Fear of Terrorists
   (C) Fear of being helpless
   (D) Fear of invasion

4. The line "must preserve the monster in order repeatedly to kill it" finds true repre station in which of the following—
   (A) Killing the enemy and preserving the body to stab at will again and again
   (B) Make the enemy a friend such that if can be deceived without suspicion
   (C) Call a true till well-equipped to defeat the enemy
   (D) Periodically subduing the so called enemy but not wiping it out in order to keep the fear alive.

Answers

1. (B) 2. (C) 3. (C) 4. (C)

Passage–39

Words–915

Do Away With Subsidies

"We spend far too much money funding subsidies in the name of equity, with neither the equity nor efficiency objective being met". So said the prime minister during the course of a recent speech in Delhi. He was repeating what one of his predecessors, Rajiv Gandhi, had said several years ago - that only 17 paise out of every rupee spent on subsidies actually reached the targeted beneficiaries.

Manmohan Singh's remark focuses attention on a malady that has affected our package of economic policies over the years. Successive governments have doled out subsidies to satisfy some vested interest or other. Perhaps, some of these subsidies had some initial economic justification. Often, these subsidies lose all rationale after some time. But the government of the day simply cannot muscle up enough courage to remove them because that would be politically unpopular. Such examples are easy to find. I have written about a couple of them in recent columns of this newspaper. Subsidised higher education or the unrealistically low prices of petrol and LPG are a burden on government revenues, with the majority of the beneficiaries being relatively well-off people. And there are many others. For instance, the subsidy on fertilisers has gradually assumed mammoth proportions over the years. However, rich farmers benefit significantly more than small ones from the fertilizer subsidy since they buy substantially higher quantities. Until recently, the urban middle classes were the biggest beneficiaries of the public distribution system. Many of the really poor do not have any fixed address, and so do find it difficult to obtain ration cards. Even if they have ration cards, they do not have money to purchase their weekly quota of food grains on anyone day of the week.

Another major reason for the mounting subsidy bill has been the support given to those public sector enterprises that run at a loss.

These enterprises would have to fold up unless they receive financial support from the central or state governments. Of course, the closure of such companies mean that the worker employed in these units would be unemployed.

While some form of safety net to mitigate their suffering needs to be evolved, there is rid rationale of continuing to operate chronically loss-making enterprises.

Unless the subsidy bill can be drastically pruned, there is no hope of restoring the fiscal health of the central and state governments. Perhaps, the Centre will be in a slightly better position because the booming economy and better tax compliance have ensured a healthy growth in central tax revenues. While this ensures some spin-off benefits to the states through larger transfers from the Centre, there has been a relatively small increase in the poorer states' own tax revenues. Since these are precisely the states which need to accelerate development expenditure, the case for a drastic, reduction in subsidies is stronger.

Of course, it is equally important to realise that all types of subsidies should not he tarred by the same brush. Consider, for instance, the recent National Rural Employment Guarantee Scheme. This is in principle a scheme that is targeted towards the rural poor-the relatively well off will not enrol in such schemes. The scheme has come in for a lot of criticism because corruption has resulted in some leakage. The correct conclusion to draw from this is that efforts need to be made to ensure less corruption. If schemes were to be scrapped simply because of the attendant corruption, then practically all government activity would have to be stopped.

Critics of schemes such as the rural employment guarantee scheme typically place all their bets on rapid growth to alleviate poverty. The "growth- only" school often cites the example of, China to bolster their arguments. They point out that in the post-reform period in China, rapid growth has resulted in huge reductions in poverty levels. They also emphasize that the Chinese government did not adopt any particularly pro-poor stance in their policies. If anything, there may have been a slight increase in inequality. The incidence of poverty came down because of the so-called "trickle down" mechanism. People from all income classes enjoyed the benefits of growth.

What these figures hide is the vastly superior levels of achievement in the social sectors in China before 1979, when the current phase of reforms was initiated. In the yearly'50s, India and China were at more or less comparable levels in terms of most social indicators. But, by the late, 70s, China had pulled very far ahead. China's poor were much better fed, with a drastic reduction in Chronic under nourishment. As far as comparable achievements in the health sector are concerned, sector China surged far ahead of India in terms of infant and child mortality as well as life expectancy.

For instance, life expectancy at birth in China was68 year in 1981 compared to only 54 year was 68 years in 1981, compared to only 54 years in India. The same differential pattern of achievements was also witnessed in education, with literacy figures in China being comparable to the most advanced countries in the world, while India lagged very far behind.

Walking on two legs is always better than limping along on one leg. A balanced approach to development must, incorporate policies that promote growth and prepare adequate safety nets. The latter must involve some targeted subsidy schemes. But there can be no p1ace for subsidy schemes where the major beneficiaries can afford to stand on their own feet.

1. What according to the passage would be the most suitable and appropriate title—
   (A) Subdue-subsidies
   (B) Subsidies do not reach the subjects
   (C) Subtle subsidies
   (D) Do away with subsidies
   (E) Substitute sub subsidies
2. The author is mainly concerned with which of the following—
   (A) A majority of state subsidies fail to reach the targeted beneficiaries
   (B) Neither equity nor objective are properly monitored
   (C) Bureaucratic corruption throws a spanner in the efficiency and objective of the subsidies
   (D) Some subsidies have some start- up economic logic
   (E) All of the above
3. The reasons the author puts forth as to why government of the day cannot totally eradicate subsidies is as follows 'EXCEPT'—
   (A) It would be a politically incorrect move
   (B) To satisfy and placate ulterior motives
   (C) According to the passage
   (D) The government would became highly unpopular
   (E) The targeted beneficiaries at least get some benefits
4. The reason why China has fared better than India is which one of the following:
   (A) A slight increase in inequality
   (B) Because of a so called trickle down mechanism
   (C) Superior levels of achievement in social sectors.

(D) The Chinese marched ahead in education.

(E) The Chinese did not adopt any pro-poor stance.

Answers

1. (D) 2. (A) 3. (E) 4. (C)

15. Lengthy Type Reading Comprehension

Passage 1

Words–1672

"IT is a statement I feel quite awkward in acknowledging". That is Ratan Naval Tata's immediate reaction when he is told that he has been voted India Inc's Best Brand Icon in the *Outlook Business-CNBC Universe Poll-the* result marks him out as the flag bearer of India's global aspirations. The response is not at all uncharacteristic of Tata. He is an extremely private person, and yet one who has managed to transform the Tata's from a Rs 13,000-crore asynchronous group into a Rs 129,994-crore cohesive global organisation in 16 years at the helm. "All that I have been doing, in my own little way, is to make the group more competitive and bold," he says.

Quite true to his nature, that also is an understatement. In the last few years, the group has wrapped up 35 cross border acquisitions, including same mind-blowing large deals that are now much too famous to require a mention here. International revenues now account far over 50 %of group revenues. (That figure was in single digits when he had taken charge.) And if there is one big reason why corporate decision makers judged him to be India's Best Brand Icon, it has gat to be his success in taking the group global. "Both Indian and global stakeholders are increasingly seeing us as a group that is making big plays, taking more risks than it was known to do in the past and managing large global takeovers with reasonable grace and finesse and, hopefully, success," Tata says.

The mind of A leader

There are many facets to Ratan Tata's global game plan his thought leadership-in identifying the need to go global very early on; his wisdom, in waiting to make the group more competitive before going in for the international push; his skill as a leader in making this theme resonate all over the group; the aggression with which he has won some of these cross-border deals; and his unshakable resolve never to compromise on the ethics and values that the group has cherished for over 100 years now.

The earliest evidence of Tata's thought leadership could be found in a document unofficially called the Tata Plan that he authored way back in 1983. Under the leadership of JRD Tata, the group got Ratan Tata (then Tata Industries' Chairman) to draw up a blueprint for the future. In it Tata recommended that the group 'seek substantial growth in international operations.' He also suggested restructuring the group to address the global opportunity better.

"Tata identified the theme of going global very early on, but his initial judgment was that the group was not yet ready to move on to this agenda," says Alan Rosling, Executive Director of Tata Sons. Rosling was hired personally by Tata in 2003 to lead the group's drive to internationalize. "To begin with, Tata focused on competitiveness. We have to earn the right to survive, he would say. Only when he judged that the group had moved to this position did he decide to stand up in 2003 to spell out the international agenda "Rosling adds.

A Man Of Ideas

Managers who have worked with Tata closely say that he has identified many such themes. Internationalisation was one. The push to hire young mangers at significant, decision-making levels across the group is another. The entire focus on the bottom of the pyramid-be it the one-Iakh car, budget hotels or low-end watches-is his idea. So is the focus on research and development. "No doubt, he has been a big influence on the group in the last three-four years," says R Gopalakrishnan, Executive Director of Tata Sons Rosling says, "he is a deep thinker, extremely strategic and long term. He is always two-three moves ahead."

And once he has identified a theme, he often leads by communication. He employs a very consultative style in seeding these ideas or themes into group companies. He encourages people to open their eyes to look at an opportunity and gets them to think differently about issues. but he will never tell them what to do. Often, he communicates by asking questions. "Why can't you" or "have you thought about this"- those are common phrases he employs. He will ask you questions that will lead to the theme. Rosling calls this the Tata way of "socialising ideas." Tata never imposes and never demands that people fall in line with his beliefs. Rather, he floats an idea, discusses and debates it and then allows managers to come up with what they would like to do about it. "He has had significant personal impact on the way the group has changed and internationalized, but he has done it through colleagues. That is what leadership is all about," says Rosling.

Be Bold

Admittedly, Tata has also been trying to increase the "dare quotient" of the group. He has been nudging his managers to be bold in their planning. This relatively new facet is perhaps best summed by the mindset with which Tata walked into the Corus auction. "Tata went into the auction with the intention to win," says Gopalakrishnan. "You are now beginning to see that attitude being reflected in group companies," he adds.

Tata encourages aggression among group mangers in many ways. To begin with, he is always encouraging

companies to think big and be bold enough to attempt the impossible. When such thinking leads a company to a cross-border deal, he makes himself available 24x7 to the CEO doing the acquisition. "When you come to him for a critical decision-which will always be in some negotiations-he will give you a very quick answer,' says Rosling. His responses would be crisp, leaving no room for doubt. The answers would be something along these lines: "Yes, I agree that we should offer this price" or "yes I agree we should withdraw" (the group has, done that on occasions.) "His involvement in cross- border deals could be quite significant," says Rosling. And that's precisely what gives the CEOs the confidence to move ahead without doubts.

Hands Off

Yet, in all this Tata never comes in the way of a manager functioning. Yes, he might step in to make a broad strategic adjustment, but he does not interfere in operational issues. Only if his help or input is sought in something specific does he come into the picture.

That can be said of his involvement in the global acquisitions as well. He is present as a member of the leadership team; he is not there in managing the process. He is available to CEOs as a sounding board, or to give advice. "That was precisely his role in the Corus acquisition," says Rosling. B Muthuraman, Tata Steel Managing Director was running the process, Arun kumar Gandhi, Executive Director, Tata Sons was in the negotiation and the bidding, Tata was there only to help with key decisions based on Muthuraman's recommendations.

Tata extends a similar philosophy into the way global acquisitions are managed and integrated into the group. Cultural compatibility is one big area in which due diligence is done before starting work on any cross-border deal. "This ensures that they are, in a manner of speaking, inclined to be in the Tata groove more readily," says Tata'. He prefers a non-prescriptive approach. "We do not take a William the conqueror approach to cross-border acquisitions," says Gopalakrishnan. "I have signed the cheque. So I am here to tell you how to handle things in the future -that's not what Tata believes in," he adds.

Man Of Integrity

A key issue that ensures cultural competency is ethics. This is where Tata has never diluted his value system. "I saw him stand by his principles, even though it cost him entry into the lucrative airline business," recalls a senior official of the group, referring to the jinxed Tata Airlines Singapore Airlines proposal of the late '90s.

"Tata has shown that there is no other way he will do business other than do it ethically," says Gopalakrishnan. He points to the Tata Finance episode (financial irregularities by senior company officials had led to the loss of few hundreds of crore) as an example. At that time, when the loss was yet to be ascertained (estimates ranged from Rs 500 crore to Rs 1,000 crare), Tata announced that the holding company would pump in the required money to prevent Tata Finance deposit holders or shareholders from suffering any loss. "By that one action, he gave a message that is far beyond all the speeches he could give in the next 10 years," he adds.

Tata lives by these high standards in the international arena, too. Before a meeting with the Prime Minister of a significant country, a senior group official suggested Tata lobby for a Specific proposal that could help the group in that country. "Tata declined," recalls the official who made the request. "Unlike global CEOs, who never hesitate to lobby with governments, Tata seldom asks governments for specific favours," he adds. This despite the fact that Tata is growing in global stature - among other things, he is an advisor to South African President Thabo Mbeki, the British government and Singapore's Economic Development Board on international investment related issues and he is on the board of Mitsubishi Corporation, American International Group and J.P. Morgan Chase. "He has a very positive global stature," says Gopalakrishnan. "That does help group companies." Many believe that Ratan Tata's global leadership is now reaching iconic proportions, at least among his Indian peers. Like all icons in big business, his personality is beginning to reflect on the group's reputation.

Again, true to his nature, rather than bask in the glory, he chooses to deflect the credit and attention showered on him away from himself. "I have spent a lot of time and energy trying to transform the group from a patriarchal concern into an institutionalised enterprise. It would be a mark of failure on my part if the perception gained ground that I epitomise the group's success," he says. Very few global business leaden are as selfless.

1. The author primary concern in the passage as for as Ratan Tata is concerned is—
   (A) Ratan Tata being named India Inc's Best Brand Icon in outlook Business CNBC universe poll
   (B) Ratan Tata's skill wisdom combined with his thought leadership method which made his company a global success
   (C) Tata's values and characteristic, chiefly accounts for his success
   (D) Ratan Tata having acquired 50% of group revenues in International acquisitions
   (E) None of the above
2. The title that would suit the passage beautiful and aptly—
   (A) Run Run Ratan Tata Internationally
   (B) Tata invites Globalisation
   (C) Global Leadership The Ratan Tata way
   (D) Strike while the Iron is Hot
   (E) Ratan Tata, the man of steel

3. The various sides to Ratan Tata's global game plan are all of the following 'EXCEPT'—
   (A) His thought leadership which identifying the requirement to go International
   (B) The need to wait and make the group more competitive prior to going global.
   (C) The skill to make his theme resonant thought the group
   (D) Unshakeable resolve once committed
   (E) Never to back down or compromise on ethics or values except when millions are at shake
4. Ratna Tata is a team player who encourages his mangers perform at their best. The best examples are as follows. All except one are false
   (A) The drive to his young manger
   (B) The been forces on research & development
   (C) The employment of a very consultative style
   (D) He demands that people fall in line with his beliefs
   (E) He allows his managers to debate, discuss and come up with solution to an idea he has brought up

Answers

1. (B) 2. (C) 3. (E) 4. (D)

Passage 2

Words–1973

The Forest Rights Act will come into forces on January 1, 2008. And on that sombre note the sun will rise on the dawn of the New Year, unknowing that the earth it bathes in its life-giving rays has inched a little closer to its death. The Act being yet another blow delivered on an already-Fractured scarred planet.

For the past three years , I have been writing an overview of wildlife issues that have dominated the year in this columm. Each year, I hope to strike an optimistic note in this season of goodwill and cheer; yet, on the contrary , the despondency only grows. This time I had hoped that pessimism would take a back seat with the outcome of the National Board of Wildlife meeting that took place in November 2007.

After months of dilly-dallying, a board comprising members known for waging battles for wildlife and taking on the Government- including the current one-was formed. After much delay-and attempts by the Ministry of Environment and Forests (MoEF) and the Prime Minister's Office (PMO) to preempt discussion on any critical and sensitive subject-its meeting took place on November 1.

Contrary to expectations, the outcome of the meeting was a pleasant surprise. The prime Minister took a keen interest, and many positive decisions were arrived at, an important one being the reconstitution and widening of the scope of the Standing Committee of the Board, which looks into matters concerning clearances for mining, industry, roads and other development projects in ecologically sensitive areas, including protected areas, and which had been reduced to a rubber stamp and a very diligent one at that. It was also decided to create a Tiger Protection Force and urgently appoint a senior police officer to head the dysfunctional Wildlife Crime Bureau.

Many focused sub-committees were appointed–one for the affairs of the tiger, another for the conservation of other equally endangered species like the hangul and snow leopard, and one dedicated to the conservation of marine species. A legal sub-committee to examine proposed amendments to the Wildlife (Protection) Act, and any other legislation that may have an impact on forests and wildlife, was also proposed. The Prime Minister also promised to make a special effort with State Chief Ministers to ensure the implementation of vital wildlife initiatives.

This was good, good news. A dream run. A positive meeting, with an outcome and decisions that actually held the power to-if not halt-at least mitigate the absolutely catastrophic situation of wildlife in India. One should have been glad. Or at least cautiously optimistic. I was, brushing aside my innate cynicism and worrying that there was a huge gap between intent and implementation.

We celebrated too soon. First came the news that the minutes of the meeting were being tweaked and hijacked by the MoEF to suit their interests. Then came another shock .

To elaborate, one of the most vital decisions taken at the meeting was the appointment of a committee that would look at the adverse impacts of the Forest Rights Act on forests and wildlife (which have been well-documented and established, but put in a nutshell, the Act will override all wildlife legislation and open up new channels for destructive commercial exploitation, and fragment wildlife habitats, including protected areas)

The committee was to suggested mitigating measures before the rules are notified. The Prime Minister rooted for this, even as the MoEF secretary, who in her previous stint served in the Tribal Affaires Ministry, opposed any such move.

How do we have the faith, Mr Prime Minister, when not more than a month after you promise that the impact of the Act would be studied and mitigating measures considered, the Act will now come into force-post hast-from January 1, and tigers be damned?

One can understand, of course. This was political double-speak at its best. It is the politics of votes that holds sway, not conservation. Why would the UPA delay the Act when its chairperson, Sonia Gandhi, uses it as her trump card while canvassing for tribal votes? Why would they not bring it into force when the CPI(M), without

whose support they lose power, puts pressure to enforce the Act ? They won't. Not with elections around the corner. Most political parties are squabbling over "taking credit" for the Act. The UPA chairperson, in her capacity as Congress president, campaigned in Gujarat promising tribals that they would get land under the new Act.

The Gujarat government also handed over *pattas* to 30 tribal families, promising 2,204 tribals that they would get their share soon till–fortunately–it was restrained by the Supreme Court. In the Gujarat election battle ground, the Forest Rights Act became an instrument to woo the vital tribal vote bank.

It is worse in Andhra Pradesh. The Left parties, along with hundreds of locals armed with axes, entered and vandalized Kawal Wildlife Sanctuary, indiscriminately felling trees. Around 631 acres of thick forest were destroyed and encroached upon. The rationale behind this devastation was the anticipation of the Forest Rights Act.

This sets the tone for the coming year. The battle ahead has only become tougher. And while this piece of news almost overwhelms other issues, let's do a quick, and by no means exhaustive, assessment of the issues of the past year.

Let's begin with tigers. Now we know–officially-that there are fewer than 1300 tigers in India. And declining. The tiger mortality for just this month is indication enough of the extent of damage.

A tiger was injured, and met an agonising death, after being hit by a speeding vehicle on the road that cuts through Dudhwa Tiger Reserve in Uttar Pradesh on December 4.

Only one day prior, three tiger skins and three skeletons were seized in Allahabad. Going back a little, in the space of a month, two tigers were caught in iron traps of the heart of India's 'best managed' tiger reserve, Kanha, while 20 such traps were recovered.

India's 'Tiger State', Madhya Pradesh, isn't safe for tigers either: their numbers have officially halved to a mere 300. And Panna, we regret to announce, is almost another Sariska, with no sign of a tigress in the park for the past six months. Sightings, pugmarks, scat and other clues that point to a tiger's presence are getting thinner by the day. But the Forest Department valiantly assures that 30 tigers are in the reserve.

Expectedly, man-animal conflict has been increasing. And as we encroach on, destroy, degrade and fragment habitats, the conflicts only escalate. Recently in Talodhi, near the Todaba Andhari Tiger Reserve, there were eight tiger attacks on humans-four of them fatal. So the tiger was declared a man-eater, and shot. Yet the killings continue. Another man lost his life after they slaughtered the supposed man eater. Was this a tragic comedy of errors ? The stage is now set of course for another tiger to be the target; and we will lose yet another big cat from the rapidly dwindling population.

In another macabre fallout of the man-animal conflict, a bear and her two cubs were axed to death by the irate residents of Bedar village near Poonch in Jammu & Kashmir on October 28. This was yet another bear bearing the brunt of deforestation, and consequent man-animal conflict in the State. Of leopards badgered, burnt and killed across the country, one has lost count.

The tiger, of course, has not been the only species that was the target of the poacher's gun. This year also saw a breach in the excellent protection according to rhinos in Kaziranga over the past few years. Seventeen Greater one-horned rhinos fell to the poacher's gun in 2007. Another victim was the beleaguered Asiatic lion. Barely 325 of them survive in the world today, and 17 were lost from January to April-half of them to poaching. In earlier years, lions have been killed over man-animal conflict, and the odd one to feed the domestic market for its claws. But the spate of killings this years was different-the lion was now being slaughtered and sold in the animal markets of China and the Far East in the same way the tiger is-for its derivatives to be used in traditional medicine.

On December 13,65 skins were seized-one tiger skin, 21 leopard skins and 43 otter skins-in northern Karnataka. All this only prompts a person to ask one question—

Where is the long promised Wildlife Crime Bureau ?

And what does one do when the enemy is within ?

According to reports, the Chhattisgarh Forest Department has come up with a plan to cut down over 20lakh old-growth trees and replace them with monocultures of teak and eucalyptus. This has already started in Sarguja and Koria districts.

Another disastrous decision that escaped public eye was the denotification of the Saraswati Wildlife Sanctuary, a rare haven in Haryana of about 200 hog deer and other species. They wanted to build a canal through it, and when that was proving difficult because of the laws, the sanctuary was denotified. In another part of the country-Madhya Pradesh-the forest Department wants to start trophy hunting. How they propose to issue licenses a country where the state of wildlife monitoring and the enforcement of its laws is abysmal is not known.

Another very distressing story was the proposed thermal power plant in Chamalapura in south Karnataka. This borders Bandipur Tiger Reserve and Nagarhole National Park-prime tiger habitat, and home to other rare flora and fauna. The plant is a disaster- it will require a million litres of fresh water from the Kabini daily, and will discharge huge amounts of fly ash, sulphur dioxide nitrogen dioxide and carbon dioxide. The impact will also be felt in Mysore, 35km away. It will also displace over 20,000 people. The Karnataka Government seems to be on a suicidal path-they have proposed a network of 73 mini hydel projects across the dense forests in the Western Ghats. The destruction is evident in the Kempholey

Reserved Forest, Where three major roads and a concrete weir dam has been constructed in the biodiversity-rich area. Unfortunately, there is no dearth of such destruction: construction of ports and shipping channels threaten myriad marine species and roads, railway lines and canals cut through protected areas. The list is endless.

But there has been good news as well. This year saw the discovery of some new wildlife species.

A new bird *Liocichla bugunorum,* a kind of babbler, was discovered in May in the Eaglenest Wildlife Sanctuary in Arunachal Pradesh. Two new caecilians-important indicators of healthy ecosystems and the least know and studied of the amphibians-were discovered in the Mahadeyi forests of Goa. These are the *Gegeneophis goaensis* and *Gegeneophis mhadeiensis*. Incidentally, the region it was found is now threatened by a proposed dam. A new species of spider, yet unnamed, was found in Melghat. All of which only serves to underline the importance of preserving our last remaining wild habitats.

Viable tiger populations and habitats have also been mapped-Corbettrajaji, Karanataka-Kerala-Tamil Nadu which includes the protected areas of the Bhadra-Bandipur-Bhimgrir-Nagarhole-Madumalai-Wayanad corridor, and the Satpura belt whichb connects Melghat-Tadoba-Satpura-Pench-Kanha.

These need to be protected against all odds, and kept free from any biotic pressures and encroachment. There are solution as has been demonstrated in Bhadra and Melghat Tiger Reserves, and Rajaji National Park, where forest dwellers have been successfully rehabilitated in a painstaking process.

Finally, it was Al Gore and the International Panel for Climate Change jointly winning the Nobel Prize for Peace, indicating that we had finally taken in the horrific impacts of global warming, and recognized those who worked for its awareness and showed the path for mitigation .

We cannot reverse the clock, cannot wipe away the horrors on the earth. But surely we can start to heal.

Else soon in the future, we do not live to see another day.

1. Choose the most suitable and appropriate title from the following—
   (A) Forest Act, Jungle Law
   (B) Wildlife Sorry State
   (C) Cat Act –Reaction less
   (D) A Catty Affair
   (E) Cat and Mouse Politics

2. Why is the author dismayed and saddened by the Forest Act which comes into force on 1 Jan., 08 ?
   (A) The Act will provide loopholes in all wildlife legislation leaving it wide open for destructive commercial advantage in addition to scattering wildlife habitats protected and unprotected
   (B) The P.M. took an eager interest, and several positive decisions were made
   (C) The reconstitution and broadening of the standing committee of the board
   (D) The creation of a Tiger Protection Force
   (E) Sub-committees were formed for the affairs of the tiger and the conservation of other equally endangered species like the Han gal and show leopard

3. When the author mention that it was “political double–speak at its best” he implies that:
   (A) The MOEF tweaked and hijacked the meeting minutes
   (B) The Wildlife Act was just another political lie
   (C) The Forest Act was merely a pawn in the game of politics to be used as a deceptive trump card to gain tribal votes and please their coalition partners
   (D) The MOEF and the PMO transpired to destroy all hopes saving Wildlife
   (E) None of the above

4. The tone of the passage can best be described by which of the following—

   1. Cynical 2. Instructive
   3. Disheartening 4. Approving

   (A) 1 and 2 (B) 2 and 4
   (C) 1 and 3 (D) All of the above
   (E) 1, 3 and 4

Answers

1. (A) 2. (A) 3. (C) 4. (C)

Passage 3

Words–1371

It’s 10 in the morning, and at the historic Azad Maidan, where The Mahatma used to address swelling crowds during the Civil Disobedience movement, another movement is slowly gathering steam. It's Mumbai's first major protest rally against corporatisation of the retail sector. 'Mike testing' is on as a sprinkling of protestors waits patiently under the sweltering sun. There are more television reporters and cops (two truckloads of the latter) than protestors waiting for the action' to begin. In sync with the historic nature of the venue, there are banners bearing the images of the founding fathers of the country-Gandhi, Jawaharlal Nehru, Bal Gangadhar Tilak, Subhash Chandra Bose and even Bhagat Singh. The organisers of the event had 'promised' 50,000 protestors but less than 50 are around. Half an hour later, 50 more join exerting their vocal cords. Just as the camera crews were getting restive, the 'action' begins.

Almost out of the blue, they came-petty shopkeepers, hawkers, pushcart vegetable vendors, chemists, onion

merchants and manual labourers. They came packed in buses and trucks, in motorcycle cavalcades from all over Mumbai and nearby districts like Thane, Pune, Raigad. Passengers in Mumbai

local trains complained of surging crowds and the occasional unruly behaviour. At the venue, it was 'byte hunting' time. A local leader spews hyperbole and spittle, in equal measures. "East India Company came to India as a trading company and then they took over the country. Today, we are an independent country and are inviting MNCs like Wal-Mart to take over the country," he proclaims.

But that was just the beginning. Nearly two dozen people mount the makeshift dais, including the anti-retail movement "champion" from Uttar Pradesh and Samajwadi Party MP, Banwari Lal Kanchal. "Why do these companies that own

refineries and sell cloth, petrol and cellphones want to sell onions and potatoes? thunders Shyam Bihari Mishra, a former BJP MP in a not-so veiled reference to Reliance Industries and Bharti Airtel. As the decibel levels of the speakers soar, impassioned zindabads fill the air. "People ask me why we use violent methods," asks Kanchal who trashed- Reliance Fresh stores in Uttar Pradesh. "Killing somebody is violence. Breaking the door of a shopping mall is not violence. Until you break their malls and burn their goods, nobody is going to listen to you," rationalises Kanchal.

By the time the speeches came to end, more than 5,000 people had participated in the protest-a far cry from the 50,000 mark but still ominous enough. The protest has been organised by Vyapar Rozgar Suraksha Kruti Samiti, in association with a dozen other associations such as Federation of Association of Maharashtra (FAM) and NGOs like India FDI watch. These bodies are protesting a list of things-from the entry of corporate in retail to Foreign Direct Investment (FDI) in retail to implementing a national policy *on* hawkers. Another demand of the protestors is scrapping the Model APMC Act. Like the President of Federation FAM, Mohan Gurnani, says: "We want the amendments to the Agricultural Produce Marketing Committee Act that allows corporate to buy products from farmers and sell them directly to the consumers."

Big is Bad

Having said that, the protestors' ire seems to largely focus around the two corporate - Reliance's Fresh outlets and Bharti's tie-up with the international poster boy of Big Bad Retail Business, Wal-Mart. Reliance has had its stores vandalised in Orissa, Uttar Pradesh and Jharkhand. In UP, the company was forced to down shutters on all its 23 Reliance Fresh outlets and let go of 235 or so employees. Protests have also been held against Reliance Fresh in West Bengal, Madhya Pradesh, Jharkhand and Kerala. In all, Reliance has opened 300 stores in 30 cities and 12 states, since its launch in November 2006. The Bharti-Wal-Mart alliance is yet to start operations and the first store may be launched only in the latter half of 2008.

Other retailers like the future Group and Subhiksha that are rapidly expanding across the country have so far managed to escape the wrath of the protestors. "We are not into the fresh vegetables business in a big way. Also, as a policy we do not sell anything below the cost price," says Future Group CEO, Kishore Biyani. Chennai-headquartered Subhiksha Trading Services has faced protests in the past from chemists who claim that the retailer has been selling drugs at prices lower than the market rates. Subhiksha Managing Director R. Subramanian, however, is confident about his expansion plans, including UP where the retailer has 60 outlets. "At the end of the day, organised retail is a great thing for consumers. Of course, we have to take care of all the stakeholders involved," says Subramanian.

It's a point that RIL Chairman Mukesh Ambani was at pains to point out at the company's recent AGM. "Our organised retail initiative is configured to increase income in the hands of the farmers and serve consumers by improving supply chain and distribution efficiency. We want to achieve these twin goals by reducing wastage," he said. Considering that 30 to 35 per cent of 60 million tonnes of fruits and vegetables in the country-in value terms about Rs 58,000 crore and more than the fresh fruit produce of the UK-are wasted due to a lack of storage and other facilities, organised retail is not the evil business it is being made *out* to be. "At the end of the day, India's retail market is so huge that there is place for everybody, including smaller players," says Subhiksha's Subramanian.

The Indian retail sector is *esti*mated to be around $328 billion, with less than 4 per cent being accounted for by organized retail. Subramanian believes that the present conflict is one largely caused by perception. "When a retailer says - that they are going to make investments to the tune of several thousand crores of rupees, it raises everybody's eyebrows. That raises the fear levels of some of the smaller players," adds Subramanian.

It's a problem that Ambani seems to be conscious *of*. "We are sensitive to the interest of small shopkeepers. Our retail initiative will in no way jeopardize their interests and that of small vendors who service customers," said Ambani, who also cited the company's purchases of banana crop as an example of how Reliance retail venture could benefit farmers. "Reliance is bringing to farmers, to begin with, in Gujarat, Maharashtra and Andhra Pradesh, high quality tissue cultured banana plants that yield 35 to 40 kg per bunch of fruits as against 20 with conventional cultivation Reliance buys these banana from farmers at prices that are 10 to 15 percent higher than what they get through conventional channels headed.

But the problem for players like Reliance might lie beyond just flawed communication or perceptions it's that ugly word called politics The problems that unlike most of its other business the retail business opens up a thousand pressure points for reliance as it involves a lot of on-the–ground activity. And people (read politicians) sense an opportunity to pressurize the company and make money" says a senior executive of a retailing group, requesting anonymity.

Observe have been surprised by the lack of serious support for Reliance from other retailers. "Earlier, the industry lacked consensus and bickered on key issues like FDI. Reliance took a tough stance against FDI in retail claming that it would harm the domestic retail, especially small traders. Now they cannot just turn around and ask for support from other players or the retail industry body. As for the small traders he does not care who he is killed by- the MNC or a large Indian corporate," says the head of another retailing major. However, industry association CII has condemned closure of Reliance Fresh stores in UP and warned that such orders will adversely affect the poor.

For now the protesters have set a deadline January 26, 2008.If by then the government does not heed to their demands, they plan to take the stir nationwide and do not rule out trashing malls and retail outlet with mid-term polls still likely, this is one emotive issue that will not die silently. Unfortunately in the process, India–farmers, consumers and even small shopkeepers included–will lose time and an opportunity to benefit.

1. The most appropriate and suitable title for the passage would be—
   - (A) Retail rage
   - (B) Much ado about nothing
   - (C) A storm in a tea cup
   - (D) Politicians go vegetarian
   - (E) Small price-big scale

2. The author when he quotes an anonymous sr. executive writes, "The problem for player like (Ambani) Reliance might lie beyond communications or perceptions. It's that ugly word called politics" unquote, it implies that
   - (A) The company will be exposed to pressure which will open the gateways to bribery
   - (B) Politicians can use it as then U.S.P. for the forthcoming electrons
   - (C) Small retailers will take advantage of the situation and yell "Autocracy of Retail"
   - (D) Ultimately it is the protestors themselves who will suffer
   - (E) Politicians are braying for reliance's blood as they are jealous of his power and wealth

3. In the last paragraph of the passage the author suggest all of the following EXCEPT—
   1. In the event of mid-term polls such a contentions and emotive issue will not be solved in quick time.
   2. A nationwide stir involving violence & destruction of malls and retail outlets will follow is demands are not met.
   3. The consumers, farmers and small time shopkeeper be the brunt of the agitation.
   4. Reliance will eventually beat the odds.
   5. The issue is only a storm in a tea cup

   (A) 1 and 3 (B) 2 and 3

   (C) 4 and 5 (D) 1, 4 and 5

   (E) 3, 4 and 5

4. What are the benefits of organised retail according to Mukesh Ambani CMD of Reliance industries ?
   1. The farmer will earn less that smaller players.
   2. Farmer and consumers will benefit.
   3. Improvement and efficiency of supply & distribution.
   4. High quality issue cultured plant to upgrade yield double fold.
   5. Wastage can be reduced enormously through proper storage & like facilities.

   (A) 1, 3 and 4 (B) all except 1

   (C) 1, 2, 3, 4 (D) 1, 3, 4, 5

   (E) 1, 2, 4, 5

Answers

1. (A) 2. (A) 3. (C) 4. (B)

Passage 4

Words–778

Not until a year later, however, in March 1974, did the Pentagon finally admit to having deemed it necessary -if not nice-to fool with Mother Nature over Laos, North Vietnam and South Vietnam, from 1966 through 1972. Defence department officials made the admissions at a briefing of the Senate Foreign Relations Committee. They said that cloud seeding project-in connection with the emplacement of electronic and chemical sensing devices-had succeeded stanching North Vietnamese infiltration down the Ho Chi Minh trail especially in the summer of 1971. But they denied allegations that their cloud seeding had been responsible for the devastating flooding of North Vietnam in the fall of that year. The Pentagon people pointed out that cloud seeding had been the object of civilian R and D for many years, and that the military had simply found it to be compatible to be the cause of the war. What they did not emphasise was that "technology" has now proceeded to the point that not just

rainshowers but torrents can be triggered : that entire continents can be targeted for catastrophic cloudbursts.

During the senate debate of 1974 on the military procurement bill for fiscal 1975, Senator Gaylord Nelson of Wisconsin introduced a floor amendment prohibiting weather modification as a weapon of war. The senate passed it, but it was deleted in conference with the House. A short time later, the Senate Armed Services Committee asked Dr. Stepher J Lukasik, director of the Pentagon's Advanced Research Projects Agency (ARPA), whether the Defense Department was pursuing any R & D that might be contrary to the intent of the Nelson amendment.

"There is no such work in ARPA", said Dr Lukasik, "but I am not familiar in detail with the totality of activities in the defence department. To the best of my knowledge, there is nothing going on in the department that is in conflict with that amendment." The committee then asked Lukasik to look into the matter and provide a statement for the' record. T' statement, when it came, was a masterpiece.

"Department of defence research in weather modification is entirely unclassified and does not involve any unique techniques not known to the civilian community. Research is pursued to develop means for protecting military personnel and resources and to prevent technology surprise, this research is thus not contrary to the intent of the amendment referred to".

What it comes down to was that the Senate committee actually had no power to force the Pentagon to account fully for its actions with respect to an amendment which had not become law. And the Pentagon did not. It is questionable in fact, that the Pentagon would have responded fully even if the amendment had become law. There is plenty of precede for the Pentagon not having done so.

In mid-October 1974, the Soviet Union introduced a little noticed draft convention at the United Nations "banning the modification of the environment and the climate for military and other purposes incompatible with the interests of international security, the well-being and health of people". More specifically, the Soviets proposed outlawing all modifications by man of "the surface of the land, the floor of seas and oceans, the earth's interior, water, the atmosphere or any other element of the natural environment". This would include, they said, cloud seeding or any other means of inducing precipitation or redistributing water resources : artificial engendering of seismic waves : creating electromagnetic and acoustic fields in the ocean : disturbing the natural heat and gas exchanges between the hydrosphere and the atmosphere, or the heat al radiation balance among earth, atmosphere and sun.

To the Pentagon, this was nothing more than a typical Russian ploy. Whether the Soviets were ahead or behind in the research and development of ways to prostitute the elements, they stood to gain. If they were ahead, the enactment of prohibitive international law would permit them to stay ahead. If they were behind-as was probably the case-they could simply break the law and work to catch up. The latter hypothesis presupposed, of course, that the Pentagon would not break the law and would disband its R & D.

So far, there is nothing at all to suggest that either the US or the Russians or the Chinese have halted their weather experiments. While it does not mean that the longest drought that India faced at the turn of the decade was not because of the EI Nino effect but because of the S-curve in a wave unleashed by a Chinese "Weatherman" missile, it does mean that by the end of this decade it could really happen.

1. *The term "ploy" in the passage specifically refers to—*
   - (A) The US undertaking.
   - (B) Prohibitive international law.
   - (C) A typical Russian strategy.
   - (D) Pentagon's defence R&D.
2. *The author is least likely to agree with the statement that—*
   - (A) On the military procurement bill for fiscal 1975, senator Gaylord Nelsons introduced an amendment prohibiting weather modification as a weapon of war.
   - (B) Dr. Lukasik of ARPA was asked whether Defence Department was pursuing any R&D work contrary to Nelson amendment.
   - (C) The Senate Committee had no power to force the Pentagon to account for its actions in respect of an amendment which had become law
   - (D) It is questionable that the Pentagon would have responded fully even if the amendment had become law.
3. *The term "engendering" in the passage specifically refers to—*
   - (A) Electromagnetic and acoustic fields
   - (B) Inducing precipitation or redistributing water resources
   - (C) The act of inducing earthquakes
   - (D) Gas exchanges between the hydrosphere and atmosphere
4. *The authors personal opinion about the statement of Dr Lukasik is—*
   - (A) Extremely sarcastic
   - (B) Extremely positive and benevolent
   - (C) Very cautious and circumlocutions
   - (D) None of the above

Answers

1. (C) 2. (C) 3. (B) 4. (A)

Passage 5

Words–1226

Naveen Jindal, 37, gives off a quiet aura. Yet, beneath the exterior lies the young politician's urge to transform India into a wealth of opportunities. His Lok Sabha ticket ensures that he is in the right profession to carry out the dreams of millions. To boot, Jindal, is the Executive Vice Chairman and Managing and Director of Jindal Steel and Power (JSPL), part of the $ 6 billion Jindal Organisation.

With his credentials in place, his belief that the time is ripe for the nation's youth to have their tryst with destiny, will, no doubt, find many takers. Standing in the way of Jindal are, however, only achievable milestones-no hurdles. The way Jindal sees it, there is no room for any morale-busting shortcomings like corruption or poor infrastructure, precisely because they ought to be the last thing on the mind of the youth-a motto that is borne out by examples from his own political career.

After joining the Lok Sabha, representing Kurukshetra constituency from Haryana, he played an active role in having smoking banned in the Central Hall of Parliament in July 2004. He is also responsible for duty-free shops at the country's international airports accepting Indian currency from September 2005. Jindal reckons that efforts directed towards the people of the country should not be confined to the upper crust.

The country's booming growth, he believes, should reach all the way down to the underprivileged classes as well. For the youth living on this side of wealth, "good quality education," especially primary education, should be given. Says Jindal : "The need is not for more policy decisions but for streamlining implementation. Our delivery mechanism has failed us completely and we have to focus on making it more efficient." The school-level education is the most important, as this is where the foundation is laid, and after school the youth should be in a position to decide on what to do next. Therefore, there should be a number of vocational and technical training courses provided for the next stage, Jindal feels. "We have to also focus on developing our infrastructure," he adds. It is a question of becoming self-reliant at a young age and standing up for what one represents. As a member of the Congress youth brigade that includes Rahul Gandhi, Sachin Pilot and Jyothiraditya Scindia, Jindal is all too aware of the responsibilities of a young politician in carrying out his agenda to a picture-perfect finish.

Initial Trigger

Jindal's decision to enter politics may have had its origins in an amusing incident that was played out over 10 years-which Jindal, however, puts down to his father's influence. His student years in the USA, where Jindal did his MEA from the University of Texas and Dallas, were the most inspiring of days. Taking a cue from the Americans whom Jindal found to have a weakness for raising their national flag, Jindal sought to do up his own company building in Raigarh with the tricolour in 1992. The result was anything but mild.

The next thing he knew the government had slapped a case on him, as the Flag Code of India didn't allow the flying of the flag on non-governmental institutions. Undeterred, Jindal fought to defend the rights of a citizen who was only raising his national flag in respect. After all, he was hoisting it in his own country. Jindal was to have the final laugh in the matter. After nearly 10 years of waiting the Supreme Court handed out a verdict in Jindal's favour. Jindal had scored a major and crucial victory. Lessons learnt ? Well, for one thing, Jindal found himself at the receiving end of a groundswell of support. It convinced him more than ever before that if he held on to what he believed in, then he might not be alone in his mission.

If there is another lesson that Jindal has learnt, it is from his father who had entered politics at the age of 60. Unlike his father, he wasted no time in giving himself up for public service. If politics, which Jindal has identified as giving him the right platform to launch his reforms, is the way forward, then he believes in joining when one is young. His desire to reach Parliament through the Lok Sabha, rather than the Rajya Sabha, was a reflection of his resolve and capacity for hard work.

Business As Usual

On the business front, Jindal sees rapid growth with constantly changing paradigms. There is growth in the industry and the challenge is to keep on growing and streamlining all aspects of business, be it sourcing of raw materials or logistics, having a really good team, adopting the latest technology or an efficient production system. People have to be ready to embrace changes. They have to constantly innovate and adapt to the changing times. "But at the same time we must focus on keeping one's core competence, core values and impeccable reputation untouched," he warns. Incidentally, when Jindal had taken over the reigns of Jindal Organisation's Raigarh operations, which had sponge iron as its backbone, it was in the red. Soon, by adopting global economies of scale, Jindal was able to ring in changes and make it the largest coal-based sponge iron manufacturing capacity in the world. It is to Jindal's credit that his steel company is a producer of low-cost sponge iron. Taking the forward integration path, he set up the Rail and Universal Beam mill.

Right now, JSPL has a capacity of 3 million tonne and Jindal wants it to go up to I5mt in the next 10years. Future plans include setting up 6 mt plants in Orissa and Jharkhand. Besides having a number of other projects in the pipeline, Jindal already has his hands full with his $2. I-billion iron and steel project in Bolivia. Jindal's power business has a capacity of producing 1,500 MW which he is keen on increasing to 10,000 MW in the next decade.

Being part of a traditional family-run business, he says that his approach has been to change with the evolving competitive scenario. He understands that though in the early days family-run businesses ran a tight ship, now there has to be more delegation of authority if systems are to work properly. More importantly, corporate governance should be given more elbow room and there has to be a better team effort. As a polo player, Jindal knows all too well what it takes to collectively work for a common goal.

The Divide

Shuttling back and forth between politics and business is hard. But helping Jindal manage both ends of a difficult career is his devoted team. He concedes that more than half of his time is taken up by politics and what keeps him going is an undying passion for the people of his constituency. One of his priorities has been on providing toilets to each household under the total sanitation scheme.

"This, I believe, provides dignity to the people, especially women. Apart from this, I am focused on providing health care. I have lately got involved in helping the Bharti Group's corporate social responsibility initiative of opening 40 primary schools in my constituency. My role has to be in facilitating their mission," he says. Besides the hard-nosed drive of a businessman, there is a humanist in Jindal that is trying to break the mould of a conventional do-gooder"

1. From the passage the reader can conclude that the author is chiefly concerned about—
   (A) Corruption and pathetic infrastructure
   (B) The country's flourishing growth should reach the underprivileged classes too.
   (C) A young politician dream to transform India into a treasure trove of opportunities
   (D) Increasing vocational and technical training course.
   (E) The nation's youth trust with destiny
2. Naveen Jindal believes in all of the following measures for the underprivileged to get the benefits of India's Booming growth 'Except'—
   (A) Our delivery system should be made more efficient-productive
   (B) Good quality education must be provided
   (C) They should be groomed to carry out responsibilities to a picture perfect finished responsibilities to a picture perfect finished
   (D) Vocational and technical training must be provided after school level education
   (E) Laying emphasis on developing our infrastructure
3. The most appropriate title for the passage would suitable be—
   (A) Stand up for your rights
   (B) Youth + opportunities = success
   (C) Equip the youth-Conquer the World
   (D) Stand up for what you are
   (E) Charge of the youth Brigade
4. Naveen Jindal has marked his presences as a serious politician By fighting against all odds and succeeding by a standing up for what he represent a few examples are—
   1. He played an active role in having smoking banned in the Central Hall of Parliament.
   2. Winning a case against the government which 10 years later changed the flag code Allowing citizen of India to raise the National flag in non–government institution
   3. His desire to reach parliament through the lok Sabha rather than the Rajya Sabha.
   4. He took over the Raigarh operations which was in the red and made it the largest coal-based sponge iron manufacturing capacity in the world.

   Choose the correct sequence—
   (A) 1, 2, 3 (b) 1, 2, 3
   (C) 2, 3, 4 (d) 1 and 4
   (E) 2 and 3

Answers

1. (C) 2. (C) 3. (D) 4. (B)

Passage 6

Words–1810

In explaining Hume's critique of the belief in miracles, we must first understand the definition of a miracle. The Webster Dictionary defines a miracle as: a supernatural event regarded as to define action, one of the acts worked by Christ which revealed his divinity an extremely remarkable achievement or event, an unexpected piece of luck. Therefore, a miracle is based on one's perception of past experiences, what everyone sees. It is based on a individual's own reality, and the faith in which he/she believes in, it is based on interior events such as what we are taught, and exterior events, such as what we hear or see first hand. Hence studying Hume's view of a miracle, he interprets or defines a miracle as such; a miracle is a violation of the laws of nature, an event which is not normal to most of mankind. Hume explains this point brilliantly when he states, "Nothing is esteemed a miracle, if it has ever happened in the common course of nature. It is no miracle that a man seemingly in good health should die on a sudden." Hume states that this death is quite unusual, however it seemed to happen naturally. He could only define it as a true

miracle if this dead man were to come back to life. This would be a miraculous event because such an experience has not yet been commonly observed. In which case, his philosophical view of a miracle would be true. Hume critiques and discredits the belief in a miracle merely because it goes against the laws of nature. Hume defines the laws of nature to be what has been "uniformly" observed by mankind, such as the laws of identity and gravity. He views society as being far too liberal in what they consider to be a miracle. He gives the reader four ideas to support his philosophy in defining a true miracle, or the belief in a miracle. These points lead us to believe that there has never been a miraculous event established. Hume's-first reason in contradicting a miracle is, in all of history there has not been a miraculous event with a sufficient number *of* witnesses. He questions the integrity of the men and the reputation which they hold in society. If their reputation holds great integrity, then and only then can we have full assurance in the testimony of men. Hume is constantly asking questions to support proof for a miracle. He asks questions such as this; Who is qualified ? Who has the authority to say who qualifies ? As he asks these questions we can see there are no real answers, in which case, it tends to break the validity of the witnesses to the miracle.

Hume's second reason in contradicting the validity of a miracle is that he views all of our beliefs, or what we choose to accept, or not accept through past experience and what history dictates to us as discreditable. Furthermore, he tends to discredit an individual by playing on a human beings' consciousness sense of reality. An example is; using words such as, the individuals need for "excitement" and "wonder" arising from miracles. Even the individual who cannot enjoy the pleasure immediately will still believe in a miracle, regardless of the possible validity of the miracle. With this, it leads the individual to feel a sense of belonging and a sense of pride. These individuals tend to be the followers within society. These individuals will tend to be believers rather than be the leaders in the society. With no regard to the miracles validity, whether it is true or false, or second hand information. Miracles lead to such strong temptations, that we as individuals tend to lose sense of our own belief of fantasy and reality. As individuals we tend to believe to find attention, and to gossip of the unknown. Through emotions and behavior, Hume tends to believe there has been many forged miracles: regardless if the information is somewhat valid or not.

His third reason in discrediting the belief in a miracle is testimony versus reality. Hume states, "It forms a strong presumption against all supernatural and miraculous events, that they are observed chiefly to abound among ignorant and barbarous ancestors; or if civilized people has ever given admission to any of them, that people will be found to have received them from these barbarous ancestors, who transmitted them with that inviolable sanction and authority, which always attend perceived opinions." In any case many of the miraculous events which happened in past history would not be considered a miracle in today's world, or at any other time in history. The reality most people believed at that period, as a result can be considered lies or exaggerations. Hume discredits the miracle as to the time period in which the miracle is taking place, the mentality, or the reality of individuals at that given time. Hume suggests that during certain times in history we are told of miraculous accounts of travellers. "Because we as individuals love to wonder, there is an end to common sense, and human testimony, in these circumstances, loses all pretensions to authority."

The final point Hume gives to discredit the validity of a miracle is that there must be a number of witnesses to validate the miracle. "So that not only the miracle destroys the credit of testimony, but the testimony destroys itself". This basically means that the witnesses must all give the exact same testimony of the facts of the event. Hume finds difficulty in the belief or integrity of any individual, and the difficulty of detecting falsehood in any private or even public place in history. "Where it is said to happen much more when the scene is removed to ever so small a distance." A court of justice with accuracy and judgment may find themselves often distinguishing between and true and false. If it is trusted to society through debate, rumors, and man's passion it tends to be difficult to trust the validity of the miracle. Throughout the rest of the readings, Hume states a few events which many believe, are miracles. He discredits many of these miracles through his critiques. I have chosen to illustrate two "so-called" miracles from the New American Bible and to show how Hume would view these miracles. The stories are of Noah's Ark and The Burning Bush. The story of Noah's Ark took place when the Lord began to realize how great man's wickedness on earth had become. He began to regret the fact that he had created man on earth. The lord decided the only way to rid earth of the wickedness would be to destroy all men, and all living creatures living on the earth. The only men which he would not destroy were to be Noah, his sons, Noah's wife and his son's wives. He also would save a pair of animals of each species. The rest were to perish from the earth. He chose Noah to be the favored one to carry out the task. The Lord requested Noah to build an ark and explained exactly how it was to be made. Noah spent six hundred years of his life building the ark which God insisted upon. When the ark was finally complete The Lord told Noah it was time to gather the selected few as the floods were about to come. These floods lasted forty days and forty nights. The floods wiped out all living creatures on earth, except all on the ark. In the six hundred and first year *of* Noah's life the floods stopped and the earth began to dry. Noah then built an altar to the Lord and choosing from every clean animal he offered holocaust on the altar. As God states "Never again will I doom the earth because of

man, since the desires of man's heart are evil from the start; nor will I ever strike down all living beings, as I have done."

In deciding upon whether this is a valid miracle in Hume's opinion of miracles, I believe he would consider it to be a miracle but, would have a hard time validating the testimony of it. The reasons in which he would criticize the validity together the testimony would be as follows. The testimony versus the reality. To further support the theory he would argue the time period in which the miracle had taken place. And would find it difficult to believe without a reasonable doubt. There is a question to whether it could be lies or exaggerations. Furthermore, it could not possibly be a validated miracle considering the amount of men who witnessed the event, as well as the integrity of the men. Although this mirade was an act of God we can still question the validity of our ancestors or God for that matter. Hume would not be satisfied not only with the integrity of the individuals but the amount of witnesses at the given time. Therefore, we can only view this as a miracle depending upon our own individual perceptions of what we believe to be true. This leads to a non uniform event, since we as individuals hold different beliefs of what we hold true, and false.

The second miracle which I will discuss was that of Moses and the burning bush. As Moses was working in the fields an angel of the Lord appeared to him in a fire flaming out of the holy bush. Almost amazing the bush was full of flames but was not yet consumed. As he walked closer he heard the voice, the voice of God telling Moses he was the chosen one to take the Israelites out of Egypt away from the cruel hands of the Egyptians. In disbelief that he was the chosen one he set forth on his journey to Egypt with God watching over him and leading the way. As Moses leads the Israelites out of Egypt he comes to the Red Sea with the Egyptians close behind. As the bible explains, the miracle takes place when the Red Sea splits leading the Israelites to freedom. As the Egyptians were crossing the sea, it closed its gates and let them drown within the waters of the sea. In justifying whether Hume would discredit this miracle he would definitely see how one may say it is a miracle, but again would have a hard time validating the testimony of the miracle. Again we seethe pattern of the fact that there is no one to testify for the event. We can only view this as a truthful experience through our belief in God and the Bible. It is what we are taught to believe through religious texts, and our house of worship. It is the individual's perception of reality and what he or she believes to be a valid event.

1. *According to Hume, which of the following will qualify to be termed as a miracle ?*
   (A) Dead man coming alive.
   (B) The incident of Noah's ark.
   (C) The narrative about Moses and the burning bush.
   (D) All of the above.

2. *What could possibly make 'The Dead Man Coming Alive" incident unacceptable to Hume as a miracle ?*
   (A) Doubts about the integrity of the witnesses.
   (B) Differences between testimony and reality.
   (C) Less number of witnesses to the event.
   (D) All of the above.

3. *Miracles, according to Hume, are—*
   (A) Supernatural occurrences requiring divine intervention
   (B) Any occurrences that defy the laws of nature.
   (C) Extra-ordinary occurrences that defy the laws of nature
   (D) Violations of Biblical teachings

4. *Which of the following is false in context of Hume's definition of miracle ?*
   (A) Plenty of miracles have occurred of late
   (B) Our beliefs influence our judgement of miracles.
   (C) The incident of Noah's Ark can't be established as a miracle.
   (D) Human testimony may be divorced from reality.

5. *A suitable title for the passage is—*
   (A) The truth about the Story of the Ark
   (B) What are Miracles ?
   (C) Hume on Miracles
   (D) None of these

Answers

1. (D) 2. (D) 3. (C) 4. (A) 5. (C)

Passage 7

Words–3416

ALPANA Rai drummed on the glass top of her table with her nails and keenly heard unassertive hollow sounds. Soon she was unwittingly playing the parade beat they used to at school. It is all about memory….. she thought. Some how, mental actions reach out to old grooves…… and resonate...… Kress Inc., where Alpana was the HR manager, was agog with a new dilemma. A moral one, which the CEO had firmly slapped down with his verdict –sack the errant manager. And while he thought he had easily accomplished it, Alpana, said "What is the justification ?" This had thrown the whole operations committee into a frightening spasm. Alpana was not ignorant of the fact that Amol, the 'errant' manager in question, was a social failure at Kress.

A simple situation, but its genesis went far back in time. This is what had happened. Until two years ago, Kress, which was slow in delivering results, had just begun to pick tremendous energy under the new supply chain manager, Amol Dua. He verily whipped people into action. Some called him a go getter, some said 'hard-as-nails', slave driver; many epithets described him. The bosses couldn't breathe a word, for this man was singu-

larly bringing home the sales and bottom line and rescued Kress from near disaster.

When Amol was hired, Kress had been in dire straits needed a strong supply chain focus. Amol was hired from a confectionery company at level 2, thus equating him wiyh Samarth Soi, the finance manager. Where as until Amol joined supply chain was a level-4 job.

So, Samarth, who felt his position weaken, resisted Amol. As if justifying it, Amol was also tough. He had come from a blue chip MNC and was seen all round as Kress's Saviour. Today, after 24 months, Kress Inc. had recycled its past weaknesses and Kress was a force to reckon with, evidenced by competitive attacks on it by even the market leader. Therefore, what command Amol possessed was seen by Samarth as arrogance. This perception was strengthened by the ringside view of Amol by managers in sales and marketing who at one time were the propitiated Gods. Today, all devotion was to the Lord of supply chain, and rightly so, for the foods business was entirely supply-chain driven.

Amol's seniors respected him for his attitude and his ability to deliver. His peers were in awe of him, his juniors on eggshells. He was himself non-political, work driven and a bit rough at the edges, so that he said what he meant, meant what he said. On the personal side, Amol was unmarried, a conscious decision he took to look after his paraplegic father. So, it was that he had less of the distractions that his married colleagues had and a home that doubled up as an office, allowing him to continue where he left off at work.

Samarth, on the other hand, was a manager who was smart, politically correct and suave. He was someone who stirred up feelings and thoughts with his presence, and relief with his departure. Thus, you could never say he was wrong, but his being right never helped because he was rarely part-of the solution. Better still, Samarth forced his way into problems and assumed stances that were seemingly participative. But his presence ruffled everything, put orderly perspective into disorder, raise dust, where after he left leaving inane rhetorics for people to deal with. This pattern was what many called 'disruptive'.

RECENTLY, when a key raw material fell short threatening sales, Amol had suggested they source from a more expensive supplier-someone he had dealt with in his old organization. This, he said, was to be a completely temporary measure to tide the season and not lose the market. Samarth, of course, grew belligerent and threw numerous spokes in the wheel. From 'bottom line cannot bear this' to 'we are losing profit focus owing to our egotistic need for market power'. Then, he delayed the payment to the supplier and hinted at examining S.299 of Companies' Act. Could Amol be deemed interested in the transaction ? Which itself caused a mild flutter.

Amol distanced himself from all this. As far as he was concerned, the material was in, production was racing and Samarth could go read up all Acts he wanted.

In the 360-degree appraisal that year, 9 out of 10 called Samarth dispensable. He survived this using his very renowned skill of challenging and ruffling. While Amol made it to the market on time, Samarth harbored a grudge against Amol and held him singularly responsible for the acrimonious 360.

Thereafter, it became a pattern where Samarth would resist Amol, worse, not share relevant financial data with him or stall his work some more. During this period, Samarth chanced upon Amol talking to Seema Goyal, his cost accountant, by the coffee machine twice and once offering her a lift home. Samarth grew anxious. So, was Amol cutting the chain of command by simply dealing with Seema for the data he needed? Samarth began to monitor Seema even more closely. "I smell a rat," he said to Dushyant Dhir, a senior engineer in IT. "Something strange is going on there. This chap is dealing with my staff directly…... wonder what is cooking!”

Kress had a mature IT department with sophisticated monitoring tools. One of the tools allowed the IT managers to take control of any computer anywhere in the network and actually see what the user was doing. Smiling at Samarth's surprise' Dushyant said, "Imagine this, suppose you are chatting with someone, and step out for a meeting and I know you are not in your room. I can take control of your PC and read your mailbox. So, if you have serious doubts, I can take control of Seema's PC or Arnol's too, in their absence, check all the mails, chat sessions, you just tell me."

Samarth agreed. He wanted to see if Seema had been sharing classified financial data with Amol. As luck would have it, Dushyant downloaded all mails between the two persons and discovered mails between Amol and Seema which indicated a deeper, non professional relationship. The broad tone of these mails was affectionate, periodically interspersed with anxiety over not having met. Some in between were terse with an undercurrent of annoyance, sometimes frustration, and then there was one, where Seema said, "Sorry, this is not going to work, my dad's health is such and I am not pretending that he is recovering or will recover."

Samarth's head went into an overdrive. Placing an this before the operations head, Ojas Dharker, he extrapolated, "So, is this what we want to see happening in a clean company like Kress ? What about confidentiality of data ? What all has he extracted by sexually harassing her ?"

You don't know ?" Then leaning forward, Samarth said, "New developments. Seema called me this morning to say she cannot come to work for a few days. Her pretext? 'Father unwell.' The plot thickens, Dharker, did you read his last mail? This morning Seema asked to be transferred to the Delhi office. Her friends say she has been looking very harassed and disturbed. Who is to say what he has done, what he has said to her ! The head of our supply chain! I worry for the women here. Imagine

what he gained for us in the swing, we will lose in the roundabout, thanks to this news getting out! Seema will talk, I know that girl, she is a firebrand…"

Dharker winced. Things looked bleak. Even a financial scam was welcome, but not this, he thought. Without a warning, Dharker ran into Dushyant in the men's room. And typical of men's room talk, Dushyant said in an anxious whisper, "It is completely unaccept able that a senior manager is using corporate resources to conduct an illicit affair ! What an example !"

Dharker hated this kind of banter. But he was completely shaken by what he was hearing. Office gossip and speculation was a killer virus. Behind their smart clothes, people were finally petty and bored. Amol was a tough nut, yes, but he was not one you classified as indecent. Sexual harassment? Indeed!

But the restlessness did not leave him. Almost carried by the power of his anxiety, Dharker met the CEO, Triumalai Rajan, (TR) and apprised him of the findings. TR did not like any of this. He shook his head for a very longtime, looking away. How was he to have known two years ago that he was hiring a maniac? Amol was a stellar chap, but then he was not compromising Kress' dignity and purity, "For me, honesty and integrity are tops," he said to Dharker. "If this is Amol, then he goes. Simple."

Picking up the phone, he called Alpana Rai, the HR head, and said, "Sorry to bother you on your vacation. There is a terrible situation that needs your help. Please come and see me as early as you can." Alpana who had taken a four-day break to attend to her per Frodo's surgery, drove hurriedly to see TR. "Just four days and the world has chosen to change in this short time!" she mused.

In TR's office as she stood there arms crossed, head cocked to her left (she did not hear too well on her right), TR spoke gravely, updating her on the episode. He said, "Make sure he leaves the premises in the next hour and effect his settlement swiftly in two days. I don't want to see him again. And yes, a very strong message has to go out that at Kress we value decency. Let this be an example for everyone that we do not encourage such behaviour at Kress. You know better than me that sexual harassment is a punishable offence in most countries. We are in a, vulnerable situation especially since we hire women here."

Alpana, thoughtful and surprised, left TR's room, her arms yet crossed, thinking. She sat in her room, drumming on her glass table, thinking, until it was 8 p.m. Then she called TR and said, "I am not falling for this. I am not doing anything in two days. No. I am first going to meet the characters in this story." TR was agitated. "Alpana, the whole organization is upset. Do we want to be seen as someone shielding a sick man?"

"How do we know he is sick ?" asked Alpana. "Let us not get theatric, TR, hang on to the faith that he is good. Give me one week. Besides, how do we know there was sexual harassment?" TR did not like this, "I am the CEO Alpana, don't make me speak unpleasant language. Well, the language I am told was flirtatious. Let's leave it at that," he said hurriedly, as if even dwelling on the words would bring him impurity. Alpana did not understand, "OK, I have read the mails, I thought they were tender exchanges. Where did harassment come from ?"

"How would 1 know ?" thundered TR. "Seema has gone on abrupt leave, and the Dushyant, who probed the PCs says…" Alpana gasped, "He did that ? Oh, dear God… ok, give me the week I asked for.."

Early next morning Alpana called Seema and asked to meet with her. On arriving, Alpana saw Seema sitting in the lawn reading to her old father while he sat there, head hanging to one side, drooling. "He has had a mild paralytic attack again," said Seema. "He keeps remembering his sister in Delhi. If I can be transferred there for a while or if I can take a sabbatical."

Alpana took the whole setting in and wondered if she would be doing right asking the questions she planned on. Then quickly setting aside her doubts, she said to Seema, "I can see how hard you work at your life, Seema. Some choices we make or have to make will be tough, but if we are doing them for all round good, the decision is not tough at all, yes ?" Seema smiled and said, "You are right."

Then Alpana said, "I am here on a mission that is not very pleasant. But I have to do this to enable other situations." Then choosing her words, she said, "There is a belief among some people at work-and this has therefore assumed grave proportions" No, this is inappropriate, just come to the point it will be easier, she thought. Bracing herself she said, "Seema, a situation has arisen which compels me to request you to answer this question. Specifically, I wish to know if you are under pressure or undue coercion from Amol." Seema was alarmed. "Won't you explain what this is all about ?" she asked.

Alpana hated her job now. She said, "Is there a relationship between the two of you, that is..." Seema looked hard at Alpana and said, "He is 35, I am 32... full grown adults by any lab test, both of us are unmarried. I am surprised I have to answer these questions."

ALPANA agreed and said, "Forgive me, Seema, it is not easy for me either. As it happens, there is an impression doing the rounds that you are under pressure." And Alpana had no more words. The whole thing seemed so wrong and unfair and juvenile to her.

Sensing her discomfort, Seema said, "I don't owe you any of this, but let me make it easy for you. Yes, we have gone out a couple of times. But I can see there is no future in this relationship. My father needs me now…, I can not think of myself. As for Amol, no, he has not coerced me. He is a very, very decent guy. He understands, and if it helps you understand him better, he himself is nurturing his ailing father for 17 years now."

Returning to the office Alpana told TR, "There is nothing of the kind you suspect. I am not sacking Amol." TR looked concerned. "Alpana, you have gone by the statements made by these two?" Now, she was really upset. "TR, how did they suddenly become 'these two' people ? They are our employees, and among the finest we have. What is the concern here ? I have told you there was no sexual harassment." .

Samarth who was called in as 'the aggrieved party', said, "I wouldn't call them 'finest' ! Amol is setting a wrong example for his subordinates-utilising office time for personal relationships, then? And yes, maybe there was no harassment, but there is certainly cause for concern over office decorum! Also, in my opinion, there is no place for emotions at work Period. People lose objectivity which is a must in functions like finance, legal, etc. How can we afford to have judgments getting coloured by personal relationships ?

Alpana said in most bland tones, "In-office attractions are a given and 1 do not want to pretend that this is unholy. Were they betraying an existing marriage ? No. Was their conduct offensive to office decorum ? No. Now where is the objection?" Then, facing TR in a manner so she would hear his protest better, she said, "We spend close to 10 hours a day at work. This verily becomes our social environment. We bond and break here... what are we complaining about?"

Dushyant, present in this meeting in his capacity as the star detective said, "The moment a relationship becomes personal, there cannot be professionalism. You don't seem to notice that corporate assets and time was being applied for a dalliance! Others will start using office computers to surf the Net, write personal e-mails, exchange jokes! Moreover, I would have thought that as a woman, you would go out of your way to protect Seema and women kind!"

Alpana wondered what it would take to enable people to negotiate tradition a bit. She said to Dushyant, "That time when you printed out whole portions on "Trinidad & Tobago's neotropical invertebrate fauna' in colour, on glossy paper for your daughter's project... that also was office resources and time, no? The principle is the same Dushyant - sometimes we do have to apply a small bit of our office time to our personal care. Like I call my son six times during work. I think it is OK. As for personal and professional boundaries, how does the positive effect love turn into professional transgression? Or are we all supposed to turn into zombies when we walk into our offices ? And then, professional transgression can also happen between two rivals who dislike each other for mutual gain... are we noticing that ?"

THEN when silence fell all round, Alpana said, "And if you must sack won't you also have to sack Seema? Why only Arnol? How about also imagining that she too was sexually harassing him? Possible ,no? I mean, both persons have admitted to sharing a relationship. So, what is good for the goose is also good for the gander, isn't it? You want objectivity? Then here it comes- Seema goes too! But you want to make an example out of Amol, and you Dushyant, you want to paint a halo around having saved a woman, whereas Seema needs no saving! She has said in very balanced terms that she was not being harassed. If we have her word on this, what is the issue?"

TR made a sound that was a cross between exasperation and disbelief and said, "Alpana, as a woman, I would have expected some grace and sensitivity from you. This situation is explosive. We owe our women a value-based workplace. You shock me with your blasé worldview!"

"Do I ?" asked Alpana, now obviously angry. "How did values become a feminine attribute and need? Values are for everyone and values include respect for colleagues, their personal lives and their private space! Yet here we are, gods of values, merrily trodding on their private lives!"

TR, deeply confused and disturbed spoke, "I want Kress to be known for its purity, for its decency. You tell me, was it 'ethical' to have indecent chats with someone on the office machine during office hours? Whatever you may say Alpana, I have serious objection to this. Tomorrow you will have others indulging in all this, then? Is there any sanctity to a workplace or not ?" Samarth nodded briskly, "Organizations need walls to protect sensitive information, budget numbers and the like, lest they be misused say for insider trading, manipulating sales performance, etc. So, such inside dalliances make this internal water- tight information 'protection' system weak!"

Alpana nodded, then to TR she said, "I have seen the chats, much as I hated doing it; six in all and not one indecent expression or innuendo. Our over zealousness can ruin career, two good careers, simply because you think this is unclean."

Then Alpana said something that made them all turn pale, "If anyone has to be sacked for protecting the purity of Kress, it should be Dushyant and Samarth." TR was now really annoyed. "Miss, you are crossing all limits!" he said, his voice cracking in emotion. Alpana spoke her mind, uninhibitedly. "You don't pry into your employee's private life, unless you think he is defrauding the company. What these two have done is flagitious and diabolical. How dare anybody misuse their functional position and peep into another's computer? How dare!

"Tell me, TR, tomorrow if Dushyant thinks it fit that it is necessary for the company to know what takeovers you are planning, and so he simply seizes control of your machine… see ? Nobody here has any business to misuse a technology. I cannot trust Dushyant anymore, nor can I trust Samarth. You don't need to have an affair to trade confidential information, Samarth, a good friendship like yours and Dushyant's did like-wise, no ? TR, this is abuse of power. That's what my HR manual tells me. It's like someone having the master key to your house... I don't

feel safe working here anymore! What if they tamper with my appraisals tomorrow ?"

TR was pained. He was losing control. Suddenly, the headlines had changed, rendering the last few weeks' news fraudulent. Almost.

"Amol and Seema are not the issue," said Alpana. "The real issue is one of feeling safe in this organization. So, I ask how did this act of Dushyant's get approved ? And how many such computers has he been peeking into? And for the future, how do we prevent this? This is serious!"

1. What would be the best and most appropriate title for the passage ?
   (A) Revenge is sweet
   (B) Women appraised
   (C) Respect for privacy
   (D) Feminine values *Vs*. masculine ones
   (E) Jealously gone haywire
2. What arguments were put forward to extradite the sacking of Amol by samarth and Dushyant ?
   All the following are false- EXCEPT—
   (A) The extraction of classified material through coercion
   (B) Sexual harassment and improper use of office machinery & time
   (C) Misbehavior with a female worker and the use of expletives
   (D) Arrogance and disrespect for senior mangers
   (E) Failure to achieve deadlines and target sales
3. The attitude of the author in the passage can best be deciphered as one of—
   (A) Outrage and disbelief
   (B) Approval and encouragement
   (C) Didactic and Utilitarian
   (D) Sober and Practical
   (E) Philosophical
4. Samarth's angst towards Amol can be traced down to which of the following reasons
   1. Jealousy 2. Inferiority complex
   3. Inadequacy 4. Under achieving
   5. Ego
   (A) 1, 2, 5 (B) 1, 2, 3, 5
   (C) 2, 3, 4, 5 (D) 1, 2, 3, 4, 5
   (E) 1 and 5

Answers

1. (C) 2. (B) 3. (A) 4. (E)

Passage 8

Words–386

The public distribution system,. Which provides food at low prices, is a subject of vital concern. There is a growing realization that though India has enough food to feel its masses two square meals a day, the monster of starvation and food insecurity continues to haunt the poor in our country.

Increasing the purchasing power of the poor through providing productive employment leading to rising income, and thus good standard of living is the ultimate objective of public policy. However, till then, there is a need to provide assured supply of food through a restructured, more efficient and decentralized public distribution system (PDS).

Although the PDS is extensive-it is one largest system in the world it has yet to reach the rural poor and the far-off places. It remains an urban phenomenon, with the majority of the rural poor still out of its reach due to lack of economic and physical assess. The poorest in the cities and the migrants are left out, for they generally do not possess ration cards. The allocation of PDS supplies in big cities is larger than in rural areas.

In view of such deficiencies in the system, the PDS urgently need to be streamlined. Also considering the large foodgrains production combined with food subsidy on one hand and the continuing slow starvation and dismal poverty of the rural population on the other, there is a strong case of making PDS target group oriented.

The growing salaried class is provided job security, regular income, and social security. It enjoys almost hundred percent insulation against inflation. The gains of development have not percolated down to the vast majority of our working population if our compares only dearness allowance to the employees in public and private sector and looks at its growth in the past few years, the rising food subsidy is insignificant to the point of inequity. The food subsidy is a kind of D.A. to the poor the self-employed and those in the unorganized sector of the economy. However, what is most unfortunate is that out of the large budget of the so called food subsidy, the major part of it is administrative cost and wastage. A small portion of the above budget goes to the real consumer and an even lesser portion to the poor who are in real need.

1. Which of the following, according to the passage, is true of public distribution system ?
   (A) It is unique in the world because of its effectiveness
   (B) It has remained effective only in the cities
   (C) It has reached the remotest corner of the country
   (D) It has improved its effectiveness over the years
   (E) It develops self-confidence among the people
2. Which of the following, according to the passage, is the main reason for insufficient supply of enough food to the poorest ?
   (A) Production of food is less than the demand
   (B) Government's apathy towards the poor
   (C) Absence of proper public distribution system

(D) Mismanagement of food stocks

(E) None of these

Answers

1. (B) 2. (C)

Passage 9

Words–924

There is no such thing as 'brain-fag'. Thinking that long concentrated mental effort produced tiredness in the brain itself. Yet scientists believe that this state cannot exist. Your brain is not like your muscles. Its operations are not muscular but electrochemical in character.

When your brain appears to be tired after hours of mental work, the fatigue is almost certainly located in other parts of the body, your eyes or muscles of the neck and back. The brain itself can go on almost indefinitely.

A young woman undertook as an experiment to multiply in her head a series of two four digit numbers one after the other as rapidly as possible. She went on doing this for twelve hours.

During that time there was only a slight decrease in her efficiency, measured by speed and accuracy. At the end of twelve hours she stopped only because of body fatigue and hunger.

What seems like mental fatigue is often merely boredom. In reading a difficult book, for example, you are torn between the desire to go on and the impulse to stop. According to an eminent psychologist, it is often not fatigue that you feel but inattention and the inability to ignore distracting thoughts.

The brain capacity is almost inexhaustible. That part of your brain involved in thinking and memory, and all your conscious activities, has at its most Important part ten or twelve thousand million minute cells. Each of these has a set of tiny tendrils by means of which an electrochemical message can pass from one cell to another. Thinking and memory are associated with the passage of these electrical currents. The wisest man who ever lived comes nowhere near using the full capacity of his wonderful mental storehouse. Quite possibly, people in general employ only ten to twelve percent of the capabilities of their brains.

Your I. Q. is less important than you probably think. Many of us have unnecessary inferiority complex about our I. Q's–the figure that represents native intelligence as compared to that of the average individual. It is easy to score lower in such a test than you deserve. This might result from temporarily ill. health or emotional disturbances. So, if you have ever seen your score on an I.Q. test, you can reasonable be sure that-your I.Q. is at least that high.

What is the physical basis of high intelligence? Contrary to a common belief, it does not require an unusually large skull and is likely to be associated with especially -large-number of surface convolutions in the cerebral cortex, the great top part of the brain, Highly intelligent people also have good blood circulation to the brain, bearing oxygen, glucose and certain other important chemicals. It is possible that a person with very special talent-a mathematical or musical genius, for example, may have an unusually thick bundle of nerve fibers in one particular place in the brain.

But the physical endowment of your brain- is far less important than what you do with it. The number of brain cells in an individual with an I.Q. of 100 (which is average) is large enough so that, if used to the full, it could far exceed the record, so far as memory is concerned, of the greatest genius who ever lived. A person of average I.Q. who industrially stores up knowledge and skills year after year, is better than a person -with very high. I. Q. who refuses to study. Research has indicated that some of the most important men in history had no more than ordinary IQs.

Among them are statesmen such as Cromwell, John Adams and Lincoln; military heroes like Drake, Napoleon and Nelson, writers like Goldsmith, Thackeray and Emerson. All these men, to be sure, were above the average in intelligence; yet they ranked far below the most brilliant of the individuals studied. What they possessed in high degree was character, and the ability to keep plodding ahead until they achieved what they had set out to do.

Age need not prevent your learning. One of the commonest misconceptions about the brain is that as you grow older something happens to it causing the learning process to become more difficult. This is true only to such a minute extent that for most of us, if is of no practical importance.

Learning is associated with ability to create new reverberating electric circuits in the brain and as long as that power remains you can continue to acquire new knowledge and skill - even at ninety.

It is true that all old people suffer impairment of their physical powers and that some experience a decline of mental power. The best current medical opinion is that, in both cases, what happens is a series of minor 'accidents' to various parts of our marvelously complicated psychological mechanism. None of these may be serious by itself, but the total effect, can be severe. Impairment of the brain in the aged is associated with decreased circulation of the blood and the precious substances it carries, especially oxygen and glucose. This is probably why old people remember happenings of their youth more vividly than those of the recent past; the youthful memories were implanted when blood circulation was better.

Yet, severe mental impairment occurs only in some, elderly people. Everyone knows of men and women who are vigorous and alert mentally into the ninth or even tenth decade of life. Their existence proves that impaired mental powers are not an inevitable accompaniment of passing years, but a result of disease processes.

1. *Which of the following can said to be true? Our brain can function continuously for—*
   (A) twelve hour and after that it gets tired
   (B) an indefinite period of time
   (C) along period, provided the mental exercise is interesting
   (D) None of the above
2. *The author is qf the opinion that* a *person with average intelligence—*
   (A) cannot expect to achieve as high as a genius can do
   (B) usually has thin bundle of nerve fibres in a particular place of the brain.
   (C) usually possesses high perseverance to achieve what he sets to do.
   (D) can exceed the record of the greatest genius if he uses his intelligence in full.
3. *Which one of the following can be concluded from the passage* ?
   (A) Highly intelligent persons usually have large skull
   (B) As man grows old, he loses the capacity of learning very rapidly
   (C) Even the wisest man of the world does not utilise the full capacity of the brain
   (D) The most intelligent men of the world are likely to achieve little.
4. *The author is most likely to agree with which of the following—*
   (A) success depends much upon the way we utilise mental-capacity
   (B) Impairment of physical powers due to old age does not hamper its apacity of learning
   (C) only person with average intelligence have tenacity to overcome the difficulties of life and that is
   (D) why most of the great men are of average intelligence

   In order to achieve success in life, the main factor is a high level intelligence.

Answers

1. (B) 2. (D) 3. (C) 4. (A)

Passage 10

Words–820

As managers become enchanted with the potential of the Internet and the interactive marketplace, many Show evidence of forgetting some basic lessons of

Marketing strategy that they learned the hard way over the past several decades. The most central of these are the importance of defining and understanding the customer, the essential efficiency of market segmentation and targeting, and the life-or-death importance of product positioning and the value proposition. Imply put, there is a real and persistent danger that, caught up excitement and hype of a new technology, marketers will once again let attention to the short-term and tactical overwhelm consideration of the long-term and strategic. In the new world of interactive marketing, tactics often precede strategy.

How many companies have created their Web site without thinking about how to maintain it or how to serve the customer ? How do the capabilities of interactive marketing fit with the company's strategy of value delivery ? Have salespeople been brought on-line with portable computers, customer databases, and Internet connections before the company has improved any of its core capabilities or processes for value delivery ? Has any provision been made for screening and qualifying people who request information ? As in the other national pastime, some hits are better than others.

Statistics that one hears cited about the growth of the Internet strains one's imagination and credulity: 2 million Web sites; 22 million users in North America; new users being added at a rate of 12,000 per day; a projection of 1 billion users the year 2000; a typical user spending 20 hours each week on the Internet. (Whatever be the numbers, they are obsolete, given the current rates of growth.) Who are these people ? Where do they find the time? What motivates them What will happen when they have to pay for services that are now free? How many of these Web surfers fit the definition of a potential customer-someone who is willing and able to buy the marketer's products? Anecdotal evidence suggests that very few of the thousands of marketers now on the Internet are profitable. It has been estimated that only 10% to 15% of Web surfers actually use the Web for shopping or to obtain commercial services such as travel information. But hope, in the form of hockey-stick-shaped profit forecasts, springs eternally.

Marketers who are attempting to address the strategic issues inherent in becoming a serious player, in the interactive marketplace should be wary. Old, familiar paradigms of mass-marketing strategy can be dangerous in this new and unfamiliar setting. Concepts as simple as the four Ps of Product, Price, Promotion, and Place and the traditional distinction between strategy and the organizational structure used for implementing strategy need to be seriously reconsidered and modified or abandoned altogether.

Consider the following : Every product can be thought of as information that comes in the form of promised benefits and a definition of value for the customer. Customers buy benefits and value. When I purchase Colgate toothpaste, I get a mint-flavored cream containing sodium monofluorophosphate, but what I am really buying is the promise of a clean taste, white teeth, healthy gums, and fewer cavities. The brand is the promise of value. The customer doesn't see product,

price, promotion, and place as separate variables. In the interactive marketplace, this is even more true. All the variables interact. They cannot be separated. What the customer sees is information that wraps product, price, the selling message, and the store into one big value proposition. Equally important, customers in the electronic marketplace expect that value proposition to be tailored to them personally. In fact, they want to be part of the design process. The four Ps don't fit here as they do in other contexts.

The mass marketplace emphasized transactions built on mass communications, but the interactive marketplace moves toward relationships based on tailored product offerings and messages. Marketers must stop thinking of marketing as persuasion (one-sided communication). They must develop a new set of conceptual skills that permit the design of customer-specific product offerings and messages. They must learn how to communicate with customers, listening to them as well as sending messages to them.

What does this mean in practical terms? It means thinking about strategy and organization as integral parts of a business design that is capable of responding to the customer's ever changing definition of value. It means seeing marketing as a set of skills, capabilities, and processes that must both pervade the organization and focus on the customer as the center of the business, guiding the business's evolving strategic response to the customer's changing needs, wants, arid preferences. It also means developing a flexible, responsive organizational structure that can evolve and move rapidly. Marketing as a separate function must give way to a set of customer-driven processes, some of which may be performed by strategic partners in areas such as Web site design and management, order fulfillment, database management, and credit.

1. It can be deduced that the author will agree with the statement that—
   (A) internet site is the most imaginative and marketed strategy
   (B) qualitative delivery is being neglected in pursuit advancement
   (C) promotional price in commercial use needs to considered
   (D) consumer value is sinking
2. Which of the following, according to the author, is a false statement ?
   (A) Integration of four Ps is profitable in interactive market
   (B) Communication with the customer is of paramount importance in the present scenario
   (C) Designing the product on consumer's specification leads to success.
   (D) Implementing traditional marketing strategies encourages success even in the new settings.
3. We can infer from the passage that the author finds the growth statistics of Internet as—
   (A) that which has no solid evidence.
   (B) figments of imagination.
   (C) difficult to believe.
   (D) a gimmick and fraud.
4. As per the passage, to be a successful marketer, one must:
   1. possess superb commun1cation skills and convincing capacity.
   2. be a good listener and be sympathetic.
   3. be innovative at all points of time.

   (A) Only 1 (B) Both 2 and 3
   (C) Both 1 and 3 (D) All three

Answers

1. (B) 2. (D) 3. (C) 4. (D)

Passage 11

Words–646

Houston, Texas (CAP) California businessman Dennis Tito paid about $ 20 million for an eight-day trip to space. Now, a Houston-based company can send you to space for $ 50. Well, part of you anyway. Encounter 2001 is working to build an unmanned spacecraft, fill it with DNA samples and messages from up to 4.5 million people, then blast it beyond the solar system. The company hopes to launch its spacecraft in late 2003.

"This is a chance for people to participate in a real space mission," said Charles Chafer, Encounter 2001 president. "Maybe one day it will be found." For $ 50, people can have their digitized photos and messages as well as hair samples placed on the spacecraft. Encounter 2001 is the Sister Company of Celestis Inc., which in April 1997 began using commercial rockets to launch the cremated remains of people into space. Encounter's spacecraft will be made up of a solar sail the size of a football field and a small container carrying the photos and messages, plus dehydrated hair samples with the DNA codes of 4·5 million people. The solar sail the spacecraft's power source is a very thin sheet of reflective material that will use the sun's photons to propel it forward, Chafer said. The concept is similar to a sailboat being pushed along the water by the force of the wind.

Like wind, sunlight exerts pressure and a large enough sail in space could harness this force and travel without using fuel. Although a solar sail is at first slower than a conventional rocket, it continues to accelerate over time and achieves a greater velocity. NASA and several private groups are working on plans to use solar sail technology.

The spacecraft, to be launched on an Ariane 5 rocket, will orbit Earth for three weeks so ground controllers can conduct system checks. After the spacecraft leaves Earth is orbit, it will deploy its solar sail and begin its journey.

It will take about 15 years for the spacecraft to fly past Pluto, the solar system's outermost planet. When the sailcraft leaves the solar system, it will be traveling at 7.8 miles per second. That compares with the space shuttle's on-orbit speed of 5 miles per second. The spacecraft's imaging component is scheduled to be tested during space souttle Endeavor's mission in late November. The mission is expected to cost about $ 25 million. Most of that is being paid by private investors. Some of the money, though, is coming from public participation in the project. About 67,000 people so far have paid to take part in the mission. Chafer said he expects the bulk of sales of participation kits to occur in the six months before the launch.

Jim Glock, a teacher at Fairmont Junior High School in suburban Deer Park, got 140 students in six of his sixth, seventh and eighth grade classes signed up two years ago. "I wanted them to see how vast the universe is and the time and distances there are to go from point A, to point B," Glock said. another purchaser is famed science fiction writer Arthur C. Clarke, who said he was delighted to be taking part, in some way, in technology he wrote about. His 1963 short story "The Wind from the Sun" envisioned space travel by using solar sails. "Fare well my clone!" Clarke wrote on his message, referring to his DNA on board.

"One day, some super civilization may encounter this relic form the vanished species and I may exist in another time, "Clarke told The Associated Press in an interview late last year. The work of Encounter and Celestis is a natural progression of the exploration of space, Chafer said. "Governments open the frontiers but without strong commercial components, frontiers don't go anywhere," he said. "It's sort of a natural evolution, combining real missions with the mass market."

1. Which of the following is not a characteristic of the solar sail ?
   (A) It is the space craft's power source
   (B) It is made up of very thin sheet of reflective material that will use sun's photons to propel forward
   (C) It uses sun light and does not require fuel
   (D) Its initial acceleration is very less and thus is unable to achieve a great velocity
2. Jim Glock got his students to participate in the mission because—
   (A) he was a science teacher and he wanted his students to know about this wonder of science.
   (B) he wanted them to know about the distance and time required in the universe to go from one particular point to another and about the vastness of universe.
   (C) he was amongst one of the investors.
   (D) he wanted them to know about solar sail technology used in space crafts.
3. What does the author say about 'super civilization ?
   (A) They may exist in some other time
   (B) The super civilization may get hold of the DNA sample and may try to experiment on it
   (C) They will be in a position to send his DNA samples into the space
   (D) They may find his clone in the next century.

Answers

1. (D) 2. (B) 3. (B)

Passage 12

Words–517

Few entrepreneurs start out with both a well-defined strategy and a plan for developing an organization that can achieve that strategy. In fact, many start-ups, which don't have formal control systems, decision-making processes, or clear roles for employees, can hardly be called organizations. The founders of such ventures improvise. They perform most of the important functions themselves and make decisions as they go along.

Informality is fine, as long as entrepreneurs aren't interested in building a large, sustainable business. Once that becomes their goal, however, they must start developing formal systems and processes. Such organizational infrastructure allows a venture to grow, but at the same time, it increases overhead and may slow down decision making. How much infrastructure is enough and how much is too much ? To match investments in infrastructure to the requirements of a venture's strategy, entrepreneurs must consider the degree to which their strategy depends on the following:

As a young venture grows, its founders will probably need to delegate many of the tasks that they used to perform. To get employees to perform those tasks competently and diligently, the founders may need to establish mechanisms to monitor employees and standard operating procedures and policies. Consider an extreme example. Randy and Debbi Fields pass along their skills and knowledge through software that tells employees in every Fields Cookies shop exactly how to make cookies and operate the business. The software analyzes data such as loca.l weather conditions and the day of the week to generate hourly instructions about such matters as which cookies to bake, when to offer free samples, and when to reorder chocolate chips.

Telling employees how to do their jobs, however, can stifle initiative. Companies that require frontline employees to act quickly and resourcefully might decide to focus more on outcomes than on behaviour, using control systems that set performance targets for employees, compare results against objectives, and provide appropriate incentives.

In a small-scale start-up everyone does a little but of everything, but as a business grows and tries to achieve

economies of scale and scope, employees must be assigned dearly defined roles and grouped into a propitiate organizational units. An all-purpose workshop employee for example, might become a machine tool operator, who is part of a manufacturing unit. Specialized activities need to be integrated by, for example, creating the, position of a general manager who coordinates the manufacturing and marketing functions, or through systems that are designed to measure and reward employees for cross functional cooperation. Poor integrative mechanisms are why geographic expansion, vertical integration, broadening of product lines, and other strategies to achieve economies of scale and scope often fail.

Cash-strapped businesses that are trying to grow need good systems to forecast and monitor the availability of funds. Outside sources of capital such as banks often refuse to advance funds to companies with weak controls and organizational infrastructure.

If entrepreneurs hope to build a company that they can sell, they-must- start preparing early. Public markets and potential acquirers like to see an extended history of well-kept financial records and controls to reassure them of the soundness of the business.

1. The author would agree with the statement that—
   (A) Founders are only performance driven
   (B) Segregated specialized activities are better always
   (C) Early preparation of sound long-term strategy is desirable
   (D) Weak structural decisions can be improvised.
2. The author is least likely to agree with the fact that—
   (A) Dictating jobs make it easier for the employees.
   (B) The right-incentives ate derived from clear aims.
   (C) Initiatives need to be encouraged.
   (D) None of the above.
3. As per the passage which of the following is true ?
   (A) Clear cut positions for employees is a prerequisite for any small business
   (B) Establishing working procedures is a standard rule for small businesses
   (C) Prediction of results is beneficial
   (D) Weak organizational edifice needs to be eradicated to grow into a bigger organization.
4. Which of the following terms is not mentioned in the passage ?
   (A) Protracted and steady business.
   (B) Ease of telecommunication.
   (C) Funds monitoring.
   (D) Sound control.
5. We may draw a conclusion from the passage that:
   (A) with growth, delegation of performance should take place
   (B) knowledge and skills should be shared
   (C) gross operational integration is worthy of reward
   (D) sound businesses invest in financial and organizational structitring

Answers

1. (C) 2. (A) 3. (D) 4. (B) 5. (D)

Passage 13

Words–2473

The million dollar question which is top most on everybody's mind is–what is the realistic external value of the Indian Rupee ? And who is to decide this ? Well, all governments have meant well when they have said that the Rupee should ideally find its own value and be decided by the free forces of demand and supply. But it is well said than done. Which government till today or central bank has been able to wash its hand off in deciding Rupee value ?

RBI for years has lost several billions of foreign exchange in arresting the Rupee fall though in vain. The central bank has decided that the Indian industry does not want a devaluation of the currency and a mere 7-10 per cent slow depreciation in the currency would be satisfactory as this would lead to a more realistic level. But, is this form of thinking correct, especially at a time when dollar denominated exports have under performed and trade deficit today is at a high of around \$ 9 billion ? A notionally competitive Rupee is one aspect of good trade performance. In such situations RBI should be the sole regulator of the exchange value and should not be roped in by political lobbies.

But a look at today's events will show that RBI is faced with situations of wars and the government falling on which it has no control thus adding a hurdle to their endeavour to ensure stable financial markets. RBI's role gets complicated when faced with complicated tasks of maintaining a stable Rupee and simultaneously ensuring soft interest rates. Any hike in interest rates will adversely affect the government's borrowing programme. The only option is foreign currency flows will reduce the pressure on the Rupee. This is provided the foreign fund managers are comfortable with the political situation and judge the government's intentions in the right earnest.

Relaxation in guidelines on ADRs/GDRs and allowing Indian companies to use US \$ 100 million from ADR/ GDR funds for acquisition abroad is a stepping stone for the Rupee to become convertible on the capital account. Removing the criteria of a three year track record will make more companies tap this route. The government announced these relaxations with good intentions but they haw; come at a time when global markets are fast moving and mega mergers and amalgamations worth billions of dollars are taking place the world over resulting in Indian companies standing no chance with mere offerings of \$ 109 million. Thus, at this juncture,' the dream of Indian companies becoming MNCs, is a far cry. The government

realising that the forex reserves were drying up announced the external commercial borrowings (ECB) programme which later started putting pressure as there was a drain on the forex reserves which was later replaced by the GDR programme. Pre-conditions attached only saw core companies remain outside the scheme. It was only in 1996-97 when the foreign exchange reserves started to show an improvement that the government took bold initiatives and relaxed the guidelines. But then they were faced with a bigger problem of demand recession which proved a dampener for raising equity abroad as excess capacities existed in the economy.

Why did this happen, especially at a time when growth was picking up and forex reserves were fairly comfortable ? Tight monetary policy, stringent pre-conditions attached to the ADR/GDR route, high interest rates and decline in both public and private investments was a grave mistake. This all resulted in Indian companies being denied to borrow cheap overseas funds. Public limited companies were worst affected as they were forced to borrow from the domestic market at high interest rates burdening them with equal high interest liabilities. The government also reduced plan outlays which forced public companies to heavily depend on market borrowings to meet their capital investments. What was the fallout of all this ? More public companies became red facing huge debt burdens only because of the shortsighted policies of the government.

Here again that the government gave its age-old justifications that a tight budgetary outlay was being allocated to curb fiscal deficit and to bring about monetary harmony in the economy which would 'help curtail its high external debt burden of around $90 billion. But as usual the government failed in its efforts and domestic debt shot up and the country lost the opportunity of raising cheap overseas funds.

Today, the industrialists are euphoric about the liberalised guidelines as this provides the Indian companies an opportunity to become global players. Today the world over stock markets are volatile and Indian companies have yet to penetrate listing on world stock markets. World MNCs would like to guard the interests of their own companies and would like to ensure maximum penetration in the overseas market even in the field of Information Technology where India has a clear edge. If the government has to succeed it will have to loosen their grip on the domestic money market and try and reduce the interest rates in which a beginning has been made–a one per cent reduction was announced in the PF rates which will cool off the tight monetary conditions prevailing in the economy. But with further cuts in interest rates likely it will no longer be attractive for an Indian entrepreneur to tap overseas markets as world interest rates would work out to be the same as in the domestic market. Thus, expanding the equity base would not be an attractive option for an Indian entrepreneur through the ADR/GDR route. This should have been done way back in 1995-96, when interest rates were high at 16-18 per cent. Today there is no danger of a run on forex reserves as they are quite comfortable at $40 billion and even if there is a sudden panic on Indian paper floated overseas there is no major cause for concern.

Today the government is still hesitant about moving towards capital account convertibility as it has not completed it's homework despite the fact the Indian economy is fairly stable politically, growth is picking up in most sectors and there are no major volatile shocks being experienced in the economy. Why, then, the delay ? Dilly dallying on the part of the government will once again see India losing out on this golden opportunity of improving its world rating.

But there are many complications involved in the process and unless and until the government is sure of the possibilities, it should not take a hasty decision which would prove to be expensive in the long run. Today foreign companies are allowed to be listed on the Indian bourses. This could possibly mean that the Indian Rupee is being allowed to be exported for the first time. A foreign company raising money in India can raise only Rupee funds. If they have a manufacturing unit then the intention is quite clear what if they are a non-production based company what do they do with the Rupee funds if they are not allowed to take them out of the country. Will all these are steps in the direction of Indian Rupee moving towards Capital Account Convertibility (CAC).

When the report was framed under the chairmanship of SS Tarpaper there were plenty of challenging tasks ahead of them. But, today, India has liberalised which will make it easy for the Rupee to move towards CAC. The gold regime has been liberalised, derivatives have been ushered and FDI and FIIs have been allowed in more sectors. Reducing procedural hurdles and making investments hassle free is being given top most priority with respect to ADRs/GDRs. The report on CAC set a three year time frame starting from 1997-98 to move towards, full convertibility. But, the Asian crisis spiked any discussion on the issue which resulted in the process getting delayed. Today for foreign nationals CAC exists. But, the crux of the issue was to allow some flexibility for the Indian residents in effecting outward remittances.

Currently, a lot of ground work is being done for foreign companies to get listed on the Indian bourses. India has liberalised its regime of inward flow of currencies. But, the same cannot be said about outflows. But the encouragement to expand foreign listings on the Indian stock markets is a clear signal that a move towards CAC is on the cards. Freeing FDI and encouraging Indian companies to raise equity abroad should provide some cushion in case there is an outflow of funds.

But what is necessary at the moment is, for the government to keep a check on the fiscal deficit and have a strong financial system in place so as to take a greater

strain of the larger currency inflows. Unless and until these conditions are in place, it will take a while till we see Rupee fully convertible both on the current as well as capital account. Looking at the Asian crisis, what is required is to have a proper sequencing of reforms and have overseas based financial sector wherein deficits are firmly in check. The government needs to wake up to this reality and not delay the matter as lags affect the credibility of a system. Indian markets are used to shocks and an entry will put things in place. Banks and stock markets underwent many shocks like entry of private players, FII role being enhanced and finally what happened the system absorbed the shocks and reacted positively. Fundamentals have to be in place and if the market requires a proof and although RBI is conservative at present, it will react favorably in the future and eventually put CAC in place.

Today, with globalisation and integration as the as buzzwords, dismantling economic barriers of course with adequate safeguards CAC is considered to be a sine qua non for effective economic management in the new millennium. What needs to be guarded is the activities of the currency speculators and other unscrupulous players who distort the smooth working of the markets with speculation and rumors becoming major factors influencing the market. Today, India is quite comfortable with its forex reserves, there is a control on inflation, financial sector reforms are in place, there exists a stringent fiscal policy and exchange policy is quite flexible. There cannot be a flight of capital as all is not in liquid form and a major part of it will be invested. But what Indian finance ministers have to look into is how to manage the capital inflows that will find their way into the developing countries. Unless and until well utilised at the right time, they can create overheating in the economy which would be a difficult situation if not handled in an organised manner.

But the chief architect of the report, Tarapore himself is not very confident about the financial sector of the economy and does not trust the fiscal stability. He firmly believes that the Indian financial system will have to cross many more hurdles before CAC can be ushered in. Many argue that a wait and watch approach to put things in place will only result in India losing out.

At present, there is a dire need for funds to finance several infrastructure projects which are a main foundation for future investments. Speculators and motivators will always exist and we cannot afford to waste our precious time to keep them at bay. The Asian crisis was triggered by unrestricted large inflow of foreign funds in largely unregulated and immature markets in Asia. These funds were inter alia utilised for unproductive activities, real estate boom and for creating other long-term assets. As a result, when the crisis really developed as a result of the sudden outflow of funds it led to devaluation, disruption in the growth process and social pain as assets were sold dirt cheap in distress sales. In the aftermath of the crisis, many economists have voiced very strongly against CAC. It is also argued that many companies had huge funds which they invested in these Asian countries only for speculative purposes with the sole intention of destabilising these economies which triggered the crisis.

To avoid such shocks in the future what should be done is to allow only the amount of currency to be traded that is actually used up to finance global trade and services. Today, it is estimated that around US $2 trillion is moving around in various currencies and only a small part is being used up which is where the entire problem lies as it is here that speculation creeps in. Volumes in the forex markets are so large that they say that currency trading is around 20 times of world trade. The question is not of trading volumes but is it really required. True currency trading is required for world trade but unnecessary currency trading is fraught with risk. A new code to monitor speculative funds is what is required which will reduce the shocks in the future. There has to be fair amount of transparency and disclosure practices which will make the system more healthy. A better practice to bring about stability is not to monitor currency stability but to curb short-term capital inflows which could play an important role in preventing a crisis in the future.

Despite India today having a fairly high forex kitty and CPI at lows, there are many other indicators that are flashing red for which India has to take a precaution note.

Looking at the above criteria, it is natural that at this stage India is still not ready to fully take on CAC. Short-term solutions are not the answer especially with the financial system still groping with high levels of non-performing assets, capital inflows practically drying up and the IMF quota already stretched to the .maximum.

So the need of the hour is to have a change in the conservative nature and thinking of our banking and financial policies and restructure our macro economic policies before we opt for CAC. It will take a while for things to be put in place and once a domestic financial architecture and macro policies are put in place CAC can flow in as a natural process. Volatility and speculation once again hit the India forex markets with the Rupee seeing an all time low of 44.70 against the US$. That's when the RBI moved in to check the Rupee fall and played heavily with the forex reserves to get Rupee back to 44.10/20 mark. A 50 per cent surcharge on import finance and a 25 per cent surcharge on overdue export bills which would be phased out as early as possible to some extent would help correct the distortions. However essentials and government related imports have been left out from surcharge. RBI has assured the banking and exporter community that it would meet any temporary demand/supply dollar imbalances which arise due to leads and lags in the system.

1. As per the passage, who should be the ultimate of the exchange value of the Rupee ?
   (A) The Government of India
   (B) Market Forces
   (C) The Reserve Bank of India
   (D) All of the above
2. What is the remedy proposed by the author to stabilize the forex market ?
   (A) Restrictions on FDI inflow
   (B) Curbing short-term capital inflows
   (C) Monitoring currency stability
   (D) Liberalizing FII inflows and outflows
3 . What does the passage hold as the crux of the CAC issue ?
   (A) CAC for foreign nationals
   (B) Freeing the FDI and FII regime
   (C) Allowing ECT limits to be raised
   (D) Allowing some flexibility for the Indian residents in effecting outward remittances
4. What according to the author should be learnt from the Asian crisis in lndia's movement towards CAC ?
   (A) To have a proper sequencing of reforms
   (B) Fundamental restructuring of the entire financial system
   (C) Introducing derivatives
   (D) All of the above
5. As per the passage, what relationship exists between interest rates and maintenance of the stability of the rupee ?
   (A) No relationship maintained
   (B) Interest rates when low, make rupee value unstable
   (C) Interest rates when high, make rupee value unstable
   (D) None of the above
6. Which of the following is false in relation>to the government reducing the plan outlay ?
   (A) Reduction of the planned outlay forced public companies to depend on market borrowings
   (B) Government preferred containing fiscal deficit as an explanation for the reduction
   (C) It was justified as a means of bringing about monetary harmony in the economy
   (D) None of the above
7. Why does the author want the government to loosen its grip on the domestic money market ?
   (A) To allow Indian companies opportunities to become global players
   (B) To improve forex reserves
   (C) To get over the problem of demand recessions
   (D) To reduce the volatility of stock markets

Answers

1. (C) 2. (B) 3. (D) 4. (A) 5. (C) 6. (D) 7. (A)

Passage 14

Words–645

Space exploration is awakening to the hunt for extra-terrestrial life–or at least the circumstances con-ducive to such life. The first step in this quest has been to find planets outside the solar system. This is crucial as no other celestials body is even theoretically likely to harbour organic life. For the longest time the only planets anyone could be sure of were those circling the sun. not any more. A planet’s gravitational pull-tends to shake the star that it circles. By measuring stellar wobbling, astronomers calculate the size and distance of its planets. Over 30" extrasolarplanets have been found circling sun-type stars in the past five years. However, most of these planets have been massive–Jupiter sized or bigger and too close to their star's to life friendly

Scientists recently confirmed the discovery of more extra solar planets, but ones smaller in size and further from their stars than any previously found. Several days ago, astronomers confirmed the discovery of a Jupiter clone around the star Epsilon Eridani. But this is as far from its star as Mars and the asteroids are from the sun. This would provide enough space for small, rocky planets like the earth also to orbit the star. In March, planet seekers found single planets around the stars 79 Ceti and HD46375. These planets were also remarkable because they were less than a third of the mass of Jupiter. Still too big, but a sign astronomers are closing in on their ultimate goal of earth look a likes. What is striking is Ire sheer plethora of planets. Dozens are being found within the immediate galactic neighbourhood: Epsilon Eridani is only 10 light years away. Far from being rare, planets seem to be commonplace.

Planets are one piece of the jigsaw puzzle of alien life. Another piece is determining if other planets have an essential ingredient to life as we know it water. Hence the excitement when it was announced in June that the space probe, Mars Global Surveyor, had produced two metre resolution pictures indicated liquid water had flowed on Mars's surface in the past one or two million years. There has long been evidence of watenon Mars some four or five billion years ago. Pictures from the National Aero-nautical and Space Administration point to the existence of underground water which may still be finding its way to the Martian surface. In 2003 NASA's Beagle 2 probe will be off to the red planet to look for water and life. If water, let alone life, is confirmed on two of the solar system's planets, the likelihood of a universe filled with strange and exotic beings will take a quantum leap forward.

Technology will soon give planet hunting a big boost. It is still impossible to detect the imperceptible

wobbling stars experience because of smaller planets. Astronomers are now using large telescope arrays to catch medium sized gas planets. More useful is transit photometry, where even a small planet can be tracked by the degree to which it dims a stellar brightness as it passes across the star's face. Last year, this method was used for the first time to detect a planet. NASA wants permission to launch a probe, Kepler, in 2005. Floating in space, Kepler would watch 1,00,000 stars for such transit glimmers.

Space exploration lost its direction and excitement with the Cold War's end. Budget cuts have accompanied this listlessness. The total lack of public interest in the $ 20 billion International Space Station is telling. Searching for alien life is beginning to catch the popular imagination. One example is last month's decision by Microsoft's cofounder, Mr. Paul Allen, to donate $ 11.5 million of his own money to the Search for Extraterrestrial Intelligence Institute. A quest brings out the exploratory best in mankind. And the holy grail of extraterrestrial life is becoming discernible in the heavens.

1. What ruled out the existence of life forms on the 30 extrasolar planets discovered earlier ?
   (A) The theory that organic life is not possible only on Earth
   (B) The fact that these planets did not have a central sun-like star which could provide for the energy requirement necessary for the development of life
   (C) These planets were too small
   (D) These planets were too close to their stars to
2. Which of the following is true about planets ?
   (A) The planets are visible through powerful telescopes arrays
   (B) The planets gravitational pull shakes the star that it encircles
   (C) The planets emit radiation that are detectable on advanced instruments.
   (D) All of the above.
3. Which of the following is true according to the passage ?
   (A) There are a large number of extra solar planets
   (B) There is evidence of existence of water on Mars at some point in time
   (C) It is difficult to detect the wobbling that stars experience of very small planets
   (D) All of the above
4. The central theme of the passage is—
   (A) The recent scientific developments in the field of space exploration
   (B) The recent developments in the field of extraterrestrial research
   (C) The discovery of extra solar planets
   (D) The growth of interest in space research over the years

Answers

1. (D) 2. (D) 3. (D) 4. (B)

Passage 15

Words–1831

Spiders can be found in all environments throughout the entire world, except in the air and sea. These invertebrates of the order Aranea are one of the several groups of the Class Arachnida, with about thirty four thousand species. They range in body size from only a few millimetres in length to almost five inches. All are carnivorous and have four pair of walking legs, one pair of pedipalps, and one pair of chelicerae. Each chelicerae consists of a base and a fang. The fang folds up inside of a groove in the base until needed when attacking food, then moves out to bite and releases venom from a tiny opening at its end as it penetrates the prey. They are also used to "chew", getting, digestive juices inside the body of the prey then squeezing out the liquid lunch. The pedipalps are mainly used to catch and rotate the prey while the chelicerae inject it with poison to tear down the tissue. Later the bases of the pedipalps are used as chewing parts. But in males, these palps are used to transfer sperm into the female. These twelve appendages are attached to a dorsal and a ventral plate, the carapace and sternum which cover the entire prosoma and provide attachment points.

The bodies of spiders consist of two parts, an anterior part called the prosoma and a posterior portion called the opisthosoma. These two portions are held together by a narrow stalk called the pedicel. This narrow junction allows for the spider to be very-limber and acts somewhat as a hinge between the prosoma and opisthosoma. So as a spider "moves forward creating a web, it can continue in a straight line throwing its webbing in the direction it chooses. This is how spiders create their zig-zag web formations.

Covering both the prosoma and the opisthosoma is a waxy covering that enables the spider to be a very efficient water conserver. This is one of the characteristics that spiders evolved to adapt to the harsh conditions of terrestrial life. There are eight eyes located in the head region usually in two rows, varying among families. Spiders that wait for and lunge at its prey will have a row of very large eyes well adapted at detecting the precise distance it is from its prey. Yet those spiders that make webs. do not have as great a need for such advanced sight and have smaller eyes. But not all spiders have eight eyes. There are some spitting spiders that have only six, and there are some with only two or four eyes. Some cave spiders have no eyes at all and rely only on vibration. There are great differences in the ways which spiders capture prey. Some may stalk their prey, while others may lie in wait and ambush it. Other spiders may weave various types of webs used to capture passing prey, and there are some smaller commensal spiders that live in

larger spiders' webs and feed on the smaller insects neglected by their host.

All spiders spin silk, though not all of them weave webs. Silk is most commonly seen used in forming webs, which may vary from a highly elaborated orb of spiraling threads to a single sticky string. Most webs can be placed into one of four different types: the orb webs, the funnel webs, tangle webs, and the sheet webs. The main purpose of a web is for catching prey. With orb weavers (Araneidae), the spider will first form a supporting structure of frame threads to which it will then add on radial threads. These tightly strung threads provide quick access to anywhere on the web, and also carry any vibrations from the outer perimeter to the center. After the initial threads are placed, the spider will build on a catching spiral made of sticky silk. These spirals will be what capture and snare prey until the spider is able to reach it and inject it with its venom. Orb webs are very delicate and lose their stickiness after a short period. So many orb weavers take down and replace their old webs daily. They recycle the old silk by eating it as they layout the new silk. Orb weavers must also consider orientation with respect to where the wind is coming from, because they will also snag leaves and blowing debris. When the orb is completed, many orb weavers remain in the center of the web called the hub. They will wait here for their prey. When the web is hit by an insect, the spider turns in the hub to face the direction from where the vibration came. It will then jerk the web sharply to entangle the victim by rapidly flexing one of its front legs. Eventually after it is sure that the prey is stuck in the web, the spider will follow down the strand. Once it is at a close enough distance to make contact, the spider will rush at and quickly bite its victim, then retreat away until the venom has taken affect. After subduing the prey, the spider will wrap it in silk before or after carrying it back to its hub or the site it may choose to hide.

There are more than 2000 orb weaving species and no two species build exactly the same web. But in most cases, the differences are very minor and only concern the symmetry of the web. But there are three dimensional orb weavers that add extra threads from the center to an outside support, thus pulling out the web into a cone shape. This enables the spider to wait at the new attachment sometimes being the attaching bridge. When an insect flies into the web area the spider may cut or simply release the web so that it goes back and ensnares the flying victim.

Funnel web spiders (Agelenidae) are also common spiders. They can be easily found outdoors in short grasses or small bushes, to large vegetation, and even between building edges. Their flat web narrows into a funnel like closure at one end where the spider hides and waits for victims. This funnel is the spider’s retreat, and is opened at both ends. With its legs feeling for any vibration, the spider can quickly ambush any insect that may blunder into its web, darting out and biting it. The insect will not be eaten where it is captured, but will be taken back into the spider's retreat where the feeding process will actually take place.

Sheet web do not have any stickiness to them nor is there a fixed pattern by which they are placed. Instead, an insect that may pass by will become entangled in the vertical strands that act like a tripping line, connected to the spider's sheet web underneath. Sheet web spiders (linyphiids) always hang beneath their dome web, and when there is prey trapped in the vertical strands, they will shake the web so that it will fall onto the sheet. The spider will then pull its victim down through the web while biting and poisoning it.

The tangle web spiders are much like the linyphiids, but their sheet has a much more loose and irregular pattern. Extending down from the sheet are vertical strands that are loosely connected to the ground, and are covered with sticky droplets a few millimeters from the ground. An insect passing by that touches one will stick to it and break it from the ground. While trying to pull free it will tangle itself up in more similar strands while the spider drops down to subdue it. Some of these spiders build retreats that they cover with dirt and pieces of leaves that they will hide in and carry prey into to eat. A variation of this retreat is that of the purse web spider (Atypus). This spider has a silken retreat that is mostly buried underground but has a balloon like tube outside that is covered with soil and bits of debris to appear like normal ground. When an insect walks across or lands on it the spider will bite it from beneath and pull it through the web.

Spiders do not only use threads to make webs and bind prey. Non-web weavers use silk threads to climb up and down with, as well as for draglines. These latter threads are used to both help a spider slow-down and to catch it in case it falls as it leaps from one place to the next, such as from flower to flower. Jumping spiders, most common to the class Salticidae, are known for using draglines foranchoring and quick stops. These spiders use their last pair of legs to propel them from the ground in long or short leaps. Salticids use this jumping ability not only to catch prey but also to escape danger. These spiders can jump up to twenty five times their body length, which is very long for an insect with out any specialized jumping legs. As mentioned earlier, jumping spiders have larger eyes for being able to distinguish visible objects at greater distances. This makes good sense, because they have no other way to obtain prey but with their own stealth and accuracy. They react very acutely to any visual stimulus. First they will turn to face the stimulus and then walk closer towards it. They will stalk their prey until within at least ten centimeters to be able to completely identify it then attack. Once the victim is captured, it is usually con5umed right where it is. Their front legs are stronger so that they may seize prey, and

they have strong perpendicular fangs to penetrate and hold prey firm.

A similar spider to the jumping spiders is the wolf spider. These spiders lie in ambush and attack their prey. They too have a large set of eyes on their upper posterior row, above a row of four generally small eyes. Although wolf spiders have well developed eyes, they react mainly to vibrations received from beating wings or movement from insects on the ground. As with the jumping spiders, there are a large pair of fangs that extend down to help assist in seizing prey. The most well known wolf spider is the tarantula. These spiders can reach up to ten inches in their complete lengths. And although, lore has it that they are one of the most poisonous spiders, their bites are only painful to humans, not deadly.

Though feeding habits vary with spiders their methods of reproduction are all relatively similar, though each species has its own specific ritual. Because spiders are cannibalistic, the much smaller male must be very cautious in 'approaching a potential mate. If he simply rushes in towards the female, the chances are that he will be seen only as food and consumed: So, spider courtship has evolved into a special complex pattern that varies in each species. This variation allows for species recognition, so no gametes are wasted.

1. What can be concluded from the web making habits of spiders ?
   (A) It is essentially reflective of the spider's eating habits
   (B) It is in accordance with the spider's habitat
   (C) It is triggered in response to different types of enemies
   (D) None of the above
2. What is false about the salticidaces ?
   (A) They have large eyes
   (B) They are jumping spiders and can jump up to
   (C) They are cannibalistic
   (D) None of the above
3. Which if the following is an accurate generalization for all spiders ?
   (A) They all have eyes, and use it to spot the food
   (B) They all spin webs in order to catch the prey
   (C) They all spin silk
   (D) They all jump over the prey and overpower them
4. An ideal title for the passage would be—
   (A) Spiders (B) Arhanids and habits.
   (C) Charolette's Web (D) Spiders and food.

Answers

1. (D) 2. (D) 3. (C) 4. (A)

10 Revision Test of Reading Comprehension

Test–1

Words–422

Real-time data is vital for most business. Fast moving consumer goods (FMCG) companies have, in a study, found that over 5-10 % of their sales are lost from the slow replenishment of fast moving items. Similarly, a reduction in pilferage would add to the bottom line of an enterprise. Technology is integral to logistics.

We, at Blue Dart, realised this quite early. Since consignment passes through several hands, modes of transport, billings and labelling in the long chain from pick-up to delivery, effective control over it was often not possible. So Blue Dart formed an in-house systems team for developing information technology (IT) solutions, 17 years ago. Till date, Blue Dart has spent over Rs 55 crore on technology, with no regrets.

The extensive use of IT affords Blue Dart a premium positioning. Over 79 % of its customers (in revenue terms) avail of its technology offerings and derive value out of them. Last year, 58 million shipments were carried across India on Blue Dart's network, with a reliability level of 99.96 %. It had introduced an online track and trace system for international shipments as early as 1988. Blue Dart's order-tracking tool, Track Dart, enables clients to track online the status of each order. It helped automate the pick-up process and eliminate the time--consuming, manual labelling of consignments; details fed into the computer from the airway bill serve as the label on each carton.

Access to real-time information helps its customers as much as the company. Leading pharma firms rely on the Blue Dart technology. A diagnostic centre uses Blue Dart's tracking tool, Internet Dart, to monitor the daily traffic of samples. A pharma major uses Track Dart to replenish the stock of its distributors. TrackDart enables the logistics personnel at the client's office and fill demand gaps accordingly.

Blue Dart's another unique initiative is the backup technology hub set up in Bangalore, a part of its Business Contingency Continuity Plan. Through this, its Bangalore office serves as the hot standby site with real time switchover in the event of any failure at the Mumbai headquarters, which hosts the entire centralised applications and the Bluedart.com website.

ED! (electronic data interchange), e-commerce, ERP (enterprise resource planning) are buzzwords in the business community and the trend looks set to continue attracting attention with the arrival of newer innovations focusing on customer service. In the logistics business, IT makes all the difference, as it can deliver the advantage to the company and the client, in a field which is faced with ever increasing competition

1. The author is primarily concerned with which of the following in the passage—
   (A) Real time data which is integral to logistics
   (B) Blue dart's unique back up technology
   (C) EDI (electronic data inter change)
   (D) ERP (enterprise resource planning)
   (E) All of the above
2. Blue Dart's order tracking tool track dart enables its clients to do all of the following 'EXCEPT'—
   (A) To track online the status of every individual order
   (B) Automates the pick up process which eliminates the time consuming manual labelling of goods.
   (C) Real time switch over in case of any failure at blue dart's headquarters.
   (D) Details fed into the computer can serve as a label on each carton
   (E) None of the above
3. The most appropriate title for the passage would be which one of the following—
   (A) Darting along ahead of the competition
   (B) A dart that's gone a long way
   (C) Blue Dart's Bull's eye
   (D) Signed, sealed and delivered
   (E) Up, up and away
4. The Authors tone in the passage can best be described as one of—
   (A) Dismay (b) Approval
   (C) Annoyance (d) Panegyric
   (E) Amusement

Answers

1. (A) 2. (C) 3. (B) 4. (B)

Test–2

Words–495

On 21 may, reports linked UK drug maker GlaxoSmithK1ine Pharmaceuticals' Avandia, a brand of diabetes drug rosiglitazone, to higher rates of heart ailments and cardiac death in western countries. There could be ethnic or other differences in the way Indians - large consumers of medicines discovered in the US, Europe and Japan - react to the same drug but some years ago there was no way of knowing. Now, there is a ray of hope.

Under a two-year-old drug safety programme launched by the health ministry, a network of well-known hospitals is tracking and reporting suspected drug side-effects in Indian patients that they treat. Also, an expert group evaluates the data and advises the Drugs Controller General of India (DCGI) on the course of action. Some of these experts will meet in New Delhi soon, says Y.K. Gupta, head, department of clinical pharmacology, All India Institute of Medical Sciences. Gupta is a member of this group and his hospital is part of the side-effect tracking network. After they confer, the DCGI will likely call a meeting of the entire group.

The group-National Pharmacovigilance Advisory Committee-might request that the designated hospitals keenly track side-effects in patients prescribed rosiglitazone. "It has to be a focussed effort," says Gupta. The drug is not currently part of a list that the DCGI asked these hospitals to take special note of about a year ago. That list included medicines like sildenafil citrate (Viagra), where loss of vision is a suspected side-effect, and painkiller nimesulide, which could harm the liver.

But it is not that simple. For one, hospitals have to prescribe a drug before they can report side-effects. This may not always be the case. Some government hospitals don't prescribe the newer medicines, like rosiglitazone, since they are more expensive than the older ones. "Our patient population is not rich. So, our tendency is to prescribe standard drugs in the hospital list," says Meena Shrivastava, head, department of pharmacology, Indira Gandhi Government Medical College in Nagpur, Maharashtra. Secondly, doctors have to be trained in the science of reporting. Hypothetically, a so-called accident-like a car crash-could be the result of a person on a cold medicine falling asleep at the wheel because of its sedating effect. In such a case, a medicine might have to carry strong warnings on its label not to drive or operate machinery after taking it. On the flipside, a perceived side-effect such as nausea after popping a pain killer actually may have nothing to do with the drug.

There has to be sustained effort in training and raising awareness among doctors in the network. "It is when the network is active that (the committee) has things to do," says Ranjit Roy Chaudhury, another member and emeritus scientist at National Institute of Immunology, New Delhi. The committee, he says, meets only twice a year on average. "I would like us to meet more," Chaudhury says. Clearly, there's miles to go.

1. The author is chiefly concerned with which issues in the passage—
   (A) The discovery of a new drug for Indian diabetics
   (B) A no side effects drugs for diabetics for Indian patients
   (C) The side effects of foreign manufactured drugs that have effected U.K. user adversely and could affect Indians who use the same drugs
   (D) Rosiglitazone a brand of medicine for diabetics should be banned in India
   (E) None of the above
2. The two year old drug safety programme launched by the health ministry, where a network of well known hospital are involved in—
   (A) The group tracks and records data of foreign manufactured drugs
   (B) The network tracks and records suspects side effects in Indian patients that they treat and

advise the DCDI on the course of action to be taken especially in regard to drugs like Rosiglitazone

(C) To monitor and ban the use of drugs which can cause heart ailments and cardiac death

(D) Scan, research all foreign drugs in order to create a drug for diabetic to suit Indian patients

(E) All of the above

3. The most appropriate title for the passage would be

(A) Diabetic cure or death cause

(B) Death import

(C) A challenging task

(D) Pill to hill

(E) Flatter to deceive

4. The tone of the passage can best be defined as one of

(A) Approval (B) Caution

(C) Disapproval (D) Alarm

(E) Panegyric

Answers

1. (C) 2. (B) 3. (C) 4. (B)

Test–3

Words–503

Forget year-end bonuses. Indian professionals have capitalized on the stock market boom to make some serious money in the last quarter of 2007. With the markets at record levels, a number of corporate big shots have made crores by enchased a part of their stock options over the last three months. While there are numerous executives who have pulled out a few lakhs, some hotshots, including Tec Mahindra managing director Vineet Nayyar, HDFC chairman Deepak Parekh and executive director Renu Karnad, Larsen and Toubro (L and T) chairman and managing director AM Naik and senior executive vice-president VK Magapu, and Ambuja Cements whole time director PB Kulkatni, among others, have grossed crores in the run-up the New Year.

The biggest gainer of them all was Mr. Nayyar, who enchased Rs. 23 crore last week in two tranches. The head of the IT outsourcing firm, which went public in 2006, is still sitting on a neat pile. Given the outstanding shares in his name, Mr. Nayyar currently owns Tech Mahindra shares worth a little over Rs. 140 crore, or roughly $ 35 million, which would place him among the best-compensated chief executives in the country.

Among other well-known names, Mr. Parekh and Ms. Karnad sold shares worth Rs. 9.3 crore each in mid-December. They had earlier sold shares worth about Rs. 5 crore each in late September. And that's not all. The current value of the shares that Ms. Karnad holds stands at Rs. 45·45 crore, or more than $ 11 million while that of Mr. Parekh stands at Rs. 60 crore, or about $ 15 million.

Some of the other board members of HDFC also encased a part of their holding. For instance, SB Patel has encashed about Rs. 1.38 crore at the fag end of October while NM Munjee, an independent director in HDFC and currently chairman of Development Credit Bank, encashed about Rs. 92 lakh through multiple transaction over the last eight weeks.

It's not just service sector companies whose top exes have hit the bull's eye. Take engineering and construction giant L and T, which has been facing succession issues. The company, which had to extend the retirement age of its top management, including the chairman and managing director has created quite a few millionaries.

Mr. Naik sold shares worth Rs. 8.6 crore between end of October and early November. Among other Mr. Magapu encashed Rs. 8·5 crore while R.N. Mukhija and K.V. Ramaswami pulled out approximately Rs. 87 lakh and Rs. 84 lakh, respectively

The top shots of L and T are still sitting on big money, given the value of their unsold shares. For instance, Mr. Naik's existing L and T shares are worth Rs. 420·4 crore, or a whopping $ 105 million, While Mr. Mukhija's shares are worth Rs. 168·55 crore.

Cement major Ambhuja Cements also proved to be a gold mine for its top executives. For instance, Mr. Kulkaeni encashed close to Rs. 1.46 crore through multiple transaction since mid-November. Among others, Ambuja Cements managing director Al Kapur sold shares worth Rs. 65 lakh and wholetime director BL Taparia encased shares worth about Rs. 90 lakh as per recent disclosures to the stock exchange.

Among other companies ITC director SSH Rehman and Anup Singh sold shares worth Rs. 1·68 crore and 1.15 crore, respectively, over the last two months alone.

1. Who have been the top grosser in the run-up to the new year (2008) ?

(A) Company executives

(B) Mid- level investors

(C) Top-notch executives, V.P.'s and hotshots of Industry

(D) The modest and careful investors who took big risks

(E) All blue-chip companies

2. Who is/was the maximum gainer and top compensated CEO in the country ?

(A) Exec Director Renn karnad (HDFC)

(B) HDFC chairman Deepak prarekh

(C) Executive director

(D) Larsen and Toubro Chairman-Managing Director A.N. Naik

(E) Tech Mahindra managing director Vineet Nayyar

3. The most appropriate title for the passage would be

(A) Top- gums blaze

(B) Sensex boom, top cannons thunder

(C) India Inc's top gums make crores or sensex boom

(D) Boom, boom certain sex at sensex Boom

(E) None of these

4. The author is most likely to agree to which of the following statements—
   (A) The stock market is ripe for small time investors
   (B) The sensex will most likely drown those who are sailing on it
   (C) The heavy weights will benefit greatly from the boom
   (D) The top guns will soon fall through the roof
   (E) The stock market is highly unpredictable

Answers

1. (C) 2. (E) 3. (E) 4. (C)

Test–4

Words–554

In some ways they are ahead of their compatriots in other religious communities, while in others they seem to grapple with the same shackles. Compared to other communities, Christians are better educated, economically better off and adopt a more equitable attitude towards women. Yet, they have also adopted the caste hierarchy though in a mellowed form. And, they are struggling with growing unemployment. Whatever be the reasons, the relatively small Christian community of India shares a complex struggle to shed social and economic backwardness.

Christians have the highest literacy rate among all religious communities. For men, it is 80% in rural areas and 96% in urban areas. For women, it is 69% in rural areas and 89 % in urban areas. This is way ahead of other communities, especially for women. Among Hindus and Muslims, only about 41 % of the women are literate in the rural areas. In urban areas, 73% of Hindu women and 60% of Muslim women are literate.

This is not just a bureaucratic statistic. Detailed data provided by a National Sample Survey report in 2004-05 shows that a larger proportion of Christian children start attending educational institutions earlier, and continue till later. Moreover, there are proportionately more graduates among Christians than in any other community.

The Christian community has the highest proportion of the elderly nearly 20% of the total. Among Hindus it is 14%, while among Muslims, it is 11 %. This may be because of better economic status and educational levels, which would tend to lower birth rates and increase longevity, thereby skewing the age structure upwards compared to other communities.

The survey also shows that the community treats its women better its sex ratio is the highest among all communities in India. This can also be partly due to the fact that a significant segment of the Christian population belongs to the tribal areas of the North East, and the tribals do not endorse the inhuman practice of female foeticide or discrimination against the girl child.

Sustained educational levels have led the Christians to a better economic status in India. According to the NSS Report, 47% of Christians in urban areas and 38% in the rural areas come within the top third of monthly earning categories, much ahead of Hindus (24%) and Muslims (20%). Christians have the lowest proportion in the bottom third with only 8% belonging to this category, compared to 12% Hindus and 25% Muslims.

However, education cannot by itself be the path for economic progress this is also brought out by their experience. Their participation in the workforce is roughly the same as for other communities among men and slightly higher among women. But compared to five years ago, this represents a slowing down of their economic contribution, and hence implies lesser opportunities. This is further confirmed by unemployment rates. Among Christians, the extent of unemployed has increased from 4 % to 4.4% in rural areas, and from 7% to 9% in urban areas between 1999-2000 and 2004-05.

While there is cause for anger among Christians, there is courage engendered by education and culture. And, as St. Augustine wrote one and a half millennia ago, Hope has two beautiful daughters. Their names are anger and courage; anger at the way things are; courage to see they do not remain the way they are.

1. It would be fairly accurate to say that the author's main concern is—
   (A) Christians dwindling socio-economic status
   (B) Though better off and comparatively more educated than other community Christians are not getting their due
   (C) Christians are not contributing economically
   (D) Christians do not endorse female foeticide.
   (E) Christians have anger and courage

2. The writer backs his statements with—
   (A) Statistics
   (B) Example
   (C) National sample survey report
   (D) Assumption
   (E) St. Augustine view

3. The attitude of the passage can best be described as—
   (A) Didactic (b) Cynical
   (C) Condescension (d) Encouraging
   (E) Informative

4. The writer infers that—
   (A) The Christian community will overcome all the odds
   (B) It's downhill all the way for the Christians
   (C) Christians are oppressed
   (D) Other communities are envious of their high literacy rate
   (E) There are no grounds for inference in the entire passage

Answers

1. (A) 2. (C) 3. (E) 4. (A)

Test–5

Words–502

Having a lithe and supple body followed a purse throbbing with currency notes are the most popular new year resolutions taken by Indians, an online survey says.

The survey conducted global information and media company Nielsen India showed that about 58% of Indians surveyed online have improved their fitness levels in the new year.

"This year, 'remaining fit and healthy' has all of a sudden taken the lead in people's resolution list compared to last year, taking over better time management, saving money, getting organised, and reading more books that ranked the top last year," The Nielsen company (India) Directed of online Panel N.S. Muthukumaran said in a statement.

According to the survey, 55% of India's online population have resolved to save money while 51% have taken the resolution to manage their time better. Interestingly, losing weight and improving fitness and health were not ranking high in new years resolutions made by people in the past year, Nielsen said in the statement. Remaining fit and healthy is a more popular resolution with 73% of people aged between 35 and 44 years.

Saving more money is popular among 62% of people in the 25-34 years age group while only 26% in the age group of 45 years and above have resolved to save more money in 2008.

Nielsen conducted the surveys using their online research panel your voice. About 292 people aged 15 years and above were surveyed understand what resolution people have made for 2008 and how they have fared on resolutions made in the past years.

"People are more health conscious today. They want to remain fit not only to look good physically, but also to protect themselves from diseases. Eating healthy and regular exercise is the way to achieve optimum health and fitness," Muthukumaran added.

In the age of 15-24 years, 14% respondents have taken the resolution to pursue higher education and 13% would like to manage their time in a better way. Meanwhile, among the people surveyed, 41% have been making resolutions for the past six to ten years, a quarter have been making such resolution for over ten years and 27% respondents make a new years.

Further, amongst those who have made new year's resolutions in the past, 27% have fulfilled their resolutions successfully while 18% have been successful for more than six months. About 23% have kept their resolutions for about three months.

Lack of planning (44%) and commitment (37%) are the key reasons mentioned by respondents for not keeping their new year resolutions while other factors include lack of time, support, and energy. People also have different ways of making resolutions. While 79% of the respondents make a mental note of the resolution that they keep, 28% tell a friend or a family member about their resolution, and 21% write it down.

Among those who share their resolution with someone, 66% share it with their close friend, 55% share it with their spouses, and 32% tell their parents about their resolution for the new year.

1. The main idea the author is concerned about in the passage can be best summed up by which of the following—
   (A) On line Indians surveyed have resolved to manage their time better
   (B) Indians surveyed this year have resolved to save more money
   (C) The online survey on Indians showed that they have made resolution to lose weight and improve fitness
   (D) That Indians surveyed online have resolved to eat healthy
   (E) This year Indians have decided to protect themselves from diseases

2. Among the people surveyed lately the main reasons for not keeping their resolution are :
   1. Lack of planning
   2. Weak commitment
   3. No support nor energy
   4. In sufficient time
   5. All of the above

   (A) 1 and 4 (B) only 5
   (C) 1 and 2 (D) 2 and 3
   (E) 3 and 4

3. According to the writer last years survey conducted by Nielsen India showed which of the following trends as compared to this years survey top priority—
   (A) Better time management
   (B) Saving more money
   (C) Reading more books
   (D) Being more organised
   (E) All of the above

4. In regard to making resolutions which of the following ranks highest on the resolutions scale—
   (A) Making new resolutions every year
   (B) Repeating the same resolutions for the past six to ten years
   (C) Repeating the same resolutions for over ten years
   (D) Breaking new resolutions with in six months
   (E) Unable to sustain their resolution for even 2-3 months

Answers

1. (C) 2. (C) 3. (E) 4. (B)

Test–6

Words–891

After coming across a four year-old marathon runner, all other records relating to age seem superfluous. Still, if you were surprised to know that CEOs a young as 17 year

exist, Silicon Valley has some news for you. Anshul Samar, a 13-year-old, runs a company called Elementeo there. The Samars of the world are extreme examples but they are symbolic of how age no longer matters for business leadership. India Inc presents more moderate examples of this trend.

Even among the 30 Sensex companies, where the top post is still the domain of those over 50 years (not counting a few such as Malvinder Singh, who, at 34, heads Ranbaxy), the next generation is already part of the leadership team. For instance, Tata Steel's VP (Finance) Koushik Chatterjee, ICICI Bank's Executive Director Vembu Vaidyanathan and Bharti Airtel's CFO Sarvjit Dhillon are all aged 39. Aseem Dhru, a vertical head in HDFC Bank, is 36.

One feature that unites business leaders in their 30s and early 40s is that they were groomed in the post-liberalisation era. "What is refreshing is that the young guns do not carry the legacy of the license raj," says Prof S Sriram, Executive Director of Chennai-based Great Lakes Institute of Management. "So, they think free and think big. They are confident, act fast and are ambitious," says he. These are vital qualities in a marketplace that's been turned on its head by the same force of liberalization. "It was earlier easier to compete because of a closed environment," says Sulajja Firodia Motwani, the 37-year-old Managing Director of Kinetic Motor Company. "Today, the speed at which you do business is important. Sulajja feels the customer has turned so savvy that a CEO has to get the organisation to generate the necessary energy to meet expectations. "The younger generation CEO is more pragmatic in decision making," says she. Agrees India bulls CEO Gagan Banga: "We are younger and more aggressive. The whole company works around facts and data more than anything else."

Tech Check

A favourable business environment and the democratizing role of technology have helped create young entrepreneurs who are ready to be unconventional. Take the case of 42 year-old Shantanu Prakash, who decided against a corporate job after passing out of the IIM, Ahmedabad. Instead, in 1994-95, he started setting up computer labs in schools, stepping into a realm where NIIT and Aptech were ruling the roost. His Educomp is now a $ 26-million company.

New values are coming up wherever the old guard has given way to the new, says Ganesh Chella, founder and CEO of HR consulting firm Totus. His list : there's a far higher level of transparency, and the culture of 'confidentiality has gone; there's a process-based way of working, which is more structured; finally, young leaders try to be inclusive and don't take people for granted, as they themselves have grown up in a competitive environment.

Infusing Young Blood

Thirty-year-old Kushagra Nayan Bajaj provided a good measure of those qualities while transforming his family sugar company into a formidable player. He joined Bajaj Hindusthan in 2001, when its turnover was about Rs 200 crore. It is now eight times that and he wants the company to be among the global top three. His approach is to set targets and let professionals take care of the rest. In the process, he has largely done away with hierarchical boundaries and keeps everyone informed of his actions.

Down South, a non-family Old Economy player is rooting for the young in a different way. Ashok Leyland doesn't have a youngster at the helm but its 59-year-old Managing Director R. Seshasayee reckons there's immense value in tapping the skills of its young executives. Last year, for instance, he got them to work on the exercise of management planning and budgeting, usually the preserve of the top management. "They did a brilliant job of challenging the management," says Seshasayee. Ashok Leyland, in an ongoing programme, tries to spot "promising employees" and mentor them. The objective of all these measures, Seshasayee says, "is that we become more youthful and therefore more speedy and innovative in responding to competition."

A company that has spotted a young leader is Cognizant Technology Solutions. What does the $ 2-billion company's 39-year-old CEO Francisco D'Souza stand for ? "I don't think it's about age. Growth drives the company and is the fuel that keeps it going." But he isn't smug. "The biggest issue is individuals or organisations becoming complacent, falling into the trap of believing success of the past guarantees success of the future. If there's one thing we have learnt, it's that in today's world things change too quickly." Amid speed, growth, aggressiveness and all those positives attributed to young leaders, industry observers feel they might need to build certain softer qualities. Sriram says, "young leaders generally think people contribute purely for monetary rewards. They fail to understand the importance of non-monetary things such as loyalty and commitment. And tend to run businesses with their brains and not their heart." Chella also reckons young leaders lack the aspect of genuinely engaging with people. Those gaps could be plugged if leaders start to delegate better and there's a support system to help them out, says he. Many believe the biggest test for them is yet to happen, what with the economy being so vibrant over the last decade .

1. The author if the passage is primarily concerned with which of the following—
   (A) Hierarchical boundaries are being done away with
   (B) Young leaders try to be more inclusive and are against taking people for granted
   (C) The youth are the new leaders of India In
   (D) Vim-vigonus and stamina are needed to steer companies
   (E) None of the above
2. The author cites many example to explain the success stories of young leaders of successful companies-'EXCEPT'—
   (A) The younger lot are more aggressive
   (B) They are more pragmatic in decision making and have high energy levels

(C) The youngset6 works around facts-data above all else
(D) They work mainly to achieve monetary gains
(E) They work more with the head than the heart

3. The following factors have helped create young entrepreneurs—
   1. Conducive business environment.
   2. They were groomed in the post liberalization era.
   3. Democratising role of technology.
   4. New values are sprouting up whenever the old guard has given way to new.
   5. They do not carry the legacy of the license raj.

   (A) 1, 3, 5 (B) 1, 2, 3, 5
   (C) 1, 2, 3, 4, 5 (D) 1, 2, 5
   (E) 2, 3, 4

4. The attitude of the author can best be described as one of
   (A) Approval (B) Disdain
   (C) Cynicism (D) Skepticism
   (E) Encouragement

Answers

1. (C) 2. (D) 3. (C) 4. (E)

Test–7

Words–961

"It is the return season for the prodigal sons and daughters who had left the country for better prospects. And they are a legion. It is borne out by the fact that India is among the largest remittance economies, topping almost $ 24·5 billion last year. Promod Haque's is a case in point. When his Indian engineering degree wasn't enough to land him a proper job, Haque, now Managing Director of Norwest Venture Partners, decided to pack his bags for the US in the 1970s for studies. "I was looking for an electrical engineering job in India but ended up working as a salesperson selling electronics," remembers Haque. A similar compulsion saw Reddy Penumalli, Managing Director of Analog Devices, book his berth on a US-bound airline. The tide is turned. India's free economy has put the country on the map of preferred investment hotspots and as an incentive to boost its burgeoning growth, it has rolled out the welcome mat to the Indian diaspora.

Key Triggers

Driving this reverse migration back to India are triggers professional and emotional. The IT and technology boom in India has made private equity, hedge funds and venture capitalists look towards India. Saurav Srivastava, Kanwal Rekhi and Vinod Khosla are just a few of them warming up to the India Story. After spending 30 years in the US, ex-McKinsey Chief Rajat Gupta has set up a $ 1·5-billion fund under New Silk Route Ventures with his entire team in India. Even the new graduates fresh from management institutes prefer being part of the growth here and look for placements. "Professionals are willing to take a salary cut to be in India and join the excitement," says 28-year-old Abhi Shah, who after passing out of Harvard Business School joined Anderson Consulting and was part of the Jerry Rao team when MphasiS was sold. For Chandra Kopparapu, Vice President, Asia-Pacific of Foundry Networks who relocated to India in 2004 after a 14-year stint in the US, it was a brighter career prospect that was attractive. "India is a great learning experience because the way business is done here is very different from what we are used to in the US. You have to learn your way up," says Kopparapu.

Not Just Money

It is not money alone that is luring Indians back to their roots. India offers an emotional pull. For the 47-year-old Chief of Jakarta-based Universal Success Enterprises, Prasoon Mukherjee, who grew up in Kolkata, nothing could be more rewarding than doing business in his homeland. His career, which began with Great Eastern Hotels, saw Mukherjee climb up the ladder with $25-billion Salim Group of Indonesia as a partner. "Emotional bondages have grown over the years. Opportunities in India are much more now than in the time I left. In fact, it was only after I went abroad that the whole idea of doing business came up," says Mukherjee.

His nostalgic connect with India is nowhere as keenly illustrated as in the investments he has lined up over five years : $ 2 billion. Mukherjee belongs to the new breed of non-resident Indian businessmen, led by stalwarts like LN Mittal or Lord Swraj Paul who are on a comeback trail. Even business houses that had kept a low profile until now are plotting a return-the Hinduja Group with stakes in Ashok Leyland, Ennore Foundries, Induslnd Bank and Hinduja Ventures are giving India a shot again. Says Dheeraj Hinduja, "There will be a definite change. We are looking at sector like insurance, real estate, health care and energy."

For still others, who are venturing back to India, it is a question of being helpful to the Indian poor who represent the other side of the prosperous facade_,For Vikram Akula, Chief Executive Officer and Founder of SKS Microfinance, his commitment is towards the millions who live in rural India. He feels that NRIs have an obligation to the country, as many of them have benefited from state-subsidised higher education. "There needs to be an increase in initiatives by NRIs to bridge the social divide-whether that be in education or health or economic development," says Akula.

One upside of such a reverse brain drain is that even multinationals like Microsoft and IBM, which have a sizable presence in India, have hired Indians to head their operations here. These executives are moved back to India perhaps for their better understanding of the working culture and government policies. "Any multinational would be governed by global policies and philosophy. However, having a local team always helps in maneuvering around the system," says Rohit Kochar, National Chairman and Managing Director of Kochar & Co, a legal services company.

Even though systems are more streamlined in developed countries like the US, one gets used to the 'Indian way' of working over time. The influx of Indians with global skills into India will make their home-grown

Indian companies rethink and strive to be more competitive.

"The Indian companies have to transform themselves and compete in the changing dynamics or become obsolete and exit the market," says Vikas Vasal, Executive Director of KPMG. Now, with India's growth closing in on double digits, it is a rosy scenario. But will the global Indian leave India if recession sets in ? Some feel that the Indian who followed opportunity outside and came back for a more vibrant India could do this again. Though, investments having sunk in the creation of assets and infrastructure-from IT parks and SEZs to manufacturing plants-a. downturn might make it difficult to take away these assets. Also, no one expects the boom phase to be temporary. Even if there are minor setbacks, the objective is the long-term growth and there is unlikely to be any knee-jerk reactions. It looks like, for the global Indians, the good times will roll.

1. The most appropriate title for the passage would most likely be—
   (A) The Root cause
   (B) Return of the prodigals
   (C) The tide has turned
   (D) India sons and daughter Return
   (E) India shores show the way to wealth
2. The main reason/s why India are returning back to India can best be described by—
   (A) Opportunities are more now
   (B) India's among the largest remit ten economics
   (C) India's free economy has turned the tide making out bound Indians turn home word
   (D) Professional emotional quotients
   (E) All of the above
3. The attitude of the author in the passage can best be described as one of
   (A) Enthusiasm (b) Cynicism
   (C) Optimism (d) Pessimism
   (E) Anger
4. In the passage some have agreed that Indians could do a turnaround the resons stated being—
   (A) Lesser pay packets and perks
   (B) More opportunities and better environment abroad
   (C) If a recession sets in they will leave
   (D) Stiff government policies and high interest rates
   (E) All of the above

Answers

1. (A) 2. (E) 3. (C) 4. (C)

Test–8

Words–1509

For me(Bill Gates) and anyone else who is passionate about using technology to help create opportunities for people trends in India today are tremendously exciting and encouraging.

As everyone knows, the nation has become a global leader in information technology and other high- tech fields such as pharmaceuticals telecommunication and telecom based business services. These sectors have contributed to the economy's rapid growth since 2003 which has lifted many millions of people out of poverty; continued growth could alleviate suffering and expand opportunities for millions more. One day, we may look back on India's progress during this decade as one of the great humanitarian achievements of our time.

Equally exhilarating is how India's rise may influence the global community. The world will be a safer place if other nations can learn from the achievements of what is not only the largest democracy, but also one of the most pluralistic cultures. The prime minister culture, Dr Manmohan singh, has said it well : "India's success will renew humanity's faith in liberal democracy, in the rule of low, in free and open societies." The entire world has a big stake in India's future.

The power of Indian skills and Talent. It seems to me that the India miracle, if you will, demonstrates the wisdom of sustained investment in the primary asset of any modern economy: people. During the nearly 60 years since independence, India's investments in human developments have reduced hunger, increased literacy and improved health conditions. Education investments have produced world-class scientists, engineers and technicians. They in turn have fuelled the growth of Indian technology companies and attracted many global technology leaders, including Microsoft.

People have been the key to Microsoft's success in India, and our experience may be illustrative. We entered the country 17 years ago working closely with the government, IT industry, academia and the local developer community. Over the years, the people of Microsoft India have had end-to-end responsibility for the development of many Microsoft technologies. They have made important contributions to many other products, including windows Vista and the 2007 Microsoft office system.

We currently employ more than 4,000 people across six business units in Delhi, Banglore, Mumbai, Hyderabad, Kolkata, Pune and Chennai and we continue to expend our presence. Outside the united state, Microsoft's India Development centre is our second largest product development facility. Two years ago, we opened Microsoft research India, where scientists and engineers work to advance the frontiers of knowledge in computer science and related fields, often in collaboration with India's academic community. These teams have demonstrated India's great capacity for innovation by filing for 100 patents during the past two years. Other India units play major roles in our world wide customer support, consulting services and management of our internal information systems.

Beyond our direct presence, Microsoft also contributes to India's growth through the thousands of local partners, large firms and small, that develop and sell products and services based on our software platform. This year, 35 Indian companies qualified for the Forbes 2000 list of the world's biggest corporate giants. Among them were four valued Microsoft partners : Tata Consul-

tancy, Infosys Technologies Wipro and Satyam Computer Services. Microsoft is extremely proud to be a part of the economic trans formation that these and other highly successful Indian companies have helped bring about.

Sustaining Growth, Broadening The Opportunity

How can India best sustain its rapid growth and broaden opportunity for all its people? Much has been written about the need for sharply increased investments in highways, airports, power plants and other infrastructure. Economists also point to a need for regulatory reforms and better ,public services provided more transparently. These are important challenges.

Also, from my perspective, investment in human capital should continue to be a high priority, especially efforts to further alleviate hunger, reduce illiteracy and improve public health. Threats to healths such as HIV/AIDS, for example, could upset much of India's recent progress. The estimate is that less than one per cent of adults are infected, but because of India's large population, the number is among the highest in the world.

Education at every level remain crucial for continued growth. Output of college and university graduates is impressive in absolute terms, and has been a great source of economic strength, but India cannot afford to become complacent. The nation now faces an acute shortage of skilled workers, as Infosys and other employers have warned recently. Education spending as a percentage of GDP lags far behind that of countries such as South Korea and Taiwan. Yet, one could argue that India needs a skilled and educated workforce even more than the so-called Asian Tigers do. They accelerated their development through manufacturing, primarily, while India's focus on services and technology makes its workforce skills especially critical.

As many others have said and as the government has recognised in its budget plans, India urgently needs to build more primary and secondary schools, improve teaching and ensure that more children attend school, especially in rural areas. Higher education needs to be expanded and upgraded. Top-tier institutions are overrun with applicants, while skill levels among graduates of some other colleges do not meet world standards or the needs of employers. By one estimate, 25 % of all new engineering graduates lack the skills to be employable in the IT industry, despite its dire need for workers.

Microsoft is committed to helping improve Indian education. Over the past several years, we have been engaged in many collaborative efforts, mainly focused on advancing the instructional uses of technology and expanding access to computers and computer skills. For example, our Project Shiksha currently works with more than 10 state governments, bringing computer skills training to more than 120,000 teachers so far. We have helped enhance learning opportunities available to students in slum and rural schools through support for Digital Study Hall, a project that records and distributes DVDs of classes led by India's best grassroots teachers. And to help overcome a scarcity of classroom computers, Microsoft Research India has developed Windows Multi Point, a technology that enables several students to work on a single PC.

In higher education, our efforts have included the Developer Platform Evangelism Academy, which has provided professional development to more than 1,000 IT and engineering faculty members at 51 Indian colleges. To help recent engineering graduates transition from school to careers, we recently began working with the Indian government and industry on an online employability portal. It will enable graduates to assess their skills, complete appropriate training and connect with prospective employers.

Technology And India's Future

Besides being an important tool in education and a growth sector of the Indian economy, information technology can aid social and economic development in many ways. Wide deployment of computers, software and telecommunications helps boost productivity and reduce transaction costs in many sectors, strengthening economic growth. Computers, mobile devices and software can help expand the quality and availability of health care and other public services, as well as education.

A lack of access to technology, on the other hand, can hinder development. More than 30 years after the invention of one of the most versatile and empowering technologies of our time, the personal computer is readily available to only 1 billion of the world's more than 6 billion people. Microsoft's founding vision of "a computer on every desk and in every home" is a reality for the roughly 1 billion people living near the top of the global economic pyramid. But the digital revolution has yet to spread very far in many rural areas, impoverished communities and developing countries, including India.

Disparities in technology access are troubling, for as the global economy is increasingly computerised and moves online, social and economic development becomes even more difficult in the places and for the people left behind, on the less fortunate side of the digital divide. This is a problem that Microsoft and others in the information technology industry have been working to address.

Microsoft's ultimate goal is to bring the benefits of technology to every person. Toward that end, we have set our sights on an ambitious milestone: With governments and other partners, we aim to deliver the power of information technology to 1 billion more people worldwide by the year 2015. We are expanding several technology training and assistance programs.

And we recently introduced the low-cost Microsoft Student Innovation Suite of software product including versions of Windows, Microsoft Office, Learning Essentials and Microsoft Math. Although we invested many millions of dollars to develop these products, the suite will be available to students for about Rs 127, through government programmes in India and many other developing countries, and in developed countries as part of targeted programmes that provide PCs to disadvantaged students.

We are taking these and other steps because, as industry leaders and simply as human beings, we believe that all 6 billion people who share this planet deserve a chance to realise their full potential. We are especially excited to be working toward realising this vision in

India, where progress on many fronts is already well underway .

1. The author's chief concern in the passage is directly linked to—
   (A) The highly exciting technological trends in India contribute to the economy's repaid growth
   (B) Pharmaceutical's upsurge in India sends out globally encouraging signals
   (C) India's success renews humanity's faith in liberal democracy
   (D) India is the largest democracy and has the most pluralistic of cultures
   (E) None of the above
2. According to the author which of the following has contributed the most in reducing hunger and improving education and health in India—
   (A) The sustained investment in the main asset of India-people.
   (B) Investment in rural development
   (C) Investment in adult literacy
   (D) Investment in health
   (E) All of the above
3. As the global economy moves online development for any country lies in investing in the following:
   (A) Literacy, health education and facilities and repaid technological growth
   (B) Rural development and infrastructure
   (C) Adult education
   (D) Computer education
   (E) All of the above
4. The author's tone the passage is—
   (A) Encouraging (B) Disapproving
   (C) Instructive (D) Cautious
   (E) Condescending

Answers

1. (A) 2. (A) 3. (A) 4. (A)

Test–9

Words–1234

Rajesh Srinivasan, a software programmer in New Jersey, regularly scans Naukry.com looking for possible opportunities in India. Srinivasan came to the US in 1999 to pursue a master's degree in computer engineering. Eight years later, this lawabiding, regular taxpayer has still not received his permanent residency–the elusive green card. His application lies somewhere in the backlog of 500,000 applicants, and it could be another couple of years before he gets the green card. Meanwhile, a proposed Immigration Bill by the Bush administration lays ,out an elaborate plan to grant legal status to the 12 million illegal immigrants in the country, while not providing any clear timelines to legal, skilled workers like Srinivasan, who are in the long queue for permanent residency.

Basically, Indians who come to work in the US do so either on an H-lB visa or anL-1 visa. They can then apply for permanent residency through the employer, a process that can take anywhere between six and 10years. Working on an H-lB visa - while the green card application is being processed by the employer - means that one can not switch jobs or be up for a promotion. Also, long-term decisions like buying a house or other investments are normally put on hold until one is certain of his status in the country. "It is like being in a state of limbo," says Nanda Kulkarni, who has been in the US for more than seven years.

The bill hopes to transition the green card process into a merit-based points system, like the way it is done in Canada and Australia. If rolled out, this system would remove the dependency on the employer and award green cards based on individual merit. While this would stand to benefit many employees, employers are not too pleased as they lose control over who they want to retain in the company. An Indian business owner in New Jersey, speaking on condition of anonymity, says, "The points system will judge paper resumes, not hands-on skills, that is not always an accurate estimate of a good employee."

The details of the points system will be formalised only after the bill is passed. But the tentative version that is put forward is skewed toward illegal immigrants, according to some observers. "Undocumented workers get points for owning property as that means they have contributed to the economy, but legal, skilled worker don't get any points for that," says Vikas Chowdhry, a software professional, who is also caught in the green card quagmire.

More importantly, the proposed reform dose not factor in the backlogged applicants who have entered the country legally and are in line for their green cards. "India has the highest number of people in this backlog," says Jwalant, who represents advocacy group Immigration Voice.

In a bid to move to the new points system and also to accommodate illegal immigrants into the fold, the number of green cards available for backlogged applicants in the old system has been set at 90,000. "This is a reduction from the previous number of 140,000 available to us," says Aman Kapoor, president of Immigration Voice. He adds that when the cap was 140,000, applicants had to wait for 6-10 years to get the green card. Now that the numbers available to this backlogged group has further decreased, the wait time could be anywhere 12-15 years. The proposed points system, if passed, would go into effect only one financial year later. Moving to that system would mean going back to the bottom of the queue. Besides, given that there are uncertainties involved with how the system will pay out, not many are willing to make the shift.

There is also a per-country limit on the number of green cards available. A little ironical, considering that there is little ironical, considering that there is no country cap on the number of H-1b visas issued. Typically, a larger number of H-1B visas go to high skilled workers from India, China and Russia than other countries. In that case, having a standard cap of 10 percent per year for each country will skew the distribution resulting in "some people being forced to go back," says a visibly frustrated Kapoor, who feels the proposed Bill has implications that are unfair to the H -1 B visa holders.

Several Indians share his sentiment. Having waited patiently for immigration reform to make things easier for them in the country, the currently proposed Bill leaves many of them disappointed. "It would have been much easier for me if I had entered the country illegally," says Abhishek Katra, who faces the possibility of being sent back to India because his green card application is not more than a year old, which is required for H-lB workers who have been in the US for over six years.

Massachusetts Senator Edward M. Kennedy issued a lengthy statement on his website supporting the proposed Bill. Regarding the inflow of skilled labour, he said, "Our plan recognises that our economy will continue to need hardworking people who are willing to come here for a few years... . We need computer programmers, scientists and engineers. So our programme will allow them to come as guest workers under a programme with strong labour laws that protect American jobs and wages."

The Bill does suggest increasing the number ofH-lB visas to 115,000 annually from the present cap of 65,000, with a possibility of increasing the cap to 180,000. However, it has raised the fee for filling an H-lB visa from $1,500 (61,500) to $5,000 (Rs 2,05,000) and added some stringent guidelines on the use of the H-IB visa. Employers with more than 50 employees may only hire 50 per cent of its employees on an H-lB visa. Also, a company will be restricted from hiring H -lB visa holders and placing them as consultants in other companies. Students, however, would be allowed to work for two years after graduation on optional practical training, instead of one.

"They are basically trying to make it difficult for employers to hire people on H -lB visas," says New York-based immigration attorney Kavitha Ramasami, who advises several Indian companies on their immigration policies. The Bill is, in a way, setting the stage for more off, shoring. As companies will be unable to afford the expensive H-IB visas, they will star sending more work to off shore locations. "Capital is always going to follow skill," say : Kapoor, who is certain that many Indians will leave the country if positive changes to immigration don't happen soon. "Ten years ago, Indians would probably have stuck it out rather than go back, but today India has much more to offer," says Voice's Pradhan.

The American Immigration Lawyers Association says, "Increased H-IB fees are nothing more than a tax on innovation that will end up driving US jobs; overseas by making it more difficult to hire the highly educated talent America needs."

While the Bill has yet to pass in Congress, lots of changes can be expected as the US tries to clean up its immigration system. Some senators have proposed amendments based on the feedback from their constituents. "There will be changes before the Bill finally comes out, hopefully they are positive," says attorney Allen Kaye, a New York attorney and ex-president of the American Immigration Lawyers Association.

Meanwhile, those like Srinivasan, who fear being lost in transition, are exploring their back-up options. .

1. The most beautiful headline for the passage would most likely be—
   (A) green card conundrum
   (B) H- 1B or Bye
   (C) Green card is red hard
   (D) Green stop
   (E) Green card- stop or Go
2. The proposed immigration bill by the bush administration according to the author liberates which confining is best explanation by which of the following ?
   (A) It plans to grant legal status to the 12 million illegal immigrants in the country
   (B) Through it plane to grant legal status to illegal immigrant it does not define a lucid time from to legal entrants
   (C) The underlying catch is that they have to apply for legal status solely through their employees
   (D) The problems that lies in the bill is that one connect switch jobs or get promoted while the employer is processing the green card application
   (E) All of the above
3. The main that legal immigrants applying for permanent residency has—
   (A) Undocumented workers get points for owing property
   (B) The proposed reforms do not factor backlogged applicants
   (C) Reduction in the number of green card as composed to previous years
   (D) The irony of the per country limit on the number of cards issued as there are no country caps on the number of H –1 B visas issued
   (E) All of the above

Answers

1. (A) 2. (B) 3. (E)

Test–10

Words–1233

It is late in the evening your wife wants to eat Chinese food, but you cannot seem to remember where that restaurant your friend was talking about is located. So you go to Yahoo's Our City service, select your city and type in "Chinese food". Immediately a list of restaurants comes up, along with their locations on map. Result: great meal and a happy spouse. All this, thanks to global Internet giants, who are now localising their applications for Indian consumers.

Two years ago, the research and development (R&D) centres of global Internet giants in India worked almost entirely on developing products for other developed markets. This was done for the same reasons as the software outsourcing sector-cost efficiency and availability of talent. But the twist in the outsourcing tale for the Internet space is India's growing importance as a market by itself.

Increasingly, global players are focussing on developing products and applications specific to India. While the current Internet user base (about 50 million) is not very large relative to the population, India is now the fastest growing Internet market in the world. With the potential to go over 200 million users over the next several years (behind only China and the US), India is a market worth pursuing early on for these players. Traditional markets such as the US and UK are getting saturated in terms of growth in numbers of Internet users. Today, 80 per cent of new Internet users come from outside the US with emerging markets adding numbers rapidly. And India is right in the middle of this growth, making the country a valuable market for Internet companies over the next five to ten years.

For companies such as Yahoo! and Google, which already have large R&D facilities here, there is no better way to corner the India market than using local talent to work on local products. Almost all of the company's India-specific applications are led by, if not entirely developed in, their centres here. In the past two months alone, the India R&D centres of Yahoo! and Google developed at least two products each for the India market. Yahoo! launched Our Cities and Yahoo! Maps, while Google launched Hindi Blogger and a Hindi news service on Google News.

As India's Internet market expands, products from the India centres will focus on addressing the needs of these users. Why is this a big deal? India differs from other Internet markets due to its regional and linguistic diversity, which poses some unique challenges from a technology standpoint. If Internet companies can find a way to overcome the barriers here, it will help them understand users in other countries where Internet is yet to take off. Technology platforms developed to tackle challenges here could then be extended to other regions with conditions similar to ours. This means that India could lead the development of technology to crack other markets where growth is possible. India is also one of the few large markets where the mobile revolution took off before the Internet If that route to Internet access is exploited well, it could become a model for regions such as Africa.

Yahoo! recently made its Bangalore centre the R&D hub for emerging markets. That Includes India, Latin America and other Asian markets, like Vietnam and Indonesia. Sharad Sharma, the newly appointed head (R&D) at Yahoo! India, says a key part of his mandate is to build more products specifically for emerging markets. "There is definitely a greater emphasis on the Bangalore centre because of this," he says.

Challenges Ahead

If the number of Internet connection in India have to increase substantially, there must be growth beyond the metros. Internet services need to become more regional more local and easily accessible. They need to be in regional languages and the options need to become more localised. For example, a user should be able to locate the nearest auto showroom to his house, online.

Take languages. As the user base inches closer to the 100- million mark, Internet companies will have to reach out to the non-English speaking populace for further growth. India's regional diversity requites offering the same vices in different languages. Rediff, which offers its mail service in 11 Indian languages, uses predictive text to help users type in the local language. "Since the lack of local language hardware is a limitation, we have to work around the software," says Manish Agarwal, V.P. (marketing), Rediff.

What makes this difficult is that Indian languages are phonetic. The text is unlike Roman script, which is the basis for most European languages. Indian languages also do not have established transliteration rules, like Japanese or Chinese. That is, one Hindi name can be spelled differently in English - Mansi, Manasi, Maanasi, and so on. Google's recently launched Blogger, in Hindi, was led by its India centre. Prasad Ram, head (R&D), Google India, in Bangalore, says "The product is an example of technology contribution that can be leveraged in other non-Roman languages."

Secondly, the Internet will now become more local. In developed countries, people use it to locate places- the nearest bus stop, a convenient Wi-fi spot, and so on. This requires accurate mapping. The problem in India is that there is no reliable geographical data available. In most countries, such data comes from sources such as a government database. Here, companies have to resort to satellite imagery to get their information, which is not an easy task. These companies are now trying out different ways to create an online presence for previously unmapped locations. Home-grown search company Guruji. com, for example, has ,tied up with Infomedia for access to local data bases.

Thirdly, given the large scale adoption of mobile telephony in the country, firms are devoting significant resources towards making Internet access available across platforms. "The Internet is becoming device agnostic," says Jaspreet Bindra, country manager, MSN India in Mumbai. MSN is looking at the mobile platform as the key to cracking

India's market. Microsoft's research lab in Hyderabad is developing specific products for mobile devices. One such product is an "on-deck" search application, which searches for items in the mobile operator's domain. For example, if you are a Hutch user, you could use its database for all available Kishore Kumar song downloads.

While R&D for the Indian market has begun, it is still only a small portion of the work that the Indian centres do. About 80 per cent of the product development is still for foreign markets, But this will gradually decrease over the next few years.

Companies are ramping up R&D operations in India like never before. Last year, Yahoo! doubled its R.&D workforce to 1,000. It had taken them four years to reach 400. And the recruitment is not just happening at the engineering level. Yahoo! also made several key appointments at the top, even pulling people from its global headquarters. Pranesh Anthapur moved from Yahoo's Sunnyvale office to take over as COO (R&D), Yahoo! India. With the Bangalore centre, Yahoo's global R&D

capabilities have nearly doubled, claims Anthapur. like Yahoo!, search giant Google is also recruiting developers in large numbers. For both companies, the Indian market represents their largest R&D operations outside the US.

As India's Internet base becomes more active, there will be more online services catering to the needs of India's market. Dinners at bad restaurants will soon be a thing of the past.

1. According to passage the author would most likely agree to which of the following suggestions. If the number of internet connections have to increase substantially ?
   1. Internet services need to become a lot more regional.
   2. The internet services should be in regional languages.
   3. Options need to be more localised.
   4. Indians languages are phonetic and do not have transliteration rules which make it difficult for internet users.

   (A) 1, 2, 4 (B) 1, 2, 3
   (C) 1 and 4 (D) 1, 2 and 4
   (E) 1, 2, 3, 4

2. The most catchy and apt title for the passage would most likely be—
   (A) On Dot Indian
   (B) Localising the Net
   (C) Spread the Net inwards
   (D) WWW. Internet .IN
   (E) Indian spiders crawling on the NET

3. 'Global Internet giants are localising their applications' in India implies—
   (A) Indias' growing importance as a market
   (B) India is presently the fastest growing internet market in the world
   (C) India has the potential to exceed 200 million users over the next coming years
   (D) US and UK market are getting saturated in terms of numbers of internet user
   (E) All of the above

4. Which of the following road blocks that are being faced by Indian internet companies 'EXCEPT'—
   (a) As Indian languages do not have established transliteration rules. Hence, the use of predictive text
   (b) Accurate mapping is difficult as there is no reliable geographical data available
   (c) There is a huge non English speaking populace
   (d) Red tapism obstructs and hinders any such ventures

   (A) 1, 2, 3 (B) 3, 4 only
   (C) Only 4 (D) Only 1
   (E) 3 and 4

Answers

1. (B) 2. (D) 3. (E) 4. (C)

2 Synonyms

Synonyms are words of the same grammatical class that have a similar, but not an identical meaning—

Words	Synonyms
A	
Abandon	Forsake, Give up, Abdicate, Relinquish
Abandoned	Deserted, Vacant
Abdicate	Give up, Vacate
Ability	Capacity, Aptitude, Capability, Expertness
Abnormal	Unusual, Unnatural
Abound	Plentiful, Overflow, Abundant
Abrupt	Sudden, Hasty
Absorb	Assimilate, Merge, Take in
Absurd	Silly, Ridiculous
Abundant	Ample, Plentiful
Accelerate	Quicken, Hasten, Speed up
Active	Alert, Agile, Alive
Acute	Keen, Piercing, Sharp
Adept	Expert, Apt, Skilful, Dexterous
Adjust	Regulate, Accommodate
Admirable	Excellent, Commendable, Praiseworthy
Admit	Own up, Concede, Acknowledge, Accept
Agree	Concur, Consent, Assent
Aid	Help , Relief, Support, Assistance
Aimless	Random, Wandering, Purposeless
Allow	Grant, Admit, Permit
Amateur	Volunteer, Novice
Ambition	Aspiration, Longing
Anger	Arouse, Inflame, Annoy, Provoke
Anger	Resentment, Wrath, Rage, fury
Apology	Excuse, Pardon, Regret, Amends
Appetite	Want, Hunger, Longing, Passion
Apt	Suitable, Qualified, Appropriate
Arrogance	Pride, Insolence, Haughtiness
Artistic	Beautiful, Graceful
Ascent	Rise, Lift, Elevation, Upswing
Assist	Help, Support
Atrocity	Outrage, Enormity, Brutality
Attack	Assault, Invasion, Aggression, Onslaught
Attain	Win, Gain, Achieve, Succeed
Average	Medium, Ordinary, Fair
Awkward	Clumsy, Ungainly, Ungraceful, Embarrassing
B	
Bane	Curse, Mischief, Harm, Scourge
Banish	Exile, Dismiss, Expel
Bard	Singer, Minstrel
Base	Vile, Low, Mean, Ignoble, Unworthy
Bashful	Timid, Shy, Diffident, Constrained
Bear	Endure, Suffer, Tolerate, Sustain
Beautiful	Pretty, Lovely, Graceful, Handsome, Charming, Elegant
Beg	Implore, Beseech, Ask, Solicit
Belief	Faith, Trust, Confidence, Credence
Benevolent	Generous, Philanthropic, Charitable
Blithe	Joyous, Gay, Light-hearted, Merry
Bogus	Sham, Counterfeit, Spurious, False
Bold	Courageous, Undaunted, Intrepid, Impudent
Bounty	Generosity, Gift
Brave	Courageous, Fearless, Valiant, Bold
Breathe	Respire, Inhale, Exhale
Bright	Lustrous, Luminous, Radiant, Glowing
Build	Make, Construct, Fashion, Erect
Burning	Flaming, Raging, Blazing
Busy	Engaged, Occupied, Engrossed, Employed
C	
Callous	Hardened, Insensitive, Stiff
Calm	Impassive, Placid, Serene, Cool-headed, Tranquil
Candid	Frank, Blunt, Outspoken, Straight forward
Capture	Arrest, Nab, Seize, Apprehend
Champion	Supporter, Defender, Protector, Victor
Chaos	Confusion, Disorder
Chaste	Pure, Clean, Virtuous, Undefiled
Cheat	Defraud, Swindle, Dupe, Deceive
Chronic	Habitual, Confirmed
Clear	Transparent, Lucid, Distinct
Clever	Able, Skilful, Adroit, Intelligent

Cold	Frigid, Cool, Chilly, Frosty, Indifferent, Reserved
Commend	Praise, Applaud, Approve, Compliment
Conceal	Hide, Veil, Shroud, Disguise
Concern	Anxiety, Solicitude, Worry
Confess	Admit, Acknowledge, Avow, Own
Confidence	Reliance, Faith, Dependence
Conflict	Fight, Clash, Strife, Collision
Conjecture	Guess, Speculation, Inference, Surmise
Conquer	Win, Overcome, Vanquish, Triumph
Considerate	Thoughtful, Kind, Sympathetic, Humane
Consolation	Solace, Comfort, Sympathy, Hopefulness
Contain	Receive, Include, Hold, Comprise
Contemptible	Despicable, Despised, Vile, Mean, Low
Contribute	Give, help, Assist, Subscribe
Control	Direct, Manage, Regular, Master
Convict	Criminal, Prisoner, Captive
Copious	Plentiful, Ample, Abundant, Adequate
Corrupt	Dishonest, Demoralised, Debase
Courage	Boldness, Valour, Bravery, Audacity
Covet	Long for, Want, Desire, Crave
Cruelty	Brutality, Barbarity, Persecution, Torture
Cultivate	Grow, Farm, Develop
Cunning	Craft, Shrewd, Sly, Wily
Curtail	Limit, Shorten, Abate, Reduce, Diminish
Cut	Hew, Chop, Carve, Slit, Slash, Divide, Curtail, Reap

D

Damage	Loss, Harm, Injury, Hurt, Impair
Dangerous	Hazardous, Perilous, Unsafe
Dead	Deceased, Lifeless, Extinct, Obsolete
Dear	Cherished, Precious, Costly, Valuable
Decide	Settle, Finalize, Fix, Determine
Deduce	Derive, Infer, Conclude, Reckon
Deed	Act, Action, Work
Defeat	Conquer, Overcome, Foil, Frustrate
Defend	Protect, Shield, Guard, Safeguard, Support
Defer	Delay, Suspend, Postpone, Stay
Dejection	Depression, Grief, Sorrow, Despondency
Delay	Put off, Dally, Postpone, Procrastinate
Deliberate	Intentional, Studied, Thoughtful
Delightful	Pleasing, Enjoyable, Charming, Alluring
Deliver	Relieve, Rescue, Redeem, Discharge
Demolish	Destroy, Raze, Wreck, Wipe out
Denounce	Decry, Censure, Accuse, Curse
Depict	Show, Exhibit, Portray, Delineate
Derisive	Sarcastic, Contemptuous, Scornful, Mocking
Derogatory	Defamatory, Humiliating, Discreditable
Desolate	Bleak, Barren, Lonely, Forlorn
Despise	Scorn, Disdain, Dislike, Detest
Destination	Goal, End, Objective, Target
Destroy	Demolish, Ruin, Devastate, Raze
Determination	Resolution, Will, Firm, Decision, Judgement
Determined	Resolute, Resolved, Decided, Firm
Detest	Loathe, Abhor, Despise
Devastate	Ravage, Destroy, Pillage, Ruin
Develop	Grow, Cultivate, Produce, Evolve
Devotee	Votary, Disciple, Worshipper, Enthusiast
Dexterous	Skilful, Deft, Clever, Adept
Dignified	Stately, Lofty, August, Noble, Majestic
Dim	Dusky, Dull, Dingy, Hazy, Vague
Diminish	Lessen, Reduce, dwindle
Disaster	Misfortune, Calamity, Tragedy, Catastrophe
Discard	Reject, Abandon, Eliminate, Repudiate, Remove
Discord	Strife, Dissension, Contention
Discover	Find, Disclose, Reveal, Discern
Disfigure	Impair, Deface, Mar, Mutilate
Disgrace	Insult, Dishonour, Humiliation
Disinterested	Indifferent, Detached, Unconcerned
Display	Show, Exhibit, Disclose
Dispute	Quarrel, Contention, Controversy
Distasteful	Unpalatable, Unsavoury, Bitter
Distinguished	Celebrated, Famous, Eminent, Illustrious, Renowned, Noted
Distress	Discomfort, Trouble, Grief, Anxiety
Distribute	Divide, Classify, Scatter, Apportion
Diversity	Variety, Variation, Difference
Divine	Holy, Godlike, Celestial, Superhuman
Docile	Gentle, Submissive, Obedient, Teachable
Dogmatic	Bigoted, Pinioned
Dreadful	Fearful, Awful, Dire, Horrible
Dynamic	Forceful, Vigorous, Impelling

E

Eager	Ardent, Zealous, Fervent, Earnest
Earnest	Serious, Solemn, Fervent, Determined

Economical	Frugal, Careful, Thrifty, Saving
Ecstasy	Joy, Bliss, Rapture, Enthusiasm
Effort	Attempt, Endeavour, Trial, Venture
Emerge	Issue, Appear, Arise, Come Forth
Eminence	Distinction, Importance
Empty	Blank, Vacant, Void, Hollow
Encourage	Inspire, Animate, Strengthen, Embolden
Endeavour	Attempt, Struggle, seek, Strive
Endure	Continue, Last, Abide, Sustain
Enemy	Foe, Opponent, Adversary, Antagonist
Enormous	Huge, Excessive, Tremendous, Colossal, Gigantic
Enthusiasm	Zeal, Spirit, Force, Fervour
Entreat	Request, Implore, Plead, Solicit
Eradicate	Abolish, Eliminate, Exterminate
Erect	Raise, Build, Construct, Setup
Essential	Indispensable, Necessary
Esteem	Respect, Regard, Honour
Eternal	Endless, Everlasting, Immortal, Perpetual
Examine	Test, Inspect, Scrutinize, Review
Expedient	Advisable, Desirable, Advantageous, suitable, proper
Expensive	Costly, Dear, High-priced, Prodigal
Extravagant	Lavish, Excessive, Absurd

F

Fabricate	Build, Construct, Invent, Make-up
Fade	Pale, Dim, Vanish
Faint	Swoon, Fade, Weaken, Dim
Fair	Beautiful, Comely, Spotless, Unblemished
Fame	Repute, Eminence, Renown, Distinction
Fatal	Deadly, Lethal, Mortal, Fateful
Fear	Timidity, Diffidence, Apprehensiveness, Dread, Fearfulness
Fierce	Violent, Aggressive, Savage, Ferocious
Fit	Appropriate, Expedient, Suitable, Proper
Flaw	Defect, Fault, Mistake, Imperfection
Forbid	Prohibit, Inhibit, Ban
Formal	Conventional, Ceremonious, Ritualistic
Fortitude	Courage, Patience, Endurance
Fragrant	Aromatic, Balmy, Scented, Perfumed
Frail	Fragile, Infirm, Feeble
Free	Unconfined, Unchecked, Unhindered, Liberal
Frenzy	Fury, rage, excitement, Mania, Agitation
Friendly	Amicable, Sociable, Cordial, Warm-hearted
Furious	Angry, Fuming, Raging, Turbulent, Wild

G

Gay	Blithe, Lively, Merry, Jolly, Bright
Generous	Magnanimous, Unselfish, Liberal, Lavish
Genuine	Sincere, Frank, Unaffected, Honest, Sterling, Authentic
Gift	Present, Donation, Bounty, Contribution
Glad	Pleased, Gratified, Happy, Joyful
Gloom	Dejection, Shadow, Obscurity, Darkness
Greedy	Grasping, Avaricious, Rapacious
Grief	Sorrow, Distress, Affliction, Tribulation, Anguish
Habitual	Customary, Usual, Routine, Frequent, Permanent
Hallucination	Illusion, Delusion, Mirage
Hamper	Obstruct, Impede, Restrict
Hard	Difficult, Stern, Firm, Cruel
Hardship	Difficulty, Adversity, Calamity, Affliction
Harmful	Injurious, Detrimental, Damaging
Hasty	Quick, Speedy, Expeditious, Swift, Rapid, Hurried
Haven	Refuge, Shelter, Protection, Sanctuary
Helpful	Useful, Cooperative, Assisting
Holy	Pious, Blessed, Saintly, Godly
Homage	Respect, Tribute, Reverence
Honest	Open, Sincere, Frank
Horrible	Dreadful, Appalling, Frightful
Horror	Dread, Terror, Disgust, Aversion
Hostage	Security, Pledge, Bond, Guarantee
Humane	Kind, Merciful, Tender, Sympathetic
Humble	Modest, Meek, Submissive
Hypocritical	Insincere, Affected

I

Ideal	Model, Perfect, Example, Paragon, Epitome
Idle	Inactive, Useless, Futile, Unemployed
Ignoble	Vile, Lease, Low, Detestable
Illegal	Unlawful, Illegitimate, Illicit
Illustrious	Famous, Distinguished, Eminent
Imaginative	Original, Inventive, Creative, Idealistic, Romantic
Immense	Vast, Great, Tremendous, Huge
Immoral	Corrupt, Licentious, Characterless, debase
Impossible	Unlikely, Unreasonable, Incredible, Unfeasible

Impure	Foul, Dirty, Filthy, Debased, Mixed, Lewd, Unchaste
Incapable	Incompetent, Unable, Inept
Incite	Stir, urge, Provoke, Instigate
Incredulous	Sceptical, Distrustful, Doubtful, Unbelievable
Independence	Freedom, Liberty, Self-reliance
Indifference	Apathy, Unconcern, Coldness, Neutrality
Industrious	Diligent, Busy, Hard-working , Active, Assiduous
Infinite	Endless, Timeless, Limitless, Boundless
Ingenuous	Frank, Open, Candid
Insignificant	Petty, Unimportant, Trivial, Small
Intellectual	Mental, Scholarly, Learned, Well-informed, intelligent
Intemperance	Excess, Unrestricted, Dissipation
Interesting	Exciting, Alluring, Fascinating
Irritable	Ill-humoured, Peevish, Petulant

J

Jealous	Envious, Distrustful, Suspicious
Jolly	Merry, Jocose, Exuberant
Junk	Rubbish, Waste, Refuse, Trash
Just	Upright, fair, Proper, Right
Justify	Defend, Prove, Vindicate, Explain
Juvenile	Youthful, Immature, Pubescent, Junior, Young

K

Keen	Piercing, Penetrating, Cutting; Quick, Eager
Kind	Benign, Gracious, Obliging, Type, Sort, Character, Nature
Kindle	Ignite, Set on fire, Light, Stir, Arouse
Knave	Rogue, Rascal, Miscreant, Scamp
Knowledge	Information, learning, Understanding

L

Laborious	Toilsome, Hard-working, Industrious
Lack	Want, Need, Shortage, Deficiency
Languish	Weaken, Wither, Droop, Fade, Decline
Lasting	Enduring, Permanent, Continuing
Lazy	Slow, Sluggish, Indolent, Slothful, Idle
Liable	Subject to, Open to, Exposed to, Accountable
Liberty	Freedom, Independence, Emancipation
Loathe	Despise, Abhor, Detest, Hate
Loquacious	Talkative, Chatter box, Garrulous, Chatty
Lucid	Plain, Clear, Understandable, Cogent, Comprehensible
Lurid	Ghastly, Grim, Gruesome, Pale, Bright, Brilliant
Luxuriant	Lush, Lavish, Profuse, Fertile

M

Merciless	Ruthless, Unfeeling, Relentless, Pitiless
Mad	Crazy, Insane, Frenzied, Wild
Magnificent	Grand, Splendid, Noble, Superb
Manifest	Show, Display, Disclose, Bring forward
Marvel	Wonder, Surprise, Astonishment
Marvellous	Wonderful, Incredible, Miraculous
Massacre	Killing, Butchery, Slaughter, Carnage
Meek	Mild, Submissive, Humble, Modest
Melancholy	Sad, Dejected, Depressed, Dispirited
Miserable	Wretched, Forlorn, Worthless, Pathetic
Mitigate	Lessen, Alleviate, Soften, Abate
Monotonous	Humdrum, Tedious, Tiresome, Unvaried
Mourn	Lament, Grieve, Bewail, Bemoan
Mutiny	Rebellion, Revolt, Uprising, Insurgency
Mysterious	Baffling, Obscure, Enigmatic, Inscrutable
Mysterious	Obscure, Occult Incomprehensible, Hidden

N

Narrate	Relate, Tell, Recite, Describe
Naughty	Mischievous, Wayward, Defiant
Neat	Tidy, Trim, Spick-and-span, Orderly
Nefarious	Evil, Wicked, Shameful, Unlawful
Nimble	Spry, Agile, Alert, Brisk
Novel	New, Different, Unusual, Unique
Nuisance	Annoyance, Bother, Pest, Bore

O

Oath	Pledge, Vow, Promise, Avowal
Objectionable	Obnoxious, Offensive, Undesirable
Obligatory	Compulsory, Imperative, Necessary
Obscene	Immoral, Corrupt, Indecent, Unchaste
Obsolete	Antiquated, Discarded, Outworn, Extinct
Obstinate	Tenacious, Stubborn, Obdurate, Dogged
Obtain	Get, Attain, Acquire, Procure
Obvious	Manifest, Clear, Evident, Plain
Offence	Insult, Affront, Attack, Wrong
Old	Aged, Elderly, Ancient, Antique, Outdated
Oppressive	Stifle, Tyrannical, Overbearing
Ostentatious	Pretentious, Showy, Glittering
Outlaw	Criminal, Bandit, Fugitive, Desperado
Outrage	Insult, Wrong, Affront, Indignation, Fury, Anger
Outstanding	Prominent, Eminent, Remarkable

Overcome	Conquer, Defeat, Overthrow, Surmount
Overlook	Oversee, Skip, Miss, Forget

P

Pact	Treaty, Accord, Agreement, Bargain
Paralyze	Disable, Incapacitate, Cripple, Unnerve
Pardon	Forgive, excuse, Absolution, Overlook
Pathetic	Touching, Sad, Pitiful, Distressing, Heartrending
Peak	Top, Apex, Summit, Pinnacle
Pensive	Melancholy, Meditative, Reflective
Pensive	Thoughtful, Sad, Dejected, Reflective
Perennial	Lasting, Endless, Persistent
Peril	Danger, Risk, Hazard, Chance
Pernicious	Harmful, Detrimental, Ruinous, Sinister
Perpetual	Eternal, Everlasting, Ceaseless, Incessant
Persecute	Oppress, Annoy, Trouble, Molest, Maltreat
Philosophical	Calm, Rational, Imperturbable, Deliberative, Speculative
Pierce	Bore, Puncture, Penetrate
Pious	Religious, Devout, Holy, Reverent
Pithy	Brief, Terse, Succinct, Forceful, Powerful
Placid	Calm, Peaceful, Gentle, Serene
Plead	Assert, State, Urge, Allege
Pleasure	Delight, Gladness, Joy, Enjoyment
Pledge	Promise, Vow, Oath, Security
Poisonous	Virulent, Deadly, Toxic, Venomous
Polite	Civil, Courteous, Polished, Refined
Ponder	Think, Muse, Reflect, Deliberate
Postpone	Delay, Defer, Shelve
Praise	Applause, Approval, Acclaim
Pray	Implore, Beg, Request, Entreat
Precious	Priceless, Costly, Invaluable, Dear
Prevent	Hinder, Stop, Check, Restrain
Prize	Award, Medal, Trophy, Laurels
Probe	Search, Investigate, Explore, Verify
Proceed	Advance, Progress, Move, Continue
Proficient	Expert, Adept, Well-versed, skilful
Proficient	Expert, Adept, Masterly, Dexterous, Adroit
Prominent	Notable, Salient, Eminent, Distinguished
Proper	Right, Just, Equitable, Meet, Suitable
Protect	Defend, Guard, Save, Shield
Prudence	Discretion, carefulness, caution
Prudent	Careful, Wary, Heedful, Cautious, Discreet, Circumspect

Q

Quack	Impostor, Charlatan, Fraud
Qualified	Fit, Capable, Able, Accomplished, Eligible
Quarrel	Dispute, Controversy, Wrangle
Queer	Quaint, Strange, Odd
Quick	Prompt, Alert, Ready, Rapid, Keen Intelligent
Quiet	Peaceful, Silent, Serene, Tranquil
Quit	Leave, Abandon, Resign, Stop

R

Radiant	Bright, Brilliant
Rage	Fury, Wrath, Frenzy, Craze
Raise	Lift, Stir up, Incite, Increase
Rapid	Swift, Speedy, Quick, Fleet, Fast
Rash	Incautious, Indiscreet, Hasty, Wild, Foolhardy
Raze	Demolish, Destroy, Level, Obliterate
Reasonable	Cheap, Low-Priced, Moderate, Fair, Credible, Plausible
Recover	Regain, Redeem, Salvage, Recuperate
Refined	Cultivated, Cultured, Well-bred
Regard	Respect, Esteem, Consider
Release	Free, Liberate, Relinquish
Reliable	Responsible, Trustworthy, Dependable
Reliance	Assurance, Confidence, Credence, Trust
Remarkable	Distinguished, Noteworthy, Rare, Uncommon, Unusual
Rescue	Liberate, Set free, Deliver, Save
Rigid	Stiff, Strict, Inflexible, Unyielding
Rival	Competitor, Opponent, Contender, Antagonist
Rival	Competitor, Contestant, Antagonist Adversary
Robust	Strong, Hardy, Seasoned, Tough, Sound
Romantic	Idealistic, Fanciful, Poetic, Imaginary, Sentimental
Rude	Barbaric, Uncivil, Insolent, Discourteous
Ruin	Destruction, Downfall, Wreck, Perdition
Ruthless	Cruel, Pitiless, Merciless, Relentless

S

Sacred	Holy, Divine, Hallowed, Blessed, Consecrated, Sanctified, Sacrosanct
Sad	Gloomy, Woeful, Melancholy, Mournful
Sarcastic	Satirical, Cynical, Biting
Savage	Wild, Rude, Barbarous, Uncivilized
Scanty	Scarce, Insufficient, Meagre, Limited

Scorn	Contempt, Despise, Derision, Disdain
Scrupulous	Strict, Careful, Conscientious
Secret	Clandestine, Hidden, Confidential
Seize	Grasp, Capture, Arrest, Confiscate
Sensational	Electrifying, Exciting, Stirring
Sentimental	Romantic, Emotional, Maudlin
Shake	Tremble, Flutter, Quiver, Quake, Vibrate
Sham	Imitation, Counterfeit, Make-Believe, Spurious
Shame	Disgrace, Dishonour, Humiliation, Ignominy
Shun	Avoid, Evade, Elude, Eschew
Solitary	Lonely, Isolated, Sole, Lone
Soothe	Quiet, Assuage, Console, Comfort
Sorry	Grieved, Sorrowful, Remorseful, Apologetic
Splendid	Gorgeous, Glorious, Magnificent
Spurious	False, Counterfeit, Sham, Specious
Strange	Odd, Queer, Unusual, Unfamiliar
Strenuous	Energetic, Ardent, Zealous; Persevering
Strife	Contention, Rivalry, Discord, Dispute
Struggle	Fight, Battle, Strive, Endeavour
Summit	Top, Apex, Peak, Zenith
Superfluous	Unnecessary, Needless, Useless, Extra, Redundant
Swift	Rapid, Speedy, Quick, Fleet

T

Tale	Story, Anecdote, Fable
Target	Aim, Goal, Object, Mark
Teach	Instruct, Educate, Train, Guide
Tedious	Dull, Dry, Wearisome, Uninteresting
Tepid	Lukewarm, Warm, Mild
Tidy	Neat, Spruce, Trim, Methodical
Timid	Cowardly, Shy, Diffident, Timorous
Transient	Momentary, Brief, Passing, Not lasting
Tremendous	Huge, Colossal, Stupendous
Trifling	Trivial, Frivolous
Trust	Faith, Belief, Reliance, Confidence
Tyranny	Despotism, Absolutism, Autocracy, Cruelty

U

Ugly	Unlovely, Coarse, Plain, Shapeless, Ill-made, Surly
Uncouth	Rough, Rude, Unpolished
Understanding	Insight, Perception, Discernment
Unique	Singular, Matchless, Unprecedented
Unmistakable	Certain, Sure, Positive, Clear
Uphold	Approve, Support, Maintain, Champion
Uproar	Tumult, Hubbub, Bedlam
Urbane	Polite, Courteous, Civil, Mannerly
Urge	Exhort, Press, Plead, Solicit

V

Vague	Unclear, Obscure, Indefinite, Indistinct, Ambiguous
Valiant	Brave, Valorous, Undaunted
Vanish	Disappear, Fade out, Dissolve
Venture	Dare, Risk, Hazard, Undertake
Vessel	Utensil, Jar, Vase, Bowl, Sailing Vessel, Ship
Vice	Wickedness, Sin, Frailty
Victory	Conquest, Triumph, Success, Winning
Vigilant	Alert, Wary, Watchful, Cautious
Vigilant	Cautious, Heedful, Chary, Prudent, Alert, Attentive
Vigour	Energy, Stamina, Force, Vitality
Vindictive	Revengeful, Resentful, Avenging
Vivacious	Sprightly, Animated; Lively, Gay, Brisk
Voluble	Talkative, Fluent, Glib, Loquacious

W

Wane	Decrease, Decline, Ebb, Lessen, diminish, fade
Want	Wish, Crave, Covet, Lack, Need
Wanton	Capricious, Wayward, Perverse, Unchaste, Immoral
Wedlock	Marriage, Wedding, Matrimony, Pledge
Wicked	Evil, Bad, Cruel, Heartless
Wily	Crafty, Artful, Sly, Subtle
Wisdom	Learning, Prudence, Judgement
Withhold	Restrain, Detain, Hold back, Suppress
Witty	Clever, Smart, Humorous
Wordy	Verbose, Prolix, Diffuse, Digressive
Worldly	Earthly, Mundane, Carnal
Wretched	Mean, Miserable, Unfortunate, Deplorable

Y

Yearn	Pine, Long, Hanker
Yield	Product, Produce, Give in, Succumb
Young	Youthful, Juvenile, Childlike, Immature

Z

Zeal	Enthusiasm, Eagerness, Earnestness, Passion
Zenith	Summit, Top, Apex, Climax
Zest	Enthusiasm, Zest, Devotion to a cause

3 Antonyms

An antonym is a word that has a meaning opposite to that of another word. English Language is very rich in vocabulary *i.e.,* in synonyms, antonyms, homophones, etc. Below is given a list of antonyms. An exhaustive list of antonyms of words is given. Antonyms of other important words have also been mentioned.

A

Abandon	Join, Engage, Unite, Embrace, Retain
Abandoned	Righteous, Virtuous
Abbreviate	Lengthen, Extend, Augment
Able	Unable, Incompetent, Incapable
Abnormal	Normal, Average, Usual
Abolish	Establish, Keep, Retain
Abominable	Admirable, Fine, Noble
Above	Below, Under, Beneath
Abridge	Expand, Extend, Increase
Abrupt	Gradual
Absence	Presence, Existence
Absent-minded	Alert, Attentive, Observant, Aware
Absolute	Partial, Limited, Fragmentary, Incomplete
Absolutely	Doubtfully, Questionably
Absolve	Involve, Blame, Accuse, Charge
Absorb	Leak, Drain
Abstract	Concrete, Real, Substantial
Abundance	Scarcity, Want, Dearth, Absence
Abundant	Scarce, Scant, Absent, Rare, Uncommon, Insufficient
Accelerate	Slow down
Accept	Refuse, Reject, Ignore
Accompany	Abandon, Leave, Forsake
Accord	Disagreement, Difference, Quarrel, Discord, Dissent
Accurate	Inaccurate, Inexact, Mistaken, Wrong
Accuse	Absolve, Clear, Discharge, Acquit
Acknowledge	Deny, Refuse, Reject
Acquire	Lose, Forfeit
Acquit	Condemn, Sentence
Active	Lazy, Lethargic, Inactive, Apathetic
Actual	Unreal, Pretended, Fake, Bogus, False
Acute	Insensitive, Dull, Blind, Deaf
Add	Subtract, Remove, Withdraw
Addicted	Averse, Disinclined
Adequate	Inadequate, deficient
Adhere	Separate, Loosen
Adherent	Defector, Renegade, Dropout
Adjacent	Apart, Separate, Distant
Adjoining	Separate, Distant, Remote
Adjourn	Assemble, Convene, Begin
Admirable	Contemptible, Despicable, Sorry
Admiration	Contempt, Disdain, Disrespect
Admire	Loathe, Despise
Admit	Deny, Obstruct, Reject
Admonish	Praise, Commend, Extol
Ado	Quietness, Composure, Tranquillity
Adore	Hate, Despise, Loathe
Adorn	Strip, Bare, Denude
Adroit	Clumsy, Awkward, Graceless
Adult	Immature, Infantile
Advance	Retreat, Withdraw, Flee, Retard, Obstruct, Decline
Advantage	Disadvantage, Drawback, Obstacle, Hindrance
Adversary	Friend, Ally
Adverse	Beneficial, Favourable
Adversity	Prosperity, Felicity, Bliss, Fortune, Happiness, Benefit
Advisable	Inadvisable, Ill-considered, Imprudent
Advocate	Oppose, Withstand, Resist, Opponent, Foe, Adversary
Affection	Dislike, Aversion, Antipathy
Affirm	Deny, Contradict, Repudiate, Disclaim
Affliction	Benefit, Gain, Relief, Comfort
Afraid	Unafraid, Bold, Cool, Confident
Against	For, In favour of, Pro, In support of, With
Aggravate	Ease, Relieve, Soothe
Aggressive	Peaceful, Friendly, Amicable, Passive, Shy, Timid

Agile	Clumsy, Awkward, Inept
Agitate	Calm, Soothe
Agree	Differ, Disagree, Argue, Refuse
Agreement	Disagreement, Discord, Misunderstanding
Aid	Impede, Obstruct, Hinder
Ailing	Well, Hale, Hearty
Alarm	Calm, Soothe, Comfort
Alert	Listless, Dulled, Sluggish
Alien	Familiar, Commonplace, Accustomed
Alight	Board, Embark
Alike	Different, Distinct, Separate
Alive	Dead, Moribund, Inactive, Insensible
Allay	Arouse, Worsen, Intensify
Allege	Deny, Refute, Contradict
Allegiance	Disloyalty, Treachery, Infidelity
Alleviate	Aggravate, Embitter, Heighten
Allow	Forbid, prohibit, Disallow, Deny, Inhibit
Ally	Enemy, Foe, Adversary
Alone	Accompanied, Together
Aloof	Friendly, outgoing, Cordial, Warm
Alter	Keep, Preserve, Maintain
Always	Never, Rarely
Amateur	Professional
Amaze	Bore, Tire, Weary
Ambiguous	Clear, Unmistakable,
Ameliorate	Worsen
Amiable	Disagreeable, Ill-tempered
Amiably	Surly, Displeasing
Amiss	Properly, Correctly, Rightly, Right, Correct
Ample	Insufficient, Inadequate
Amplify	Abridge, Condense
Amuse	Bore, Tire
Amusement	Boredom, Tedium
Ancestor	Descendant
Ancestry	Posterity, Descendants
Ancient	New, Fresh, Recent, Current
Angel	Devil, Demon
Anger	Forbearance, Patience
Angry	Happy, Content, Peaceful, Tranquil
Animate	Inanimate, Dead
Announce	Suppress, Stifle, Censor
Annoy	Comfort, Soothe, Please
Answer	Question, Query, Inquiry
Antagonism	Friendliness, Geniality, Cordiality
Anxiety	Peacefulness, Placidity, Calmness, Tranquillity
Anxious	Calm, Composed
Apathetic	Interested, Concerned
Apologetic	Unrepentant, Excited, Stubborn, Obstinate
Appal	Please, Edify, Comfort
Apparent	Real, Actual
Appeal	Repel, Repulse
Appear	Disappear, Vanish, Evaporate
Appetite	Distaste, Surfeit
Applaud	Disapprove of, Criticize, Condemn, Denounce
Appreciate	Take for granted, Scorn, Depreciate, Undervalue
Apprehension	Composure, Confidence
Apprehensive	Calm, Composed
Approach	Recede, Go away
Appropriate	Inappropriate, Unfit, Inapt
Approve	Disapprove, Oppose
Approximate	Exact, Precise, Perfect
Apt	Unfit, Ill-becoming, Unsuitable, Inapt, Unlikely
Ardent	Cold, Unemotional, Feeble
Arduous	Easy
Argue	Agree, Concur
Argument	Agreement, Harmony, Accord
Arouse	Calm, Settle, Soothe, Dull
Arrange	Disarrange, Disturb, Disorder
Array	Disarray, Disorder
Arrest	Liberate, Release, Set free
Arrive	Leave, Depart
Arrogance	Humility, Modesty
Arrogant	Humble, Modest
Artful	Artless, Simple, Naive
Artificial	Real, Genuine, Authentic
Artistic	Tasteless, Dull, Flat
Ascend	Descend, Go down
Ashamed	Proud, Self-respecting
Asleep	Awake, Alert
Assemble	Disassemble, Scatter, Disperse
Assent	Refusal, Disapproval
Assert	Deny, Contradict, Decline, Reject
Assertion	Denial, Contradiction
Assist	Hinder, Obstruct, Impede, Thwart

Assistance	Obstruction, Interference
Associate	Separate, Divide, Avoid
Assorted	Same, Alike
Assure	Deny, Equivocate
Astonish	Bore, Tire
Astute	Slow, Duel
Attach	Detach, Unfasten, Loosen
Attachment	Detachment
Attack	Withdraw, Retreat
Attention	Inattention, Absent-minded
Attract	Repel, Repulse
Attraction	Repulsion
Attractive	Unattractive, Plain, Ugly
Audible	Inaudible, Indistinct
Austere	Lenient, Permissive, Soft, Luxurious, Fancy, Opulent
Austerity	Comfort, Luxury
Authentic	Fake, Bogus, Imitation, Counterfeit
Authorize	Prohibit, Forbid
Autonomy	Dependence
Available	Unavailable, Unobtainable
Aversion	Liking, Affinity, Attraction
Avoid	Meet, Confront, Face, Encounter
Aware	Unaware, Unconscious
Awe	Scorn, Contempt
Awful	Wonderful, Delightful
Awkward	Graceful, Deft, Elegant, Skilled, Skilful

B

Barbaric	Civil
Barren	Fertile
Beautifu1	Ugly, Repulsive
Begin	End, Conclude, Stop
Belief	Disbelief
Belief	Disbelief
Beneficial	Injurious, Harmful
Benevolent	Malevolent
Better	Worse
Bind	Loosen, Set free
Bleak	Cheerful, Bright, Sheltered
Blessing	Curse
Bliss	Agony, Sorrow
Blunt	Sharp
Boon	Bane
Brave	Cowardly
Bright	Dim, Dark, Dull, Stupid

C

Calm	Stormy, Boisterous
Candid	Evasive, Foxy, Tricky
Care	Neglect, Carelessness, Indifference, Heedlessness
Caution	Negligent, Reckless
Cavity	Bulge, Protuberance
Cheerful	Gloomy, Pensive
Civilized	Savage, Wild, Uncivilized
Comfortable	Uncomfortable, Miserable, Cheerless
Complicated	Simple
Compose	Contrast
Conclude	Begin
Concord	Discord
Confess	Deny, Renounce
Confidence	Diffidence
Conscious	Unconscious, Unaware, Oblivious
Consent	Dissent
Contempt	Respect, Esteem, Reverence
Contrast	Comparison
Converge	Diverge
Cooperate	Counteract, Nullify
Country	Town
Courageous	Timid
Courtesy	Rudeness
Creation	Destruction
Criminal	Innocent, Moral
Cunning	Artless, Naïve, Simple

D

Dead	Alive
Deep	Shallow
Defeat	Victory
Defensive	Offensive
Deficit	Surplus
Delay	Haste
Delicious	Bitter, Nauseous, Bland
Delightful	Distressing, Horrid
Demand	Supply
Demolish	Construct, Restore
Deposit	Withdraw
Desolate	Inhabited
Despair	Hope
Difference	Resemblance, Likeness, Identity
Difficult	Easy, Obliging
Diligent	Lazy

Diminish	Increase
Discourage	Encourage, Hearten, Cheer
Distress	Comfort, Safe
Docile	Ungovernable, Headstrong
Domestic	Wild, Untamed
Drunk	Sober, Temperate
Dwarf	Giant
Dynamic	Static, Still

E

Eager	Listless, Apathetic, Phlegmatic
Earn	Spend
Easy	Difficult
Ebb	Flow
Economical	Extravagant
Economy	Extravagance
Egoism	Altruism
Encourage	Discourage
Endure	Perish, Reject
Enemy	Friend, Ally, Champion
Enjoyment	Suffering, Anguish
Entrance	Exit, Departure
Excess	Deficiency, Dearth
Expedite	Delay

F

Fact	Fiction
Failure	Success
Faithful	Faithless, Untrue
Falsehood	Truth
Fame	Infamy, Shame, Dishonour
Famous	Notorious
Fickle	Constant, Steady, Reliable
Fine	Coarse, Rough
Firm	Wavering, fickle
Flexible	Rigid
Foreign	Native
Formidable	Weak, helpless
Fortunate	Unfortunate, disastrous
Fortune	Misfortune
Frank	Reserved
Freedom	Slavery
Fresh	Stale
Friend	Enemy, Foe
Frown	Smile
Full	Empty
Fuzzy	Clear, lucid, Well-defined

G

Generous	Stingy, Ignoble
Genuine	Spurious, Counterfeit
Glory	Humiliation, Shame
Gorgeous	Plain, Simple, Ugly
Greatness	Smallness
Grief	Comfort, Solace, Joy, Exultation
Guest	Host

H

Happiness	Sadness, Unhappiness, Adversity
Harmony	Discord, Conflict
Help	Hinder, Hamper, Weaken
Heredity	Environment
High	Low, Mean, Degraded
Honest	Dishonest, Corrupt
Honour	Shame, Dishonour
Hope	Despair
Humble	Proud, Conceited
Humility	Insolence
Hurry	Delay
Hypocrisy	Sincerity, Frankness, Candour

I

Idle	Busy
Ignorant	Wise, Learned, Aware, Informed
Impetuous	Careful, Cautious, Thoughtful, Prudent
Import	Export
Important	Unimportant, Obscure
Include	Exclude
Increase	Decrease
Initial	Final
Innocent	Guilty

J

Join	Separate, Sever, Disconnect
Joy	Sorrow
Junior	Senior
Justice	Injustice, Inequity

K

Keen	Unwilling
Kindle	Extinguish
Knowledge	Ignorance

L

Lad	Lass
Large	Small, Little, Petty
Lenient	Strict

Liberty	Restraint
Liberty	Bondage, Servitude, Slavery
Light	Darkness, Shade
Like	Unlike, Dislike
Literate	Illiterate
Logical	Illogical, Absurd
Loyalty	Disloyalty, Treachery, Treason
Luscious	Sour

M

Major	Minor
Make	Mar, Destroy
Meek	Arrogant, Domineering, Proud, Blustering
Memory	Forgetfulness, Oblivion
Merit	Demerit
Miser	Spendthrift, Extravagant
Modest	Immodest, Ambitious, Indecent, Conceited
Mortal	Immortal
Motion	Rest
Mourn	Rejoice

N

Native	Alien, foreign
Natural	Unnatural, Artificial
Neat	Filthy, Disorderly, Slovenly
New	Old, Familiar, Common, Obsolete, Antiquated
Noble	Ignoble, Mean

O

Obstinate	Pliable
Optimistic	Pessimistic
Oral	Written
Overt	Covert, Secret

P

Pain	Pleasure
Peace	War
Plenty	Scarce
Polite	Impolite, Rude
Positive	Negative
Poverty	Affluence
Praise	Blame
Precarious	Safe
Prefix	Suffix
Pretentious	Humble, Simple
Pride	Humility, Modesty
Progress	Retrogress, Halt, Stop
Progressive	Orthodox, Reactionary
Prohibit	Permit, Allow, Sanction
Prose	Poetry
Proud	Humble
Prudence	Imprudence, Folly, Indiscretion
Punishment	Reward
Pure	Impure, Adulterated

Q

Queer	Common, Usual, Regular
Quiet	Noisy, Occupy

R

Rabid	Rational, Logical
Rare	Common, Ordinary
Rational	Irrational, Emotional
Raw	Ripe
Real	Unreal, Apparent, Imaginary
Rear	Front
Rebellious	Submissive, Acquiescent, Conforming, Docile, Manageable
Recede	Advance
Recovery	Relapse
Regular	Irregular
Religious	Secular, Irreligious
Reluctant	Eager, Avid, Desirous
Remember	Forget
Renounce	Assert, Maintain
Rest	Bustle, Motion, Commotion, Disturbance
Restrain	Incite, Impel, Loose, Liberate
Reveal	Conceal
Rigorous	Simple, Easy
Ruthless	Merciful

S

Sacred	Profane
Safety	Danger
Savage	Tame, Civilised
Sensibility	Insensibility, Deadness, Numbness
Severe	Tolerant, Lenient, Lax
Shallow	Deep
Skilful	Unskilful, Clumsy, Inept
Slow	Fast, Alert, Lively
Slow	Fast
Smile	Frown
Solid	Liquid
Solid	Liquid, Hollow
Steadfast	Fickle

Steep	Gradual, Flat
Stop	Start, Begin, Initiate
Stupidity	Intelligence, Acuteness, Keenness
Summit	Base
Superiority	Inferiority
Supernatural	Natural, Usual, Common place
Sweet	Sour
System	Chaos, Disorder, Confusion

T

Take	Give
Talkative	Silent
Tardy	Prompt, punctual, timely
Terse	Diffuse, Wordy
Thrifty	Spendthrift, Prodigal
Tragedy	Comedy
Transient	Perpetual, Permanent, Lasting
Transplant	Opaque

U

Unity	Diversity
Universal	Regional
Unruly	Orderly

V

Vague	Definite, Precise
Valiant	Cowardly, Afraid, Fearful
Valid	Invalid, Illogical, Fake
Victor	Vanquished
Violent	Compassionate, Kind
Virtue	Vice

W

Win	Lose, Fail, Forfeit
Wisdom	Folly

Y

Young	Old, Mature, Grown

Z

Zealous	Indifferent, Apathetic

Test Paper–1

Below in each set the lead word is followed by four options. Pick out the most suitable ANTONYM from the options.

1. UNREALISTIC
(A) Natural (B) Visionary
(C) Reasonable (D) Actual

2. DEPRAVED
(A) Great (B) Enhanced
(C) Moral (D) Prosperous

3. METICULOUS
(A) Slovenly (B) Meretricious
(C) Shaggy (D) Mutual

4. AMICABLE
(A) Cunning (B) Shy
(C) Hostile (D) Crazy

5. CLARITY
(A) Exaggeration (B) Candour
(C) Confusion (D) Reserve

6. OPULENT
(A) Wealthy (B) Poor
(C) Sumptuous (D) Drooping

7. SANCTIFY
(A) Dedicate (B) Patronise
(C) Venerate (D) Pollute

8. CHALLENGE
(A) Admire (B) Accept
(C) Favour (D) Praise

9. TEDIOUS
(A) Pleasant (B) Lovely
(C) Lively (D) Gay

10. COMMEND
(A) Censure (B) Condemn
(C) Defy (D) Defame

11. SUPERFICIAL
(A) Artificial (B) Deep
(C) Shallow (D) Real

12. EFFETE
(A) Adamant (B) Strong
(C) Courageous y (D) Bold

13. COMMEND
(A) Suspend (B) Admonish
(C) Hate (D) Dislike

14. SERENE
(A) Jovial (B) Moving
(C) Agitated (D) Nervous

15. ANTIPATHY
(A) Fondness (B) Obedience
(C) Agreement (D) Admiration

16. DAUNTLESS
(A) Cautious (B) Thoughtful
(C) Weak (D) Adventurous

17. CHAFFING
(A) Expensive (B) Achieving
(C) Capitalistic (D) Serious

18. TRAGIC
(A) Funny (B) Comic
(C) Light (D) Humorous

19. EXODUS
(A) Restoration (B) Return
(C) Home-coming (D) Influx

20. PREVENT
(A) Excite (B) Support
(C) Invite (D) Incite

Test Paper–2

1. RUGGED
(A) Delicate (B) Coarse
(C) Tough (D) Timid

2. EDIFICATION
(A) Lamentation (B) Annotation
(C) Corruption (D) Segregation

3. INNOCENT
(A) Sinful (B) Guilty
(C) Deadly (D) Corruption

4. PACIFY
(A) Quarrel (B) Challenge
(C) Threaten (D) Darken

5. FURTIVE
(A) Straight (B) Obvious
(C) Unambiguous (D) Open

6. SECULAR
(A) Righteous (B) Religious
(C) Spiritual (D) Moral

7. DEAR
(A) Cheap (B) Worthless
(C) Free (D) Priceless

8. ASCETICISM
(A) Bliss (B) Pleasure
(C) Joy (D) Trance

9. PREDILECTION
(A) Denial (B) Concealment
(C) Aversion (D) Attraction

10. APPOSITE
(A) Inappropriate (B) Intemperate
(C) Inconsistent (D) Irregular

11. JETTISON
(A) Rejoice (B) Surrender
(C) Accept (D) Defend

12. PERSISTENT
(A) Wavering (B) Obstinate
(C) Enduring (D) Steady

13. SCOLD
(A) Enamour (B) Rebuke
(C) Criticise (D) Praise

14. PODGY
(A) Short (B) Thin
(C) Weak (D) Slim

15. SEGREGATION
(A) Appreciation (B) Cohesion
(C) Integration (D) Union

16. JITTERY
(A) Profuse (B) Tense
(C) Bold (D) Shaky

17. VIRTUOUS
(A) Scandalous (B) Vicious
(C) Wicked (D) Corrupt

18. EXASPERATE
(A) Belittle (B) Annoy
(C) Please (D) Tarnish

19. SORDID
(A) Steady (B) Enthusiasti
(C) Generous (D) Splendid

20. GRIM
(A) Serious (B) Satisfying
(C) Delightful (D) Painful

Answers

Test Paper 1

1. (C)	2. (C)	3. (A)	4. (C)	5. (C)	6. (B)
7. (D)	8. (B)	9. (A)	10. (A)	11. (B)	12. (B)
13. (B)	14. (C)	15 (A)	16. (D)	17. (B)	18. (B)
19. (D)	20. (B)				

Test Paper 2

1. (A)	2. (C)	3. (B)	4. (A)	5 (D)	6. (B)
7. (A)	8. (B)	9. (C)	10. (A)	11. (C)	12. (a)
13. (D)	14. (B)	15 (C)	16. (C)	17. (B)	18. (C)
19. (C)	20. (C)				

4 One Word Substitution

Abdicate — Renounce formally or by default power, right, office.

Aborigines — Original or first inhabitants.

Accelerate — To increase the speed.

Acoustics — Relating to the sense of hearing.

Affidavit — A statement on oath.

Agnostic — One who holds the view that nothing can be known about the existence of God.

Alimony — Allowances due to a wife from her hus-band on separation.

Allegation — A charge or statement against a person.

Allegorical — A narrative describing one subject under the guise of another

Allergy — Sensitiveness to the action of a particular food or other things.

Alleviate — To mitigate or lesson the pain or suffering.

Altruist — A person who has regard and concern for others, unselfish.

Amateur — A person who engages in a pursuit as a pastime rather than a profession, a person who does something unskil-fully.

Ambassador — A person who represents the interest of his country's Government abroad and acts in pursuance of the policy of the Government.

Ambiguous — Having more than one interpretation.

Anachronous — Anything out of harmony with its period, an old-fashioned or out-of-date person or thing.

Anarchist — A person who believes that all government and authority should be abolished.

Annihilate — To destroy completely.

Anonymous — That which does not bear a name of the writer/creator.

Antidote — Anything which destroys the effect of poison.

Antonym — A word opposites in meaning to another

Apiculture — Bee-keeping.

Apostle — A messenger to preach gospel.

Aquatic — Animals living in water.

Arbiter — One who is appointed by two parties to settle a dispute.

Archaeology — Study of antiquities (historic and prehistoric times).

Arsonist — A person guilty of maliciously setting on fire of property etc.

Ascetic — One who practices severe self-discipline.

Atheist — One who does not believe in God

Auction — A bargain Where things are sold to the highest bidder..

Audible — A thing which can be heard.

Audience — An assembly of listeners.

Authentic — A reliable piece of information.

Autocracy — Absolute government.

Aviary — A place where birds are kept.

Bankrupt — One who cannot pay his debts.

Bathos — A fall from the sublime to the ridiculous.

Bellicose — Given to fighting or fond of fighting.

Belligerent — Aggressive, eager to fight, engaged in war.

Bibliophile — A person who is a great lover of books.

Biennial — Happening or appearing once in two years.

Bigamist — One who has two wives or husbands at one time.

Bigamy — The state of having two wives.

Bigot — One blindly devoted to a particular creed or party.

Bilingual — Containing or speaking two languages.

Biography — A written account of the life of a person.

Bizarre — Strange in appearance or effect, eccentric, grotesque.

Blasphemy — The act of speaking disrespectfully about holy and sacred things.

Blonde	—	A person having a fair complexion and light golden hair.
Bohemian	—	A man of free and easy habits-socially unconventional.
Boor	—	Clumsy or ill-bred fellow.
Botany	—	Science of the structure of plants.
Bourgeoisie	—	A conventionally middle-class, selfishly materialistic and upholding the interests of the capitalist class.
Bovine	—	Pertaining to cows or cattle.
Brittle	—	Liable to be easily broken.
Coche	—	Gold or silver in bulk or bars before coining or manufacture.
Cadaverous	—	Looking like a dead body.
Calligraphy	—	Beautiful handwriting.
Cannibal	—	One who eats human flesh or animals, that eat their own species.
Cant	—	Affected manner of speech.
Cardiologist	—	Specialist in heart diseases.
Carnivorous	—	Flesh-eating.
Carnivorous	—	One who lives on human flesh.
Catalogue	—	A list of names, books, etc.
Cosmopolitan	—	Of interest or use of all; universal, all-embracing, of wide sympathies or interests.
Caucus	—	A small group within a political party, influencing party decision and policy etc.
Celibate	—	The state of being without a wife, abstaining from marital relationship.
Centenarian	—	A person who is hundred years old.
Centenary	—	Hundredth anniversary.
Charlatan	—	Empty or shallow pretender of knowledge or skill.
Chauvinism	—	Bellicose patriotism or nationalism.
Chronic	—	extending over a long time.
Circumlocution	—	A round about way of expression.
Cliche	—	Hackneyed phrase or opinion.
Colleague	—	one associated with others in some employment.
Colloquial	—	Belonging to or proper to ordinary or familiar conversation; not formal or literary.
Combustible	—	liable to take fire and bum.
Compensation	—	To give something to someone to make good for loss, injury or damage.
Competence	—	Sufficient income to live on in comfort.
Compromise	—	(to settle a dispute by) mutual give and take.
Confiscate	—	To seize something by authority.
Congenital	—	Acquired at the time of or before birth. (esp. a disease, etc.)
An expert	—	a person who understands profoundly the value of art, antiques, etc.
Connoisseur	—	One who is will versed in any subject; a critical judge of any art, particularly of fine arts.
Conservative	—	Disposed to maintain existing institutions.
Constellation	—	A group of stars.
Contagious	—	Spreading through actual contact, disease, etc.
Contemporary	—	One who is living or lived at the same time.
Contiguous	—	Touching along a line; in contact.
Convalescent	—	(one) gradually recovering health.
Cosmo	—	One who is at home in any country of the world, a citizen of the world very broad minded.
Credulous	—	On who easily believes everyone and everything.
Culmination	—	Reaching the highest or the final point.
Cynic	—	One who is a sneering critic of everything.
Cynosure	—	Centre of attraction or interest.
Declamation	—	Act of speaking oratorically or forcefully.
Defector	—	One who leaves or breaks his allegiance to a party.
Defensible	—	Capable of annulment, liable to forfeiture.
Deist	—	One who believes in God but not in the Revelation.
Delegate	—	one to whom full authority to act has been given.
Delinquent	—	A person who fails in the performance of his duty or a person who commits an offence.
Deluge	—	A large flood, a heavy fall of rain.
Demagogue	—	A political orator - agitator appealing to the basest instincts of a mob.
Democracy	—	Government in which supreme power is vested in the people collectively.
Dermatologist	—	Specialist in skin diseases.
Dessert	—	Fruit or sweet dish usually eaten after main meals
Destitution	—	The state of being miserable, bereft of all material possessions, the state of extreme poverty.

Detest	—	Hate, loathe
Despot	—	A ruler who uses force in order to subjugate the people to obey him.
Digress	—	To wander away from the main point. Dilettante : an amateur, lover of fine arts.
Dilettante	—	A dabbler in art and literature, a person who studies a subject or area of knowledge superficially.
Diplomacy	—	The art of negotiation, specially between states.
Diplomat	—	One who is engaged in the diplomatic service of one's country.
Domestic	—	belonging to the house.
Drawn	—	A contest in which neither party wins.
Duplicity	—	The nature of a double-dealer or impostor.
Eccentric	—	not conforming to common rules.
Eccentricity	—	A personal peculiarity of temperament, an unusual trait.
Edible	—	that may be eaten.
Effeminate	—	One who is like a woman, unmanly.
Efficacious	—	able to produce quick and intended result.
Egotism	—	talking too much about oneself.
Elegy	—	a song of mourning.
Elite	—	the best or choice part of a larger body or group, a select group or class elect.
Emancipate	—	To free from the bondage of something, to liberate from.
Emeritus	—	A professor who has honourably retired from service.
Emigrant	—	one who leaves his own country and settles down in another.
Empiric	—	One who relies on experience and observation,
Endemic	—	Something regularly found in a particular area or among a particular people or community.
Ennui	—	mental weariness from want of employment, A state of mental weariness from lack of occupation of interest .
Entomologist	—	scientist who studies insects. .
Entomology	—	The study of insects.
Ephemeral	—	lasting only for a day
Epicurean	—	one devoted to luxuries or sensuous enjoyment or one who believes in the philosophy of pleasure
Epidemic	—	A disease which attacks a large number of people simultaneously,
Epigram	—	A short but pithy and weighty saying, a short witty phrase
Epitaph	—	Words inscribed on a tomb.
Epitome	—	an abridgment or short summary.
Equestrian	—	pertaining to horses or horsemanship.
Equivocal	—	Ambiguous, of uncertain nature.
Eradicate	—	To get rid of something and remove all traces of it.
Erratic	—	Without a fixed or regular course, inconsistent, unconventional, eccentric.
Escapist	—	One who is always seeking to escape from the harsh and -bitter realities of life. established conventions, beliefs. customs, etc.
Ethics	—	Science of morals or rules of conduct.
Etymology	—	The study of the origin of words
Eugenics	—	Science of production of healthier and finer children.
Euphemism	—	Substitution of a mild for a very blunt expression or pointed phrase.
Excavate	—	to unearth by digging.
Exchange	—	give or receive one thing in place of another.
Encyclopaedic	—	That which is all-inclusive or deals comprehensively with a subject.
Exhibitionism	—	Perverted mental condition characterized by indecent exposure of the-person
Exonerate	—	to free from the burden of blame or obligation.
Expiate	—	To atone for one's sins.
Explicit	—	not implied merely ,but distinctly stated.
Export	—	send out goods to another country.
Expurgate	—	to remove objectionable matter from a book;
Extempore	—	A speech made on the spur of the moment, without proper preparation.
Fanatic	—	One filled with excessive and mistaken enthusiasm in religions.
Factitious	—	Artificial, false.
Fallible	—	One who is subject to failure, or committing mistakes.
Fastidious	—	Careful in all details, exacting, meticulous, difficult to please, easily disgusted.
Fatalist	—	A person who believes in fate having firm belief that all events are predetermined by fate.

Fauna — The animals of a particular region or epoch.

Feasible — That which is practicable or possible.

Feminist — A man who is genuinely interested in the welfare, betterment and emancipation of women.

Fictitious — As opposed to realistic; imagined, not real.

Flamboyant — Extremely showy and colourful personality.

Foster — Child a child brought up by one who is not the real parent.

Fratricide — One who kills one's brother or sister; killing one's brother or sister.

Frugal — Avoiding unnecessary expenditure of money, thrifty.

Garage — A building where motor vehicles are housed.

Garrulous — given to excessive talking, talkative.

Genealogy — Plant or animals line of development from earlier forms.

Genocide — Act of killing one's clan, family or community.

Genocide — Extermination of a race or community by mass murders.

Geologist — Scientist who studies the composition of the earth.

Geology — The science of the study of the origin, history and structure of

Germicide — medicine that kills germs.

Gerentology — Scientific study of old age and its disease.

Glutton — one who eats excessively.

Gourmet — A connoisseur of table delicacies and wine.

Gregarious — Animals which live in groups.

Gullible — one who can be fooled easily.

Gymnasium — a large room or hall with apparatus for physical training.

Gynaecology — science of the diseases of women.

Hedonism — Doctrine that the pleasure is the chief good.

Herbivorous — (animals) that feed on herbs.

Hereditary — Traits, physical or mental-received from forefathers by birth.

Heretic — One who expresses ideas which are, not in conformity with conventional reli-gious teachings.

Hermit — One who lives in seclusion with thoughts of God.

Heterodox — holding an unconventional opinion.

Heterogeneous — Things which have diverse elements.

Homicide — murder or murderer of man

Homogenous — composed of similar elements.

Honorarium — Voluntary fees paid for professional services which carry no salary.

Honorary — (an office) without performing any service or receiving any reward.

Hygienist — One who is very careful about one's health.

Hymeneal — pertaining to marriage;

Hyperbole — Rhetorical exaggeration in speaking or in a piece of writing.

Hypochondriac — One who suffers from mental depression caused by imaginary fear of some disease or aliment.

Iconoclast — One who breaks images or idols a man who assails cherished beliefs and ideas a breaker of idols or images; one woo is opposed to well

Idiosyncrasy — A mental constitution, view, or feeling or mode of behaviour peculiar to a person, anything highly eccentric.

Idolater — a worshipper of idols.

Ignominy — Dishonour , infamy.

Illegible — that cannot be read or deciphered. Illicit: not allowed by law.

Illegitimate — Born of parents not married to each other, not authorised by law, abnormal, improper.

Illiterate — One who can neither read nor write.

Illusion — A deceptive appearance, statement or belief.

Immutable — that which is not subject to, any change or alteration.

Implacable — One who cannot be soothed or calmed or pacified.

Implicit — not distinctly stated but implied.

Import — bring goods from a foreign country.

Impostor — one who pretends to be what he is not or assumes some body else's name or title in order to deceive others.

Impracticable — That which cannot be put into practice. not able to be done.

Impregnable — That cannot be seized. or taken by force; able to resist all attacks.

Imprudent — who lacks foresight; indiscreet

Imputation — Attributing or ascribing to something or someone.

Inaccessible — That cannot be easily approached.

Inaudible	—	That which cannot be heard.
Incendiaries	—	Wilfully setting fire to buildings etc.
Incentive	—	Something which provides a person attraction or interest.
Incognito	—	concealed under a disguised character and assumed name.
Incompatible	—	incapable of existing together in harmony .
Incorrigible	—	One who cannot be corrected or reformed.
Indescribable	—	that cannot be described.
Inimitable	—	That which cannot be easily imitated.
Innocuous	—	that is harmless or without effect.
Innuendo	—	An implied (generally deprecatory) remark.
Insomnia	—	A disease in which one suffer from sleeplessness. inability to sleep.
Instigate	—	The act of provoking a person.
Insurgent	—	Rising in active revolt, a rebel, a revolutionary.
Insurmountable	—	That cannot be overcome.
Intermittent	—	ceasing at intervals.
Intestate	—	(one who dies) without making a will.
Intimidation	—	Language or gesture which implies threat to the other.
Invincible	—	that cannot be overcome.
Invulnerable	—	that cannot be wounded or hurt.
Irrational	—	Not rational, opposed to reason.
Irreconcilable	—	Incapable of being reconciled.
Irrefutable	—	That cannot be proved false.
Irrelevant	—	Not to the point.
Irrevocable	—	That cannot be recalled.
Irritable	—	Easily provoked or irritated.
Itinerant	—	Travelling from one place to another.
Jingoism	—	Blustering or blind patriotism and nationalism.
Jurist	—	A person who is well-versed in law.
Kleptomania	—	An excessively morbid desire to steal.
Lexicographer	—	One who makes or compiles dictionary.
Libertine	—	A free thinker on religion, a dissolute or licentious person.
Licentious	—	Extremely extravagant in manners and morals.
Linguist	—	Proficient in many languages.
Loquacious	—	One who talk continuously, talkative, chattering, babbling.
Ludicrous	—	Laughable, absurd, ridiculous, preposterous.
Lunar	—	pertaining to the moon.
Macafee	—	A scene or situation which is gruesomely imaginative full of gruesome details.
Machiavellism	—	Philosophy of practising duplicity in statecraft.
Majority	—	The greater number.
Malady	—	Feeling of bodily discomfort without clear signs of a particular disease.
Malapropism	—	A ridiculous confusion and misuse of words.
Malleable	—	Any metal that can be spread out in sheets. Animals which feed theyoung with milk from their breasts.
Manuscript	—	Hand-written matter.
Materialistic	—	Concerned solely with material objects.
Matriarchy	—	A society in which mother is head of family.
Matricide	—	The murder or murderer of mother.
Matrimony	—	State of being married.
Mediator	—	One who plays the role of bringing two antagonistic part together
Melodrama	—	A drama which is marked by very crude appeal to feelings and emotions.
Mercenary	—	A person working or a soldier fighting merely for money.
Metamorphosis	—	The state of being changed or transformed by natural supernatural means.
Metaphysics	—	philosophy dealing with the ultimate truth.
Meticulous	—	Giving great or excessive attention to details, very careful and precise .
Migratory	—	Birds moving from one place to another.
Millennium	—	A golden age, a period of 1,000 years.
Minority	—	The smaller number.
Misanthrope	—	Hater of mankind
Misanthropist	—	One who is a hater of mankind.
Misogynist	—	hater of Women.
Monarchy	—	government by a king.
Monogamist	—	A person who believes in being married to one woman or one man at a time
Monogamy	—	practice of being' married to only one person at a time.
Monotheism	—	Doctrine that there is only one God.

Monotony — The state of being monotonous, dull or tedious routine

Morphology — Study of the forms of animals, plants or words.

Mortuary — A place where dead bodies are kept until burial.

Mundane — Belonging to the earthly world.

Narcotic — Drug that in produces sleep or insensible condition.

Naturalization — adapted to different conditions. granted the rights of natural born citizens.

Navigable — that may be passed by ships.

Nepotism — undue favour to one's relations by appointing them to high and lucrative posts.

Neurosis — nervous disorder.

Neurotic — (a person) suffering .from disordered nerves.

Nihilism — The rejection of all religious and moral principles, an extreme form of skepticism characterized by the assertion that nothing really exists.

Numismatics — Science and study that treats coins and medals.

Oasis — Fertile spot in a desert, where water is found.

Obituary — Notice of death especially in newspaper.

Obsolete — That which is no longer in use.

Octagon — A figure having eight sides. Octogenarian: one who is eighty years old.

Oligarchy — government by a small exclusive class.

Omnipotent — all powerful.

Omnipresent — Present everywhere.

Omniscient — having knowledge of all things.

Omnivorous — Feeding on anything

Opaque — Not reflecting or transmitting light.

Ophthalmologist — specialist in eye diseases.

Opportunist — An unscrupulous person who puts expediency before principle.

Optimist — One who habitually looks at the brighter side or as things.

Ornithologist — scientist who studies birds.

Orthography — The art of spelling words correctly.

Ostentatious — Extremely loud and showy as opposed to reserved and modest.

Ostracise — To banish or turnout of society and fellowship.

Pacifist — one who believes that war should be abolished.

Palaeography — Study of ancient writings and documents.

Panacea — A universal remedy for all ailments or diseases.

Pandemonium — A disorderly assembly or tumultuous noise.

Pantheism — Doctrine that God is everything and everything is god

Parasite — one who lives upon another

Parasol — A lady's umbrella.

Pathology — science of diseases.

Patricide — the murder or murderer of father.

Patrimony — right or estate inherited from one's father.

Patriot — A lover of one's country.

Pawn — A person used by others for their own purposes.

Pedagogue — A school master who is very strict and teaches in a pedantic manner.

Pedant — One who makes a show of one's knowledge.

Paediatrician — Specialist in children's diseases.

Penology — The study of the punishment of crime and of prison management

Perception — Understanding, comprehension

Peroration — A passage marking the close of a speech.

Pessimist — One who takes a gloomy view of life.

Phil1uithropist — A man who generously donates and gives help to welfare projects.

Philanderer — One who makes love in a light and non-serious manner.

Philanthropy — A lover of mankind.

Philatelist — One who collects stamps.

Philately — Science dealing with stamp collection.

Philistine — A person who is hostile or different to art and culture.

Philologist — One who studies the history and growth of language. .

Plagiarism — Act of stealing from the writings or ideas of others.

Plagiarist — One who steals from the writings or ideas of others.

Platitude — A commonplace or an oft-repeated statement or remark. Plutocracy: government of the rich and powerful people.

Plutocracy — Government by the rich.

Polyandrous — Woman who has more than one husband at a time.

Polyandry — The custom of having several husbands at a time.

Polygamy — The practice of having more than one husband or wife.

Polyglot — One who knows many languages.

Polytheism — Belief in or worship of more than one god.

Post mortem — Operation of the body after death.

Posthumous — born after the father's death; published after the author's death.

Post-mortem — that which is held after death.

Pragmatism — Philosophy of judging the truth or validity of one's actions solely on their practical success.

Precedent — Some previous example from the past.

Precedent — A former holder of an office or position with respect to a later holder.

Prevaricate — To make evasive or misleading statements.

Primogeniture — right of succession belonging to the first born son.

Prophecy — Statement showing remarkable degree of prediction

Pseudonym — A fictitious name assumed by a writer

Pseudonym — an imaginary name assumed by author.

Psychiatrist — specialist in mental and emotional disorders.

Psychologist — one who studies the human mind.

Pugnacity — readiness or inclination to fight.

Purist — one who pays great attention to the correct use of words language. etc.

Purl tan — Person affecting extreme strictness in religion or morals.

Quack — A person who pretends to have knowledge or skill especially in medicine

Quinquennial — occurring once in five years.

Radical — Far-reaching, thorough, advocating thorough reforms, holding extreme views.

Rebel — one who takes up arms against a government.

Recalcitrant — Obstinately disobedient.

Red-tapism — Strict adherence to all forms of official formalities

Regicide — the murder or murderer of king.

Reminiscent — That which reminds something, tending to recall or talk of the past

Remit — to send money by post. '

Remuneration — compensation rendered as equivalent of a service.

Renegade — A deserter of party, principles or an apostate.

Renounce — To give up entirely.

Republic — A state where the Head of the State is elected and sovereignty resides in the people.

Reticent — reserved in speech.

Retrospective — which has reference to what is past.

Sacrilege — The act of violating the sanctity of a holy place.

Sadist — One who takes pleasure in cruel, inhuman acts.

Samaritan — one who helps the poor and helpless in trouble.

Sanatorium — A health resort, an institution for the treatment of chronic diseases.

Sartorial — pertaining to tailors or clothes.

Savvy — Know, knowingness, shrewdness, understanding.

Scapegoat — Someone on whom blame of others is fixed.

Simultaneously— at one and the same time.

Sinecure — an office or post with salary but without work.

Sociologist — one who studies human society;

Solar — pertaining to the sun.

Soliloquy — Talking loudly when alone, a speech addressed to self on a dramatic stage.

Somnambulist — One who walks in sleep.

Sophism — False arguments intended to deceive

Sophisticated — Extremely refined in dress, conduct and speech.

Sordid — Dirty or squalid, ignoble, mean, mercenary.

Spinster — An elderly unmarried women.

Sterilize — to free anything of germs.

Stoic — A person not easily moved by pleasure or plain.

Suicide — murder of self.

Superannuation — to al1ow to retire from service on a pension on account of o1d age or infirmity.

Supercilious — Overbearing, haughty, arrogant.

Sycophant — A person who flatters other for personal motive.

Synchronize — To take place at the same time as another.

Syndicate — an association of persons formed with a view to

Synopsis — a summary giving a general view of some subject.

Taxidermist — one who skins animals.

Teetotaller — Art of preparing and mounting skins of animals in life like manner.

Teetotaler — A person advocating or practising abstinence from alcoholic drinks.

Telepathy — Communication between minds. -A person advocating or practising abstinence

Tempest — A violent storm.

Termagant — a woman of over bearing nature.

Terminus — the extreme station at either end of a railway or railroad.

Theist — One who believes in God .

Theocracy — government based on religion.

Tirade — A long vehement speech or reproof.

Titanic — enormous in size and strength.

Toddler — one who walks with short steps in a tottering .way as a child or an old man does.

Transmigration — the passing of soul after death from one body to another.

Transparent — that can be seen through.

Treason — disloyalty or treachery to the state.

Triennial — that which happens once in three years.

Truant — a student who runs away from the class or school without permission.

Truism — an oft-repeated truth.

Turncoat — One who easily gives up his party or principles

Ubiquitous — present everywhere at the same time.

Unanimous — With everyone agreeing

Underhand — in a clandestine manner and often with a bad design.

Unintelligible — that which cannot be understood.

Unsociable — indisposed to society.

Usrer — The practice of charging exorbitant or excessive. interest on money

Utilitarianism — the doctrine that actions. should be judged as right or wrong solely by their utility or promoting the happiness and good of the greatest number.

Utopian — admirable but impracticable.

Vacillation — wavering between different courses.

Valetudinarian — one who always thinks that he is ill.

Vamp — an adventurer who extracts money from men by means of sex appeal.

Vandal — One who destroys all work of art.

Vandalism — wilful destruction of works of art,

Vegetarian — one who lives on vegetables only.

Venial — a fault or sin which may be pardoned.

Ventriloquist — one who can produce sounds and words without any motion of the mouth.

Venue — the place where an action is laid or the trial of a case takes place.

Verbatim — Repetition of a writing or conversation word by word.

Verbose — Using more words than necessary, wordy, prolix.

Verisimilitude — likeness to truth.

Versatile — having ability of many kinds.

Vesper — Evening prayers in the church.

Veteran — One who has long experience or expertise.

Veteran — one having a long experience of something.

Voluntary — offered of one's own accord.

Vulnerable — capable of being wounded or criticised.

Wardrobe — a place where clothes are kept.

Whirligig — anything that revolves rapidly (like fortune).

Widow — a woman whose husband is dead.

Widower — a man whose wife is dead.

Zeal — Extreme enthusiasm for a cause.

Zealous — Ardent, eager, enthusiastic.

Zenith — The highest point.

Zest — Spice, relish, tang; gusto

Zone — Area, region, district, section.

Zoo — a place where animals are kept.

Zoology — science of animal life.

5 How To Use Foreign Expression

A fortiori—with strong reason.

A posteriori—form the effect to the cause.

Abandon—to leave, to give up, to surrender.

Abate—to put end.

Ab antique—form ancient time.

Ab antique—form olden times.

Ab initio—form the very beginning.

Abduction—this is an offence committed against a person of any age. Section 352 of the India Penal Code deals with this offence.

Abide—to act in accordance with rule of law.

Abiure—to renounce on oath or affirmation.

Abrogate—to repeal.

Absolvitor—an acquittal; a decree in favour of the dafander.

Accession cedit principali—an accessory thing when annexed to a principal thing becomes part of the principal thing.

Acta exoriora indicant interiora secreta—an indicate the intention.

Action personalis mortar cum personal—a personal actiondies with the promisor.

Actus dei nemini facit injuriam—law holds no man responsible for the Act of God.

Actus legis nemini facit injurium—law wrongs no man.

Actus non facit reum, nisi mens sit rea—an act does not constitute guilt unless it is done with a guilty intent.

Actus rea—physical involvement in crime.

Actus reus—the physical acts involving or constituting occurrence of crime.

Ad arbitrium—at will.

Ad ea Quae Frequentius Accident Jura Adaptatur—law are adapted to those cases which more frequently occur.

Ad hoc—for this purpose.

Ad idem—of the same mind; agrees.

Ad infinitum—for ever.

Ad interim—in the meanwhile.

Ad largum—at large; used in the following and other expression: title at large, commom at large, assize at large, verdict at large, to vouch at large.

Ad naseam—to a disgusting extent.

Ad quem—to whom.

Ad referendum—for further consideration.

Adjourn—to postpone hearing of a case for some future date.

Ad rem—to the point.

Ad valorem—according to value. For example, stamp fee on sale of land is charged is charged according to the value of the land, or value added price or tax included in price.

Addenda—list of additions.

Advers possession—a person not being the owner of the property is in possession of the property.

Advocare—to defend, to call to one's aid also to vouch to warranty.

Advocate general—a person appointed by the state to represent state in cases.

Advocatus diabolin—the devil`s advocate, an officer of the sacred congregation of rites at Rome, whose duty is to prepare all possible argument against the admission of any one to the posthumous honouree of beatification and canonization.

Aequitas—equity, *i.e.,* fair of just according to natural law.

Affidavit—a written statement under an oath, which is sworn to and signed by person making it, as true.

Agent—a person employed to act on behalf of another; an act of an agent done within the scope of his authority, binds his principal.

Agreement—every promise and every set of promise, either written or oral for the purpose of contract forming the consideration for each other.

Allegans contraria non set audiendus—a person alleging contradictory facts should not be heard.

Alias—otherwise calls.

Alibi—a plea taken by the accused to prove that the he has not committed the crime.

Alientio Rei Preferature Juri Accrecendi—alience is favoured by the law itself.

Alimony—a maintenance given by a husband to his divorced wife.

Amicus cuiae—literally 'friend pf Court'. The name is given to a lawyer appointed by a Court to represent a poor litigant.

Amnesty—a general pardon for political offences.

Amnesty international—an organization which fights for human rights.

Animus atestandi—the intention of attesting.

Animus deserendi—the intention of deserting.

Annuity—an amount that is payable yearly.

Annul—to deprive a judicial proceeding of its operation, either retrospectively or only as to future transactions.

Antenuptial—before marriage.

Appeal convict—an application made to a superior court/higher court against the decision of a lower court.

Apriori—form the cause to the effect

Arbitration—setting disputes by referring them to independent third party, as an alternative to the court

Arbitrator—he is a person appointed by parties to decide any difference between them.

Argumentum As Inconvenient Plurimum Valet in lega—an argument drawn form inconvenience is forcible in law.

Arrest—this is a deprivation of personal liberty of a person so that he become available during the trial of any offence to which he is involved.

Assault—striking or attempting to strike another person.

Assignatus Utitur Jure Auctoris—an assignee is clothed with the right of his principal.

Assumpsit—he promised or undertook.

Attorney—one duly appointed or constituted to act for another in business or legal matters. A properly qualified legal agent who conducts litigation.

Au fait—conversant with.

Audi alteram partem—hear the other side.

Authority—a judicial decision cite d as a statement of law [also called precedent].

Autrefois—nobody cannot be tried twice for the same offence.

Autrefois acquit—formerly acquitted.

Autrefois convict—formerly convicted.

Bail—reease of arrested person on furnishing surety bonds.

Bailiff—a subordinate officer of the court who executes writ and other court orders, such as serving summons.

Ballot—a system of secret voting.

Bar—it is a collective trem used for lawyers as a body.

Bench—term is used to explain judges physical force on a person.

Bigamy—marrying again in the life time of another spouse without getting divorce.

Bona fide—in good faith, honestly, without fraud, collusion or participation in wrongdoing.

Bona gestura—good behaviour.

Bona mobilia—movable effects and goods.

Bona vacantia—goods that do not have an owner. Generally they goto the finder.

Boni judici est ansliare jurisdictioness—it is the part of a good judge to enraged his jurisdiction.

Breach—breaking of law, contract.

Bye-law—regulation made by the local authority or corporation or company or society for its members for their day to day operation. They are provided in their principal acts.

Caeteris paribus—other things being equal.

Carte blanche—full discretionary power.

Capital punishment—death sentence.

Casus ommissus—a matter which should have been has not been, provided for in a statute or in statutory rules.

Causa causans—the immediate cause, the last link in the chain of causation.

Causa celebre—a celebrated case.

Causa sine qua non—factor essential to the occurring of event.

Caveat—an order which says "lethim beware". Warning [lat. Let him take heed].

Cvceat emptor—let the buyer beware.

Caveat venditor—let the seller beware.

Caveat viator—let the traveller beware.

Censuer—a reprimand form a superior.

Certiorari—a writ of a superior court calling forth the records and entire proceeding of an inferior court or a writ by which causes are removed form an inferior court into a superior court.

Certum est quod certum redid potest—that is certain which can be rendered certain.

Cessante ratione legis cessat ipsa lex—reason is the soul of law, and when the reason of any particular law ceases, so does the law itself.

Cestui que trus—a person for whom another is trustee : a beneficiary.

Chattel—movable property.

Cognate—blood relation other than agnate and includes a female relation.

Cognizance—judicial knowledge. Thus to take cognization of an offence is to bring to the knowledge of law enforcers or to proceed.

Compos mentis—of sound mind.

Consensus ad idem—agreement as to the same thing.

Consensus, non concubitus, facit matri-monium—it is the consent of the parties, not their cohabitation, which constitutes a valid marriage.

Consummation—voluntary sexual intercourse.

Contea bonos mores—against good morals.

Contemporanea ezpositio est optima et fortissimo in lege—the best way to construe a document is to read it as it would have read made.

Contra pacem—against the peace.

Coram—in the presence of.

Corner—an officer who in enquires into any unnatural death.

Coup d`etat—violent or illegal change.

Coup d`grace—finishing stroke.

Criminology—this refer to a science that deals with crimes as well as criminals.

Culpa—wrongful default.

Curia Regis—the King`s Court.

Damage—a sum of money which the court orders the defendant to pay to the plaintiff as compensation for breach of contract or tort [i.e., civil wrong].

Damnum absque injuria—loss or damage for which there is no legal remedy.

Damnum sine injuria—damage without a legal wrong.

De die in diem—form day to day.

De facto—in fact.

De hors—outside the scope of.

Deed—an instrument in writing which is signed, sealed and delivered.

De jure—by virtue of law.

De minimias non curat lex—law dose not concern itself with trifles.

Demise—the grant of lrase, fallen.

De novo—a new.

De nisi—a conditional decree, not absolute.

Defeasible right—right with can be defeated.

Dei gratia—by god`s geace.

Delegates non potest delegare—a delegate cannot further delegate.

Demission Regis vel coronoe—transfer of property.

Deo volente—if nothing prevents.

Doctrine of harmonious construction—a law is so interpreted to give effect to all its parts and the presumption is taken for various provisions of statutes.

Doctrine of pith and substance—true subject matter of legislation.

Doli capax—capable of crime.

Doli incapax—incapable of crime.

Domicile—a place where a person has his permanent home with intention to return, or a place where a person is ordinarily resident.

Domus sua cuique est tutissinum refugium—to everyone his own house in the safest refuge.

Donation mortis causa—a gift personal property made in contemplation of death.

Donee—one to whom a gift is made.

Donor—one who makes a gift.

Double jeopardy—a second prosecution after a first trial for the same offence.

Droit administrative—it is an ordinance or process where la court are deprived of their jurisdiction in administrative matters.

Durante absentia—during absence.

Durante vita—during life.

Ejusdem generic—of the same kind or nature; the rule that where particular words are followed by general words, the general words are limited to the same kind as the particular words.

Emeritus—retired after long service.

Endorsement—a writing on the back of a document and includes endorsement in negotiable instrument and also writing in evidence of payment of any amount or portion of amount due on a document.

Eminent domain—power of the state to acquire private for public use.

En bloc—all at the same time

En masse—in a body.

En route—on the way.

End of justice—objective of justice.

Eo nomine—by that very name.

Equali Jure Melior Conditio Possidentis—if right is equal, the claim of party is actual possession shall prevail.

Easaped convict—a convict who was run away form lawful custody.

Escheat—the lapsing of property to the sovereign or state on the death of the owner intestate and without heir.

Esprit de corps—regard for honour of body one belongs to.

Estoppel—rule of evidence in which a person is stopped form denying what he has said earlier.

Et incumdit probation qui dicit, non qui negat—the burden of proof is on him who alleges, and not on him who denies.

Evidence—a document, a preventing a person either oral or written or any other thing which the court bylaw is permitted to take into consideration for making clear or ascertaining the truth of the fact or point in issue.

Ex aequo et bono—in justice and good faith.

Ex contractu—arising out of contract.

Ex curia—out of court.

Ex debito justitive—prayer is grantable.

Ex debito—arising out of wrong.

Ex Dolo Malo Non Actio—right of action cannot arise out of fraud.

Ex done—as a gift.

Executor—a representative appointed in a will to execute after the testator's death.

Ex gratia—as an act of grace or favour.

Ex nudo pacto—non oiur action. An action dose not arise form a bare promise.

Ex nudo pacto Non Oritur Actio—no cause of action arise form a bare promise.

Ex officio—by virtue of pervious office.

Ex parte—an order granted after hearing one party only.

Ex post facto—by a subsequent act.

Ex turpi causa non oritur catio—no action arises form a base cause.

Expression Unius Est Exclusio Alierius—express mention of one thing implies the exclusion of another.

Expression unius personae vel rei, est exclusion—special or express mention of one thing or person implies exclusion of another.

Extra vires—beyond powers.

Facsimile—make it like. An exacta copy preserving all the marks of the original.

Factum probanda—facts which are required to be proved.

Fait accompli—a thing already done.

Faux pas—tactless mistake.

Feme sole—an married women.

Ferae naturae—of a wild or ferocious nature.

Fiat—a command.

Fiduciary—relationship based on good faith or trust.

FIR—first information report of grievance which given to police.

Flagrante delicto—in the commission of the offence.

Force majeure—irresistible compulsion.

Freehold—the absolute ownership of land.

Functus offcio—having discharged his duty. Thus once an arbitrator has given his award, he is functus offcio, and cannot revoke the award and re-try the case.

Furiosis nulla voluntas ets—Having discharged his duty.

Generalia specialibus non derogant—general things do not derogate form special things.

Generalibus specialia derogant—special things derogate form general things.

Gestis pro haerede—behaviour as heir.

Gratis dictum—mrer assertion.

Gratuitous—without valuable or legal consideration.

Habeas corpus—a prerogative writ to a person who detains another in custody and with and which commands him to produce or have the body of that person before him.

Hiba—agift in mohammaden law accompanied by delivery and acceptance.

Holograph—a document written in the maker`s own handwritten.

Hypothecation—a pledge in which the pledges retains the possession of the thing pledged.

Ibid—in same place.

Impasse—an insoluble difficulty.

In curia—in open court.

In fra—below.

In futuro—in the future.

Injuria—a legal wrong.

In lieu of—in place of.

In limine—on the threshold.

In loco parentis—in the place of a parent.

In memoriam—in memory of.

In pais—in the country.

In pari delicto—where both parties are equally in fault.

In pari material—where two enactments have a common purpose.

In personam—an act, proceeding or right done or directed against or with reference to a specific person, as opposed to in rem.

In re—in the matter of.

In rem—an act, proceeding or right available against the world at large, as opposed to in personam.

In situ—in its original situation.

In toto—totally, wholly.

Infra—below.

Ignoratia juris non excusat—ignorance of law is no excuse.

Injuria sine damnum—legal wrong without a damage.

Insolvent—a person whose assets are insufficient to pay his debts.

International law—the which regulates relations between countries.

Inter alia—among other things.

Inter se—amongst themselves.

Inter vivos—during life, between living persons.

Intra vires—within power [as opposed to 'ultra vires' which is beyond power].

Intestate—dying with leaving a will.

Ipso facto—by reason of the fact.

Issue—matter or fact that raises the cause of action.

Judgment—a decision of the court with legal reasoning for the same.

Judicial review—the process of review of executive action by court of law.

Jurisdiction—the extent or territorial area over which authority of court runs.

Jurisprudence—the philosophy of law, which relates to fundamental questions, criticisms, etc.

Jus—law.

Juvenile delinquency—an offence committed by a person who is under 16 years of age.

Kin—bool relatives.

Lex—statute.

Lexfori—the law of the forum or court in which a case is tried. More particularly the law relating to procedure or the formalities in force [adjective law] in a given place.

Lex loci—the law of a place.

Lex mercatoria—the law merchant.

Lis pendens or lite pendente—a pending suit, action, petition or matter, particularly one relating to land.

Litera legis—letter of the law.

LL.B.—[legume baccalaureus] bachelor of law.

Locus standi—right to be heard in court.

Mala fide—bad faith, bad intention.

Malfeasance—the doing of an unawtulact e.g. a trespass.

Mandamus—[we command] a prerogative writ or command issued by a higher court to a lower court.

Mansuetae naturae—harmless or tame by nature.

Mc Naghten rule—classic rule which determine whether a person is legally insane.

Mens rea—guilty mind, an evil intention, or a knowledge of the wrognfulness of an act.

Mesne—middle, intervening or intermediate.

Modus operandi—mode of operating.

Mortgage—a loan of money taken after giving security of immovable property.

Mutatis mutandis—whit necessary changes being made.

Nisi—ineffective unless person affected fail to show cause.

Non assumpsit—he did not promise.

Non compos mentis—not of sound mind.

Non-cognizable offence—this is an offence where the police has no legal authority to arrest an offender without a warrant form court.

Nocumentum—an annoying, unpleasant or obnoxious things or pratice.

Non obstante—nothwithstanding.

Noscitur a sociis—a word is known by its associates, *i.e.*, the meaning of a word can be gathered form the context.

Nota bene (NB)—take notice.

Nudum pactum—a nude contract i.e. unenforceable.

Obiter dictum—[a saying by the way] an incidential and collateral opinion uttered a by a judge while delivering a judgment and which is not binding.

Obligee—one to whom a bond is made.

Obligor—one who binds himself by bond.

Ombudsman—a parliamentary commission or a grievance man of administration.

Onus—burden of proof.

Op. cit—the book previously cited.

Pacta sunt servanda—states are found to fulfill in good faith the obligations assumed by them under agreements.

Par excellence—eminently.

Parens Patrice—protector of rights.

Pari material—on the same subject.

Pari passu—equally, without preference.

Passim—in various places.

Patent—exclusive privilege granted by the sovereign authority to an inventor with respect of his invention.

Per—as stated by.

Per annum—by the year.

Per capital—individually.

Per curiam—by the court.

Per mensem—by the month.

Per se—by itself.

Penal—relating to punishment.

Pendente lite—during the process of litigation.

Perjury—an offence of giving a false statement before the court which a person knows to be false.

Prima facie—on the face of it.

Pro bono publico—for the good of the public.

Pro et contra—for that exent.

Pro rata—in proportion.

Pro tanto—to that exent.

Pro tempore—for the time being.

Procedure—mode or form of conducting judicial proceedings.

Proviso—a condition or a clause.

Puberty—earliest age of being capable of bearing child.

Qua—with respect to.

Quasi—as if, as it were analogous to.

Qui facit per alium facit per se—he who acts through another is deemed to act in person.

Qui prior est tempore potior est jure—he who is first in time has the strongest claim.

Qui sentit commodum debet et onus—he who accepts the benefit of a transaction must also accept the burden of the same.

Quia timet—because he fears.

Quorum—minimum number of person necessary for conduct of proceedings.

Quo warranto—[by what authority] a prerpgative writ requiring a person to show by what authority he exercise a public office.

Raison d'etre—reason for the existence of a thing.

Ratio legis—according to spirit of law.

Ratio decidendi—the reason or groumd of a judicial decision. It is the ratio deciendi of a case which make the decision a precedent for the future.

Re—in the matter of.

Remand—the committal of an accused to prison.

Res communes—common things; things common to al by the law of nature.

Res extra commercium—things thrown out of commerce.

Res gestae—all facts so connected with a fact in issue as to introduce it, explain its nature, or form in connection with it one continuous transaction.

Res ipsa loquitur—the thing speaks for itself.

Res judicata—a case or suit already decided.

Res sub judice—a matter under judicial consideration.

Scienter—knowledge; an allegation in a pleading that the thing has been done knowingly.

Scienti non fit injuria—an injury is not done to one who knows.

Sic utere tuo ut alienum non laedas—live and let live and hbe reasonable as to your acts in regard to your neighbours.

Spes successionts—a mere hope of succeeding to property.

Stare decisis—adherence to earlier precedents as authoritative and binding.

Status quo—the state in which thing are, or were.

Status quo ante—the state in which thing were

Sub silentio—under silence.

Suit—a process instituted in a court of justice for protection recovery or reinstatement of a right.

Summons—an order by which a person is called to appear before a court, judicial officer etc.

Supra—above.

Testimony—the statement made by a witness under oath.

Tort—wrong conduct.

Ubi jus ibi remedium—where there is a right, there is a remedy.

Ut lite pendente nihil innovateur—nothing new should be introduced during the pndency of a suit.

Ut res margis valeat quam pereat—it is better for a thing to have effect than to be made void, i.e., the words of a statute must be constructed soas to give a sensible meaning to them.

Void ab intio—not valid form the very beginning.

Void—not valid.

Vigilantibus non dormientibus leges subveniunt—the law aids the diligent and not the indolent.

Volenti non fit injuria—where there is consent, there is no injury.

Volkogeist—general awareness of the people.

Vs—versus or aganst or prefix symbol of opposite party. Also written as ,v,. for example, Maneka Gandhi v. Union of India.

Wager—a bet.

Writ—a judicial process of written command or order by court, by which any one is summoned or directed; a legal instrument to enforce obedience to the order for restraining to do some act.

6 Drills

Direction—In the following DRILLS you are asked to select the definition *closest* in meaning. The correct answer is not necessarily, an exact equivalent or even a very good definition. But it is the one choice that comes *closest* in meaning to the word .

Drill—One

1. abash
   (A) squash (B) embarrass
   (C) amaze (D) refuse
2. abate
   (A) aid (B) remove
   (C) lessen (D) howl
3. abominable
   (A) unfortunate (B) loathsome
   (C) cheap (D) stormy
4. acclaim
   (A) demand (B) applaud
   (C) surpass (D) elect
5. addicted
   (A) strongly disposed to
   (B) mad
   (C) increased
   (D) sentenced
6. affront
   (A) insult (B) projection
   (C) invasion (D) success
7. altercation
   (A) drastic change (B) angry dispute
   (C) noisy dialogue (D) loud explosion
8. anomaly
   (A) abnormality (B) ignorance
   (C) accident (D) rarity
9. arboreal
   (A) holiday (B) bower
   (C) treelike (D) shady
10. askew
   (A) turned to one side (B) direct
   (C) doubtful (D) wide open
11. avowal
   (A) sacred oath (B) open declaration
   (C) harsh sound (D) sterndenial
12. berate
   (A) deny (B) downgrade
   (C) scold (D) judge
13. bicameral
   (A) meeting twice a year
   (B) having two legislative branches
   (C) having twin lenses
   (D) published every two years',
14. blatant
   (A) tardy (B) futile
   (C) depressed (D) noisy
15. capitulate
   (A) summarize (B) execute
   (C) withdraw (D) surrender
16. Careen
   (A) secure (B) sway
   (C) decay (D) fondle
17. cauterize
   (A) sear (B) warn
   (C) cut away (D) bind
18. cherubic
   (A) mischievous (B) expensive
   (C) rustic (D) angelic
19. compliance
   (A) flexibility (B) spite
   (C) obedience (D) weakness
20. compunction
   (A) remorse (B) conscience
   (C) piercing blow (D) satisfaction

Answers

1. (B)	2. (C)	3. (B)	4. (B)	5. (A)	6. (A)
7. (B)	8. (A)	9. (C)	10. (A)	11. (B)	12. (C)
13. (B)	14. (D)	15. (D)	16. (B)	17. (A)	18. (D)
19. (C)	20. (A)				

Drill—Two

21. consternation
    (A) group of stars (B) humble service
    (C) large display (D) great amazement
22. corrosive
    (A) polishing (B) acid-forming
    (C) hiding (D) eating away
23. covert
    (A) patent (B) secret
    (C) ditch (D) greedy
24. covetous
    (A) sheltered (B) hidden
    (C) grasping (D) thrifty
25. cumbersome
    (A) heavy (B) sorrowful
    (C) unwieldy (D) laborious
26. debility
    (A) debit (B) instability
    (C) pain (D) weakness
27. decor
    (A) dramatic presentation
    (B) showpiece
    (C) ornamental setting
    (D) rich furniture
28. derisive
    (A) mocking (B) copied
    (C) limited (D) borrowed
29. derogatory
    (A) questionable (B) inquisitive
    (C) humble (D) depreciating
30. devious
    (A) multitudinous (B) guessing
    (C) circuitous (D) premature
31. dilatory
    (A) expanded (B) casual
    (C) slow (D) amateurish
32. discursive
    (A) profane (B) rambling
    (C) detailed (D) extraneous
33. disparage
    (A) separate (B) discourage
    (C) compare (D) belittle
34. diurnal
    (A) news account (B) solar
    (C) daily (D) everlasting
35. dolorous
    (A) sorrowful (B) financial
    (C) sacred (D) parsimonious
36. dowdy
    (A) corpulent (B) rakish
    (C) elegant (D) unstylish
37. dulcet
    (A) melodious (B) zither
    (C) pastry (D) twofold
38. echelon
    (A) level of command (B) squadron leader
    (C) summit (D) battleground
39. edify
    (A) amuse (B) satisfy
    (C) consume (D) instruct
40. engender
    (A) maneuver (B) cause
    (C) fertilize (D) incite

Answers

21. (D) 22. (D) 23. (B) 24. (C) 25. (C) 26.(D)
27. (C) 28. (A) 29. (D) 30. (C) 31. (C) 32. (B)
33. (D) 34. (C) 35. (A) 36. (D) 37. (A) 38. (A)
39. (D) 40. (B)

Drill—Three

41. epithet
    (A) inscription (B) shoulder piece
    (C) descriptive term (D) honorary award
42. expedient
    (A) advantageous (B) free
    (C) fatigued (D) rapid
43. expiate
    (A) expire (B) sanctify
    (C) demolish (D) atone
44. exude
    (A) evaporate (B) overflow
    (C) wither away (D) ooze out
45. facet
    (A) gem (B) aspect
    (C) spout (D) trait pertaining to a

46. filial

(A) parent (B) son

(C) duty (D) wise man

47. fillip

(A) beverage (B) acrobatic trick

(C) large dose (D) stimulus

48. flippancy

(A) levity (B) dexterity

(C) heaviness (D) clumsiness

49. germane

(A) bacterial (B) Teutonic

(C) relevant (D) microscopic

50. gratuitous

(A) thankful (B) reproachful

(C) satisfactory (D) uncal1ed for

51. guise

(A) deceit

(B) malice

(C) protection

(D) appearance composed of

52. heterogeneous

(A) similar parts (B) unlike elements

(C) smooth surfaces (D) complex problems

53. idiosyncrasy

(A) personality (B) lack of intelligence

(C) absolute rule (D) distinctive characteristic

54. impinge

(A) paint (B) constrict

(C) steal (D) encroach

55. incisive

(A) penetrating (B) short

(C) compendious (D) assured

56. incongruous

(A) unofficial (B) incompatible

(C) poorly timed (D) uneven

57. incumbent

(A) obligatory (B) dutiful

(C) weak (D) slanting

58. ineptitude

(A) dullness (B) vacillation

(C) awkwardness (D) inexperience

59. insinuate

(A) spy upon (B) suggest slyly

(C) set free (D) cause injury

60. insipid

(A) tasteless (B) animated

(C) interminable (D) unplanned

Answers

41. (C)	42. (A)	43. (D)	44. (D)	45. (B)	46. (B)
47. (D)	48. (A)	49. (C)	50. (D)	51. (D)	52. (B)
53. (D)	54. (D)	55. (A)	56. (B)	57. (A)	58. (C)
59. (B)	60. (A)				

Drill—Four

61. interloper

(A) acrobat (B) intruder

(C) slanderer (D) malingerer

62. jocose

(A) trite (B) playful

(C) useless (D) illusory

63. malign

(A) disapprove (B) mistreat

(C) curse (D) slander

64. manifesto

(A) cargo list (B) secret treaty

(C) revolutionary plot (D) public declaration

65. maudlin

(A) overwrought (B) weakly sentimental

(C) exceedingly sad (D) dispirited

66. morose

(A) quick-tempered (B) miserly

(C) illhumored (D) despondent

67. mutation

(A) silence (B) severance

(C) display (D) variation to make:

68. obviate

(A) unnecessary (B) clear

(C) sure (D) difficult

69. ostentatious

(A) modest (B) flagrant

(C) showy (D) diligent

70. perfunctory

(A) lazy (B) official

(C) mechanical (D) impromptu

71. plaudit

(A) expression of approval

(B) consent

(C) detonation

(D) pleasure,

72. prevaricate
(A) authenticate (B) delay
(C) lie (D) anticipate

73. pristine
(A) meritorious (B) original
(C) expensive (D) traditional

74. privation
(A) seclusion (B) sloop
(C) security (D) hardship

75. proton
(A) tribal leader (B) meat substitute
(C) food element (D) positive particle

76. protrude
(A) stick out (B) insult
(C) act discourteously (D) emigrate

77. raffish
(A) made of straw (B) ludicrous
(C) disreputable (D) due to chance

78. rampant
(A) forbidding (B) lion like
(C) protective (D) raging unchecked

79. reiterate
(A) stutter (B) repeat
(C) rewrite (D) reassess

80. replica
(A) mythical creature (B) answer
(C) copy (D) public building

Answers

61. (B) 62. (B) 63. (D) 64. (D) 65. (B) 66. (C)
67. (D) 68. (A) 69. (B) 70. (C) 71. (A) 72. (C)
73. (B) 74. (D) 75. (D) 76. (A) 77. (C) 78. (D)
79. (B) 80. (C)

Drill—Five

81. retrospect
(A) brief summary (B) survey of the past
(C) close examination (D) full payment

82. rhapsodic
(A) ecstatic (B) bombastic
(C) tightly knit (D) fervent

83. roster
(A) nesting place (B) professional team
(C) speaker's platform (D) list of persons

84. ruminate
(A) slander (B) digest
(C) meditate (D) remove

85. salacious
(A) briny (B) purchasable
(C) obscene (D) flavored

86. savant
(A) cleansing agent (B) learned person
(C) young student (D) French courtier

87. scrutinize
(A) erase completely (B) turn aside
(C) examine closely (D) read aloud

88. silo
(A) sandy surface
(B) water tower
(C) structure for storage
(D) musical notes

89. subsidy
(A) replacement (B) financial aid
(C) public funds (D) depth charge

90. torpid
(A) stormy (B) hibernating
(C) warm (D) inactive

91. travesty
(A) garment (B) long journey
(C) parody (D) deterioration

92. tussock
(A) soft cushion (B) low hammock
(C) bunch of grass (D) small hill

93. tycoon
(A) labor leader (B) autocratic ruler
(C) mystic prophet (D) industrial magnate'

94. upbraid
(A) plait (B) reproach
(C) elevate (D) foster

95. vapid
(A) spiritless (B) foggy
(C) accelerated (D) shapeless

96. venerable
(A) antique (B) retired
(C) inimitable (D) worthy of respect

97. vernacular
(A) native speech (B) slang
(C) local custom (D) uneducated group

98. vituperation

(A) wordy abuse (B) poisonous liquid

(C) bombast (D) violent action

99. winnow

(A) blow (B) fish

(C) separate (D) minimize

100. wry

(A) sad (B) smiling

(C) U1ldeserved (D) twisted

Answers

80. (B)	82. (A)	83. (D)	84. (C)	85. (C)	86. (B)
87. (C)	88. (C)	89. (B)	90. (D)	91. (C)	92. (C)
93. (D)	94. (B)	95. (A)	96. (D)	97. (A)	98. (A)
99. (C)	100. (D)				

Drill—Six

1. Loquacity is an inordinate amount of—

(A) singing (B) attention to details

(C) talking

2. Gullible people fall easy prey to—

(A) doctors (B) used-car salesmen

(C) teachers

3. Suave men are experts at—

(A) home repair (B) surfing

(C) getting along with women

4. Pomposity probably comes from—

(A) fear (B) obesity

(C) vanity

5. Most likely to be esthetic is an—

(A) electrician (B) aviator

(C) artist

6. Taciturnity would likely be found in—

(A) salesmen (B) public speakers

(C) hermits

7. Opinionated assertions may likely lead to—

(A) marriage (B) arguments

(C) truth

8. A phlegmatic person—

(A) sheds tears at an emotional play

(B) becomes hysterical in a crisis

(C) does not become easily emotional

9. Erudite men are most interested in—

(A) scholarly books (B) light fiction

(C) the comics

10. People who are complacent about their jobs will—

(A) take it easy

(B) worry about their future

(C) keep an eye on the help-wanted ads

11. A punctilious person is a stickler for—

(A) originality (B) courage

(C) proper etiquette

12. To be indefatigable, one usually needs a great amount of—

(A) money (B) energy

(C) education

13. Vapid people are—

(A) boring (B) successful

(C) quarrelsome

14. Iconoclasts are opposed to—

(A) change (B) tradition

(C) reform

15. A misanthrope dislikes—

(A) people (B) good food

(C) literature

16. Men are most likely to be puerile when—

(A) they don't get their own way

(B) they are reading

(C) they are eating

17. Most ascetics prefer to—

(A) drink excessively

(B) eat sparingly

(C) participate in orgies

Answers

1. (C)	2. (B)	3. (C)	4. (C)	5. (C)	6. (C)
7. (B)	8. (C)	9. (A)	10. (A)	11. (C)	12. (B)
13. (A)	14. (B)	15. (A)	16. (A)	17. (B)	

7 Test Papers

Test Paper 1

1. The manager tried hard to his men to return to work before declaring a lockout—
(A) motivate (B) persuade
(C) encourage (D) permit

2. There was so much material in the speech that it was difficult to know what the peaker wanted to stay—
(A) banal (B) extraneous
(C) superficial (D) variegated

3. Our flight wasfrom Jaipur to Agra air port—
(A) deflected (B) shifted
(C) diverted (D) reverted

4. His handling resulted in all that destruction and damage—
(A) inept (B) skilful
(C) sophisticated (D) uncouth

5. Once I forgot the piece of paper on which the Dame of the hotel was written, I was as ………… as lost.
(A) much (B) sure
(C) good (D) bad

6. Infant mortality rate in China has from 200 per thousand to 14 per thousand—
(A) retarded (B) declined
(C) contracted (D) minimised

7. Time once lost cannot be …………
(A) gained (B) recalled
(C) renumerated (D) recovered

8. He is very on meeting foreigner and befriending them—
(A) anxious (B) fond
(C) insistent (D) keen

9. Laser is a (an) of Light Amplification by stimulated Emission of Radiation.
(A) homonym (B) acronym
(C) malapropos (D) collocation

10. The exhibition was a curious of the ok and the new—
(A) melange (B) fervour
(C) mixture (D) blend

11. The Hubble Space Telescope will search for planets around other stars, a key to the ………… for extra terrestrial life.
(A) discovery (B) quest
(C) perception (D) enquiry

12. Colgate has also set an ambitious aim of an eight per cent value share of the tooth paste market by the end of the first year—
(A) keeping (B) distributing
(C) cornering (D) soliciting

13. Marie Curie was excited when she knew that she was on the …………, of a new discovery.
(A) outskirts (B) frontier
(C) threshold (D) gateway

14. Many women in developing countries experience a cycle of poor health that ………… before they are born and persists through adulthood passing from generation to generation.
(A) derives (B) establishes
(C) begins (D) originates

15. I have for one month's leave—
(A) demanded (B) requested
(C) wanted (D) asked

16. A man remains narrow-minded, self complacent and ignorant unless he visits other people and from them—
(A) hears (B) earns
(C) learns (D) borrows

17. His companions prevailed upon him not to ………… to violence.
(A) refer (B) resort
(C) prone (D) provoke
(E) pertain

18. No country can to practice a constant, rigid foreign police in view of the world power dynamics—
(A) obligate (B) anticipate
(C) afford (D) envisage
(E) visualise

19. If the President does not grant in his case, he will be hanged shortly—
(A) parole (B) release
(C) freedom (D) gratitude
(E) clemency

20. The cancer patients are into their mode of life for research purposes—
(A) diagnosed (B) checked
(C) probed (D) examined
(E) investigated

Answers

1. (B)	2. (C)	3. (C)	4. (A)	5 (A)	6. (B)
7. (B)	8. (D)	9. (B)	10. (A)	11. (B)	12. (C)
13. (C)	14. (D)	15 (D)	16. (C)	17. (B)	18. (C)
19. (C)	20. (E)				

Test Paper 2

1. I request you to your crime—
   (A) apologise (B) agree
   (C) confess (D) pardon
   (E) submit

2. The payment was delayed this time because some of the supporting documents for the claim were missing—
   (A) unduly (B) unjustifiably
   (C) ul1foreseenly (D) undeservedly
   (E) unquestionably

3. The prisoner was released on for good behaviour—
   (A) probation (B) bail
   (C) parole (D) grounds
   (E) guarantee

4. Although religion does not inhibit acquisition of wealth, the tenor of its teaching is to induce an attitude ofto worldly things—
   (A) indifference (B) hostility
   (C) affinity (D) immunity
   (E) attachment

5. ………… eyewitnesses, the news reporter gave a graphic description of how the fire broke.
   (A) Reporting (B) Observing
   (C) Seeing (D) Quoting
   (E) Examining

6. The library expects you to return each and every book that you have ……………
   (A) demanded (B) taken
   (C) lent (D) handed
   (E) given

7. …………… between labour and management is inevitable hi any industrial society.
   (A) Controversy (B) Friction
   (C) Association (D) Competition
   (E) Coordination

8. This article tries to ………… us with problems of poor nations so that we help them more effectively.
   (A) allow (B) enable
   (C) convince (D) project
   (E) acquaint

9. One should develop a ……… habit of going through a newspaper, some selected magazines and general literature.
   (A) continuous (B) constant
   (C) regular (D) persistent
   (E) recurring

10. The current ……… in global negotiations over the ban on unclear test does not augur well for the future—
    (A) moratorium (B) controversy
    (C) insight (D) stalemate
    (E) rapprochement

11. Improvement in efficiency and productivity has to be the key ……… of policy in respect of both the public and the private sectors in the Five Years Plans—
    (A) ingredient (B) purpose
    (C) role (D) platform
    (E) criterion

12. The skill and ease with which he repaired the machine proved that he is a/an ………… mechanic.
    (A) able (B) handy
    (C) nimble (D) maladroit
    (E) competent

13. The chemical warfare capability of Iraq is a ……… to aggression even by the Super powers.
    (A) pretext (B) deterrent
    (C) constraint (D) detriment
    (E) precaution

14. Though the issue of bonus provision was not on the agenda of the meeting, the Chairman was sure that this would up—
    (A) bring (B) shoot
    (C) trickle (D) crop
    (E) heat

15. They have decided to meet the Prime Minister in order to have their heard—
    (A) agony (B) apathy
    (C) woes (D) sorrow
    (E) sufferings

16. The foundation of all civilizations and societies is the ability of humans to with each other—
    (A) bear (B) dispens1
    (C) unite (D) collaborate
    (E) exchange

17. The childrencrackers to celebrate the victory of their team—
    (A) burst (B) fired
    (C) shot (D) broke
    (E) released

18. The ……… council has passed against the magazine for their irresponsibility in reporting are justified—
    (A) laws (B) ban
    (C) penalty (D) codes
    (E) strictures

19. After a recent mild paralytic attack, his movements are …… restricted, otherwise he is still very active—
    (A) entirely (B) nowhere
    (C) not (D) slightly
    (E) frequently

20. His of the topic was so good that students had few doubts to raise at the end.
    (A) handling (B) clarity
    (C) exposure (D) exposition
    (E) picturisation

Answers

1. (C)	2. (A)	3. (C)	4. (A)	5. (D)	6. (B)
7. (B)	8. (E)	9. (C)	10. (D)	11. (C)	12. (E)
13. (B)	14. (D)	15 (C)	16. (D)	17. (A)	18. (E)
19. (D)	20. (D)				

Test Paper 3

1. I request you to your crime—
   (A) apologise (B) agree
   (C) confess (D) pardon
   (E) submit
2. The payment was delayed this time because some of the supporting documents for the claim were missing—
   (A) unduly (B) unjustifiably
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   (C) Association (D) Competition
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   (C) regular (D) persistent
   (E) recurring
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    (C) insight (D) stalemate
    (E) rapprochement
11. Improvement in efficiency and productivity has to be the key ……… of policy in respect of both the public and the private sectors in the Five Years Plans.
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    (A) handling (B) clarity
    (C) exposure (D) exposition
    (E) picturisation

Answers

1. (C)	2. (A)	3. (C)	4. (A)	5 (D)	6. (B)
7. (B)	8. (E)	9. (C)	10. (D)	11. (C)	12. (E)
13. (B)	14. (D)	15 (C)	16. (D)	17. (A)	18. (E)
19. (D)	20. (D)				

Test Paper 4

1. They decided to ……… down their original plans for the bigger house and make it smaller.
   (A) rule (B) turn
   (C) change (D) scale
   (E) play
2. My father keeps all hispapers in a lock and key—
   (A) required (B) necessary
   (C) useful (D) confidential
   (E) enclosed
3. Everyone knows that he is not ……… to hard work.
   (A) trained (B) accustomed
   (C) willing (D) suitable
   (E) addicted
4. The Supreme Court had recently ……… the government from implementing the Finance Commission Report in view of adverse economic situation in the country.
   (A) abstained (B) avoided
   (C) directed (D) withheld
   (E) restrained
5. He made a slight of judgment for which he had to repent later—
   (A) error (B) slip
   (C) mistake (D) blunder
   (E) inexactness
6. Freedom is not a ………… but our birth right.
   (A) sin (B) gift
   (C) farce (D) illusion
   (E) presentation
7. Macbeth is a tragedy of a man who was with great qualities—
   (A) possessed (B) empowered
   (C) privileged (D) endowed
   (E) obsessed
8. Mounting unemployment is the most seri ous and ………… problem faced by India today.
   (A) dubious (B) profound
   (C) unpopular (D) intractable
   (E) unattainable
9. Democracy ………… some values which are fundamental to the realization of the dignity of man.
   (A) cherishes (B) nourishes
   (C) espouses (D) nurtures
   (E) harbors
10. Successful people are genuinely very efficient in their tasks—
    (A) making (B) attaining
    (C) achieving (D) completing
    (E) accomplishing
11. Automobile manufacturers are revving up to launch a campaign designed to increase consumer about the new emission control—
    (A) production (B) education
    (C) capacity (D) knowledge
    (E) awareness
12. To break the stalemate over the controversial issue, the Prime Minister held discussions today with four other leaders to ………… a consensus.
    (A) evolve (B) win
    (C) capture (D) emerge
    (E) develop
13. Forests on the whole are less ………… than farms to flood damage.
    (A) exposed (B) affected
    (C) destroyed (D) vulnerable
    (E) destructible
14. His logic ……… everyone, including the experts.
    (A) teased (B) defied
    (C) surprised (D) confounded
    (E) overwhelmed
15. The factory went into a state of suspended ……… today with. all its workers on strike.
    (A) symbiosis (B) animation
    (C) ways (D) condition
    (E) mortification
16. It is not fair to cast ………… on honest and innocent persons.
    (A) aspiration (B) aspersions
    (C) inspiration (D) adulation
17. The code of Manu from the theological aspect is regarded as ………… from God.
    (A) originating (B) issuing
    (C) generating (D) emanating
    (E) coming forth
18. This book is about a man who ………… his family and went to live in the Himalayas.
    (A) exiled (B) deserted
    (C) banished (D) expelled
    (E) admonished
19. You must ………… your career with all seriousness.
    (A) direct (B) complete
    (C) follow (D) manage
    (E) pursue
20. The affluent life styles of contemporary politicians are in sharp contrast to the ……… ways of living of the freedom fighters.
    (A) austere (B) agnostic
    (C) stingy (D) extravagant
    (E) disciplined

Answers

1. (D)	2. (D)	3. (B)	4. (E)	5 (A)	6. (B)
7. (D)	8. (D)	9. (A)	10. (E)	11. (E)	12. (B)
13. (D)	14. (C)	15 (B)	16. (B)	17. (D)	18. (B)
19. (E)	20. (A)				

Test Paper 5

1. The villagersthe death of their leader by keeping all the shops closed—
(A) announced (B) protested
(C) mourned (D) consoled
(E) avenged

2. Everyone of us should endcavour to .... thc miseries of thc poor.
(A) diffuse (B) mitigate
(C) condemn (D) suppress
(E) acknowledge

3. The government willall resources to fight poverty—
(A) move (B) collect
(C) harness (D) exploit
(E) TIuster

4. These medicines are......for curing cold.
(A) proper (B) real
(C) effective (D) capable
(E) powerful

5. It is easy to .... but impossible to replace English medium education.
(A) deny (B) approve
(C) propagate (D) castigate
(E) eliminate

6. Since she is a teacher of language, one would not expect her to be guilty of a/am ....
(A) aberration (B) solecism
(C) schism (D) bombast
(E) stanchion

7. The poor ones continue to .... out a living in spite of economic liberalisation in that country.
(A) find (B) go
(C) eke (D) bring
(E) manage

8. I will write a letter to you tentatively .... the dates of the programme.
(A) involving (B) urging
(C) guiding (D) indicating
(E) propagating

9. He should be dismissed for his remarks about his superiors—
(A) critical (B) depreciatory
(C) scurrilous (D) laudatory
(E) impeccable

10. Contemporary economic development differs .... from the Industrial Revolution of the19th cenury.
(A) naturally (B) usually
(C) literally (D) specially
(E) markedly

11. After discussing the matter for about an hour, the committee .... without having reached any decision.
(A) dispersed (B) dissolved
(C) postponed (D) withdrew
(E) adjourned

12. The word gharana points to the concepts of stylistic individuality and handing down of tradition within family confines.
(A) joint (B) conflicting
(C) dual (D) contradictory
(E) extraordinary

13. The Government is certain to the publication of any details of this fraudulent research—
(A) retain (B) restrict
(C) delay (D) prohibit
(E) conceal

14. It was the help he got from his friends which .... him through the tragedy.
(A) helped (B) boosted
(C) perked (D) supported
(E) sustained

15. The security for the Ministers has been .... up following the attack at a public meeting last evening.
(A) steered (B) geared
(C) speeded (D) bloated
(E) beefed

16. The criminals managed to escape from the prison even though two armed policemen were .... vigil over them.
(A) taking (B) putting
(C) guarding (D) keeping
(E) looking

17. General awareness and education facilitate the .... of specific skills.
(A) creation (B) requirement
(C) acquisition (D) procurement
(E) organisation

18. The speaker did not properly use the time as he went on .... on one point alone.
(A) dilating (B) devoting
(C) deliberating (D) diluting
(E) distributing

19. A number of advances in medicine would have been .... sooner if free enquiry had been common and orthodox thinking habits had been rare.
(A) persisted (B) inducted
(C) secured (D) achieved
(E) propagated

20. The final electoral rolls have been intensively revised through house to house ....
(A) investigation (B) enunciation
(C) enumeration (D) documentation
(E) categorization

Answers

1. (C)	2. (B)	3. (E)	4. (C)	5. (D)	6. (B)
7. (C)	8. (D)	9. (C)	10. (E)	11. (E)	12. (C)
13. (D)	14. (A)	15. (E)	16. (D)	17. (C)	18. (C)
19. (D)	20. (C)				

Test Paper 6

1. Ravi had to drop his plan of going to picnic as he had certain ……….. to meet during that period.
   (A) preparations (B) observations
   (C) urgencies (D) transactions
   (E) commitments
2. The unruly behavior of the students ……….. their teacher.
   (A) tempered (B) incensed
   (C) aggrieved (D) clashed
   (E) impeached
3. Although it is two years since this book was first published, its Indian edition has just been ………..
   (A) sold (B) started
   (C) published (D) launched
   (E) marketed
4. Even in today's modern society people ……….. god to bring rains.
   (A) provoke (B) evoke
   (C) appeal (D) propitiate
   (E) superimpose
5. The Union leader assured the workers that their grievances could be ……….. through negotiations.
   (A) attended (B) heard
   (C) settled (D) answered
   (E) satisfied
6. The good is oftenwith their bones.
   (A) buried (B) covered
   (C) exhumed (D) interred
   (E) fleshed
7. If this interpretation is held valid, then the states are ……….. of power to plan, implementand monitor their schemes.
   (A) awarded (B) invested
   (C) relieved (D) delegated
   (E) divested
8. He knew that social evils were only of deeper maladies.
   (A) cause (B) indications
   (C) part (D) consequences
   (E) manifestations
9. Eight scientists have the national awards for outstanding contribution and dedication to the profession.
   (A) bestowed (B) picked
   (C) bagged (D) conferred
   (E) discovered
10. The judge complimented the young witness for standing upto the ……….. cross examination.
    (A) terrible (B) tedious
    (C) arduous (D) lengthy
    (E) gruelling
11. Defection is an unprincipled practice which can do ……….. damage to the democratic process.
    (A) incalculable (B) inalienable
    (C) intolerable (D) infallible
    (E) indispensable
12. With the, growing in the country the Government is gearing itself to quell there bellion—
    (A) disturbances (B) tension
    (C) unrest (D) insurgency
    (E) coup
13. It was ……….. hot that day and the cable suffered the brunt of the heat.
    (A) treacherously (B) acceptably
    (C) unfailingly (D) unbelievably
    (E) uncompromisingly
14. Sachin was to reach that afternoon but was ……….. up at Delhi for some personal work
    (A) kept (B) held
    (C) delayed (D) stayed
    (E) detained
15. I do not think the evidence you have heard ……….. you opinion.
    (A) promotes (B) accuses
    (C) commits (D) warrants
    (E) convinces
16. He his shoes till they shone—
    (A) brushed (B) scrubbed
    (C) shined (D) polished
    (E) wiped
17. The Hubble Space Telescope will search for planets around other stars, a key to the ……….. for extra terrestrial life.
    (A) quest (B) perception
    (C) discovery (D) inquiry
18. Colgate has also set an ambitious aim ……… of an eight per cent value share of the tooth paste market by the end of the first year.
    (A) keeping (B) distributing
    (C) cornering (D) soliciting
19. Marie Curie was excited when she knew that she was on the …………, of a new discovery.
    (A) outskirts (B) frontier
    (C) threshold (D) gateway
20. Many women in developing countries experience a cycle of poor health that ……….. before they are born and persists through adulthood passing from generation to generation.
    (A) derives (B) establishes
    (C) begins (D) originates

Answers

1. (E)	2. (B)	3. (D)	4. (B)	5. (C)	6. (D)
7. (E)	8. (E)	9. (C)	10. (E)	11. (A)	12. (C)
13. (D)	14. (B)	15 (D)	16. (D)	17. (B)	18. (C)
19. (C)	20. (D)				

8 Use of Proper Vocabulary

Accept; Except

We **accept** your offer; All will be present, not even your friend **excepted**.

Acceptation; Acceptance

We do not take the word in that **acceptation** (meaning); The bill was sent for our **acceptance**.

Adverse; Averse

In the most **adverse** conditions, he never ceased to pursue his great object in life; You are not **averse** to a little recreation, are you ? He is **averse** to taking my advice.

Affect; Effect

Does this **affect** you in any way ? What will be the **effect** of their decision ?

Apposite; Opposite

The reply was not **apposite** (to the point). The house is on the **opposite** bank of the river.

Appreciative; Appreciable

He did not show himself sufficiently **appreciative** of my kindness. The difference will be **appreciable**.

Beneficent; Benevolent

He is the most **beneficent** supporter of the hospital. Although poor, he has a **benevolent** heart.

Canvas; Canvass

A **canvas** tent was erected on the lawn. Will your friend be prepared to **canvass** for orders ?

Childish; Childlike

This was, on the part of Mrs. Deepika, a very **childish** remark This great man had a **childlike** simplicity.

Contemptous; Contemptible

They showed themselves **contemptuous** of our offers of help. What a mean and **contemptible** trick !

Continual; Continuous

We suffered from **continual** interruptions. The war-ships of the battle-squadron formed a **continuous** line.

Council; Counsel

The common **council** approved of the scheme. He would not follow our **counsel**. Counsel was of opinion that they would lose the case.

Deficient; Defective

He is **deficient** in politeness. My typewriter is very **defective**.

Definite; Definitive

Will he give us a **definite** (precise) answer ? The edition of the works of the great poet must be regarded as **definitive**.

Deprecate; Depeciate

I strongly **deprecate** the suggestion that I am not impartial in this matter. You always **depreciate** my efforts. The shares have **depreciated** during the last few days.

Distinct; Distinctive

Although speaking the same language, the two peoples have a **distinct** origin. Each of the guests wore the **distinctive** emblems of the order.

Efficient; Effectual; Effective

She is an **efficient** shorthand–typist. I found this an **effectual** method of preventing waste. Some of the clauses of the Education Act are not intended to become **effective** immediately.

Emergence; Emergency

Owing to the **emergence** of unexpected difficulties, the plan must be abandoned. The Government has proclaimed a state of national **emergency**.

Eminent; Imminent

Mr. S.K.F. Perumal–is one of our most **eminent** barristers. The catastrophe is **imminent** ; we may expect it in a few hour.

Eruption; Irruption

We witnessed the **eruption** of Vesuvius. The Enemy made an **irruption** (inroad, invasion) into the island.

Exceedingly; Excessively

They feel **exceedingly** (greatly) obliged. I think the price is **excessively** high.

Factitious; Fictitious

Their indignation was entirely **factitious** (affected). The shares in this company have only a **fictitious** (imaginary) value.

Gourmand; Gourmet

Mr. Jagan Mohan–is a well know **gourmand** (greedy, gluttonous man). Mr. Perumal–is a well known **gourmet** (epicure).

Immigrant; Emigrant

All the **immigrants** were detained twenty–four hours on Ellia Island. England does not want to send **emigrants** to Brazil.

Ingenious; Ingenuous

This young man is a very **ingenious** mechanic. I was amused by the child's **ingenuous** (frank) remarks.

Intelligent; Intellectual

The boy is ignorant, but he seems **intelligent**. The writer of this book must have **intellectual** powers of the highest order.

Judicious; Judicial

In this difficult situation, his answer was very **judicious** (marked by wisdom). The Government will set up a **judicial** body to settle the conditions in the industry.

Luxuriant; luxurious

She had her **luxuriant** hair cut yesterday. Nothing can be more **luxurious** than their town residence.

Metal; Mettle

Gold is a heavy **metal**. I will pay you according to your **mettle**.

Notable; Notorious

Your book is a very **notable** one. He was a **notorious** swindler.

Observation; Observance

Her gift for **observation** was very remarkable. They did not attach much weight to **observance** of that kind.

Official; Officious

I am not speaking to you in my **official** capacity. He annoyed me by his **officious** manners.

Practise; Practice

Do you **practise** the piano every day ? I have given up this **practice** altogether.

Primary; Primitive

This is only a **primary** (elementary) schools. He follows his **primitive** instincts.

Principal; Principle

My **principal** (chief, employer) is Mr.Rajiv.

His **principal** object is to make money. The **principle** is a very sound one.

Punctual; Punctilious

You will always find me very **punctual**. He is **punctilious** (strictly observant of nice points) in his treatment of the matter.

Salutary; Salubrious

This taught him a **salutary** lesson. The air is very **salubrious** there.

Stationary; Stationery

The motor remained **stationary**. We want some **stationery** at our office.

Summons; Summon

I sent him a **summons**. I **summoned** him.

Track; Tract

We travelled out of the beaten **track**. They bought a large **tract** of land.

He presented me with his **tract** (small book).

Exercises

Q. 1

1. The prince wanted to ascent (A) ascend (B) to the throne
2. The doctor did not expect (A) except (B) the patient to die during the operation.
3. He delivered a speech on India shining with panacea (A) Panacea (B) panache
4. The accused swore the he did not steel (A) steal (B) the dead body of the saint
5. The customer could not gain accession (A) access (B) to his ATM account

(1) BABBA (2) ABBBA
(3) AAABB (4) AAABA
(5) BBABA

Q. 2

1. This situation will not effect (A) affect (B) the results.
2. He made several illusions(A)allusions (B) to the murder of the prime minister
3. There is little reason to altar (A) alter (B) my decision
4. The chairman remarks were not very (A) Apposite (B) opposite
5. A printer is an (A) Artisan (B) Artist

(1) ABBAB (2) BABBA
(3) BABAB (4) BBBAB
(5) BBABB

Q. 3

1. He accepted (A) excepted (B) my reason for being late
2. The ministers drank to access(A) excess(B)
3. That was a terrible accident (A) incident (B) in his life
4. We must adopt (A) adapt (B) ourselves to the situation.
5. They must adapt (A) Adapt (B) the child if it is to be saved

(1) ABABA (2) BABBA
(3) ABBBA (4) BBBBA
(5) BAABA

Q. 4

1. He used to be audited (A) addicted (B) to drinking but now he is devoted to his studies
2. He acted on his lawyers (A) advise (B) advice
3. The lawyer nodded in a/an assent (A) ascent (B) at 6.p.m
4. We will avenge (A) revenge (B) the helpless and poor people
5. He stepped on the breaks(A) brakes (B).

(1) BBAAB (2) ABBAB
(3) BAABA (4) BBABA
(5) BBAAA

Q. 5

1. My brother's vocation (A) vacation (B) is banking
2. These customs are a relics of barbarisms (A) barbarity (B)
3. He could not bare (A) bear (B) such hardship
4. Water ran down the creak (A) creek(B) rapidly
5. His acts were (A) beneficent (B) beneficial to all.

(1) AABBB (2) ABAAB
(3) BABAB (4) ABBAB
(5) BAABA

Q. 6

1. Kindly reserve a birth (A) berth (B) for me
2. Shakespeare was borne (A) born (B) in Stafford
3. The bridle (A) bridal (B) ceremony lasted for two hours
4. They canvas (A) canvass (B) strongly for their party.
5. There is a special cell (A) sell (B) for hard core criminals.

(1) ABABA (2) BBBBA
(3) AAAAB (4) BAABA
(5) BBABA

Q. 7

1. His manner was rather (A) ceremonious (B) ceremonial
2. The cession (A) cessation (B) of the territory was demanded by the British
3. The bill will be introduced in the current (A) cession (B) session
4. The teller was asked to cheque (A) check (B) his accounts
5. The mans attitude was too childish (A) childlike (B) for his age

(1) ABBBB (2) AABBA
(3) ABABA (4) ABBBA
(5) BBABB

Q. 8

1. The speaker will cite (A) site (B) many reasons for our failure
2. My work is the compliment (A) complement (B) of his
3. Her speech was scarcely comprehensible (A) comprehensive (B)
4. He spoke with baited bated(A) breath(B).
5. The young man made his uncle confidant (A) confident (B).

(1) ABBBA (2) ABABA
(3) BABBA (4) BAABA
(5) AAABA

Q. 9

1. I will keep your counsel (A) council (B) in mind
2. She was conscientious (A) conscious (B) of her faults
3. He is a contemptible (A) Contemptuous (B) chap
4. He worked continuously (A) Continually (B) from morn till night
5. Corporal (A) Corporeal (B) Punishment is forbidden in schools..

(1) ABBAA (2) BAAAA
(3) ABAAA (4) ABBBB
(5) BBABA

Q. 10

1. This coarse (A) Course (B) of action will eventually ruin us.
2. The rumour was to far fetched to be credible (A) creditable (B)
3. Drunkenness is not among his crimes (A) vices(B)
4. This custom habit (A) habitual (B) still exists among savages
5. The deceased (A) diseased (B) sheep were examined by the vet

(1) BAABA (2) ABBAA
(3) AAABB (4) BABAB
(5) BBBBA

Q. 11

1. His manner can hardly be called descent (A) decent (B)
2. The dissent (A) descent (B) among the party workers was noted by the Gen. Sec.
3. His essays were deficient (A) defective (B) in common sense
4. We are dependent (A) depended upon his support
5. The new car will depreciate (A) deprecate (B) in value soon

(1) BABAB (2) BABBA
(3) AAABA (4) BAAAA
(5) BAABA

Q. 12

1. A judge must be interested (A) disinterested (B) in a case.
2. If you sit in a draught (A) draft (B) you will catch a cold.
3. The boat slunk (A) sunk (B) in a river.
4. "Has the doctor found any efficient (A) efficacious (B) treatment for your complaint ?"
5. The problem is elemental (A) elementary (B).

(1) AAABB (2) BABAB
(3) AABAB (4) ABABB
(5) BBBBA

Q. 13

1. He failed to elicit (A) illicit any useful information.
2. Even women are illegible (A) eligible (B) for the post.
3. He is an emigrant (A) eminent (B) scientist
4. The building is insured (A) ensured (B) for Rs. 1 crore.
5. The irruption (A) eruption (B) of the Chinese into Tibet was condemned by most countries.

(1) BABBA (2) ABABA
(3) BBAAA (4) ABBAA
(5) AABBA

Q. 14

1. The holy man was notorious (A) famous (B) for his good deeds
2. Go and fetch (A) bring (B) a doctor
3. I have no love for official formalism (A) formality (B)
4. Tom can be recognized by his gate (A) gait (B)
5. The girls gambol (A) gamble (B) happily in the garden

(1) ABABA (2) BAABA
(3) BAAAB (4) BBABB
(5) BBAAA

Q. 15

1. They played a ghastly (A) ghostly (B) trick on him
2. To forgive an injury is godly (A) god like (B)
3. The host was graceful (A) gracious (B) to all his guests
4. He received an honourable (A) honorary (B) degree from Harvard
5. I believe in the human(A) humane (B) treatment of prisoners

(1) ABABA (2) BAABA
(3) BAAAB (4) BBABB
(5) AABBB

Q. 16

1. Humiliation (A) Humility (B) is a good virtue
2. A unicorn is an imaginary (A) imagery (B).
3. If a man is not industrious (A) industrial (B) he can hardly except to succeed
4. No one will deny that he is ingenuous (A) ingenious (B) and truthful
5. John was a Zealous (A) jealous (B) worker in the cause of education

(1) AAAAA (2) ABAAA
(3) BABAB (4) BBABA
(5) BBBBA

Q. 17

1. He made a judicial (A) judicious (B) selection of books.
2. The prisoners was set at library (A) liberty (B)
3. He was advised not to loose (A) lose (B) his temper
4. She was of a lovely (A) lovable (B) nature
5. My boss lead a luxurious (A) luxuriant (B) life

(1) BBBBA (2) ABABB
(3) BABBA (4) ABABB
(5) BABAA

Q. 18

1. A memorable (A) Memorial (B) was erected to the memory to the memory of Pt.Nehru
2. The grand event was a momentary (A) momentous (B) one
3. The servant was a negligible (A) negligent (B) of his duties
4. Ravi was a notable (A) notorious (B) officer in the indo pak war.
5. He is an official (A) Officious (B) person nobody like him.

(1) BABAB (2) ABBAB
(3) BBBAB (4) ABAAB
(5) AABBA

Q. 19

1. The doctor is a man of peaceable (A) peaceful (B) disposition
2. We know every tryst (A) twist (B) and turn of ranipur more.
3. He received a pitiful (A) pitiable (B)amount for all his labours
4. They took a tour of the back land (A) planes (B) plains.
5. India is a popular (A) populous (B) country

(1) BAAAB (2) AABAB
(3) ABBBA (4) ABBBB
(5) BBABA

Q. 20

1. Your plan is not practical (A) practicable (B)
2. The vulture is a bird of prey (A) pray (B)
3. Let us precede (A) proceed (B) with the lesson.
4. In the past history of Pakistan rulers have often proscribed (A) prescribed all religions other than their own.
5. I refuse to play cards on principle (A) principal (B)

(1) ABBAA (2) AABAA
(3) ABBBA (4) BBBAA
(5) BBBBA

Q. 21

1. He took refuse (A) refuge (B) in an Arabic state
2. They warned him not to temper (A) tamper (B) with the riddance.
3. Social (A) sociable (B) people do not like living alone.
4. The stationary (A) stationery (B) was laid out on the table
5. The monarch's scooter is the symbol of temporal (A) temporary (B) power

(1) ABBBB (2) BABAB
(3) BABBAA (4) BBBBA
(5) AABBA

Q. 22

1. His feelings were easily hurt as he was a sensitive (A) sensible (B) man
2. God's laws are highly spirituous (A) spiritual (B)
3. The statue (A) statute (B) of the lord is simple
4. The house we saw is still empty (A) vacant (B)
5. He was ordered to give a verbose (A) verbal (B) speech on education

(1) ABBBB (2) ABABA
(3) BBABB (4) BABAA
(5) ABBAB

Q. 23

1. Willing (A) wilful (B) waste make sad want
2. It is womanly (A) womanish (B) on a man's part to shed tears
3. He worked all one's (A) his (B) life to get his daughters married

4. Mahatma Gandhi tried his best for Hindu Muslim Union (A) unity (B)
5. The lord abides (A) abounds (B) in each one of us

(1) BABBA (2) BAAAB
(3) AABBA (4) ABBAB
(5) BBBAB

Q. 24

1. We shall ensure (A) Insure (B) that you get selected
2. The cook tried to prize (A) prise (B) open the container
3. The teacher asked the students to commit (A) commute (B) the lesson to memory
4. I abhor (A) adhere (B) people who are smart Alexis
5. Grapes are grown in vineyards (A) wine yards(B)

(1) ABBAA (2) AABAA
(3) ABAAA (4) BAAAB
(5) AABBA

Q. 25

1. Anybody who tress passes on this premises (A) premise (B) will be prosecuted
2. The prime minister along with his minister is (A) are (B) coming to attend the event
3. His wife who continuously quarrelled with him was the boon (A) bane (B) on his life
4. His neighbour was filled with jealousy (A) zealous (B).
5. He had to roam (A) room (B) about the town for several house before h found a suitable peace.

(1) BBAAA (2) AABBA
(3) ABABA (4) BABBA
(5) AABBA

Q. 26

1. There was a fierce dual (A) duel (B) between the brothers.
2. "Don't jump to conclusions" (A) Allusions (B) he advised his juniors
3. The police officers will alloy (A) allay (B) your fears.
4. The custom officials frisked (A) fleshed (B) the passengers
5. He has been working (A) is working (B) for 6 hours

(1) BABAA (2) BAABA
(3) BBBAA (4) ABABA
(5) AABBA

Q. 27

1. He mediates (A) meditates (B) in the action to be taken
2. A drawing man will catch (A) cling (B) on to a straw.
3. Search (A) examine (B) the thief for the jewellery.
4. The man died of a strike (A) stroke (B)
5. Her absence rued (A) ruined (B) his day

(1) BBAAA (2) ABBAA
(3) ABABB (4) BABBA
(5) AAABB

Q. 28

1. The side hero was simply a substitute (A) substance (B) for the main hers.
2. A breed (A) brood (B) of pigeons were placed in a pen.
3. A gaggle (A) giggle (B) of geese waddled along the banks of the rivers.
4. I have no opinion (A) option (B)but to resign.
5. The government has decided to wave (A) waive (B) bad loans

(1) BABBA (2) AAABA
(3) AABBA (4) BBABA
(5) ABABB

Q. 29

1. He stepped on (A) into (B) the hall from the bedroom
2. The guests dined (A) dinner (B) on pies & wine
3. The Gupta's live adjunct (A) adjacent (B) to the kumar's
4. There were three different causes (A) clauses (B) in the contract
5. The basis on which his arguments are based is sheer conjecture (A) conjunction (B)

(1) BABBA (2) BABAA
(3) BAAAB (4) ABBBA
(5) AABBA

Q. 30

1. The convict did not revel (A) reveal (B) his plans to escape from prison.
2. The orator had a bass (A) base (B) voice
3. Full of happiness his face looked beatific (A) beauty (B)
4. From the passage we can infer (A) inform (B) that the author dislike corruption.
5. Columbus voiced his dissent (A) descent (B) on the contentions issue.

(1) BBBAB (2) BAAAA
(3) AABAB (4) ABAAA
(5) BABAA

Answer

1. (1) BABBA	2. (4) BBBAB	3. (3) ABBBA
4. (1) BBAAB	5. (1) AABBB	6. (2) BBBBA
7. (4) ABBBA	8. (1) ABBBA	9. (3) ABAAA
10. (4) BABAB	11. (4)BAAAA	12. (3) AABAB
13. (4) ABBAA	14. (2) BAABA	15. (5) AABBB
16. (1) AAAAA	17. (1)BBBBA	18. (3) BBBAB
19. (4) ABBBB	20. (2) AABAA	21. (4) BBBBA
22. (1) ABBBB	23. (1) BABBA	24. (3) ABAAA
25. (2) AABAA	26. (1) BABAA	27. (3) ABABB
28. (5) ABABB	29. (1)BABBA	30. (2) BAAAA

PART–III : LOGIC

ONE DAY CAPSULE OF REASONING

Part A
Logical Reasoning

1. Analogy

Analogy test : Analogy literally means similar features. Question on analogy, test the ability of a candidate to understand the relationship between two given objects and apply the same relationship to find that asked in the question. It must be borne in mind that a candidate's intellectual skills is important to analyse the similarity between two or more objects, yet a rich knowledge of usage of different words adds to one's performance. This type of question cover all types of relationship that one can think of. There are many ways of establishing a relationship, some of the most common ones are given here.

S.No.	Type of Relationship	Example
1.	Cause and Effect	Fast : Hunger Mosquito : Malaria
2.	Subset of Set	Soldier : Regiment Student : Class
3.	Quantity and Unit	Area : Hectare Energy : joule
4.	Instrument and Measurement	Odometer : Speed Lactometer : Milk
5.	Worker and Tools	Author : Pen Carpenter : Saw
6.	Gender or Sex Relationship	Cow : Bull Man : Women
7.	Word and Synonym	Miracle : Surprise Muddy : Unclean
8.	Word and Antonym	Black : White Kind : Cruel
9.	Worker and Working place	Lawyer : Court Secretary : Office
10.	Study and Terminology	Numismatic : Coin Paleontology : Fossil
11.	Product and Raw Material	Grape : Wine Pulp : Paper
12.	Worker and Product	Author : Book Painter : Painting
13.	Association Relationship	Dance : Dancer Melt : Liquid
14.	Product and Quality	Diamond : Hard Rubber : Soft
15.	Symbolic Relationship	Star : Rank Flag : Nation
16.	Numeric Operational Relationship	3 : 9 5 : 125
17.	Place and Famous	Haridwar : Ganga Delhi : Red fort
18.	Stages Relationship	Girl : Women Calf : Cow
19.	Limit Definition	Red : Blood Green : Sea or Military
20.	Habitual Relationship	Lion : Carnivorous Cow : Herbivorous
21.	Individual and Group	Sailor : Crew Singer : Chorus
22.	Class and Member	Sonnet : Poem Mammal : Animal
23.	Degree of Intensity	Cool : Cold Warm : Hot
24.	Time Sequence Relationship	Day : Night Winter : Summer
25.	Functional Relationship	Scissors : Cloth Axe : Wood
26.	Word Relationship	Wash : Face : : Sweep : Floor Rain : Cloud : : Smoke : Fire
27.	Relationship of Purpose	Anchor : Ship Hook : Fish
28.	Part and Whole	Skin : Body Tyre : Bus
29.	Action and Object	Kick : Football Eat : Food
30.	Sound and Object	Knock : Door Ring : Telephone
31.	Alphabet Relationship	ABC : ZYX CBA : XYZ

There are many ways in which two words can have a relationship. Some of them—commonest are discussed above.

Following example will help students to understand the pattern of such questions and also methods to solve them.

1. Product and Quality

Example : 'Food' is related to 'Grain' as 'Cloth' is related to—

(A) Cotton (B) Thread
(C) Texture (D) Polyester

Solution : The above question is based on Product and Quality relationship. 'Grain' determine the quality of 'Food' and 'Texture' determine the quality of 'Cloth'.

2. Cause and Effect

Example : 'Goiter' is related to 'Iodine' as 'Anaemia' is related to—

(A) Vitamin (B) Blood
(C) Iron (D) Weakness

Solution : The above question is based on cause and effect relationship. 'Goiter' disease is caused by the deficiency of ' Iodine' then 'Anaemia' will be related to 'Iron' as 'Anaemia' disease is caused by the deficiency of 'Iron'. Hence our answer is (C).

3. Subset of Set

Example : ' Question' is related to 'Question Hour' as 'Argument' is related to—

(A) Lecture (B) Argument Hour
(C) Debate (D) Skirmish

Solution : 'Question' is a part of 'Question Hour' and in the same way we see that 'Argument' is a part of 'Debate'. Hence, our answer is (C).

4. Quantity and Unit

Example : 'Current' is related to 'Ampere' in the same way as 'Weight' is related to—

(A) Scale (B) Pound
(C) Commodity (D) Measurement

Solution : 'Current' is measured in terms of 'Ampere' and 'Weight' is measured in terms of 'Pound'. Hence, our answer is (B).

5. Instrument and Measurement

Example : 'Seismograph' is related to 'Earthquakes' in the same way as ' Thermometer' is related to—

(A) Fever (B) Doctor
(C) Temperature (D) Mercury

Solution : Relationship given in the question is the relationship between Instrument and Measurement. 'Seismograph' measures 'Earthquakes' and 'Thermometer' measures Temperature. So our answer is (C).

6. Worker and Tools

Example : 'Doctor' is related to 'stethoscope' in the same way as 'Painter' is related to—

(A) Painting
(B) Brush
(C) Exhibition
(D) Art

Solution : 'Stethoscope' is used by the 'Doctor' as a tool to perform his work. Similarly a 'painter' uses a 'Brush' as a tool to perform his work. Hence our answer is (B).

7. Gender or Sex Relationship

Example : 'Bull' is related to 'Cow' in the same way as ' Horse' is related to—

(A) Animal
(B) Mare
(C) Stable
(D) Meat

Solution : The relationship in question is a male-female relationship. So, 'Horse' is related to 'Mare'. Hence our answer is (B).

8. Word and Synonym

Example : 'Mad' is related to 'Insane' in the same way as 'Slim' is related to :

(A) Thin (B) Healthy
(C) Sexy (D) Timid

Solution : 'Mad' is synonym of 'Insane'. In the same way 'Slim' is a word nearest in meaning to the word 'Thin'. Therefore our answer is (A).

9. Word and Antonym

Example : 'Hate' is related to 'Love' in the same way as 'Create' is related to—

(A) Make (B) Renovate
(C) Destroy (D) Build

Solution : 'Love is just opposite to 'Hate'. So the word opposite in meaning to 'Create' is 'Destroy'. Therefore, our answer is (C).

10. Worker and Working place

Example : 'Sailor' is related to 'Ship' in the same way as 'Lawyer' is related to—

(A) Legal (B) Law
(C) Court (D) Ruling

Solution : The place of work of 'Sailor' is 'Ship'. Similarly the place where 'Lawyer' works is 'Court'. So, the answer is (C).

11. Study and Terminology

Example : 'Mycology' is related to 'Fungi' in the same way as 'Vexillology' is related to—

(A) Earth (B) Soil
(C) Flag (D) Stones

Solution : Study of 'Fungi' is known as 'Mycology'. 'Vexillology' is the study of 'Flag'. Hence, the answer is (C).

12. Product and Raw Material

Example : 'Shoe' is related to 'Leather' in the same way as 'Rubber' is related to—

(A) Plastic (B) Polythene

(C) Latex (D) Chappal

Solution : Leather is a raw material used to make 'Shoes'. Similarly, 'Rubber' is made using 'Latex' as raw material. Therefore, our answer is (C).

13. Worker and Product

Example : ' Carpenter' is related to 'Furniture' in the same way as 'Blacksmith' is related to—

(A) Gold (B) Jewellery

(C) Shoes (D) Metal

Solution : 'Carpenter' makes 'Furniture'. Similarly 'Blacksmith' makes 'Metal'. So our answer is (D).

14. Association Relationship

Example: 'Melt' is related to 'Liquid' in the same way as 'Freeze' is related to—

(A) Ice (B) Crystal

(C) Water (D) Cubes

Solution : The term 'Melt' is associated with 'Liquid' because after melting the ice we obtain liquid. Similarly the state of 'Water' after freezing is 'Ice'. Hence our answer is (A).

15. Miscellaneous Relationship

Example : 'Telephone' is related to 'Ring' in the same way 'Door' is related to—

(A) Wood (B) Key

(C) Open (D) Knock

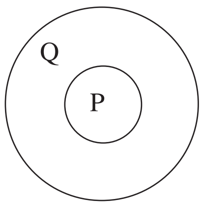
Solution : The term 'Ring' is associated with 'Telephone' and 'Knock' is related with 'Door'.

2. Logical Diagram

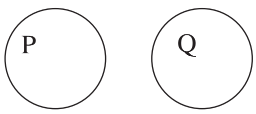
Logical Diagrams are an extension of the venn diagram concept.

Logical Diagrams for denoting propositions are namely A, E, I and O types.

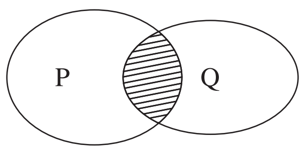
A—Proposition : An 'A' Proposition is of the type 'All Ps are Qs'. Hence the diagram of A Proposition is denoting in the following way.



E—Proposition : E Proposition is of the form , 'No Ps are Qs.' It can be drawn as :

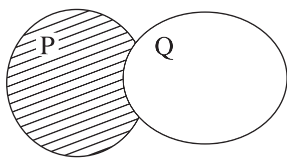


I—Proposition : I Proposition is of the form, 'Some Ps are Qs'. Hence there is some part called intersection, is drawn .



O—Preposition : The denoting of O Proposition is slightly tricky. 'Some Ps are not Qs'.

Here we obviously know that there are some Ps which are definitely not Qs. But we do not know with certainly about the remaining Ps. They might or might not be Qs. Hence the information given by an O Proposition is incomplete and therefore the figure drawn to denote an O proposition is also incomplete like following—



The dashed section of the circle denoting P is the part which denoting Ps which are not known to be Qs or non-Qs.

3. Ranking

This topic deals with the questions related with comparison of ranks. The term ranks may include various objects such as age, height, weight, marks, salary, %, etc.

Fixed Ranking : In the fixed ranking of objects, we get the position of ranks as fixed.

Variable Ranking : In varying ranking the positions of any one of the ranks keeps varying. The variable ranking is the consequences of inadequate information given in the questions. Students are therefore, required to be cautious while answering the questions those are basically designed to confuse the students, so be attentive.

Following example will help students to understand the pattern of such questions and also methods to solve them.

Example 1 : Read the information carefully and answer the questions based on it.

Five persons are sitting in a row. One of the two person at the extreme ends is intelligent and other one is fair. A fat person is sitting to the right of a weak person. A tall person is sitting to the left of the weak person and the weak person is sitting between the intelligent and fat person.

1. Tall person is at which place counting from right ?
   (A) First (B) Second
   (C) Third (D) Fourth
   (E) Cannot be determined
2. Person to the left of weak possess which of the following characteristics ?
   (A) Intelligent (B) Fat
   (C) Fair (D) Tall
   (E) Cannot be determined
3. Which of the following persons is sitting at the centre ?
   (A) Intelligent (B) Fat
   (C) Fair (D) Weak
   (E) Tall

Solution : First information given in the question that one of the two persons at the extreme ends is intelligent and other one is fair, suggest as shown in **Fig. (1) and (2).**

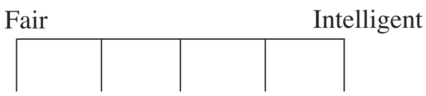


Fig. 1

Information that a tall person is sitting to the left of fair person rules out the possibility of **Fig. (1)** as no person in **Fig. (1)** can sit to the left of fair person. Therefore, only **Fig. (2)** shows the correct position of intelligent and fair persons.

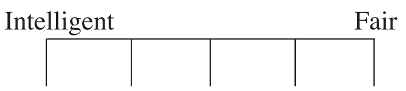


Fig. 2

Now, rest of the information regarding the position of other persons can easily be inserted. The final ranking of their sitting arrangement is as shown in **Fig. (3).**

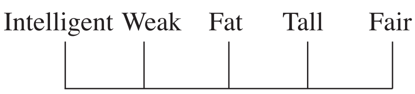


Fig. 3

Example 2 : Six persons are sitting in a circle facing the centre of the circle. Parikh is between Babita and Narendra. Asha is between Chitra and Pankaj. Chitra is to the immediate left of Babita. Who is to the immediate right of Babita ?

(A) Parikh
(B) Pankaj
(C) Narendra
(D) Chitra
(E) None of these

Solution : On the basis of two informations that Parikh is between Babita and Narendra and Asha is between Chitra and Pankaj, we can determine the exact position of persons sitting adjacent to Asha and Parikh. However, the last information that Chitra is to the immediate left of Babita fixes the position of all the six persons as the **Fig. (4)**. On the basis of which we can determine that Parikh is sitting to the immediate right of Babita. So, our answer is (A).

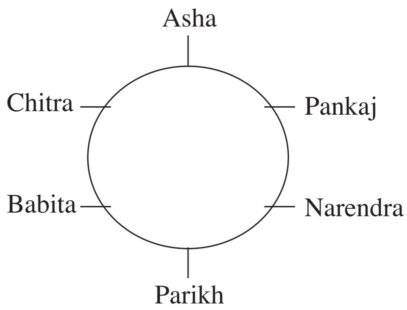


Fig. 4

Example 3 : Four men A, B, C and D and four women W, X, Y and Z are sitting round a table facing each other.

(i) No two men and women are sitting together.
(ii) W is to the right of B.
(iii) Y is facing X and is to the left of A.
(iv) C is to the right of Z.

Who are the two persons sitting adjacent to D ?

(A) W and Y (B) X and W
(C) X and Z (D) W and Z
(E) Can't be determined

Solution : Figure given here represent the exact position of all the eight persons. The sitting arrangement fulfills all the conditions given in the question. We observe from here that D is sitting between W and Y. Hence, our answer is (A).

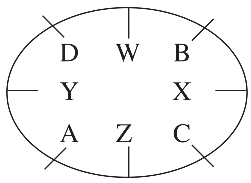


Fig. 5

4. Calendar Test

The solar year consist of 365 days, 5 hours and 8 minutes.

Some Useful Points

Calendar

1. In an ordinary year, there are 52 weeks.

2. In an Year, 52 weeks and 1 odd day mean $52 \times 7 = 364$ and plus $1 = 365$.

The number of days more than the complete weeks in a given period, are called odd days.

For Example : 2 odd days in 9 days and 3 odd days in 10 days.

And 2 odd days in leap year (367 days)

3. In a century = 76 ordinary year + 24 leap year and in century there are 5 odd days.

4. 100 years contain 5 odd days, 200 years contain 3 odd days, 300 years contain 1 odd day and 400 years contain 0 odd day. Hence, the years 400, 800,1200,1600 etc. have no odd days.

Clock

1. In every hour, both the hand coincides once.
2. In every hour, the hands are at right angles 2 times.
3. In every hour, the hands are in opposite directions once.
4. In every 12 hours, the hands are coinciding 11 times.
5. In every 12 hours, the hands of clock are in opposite directions 11 times.
6. In every 12 hours, the hands of clock are right angles 22 times.
7. In a day, the hands are coinciding 22 times.
8. In a day, the hands are at right angle 44 times.

More

1. One hour number division = 30° apart
2. One minute division = 6° apart
3. In one minute, the minute hand moves 6°
4. In one minute, the hour hand moves $\left(\frac{1}{2}\right)^\circ$
5. In one minute, the minute hand gain $\left(5\frac{1}{2}\right)^\circ$ more than hour hand
6. In one hour, the minute hand gains 55 minutes divisions over the hour hand.
7. If both hands coincide, then they will again coincide after $65\frac{5}{11}$ minutes. *i.e.* in correct clock, both hand coincide at an interval of $65\frac{5}{11}$ minutes.
8. If two hands coincide in time less than $65\frac{5}{11}$ minutes, then clock is too fast and if the two hands coincide in time more than $65\frac{5}{11}$ minutes, then the clock is too slow.
9. If both hands coincide at an interval t minutes and $t < 65\frac{5}{11}$, then total time gained $= \left(\frac{65\frac{5}{11} - t}{t}\right)$ minutes and clock is said to be fast.
10. If both hands coincide at an interval t minutes and $t > 65\frac{5}{11}$, then total time lost $= \left(\frac{t - 65\frac{5}{11}}{t}\right)$ minutes and clock is said to be slow.

5. Blood Relation Test

Problems on blood relations involve analysis of information showing blood relationship among members of a family. In the question, a chain of relationships is given in the form of information and on the basis of these informations, relation between any two members of the chain is asked from the candidate. Candidates are supposed to be familiar with the knowledge of different relationship in a family. Some examples to illustrate the pattern of such question are given below :

For the easy understanding of the candidates, a table containing few main relations is given hereunder. The study of the table will prove to be very useful for the students in solving question in blood relations.

1. Grandfather's son : Father or uncle.
2. Grandmother's son : Father or uncle.
3. Grandfather's only son : Father.
4. Grandmother's only son : Father.
5. Mother's or Father's Mother : Grandmother.
6. Mother's or Father's Father : Grandfather.
7. Grandfather's only daughter-in-law : Mother.
8. Grandmother's only daughter-in-law : Mother.
9. Mother's or Father's son : Brother.
10. Mother's or Father's daughter : Sister.
11. Mother's or Father's brother : Uncle.

12. Mother's or Father's Sister : Aunt.
13. Husband's or Wife's Sister: Sister-in-law.
14. Husband's or Wife's Brother : Brother-in-law.
15. Son's Wife : Daughter-in-law.
16. Daughter's Husband : Son-in-law.
17. Brother's son : Nephew.
18. Brother's daughter : Niece.
19. Uncle or Aunt's son or daughter : Cousin.
20. Sister's husband : Brother-in-law.
21. Brother's wife : Sister-in-law.

6. Odd One Out (Classification)

Odd one out is a process of grouping various objects on the basis of their common properties. Odd one out is a kind of Classification in which we make a homogeneous group from heterogeneous groups. Questions on odd one out or classification are designed to test candidate's ability to classify given object and find one which does not share the common property with the other objects of group. Question on Odd One Out can be asked in any form. Some of them have been given below :

1. Words Odd One Out : In this type of classification, different objects are classified on the basis of common feature/properties—name, place, uses, situations, origin, etc.

2. Alphabet Odd One Out : In this type, alphabet are classified in a group using a particular method or rule. Rules or method used for such classification are often simple and hence can easily be understood.

3. Miscellaneous Odd One Out : In this type of classification, any rule other than described above can be used for classification or grouping. Questions on such pattern do not necessarily use the alphabet and words. Here the numerics and other mathematical symbols can also be used.

All the possible classification have been illustrated in the following examples :

Following examples will help students to understand the pattern of such questions and also methods to solve them.

Directions : In each of the following questions, a group of five items is given. Four of them share the common features whereas one of them is different from other. Choose the item which is different from the other.

Example 1 : Four of the following five are alike in a certain way and so form a group. Which one does not belong to that group?

(A) Ears (B) Hands
(C) Finger (D) Eyes
(E) Legs

Solution : Except finger, all other parts of body are in pair. Hence option (C) is the correct answer.

Example 2 : Four of the following five are alike in a certain way and so form a group. Which one does not belong to that group?

(A) Bud (B) Branch
(C) Leaf (D) Root
(E) Plant

Solution : All items are the parts of a plant. Hence plant does not belong to the group. So the answer is (E).

Example 3 : Find the odd-one out.

(A) PSRQ (B) CGEF
(C) JMLK (D) VYXW

Solution : The pattern used for classification is placement of alphabets in the order (+ 3, – 1, – 1). Since option (B) does not follow the pattern, it is odd in the group.

Example 4 : Four out of the five pairs of number have the same relationship. Find the odd-one out.

(A) 4 : 63 (B) 1 : 0
(C) 5 : 124 (D) 2 : 15
(E) 3 : 26

Solution : In the above classification, second number is one less than the cubes of the first number. Option (D) does not belong to the group, as it does not follow the pattern.

Example 5 : Find the odd-one out.

(A) DEHG (B) RSVU
(C) XYBA (D) LMQP
(E) JKNM

Solution : Method used for the classification is placement of alphabet in the order (+ 1, + 3, – 1). Option (D) LMQP does not follow the method. Hence it is odd-one-out.

Example 6 : Find the odd-one out.

(A) 32 : 15 (b) 86 : 42
(C) 56 : 26 (D) 74 : 36
(E) 38 : 18

Solution : Second no. is one less than the half of first number. So option (C) is our answer.

7. Number and Alphabetical Series

Number Series : In this test, a few numbers are given according to a definite rule and one is asked to work out the next number according to that rule.

There are innumerable ways in which a series could be generated and it is impossible to even think of let alone explain all of them here.

In these questions, a number series is given and candidates are asked to either insert a missing number or find the one that does not follow the pattern of the series. Series can be of constant differences or of constant multiples or of squares or of square roots or can be mixed series. The only thing to be understood for solving these question is the pattern, on which a number series is written. A number series can be formed by using various methods. Nevertheless there are certain standard methods of generating a series which can directly or indirectly help in solving problems of this type. These are discussed below.

Type 1

Direction (1–5) : Find out the missing numbers :

1. 2, 9, 28, 65, ……………
   (A) 121 (B) 195
   (C) 126 (D) 103
   (E) 96

2. 2, 6, 14, 26 ………….., 62
   (A) 52 (B) 54
   (C) 44 (D) 42
   (E) 50

3. 101, 100,……………, 87, 71, 46
   (A) 92 (B) 88
   (C) 89 (D) 96
   (E) 99

4. 100, 50, 52, 26, 28,……………., 16, 8
   (A) 30 (B) 36
   (C) 14 (D) 32
   (E) 12

5. 4, 9, 20, 43,…………
   (A) 133 (B) 84
   (C) 96 (D) 95
   (E) None of these

Solution : 1 (C) Method used to form the series is

$(1)^3 + 1, (2)^3 +1, (4)^3 + 1$ …………

Therefore the missing number is $(5)^3 + 1 = 126$

Solution 2 : (D)

$$\begin{array}{ccccccccccc} 2 & & 6 & & 14 & & 26 & & 42 & & 62 \\ \hline & 4 & & 8 & & 12 & & 16 & & 20 & \end{array}$$

The different of each successive number is increased by 4. Hence number 42 will fill up the space.

Solution 3 : (D)

$$\begin{array}{ccccccccccc} 101 & & 100 & & 96 & & 87 & & 71 & & 46 \\ \hline & (1)^2 & & (2)^2 & & (3)^2 & & (4)^2 & & (5)^2 & \end{array}$$

Difference of each successive number is the square of natural number.

Solution 4 : (C) the second number is half of the first number, fourth number is half of the third number and so on.

Solution 5 : (A) The series follows the method :

$4 \times 2 + 1 \;=\; 9, 9 \times 2 + 2 = 20, 20 \times 2 + 3 = 43,$

$43 \times 3 + 4 \;=\; 133.$

Type 2

Direction : (6–10) : In each of the following question one number is wrong in the series . Find out the wrong number :

6. 864, 420, 200, 96, 40, 16, 6
   (A) 420 (B) 200
   (C) 96 (D) 40
   (E) 16

7. 1, 2, 6, 21, 84, 445, 2676
   (A) 2 (B) 6
   (C) 21 (D) 84
   (E) 445

8. 88, 54, 28, 13, 5, 2, 2, 2
   (A) 28 (B) 54
   (C) 13 (D) 2
   (E) 88

9. 4, 12, 30, 68, 146, 302, 622
   (A) 12 (B) 30
   (C) 68 (D) 146
   (E) 302

10. 3, 6, 9, 22, 5, 67·5, 236·25, 945
    (A) 6 (B) 9
    (C) 22·5 (D) 67·5
    (E) 236·25

Solution 6 : (C) Pattern of the series from end follows the rule $6 \times 2 + 4 = 16$, $16 \times 2 + 8 = 40$, $40 \times 2 + 12 = 92$, $92 \times 2 + 16 = 200$,…….. and so on. Therefore the number should be 92.

Solution 7 : (D) Series follows the pattern : $1 \times 1 + 1 = 2, 2 \times 2 + 2 = 6, 6 \times 3 + 3 = 21, 21 \times 4 + 4 = 88, 88 \times 5 + 5 = 445, 445 \times 6 + 6 = 2676$. Therefore, 84 should be replaced by 88.

Solution 8 : (B) Series moves from the end with a difference of 0, 3, 8, 15, 24, 35 *i.e.* with a difference of numbers which are one less than the square of natural numbers. Hence number 54 should be replaced by 53.

Solution 9 : (E) The series is written using the pattern : $4 \times 2 + 4 = 12, 12 \times 2 + 6 = 30, 30 \times 2 + 8 = 68, 68 \times 2 + 10 = 146, 146 \times 2 + 12 = 304, 304 \times 2 + 14 = 662$. Therefore, number 302 should be replaced by 304.

Solution 10 : (A) Pattern of the series is $3 \times 1{\cdot}5 = 4{\cdot}5, 4{\cdot}5 \times 2 = 9, 9 \times 2{\cdot}5 = 22{\cdot}5, 22{\cdot}5 \times 3 = 67{\cdot}5, 67{\cdot}5 \times 3{\cdot}5 = 236{\cdot}25$.................. Therefore, number 6 should be replaced by 4·5.

Alphabatical Series

In the question involving alphabetical series, a set of letters is written four or five times with blank spaces in between. The series follows a specific pattern and students are required to find out the letters which should come in place of the missing spaces. Pattern on which a series is written, is not well defined and hence can follow any method which an examiner can think of. Thus the alphabetical series may look like any of the following series :

abcde, bcdea, cdeab, deabc...........

Here in the first repetition of the sequence abcde, the first letter is taken to the end of the sequence which then becomes bcdea. Using the same procedure the next repetition yield the sequence cdeab..............

abcde, abcee, abeee, aeeee................

Here the successive letters towards the end of the series are replaced by e, the last letter of each sequence.

uvwxy, yuvwx, xyuvw...............

Here the last letter in the previous set is made the first letter of the next set and so on.

Following examples will make the students understand the method as how to approach question based on alphabetical series.

Example 1 : What are missing letters in the following series ?

Pqr—rs prs—spq–

(A) pqppq (B) spqpr

(C) sqpqr (D) ssqprq

Solution : Here the block which is repeated, consists of four letters pqrs. Rewriting the series in the block of four letters, we get pqrs, ...rsp, rs..., spq... and now filling the block space, we observe that the series must be sqpqr. Thus (C) is the correct option.

Example 2 : What are the last five missing letters in the following series ?

b–b–cab–baca–cba–a–

(A) acbbb (B) acbcb

(C) abcbc (D) accbc

Solution : The longest available chain of letters in the given series is baca. Two other such blocks are formed if we fill up the fourth space from right, so that the series now looks like.

b–bacab–baca–cbaca–

The filled up sequence now, suggest that the series will be bcba ca bcba ca bcba ca b

Hence, the missing letters are acbcb. Thus (B) is the right option.

Example 3 : Find the missing number letters in the following series ?

Uv–uww—xu–vv–

(A) vwxuu (B) vuvuv

(C) uvxuv (D) vuxuu

Solution : The presence of the term uww and then the letter next to w, which is x, in the ninth position (the blocks are of three letters) suggests the series to be uvv, uww, uxx,uuvv, u, so the missing letters are vuxuu and hence (D) is the correct alternative.

Example 4 : Find the last five missing letters of the series :

a...aabb...ab...b......

(A) baaaa (B) babab

(C) baaba (D) bbaaa

Solution : The presence of the term aabb in the middle suggests the form of the series to be ab, aabb, followed by aaabbb, aaaa. Thus (A) is the correct option.

Example 5 : Find the missing letters in the following series **:**

Adb–ac–da–addcb–dbc–cbda

(A) bccba

(B) cbbaa

(C) ccbba

(D) bbcad

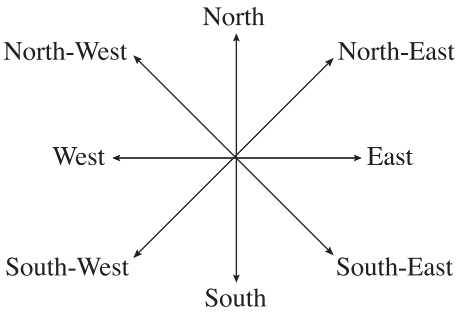
Solution : In the above series, the letters are equidistant from the beginning and end.

Adbcac b da b cddcb a dbc a cbda

Hence the missing letters are cbbaa. Therefore the correct option is (B).

8. Direction Sense

These questions are designed to test candidate's ability to sense direction. Questions on direction are, simpler than other questions, if student possess the right knowledge of the direction. Confusion is created in the question by giving frequent right and left turns to a specific direction. Students are, therefore, advise to use the diagram as given in the figure for the purpose of sensing direction.



There are two basic ideas that one should know before attempting these questions.

I. If one stands with his face towards NORTH his right hand towards EAST, his left hand points towards WEST and his back towards SOUTH.

II. Pythagoreas Theorem : Students must have done this theorem in their schools. It gives us a formula to measure the Hypotenuse of a right angled triangle if the other two sides are known.

$$H^2 = B^2 + P^2$$

Direction : Study the information and answer the questions given below :

On a playing ground Dev, Nilesh, Ankur, and Pintu, are standing as directed below facing the North :

(i) Kumar is 40 meters to the right of Ankur.

(ii) Dev is 30 meter to the South of the Kumar.

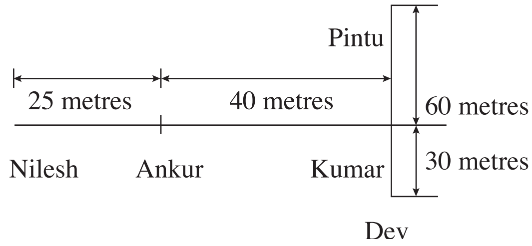
(iii) Nilesh is 25 meter to the West of the Ankur.

(iv) Pintu is 90 meter to the North of the Dev.

Example 1 : Who is the North-East of the person, who is to the left of Kumar ?

(A) Nilesh (B) Ankur
(C) Dev (D) Either Nilesh
(E) None of these

Solution : From Fig. 2 it is very clear that the person to the left to Kumar is Ankur and Pintu is to the North-East of Ankur. Therefore, Pintu is our answer. But none of the options contain Pintu. Hence, option (E) is our answer.



Example 2 : If a boy walks from Nilesh, meets Ankur followed by Kumar, Dev and then Pintu, how many metres has he walked if he has travelled the straight distance all through ?

(A) 215 metres (B) 155 metres
(C) 245 metres (D) 185 metres
(E) None of these

Solution : Following the instructions as given in the question the total distance covered by the person = 25 + 40 + 30 + 90 = 185 metres. Hence option (D) is correct answer.

Example 3 : Ankit started walking towards North. After waking 30 meters, he turned towards left and walked 40 meters. He then turned left and walked 30 meters. He again turned left and walked 50 meters. How far is he from his original position ?

(A) 50 metres (B) 40 metres
(C) 30 metres (D) 20 metres
(E) None of these

Solution : The final position of Ankit is E and starting point is A. Therefore, he is only 10 metre away from his starting point. Hence, our answer is the option (E).

Example 4 : Lakshman went 15 km. to the west from his house, then turned left and walked 20 km. He then turned East and walked 25 km. and finally turning left covered 20 km. How far is he now from his house?

(A) 15 km (B) 20 km
(C) 25 km (D) 10 km
(E) None of these

Solution : Points A and E shows the starting and end positions respectively of Lakshman. It is clear that E is 10 km away from A. Hence option (D) is the correct answer.

9. Coding and Decoding

In this type of test, secret messages or words have to be deciphered or decoded. They are coded according to a definite pattern or rule which should be identified first.

The term coding-decoding primarily relates with messages sent in secret form which cannot be understood by others easily. Coding, therefore, means rule or method used to hide the actual meaning of a word or group of words and decoding means the method of making out the actual message that is disguised in coding.

In question, word (Basic word) is coded in a particular way and candidates are asked to code other words in the same way. Question of coding-decoding are

designed to test candidate's ability to understand the rule used for the coding and then translate it quickly to find out the coding for the given word. Types of these questions are manifold which initially pose a slight problem before the students as to how to solve the question. It is therefore, required to discuss first, before we switch over to the methods or steps used in solving these questions.

As a matter of facts, there exists no uniform and particular type or category of these questions according to which we could classify question of coding-decoding. However, keeping in view the candidates convenience we have classified different types of question with illustrations and explanations under different heads.

Category I : In this category of question, a word is coded by simply changing the order of letter of the word.

Example 1 : In a code language if TRAINS is coded as RTIASN, how PISTOL will be coded in the same language ?

(A) SITLOP (B) IPSTLO

(C) SIPTLO (D) IPTSLO

Solution : If we compare the basic word {TRAINS} with the coded word {RTIANS}, we would see that the letters used in the word are same as in the basic word but their order of placement has been changed. Letter T at first position of basic word has been placed at second position in the coded word & letter R at second position has been placed at the first position.

It means that in this question, letters of the basic word have been interchanged *i.e.* first letter with second, third with the fourth and so on. And thus we get the coded word. In this code language word, PISTOL will be coded as IPTSLO. Hence, option (D) is our answer.

Category II : In this category of questions, letters of a word are substituted for either a new letter or a numeric. And the same substitution helps to find out the coding of the word in question. This substitution of letters may either be direct or in a jumbled up fashion. We shall discuss each type in detail in the following paragraphs.

(A) Direct Substitution

Example 2 : In a code language, if SUGAR is coded as PKLTN and TEA is coded as QGT, how would you code GREAT in the same code language?

(A) ENGTP (B) LNGTK

(C) LNGTQ (D) LNGQT

Solution :

Basic Word	Coded Word	Basic Word	Coded Word
S U G A R	P K L T N	T E A	Q G T

Here we see that S is substituted for P, U for K, G for L, A for T and R for N. And in the word TEA, T is substituted for Q, E for G and A for T. Now we find that coding for letter A in SUGAR is T and coding for letter A in TEA is also T. This implies that substitution of letter in coded word takes place in the same order as in basic word. In simple words, in the word SUGAR letter S in the basic word has been substituted for a new letter and coded at first place in the coded word, U at the second place, G at the third place and so on.

When letter of a word are substituted for the new letters and are placed in the coded word at the same position as in the basic word then this method of substitution is called direct substitution.

Therefore, code for GREAT will be LNGTQ. Hence, option (C) will be our answer.

Example 3 : In a code language if TEARS is coded as VWXYZ and MAN is coded as 123 then how would you code RESENTMENT in that language ?

(A) YWZW3V1W3Y (B) YWZ3WV1W3Y

(C) YWZW3V1WY3 (D) YWZW3V1W3V

Solution : This question is of direct substitution method. Letters of the basic words are substituted as under.

Basic Word : T E A R S
↓ ↓ ↓ ↓ ↓
Coded Word : V W X Y Z

Basic Word : M A N
↓ ↓ ↓
Coded Word : 1 2 3

Therefore, the coding for the word RESENTMENT will be YWZW3V1W3V. Hence option (D) is our answer.

Example 4 : If TEARS is coded as VWXYZ, then how would you code BEAS in that language ?

(A) YWXZ (B) VWXZ

(C) MWXZ (D) WVZY

Solution : Basic Word : T E A R S
↓ ↓ ↓ ↓ ↓
Coded Word : V W X Y Z

Using the same coding pattern for BEAS, we get W for E, X for A & Z for S but code for B cannot be determined as it is absent in the basic word. Now shall take the help of options. In option (A) we get W, X, Z for E, A & S respectively which is correct but coding for B cannot be coded for R. Likewise in option (B), V can not be coded for B as it is the code for T. In option (C) we see that code for B is M which seems to be correct as it has not been used as a code for any of the letters in basic word. Hence, code for BEAS would be MWXZ. Therefore, option (C) is our answer.

(B) Substitution in Mixed up Fashion

Rule applied in this type of questions follows the same pattern as used in type (A)—**Direct substitution** but with slight modification. Here also in this type each letter of basic word is substituted for a new letter, but it

may not necessarily be placed in the same position in coded word as it occupies in the basic word. In simple words, it implies that coded letter at first position in basic word does not necessarily occupy the first position in the coded word. It can be placed at any place in the coded word.

Example 5 : In a code language, if BEAT is coded as 5642 and SWEET is coded as 66912, how would you code TEASE ?

(A) 96162 (B) 56264

(C) 96625 (D) 55296

Solution :

Basic Word	Coded Word
B E A T	5 6 4 2
Basic Word	Coded Word
S W E E T	6 6 9 1 2

It is very clear from the coding pattern used in the above question that substitution for the letter has not been used directly. Code for E seems to be 6 in BEAT whereas it carries code 9 and 1 in the word SWEET which apparently appears to be incorrect because in substitution method same letter caries same code even if it appears in different words.

Now we see that in word SWEET letter E appears twice and in its coding numeric 6 appears twice. Thus it is confirmed that code for E is 6. **It may be concluded from the coding pattern used here that letters of the words BEAT and SWEET have been substituted for numeric in jumbled up from.** Word BEAT and SWEET share common letters E and T and their coding share common numeric 6 and 2. Therefore, E is coded as 6 and T is coded as 2. It means that the code for TEASE will definitely have 6, 6 & 2 for E , E and T. Code for A and S has yet to be found out.

Now look at the word BEAT and SWEET again, we see that code for A in word BEAT should be either 5 or 4 (leaving 6 & 2 for E and T). Similarly, code for S should be either 9 or 1 (leaving 6 & 2 for E and T). We shall not take the help of option to answer the code for TEASE.

(a) It cannot be the coded for TEASE as code for A (5 or 4) it cannot available.

(b) It cannot be the coded for TEASE as code for S (9 or 1) is not available.

(c) It may be the coded for TEASE as besides 662 code for EE and T we get 5 probable code for A and 9 probable code for S. Hence option (C) is our answer.

Category III : In this category, rule of coding is of entirely different type from what has been discussed so far in category I and II. In this type words are coded, replacing each letter of the word by a new word from the alphabets (A…Z) on the basis of a particular method. The 26 letters of alphabets are split into two equal groups in the following manner :

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
26	25	24	23	22	21	20	19	18	17	16	15	14
A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**
Z	**Y**	**X**	**W**	**V**	**U**	**T**	**S**	**R**	**Q**	**P**	**O**	**N**
1	2	3	4	5	6	7	8	9	10	11	12	13
26	**25**	**24**	**23**	**22**	**21**	**20**	**19**	**18**	**17**	**16**	**15**	**14**

Now A can be coded Z or 1or 26, B can be coded as Y or 2 or 25. It has further been explained in the following examples.

Example 6 : If the TEMPLE is coded as VHQURL, how would you code CHURCH ?

(A) EKYWIO (B) EKUWIO

(C) EKYWIN (D) EKYWJO

Solution :

T E M P L E V H Q U R L

Code for T is V; for E, it is H, for M it is Q. It may be noticed from here that letters of TEMPLE have been replaced by new letters from the alphabets.

There is a gap of one letter between T and V, gap of two letters between E and H, gap of three letters between M and Q and so on.

Therefore, coding for CHURCH is

C	H	U	R	C	H
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6
E	K	Y	W	I	O

Hence (A) is our answer.

Example 7 : If TEMPLE is coded as VHQNIA, how would you code CHRUCH ?

(A) EKYWI (B) EKYQZD

(C) EKYPZD (D) EKYQWD

Solution : Coding of TEMPLE is

T	E	M	P	L	E
+ 1	+ 2	+ 3	– 1	– 2	– 3
V	H	Q	N	I	A

Here the first half of the letters of word TEMPLE have been replaced with new letter from the alphabet with a gap of 1, 2 and 3 letters respectively in forward direction, and second half of the letters of the word have been replaced by new letters from the alphabets with a gap of 1, 2, and 3 in backward directions . Hence code for word CHRUCH would be EKYPZD. Hence, (C) is our answer.

If the students expose themselves to the variety of questions appearing in different magazines, then they can solve these problems with relatively little efforts. We have endeavoured here to incorporate maximum types of question in the following exercise taking into consideration the existing pattern of competitive exams.

In the following exercise, question of all types have been mixed up and given in one exercise instead of giving them separately category-wise. It has been to enable the student to use their mind to find as to what rule will be applied for each question.

Part B

Data Interpretation

All about the Data Interpretation

Graphs, tables and charts etc., that display data so that they are easier to understand, are all examples of descriptive statistics.

The Data Interpretation is a part of statistics. The word statistics comes from the Italian word STATISTA (meaning 'statisman') : IT WAS FIRST USED BY Goffried Achewall (1719–1772), a professor at Marlborough and Gottingen. Dr. EAW Zimmerman introduced the word statistics into England. Its use was popularized by Sir John Sinclair in his work Statistical Account of Scotland 1791–1799. Long before the eighteenth century, however, people had been recording and using data.

Every 5 years, Indians suffer through an affliction known as the assembly election, television, radio and newspaper broadcasts inform us that "a poll conducted by channel opinion research shows that which party candidate has the support of 50 + percent of voters and which party is losing their seat in the election. Can we rely on the truth (so called) of what they reported? Who has actually done the polling ? How many people did they interview and how many should they have interviewed to make this ascertain ? Polling is a big business in India and many channels conduct polls for political candidates, new products and even TV shows. If you have an ambition to become manager, minister, run a company, or even a star in a TV show, you need to know something about Data Interpretation.

What Skill is required for solving the Data Interpretation

No mathematics beyond simple algebra is required. If you felt reasonable and comfortable when you finished your high school algebra course, you have enough background to understand everything in this DI part of this book. Nothing beyond basic algebra is assumed or used. My goals are for you to be comfortable as you learn and for you to get a good intuitive grasp of statistical concepts and techniques. As a future manager, you will need to know when statistics can help your decision process and which tools to use.

What is the Data Interpretation

Data are collections of any number of related observa-tions. A collection of data is called a Data Set and a single observation a Data Point.

Data can come from actual observations or from records that are kept for normal purposes. Data can assist decision makers' in educated guesses about the causes and therefore the probable effected of certain characteristics in given situations. Also, knowledge of trends from past experience can enable concerned citizens to be aware of potential outcomes and to plan in advance.

Data are not necessarily information and having more data doesn't necessarily produce better decisions. The goal is to summarize and present data in useful ways to enable us to see quickly some of the characteristics of the data, we have collected. When data are arranged in compact, usable forms, then decision makers (reliable manager) use it to make intelligent decisions. This act of organising and getting meaningful information, is called Data Interpretation.

If you know the directions in advance, it will help to save your time, Read them.

Your allotted time is limited for paper. No additional time is given for reading instructions. If you spend a minute or two reading directions, you are losing points because you could be spending more time analyzing the questions. The solution of this problem is to be throughly familiar with the directions for each question type before you go to take the exam.

X-Ray of DI Questions

It is very necessary for you to know about what is in it and how to perform. For that this book provides you X-Ray of the DI QUESTIONS. Every question of DI consume your time by two ways—one is to make you confuse by its typical Foggy Language and one is to absorb time by long or difficult calculation and one thing is very clear that calculation are generally tricky, instead of difficult.

Some very important STEPS to Improve Your Calculation Speed.

Memorise the following things :

1. Memorise Squares upto 35.

Square of Numbers upto 35

1	1	**6**	36	**11**	121	**16**	256	**21**	441	**26**	676	**31**	961
2	4	**7**	49	**12**	144	**17**	289	**22**	484	**27**	729	**32**	1024
3	9	**8**	64	**13**	169	**18**	324	**23**	529	**28**	784	**33**	1089
4	16	**9**	81	**14**	196	**19**	361	**24**	576	**29**	841	**34**	1156
5	25	**10**	100	**15**	225	**20**	400	**25**	625	**30**	900	**35**	1225

2. Memorise Cubs upto 25.

Cube of Numbers upto 25

1	1	**6**	216	**11**	1331	**16**	4096	**21**	9261
2	8	**7**	343	**12**	1728	**17**	4913	**22**	10648
3	27	**8**	512	**13**	2197	**18**	5832	**23**	12167
4	64	**9**	729	**14**	2744	**19**	6859	**24**	13824
5	125	**10**	1000	**15**	3375	**20**	8000	**25**	15625

3. Memorise Tables upto 19.

2	**3**	4	**5**	6	**7**	8	**9**	10	**11**	12	**13**	14	**15**	16	**17**	18	**19**
4	**6**	8	**10**	12	**14**	16	**18**	20	**22**	24	**26**	28	**30**	32	**34**	36	**38**
6	**9**	12	**15**	18	**21**	24	**27**	30	**33**	36	**39**	42	**45**	48	**51**	54	**57**
8	**12**	16	**20**	24	**28**	32	**36**	40	**44**	48	**52**	56	**60**	64	**68**	72	**76**
10	**15**	20	**25**	30	**35**	40	**45**	50	**55**	60	**65**	70	**75**	80	**85**	90	**95**
12	**18**	24	**30**	36	**42**	48	**54**	60	**66**	72	**78**	84	**90**	96	**102**	108	**114**
14	**21**	28	**35**	42	**49**	56	**63**	70	**77**	84	**91**	98	**105**	112	**119**	126	**133**
16	**24**	32	**40**	48	**56**	64	**72**	80	**88**	96	**104**	112	**120**	128	**136**	144	**152**
18	**27**	36	**45**	54	**63**	72	**81**	90	**99**	108	**117**	126	**135**	144	**153**	162	**171**
20	**30**	40	**50**	60	**70**	80	**90**	100	**110**	120	**130**	140	**150**	160	**170**	180	**190**

4. Most important conversions of percen-tages of fraction are—

(A) Dividing 100% into 10 equal parts.

1. $\frac{10}{10} = 100\%$
2. $\frac{9}{10} = 90\%$
3. $\frac{8}{10} = \frac{4}{5} = 80\%$
4. $\frac{7}{10} = 70\%$
5. $\frac{6}{10} = \frac{3}{5} = 100\%$
6. $\frac{5}{10} = \frac{1}{2} = 50\%$
7. $\frac{4}{10} = \frac{2}{5} = 40\%$
8. $\frac{3}{10} = 30\%$
9. $\frac{2}{10} = \frac{1}{5} = 20\%$
10. $\frac{1}{10} = 10\%$

Extra Shot : $5\% = \frac{1}{20}$

(B) Dividing 100% into 8 equal parts.

1. $\frac{8}{8} = 100\%$
2. $\frac{7}{8} = 87\frac{1}{2}\%$
3. $\frac{6}{8} = \frac{3}{4} = 75\%$
4. $\frac{5}{8} = 62\frac{1}{2}\%$
5. $\frac{4}{8} = \frac{1}{2} = 50\%$
6. $\frac{3}{8} = 37\frac{1}{2}\%$
7. $\frac{2}{8} = \frac{1}{4} = 25\%$
8. $\frac{1}{8} = 12\frac{1}{2}\%$

Extra Shot : $6\frac{1}{4}\% = \frac{1}{16}$

(C) Dividing 100% into 6 equal parts.

1. $\frac{6}{6} = 100\%$
2. $\frac{5}{6} = 83\frac{1}{3}\%$
3. $\frac{4}{6} = \frac{2}{3} = 66\frac{2}{3}\%$
4. $\frac{3}{6} = \frac{1}{2} = 50\%$
5. $\frac{2}{6} = \frac{1}{3} = 33\frac{1}{3}\%$
6. $\frac{1}{6} = 16\frac{2}{3}\%$

Extra Shot : $8\frac{1}{3}\% = \frac{1}{12}$

(D) Non-Conventional Percentages.

1. $11\frac{1}{9}\% = \frac{1}{9}$
2. $9\frac{1}{11}\% = \frac{1}{11}$

Practice Calculations with time limits

These Tests are designed to help you to check and improve your level of calculation. All calculations of this part has been taken from previous tests of reputed MBA's entrance exams.

Calculation Test One

Total Ques. : 27 **Total Time : 25 Min. 40 sec**

1. $480000 \times \frac{12}{100} + 600000 \times \frac{20}{100} + 400000 \times \frac{10}{100} =$ 1 min 15 sec
2. $\frac{2370}{45} =$ 25 sec
3. $\frac{2000 \times \frac{75}{100} + 4000 \times \frac{50}{100} + 3000 \times \frac{50}{100} + 8000 \times \frac{20}{100}}{4} =$ 48 sec
4. $\frac{70 + 60 + 45 + 40 + 80 + 75 + 55 + 35 + 90 + 65 + 60 + 35}{4 \times 3} =$ 1 min 29 sec
5. $2340 - 240 + 270 =$ 48 sec
6. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} =$ 51 sec
7. $\frac{2^2 \times 3^2 \times 2^4 \times 3^5 \times 5^2 \times 7^2}{2^3 \times 3^4 \times 5^1 \times 2^4 \times 3^2 \times 5^2} =$ 1 min 5 sec
8. $\frac{2}{3} \times 114 - \frac{3}{4} \times 68 =$ 35 sec
9. $\frac{17}{3} \times \frac{19}{6} + \frac{7}{3} =$ 1 min 5 sec
10. $3189 - (9 + 180 + 2700) =$ 25 sec
11. $9 + 90 + 900 + 75 =$ 15 sec
12. $\frac{2}{3} \times \frac{3}{6} + \frac{4}{6} \times \frac{3}{6} + \frac{3}{6} \times \frac{2}{6} =$ 1 min 45 sec
13. $\frac{80}{\frac{60}{40} + \frac{20}{20}} =$ 25 sec
14. $\frac{275 \times 18}{132 \times 5} =$ 40 sec
15. $100 \times 0{\cdot}09 \times 0{\cdot}88 \times 0{\cdot}85 =$ 1 min 10 sec
16. $\frac{1{\cdot}25 \times 4 \times 90000}{5 \times 90000 \times 50000} =$ 45 sec
17. $50000 + 10000 \times 10\% + 90000 \times 20\% + 20000 \times 30\% =$ 30 sec
18. $\frac{24 \times 10 \times 18}{6 \times 10} =$ 28 sec
19. $\frac{1}{12} + \frac{1}{15} - \frac{1}{20} =$ 25 sec
20. $5{\cdot}5 \times 4 \times 405 \times 2500 \times 6 =$ 1 min 55 sec
21. $\frac{133650000}{24000} =$ 2 min 10 sec
22. $72 \times 25 \times \frac{5}{18} =$ 20 sec
23. $\frac{726 \times 60}{8250} =$ 1 min 40 sec
24. $\frac{10 \times 4 \times 270}{6} =$ 20 sec
25. $\frac{2 \times 50}{\frac{5}{6} + \frac{5}{4}} =$ 35 sec
26. $\left(2 \times \frac{22}{7} \times \frac{7}{2} + 2 \times \frac{22}{7} \times \frac{14}{2}\right) \times 10 =$ 30 sec
27. $\frac{600}{374} \times 4 =$ 1 min 45 sec

Calculation Test Two

Total Ques. : 27 **Total Time : 35 Min. 00 Sec.**

1. $\frac{4}{9} \times \frac{7}{15} + \frac{5}{9} \times \frac{7}{15} =$ 1 min 50 sec
2. $\frac{150 \times 30 - 135 + 165}{30} =$ 35 sec
3. $\frac{1200}{\frac{4000}{7{\cdot}5} + \frac{4000}{8} + \frac{4000}{8{\cdot}5}} =$ 2 min 55 sec
4. $\frac{125 + 262}{30} =$ 1 min 20 sec
5. $100000 - 10 \times 9 \times 8 \times 7 \times 6 =$ 1 min 5 sec
6. $3 \times \frac{7 \times 6}{2 \times 1} + 3 \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1} =$ 25 sec
7. $\frac{282 - 32}{40} =$ 20 sec

Solutions : Calculation Test One

1	217600	**6**	$\frac{127}{128} = 0{\cdot}99$	**11**	1074	**16**	0·9	**21**	5568·75	**26**	660
2	52·66	**7**	14·7	**12**	$\frac{5}{6} = 0{\cdot}83$	**17**	75000	**22**	500	**27**	6·42
3	1650	**8**	25	**13**	32	**18**	72	**23**	5·28		
4	59·16	**9**	$\frac{365}{18} = 20{\cdot}27$	**14**	7·5	**19**	0·1	**24**	1800		
5	2370	**10**	300	**15**	67·32	**20**	133650000	**25**	48		

8. $6 \times 10{\cdot}5 + 6 \times 11{\cdot}4 - 11 \times 10{\cdot}9 =$ 1 min 45 sec

9. $40{\cdot}2 \times 4 + 41{\cdot}3 \times 4 - 40{\cdot}6 \times 7 =$ 1 min 30 sec

10. $\frac{1}{5} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{3} =$ 30 sec

11. $\frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1} =$ 1 min 30 sec

12. $1 \times 12 \times 9 + \left(\frac{1}{2} \times 12 \times 3 \times 9\right) =$ 45 sec

13. $42 \times 35 + 2 \times \frac{1}{2} \times \frac{22}{7} \times (21)^2$

$+ 2 \times \frac{1}{2} \times \frac{22}{7} \times (17{\cdot}5)^2 =$ 2 min 30 sec

14. $\frac{1}{3} \times \frac{22}{7} \times 49 \times 51 =$ 1 min 30 sec

15. $\frac{1}{3} \times \frac{22}{7} \times 45\,(28^2 + 7^2 + 28 \times 7) =$ 2 min 10 sec

16. $196 - \frac{22}{7} \times 49 =$ 20sec

17. $\frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 7 =$ 55 sec

18. $\frac{1}{4} \times \frac{22}{7} \times 21 \times 21 =$ 40 sec

19. $\frac{2}{3} \times \frac{22}{7} \times 27{\cdot}125 =$ 2 min 20 sec

20. $\frac{6\,(3^3 + 4^2 + 5^2)}{6\,(6)^2} =$ 1 min 10 sec

21. $\frac{22}{7} \times (24{\cdot}5 + 21)(24{\cdot}5 - 21) =$ 1 min 15 sec

22. $\frac{(2{\cdot}25)^2 \times 10}{(0{\cdot}75)^2 \times 0{\cdot}2} =$ 1 min 30 sec

23. $\frac{2}{3} \times \frac{22}{7}\,(4{\cdot}5^3 - 4^3) =$ 2 min 40 sec

24. $80 \times 10 + 60 \times 10 - 10 \times 10 =$ 15 sec

25. $(47)^2 + 2 \times 1008 =$ 40 sec

26. $20\,(4 + 39 \times 4) =$ 1 min 30 sec

27. $\left(\frac{15 \times 16}{2}\right)^2 - \left(\frac{15 \times 16}{2}\right) =$ 45 sec

Calculation Test Three

Total Ques. : 25 **Total Time : 10 Min. 20 Sec.**

1. $40 + 60 + 35 + 40 + 15 + 10 =$ 6 sec

2. $40 \times 1 + 60 \times 2 + 35 \times 3 + 40 \times 4$
$+ 15 \times 5 + 10 \times 6 =$ 35 sec

3. $200 - (15 + 25 + 20 + 20 + 10 + 5) =$ 10 sec

4. $(35 - 20) + (40 - 20) + (15 - 10) + (10 - 5) =$ 9 sec

5. $\frac{10}{46} \times 100 =$ 57 sec

6. $\frac{14}{64} \times 100 =$ 1 min 2 sec

7. $\frac{18}{71} \times 100 =$ 55 sec

8. $\frac{20}{60} \times 100 =$ 7 sec

9. $\frac{9-3}{9} \times 100 =$ 14 sec

10. $\frac{8-3}{3} \times 100 =$ 16 sec

11. $4 + 8 + 9 + 10 + 11 + 13 =$ 16 sec

12. $10 + 14 + 18 + 20 + 25 + 30 =$ 14 sec

13. $7 + 9 + 5 + 3 + 6 + 8 =$ 9 sec

14. $8 + 12 + 12 + 11 + 15 + 13 =$ 12 sec

15. $\frac{(125 - 105)}{105} \times 100 =$ 26 sec

16. $200 + 150 + 180 + 195 + 220 =$ 14 sec

17. $\frac{130}{78} =$ 50 sec

18. $\frac{220}{200} =$ 6 sec

19. $\frac{135}{90} =$ 13 sec

20. $\frac{80}{65} =$ 40 sec

21. $\frac{3}{50} =$ 16 sec

Solutions : Calculation Test Two

1	$\frac{7}{15} = 0{\cdot}46$	**6**	168	**11**	0·008	**16**	42	**21**	500·5	**26**	3200
2	151	**7**	6·25	**12**	270	**17**	89·83	**22**	450	**27**	14280
3	7·9	**8**	11·5	**13**	3818·5	**18**	346·5	**23**	56·83		
4	12·9	**9**	41·8	**14**	2618	**19**	56·83	**24**	1300		
5	69760	**10**	0·4	**15**	48510	**20**	1·88	**25**	4225		

22. $\frac{2}{45} =$ 16 sec

23. $\frac{25}{40} =$ 14 sec

24. $\frac{25}{48} =$ 54 sec

25. $\frac{58{\cdot}5}{54} \times 100 =$ 52 sec

12. $\frac{1401}{3119} \times 100 =$ 2 min

13. $\frac{428}{2432} \times 100 =$ 2 min 2 sec

14. $\frac{308}{1522} \times 100 =$ 1 min 6 sec

Solutions : Calculation Test Three

1	200	**6**	21·87	**11**	55	**16**	945	**21**	0·06
2	560	**7**	25·35	**12**	117	**17**	1·67	**22**	0·04
3	105	**8**	33·33	**13**	38	**18**	1·1	**23**	0·625
4	45	**9**	66·66	**14**	71	**19**	1·5	**24**	0·52
5	21·73	**10**	166·67	**15**	19·04	**20**	1·23	**25**	108·33

Calculation Test Four

Total Ques. : 25 **Total Time : 30 Min. 50 Sec.**

1. $\frac{3}{35} \times 100$ 56 sec

2. $\frac{0{\cdot}55}{20} \times 100 =$ 10 sec

3. $\frac{2}{45} \times 100 =$ 25 sec

4. $\frac{4}{75} \times 100 =$ 27 sec

5. $\frac{4}{58} \times 100 =$ 46 sec

6. $\frac{4741}{4136} =$ 50 sec

7. $\frac{109292 - 97500}{97500} \times 100 =$ 1 min 45 sec

8. $\frac{97500 - 39303}{39303} \times 100 =$ 4 min

9. $\frac{39426 - 39303}{39303} \times 100 =$ 1 min 13 sec

10. $\frac{372}{5933} \times 100 =$ 1 min 29 sec

11. $\frac{811}{4730} \times 100 =$ 2 min 7 sec

15. $\frac{117}{933} \times 100 =$ 1 min 28 sec

16. $\frac{22108 - 18669}{18669} =$ 2 min 4 sec

17. $\frac{14308 + 16188 + 16798 + 19145}{4} =$ 2 min 2 sec

18. $\frac{4240 + 4010 + 4160 + 3700 + 3930}{5} =$ 35 sec

19. $\frac{90{\cdot}50}{367{\cdot}50} =$ 1 min 7 sec

20. $\frac{18{\cdot}50 + 15 + 16{\cdot}5 + 14{\cdot}5 + 50}{5} =$ 42 sec

21. $\frac{15 - 11{\cdot}50}{11{\cdot}50} \times 100 =$ 59 sec

22. $\frac{50 - 14{\cdot}5}{14{\cdot}5} \times 100 =$ 1 min 41 sec

23. $\frac{48 - 36}{36} \times 100 =$ 24 sec

24. $\frac{14 + 46 + 58}{400} \times 100 =$ 20 sec

25. $\frac{48 + 22 + 6}{400} \times 100 =$ 13 sec

Solutions : Calculation Test Four

1	8·57	**6**	1·51	**11**	17·14	**16**	0·18	**21**	30·43
2	2·75	**7**	12·09	**12**	44·91	**17**	16609·75	**22**	244·82
3	4·44	**8**	148·07	**13**	17·59	**18**	4008	**23**	33·33
4	5·33	**9**	0·312	**14**	20·23	**19**	0·24	**24**	29·5
5	6·89	**10**	6·27	**15**	12·54	**20**	22·9	**25**	19

Calculation Test Five

Total Ques. : 25 **Total Time : 30 Min. 50 Sec.**

1. $\frac{58 + 76 + 68 + 62 + 48}{400} \times 100 =$ 45 sec
2. $\frac{526 + 620 + 674 + 717 + 681}{5439} =$ 2 min 51 sec
3. $\frac{14 + 46 + 58}{62 + 48 + 22 + 6} =$ 1 min 12 sec
4. $\frac{717 + 681 + 612 + 540 + 517}{5439} \times 100 =$ 2 min 40 sec
5. $\frac{517}{5439} \times 100 =$ 1 min 41 sec
6. $\frac{33659 - 16613}{16613} \times 100 =$ 2 min 33 sec
7. $\frac{31795 - 18537}{18537} \times 100 \times \frac{1}{4} =$ 4 min 6 sec
8. $\frac{201036}{232992} \times 100 =$ 3 min 8 sec
9. $\frac{16613}{6 \cdot 1} \times 100 =$ 2 min 17 sec
10. $\frac{8 \cdot 191 - 6 \cdot 23}{6 \cdot 23} \times \frac{100}{10} =$ 56 sec
11. $\frac{57 \cdot 75}{185 \cdot 5} \times 100 =$ 1 min 8 sec
12. $\frac{1 \cdot 79 + 1 \cdot 50}{13 \cdot 02 + 8 \cdot 44} \times 100 =$ 1 min 45 sec
13. $39 \cdot 7 \times 1 \cdot 3\% + 35 \cdot 7 \times 3\% + 11 \cdot 3 \times 0 \cdot 3\% + 10 \cdot 2 \times 1 \cdot 3\% + 36 \cdot 1 \times 12 \cdot 8\% + 30 \cdot 3 \times 30 \cdot 3\% + 11 \cdot 7 \times 7\% + 10 \cdot 5 \times 13 \cdot 1\% =$ 5 min 44 sec
14. $12 \cdot 5 + 14 \cdot 56 + 1 \cdot 72 + 1 \cdot 9 + 3 \cdot 83 + 4 \cdot 24 + 0 \cdot 68 + 0 \cdot 56 =$ 1 min 5 sec
15. $\frac{39 \cdot 99}{185 \cdot 5} \times 100 =$ 1 min 50 sec
16. $\frac{483 - 388}{483} \times 100 =$ 1 min 45 sec
17. $\frac{656 - 582}{656} \times 100 =$ 1 min 9 sec
18. $\frac{506 - 447}{506} \times 100 =$ 57 sec
19. $\frac{32 - 25}{32} \times 100 =$ 57 sec
20. $\frac{112 - 64}{64} \times 100 =$ 20 sec
21. $\frac{5 + 9 + 74 + 8 + 4}{5} =$ 21 sec
22. $12 + 23 + 6 + 6 + 6 =$ 10 sec
23. $\frac{35}{53} \times 100 =$ 41 sec
24. $25 \times 12 \times 12650 =$ 18 sec
25. $(13500 - 12500) \times 20 \times 12 =$ 9 sec

Calculation Test Six

Total Ques. : 25 **Total Time : 23 Min. 10 Sec.**

1. $15 \times 12 \times 10300 =$ 24 sec
2. $10 \times 12 \times 19400 =$ 17 sec
3. $\frac{211}{577} \times 100 =$ 1 min 32 sec
4. $\frac{340 - 140}{140} \times 100 =$ 20 sec
5. $\frac{351 - 150}{150} \times 100 =$ 15 sec
6. $\frac{350 - 156}{156} \times 100 =$ 1 min 21 sec
7. $\frac{14400}{185000} \times 100 =$ 1 min 21 sec
8. $\frac{30 - 25}{30} \times 100 =$ 7 sec
9. $\frac{25 - 22}{25} \times 100 =$ 5 sec
10. $\frac{22 - 20}{22} \times 100 =$ 7 sec
11. $\frac{20 - 15}{20} \times 100 =$ 4 sec
12. $\frac{70}{203} \times 100 =$ 33 sec
13. $\frac{85}{118} \times 100 =$ 36 sec
14. $\frac{216}{408} \times 100 =$ 1 min 5 sec
15. $\frac{227}{97} =$ 39 sec

Solutions : Calculation Test Five

1	78	**6**	102·6	**11**	31·13	**16**	19·69	**21**	20
2	59·16	**7**	17·88	**12**	15·33	**17**	11·28	**22**	53
3	0·85	**8**	86·28	**13**	17·74	**18**	11·66	**23**	66·03
4	56·40	**9**	272344·26	**14**	39·99	**19**	21·87	**24**	3795000
5	9·5	**10**	3·14	**15**	21·55	**20**	75	**25**	240000

16. $\frac{16149 - 15308}{15308} \times 100 =$ 1 min 51 sec

17. $\frac{24941 - 19474}{19474} \times 100$ 1 min 41 sec

18. $\frac{6057 - 4123}{4123} \times 100 =$ 1 min 51 sec

19. $\frac{0{\cdot}58}{3{\cdot}01} \times 100 =$ 1 min 21 sec

20. $\frac{0{\cdot}60}{3{\cdot}06} \times 100 =$ 1 min 11 sec

21. $\frac{0{\cdot}55}{2{\cdot}05} \times 100 =$ 1 min 31 sec

22. $\frac{0{\cdot}54}{1{\cdot}70} \times 100 =$ 1 min 27 sec

23. $(13500 \times 20 \times 12) - (12500 \times 20 \times 12) =$ 58 sec

24. $\frac{170000 - 120000}{120000} \times 100 =$ 1 min 13 sec

25. $\frac{1275200 - 956400}{956400} \times 100$ 2 min 21 sec

13. $\frac{180 - 120}{180} \times 100$ 10 sec

14. $(250 + 140 + 80 + 60)$ 30 sec

15. $\frac{68718 - 42137}{42137} \times 100$ 1 min 45 sec

16. $\frac{82175 - 65303}{65303} \times 100$ 45 sec

17. $\frac{20177 - 8820}{8820} \times 100$ 40 sec

18. $0{\cdot}23 \times 25800$ 30 sec

19. $\frac{4928}{2240}$ 25 sec

20. $\frac{1}{3} \times 17472$ 20 sec

21. $\frac{8514 - 5824}{5824}$ 25 sec

22. $(20 + 30 + 30 + 10 + 10) \times 1000 \times 1{\cdot}5$ 25 sec

23. $\frac{79}{173} \times 100$ 1 min

Solutions : Calculation Test Six

1	1854000	**6**	124·35	**11**	25	**16**	5·49	**21**	26·82
2	2328000	**7**	7·78	**12**	34·48	**17**	28·07	**22**	31·76
3	36·57	**8**	16·67	**13**	72·03	**18**	46·9	**23**	240000
4	142·85	**9**	12	**14**	52·94	**19**	19·26	**24**	41·66
5	134	**10**	9·09	**15**	2·34	**20**	19·6	**25**	33·33

Calculation Test Seven

Total Ques. : 48 **Total Time : 30 Min. 00 Sec.**

1. $\frac{1}{10} \times 100$ 1 sec

2. $\frac{1{\cdot}5}{20} \times 100$ 2sec

3. $\frac{2}{33{\cdot}3} \times 100$ 30 sec

4. $\frac{22}{30}$ 30 sec

5. $6 \times 5 \times 20$ 35 sec

6. $600 \times 600 \times 120 \times 120$ 37 sec

7. $2 \times (7 \times 6 \times 20) + 2 \times (3 \times 6 \times 10)$ 26 sec

8. $2 \times (6 \times 6 \times 20) + 2 \times (4 \times 6 \times 10)$ 18 sec

9. $2 \times (8 \times 6 \times 20)$ 15 sec

10. $2 \times (6 \times 6 \times 25) + 2\,(4 \times 6 \times 12{\cdot}5)$
$- 6 \times (4 \times 5 \times 2 + 6 \times 5 \times 2)$ 1 min

11. $(1440 + 2440 + 1800 + 4320)$ 20 sec

12. $(180 + 130 + 70 + 40)$ 22 sec

24. $\frac{173 \times 169}{169}$ 25 sec

25. $173\left(1 + \frac{2{\cdot}36}{100}\right)$ 55 sec

26. $\left(\frac{103 \times 100}{20}\right)$ 20 sec

27. $\frac{32{\cdot}5 - 32}{32{\cdot}5}$ 10 sec

28. $\frac{245}{290} \times 100$ 50 sec

29. $\frac{400}{535} \times 100$ 30 sec

30. $(80 + 140 + 70 + 245 + 400)$ 20 sec

31. $1{\cdot}5\,(30000 \times 30\% + 30000 \times 30\%$
$+ 10000 \times 40\% + 10000 \times 90\%)$ 30 sec

32. $\frac{790}{6435}$ 20 sec

33. $\frac{9810 - 5450}{5450} \times 100$ 25 sec

34. $\frac{6380 + 6390 + 6440}{3}$ 20 sec

35. $(815\cdot2 + 632\cdot4 + 2065\cdot8 + 1232\cdot7)$ 45 sec

36. $\frac{8352 - 7081}{7081} \times 100$ 1 min 30 sec

37. $\frac{986 - 745}{745} \times 100$ 1 min 55 sec

38. $\frac{100}{64\cdot8} \times 100$ 35 sec

39. $\frac{2\cdot97}{76\cdot5} \times 100 - 2\cdot97$ 30 sec

40. $\frac{100}{61\cdot3} \times 11\cdot6$ 50 sec

41. $\frac{42\cdot2}{132\cdot8} \times 100$ 1 min 30 sec

42. $\frac{120 + 130 + 145 + 165 + 185 + 200 + 220}{7}$ 30 sec

43. $(70 + 70 \times 16\cdot66\%)$ 20 sec

44. $(260 \times 98 \times 243)$ 1 min

45. $\frac{34\cdot54}{40\cdot73}$ 1 min

46. $(2520 + 4485 + 6760 + 25480 + 38478 + 174240)$ 1 min 10 sec

47. $\frac{43 \times 915\cdot7}{149}$ 1 min 45 sec

48. $\frac{211600 - 18300}{18300} \times 100$ 1 min 45 sec

4. $\left(597\cdot19 \times \frac{149}{131}\right)$ 2·40 sec

5. $\left(1750 \times \frac{15}{20}\right)$ 30 sec

6. $\frac{415 + 432 + 441 + 451}{4}$ 20 sec

7. $\frac{472 + 468 + 478 + 470}{4}$ 20 sec

8. $\frac{440 + 427 + 439 + 446}{4}$ 17 sec

9. $\frac{15 + 25 + 20 - 30 + 15}{330 + 290 + 90 + 260 + 45}$ 30 sec

10. $\frac{5}{(34 + 36)}$ 31 sec

11. $(320 + 0\cdot93 + 160 \times 0\cdot37)$ 50 sec

12. $1282\cdot6 + 522\cdot5 + [250 \times (1\cdot1)^3]$ 55 sec

13. $\frac{[(4 \times 5\cdot26)] + (3 \times 14\cdot28) + (3 \times 20)}{(4 + 3 + 3)}$ 50 sec

14. $\frac{(4 \times 5\cdot26 \times 1\cdot05) + (3 \times 14\cdot28 \times 1\cdot01)(3 \times 20 \times 1\cdot1)}{(4 + 3 + 3)}$ 1·50 sec

15. $\frac{400 - (280 + 70)}{20}$ 50 sec

16. $\frac{680 - (476 + 70)}{34}$ 30 sec

17. $\frac{800 - (580 + 70)}{45}$ 20 sec

18. $\frac{600 - (420 + 70)}{30}$ 40 sec

Solutions : Calculation Test Seven

1	10	**11**	10000	**21**	0·46	**31**	46500	**41**	31·77
2	7·5	**12**	420	**22**	150000	**32**	0·12	**42**	166·42
3	600	**13**	33·33	**23**	45·66	**33**	80	**43**	81·66
4	0·73	**14**	530	**24**	0·02	**34**	6403·33	**44**	619164
5	600	**15**	63·08	**25**	176·46	**35**	4746·1	**45**	0·84
6	1440	**16**	25·83	**26**	515	**36**	17·94	**46**	251963
7	2040	**17**	128·76	**27**	0·01	**37**	32·34	**47**	264·26
8	1920	**18**	5934	**28**	84·48	**38**	2·53	**48**	1056·28
9	1920	**19**	0·22	**29**	74·76	**39**	0·91		
10	1800	**20**	5824	**30**	935	**40**	18·92		

Calculation Test Eight

Total Ques. : 51 **Total Time : 25 Min. 40 Sec.**

1. $\left(149 \times \frac{105}{43}\right)$ 1·50 sec

2. $\left(78 \times \frac{105}{43}\right)$ 1 min

3. $(605 \times 288 \times 567)$ 45 sec

19. $\frac{2476}{2410}$ 45 sec

20. $\frac{(1\cdot02 - 0\cdot75)}{0\cdot75}$ 30 sec

21. $\frac{y}{\frac{15}{41}y + y} \times 11$ 50 sec

22. $\frac{25 + 4 - 0\cdot3 - 7\cdot8}{400} \times 100$ 55 sec

23. $(40779 \times 0{\cdot}09 - 33979 \times 0{\cdot}19)$ 45 sec

24. $\frac{6 - 4{\cdot}5}{4{\cdot}5} \times 100$ 1·1 min

25. $\frac{12 - 8{\cdot}5}{8{\cdot}5} \times 100$ 40 sec

26. $\frac{(65{\cdot}32 - 64{\cdot}25)}{64{\cdot}25} \times 100$ 1·30 min

27. $\frac{75{\cdot}04 - 60}{60} \times 100$ 2·00 min

28. $\frac{6{\cdot}3 - 5{\cdot}1}{5{\cdot}3} \times 100$ 1·3 min

29. $\frac{6{\cdot}3 - 5{\cdot}9}{5{\cdot}3} \times 100$ 1·15 min

30. $\frac{2{\cdot}08x - 1{\cdot}39x}{1{\cdot}39x} \times 100$ 2·05 min

31. $\frac{63{\cdot}8}{46{\cdot}8}$ 1·50 min

32. $\frac{668}{2041} \times 100$ 1·50 min

33. $(10{\cdot}3 + 8{\cdot}2 + 11{\cdot}2 + 28{\cdot}6 + 23{\cdot}4)$ 25 sec

34. $\frac{17{\cdot}6 + 9{\cdot}8 + 15{\cdot}7}{3}$ 40 sec

35. $\frac{10{\cdot}3 + 8{\cdot}2 + 11{\cdot}2}{3}$ 15 sec

36. $\frac{43}{114} \times 405$ 40 sec

37. $\frac{140}{140 + 140 + 153} \times 100$ 40 sec

38. $288 + 430 \times \frac{50}{100}$ 30 sec

39. $293 + 270 \times \frac{50}{100}$ 25 sec

40. $\frac{62}{(77 + 72 + 63 + 62 + 60)}$ 45 sec

41. $200080 - \frac{600}{10}$ 2 sec

42. $\frac{2832}{1372}$ 55 sec

43. $\frac{5760}{1055}$ 40 sec

44. $(10{\cdot}95 + 10{\cdot}85 + 10{\cdot}58 + 10{\cdot}63)$ 30 sec

45. $(10{\cdot}78 + 10{\cdot}75 + 10{\cdot}94 + 10{\cdot}36)$ 17 sec

46. $[(104 - 63{\cdot}795) \times 100] \div 104$ 1 min

47. $\frac{800 + 1700 + 2700}{3}$ 21 min

48. $\frac{900 + 1600 + 2300}{3}$ 16 min

49. $\frac{400 + 1100 + 1900}{3}$ 4 min

50. $\frac{1100 + 2200 + 1600}{3}$ 15 min

51. $\frac{700 + 1100 - 300}{3}$ 5 sec

Types of the Data Interpretation Questions

1. Table form or Tabular or Data Table
2. Line Graph or Cartesian
3. Bar Graph
4. Pie Charts
5. 3-D Diagram
6. Venn Diagram
7. Pyramid Graph
8. Triangular form Graph
9. Case let
10. Miscellaneous form or multiform or mixed Graphs

Solutions : Calculation Test Eight

1	363·83	**12**	2137·85	**23**	2785·9	**34**	14·36	**45**	42·83
2	190·46	**13**	12·38	**24**	33·33	**35**	9·9	**46**	38·65
3	98794080	**14**	13·13	**25**	41·17	**36**	152·7	**47**	1733·33
4	679·24	**15**	2·5	**26**	1·66	**37**	32·33	**48**	1600
5	13125	**16**	3·94	**27**	25·06	**38**	503	**49**	1133·33
6	432·5	**17**	3·33	**28**	22·64	**39**	428	**50**	1633·33
7	472	**18**	3·66	**29**	22·64	**40**	0·18	**51**	500
8	438	**19**	1·02	**30**	49·63	**41**	200020		
9	0·04	**20**	0·36	**31**	1·36	**42**	2·06		
10	0·07	**21**	8·05	**32**	32·72	**43**	5·46		
11	88·96	**22**	5·22	**33**	81·7	**44**	43·01		

NB : These calculations are not mind games but taken from real papers.

1.Tabular or Table DI

The number of table-based questions is coming in previous years CAT papers

Construct	No. of Ques.
Tables	3
Bar Chart	3
Scatter Diagram	2
Table- Missing Data	1
Line Graph or Cartesian	2
3 – D Diagram	1
Pie Geometrical diagram	3
Case let	2
Miscellaneous	4

"A compact, systematic list of details, contents etc."

"A compact arrangement of related facts, figures, values etc. in orderly sequence and usually in rows and columns for convenience of reference."

The logical listing of data in vertical columns and horizontal rows of number with sufficient explanatory and qualifying words, in the form of titles, heading and explanatory notes to make clear the full meaning and context of the Data.

Essential parts of A Table

The following are the essential part in a good statistical table—

(A) Title

(B) Explanatory Notes

(C) Captions and Stubs

(D) Body of Table

(E) Foot note

Making of A Table

Practice 1 : Present the following information in a suitable tabular form, supplying the figure not directly given. In 2005, out of a total of 2000 students in The IOP (Institute of Perfection), 1550 were preparing for CAT. The number of girls students was 250 out of which 200 did not prepare for CAT.

In 2004, the number of students in the CAT class was 1725, of which 1600 were boys; the number of MAT aspirants was 380 of which 155 were girls. Those who were not preparing for CAT, mean they were preparing for MAT.

Explanation 1 : Table showing Aspirants of CAT and MAT by gender.

Aspi-rants of …..	2004			2005		
	Male	Female	Total	Male	Female	Total
CAT	1600	125	1725	1500	50	1550
MAT	225	155	380	250	200	450
Total	1825	280	2105	1750	250	2000

Practice 2 : Industrial finance in India showed great variation in respect of source during the First, Second and Third Plans. There were two main sources, *viz.,* internal and external. The former had two sources—Depreciation, and Free reserves and surplus. The latter had three sources—Capital issues, Borrowings and other sources.

During the First Plan internal and external sources accounted for 62% and 38% of the total and in this depreciation, fresh capital and Other sources formed 29%, 7% and 10.6% respectively.

During the Second plan internal sources decreased by 17·3% compared to the first plan, and depreciation was 24·5%. The external finance during the same period consisted of fresh capital 10·9% and borrowings 28·9%.

Compared to the Second Plan, during the Third Plan external finance decreased by 4·4% and borrowings and Other sources were 29·4% and 14·9% respectively. During the Third Plan internal finance increased by 4·4% and free reserves and surplus formed 18·6%.

Tabulate the above information with the above details as clearly as possible.

Explanation 2 : Table Showing Pattern of Industrial Finance (In Percent)

Plan	Sources						
	Internal			External			
	Depreciation	Free reserves & Surplus	Total	Capital Issues	Borrowings	Other sources	Total
First	29	33	62	7	20·4	10·6	38
Second	4·5	20·2	44·7	10·9	28·9	15·5	55·5
Third	30·5	18·6	49·1	6·6	29·4	14·9	50·9

Practice 3 : The highlights of the Railway Budget for 2010-11 are as follows :

The actual Gross traffic receipts for 2008-09 was Rs. 3538·24 crores. The budget estimates for 2009-10 in regard to Gross traffic receipts was Rs. 4171·8 crores and the revised budget estimate for 2009-10 for the above items is Rs. 4375·79 crores, respectively. Whereas the budget estimates for 2010-11 for Gross traffic receipts is Rs. 5343·63 crores. The actual total working expenses and the net railway revenue for 2008-09 were Rs. 3182·05 and Rs. 403·1 crores respectively. The budget estimates and revised budget estimate for 2009-10 in respect of total working expenses are Rs. 3700·90 and Rs.3892.28 crores respectively. Whereas the corresponding estimates for 2009-10 in respect of net railway revenue are Rs. 510·91 crores and Rs. 533·4 crores respectively. The estimated budget on total working expenses and net railway revenue for 2010-11 exceed the actual for 2008-09 by Rs. 1338·72 crores and Rs. 268·72 crores respectively. Regarding the dividend payable to general revenues the actual for 2008-09 are Rs. 356·47 crores, the budget estimates for 2009-10 are Rs. 405·12 crores, the revised budget estimates for 2009-10 are 458·17 crores and budget estimates for 2010-11 are 465.5 crores respectively. Present the above data in a table.

Explanation 3 :

Questions

1. What is the maximum percentage increase in price per cubic meter or per tonne over the previous year ?
   (A) 33.33%
   (B) 85%
   (C) 50%
   (D) Cannot be determined
2. Which product shows maximum percentage increase in price over the period ?
   (A) Saw Timber
   (B) Plywood
   (C) Logs
   (D) Cannot be determined
3. If cubic metre = 750 kg for saw timber, find in which year was the difference in price of saw timber and logs the least ?
   (A) 1989 (B) 1990
   (C) 1991 (D) 1992
4. If one cubic metre = 700 kg for plywood and 800 kg for saw timber, find in which year was the difference in the prices of plywood and saw timber (per cubic metre) the maximum ?
   (A) 1989 (B) 1990
   (C) 1991 (D) 1992

HighLights of Railway Budget

Number	Actual 2009-10	Budget Estimates 2009-10	Revised Estimates 2009-10	Budget Estimates 2010-11
Rupees in Crores				
1. Gross traffic Receipts	3538·24	4171·8	4375·79	5343·63
2. Total working Expenses	3182·05	3700·90	3892·28	4520·77
3. Net Railway Revenues	403·10	510·91	533·40	671·82
4. Dividend payable to General Reserves	356·47	405·12	458·17	465·5

Data Interpretation

Illusion : A : Answer the following questions based on the following information :

In the following table, the price of Logs shown is per cubic metre that of plywood and saw timber is per tonne.

Price in Rs.	1987	1988	1989	1990	1991	1992	1993
Plywood	3	3	4	5	4	6	7
Saw Timber	10	10	12	10	13	15	20
Logs	15	16	18	15	18	19	20

Solutions

1. (C) From the table, it is clear that maximum increase is registered in Plywood from 1991 to 1992 and is equal to $\frac{6-4}{4} \times 100 = 50\%$

2. (C) Percentage increase in Plywood = $\frac{7-3}{3} \times 100$ $= 133{\cdot}33\%$, in saw timber it is $\frac{19-10}{10} \times 100 = 90\%$ and in logs it is $\frac{20-15}{15} \times 100 = 33{\cdot}33\%$. Thus we see that maximum percentage increase over the period is shown by plywood.

3. (B) 1 tonne $= \frac{4}{3} = 1{\cdot}33$ m^3

Year	**Saw Timber** (Price in Rs. / Tonnes)	**Saw Timber** (Price in Rs. / Cubic meters)	**Logs** (Price in Rs. / cubic meters)	**Difference in Price**
1989	12	9	18	9
1990	10	7·50	15	7·50
1991	13	9·75	18	8·25
1992	15	11·25	19	7·75

It is hence, clear that the difference is least in the year 1990.

4. (D) As in the previous table, we can draw a similar table for saw timber and logs. (Note—1 tonne of plywood = 1·43 m^3 and 1 tonne of saw timber = 1·25 m^3).

Year	**Saw Timber** (Price in Rs. / Tonne)	**Saw Timber** (Price in Rs. / m^3)	**Plywood** (Price in Rs. / tonne)	**Plywood** (Price in Rs. / m^3)	**Difference in Price**
2002	12	9·60	4	2·80	6·80
2003	10	8·00	5	3·50	4·50
2004	13	10·40	4	2·80	7·60
2005	15	12·00	6	4·20	7·80

Hence, it can be seen that the difference is maximum for the year 2005.

Illusion B : The Table below represents sales and net profit in Rs. crore of IOP Ltd., for the five years from 1994-95 to 1998-99. During this period, the sales increased from Rs. 100 crore to Rs. 680 crore. Correspondingly, the net profit increased from Rs. 2·5 crore to Rs. 12 crore. Net profit is defined as the excess of sales over total costs.

	Sales	Net Profit
1994-95	100	2.5
1995-96	250	4.5
1996-97	300	6
1997-98	290	8.5
1998-99	680	12

Questions

1. The highest percentage of growth in sales, relative to the previous year, occurred in—
   (A) 1995-96 (B) 1996-97
   (C) 1997-98 (D) 1998-99

2. The highest percentage growth in net profit, relative to the previous year, was achieved in—
   (A) 1998-99 (B) 1997-98
   (C) 1996-97 (D) 1995-96

3. Defining profitability as the ratio of net profit to sales, IOP Ltd., recorded the highest profitability in—
   (A) 1998-99 (B) 1997-98
   (C) 1994-95 (D) 1996-97

Solutions

1. (A) Percentage growth in 1995-96—150%.

Percentage growth in 1996-97 $= \frac{50 \times 100}{250} = 20\%$

Percentage growth in 1997-98—There is no growth but decrease in sales.

Percentage growth in 1998-99 $= \frac{390 \times 100}{290} = 134{\cdot}48\%$

Hence, the growth in 1995-96 is maximum.

2. (D) Percentage growth in net profit in 1998-99

$$= \frac{3{\cdot}5 \times 100}{8{\cdot}5}$$
$$= 41{\cdot}17\%$$

Percentage growth in net profit in 1997-98

$$= \frac{2{\cdot}5 \times 100}{06}$$
$$= 41{\cdot}66\%$$

Percentage growth in net profit in 1996-97

$$= \frac{1{\cdot}5 \times 100}{4{\cdot}5}$$
$$= 33{\cdot}33\%$$

Percentage growth in net profit in 1995-96

$$= \frac{2 \times 100}{2{\cdot}5}$$
$$= 80\%$$

Hence, the highest percentage growth in net profit is in 1995-96.

3. (B) The profitability in 1998–99

$$= \frac{12 \times 100}{680} = 1{\cdot}76\%$$

The profitability in 1997-98

$$= \frac{8{\cdot}5 \times 100}{290} = 2{\cdot}93\%$$

The profitability in 1996-97

$$= \frac{6 \times 100}{300} = 2\%$$

Hence the profitabilty in 1997-98 is the highest.

Table Chart

Solved Example

Directions (Q. 1 to 4) : Answer the questions on the basis of the information given below.

The following table shows the growth of regular monthly investment at 7% return compounded annually :

Number of Years	Monthly Investment			
	Rs. 50	Rs. 100	Rs. 250	Rs. 500
Amount after X Years				
2	1292	2583	6458	12915
5	3601	7201	18003	36005
10	8705	17409	43524	87047
20	26198	52397	130991	261983

1. How much interest is earned on an investment for a 5-year period with monthly investment of Rs. 100 ?
   (A) Rs. 1,201 (B) Rs. 6,001
   (C) Rs. 7,204 (D) Rs. 608

2. Find the approximate ratio of interest earned in 10-year period to the interest earned on a 5-year period with monthly investment of Rs. 100 ?
   (A) 2 : 9 (B) 4 : 1
   (C) 4 : 9 (D) 9 : 2

3. How much less would be earned on a Rs. 500 monthly investment for 10 years than a Rs. 250 monthly investment for 20 years ?
   (A) Rs. 45,844 (B) Rs. 43,944
   (C) Rs. 35,004 (D) Rs. 37,832

4. What is the total interest earned on a 7% investment for 10-year period with monthly investment of Rs. 100 ?
   (A) Rs. 5,809 (B) Rs. 5,782
   (C) Rs. 5,652 (D) Rs. 5,409

Discussion

1. (A) Interest earned $= 17201 - 12 \times 5 \times 100$
   $=$ Rs. 1,201

2. (D) $\dfrac{\text{Interest on 10 years}}{\text{Interest on 5 years}} = \dfrac{17409 - 12000}{7201 - 6000}$
   $= \dfrac{5409}{1201} \cong 9 : 2$

3. (B) Interest on 250 monthly investment for 20 year $= 130991 - 12 \times 20 \times 250 =$ Rs. 70,991

   Interest on 500 monthly investment for 10 year $= 87047 - 60000 =$ Rs. 27,047

   How much less would be earned $= 70991 - 27047 =$ Rs. 43944.

4. (D) Interest on 10 year period with monthly investment of Rs. 100 $= 17409 - (12 \times 100 \times 10)$
   $=$ Rs. 5409.

Directions (Q. 5 to 8) : Answer the questions on the basis of the information given below.

Country	Total Emission (Tonnes in Millions)	Emission per person (In Tonnes)	Growth in emission per person (1974-78) (Per cent)
Hungary	123	0·24	– 23·5
India	1289	0·71	13
Pakistan	285	2·62	– 0·3
Japan	597	5·26	–4·4
China	90	0·96	7·1
UK	356	2·39	21·6
Sri Lanka	954	3·08	24·1

5. In 1978, emission from UK accounted for what percentage of the total emission by all the given countries ?
   (A) 10% (B) 8%
   (C) 12% (D) 6%

6. In 1978, What was the approximate difference between the population of Hungary and Pakistan ?
   (A) 410 (B) 403
   (C) 420 (D) 430

7. Which country had the maximum emission per person in 1978, out of the countries given ?
   (A) Japan (B) India
   (C) UK (D) Sri Lanka

8. What is the name of the country which stands at second position, if we arrange the countries in term of descending order of their population ?
   (A) Sri Lanka (B) Hungary
   (C) China (D) India

Discussion

5. (A) Percentage $= \dfrac{356}{3694} \times 100 \approx 10\%$

6. (B) Population of Hungary $= \dfrac{123}{0{\cdot}24} = 512$ million

   Population of Pakistan $= \dfrac{285}{2{\cdot}62} = 109$

   So, difference = 403 million

7. (A) Obviously, Japan

8. (B) Sri Lanka $= \dfrac{954}{3{\cdot}08} = 318;$

   China $= \dfrac{90}{0{\cdot}96} = 100;$

$$\text{Hungary} = \frac{123}{0{\cdot}24} = 500;$$

$$\text{India} = \frac{1289}{0{\cdot}71} = 1815$$

So, obviously Hungary is Second in population, if we arrange in descending order.

Directions (Q. 9 to 13) : Hundai and Maruti can produce either Body Case or Carburetor. The time taken by Hundai and Maruti (in minutes) to produce one unit of Body Case and Carburetor are given in the table below :

(Each machine works 8 hour per day)

Product	Hundai	Maruti
Body Case	10	8
Carburetor	6	6

9. What is the maximum number of units that can be manufactured in one day ?
   (A) 140 (B) 160
   (C) 120 (D) 180
10. If Hundai works at half its normal efficiency, what is the maximum number of units produced, if at least one unit of each must be produced ?
    (A) 96 (B) 89
    (C) 100 (D) 119
11. What is the least number of machine hours required to produce 30 pieces of Body Case and 25 Pieces of Carburetor respectively ?
    (A) 6 hrs 30 min (B) 7 hrs 24 min
    (C) 6 hrs 48 min (D) 4 hrs 6 min
12. If the number of units of Body Case to be three times that of Carburetor, what is the maximum idle time to maximize total units manufactured ?
    (A) 0 min (B) 24 min
    (C) 1 Hr. (D) 2 Hr.
13. If equal quantities of both are to be produced, then out of the four choices given below, the least efficient way would be :
    (A) 48 of each with 3 min idle
    (B) 64 of each with 12 min idle
    (C) 61 of each with 10 min idle
    (D) 71 of each with 9 min idle

Discussion

9. (B) Since time taken to manufacture Carburetor on both the machine is the last, we have to manufacture only Carburetor in order to maximize the output for the day.

   In such a case total number of units of Carburetor produced on Hundai $= \left(\frac{8 \times 60}{6}\right)$ = 80 units and that produced on Maruti $= \left(\frac{8 \times 60}{6}\right)$ = 80 Units.

   So, the maximum number of units that can be produced = (80 + 80) = 160 units.

10. (D) If Hundai works at half of its normal efficiency, time taken by Hundai to manufacture 1 unit of Body Case = 20 min. and Carburetor = 12 min.

    And now for maximum number of units, we have to produce Carburetor on Maruti first as it takes only 6 min. per piece.

    Also since at least one unit of Body case has to be manufactured and it is more efficient to do so on Maruti, we would do that.

    So time taken to manufacture 1 unit of Body case on Maruti = 8 min.

    Hence, the remaining time on Maruti = (480 – 8) = 472. In this remaining time number of units of Carburetor that can be manufactured $= \left(\frac{472}{6}\right) = 78.$

    Now since it takes less time to manufacture Carburetor on Hundai as well, we will maximize Carburetor on Hundai. Number of units that can be produced $= \left(\frac{8 \times 60}{12}\right) = 40.$

    Hence, total number of units manufactured = (1 + 78 + 40) = 119 units.

11. (A) In order to take minimum time, manufacture Body case on Maruti and Carburetor on Hundai. Number of machine hours required to manufacture 30 units of Body case on Maruti = (30 × 8) = 240 min = 4 Hrs. Number of Factory hours required to manufacture 25 units of Carburetor on Hundai = (25 × 6) = 150 min = 2·5 Hrs.

    So total time taken = (4 + 2·5) = 6·5 hrs.

12. (A) Given condition, the number of units of body case to be three times that of carburetor. Maruti can make $= \left(\frac{8 \times 60}{8}\right)$ = 60 body case in one day .

    So, by condition, required. No of carburetor = one third of body case = 20.

    Time consumed by Hundai to make 20 body case = $\left(\frac{6 \times 20}{60}\right)$ = 2 Hrs. Hence the remaining time on Hundai is 8 – 2 = 6 Hrs or 360 Minutes.

    Time taken to manufacture 3 body case and 1 carburetor on Hundai is $3 \times 10 + 1 \times 6 = 36$ minutes.

    So, in 360 minute Hundai can produce 10 times what it produces in 36 minutes.

    Hence, 30 body case and 10 carburetor. So idle time is zero.

13. (C) Use option (for least efficient way)

    (a) 48 Body case = Hundai takes 48 × 10 = 480 minutes (one day)

48 carburetor = any, because both have equal efficiency. Maruti takes 48 × 6 = 288 minutes. Idle time = 480 – 288 = 192. NOT SATISFIED.

(b) 64 Body case = Hundai takes 48 × 0 = 480 minutes (one day)

Maruti takes (64 – 48) × 8 = 16 × 8 = 128 minutes

64 carburetor = Maruti takes 64 × 6 = 384 minutes. Idle time = 480 – (128 + 384) = – 32 not possible. NOT SATISFIED.

(c) 61 Body case = Hundai takes 48 × 10 = 480 minutes (one day) Maruti takes (61 – 48) × 8 = 13 × 8 = 104 minutes.

61 carburetor = Maruti takes 61 × 6 = 366 minutes. Idle time = 480 – (104 + 366) =10 minutes idle SATISFIED.

(d) If 64 is not satisfied, 71 also rejected. NOT SATISFIED.

Exercise–1

Direction (Q. 1 to 5) : Refer to the following tabular statement which records performance of department of surgery of a hospital for the period January to July.

Department of Surgery, M.G. Medical College E.N.T. and Eyes Operations

Month	Total Successful Operations		Total. Unsuccessful operations	Total no. of operations performed
	E.N.T.	Eyes		
Jan.	2	3	3	8
Feb.	3	1	4	16
March	3	2	2	23
April	4	2	3	32
May	2	2	2	38
June	1	2	3	44
July	0	2	4	50

1. How many successful operations were performed upto July ?
   (A) 32 (B) 29
   (C) 25 (D) 21
   (E) 50
2. What percentage of operations were successful during the period covered in the statement above ?
   (A) 75% (B) 60%
   (C) 58% (D) 29%
   (E) 80%
3. Which month was worst as per data given in the statement' ?
   (A) February (B) April
   (C) May (D) June
   (E) July
4. Operations performance of which months can be termed the best ?
   (A) January (B) March
   (C) April (D) May
   (E) June
5. If there are 50 more operations left to be performed, how many operations must be successful in order to make the overall percentage of successful operations to 70 ?
   (A) 39 (B) 35
   (C) 41 (D) 34
   (E) 32

Directions (Q. 6 to 9) : Study the following statement minutely and answer the questions that follow :

Year	Birth Rate	Death Rate	Child deaths
1988	80·7	71·3	20%
1989	51·8	41·7	19%
1990	54·.3	44·2	20%
1991	60·.7	50·6	19%
1992	75·2	65·1	21%
1993	79·2	63·1	17%
1994	78·3	66·4	22%

6. From the table it follows that the maximum addition of population took place during the year ....
   (A) 1990 (B) 1992
   (C) 1993 (D) 1994
7. Which year had the minimum growth in population ?
   (A) 1989 (B) 1994
   (C) 1990 (D) 1988
8. Death rate remained static during .... ?
   (A) 1989-90
   (B) 1989-92
   (C) 1991-93
   (D) It did not remain static at all
9. From overall health/welfare point, which year was the best ? (Figures per 100 persons)
   (A) 1994 (B) 1992
   (C) 1989 (D) 1993

Direction (Q. 10 to 14) : Study the data given in the following table carefully and answer the questions 10-14 given below it :

Wheat Production (in lakh tonnes)

State	2006	2007	2008	2009	2010
A	9·0	10·7	8·9	11·6	8·4
B	14·5	16·3	16·2	16·4	16·8
C	14·9	15·7	16·8	16·9	17·8
D	7·6	8·4	7·4	7·9	8·6
E	21·0	22·6	23·12	22·2	23·9

10. In 2008, which states contributed close to one-eighth of the total production of all the five states ?
(A) A (B) B
(C) C (D) D
(E) E

11. In which year did the production of state D fall for the first time ?
(A) 2006 (B) 2007
(C) 2008 (D) 2009
(E) 2010

12. In which state, the production in 2009 showed the highest increase over that in 2006 ?
(C) A (B) B
(C) C (D) D
(E) E

13. In which year does the production in state E show the higher percentage of increase over that in the previous year ?
(A) 2006 (B) 2007
(C) 2008 (D) 2009
(E) 2010

14. In which state did the production of wheat increase continuously from 2006 to 2010 ?
(A) A (B) B
(C) C (D) D
(E) E

15. How many toppers (maximum) of the entrance exam H, took training from 3 institutes ?
(A) 5 (B) 3
(C) 2 (D) 7

16. What percentage of the successful candidates enrolled for institute Q's training, are toppers ?
(A) $\frac{180}{11}$ (B) $\frac{183}{11}$
(C) $\frac{191}{11}$ (d) $\frac{193}{11}$

17. What percentage (minimum) of institute Y's students are toppers ?
(A) $\frac{6}{13}\%$ (B) $\frac{19}{39}\%$
(C) $\frac{20}{39}\%$ (D) $\frac{7}{13}\%$

18. Which of the following is/are necessarily true ?
I. Out of the students who are trained with either institute Y or P, the equal numbers obtained ranks in the Civil Service Entrance Exams.
II. Institute P trained maximum number of Bank PO exam. toppers.
III. Out of the toppers who are trained with institute Y, maximum number are toppers of the Bank PO exams.
(A) I and II (B) II and III
(C) I and III (D) None of these

Directions (Q. 15 to 18) : Answer the questions on the basis of the information given below :

The table (I) shows the number of toppers in the various entrance exams and the other table (II) shows the number of students enrolled at different institutes. Refer to the tables to answer the questions that follow :

Table–I

Institute where toppers are trained	Number of toppers in the entrance exams for—								
	Civil Services			Bank P.O.			SSC		
	F	G	H	I	J	K	L	M	N
X	4	3	7	2	3	4	4	4	4
Y	2	1	8	5	6	2	6	2	1
Z	3	1	2	6	4	3	7	1	2
P	6	4	1	7	1	5	5	3	4
Q	5	3	1	3	6	5	1	1	2

Table–II

Institute's name	Enrollment for training at different institutes for various entrance exams—					
	Civil Services		Bank P.O.		SSC	
	Enrollment	Success Ratio	Enrollment	Success Ratio	Enrollment	Success Ratio
X	800	01 : 10	1500	01 : 10	1000	01 : 10
Y	1000	01 : 10	2100	01 : 15	800	01 : 10
Z	500	01 : 08	1500	01 : 12	1000	01 : 10
P	900	01 : 10	1800	01 : 12	500	01 : 10
Q	600	01 : 15	1000	01 : 20	900	01 : 12

Directions (Q. 19 to 22) : Answer the questions on the basis of the information given below.

The following table gives the production of cloth in meters by 5 workers on 6 consecutive days in a textile mill.

Worker	Mon.	Tue.	Wed.	Thr.	Fri.	Sat.
1	3000	3200	3100	3250	3300	3650
2	4008	3850	3900	4050	4100	4300
3	4320	3900	4000	4200	4300	4400
4	2820	2900	3030	3100	3240	3500
5	4550	4350	4400	4500	4100	4650

19. The production of which worker shows highest increase on Saturday over his production on Monday ?
    (A) Worker 1 (B) Worker 5
    (C) Worker 4 (D) Worker 2

20. On Wednesday, the production of worker 3 was what % of production of worker 1 ?
    (A) 29·03%
    (B) 129·03%
    (C) 78·3%
    (D) 86·49%

21. On which of the following days did the production of worker 4 show the highest increase over his production on the preceding day ?
    (A) Monday (B) Friday
    (C) Wednesday (D) Saturday

22. For each of the given days, the ratio of number of workers, having their production above average, to those, having below the average, is :
    (A) 3 : 2 (B) 2 : 3
    (C) 4 : 1 (D) 1 : 4

Directions (Q. 23 to 26) : The following table gives the sales details for Nuts and Bolts of Car, Bike, Scooter and Bus.

Year	Car	Bike	Scooter	Bus
2005	42137	8820	65303	25343
2006	53568	10285	71602	27930
2007	58770	16437	73667	28687
2008	56872	15475	71668	30057
2009	66213	17500	78697	33682
2010	68718	20177	82175	36697

23. What is the growth rate of sales of Nuts and Bolts at Car 2005 to 2010 ?
    (A) 29% (B) 51%
    (C) 63% (D) 163%

24. Which of the categories shows the lowest growth rate from 2005 to 2010 ?
    (A) Car (B) Bike
    (C) Scooter (D) Bus

25. Which category had the highest growth rate in period ?
    (A) Car (B) Bike
    (C) Scooter (D) Bus

26. Which of the categories had either a consistent growth or a consistent decline in the period shown ?
    (A) Car (B) Bike
    (C) Scooter (D) Bus

Direction (Q. 27 to 30) : Refer the information in the table below about TCIL as on 31st March : (All figures in Rs. Crores)

	1991-92	1992-93	1993-94	1994-95	1995-96
Share Capital & Reserves	50·68	61·44	67·16	69·97	69·97
Turnover	61·72	44·86	26·24	34·75	40·62
Cash Losses	7·94	14·80	18·82	16·78	11·57
Net Losses	16·69	25·47	31·28	44·50	26·58

27. If accumulated losses on 31st March 1996 were Rs. 214·74 crore, what was the cumulative loss as on March 31st 1992.
    (A) 144·83 cores (B) 70·22 crores
    (C) 152·77 crores (D) 86·91 crores

28. In which year was TCIL's net worth (share Capital + profit) eroded ?
    (A) 1991-92
    (B) 1992-93
    (C) 1994-95
    (D) Can't be determined

29. Cash loss as a %age of turnover was minimum in—
    (A) 1991-92 (B) 1994-95
    (C) 1993-94 (D) 1992-93

30. If capacity of TCIL is 60,000 tyres/month and is currently survivng on job work that generates revenues of Rs. 1100 per tyre. What was capacity utilization in 1995-96 ?
    (A) 48
    (B) 51
    (C) 61
    (D) Can't be determined

Directions (Q. 31 to 34) : Answer the questions on the basis of the information given below :

Demand and Capacity Projections

Intermediates	Demand			Capacity		
	1989-90	**1994-95**	**1999-00**	**1989-90**	**1994-95**	**1999-00**
Ethylene Oxide (non MEG)	19,000	23,990	29,350	12,000	17,000	34,000
MEG	146,300	199,800	268,000	23,500	140,000	163,500
Propylene Oxide	17,000	95,000	115,000	—	30,000	30,000
Acrylontrile	91,200	125,600	169,000	24,000	50,000	74,000
Phenol	48,640	68,250	92,185	56,000	15,000	71,000
Maleic Anhydride	6,500	13,850	24,237	6,000	58,700	65,700
Benzene	412,000	626,000	885,000	234,500	238,000	472,500
Toluene	51,270	62,220	76,430	45,000	—	45,000
Paraxylene	225,000	301,000	396,000	46,000	—	237,100
Orthoxylene	67,000	106,000	151,200	27,000	67,000	118,400

31. Which product demand-wise has shown the highest growth rate from 1989-90 to 1999-2000 ?
    (A) Propylene Oxide (B) MEG
    (C) Maleic Anhydride (D) Orthoxylene

32. Which of the following are the products that will not have a short fall of capacity (As compared to demand) of more than 30% of demand in 1999-2000 ?
    (A) MEG (B) Toulene
    (C) Acrylonitriel (D) Phenol

33. Which of the following statement(s) is/are true ?
    I. Orthoxylene capacity will be in surplus in the year 1999-2000.
    II. Paraxylene's demand to capacity ratio is higher in 1989-90 than in 1999-2000.
    III. Total demand for all products in 1994-95 is 50% higher than in 1989-90.
    (A) I only (B) II only
    (C) III only (D) II and III

34. The demand for which product has shown the lowest growth rate from 1994-95 to 1999-2000 ?
    (A) Propylene Oxide (B) Benzene
    (C) Paraxylene (D) MEG

35. The total number of candidates scoring at last 66·67% in the written test and 20% in the interview is—
    (A) 340
    (B) 360
    (C) 350
    (D) None of these

36. If the institute has to qualify 650 candidates in the written test, what should be the cut-off ?
    (A) 50%
    (B) 160 marks%
    (C) 168 marks
    (D) In the range of 150 to 199 marks

37. Find out the approximate average marks in the written test. Mid-points can be taken of the class intervals for average calculation—
    (A) 125 (B) 130
    (C) 136 (D) 150

38. If cut-off in the written test is 200 and in interview 60, how many people would be selected ?
    (A) 100 (B) 110
    (C) 98 (D) 105

Directions (Q. 35 to 38) : Answer the questions on the basis of the information given below :

The following table gives the performance of 2,000 candidates who have taken a written test and a GD. The number of candidates who have scored in the specified range of written test scores and GD marks given in row headings and column headings respectively.

Max. marks in the written test = 300 : Maximum marks in GD = 100 marks

Written test				GD marks			
	20-29	**30-39**	**40-49**	**50-59**	**60-69**	**70**	**Total**
250	16	36	52	36	52	8	2000
200 to 249	10	8	60	44	20	18	160
150 to 199	32	20	90	112	36	18	308
100 to 149	56	84	200	380	30	10	760
50 to 99	70	230	40	16	14	10	380
Below 50	64	64	40	8	12	4	192
Total	**248**	**442**	**482**	**596**	**164**	**68**	**2000**

Solutions

1. (B) Add the number of successful operations given in the 2nd and 3rd column :

   5 + 4 + 5 + 6 + 4 + 3 + 2 = 29

2. (C) Out of the total operations performed. 29 were successful or 58%.

3. (E) In July the number of unsuccessful operations was more than successful operations performed.

4. (B) In March five operations were successful and two were unsuccessful or say 70% compared to the previous month.

5. (C) To make 70% score of successful operations. there should be 70 operations successful out of 100. Since there were already 29 successful operations performed till July. There should be 70–29 or 41 successful operations out of the next fifty to make overall percentage 70.

6. (D) 7. (D) 8. (B) 9. (C)

10. (A) Total production of all the five states in 2008 = 72·5.

    1/8th of 72·5 = 9 (approx.).

    State A contributes close to 1/8th of the total production of all the five states.

11. (C) 2008

12. (A) Percentage increase in production of state

    A in 2009 over that in 2006 $= \frac{2 \cdot 6}{9} \times 100 = 28 \cdot 8$

    Similar figures for the other states are respectively 20, 12·3, 3·94, 5·71.

13. (E) % Increase of production in 2007 over that in 2006 $\frac{1 \cdot 6}{21} \times 100 = 7 \cdot 61$. Similar figures in 2008 over that in 2007 = 2·6 and in 2010 over that in 2009 is 7·65

14. (C)

15. (C) Total number of toppers of entrance exam H = 10

    ⇒ – minimum (7 + 8) - 10 = 5 took training from both X and Y

    ⇒ Maximum 2 took training from X, Y and Z

16. (A) Successful candidates

    $$= \frac{600}{15} + \frac{1000}{20} + \frac{900}{12}$$

    $$= 40 + 50 + 75 = 165$$

    Maximum number of toppers

    $$= 5 + 3 + 1 + 3 + 6 + 5 + 1 + 1 + 2$$

    $$= 27$$

    ⇒ Required percentage

    $$= \frac{27}{165} \times 100 = \frac{180}{11}\%$$

17. (C) Maximum number of rankers = 8 + 6 + 6 = 20

    (Assuming that toppers of F and G will top H and topper of I and K will top J and toppers of M × N will top L)

    Total enrollments = 1000 + 2100 + 800 = 3900

    ⇒ Required percentage $= \frac{20}{3900} \times 100 = \%$

18. (D) Ist is not necessarily true as each topper can top in one or more of the exams F, G and H.

    IInd is not necessarily true, as each topper can top in one or more of the exams I, J and K.

    IIIrd is not necessarily true, as explained above.

19. (C) The increase of production of the workers 1, 2, 3, 4, 5 on Saturday over Monday are 650m, 300m, 80m, 68m and 100m respectively. Hence worker 4 shows the maximum increase.

20. (B) Required % $= \frac{4000}{3100} \times 100\% = 129 \cdot 03\%$.

21. (D) Take the successive differences of production by worker 4. These are 80m, 130m, 70m, 140m and 260m respectively. Hence it is highest on Saturday.

22. (A) The required ratios for Monday to Saturday are 3 : 2 for each day.

23. (C) Required percentage growth = (68,718 – 42,137) $\times \frac{100}{42137}$ = 63% (approximate).

24. (C)

Nuts & Bolts	2005	2010	Percentage Growth
Car	42137	68718	66%
Bike	8820	20177	125%
Scooter	65303	82175	26%
Bus	25343	36697	36%

25. (B) Again referring to the above table, we can see that the % growth rate is maximum for Bikes Nuts and Bolts, *viz* 25%.

26. (D) It can be seen from the given table that though car Nuts and Bolts have shown a consistent growth. On the other hand, Bike and Scooters Nuts and Bolts have shown a consistent increase except for 2007 when it had declined. But the Bus's Nuts and Bolts have shown a consistent growth over the period.

27. (D) Losses from 92-93 to 95-96 total 25·47 + 31·58 + 44·50 + 26·58 = 127·93 cumulative loss at the start of 92-93 was 214·74 – 127·93 = 86·91 crores.

28. (D) The net worth is eroded when cumulative losses total more than the share capital and reserves. Since we do not have data on commutative loss, we can not determine the year when the net worth became negative.

29. (A) Cash losses are 8/51; 15/61; 19/67; 17/70; 12/70 The lowest is 12/70 which is in 91-92, hence option 1st is the answer.

30. (B) Total Turnover = 60,000 tyres pm $\times$ 12 $\times$ 1100 = 7920 lakhs = 79·20 crore p.a.

In 95-96 job work was = 40·62 crore. Capacity utilization = $\frac{40\cdot62}{79\cdot20} \times 100 = 51\cdot28\% = 51\%$.

31. (A) From the table, the following growth ratio can be seen : Propylene Oxide = 576%, MEG = 83%, Maleic Anhydride = 273% ; Orthoxylene = 125%.

32. (D) Shortfall of capacity = Demand – Capacity

Product	Demand	Capacity	Short-fall	Shortfall as % of Demand
MEG	268	163·5	104	> 30%
Toulene	76	45	31	> 30%
Acrylo-nitrile	169	74	95	> 30%
Phenol	92	71	21	< 30%

33. (D) Ist is false from visual observation.

IInd is true because paraxylene's demand to capacity rates in 1989-90 is 225/46 = 5% in 1999-2000, it is 396/237 = 1·5.

IIIrd is true by visual observation.

34. (A) Propylene Oxide = $\frac{115000 - 95000}{95000} \times 100$

= 21%

Benzene = $\frac{885000 - 626000}{626000} \times 100$

= 41% Paraxylene

= $\frac{396000 - 301000}{301000} \times 100$

= 31·5%

MEG = $\frac{268000 - 1998000}{199800} \times 100$

= 34%

35. (B) 200 + 160 = 360

36. (D) Sum of the first three row = 200 + 160 + 308 = 668. Thus, all we can know is the cut off will be in range 150 – 199. We cannot find exact cut off.

37. (C) Average marks = [Class interval mid point $\times$ average marks for class] / total number of candidates

$$= \frac{25 \times 192 + 75 \times 380 + 125 \times 760 + 175 \times 308 + 225 \times 160 + 275 \times 200}{200}$$

$$= \frac{273200}{2000} = 136\cdot6$$

38. (C) 52 + 8 + 20 + 18 = 98.

Exercise-2

Directions (Q. 1 to 4) : Answer the questions on the basis of the information given below :

The following table gives the performance of 2,000 candidates who have taken a written test and a GD, the number of candidates who have scored in the specified range of written test scores and GD marks given in row headings and column headings respectively.

Maximum marks in the written test = 300 Maximum marks in GD = 100

Written test marks				GD marks			
	20-29	**30-39**	**40-49**	**50-59**	**60-69**	**≥ 70**	**Total**
250	16	36	52	36	52	8	2000
200 to 249	10	8	60	44	20	18	160
150 to 199	32	20	90	112	36	18	308
100 to 149	56	84	200	380	30	10	760
50 to 99	70	230	40	16	14	10	380
Below 50	64	64	40	8	12	4	192
Total	**248**	**442**	**482**	**596**	**164**	**68**	**2000**

1. What percentage of total number of students got > 250 in written test ?
   (A) 8% (B) 9%
   (C) 10% (D) None of these

2. The number of students who got 110 marks in written test is—
   (A) 740 (B) 750
   (C) None (D) Cannot be determined

3. How many candidates obtained > 50% marks in the written test as well as in the GD ?
   (A) 350 (B) 344
   (C) 360 (D) None of these

4. The total number of candidates getting at least 66[2/3]% in the written test and 50% in the interview is—
   (A) 150 (B) 160
   (C) 170 (D) 178

Directions (Q. 5 to 8) : Answer the questions on the basis of the information given below :

The table given below shows that the achievement of agriculture development programmers from1970 to 1976.

Programmes	1970-71	1971-72	1972-73	1973-74	1974-75	1975-76
Area under irrigation (Cumulative in hectares in millions)						
Major	22·05	22·70	23·20	24·00	24·60	25·32
Minor	28·60	32·77	32·77	34·20	34·00	35·14
Area with high yielding variety (Hectares in millions)						
1. Rice	2·90	3·6	4·6	4·7	5·4	5·2
2. Wheat	15·90	16·10	16·8	17·8	19·4	19·1
3. Bajra	16·90	18·20	19·7	18·7	21·7	22·8
4. Paddy	1·4	1·6	1·6	1·7	1·9	2
5. Maize	3·1	3·5	3·9	4·4	5·3	5·1
Consumption of chemical fertilizers (Tonnes in millions)						
1. Urea	1·11	1·21	1·32	1·44	1·73	1·89
2. Potash	3·42	3·68	4·07	4·22	5·20	5·49
3. Nitrogen	0·59	0·62	0·67	0·73	0·78	0·84
Gross Cropped Area (Hectares in millions)	174·8	173·1	177·0	172·6	180·4	187·8

5. Assume that all potash fertilizers were used only for high-yielding varieties of rice and bajra and nitrogen for those of maize and paddy. In which year is the difference between per hectare consumption of potash and nitrogen fertilizer maximum ?
   (A) 1973-74
   (B) 1974-75
   (C) 1975-76
   (D) 1970-71

6. Consumption of chemical fertilizer per hectare of gross cropped area is least for the year—
   (A) 1974-75 (B) 1975-76
   (C) 1970-71 (D) 1971-72

7. In 1973-74, how much more areas were brought under irrigation ?
   (A) 3·34 million hectares
   (B) 2·33 million hectares
   (C) 2·23 million hectares
   (D) 1·53 million hectares

8. If we assume that a part of the minor irrigated area can come under major area, in which year has it definitely happened ? (Assume once a area comes under irrigation, it always remains in irrigation.)
   (A) 1972-73
   (B) 1973-74
   (C) 1974-75
   (D) 1975-76

Directions (Q. 9 to 12) : Answer the questions on the basis of the information given below :

The following table gives the distance between any two godowns of a company :

	A	B	C	D
A	—	40	50	30
B	40	—	20	60
C	50	20	—	20
D	30	60	20	—

The following table shows the cost of transportation of goods (in rupees per kilometer) :

	A	B	C	D
A	—	12	15	10
B	12	—	08	16
C	15	08	—	08
D	10	16	08	—

9. What is the minimum cost of transportation for a truck starting from B and ending at C while going through both the other godowns ?
   (A) Rs. 940 (B) Rs. 1,060
   (C) Rs. 2,010 (D) Rs. 2,400

10. What is the lowest cost of transportation for a truck starting from C and visiting all the other three godowns ?
    (A) Rs. 920 (B) Rs. 940
    (C) Rs. 960 (D) Data insufficient

11. What is the total cost of transportation for a truck that goes from A to B, B to D and then returns to A ?
   (A) Rs. 1,950 (B) Rs. 1,100
   (C) Rs. 2,010 (D) Data insufficient

12. What is the shortest distance between A and B, if each of the other two godowns have to be visited in between ?
   (A) 50 km (B) 60 km
   (C) 70 km (D) Data insufficient

17. If a man wants to buy all the goods listed in the table and travelling between any two countries costs $20, then what is the lowest price at which he can procure all the goods ?
   (A) $ 14565·56 (B) $ 14546·43
   (C) $ 14651·86 (D) None of these

18. If Common Currency (CC) were introduced by averaging out the given five costs across the countries, then how much would a compaq presario 2240 costs, if 1 CC = 2 dollars. (in CC) ?

Directions (Q. 13 to 16) : Answer the questions on the basis of the information given below :

The following table gives the readership of different categories of magazines (in hundreds) from 1980 to 1995.

	1980		1985		1990		1995	
Category	E	H	E	H	E	H	E	H
Business	300	100	400	150	600	250	800	500
Film	600	800	900	1200	1000	1500	1200	1800
Sports	400	200	600	400	1000	700	1200	800
General	1000	1200	1500	1600	2000	1800	2500	2000

The following table gives the number of magazines published in different categories during the period 1980 to 1995 :

	1980		1985		1990		1995	
Category	E	H	E	H	E	H	E	H
Business	4	2	5	3	6	4	7	8
Film	3	4	5	6	5	7	6	8
Sports	3	2	5	3	7	4	8	6
General	12	10	15	14	17	15	29	16

E : English H : Hindi **Note :** No magazine was discontinued during the given period.

13. In 1985, as against in 1980, readership per magazine declined for the.........category.
   (A) Business (B) Film
   (C) Sports (D) General

14. How many new magazines were started between 1980 and 1990 ?
   (A) 22 (B) 24
   (C) 25 (D) 28

15. Between 1980 and 1985 what was the ratio of new Hindi magazines, to new English magazines ?
   (A) 1 (B) 2
   (C) 3 (D) more than 3

16. What is the average (approximate) readership per magazine, in 1990 ?
   (A) 13,000 (B) 13,600
   (C) 14,000 (D) 14,600

   (A) 605·6 CC (B) 1211·2 CC
   (C) 842·1 CC (D) 607·3 CC

19. If a person buys all the goods and uses all the services listed, then between which 2 countries is the absolute difference the most ?
   (A) China and Malaysia
   (B) Malaysia and Sri Lanka
   (C) Sri Lanka and China
   (D) China and India

20. How much does a person spend if he makes a round trip covering all the 5 countries, using up 84 liters of Petrol in each country ? Each country takes 1 day to cover and no country permits another country's car to enter.
   (A) $ 727·5 (B) $ 1101
   (C) $ 988 (D) $ 1255

Directions (Q. 17 to 20) : The following tables shows the costs of various good/services in the given countries. Refer to the table to answer the questions that follow : (All the values are in $)

Goods/Services	China	Korea	India	Sri Lanka	Malaysia
Pepsi (1·5 litre)	2·05	1·05	1·89	1·65	1·14
Pizza (8")	2·86	3·08	2·67	2·48	2·38
New Port Jeans	71·00	83·00	81·00	69·00	70·00
Compaq Presario 2240	1316·00	1348·00	917·00	1208·00	1267·00
Petrol (1 litre)	0·93	1·03	0·87	0·94	0·73
Dry Cleaning (Shirt)	3·68	4·67	2·43	2·75	2·92
Santro (1 day rental)	154·00	110·00	103·00	243·00	113·00
Volkswagen Golf GI	13553·00	16317·00	13999·00	17056·00	17356·00

Directions (Q. 21 to 23) : Study the following table carefully and answer the questions given below it :

Number of People Travelled to Five Destinations Over The Years

(In Thouands)

Destination → ↓ Years	A	B	C	D	E
2004	20	24	20	18	21
2005	36	22	16	24	23
2006	18	16	12	22	16
2007	24	30	18	20	30
2008	28	32	26	19	34
2009	22	26	28	25	38

21. In which of the following years, the number of travellers for destination C was equal to the number of travellers for destination A in 2006 ?
(A) 2004 (B) 2005
(C) 2007 (D) 2008
(E) 2009

22. What was the percent increase in the number of travellers for destination D from 2004 to 2005 ?
(A) $66\frac{2}{3}$
(B) $6\frac{1}{3}$
(C) $33\frac{1}{3}$
(D) 50
(E) None of these

23. What was the difference in the number of travellers for destination D from 2005 to 2007 ?
(A) 4,000
(B) 400
(C) 6,000
(D) 8,000
(E) None of these

24. The number of candidates appeared under Agriculture in 2007 was approximately what per cent of the number of candidates qualified under Arts in 2006 ?
(A) 20 (B) 100
(C) 400 (D) 40
(E) 200

25. What was the percent drop in the number of candidates qualified in science discipline from 2006 to 2007 ?
(A) 10 (B) 20
(C) 50 (D) 25
(E) None of these

26. In which of the following disciplines, there was a continuous increase in the number of candidates appearing over the given years ?
(A) Arts
(B) Commerce
(C) Agriculture
(D) Science
(E) None of these

27. In which of the following years, the percentage of the qualified to the appeared ones in engineering discipline was the maximum ?
(A) 2004
(B) 2005
(C) 2007
(D) 2008
(E) 2009

28. In which of the following years , the percentage of total number of candidates of all the disciplines together, qualified to the appeared, was the maximum ?
(A) 2005 (B) 2006
(C) 2007 (D) 2008
(E) 2009

Directions (Q. 24 to 28) : Study the following table carefully and answer questions given below it :

Number of Candidates Appeared and Qualified Under Various Disciplines in an Examination Over the Years

Disciplines	Arts		Science		Commerce		Agriculture		Engineering		Total	
Years	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App	Qual.
2004	850	200	1614	402	750	212	614	170	801	270	4629	1254
2005	1214	315	1826	420	800	220	580	150	934	350	5354	1455
2006	975	250	1970	500	860	260	624	160	742	300	5171	1470
2007	820	196	1560	450	842	300	490	160	850	312	4562	1418
2008	1412	378	2120	625	1105	320	760	200	642	301	6039	1824
2009	738	359	3506	880	1240	308	640	210	962	400	7086	2157

Directions (Q. 29 to 33) : These questions are to be answered on the basis of the table below giving the percentage of different factors that employees in a modern organization want.

Why employees like an organization	Factor	Why employees leave an organization
71%	Job Content	50%
57	Opportunities	57%
21%	Training	21%
29%	Compensation	36%
36%	Company's Image	7%
29%	Flexibility	0
0	Sense of Purpose	0
7%	Leadership	21%
70%	Work Culture	7%
0	Work Relationships	0
21%	Quality of Work life	0
NA	Higher Studies	42%
NA	Overseas Assignments	64%
NA	Personal Needs	29%

29. The single most popular factor to encourage employees to leave an organisation is :
    (A) Bad work culture
    (B) Not enough training
    (C) The attraction of foreign assignments
    (D) The desire to pursue higher studies

30. For how many factors listed in the table, is the percentage for employees leaving the organization greater than the percentage for employees liking an organization ?
    (A) 2 (B) 3
    (C) 4 (D) 5

31. If the job content and work culture are right, approximately what percentage of the employees would be happy in an organization ?
    (A) 90% (B) 30%
    (C) 75% (D) 50%

32. Which factor, other than work relationships, seems to have no significant bearing on the employee liking or leaving an organization ?
    (A) Work culture
    (B) Flexibility
    (C) Sense of purpose
    (D) Quality of worklife

33. The percentage for all factors contributing to an employee leaving an organization can be expressed as multiples of 7 (including 0), plus or minus 1. There is only one unit of 7, which does not occur in the data. What is that ?
    (A) 7 (B) 21
    (C) 14 (D) 56

Solution

1. (C) $\frac{200}{2000} = 10\%$
2. (D) Not possible to determine exact figure.
3. (B) All cells in the grid of first three row and last three column, would fulfill both the conditions.
   $= 36 + 52 + 8 + 44 + 20 + 18 + 112 + 36 + 18$
   $= 344$
4. (D) $36 + 44 + 52 + 20 + 8 + 18 + 178$
5. (B)
   $$1970\text{-}71 = \frac{3\cdot42}{16\cdot9 + 2\cdot9} - \frac{0\cdot59}{3\cdot1 + 1\cdot4}$$
   $$\cong 0\cdot17 - 0\cdot13 = 0\cdot04$$
   $$1973\text{-}74 = \frac{4\cdot22}{23\cdot4} - \frac{0\cdot73}{6\cdot1}$$
   $$\cong 0\cdot18 - 0\cdot12 = 0\cdot06$$
   $$1974\text{-}75 = \frac{5\cdot2}{27\cdot1} - \frac{0\cdot78}{7\cdot2}$$
   $$\cong 0\cdot19 - 0\cdot11 = 0\cdot08$$
   $$1975\text{-}76 = \frac{5\cdot44}{28} - \frac{0\cdot84}{7\cdot1}$$
   $$\cong 0\cdot19 - 0\cdot12 = 0\cdot07$$
   So, it is the highest in 1974-75.
6. (C) Or the ratio of gross cropped area to consumption of fertilizers should be highest
   $$1970\text{-}71 = \frac{174\cdot8}{1\cdot11 + 3\cdot24 + 0\cdot59}$$
   $$= \frac{174\cdot8}{5\cdot12}$$
   $$1971\text{-}72 = \frac{173\cdot1}{5\cdot51}$$
   $$1974\text{-}75 = \frac{180\cdot4}{7\cdot71}$$
   $$1972\text{-}73 = \frac{177}{6\cdot06}$$
   $$1975\text{-}76 = \frac{187\cdot8}{8\cdot22}$$
7. (C) In 1972-73 = 23·20 + 32·77 = 55·97 and In 1973-74 = 24·00 + 34·20 = 58·20
   More = 2·23 (remember these figures are cumulative figures)

8. (C) If you look at the trend, area under irrigation has been increasing over the year in minor as well as major. Only in 1974-1975, in case of minor, it has decreased which suggests that some minor area has came under major.

9. (A) For the minimum cost of transportation, starting from B and ending at C; we should move to that station from B that will incur the minimum cost. From B, moving to A will cost Rs. 12×40 = Rs. 480 while moving to D will cost Rs. 16×60 = Rs. 960.

   Thus, from B we should move to A and the desired rate is B–A–D–C at a cost = $40 \times 12 + 30 \times 10 + 20 \times 8$ = Rs. 940.

10. (B) Using the same logic as in question 13, the two possible routes are C–B–A–D and C–D–A–B because from C the minimum cost is to B and D and is equal to Rs.160. Cost of route CBAD = $20 \times 8 + 40 \times 12 + 30 \times 10$ = Rs. 940. Similarly, cost of route CDAB is Rs 940. Thus total minimum cost is Rs. 940.

11. (B) Cost of route ABCDA = $40\times12+20\times8+20\times8$ (Instead of 60×16)+30×10 =Rs.1100.

12. (C) Using the same logic as in question 13, the shortest distance will be for the route ABCD and the distance = 70 Km.

13. (B) The change in readership from 1980 to 1985 for :

    Business …… increases from to $\frac{400}{6}$ to $\frac{550}{8}$

    Films …… decreases from to $\frac{1400}{7}$ to $\frac{2100}{11}$

    Sports …… increases from to $\frac{600}{5}$ to $\frac{1000}{8}$

    General …… increases from to $\frac{2200}{22}$ to $\frac{3100}{29}$

14. (C) Total number of magazines in 1980 is 40 and in 1990, it is 65. So the number of new magazines = 25

15. (A) Between 1980 and 1985, the number of new Hindi magazines is 8 and that of English is also 8.

    Hence, required ratio = 8/8 = 1

16. (B) Average readership per magazine in 1990 = $\frac{885000}{65}$ = 13,615.

17. (D) Consider only the goods (not the services). The answer is (D) since the country from which the man starts is not given. Thus travel costs cannot be as certain .

18. (A) Average cost of company

    $= \frac{1316 + 1348 + 917 + 1208 + 1267}{5}$

    = \$ 1211·2

    In CC = $\frac{1211 \cdot 2}{2}$ \$ = 605·6

19. (A) Calculate total of all the goods and services for all the countries.

China	15,103·52
India	15,107·86
Sri Lanka	18,583·82
Malaysia	18,813·17

    Absolute difference between China and Malaysia = 18813·17 – 15103·52 = \$ 3710

    Absolute difference between Malaysia and Sri Lanka = 18813·17 – 18583·82 = \$ 230

    Absolute difference between Sri Lanka and China = 18583·82 – 15103·52 = \$ 3480

    Absolute difference between China and India = 15107·86 – 15103·52 = \$ 4

20. (B) Total cost = 1 day rent of Santro Car + Petrol Cost (in 5 countries)

    = $(154 + 110 + 103 + 243 + 113) + 84 \times (0{\cdot}93 + 1{\cdot}0) + 0{\cdot}87 + 0{\cdot}94 + 0{\cdot}73)$ = $723 + (84 \times 4{\cdot}5)$

    = \$ 1101

21. (C) 18

22. (C) $\frac{6}{18} \times 100 = 33\frac{1}{3}$

23. (A) 24,000 – 20,000 = 4,000

24. (E) Suppose 490 = X% of 250. Then

    $$X\% = \frac{490}{250} \times 100 = 200\%.$$

25. (A) $\frac{50}{500} \times 100 = 10.$

26. (E)

27. (D) Percentage of the qualified to the appeared in Engineering discipline in 2004 = $\frac{270}{801} \times 100 = 33{\cdot}71.$

    In 2005 = 37·43, In 2006 = 40·43, In 2007 = 36·71, In 2008 = 46·88 and In 2009 = 41·58

28. (C) Percentage of the qualified to the appeared in all the disciplines together in 2004 = $\frac{1254}{4629} \times 100 = 27{\cdot}09.$

    In 2005 = 27·18, In 2006 = 28·43, In 2007 = 31·08, In 2008 = 30·20 and In 2009 = 30·44.

29. (C)

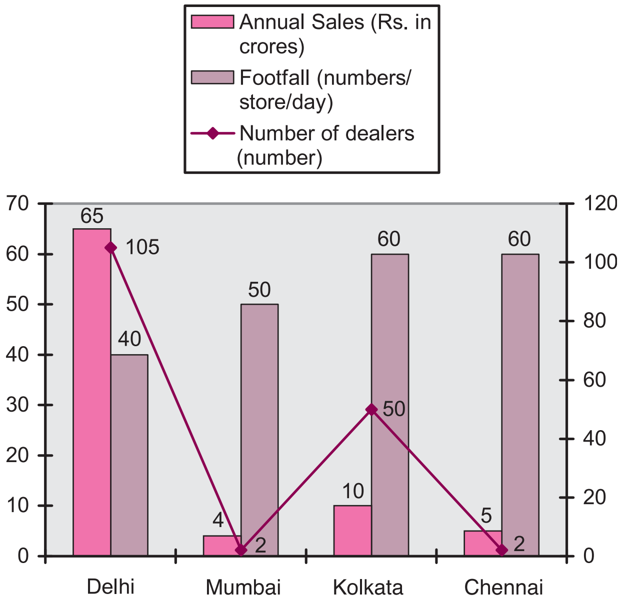
30. (C) Compensation, Leadership

31. (D) 32. (C)

33. (C) 7 & 21%, 57% (+ 1) occur. 13% or 14% or 15% does not occur.

Exercise–3

Directions (Q. 1 to 4) : The Line-bar chart below provides certain data regarding Hero Honda's dealers in four metro cities of India for the year ending March 31, 2009. Footfall is defined as the number of customers visiting each store per day. Assume each of the stores to be opened on all 365 days during the year. Also assume every customer visiting each of the stores in a day to be different.



1. Which of the four city had the highest average sales per dealer in the year ending March 31,2009 ?
   (A) Delhi
   (B) Mumbai
   (C) Kolkata
   (D) Chennai
   (E) Cannot be determined

2. Which of the four city served the maximum number of customers per day during the year ending March 31, 2009 ?
   (A) Delhi
   (B) Mumbai
   (C) Kolkata
   (D) Chennai
   (E) Cannot be determined

3. Delhi plans to expand its chain to 250 dealers by the next year end. If it continues to have the same foot-fall (as for the year ending march 31, 2009), what will be its projected annual sales for the year ending March 31, 2010 ?
   (A) Rs. 145 crore
   (B) Rs. 155 crore
   (C) Rs. 165 crore
   (D) Rs. 135 crore
   (E) Data insufficient

4. What is the approximate average amount spent by a customer per visit at a Kolkata dealer during the year ending March 31, 2009 ?
   (A) Rs. 40·20
   (B) Rs. 91·00
   (C) Rs. 67·5
   (D) Rs. 55·5
   (E) Rs. 35·8

5. What percentage of cities within 10 E to 40 E lie in the southern hemisphere ?
   (A) 15%
   (B) 20%
   (C) 25%
   (D) 30%

Directions (Q. 5 to 7) : The questions are based on the table :

Country	Capital	Longitude	Latitude	Country	Capital	Longitude	Latitude
Argentina	Buenos Aires	34 S	58 E	Ireland	Dublin	53 N	6 E
Australia	Canberra	35 S	149 E	Libya	Tripli	32 N	13 E
Austria	Vienna	48 N	16 E	Malaysia	Kuala Lampur	4 N	101 E
Bulgaria	Sofia	42 N	23 E	Peru	Lima	12 S	77 E
Brazil	Brasilia	15 S	48 E	Poland	Warsaw	52 N	21 E
Canada	Ottawa	45 N	75 E	New Zealand	Wellington	41 S	174 E
Cambodia	Phnom Penh	11 N	105 E	Saudi Arabia	Riyadh	24 N	46 E
Ecuador	Quito	0 S	78 E	Spain	Madrid	40 N	3 W
Ghana	Accra	5 N	1 E	Sri Lanka	Colombo	7 N	80 E
Iran	Tehran	35 N	51 E	Zambia	Lusaka	15 S	28 E

6. Number of capital cities name starting with consonants in the northern hemisphere in the table.
   (A) Exceed the number of cities starting with consonants in the southern hemisphere by 1
   (B) Exceed the number of cities starting with consonants in the southern hemisphere by 2
   (C) Is less than the number of cities starting with consonants in the east of the meridian by 1
   (D) Is less than the number of cities name starting with consonants in the east of the meridian by 2

7. What is the ratio of names of country starting from a vowel and situated in southern hemisphere to the number of countries whose capital begin with a vowel is—
   (A) 3 : 2 (B) 3 : 3
   (C) 3 : 1 (D) 4: 3

(A) Spain (B) Africa
(C) Far East (D) None of these

12. Which of the following is not true ?
   (A) Spain has higher efficiency in 99 then 98
   (B) North America profitability has decreased from 98 to 99
   (C) Efficiency of Far East has increased from 99 to 2000
   (D) None of these

13. For which of the following, the profitability has shown maximum % increase in year 98-99 ?
   (A) Spain (b) Africa
   (C) North Africa (D) Argentina

14. For how many countries profit before tax increase in every year in given period ?
   (A) 3 (B) 4
   (C) 5 (D) None of these

Directions (Q. 8–14) : Following table gives the data about operations performed by a company in different countries.

	Year	Total	Spain	Africa	Far East	North America	Australia	Argentina	North Sea	Other World
Income	98	3790	169	106	408	709	690	806	300	611
	99	2832	91	215	340	680	306	454	716	30
	2000	4357	324	660	1354	1094	66	224	409	216
Expenses	98	2996	129	53	340	594	409	774	82	645
	99	1372	38	142	296	450	2	298	146	44
	2000	2960	205	465	1024	818	126	115	23	184
Profit Before Tax	98	794	40	55	68	115	281	62	218	– 34
	99	1460	53	73	44	230	304	256	570	– 14
	2000	1397	119	195	230	276	– 60	119	396	30
Tax and Charges	98	487	16	30	33	63	209	31	105	—
	99	727	30	35	21	137	176	134	280	—
	2000	888	59	153	184	218	—	72	196	6
Net Profit After Tax and Charges	98	307	24	20	35	52	72	31	113	– 32
	99	733	23	38	23	93	128	122	290	– 14
	2000	509	60	42	46	58	– 60	47	200	24

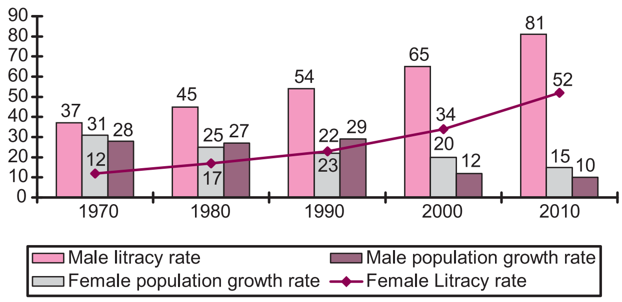
8. How many operation has increase of more than 200% in expenses from 1999-2000 ?
   (A) 3 (B) 4
   (C) 5 (D) None of these

9. If ratio of net profit to tax and charges is defined as profitability then which of the following has shown highest profitability in 99 ?
   (A) Far East (B) North America
   (C) Argentina (D) North Sea

10. How many operations have less then 5% contribution to total revenue in 98 ?
   (A) 4 (B) 5
   (C) 6 (D) None of these

11. If income to expenses ratio is defined as efficiency, which operation has the least efficiency for year 99 ?

Directions (Q. 15 to 18) : In 1960, population of the continent Oceania was 16·4 million and male to female ratio was 1050 : 1000. The following graph shows population growth rate over the pervious decade and total literacy rate in a particular year.



15. What is the total population of continent Oceania in 1990 ?

(A) 41 million

(B) 34 Million

(C) 44 million

(D) 47 million

(E) 37 million

16. In how many decades, population of female is more than that of male ?

(A) 5 (b) 1

(C) 2 (D) 0

(E) 3

17. What percentage of population was literate in 1980 ?

(A) 62·4% (B) 41·5%

(C) 31·6% (D) 26·7%

(E) 50%

18. Which of the following is true ?

(A) Male to female ratio was 15:10 in 1980

(B) Number of male illiterates is more than number of female illiterates in 1970.

(C) Population is showing an increasing trend per year for last 40 years.

(D) Male population growth is always more than female population growth

(E) None of these

Directions (Q. 19–22) : Answer the questions on the basis the following table :

Below is a table that lists countries region-wise. Each region-wise list is stored, first birth rate and than alphabetically by the name of the country. We now wish to merge the region-wise list into one consolidated list and provide overall rankings to each country based first on birth rate and then on death rate. Thus, if some countries have the same birth rate, then the country with a lower death rate will be ranked higher. Further, countries have identical birth and death rates will get the same rank. For example, if two countries are tied for the third position, then both will be given rank 3, while the next country (in the ordered list) will be ranked 5.

Rank	Country	Birth Rate	Death Rate	Region
1	South Africa	36	12	Africa
2	Egypt	39	13	Africa
3	Cameroon	42	22	Africa
4	Mozambique	45	18	Africa
5	Zaire	45	18	Africa
6	Ghana	46	14	Africa
7	Angola	47	23	Africa
8	Madagascar	47	22	Africa
9	Morocco	47	16	Africa
10	Tanzania	47	17	Africa
11	Ethiopia	45	23	Africa
12	Ivory Coast	43	23	Africa
13	Rhodesia	48	14	Africa
14	Uganda	48	17	Africa
15	Nigeria	49	22	Africa
16	Saudi Arabia	49	19	Africa
17	Sudan	49	17	Africa
18	Algeria	50	16	Africa
19	Kenya	50	14	Africa
20	Upper Volta	50	28	Africa
1	Japan	16	6	Asia
2	Korea (ROK)	26	6	Asia
3	Sri Lanka	26	9	Asia
4	Taiwan	26	5	Asia
5	Malaysia	13	6	Asia
6	China	31	11	Asia
7	Thailand	34	10	Asia
8	Turkey	34	12	Asia

9	India	36	15	Asia
10	Burma	38	15	Asia
11	Iran	42	12	Asia
12	Vicuiam	42	17	Asia
13	Korea (DPRK)	43	12	Asia
14	Pakistan	44	14	Asia
15	Nepal	46	20	Asia
16	Bangladesh	47	19	Asia
17	Syria	47	14	Asia
18	Iraq	48	14	Asia
19	Afghanistan	52	30	Asia

Rank	**Country**	**Birth Rate**	**Death Rate**	**Region**
1	Germany (FRG)	10	12	Europe
2	Austria	12	13	Europe
3	Belgium	12	12	Europe
4	Germany (DRG)	12	14	Europe
5	Sweden	12	11	Europe
6	Switzerland	12	9	Europe
7	U.K.	12	12	Europe
8	Netherlands	13	8	Europe
9	France	14	11	Europe
10	Italy	14	10	Europe
11	Greece	16	9	Europe
12	Bulgaria	17	10	Europe
13	Hungary	18	12	Europe
14	Spain	18	8	Europe
15	USSR	18	9	Europe
16	Yugoslavia	18	8	Europe
17	Czech. Rep.	19	11	Europe
18	Portugal	19	10	Europe
19	Romania	19	10	Europe
20	Poland	20	9	Europe
1	U.S.A.	15	9	N. America
2	Canada	16	7	N. America
3	Cuba	20	6	N. America
4	Mexico	40	7	N. America
1	Australia	16	8	Pacific
2	Philippines	34	10	Pacific
3	Indonesia	38	16	Pacific
1	Argentina	22	10	S. America
2	Chile	22	7	S. America
3	Colombia	34	10	S. America
4	Brazil	36	10	S. America
5	Venezuela	36	6	S. America
6	Guatemala	40	14	S. America
7	Peru	40	13	S. America
8	Ecuador	42	11	S. America

19. In the consolidated list, what would be the overall rank of the Philippines ?

(A) 32 (B) 33

(C) 34 (D) 35

20. In the consolidated list, how many countries would rank below Spain and above Taiwan ?

(A) 9 (b) 8

(C) 7 (D) 6

21. In the consolidated list, which country ranks 37th ?

(A) South Africa

(B) Brazil

(C) Turkey

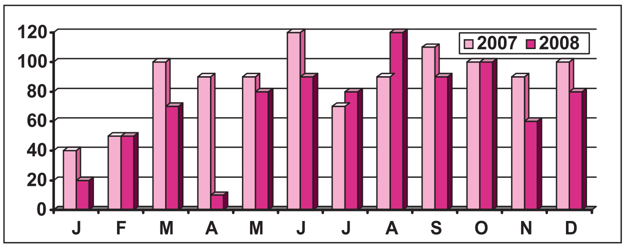
(D) Venezuela

22. In the consolidated list, how many countries in Asia will rank lower than every country in South America, but higher than at least one Country in Africa ?

(A) 8 (B) 7

(C) 6 (D) 5

Directions (Q. 23 to 26) : Production index for 2007 and 2008; Base year : 2000 index = 100



23. In how many months is the production in 2008 is greater than that in the corresponding months of 2007 ?

(A) 1 (B) 2

(C) 3 (D) 4

(E) 5

24. The largest difference between the indices of 2007 and 2008 is in the month of :

(A) January (B) April

(C) July (D) October

(E) December

25. As against 2007, production performance in 2008 is :

(A) Almost same

(B) Somewhat better

(C) Inferior

(D) Consistent

(E) Can not say

26. What is the approximate ratio of the highest and the lowest values of the index in 2008 ?

(A) 1 : 1 (B) 2 : 1

(C) 3 : 1 (D) 1 : 2

(E) None of these

27. Which of the following had the least cost per room ?

(A) Lokhandwala group

(B) Raheja group

(C) IHCL

(D) ITC

28. Which of the following has the maximum number of room per crore of rupees ?

(A) IHCL

(B) Raheja Group

(C) Lokhandwala Group

(D) ITC

29. What is the cost incurred for projects completed in 1998 ?

(A) Rs. 475 crore

(B) Rs. 500 crore

(C) Rs. 522·5 crore

(D) Rs. 502·2 crore

30. What is the cost incurred for project completed in 1999 ?

(A) Rs. 1282·6 crore

(B) Rs. 1270 crore

(C) Rs. 1805·1 crore

(D) Rs. 1535 crore

31. What is the approximate cost incurred for projects completed by 2000 ?

(A) Rs. 1785 crore

(B) Rs. 2140 crore

(C) Rs. 2320 crore

(D) None of these

Directions (Q. 27–31) : Answer the questions based on the following table.

Hotels in Mumbai

Projects	No. of Rooms	Cost (Rs. in crores)	Year of Completion	Company
Windsor Manor	600	275	1999	IHCL
Leela Hotels	310	235	1999	Leela Hotels
Mumbai Heights	250	250	1998	Bombay Hotels
Royal Holidays	536	225	1998	Lokhndwala Group
Majestic Holiday	500	250	1999	Raheja Group
Supremo Hotel	300	300	1999	ITC
Hyatt Regency	500	250	2000	Asian Hotels

Directions (Q. 32 to 36) : Answer the questions based on the following informations, which gives data about certain coffee producers in India :

	Production ('000 tonnes)	Capacity Utilisation (%)	Sales ('000 tonnes)	Total Sales Value (Rs. in crores)
Brooke Bond	2.97	76.50	2.55	31.15
Nestle	2.48	71.20	2.03	26.75
Lipton	1.64	64.80	1.26	15.25
MAC	1.54	59.35	1.47	17.45
Total (including others)	11.60	61.30	10.67	132.80

32. What is the maximum production capacity (in '000 tonnes) of Lipton for coffee ?
    (A) 2·53 (B) 2·85
    (C) 2·24 (D) 2·07

33. Which company out of the four companies mentioned above has the maximum unutilized capacity (in '000 tonnes) ?
    (A) Lipton (B) Nestle
    (C) Brooke bond (D) MAC

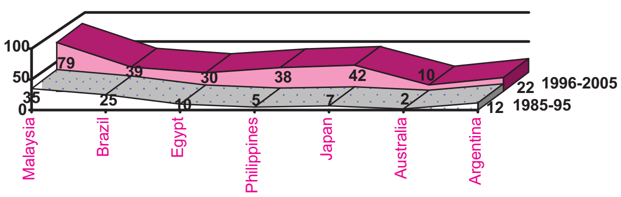
34. What is the approximate total production capacity (in '000 tonnes) for coffee in India ?
    (A) 18 (B) 20
    (C) 18·7 (D) Data insufficient

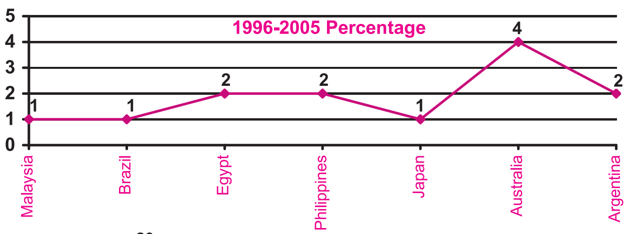
35. The highest price for coffee per kilogram is for—
    (A) Nestle (B) MAC
    (C) Lipton (D) Data insufficient

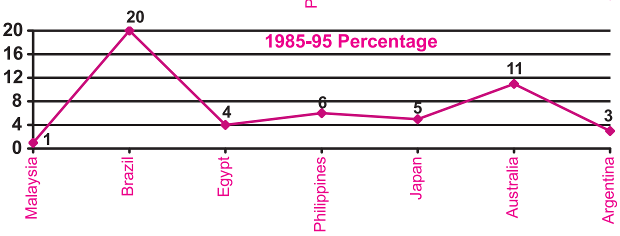
36. What per cent of the total market share (by sales value) is controlled by 'others' ?
    (A) 60% (B) 32%
    (C) 67% (D) Data insufficient

Direction (Q. 37 to 41) :

Market share of JRD International inc.







Above graph gives the market share and percentage of JRD International inc., over two different periods in different countries. The figures i n brackets give the market position of JRD International inc. during the given period for the given countries. You may assume that the company does not operate in any other countries.

37. In what percentage of countries has the market share shown a growth over the two given periods ?
    (A) 71·44% (B) 100%
    (C) 85·7% (D) 14·28%
    (E) 150%

38. In how many countries has the percentage growth rate of market share exceeded 75% over the two given periods ?
    (A) 7 (B) 5
    (C) 6 (D) 8
    (E) None of these

39. If total business done by JRD International Inc. in the period 1985-95 was Rs. 500 crore, the total market size for the given period for the given countries was—
    (A) Rs. 1,650 crore (B) Rs. 400 crore
    (C) Rs. 2,050 crore (D) Rs. 1,200 crore
    (E) Cannot be determined

40. If the business done by JRD International Inc. in Australia in 1985-95 was Rs 9 crore, total market size of Australia was—
    (A) Rs. 450 crore
    (B) Rs. 400 crore
    (C) Rs. 550 crore
    (D) Rs. 425 crore
    (E) Rs. 475 crore

41. The business volume for JRD International Inc. during 1996-2005 in Egypt and Australia were in the ratio 1 : 2; find the market size of Egypt if JRD International Inc. did a business of Rs. 5 crore in Australia.
    (A) Rs. 33·3 crore
    (B) Rs. 8·33 crore
    (C) Rs. 8 crore
    (D) Rs. 25 crore
    (E) Rs. 6·33 crore

Directions (Q. 42 to 44) : Answer the questions based on the following information :

The following table gives the tariff [in paise per kilo-watt-hour (kWh) levied by the UPSEB in 1994-95, in four sectors and the regions within them. The table also gives the percentage change in the tariff as compared to 1991-92.

	Region 1		Region 2		Region 3		Region 4		Region 5	
	p/kWh	% increase	p/kWh	% increase	p/kWh	% increase	p/kWh	% increase	p/kWh	% increase
Sector 1	425	+ 15	472	+ 5	420	– 4	415	+ 8	440	+ 10
Sector 2	430	+ 12	468	+ 8	448	+ 7	423	– 3	427	+ 11
Sector 3	428	+ 8	478	– 4	432	+ 6	441	+ 10	439	+ 8
Sector 4	434	– 5	470	+ 15	456	+ 10	451	+ 12	446	– 12

Additional directions : The UPSEB supplies power under four categories; urban (25%), domestic (20%), industrial (40%) and rural (15%). In 1994-95, the total power produced by UPSEB was 7875 megawatts.

42. If in 1994-1995, there was a 10% decrease in the domestic consumption of power as compared to that in 1991-92, what was the consumption of Power in the rural sector in 1991-92 ?
(A) 1312 megawatt (B) 1422 megawatt
(C) 1750 megawatt (D) None of these

43. In the given two years, what is the total tariff paid by the urban sector ?
(A) Rs. 22·4 lack (B) Rs. 21·6 lack
(C) Rs. 27·2 lack (D) Cannot be determined

44. Which of the following statements is true ?
(A) The average tariff in region 4 is 437·5 p/kWh.
(B) The average tariff in region 2 is greater than the average tariff in region 5
(C) In 1991- 92, the industrial sector contributed to about 42% of the total revenue from power
(D) None of these

45. The athletes from FRG and USE decided to run a 4 × 100 m relay race for their respective countries with the country having three athletes borrowing the athlete from CZE. Assume that all athletes ran their stretch of the relay race at the same speed as in Decathlon event. How much more time did the FRG relay team take as compared to the USA team ?
(A) 0·18 (B) 0·28
(C) 0·78 (D) 0·00

46. What is the least that Daley Thompson must get in score 2 that ensure him a bronze medal ?
(A) 5309 (B) 5296
(C) 5271 (D) 5270

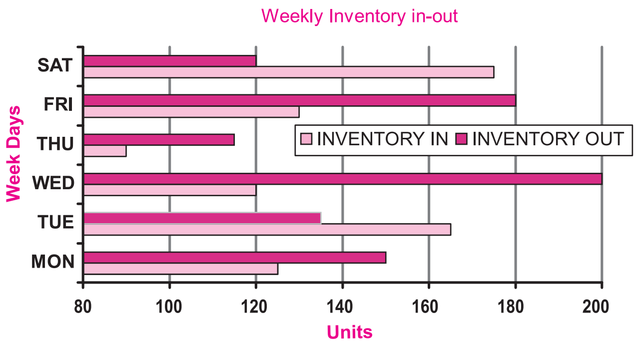
47. At least how many competitors (excluding Daley Thomson) must Michael Smith have out jumped in the long jump event ?
(A) 1 (B) 2
(C) 3 (D) 4

Directions (Q. 45 to 47) : Answer the following questions on the basis of following information :

In a Decathlon, the events are 100 m, 400 m, 100 m Hurdles, 1500 m, and High jump, Pole vault, Long jump, Discus, Shot put and Javelin. The performance in the first four of these events is consolidated into score 1, the next three into score 2, and the last three into the score 3. Each such consolidation is obtained by giving appropriate positive weights to individual's events. The final score is simply the total of these three scores. The athletes with the highest, second highest and the third highest final scores receive the gold, silver and bronze medals, respectively. The table given below gives the scores and performance of nineteen top athletes in this event.

Name	Country	Final Score	Score 1	Score 2	Score 3	100 m	High jump	Polevault
Eduard Hamalainen	BLS	8802	491	5322	2989	10.74	2.1	4.8
Michael Smith	CAN	8855	174	5274	3407	11.23	2.0	4.9
Thomas Dvorak	CZE	8796	499	5169	3128	10.63	1.9	4.7
Uwe Frrimuth	DDR	8799	441	5491	3124	11.06	2.0	4.8
Torsten Voss	DHK	8880	521	5234	5868	10.69	2.1	5.1
Erki Nool	EST	8768	408	5553	2808	10.71	2.0	5.4
Christian Plaziat	FRG	8775	563	5430	2781	10.72	2.1	5.0
Jurgen Hingsen	FRG	8792	451	5223	3033	10.95	2.0	4.9
Siegfried Wentz	FRG	8856	470	5250	3137	10.85	2.1	4.8
Guido Kratschmer	FRG	8861	575	5308	3064	10.58	2.0	4.6
Daisy Thompson	GBR	—	582	—	3003	10.55	2.1	4.6
Frank Busemann	SOV	8805	568	5392	2945	10.60	2.0	4.8
Alexander Apaichev	SOV	8803	492	5370	3115	10.92	2.0	4.8
Grigory Degtyarov	TCH	8823	339	5196	3114	11.05	2.1	4.9
Robert Zmelik	USA	8832	494	5455	2883	10.78	2.1	5.1
Dave Johnson	USA	8811	366	5370	3114	10.78	2.1	5.0
Steve Frritz	USA	8827	427	5163	3119	10.75	2.0	5.0
Bruce Jenner	USA	8846	483	5280	3200	10.94	2.0	4.8
Dan O'Brien	USA	8897	408	5331	3120	10.36	2.1	4.8

Directions (Q. 48 to 52) :



The Above Bar graph shows inventory in and out from SATYAM AUTO during any week. Sunday is a holiday and no transaction is done on that day. If on the first day of the month, *i.e.*, Monday 1st June the company has units in stock, then

48. What will be the units after one week ?
    (A) 1235 (B) 1065
    (C) 1095 (D) 1265
    (E) 1100

49. How many days will the inventory last ?
    (A) 80 (B) 79
    (C) 78 (D) 81
    (E) None of these

50. Which day shows the biggest change in inventory as compared to the previous day ?
    (a) Monday (B) Saturday
    (C) Wednesday (D) Friday
    (E) Tuesday

51. If Mr Anil Datta GM Operation said, the inventory can be supplied only on Monday, Wednesday and Friday, and inventory goes out only on Tuesday, Thursday and Saturday, then what inventory will remain after 15th June of the same year ?
    (A) 1210 (B) 1190
    (C) 1325 (D) 1335
    (E) 1265

52. If company orders a special inventory of 200 unit when the total stock goes to 600 units or below, then on which one of the following days will the company order a special inventory ?
    (A) 9 July (B) 8 July
    (C) 10 July (D) 11 July
    (E) 12 July

Directions (Q. 53 to 55) : Answer the following question on the basis of the table given below :

	1901	**1911**	**1921**	**1931**	**1941**	**1951**	**1961**	**1971**	**1981**	**1991**	**2001**
AP	985	992	993	987	980	986	981	977	975	972	978
Assam	919	915	896	874	875	868	869	896	910	923	932
Bihar	1061	1051	1021	995	1002	1000	1005	957	948	907	921
Goa	1091	1108	1120	1088	1084	1128	1066	981	975	967	960
Gujarat	954	946	944	945	941	952	940	934	942	934	921
Haryana	867	835	844	844	869	871	868	867	870	865	861
HP	884	889	890	897	890	912	938	958	973	976	970
J & K	882	876	870	864	869	873	878	878	892	896	900
Karnataka	983	981	969	965	960	966	959	957	963	960	964
Kerala	1004	1008	1011	1022	1027	1028	1022	1016	1032	1036	1058
MP	972	967	949	947	946	945	932	920	921	912	920
Maharastra	978	966	950	947	949	941	936	930	937	934	922
Orissa	1037	1056	1086	1067	1053	1022	1001	988	981	971	972
Punjab	832	780	799	815	826	844	854	865	879	882	874
Rajasthan	905	908	896	907	06	921	908	911	919	910	922
TN	1044	1042	1029	1027	1012	1007	992	978	977	974	986
UP	938	916	908	903	907	998	907	876	882	876	898
WB	945	925	905	890	852	865	878	891	911	917	934
India	**972**	**964**	**955**	**950**	**945**	**946**	**941**	**930**	**934**	**927**	**933**

53. Each of the following statements pertains to the number of states with females out numbering males in a given census year. Which of these statements is NOT correct ?

(A) This number never exceeded 5 in any census year

(B) This number registered its sharpest decline in the year 1971

(C) The number of consecutive census in which this number remained unchanged never exceeded

(D) Prior to the 1971 census, this number was never less than 4

54. The two states which achieved the largest increase in sex ratio over the period 1991-2001 are :

(A) Punjab and HP

(B) HP and Kerala

(C) Assam and J & K

(D) Kerala and J & K

55. Among the states which have a sex ratio exceeding 1000 in 1901, the sharpest decline over the period 1901-2001 was registered in the state of :

(A) Goa (B) TN

(C) Bihar (D) Orissa

56. How many international airports of type 'A' account for more then 40 million passenger ?

(A) 4 (B) 5

(C) 6 (D) 8

57. What percentage of top ten busiest airports is in the United States of America ?

(A) 60 (B) 80

(C) 70 (D) 90

58. Of the five busiest airports, roughly what percentage of passengers is handled by Heathrow airport ?

(A) 30 (B) 40

(C) 20 (D) 50

59. How many international airports, not located in the USA; handle more then 30 million passengers ?

(A) 5 (B) 6

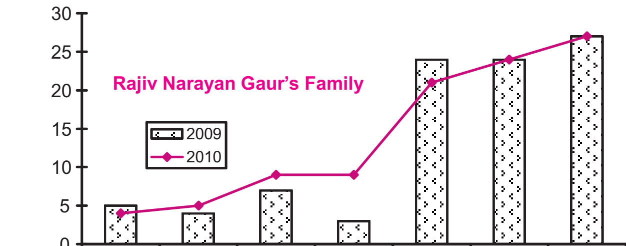
(C) 10 (D) 14

Direction (Q. 60 to 64) : The following graph shows the break-up of the expenditure of Rajiv Narayan Gaur family for 2 year. In each year, the break-up is given as a percentage of gross annual income of Rajiv Narayan Gaur family for that year.

Directions (Q. 56–59) : Answer these questions based on the table given below concerning the busiest twenty international airports in the world :

No.	Name	International Airport Type	Code	Location	Passengers
1	Hartsfield	A	ATL	Atlanta, Georgia, USA	77939536
2	Chicago – 0' Hare	A	ORD	Chicago, Illinois, USA	72568076
3	Los Angeles	A	LAX	Los Angeles, California, USA	63876561
4	Heathrow Airport	E	LHR	London, United Kingdom	62263710
5	DFW	A	DFW	Dallas/ Ft. Worth, Texas, USA	60000125
6	Haneda Airport	F	HND	Tokyo, Japan	54338212
7	Frankfurt Airport	E	FRA	Frankfurt, Germany	45858315
8	Roissy - Charles	E	CDG	Paris, France	43596943
9	San Francisco	A	SFO	San Francisco, California, USA	40387422
10	Denver	A	DIA	Denver, Colorado, USA	38034231
11	Amsterdam Schiphol	E	AMS	Amsterdam, Netherlands	36781015
12	Minneapolis - St. Paul	A	MSP	Minneapolis-St. Paul, USA	34216331
13	Detroit Metropolitan	A	DTW	Detroit, Michigan, USA	34038381
14	Miami	A	MIA	Miami, Florida, USA	33899246
15	Newark	A	EWR	Newark, New jersey, USA	33814000
16	McCarran	A	LAS	Las Vegas, Nevada, USA	33669185
17	Phoenix Sky Harbor	A	PHX	Phoenix, Arizona, USA	33533353
18	Kimpo	FE	SEL	Seoul, Korea	33371074
19	George Bush	A	IAH	Houston, Texas, USA	33089333
20	John F. Kennedy	A	JFK	New York, USA	32003000

In 2009 gross annual income of Rajiv Narayan Gaur family = Rs. 5 lakh. Also the savings of Rajiv Narayan Gaur family decreased by 20% in 2010 as compared to its value in 2009.



	Auto-mobile Expenses	Entertainment	Food	Miscellaneous	Mortgage and property	Savings	Income taxes
2009	5	4	7	3	24	24	27
2010	4	5	9	9	21	24	27

60. If mortgage accounts for 20% of mortgage and property taxes in 2009, then what is the expenditure on mortgage in 2010 ?
    (a) Rs. 21,200 (B) Rs. 16,800
    (C) Rs. 19,200 (D) Rs. 18,000
    (E) Cannot be determined

61. In 2009, Rajiv Narayan Gaur family used 39% of its gross annual income for two of the categories listed. What was the total expenditure of Rajiv Narayan Gaur family for those same two categories in 2010 ?
    (A) 1·44 Lakh (B) 3·24 Lakh
    (C) 2·89 Lakh (D) 1·96 Lakh
    (E) Unchanged

62. Rajiv Narayan Gaur Family's gross annual income is the sum of only the income of Mr. Rajiv Narayan Gaur and Mrs. Rajiv Narayan Gaur (Renu Gaur). If in 2009 the ratio of gross annual income of Mr. Rajiv Narayan Gaur and Mrs. Renu Gaur was 3 : 1 and in 2010, the income of Mr. Rajiv Narayan Gaur decreased by 33.33% from its value in 2009, then by what per cent did Mrs. Rajiv Narayan Gaur' income increase for the same period ?
    (A) 26%
    (B) 19·25%
    (C) 20%
    (D) 22·22%
    (E) Unchanged

63. Which category has shown the greatest percentage change in expenditure from 2009 to 2010 ?
    (A) Mortgage and property taxes
    (B) Income taxes
    (C) Miscellaneous
    (D) Food
    (E) Automobile expenses

64. Expenditure on food, entertainment and automobile expenses in 2009 is what per cent of the same in 2010 ?
    (A) 112·2% (B) 1·1111%
    (C) 111·11% (D) 13·31%
    (E) 11·111%

Directions (Q. 65 to 68) : The table provides information about the salary and the number of working days of employees in a company. Employee will be paid only if he works with minimum required efficiency.

Employe's Code	Total Salary (in Rs.)				No. of Working Days			
	Complex	Medium	Simple	Total	Complex	Medium	Simple	Total
200040	149	—	50	199	10	0	3	13
200050	236	536	—	722	9	11	00	20
200060	350	—	100	450	11	0	4	15
200070	500	405	76	981	13	6	4	23
200080	600	—	20	620	10	0	11	21
200090	450	700	120	1270	8	10	6	24
200100	550	377	200	1127	9	7	9	25
200110	140	50	176	366	11	2	9	22
200120	250	—	126	376	4	0	1	5
200130	330	100	86	516	9	6	2	17
200140	390	—	56	446	10	0	10	20
200150	360	166	46	572	6	1	1	8
200160	160	—	89	249	15	0	6	21
200170	490	120	129	739	8	5	6	19
200180	1234	600	300	2134	19	2	1	22

65. How many employees are getting more then Rs. 50 per day in complex work ?

(A) 2 (B) 5 (C) 7 (D) None of these

66. Which employee has received maximum salary per day in complex work ?

(A) 200180 (B) 200080 (C) 200170 (D) 200040

67. How many employees are having more then 80% attendence and earning more then Rs. 600 in the month of June which consist of 25 working days ?

(A) 5 (B) 7 (C) 8 (D) 10

68. How many people worked for complex and medium both and earn more in complex work then in medium work ?

(A) 7 (B) 4 (C) 5 (D) 9

Directions (Q. 69 to 76) : Answer the following questions based on the table :

Year	Loans from rural banks			Agricultural loans		
	Number of rural banks	Average number of loans	Average loans (in Rs.)	No. ('000)	Value (Rs. in millions)	Consumer price index
1970	90	28	109	18·3	2·00	43
1971	115	39	133	20·4	3·58	49
1972	130	52	178	25·1	6·26	55
1974	260	98	243	41·2	34·54	70
1975	318	121	283	51·4	52·21	78
1980	605	288	567	135·7	498·4	131
1981	665	312	622	152·8	612·4	137
1983	840	380	711	211.6	915.7	149

69. In 1974, the agricultural loans formed what percent of total loans ?

(A) 85% (B) 71%
(C) 77% (D) Cannot be determined

70. Form the given Data, the number of rural loans upto 1980 formed approximately what percent of those in 1983 ?

(A) 112% (B) 80%
(C) 97% (D) Cannot be determined

71. Which of the following pairs of year showed the maximum increase in the number of rural bank loans ?

(A) 1971-72 (B) 1974-75
(C) 1990-91 (D) 1980-81

72. What is the value of the agricultural loan in 1983 at 1970 price ?

(A) Rs. 326 (B) Rs. 264
(C) Rs. 305 (D) None of these

73. In which year was the number of rural bank loans per rural bank least ?

(A) 1974 (B) 1971
(C) 1970 (D) 1975

74. What is the simple annual rate of increase in the number of agricultural loans 1970 to 1983 ?

(A) 132% (B) 81%
(C) 75% (D) 1056%

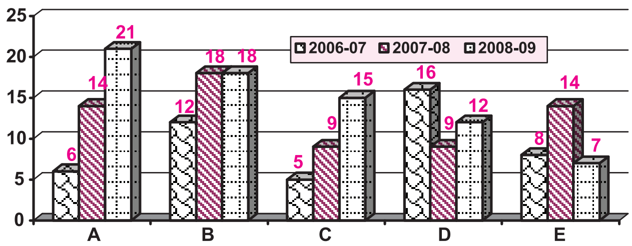
75. By roughly how many points do the indices for the year 1983 and 1975 differ ?

(A) 174 (B) 180
(C) 188 (D) 195

76. What is the value of the loans in 1980 at 1983 price ?

(A) Rs. 570 million (B) Rs. 680 million
(C) Rs. 525 million (D) Rs. 440 million

Directions (Q. 77 to 81) : PRODCTION OF COTTON FOR FIVE STATES A–Rajasthan, B–Karnataka, C–West Bengal, D–Orrisa, E–Assam FOR THE GIVEN PERIOD IN THOUSANDS OF TONS



77. The production of cotton in Orissa in 2007-08 is how many times its production in 2008-09 ?

(a) 1·33 (B) 0·75
(C) 0·56 (D) 1·77
(E) 87·5

78. In which of the state is there a steady increase in the production of cotton during the given period ?

(A) Rajasthan and Karnataka
(B) Rajasthan and West Bengal

(C) Karnataka only

(D) Orissa and Assam

(E) West Bengal only

79. How many quintals of cotton was produced by Assam during the given period ?

(A) 29000 (B) 290000

(C) 2900 (D) 2900000

(E) 290

80. How many states showing below average production in 2006-07 showed above average production in 2007-08 ?

(A) 4 (B) 2

(C) 3 (D) 1

(E) 5

81. Which of the following statements is false ?

(A) Rajasthan and Assam showed the same production in 2007-08

(B) There was no improvement in the production of cotton in state Karnataka in the year 2008-09 compared to that of 2007-08

(C) Rajasthan has producted maximum cotton during the given period

(D) Production of West Bengal and Orissa together in 2007-08 is equal to that of Karnataka during the same period

(E) None of these

82. What was the maximum percentage of apples supplied by any state in any of the month ?

(A) 99% (B) 95%

(C) 88% (D) 100%

83. Which state supplied the maximum apples ?

(A) UP (B) HP

(C) J & K (D) Cold storage

84. Which state supplied the highest percent of apples supplied ?

(A) HP

(B) UP

(C) J & K

(D) Cannot be determined

85. In which of the following period was the supply greater than the demand ?

(A) August-March

(B) June-October

(C) May- September

(D) Cannot be determined

86. If the yield per tree was 40 kg then from how many trees were the apples supplied to New Delhi (in million) during the year ?

(A) 11.5 (B) 12.5

(C) 13.5 (D) Cannot be determine

87. Using in data in question 86, if there was 250 trees per hectare then how many hectares of land was used ?

(A) 9,400 (B) 49,900

(C) 50,000 (D) 49,450

Directions (Q. 82–87) : The following table gives the quantity of apples (in tonnes) arriving at New Delhi market from various states in a particular year. The month, in which demand was more than supply, the additional demand was met by the stock from cold storage.

Month	State			Cold Storage	Total
	HP	UP	J & K		
April	7	0	7	59	73
May	12	1	0	0	13
June	9741	257	8017	0	18015
July	71497	0	18750	0	90247
August	77675	0	20286	0	97961
September	53912	0	56602	0	110514
October	12604	0	79591	24	92219
November	3499	0	41872	42	45413
December	1741	0	14822	15	16578
January	315	0	10922	201	11438
February	25	0	11183	77	11285
March	0	0	983	86	769

Directions (Q. 88 to 91) : Answer the questions based on the table given below :

The following is a table describing garments manufactured based upon the colour and size for each. There are four sizes : M–Medium, L–Large and XXL–Extra-Extra Large. There are three colours: Yellow, Red and White.

Lay	Number of Garments											
	Yellow				Red				White			
Lay No.	**M**	**L**	**XL**	**XXL**	**M**	**L**	**XL**	**XXL**	**M**	**L**	**XL**	**XXL**
1	14	14	7	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	42	42	21	0
3	20	20	10	0	18	18	9	0	0	0	0	0
4	20	20	10	0	0	0	0	0	30	30	15	0
5	0	0	0	0	24	24	12	0	30	30	15	0
6	22	22	11	0	24	24	12	0	32	32	16	0
7	0	24	24	12	0	0	0	0	0	0	0	0
8	0	20	20	10	0	2	0	1	0	0	0	0
9	0	20	20	10	0	0	0	0	0	0	0	11
10	0	20	20	10	0	26	26	13	0	22	22	10
11	0	0	0	0	0	26	26	13	0	20	20	11
12	0	22	22	11	0	0	0	0	0	22	22	0
13	0	0	2	2	0	0	0	0	0	0	0	20
14	0	0	0	0	0	0	0	0	0	0	20	22
15	0	0	0	0	0	0	2	2	0	0	22	22
16	0	0	10	10	1	0	0	0	1	0	22	0
17	0	0	0	0	0	5	0	0	0	0	0	0
18	0	0	0	0	0	32	0	0	0	0	0	0
19	0	0	0	0	0	32	0	0	0	0	0	0
20	0	0	0	0	0	5	0	0	0	0	0	0
21	0	0	0	0	18	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	26	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	22
24	0	0	0	0	8	0	0	1	0	0	0	0
25	0	0	0	0	8	0	0	0	0	0	0	12
26	0	0	0	0	0	0	0	1	0	0	0	14
27	0	0	0	0	8	0	0	2	0	0	0	12
Production	76	162	136	136	97	194	89	59	135	198	195	156
Order	75	162	135	135	97	194	89	59	135	197	195	155
Surplus	1	0	1	1	0	0	0	0	0	1	0	1

88. How many lays are used to produce Yellow coloured fabrics ?

(A) 10 (B) 11 (C) 12 (D) 14

89. How many lays are used to produce Extra-Extra Large fabrics ?

(A) 8 (B) 16 (C) 17 (D) 18

90. How many lays are used to product Extra-Extra Large Yellow Extra-Extra Large White fabrics ?

(A) 8 (B) 9 (C) 10 (D) 15

91. How many varieties of fabrics, which exceed the order, have been produced ?

(A) 3 (B) 4 (C) 5 (D) 6

Directions (Q. 92 to 95) : Answer the questions on the basis of the following information :

The following is the Wholesale Price Index (WPI) of a select list of items with the base year of 1993-94. In other words, all the item prices are made 100 in that year (1993-94). Price in all other years for an item is measured with respect to its price in the base year. For instance, the price of cement went up by 1% in 1994-95 as compared to 1993-94. Similarly, the price of power went up by 3% in 1996-97 as compared to 1993-94.

	1993-94	1994-95	1995-96	1996-97	1997-98	1998-99	1999-00	2000-01	2001-02	2002-03
All Items	100	102·0	102·5	104·00	103·00	105·00	106·00	108·00	107·00	106·00
Cement	100	101·0	100·5	103·00	102·50	103·50	103·10	103·80	103·70	104·00
Limestone	100	102·0	102·5	102·75	102·25	103·00	104·00	105·00	104·50	105·00
Power	100	101·5	102·5	103·00	103·50	104·00	106·00	107·00	107·50	108·00
Steel	100	101·5	101·0	103·50	104·00	104·25	105·00	105·50	106·00	105·50
Timber	100	100·5	101·5	102·00	102·00	102·00	103·00	103·50	104·00	104·50
Wages	100	101·5	103·0	103·50	103·50	104·25	104·00	104·75	104·90	105·30

92. Let us suppose that one bag of cement (50 kgs) consumes 100 kgs of limestones and 100 unit of power. The only other cost item in producing cement is in the form of wages. During 1993-94, limestone, power and wages contribute, respectively, 20%, 25%, and 15% to the cement price per bag. The average operating profit (% of price cement bag) earned by a cement manufacturer during 2002-03 is closest to—

(A) 40% (B) 39·5%

(C) 38·5% (D) 37·5%

93. Steel manufacturing requires the use of iron ore, power and manpower. The cost of iron ore has followed the All Item index. During 1993-94 power accounted for 30% of the selling price of steel, iron ore for 25%, and wages for 10% of the selling price of steel. Assuming the cost and price data for cement as given in the previous question, the operating profit (% of selling price) of an average steel manufacturer in 2002-03—

(A) Is more than that of a cement manufacturer

(B) Is less than that of a cement manufacturer

(C) Is the same as that of a cement manufacturer

(D) Can not be determined

94. Which item experienced continuous price rise during the ten-year period ?

(A) Power (B) Cement

(C) Wages (D) Limestone

95. Which item (s) experienced only one decline in price during the ten-year period ?

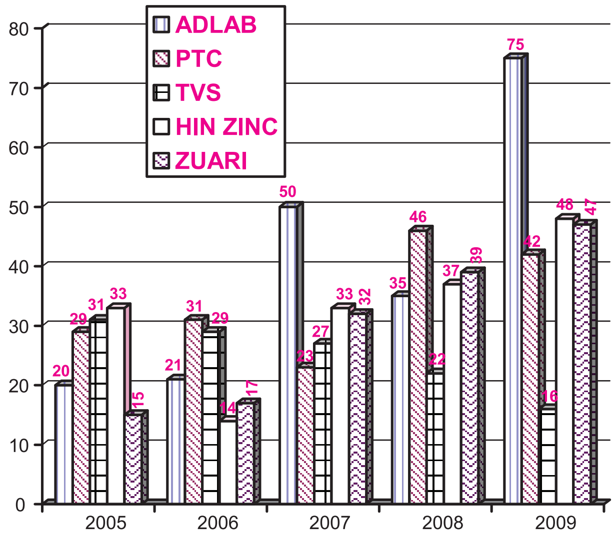
(A) Steel and Limestone

(B) Steel and Timber

(C) Timber

(D) Timber and Wages

Directions (Q. 96 to 100) : Following table showing investments made by five companies over the year (amounts for various year in lakh of rupees) :



96. Which Company's investment is more than 25 per cent of the total investment made by all companies in 2008 ?

(A) Adlab (B) PTC India Ltd

(C) TVS Motors (D) Hindustan Zinc

(E) Zuari Industries Ltd

97. For which company has amount of investment made increased continuously over the years ?

(A) Zuari Industries Ltd

(B) PTC India Ltd

(C) TVS Motors

(D) Hindustan Zinc

(E) Adlab

98. For which company has amount of investment made decreased continuously over the years ?

(A) Adlab (B) PTC India Ltd

(C) TVS Motors (D) Hindustan Zinc

(E) Zuari Industries Ltd

99. In which year was the amount of investment least, as compared to average investment made by all the companies over the years ?

(A) 2005 (B) 2006

(C) 2007 (D) 2008

(E) 2009

100. In which year is the percentage change in the total amount of investments made the highest over its immediately preceding year ?

(A) 2006 (b) 2007

(C) 2008 (D) 2009

(E) 2005

Directions (Q. 101 to 104) : Answer the ques-tions based on the table.

The table shows trends in external transactions of Indian corporate sector during the period 1993-94 to 1997-98. In addition, following definitions hold good :

Sales, Imports and exports respectively denote the sales; Imports respectively denote the sales import and exports in year.

Deficit for year I, Deficit = Imports – Exports

Deficit intensity in year i, DIi = Deficit/Sales

Growth rate of deficit intensity in year I, GDIi = (DIi – Dii – 1)/DIi - 1

Further, note that all imports are classified as either raw material or capital goods.

Trends in External Transactions of Indian Corporate Sector

Year	1997-98	1996-97	1995-96	1994-95	1993-94
Export intensity*	9·2	8·2	7·9	7·5	7·3
Import Intensity*	14·2	16·2	15·5	13·8	12·4
Import raw material / total cost of raw material	20·2	19·2	17·6	16·3	16
Imported capital goods/gross fixed assets	17·6	9·8	11·8	16·3	19·5

101. The highest growth rate in deficit intensity was recorded in—

(A) 1994-95 (B) 1995-96

(C) 1996-97 (D) 1997-98

102. The value of highest growth rate in deficit intensity is approximately—

(A) 8·45% (B) 2·15%

(C) 33·3% (D) 23·5%

103. In 1997-98, the total cost of raw materials I estimated as 50% of sales of that year. The turnover of gross fixed assets, defined as the ratio of sales to gross fixed assets, in 1997-98 is approximately—

(A) 3·3

(B) 4·3

(C) 0·33

(D) Not possible to determined

104. Which of the following statements can be inferred to be true from the given data—

(A) During the 5-year period between 1993-94 and 1997-98, export have increased every year

(B) During the 5-year period between 1993-94 and 1997-98, import have decreased every year

(C) Deficit in1997-98 was lower than that in 1993-94

(D) Deficit intensity has increased every year between 1993-94 and 1996-97

Directions (Q. 105 to 109) : Answer the Questions based on the following information :

Mulayam Software Co., before selling a package to its clients, follows the given schedule.

Month	Stage	Cost (Rs. '000 per man/month)
1–2	Specification	40
3–4	Design	20
5–8	Coding	10
9–10	Testing	15
11–15	Maintenance	10

105. Due to overrun in 'design', the design stage took 3 month, *i.e,* month 3, 4, and 5. The number of people working on design in the fifth month was 5. Calculate the percentage change in the cost incurred in the fifth month. (Due to improved in coding technique, this stage was completed in month 6-8 only)

(A) 225% (B) 150%

(C) 275% (D) 240%

106. With reference to the above question, what is the cost incurred in the new 'coding' stage? (Under the new technique, 4 people work in the sixth month and 5 in the eighth)—

(A) Rs. 140000 (B) Rs. 150000

(C) Rs. 160000 (D) Rs. 190000

107. What is the difference in cost between the old and the new techniques ?

(A) 30000 (B) 60000

(C) 70000 (D) 40000

108. Under the new technique, which stage of software development is most expensive for Mulayam Software Co. ?

(A) Testing (B) Specification

(C) Coding (D) Design

109. Which fiv Consecutive Month have the lowest average cost per man-month under the new technique ?

(A) 1-5 (B) 9-13

(C) 11-15 (D) None of these

Directions (Q. 110 to 114) : Answer these questions with reference to the table :

Information Technology Industry in India

		1994-95	1995-96	1996-97	1997-98	1998-99
Software						
	Domestic	350	490	670	950	1250
	Exports	485	734	1083	1750	2650
Hardware						
	Domestic	590	1037	1050	1205	1026
	Exports	177	35	286	201	4
Peripheral						
	Domestic	148	196	181	229	329
	Exports	6	6	14	19	18
Training		107	143	185	263	302
Maintenance		142	172	182	221	236
Networking and other		36	73	156	193	237
Total		2041	2886	3807	5031	6052

110. The total annual exports lay between 35and 40 per cent to the total annual business of the IT industry, in the year—

(A) 1997-98 and 1994-95

(B) 1996-97 and 1997-98

(C) 1996-97 and 1998-99

(D) 1996-97 and 1994-95

111. The highest percentage growth in the total IT business, relative to the previous year was achieved in—

(A) 1995-96 (B) 1996-97

(C) 1997-98 (D) 1998-99

112. Which one of the following statements is correct ?

(A) The annual software exports steadily increased but annual hardware exports steadily declined during 1994-99

(B) The annual peripheral exports steadily increased during 1994-99

(C) The IT business in training during 1994-99 was higher than the total IT business in maintenance during the same period

(D) None of the above

113. For the IT hardware business activity, which one of the following is not true ?

(A) 1997-98 dominates 1996-97

(B) 1997-98 dominates 1995-96

(C) 1995-96 dominate 1998-99

(D) 1998-99 dominate 1996-97

114. For the two IT business activities, hardware and peripherals, which one of the following is true ?

(A) 1996-97 dominates 1995-96

(B) 1997-98 dominates 1995-96

(C) 1997-98 dominates 1998-99

(D) None of these

Directions (Q.115 to 119) : Answer the questions based on the following information :

The table below presents data on percentage population covered by drinking water and sanitation facilities in selected Asian countries.

Population covered by Drinking Water and Sanitation Facilities

Percentage Coverage

	Drinking Water			Sanitation Facilities		
	Urban	Rural	Total	Urban	Rural	Total
India	85	79	81	70	14	29
Bangladesh	99	96	97	79	44	48
China	97	56	67	74	7	24
Pakistan	82	69	74	77	22	47
Philippines	92	80	86	88	66	77
Indonesia	79	54	62	73	40	51
Sri Lanka	88	52	57	68	62	63
Nepal	88	60	63	58	12	1

115. Which are the countries on the coverage frontier ?
(A) India and China
(B) Sri Lanka and Indonesia
(C) Philippines and Bangladesh
(D) Nepal and Pakistan

116. Which of the following statements are true ?
1. India > Pakistan and India > Indonesia
2. India > China and India > Nepal
3. Sri Lanka > China
4. China > Nepal
(A) 1 and 3 (B) 2 and 4
(C) 1, 2 and 4 (D) 2, 3 and 4

117. Using only the data presented under 'sanitation facilities' columns, it can be concluded that rural population in India, as a percentage of its total population is approximately :
(A) 76 (B) 70
(C) 73 (D) Cannot be determined

118. Again, using only the data presented under 'sanitation facilities' columns sequence; China, Indonesia and Philippines in ascending order of rural population as a percentage of their respective total population. The correct order is—
(A) Philippines, Indonesia and China
(B) Indonesia, China and Philippines
(C) Indonesia, Philippines, China
(D) China, Indonesia, Philippines

119. India is not on the coverage frontier because—
1. It is lower than Bangladesh in terms of coverage of drinking water facilities.
2. It is lower than Sri Lanka in terms of coverage of sanitation facilities.
3. It is lower than Pakistan in terms of coverage of sanitation facilities.
4. It is dominated by India.
(A) 1 and 2 (B) 1 and 3
(C) 4 (D) None of these

120. Suppose the average employment level is 60 per factory. The average employment in 'wholly private' factor is approximately—
(A) 43 (B) 47
(C) 50 (D) 54

121. Among the firm in different sectors, value added per employee is highest in—
(A) Central Government
(B) Central and state/ local governments
(C) Joint Sector
(D) Wholly private

122. Capital productivity is defined as the gross output value per rupee of fixed capital. The three sectors with the higher capital productivity, arranged in descending order are—
(A) Joint, wholly private, central and state/ local
(B) Wholly private, joint, central and state/local
(C) Wholly private, central and state/local, joint
(D) Joint, wholly private, central

123. A sector is considered 'pareto efficient' if its value added per employee and its value added per rupee of fixed capital is higher than those of all other sectors. Based on the table data, the Pareto efficient sector is—
(A) Wholly private
(B) Joint
(C) Central and state/local
(D) Other

124. The total value added in all estimated at Rs. 1,40,000 crore. Suppose the number of firms in the joint sector is 2,700. The average value added per factory, in Rs. crore, in the Central Government is—
(A) 141
(B) 14·1
(C) 131
(D) 13·1

Directions (Q. 120 to 124) : Answer these questions based on the data provided in the following table below :

Factory Sector by Type of Ownership

Sector	Factories	Employment	Fixed Capital	Gross Output	Value Added
Public :	7	27·7	43·2	25·8	30·8
Central government	1	10·5	17·5	12·7	14·1
State local government	5·2	16·2	24·3	11·6	14·9
Central and State/local government	0·8	1·0	1·4	1·5	1·8
Joint Sector	1·8	5·1	6·8	8·4	8·1
Wholly Private	90·3	64·6	46·8	63·8	58·7
Others	0·9	2·6	3·2	2·0	2·4
Total	**100**	**100**	**100**	**100**	**100**

Directions (Q. 125 to 128) : These question are based on the following table giving the number of fatal accidents in different Power Projects of India in 2004 -2009 as reported in Economic Times (29 Feb., 2008) :

	2004	2005	2006	2007	2008	2009
NHPC	107	20	22	23	22	24
BHEL	37	131	36	29	26	27
ONGC	23	2	18	2	10	9
NTPC	5	5	2	3	5	3
JPEE	18	10	9	11	14	10
SUZLON	23	23	13	17	12	20
RELIANCE ENERGY	1	9	7	8	8	6
TATA POWER	0	0	3	1	2	1

125. For which power project has the percentage decrease in accidents been maximum from one year to the next ?
(A) NHPC (B) BHEL
(C) ONGC (D) TATA POWER
(E) JPEE

126. The number of power project for which the percentage increase in accidents from one year to the next is maximum is—
(A) 4 (B) 3
(C) 2 (D) 1
(E) None of these

127. The power project which has the maximum number of accidents consistently for all the six year compared to any other is—
(A) BHEL (B) NHPC
(C) SUZLON (D) ONGC
(E) None of these

128. Consider the power project with the least Total number of accidents during 2004-2009 for a certain year, a particular power project might have had a fewer accidents than this power project in the corresponding year : which one is it ?
(A) NTPC (B) ONGC
(C) TATA POWER (D) JPEE
(E) None of these

Directions (Q. 129 to 132) :

Country	Laptop Sales in 2008 in (000's)	Installed Base in Millions	Laptop Penetration Per 1000	IT Spending % of GDP	Per capita GDP in US$
China	7168	26.3	21.9	1.1	793
India	1880	6.2	6.2	0.8	461
Indonesia	417	2.8	11.2	1.0	881
Malaysia	670	1.9	69.4	1.3	3288
Philippines	279	1.7	19.1	0.6	981
Singapore	490	2.4	700.0	2.5	26360
Thailand	525	1.9	22.0	0.5	2008
UK	6000	19.2	296.0	3.7	23238
USA	48620	153.8	500.0	4.3	31915

129. Which country has the lowest spending on Infotech ?
(A) Indonesia (B) Malaysia
(C) Philippines (D) India
(E) Singapore

130. If you add Singapore's population to that of Thailand, then it equals the population of which of the following countries, approximately ?
(A) India (B) Malaysia
(C) Philippines (D) Indonesia
(D) China

131. Which country has the highest growth rate in the number of Laptop's in 2008 ?
(A) Malaysia (B) China
(C) USA (D) UK
(E) Cannot be determined

132. India is the 4th largest market on which parameter ?
(A) Per capita GDP
(B) Laptop penetration per 1000
(C) IT spending % of GDP
(D) Laptop sales in 2008
(E) None of these

Directions (Q. 133 to 135) : The following table gives the number of dolmite produced by the employees in a National Mineral Development Corporation Ltd. in January 2008. The total number of dolomite produced during the mouth was 45,000.

Number of Dolomite produced by each employee per day	Number of employees
0–49	20
50–99	60
100–199	80
200–499	30
500–800	10

133. What is the average number of dolomite produced per employee during the month of Jan. 2008 ?

(A) 175 (B) 200

(C) 225 (D) 275

(E) None of these

134. The percentage of workers producing 500 or more dolomite in January 2008 is—

(A) 30% (B) 25%

(C) 20% (D) 15%

(E) None of these

135. The number of employees producing less than 50 dolomite is less than that producing 200 or more by—

(A) 20% (B) 30%

(C) 40% (D) 50%

(E) 60%

Directions (Q. 136 to 140) :

Industry				Indian Brands	MNC Brands	Others (Assembled)
Laptop Sales	**Contribution (%)**	**Current Year Volume (000's)**	**Last Year Volume (000's)**	**Market Share (%)**	**Market Share (%)**	**Market Share (%)**
World	100	54813	54691	38.1%	4.0	57.9
Africa	13.2	7261	7002	36.2%	13.2	50.6
Asia	34.9	19117	19319	47.1%	10.3	42.5
Europe	14.6	7984	7700	37.8%	1.7	60.3
Latin America & Caribbean Island	21.2	11617	11743	44.0%	0.7	55.2
North America & Oceania	16.1	8833	8927	24.9%	1.5	73.6

136. Which region shows the strongest rate of growth ?

(A) Latin America & Caribbean Island

(B) Asia

(C) Europe

(D) Africa

(E) North America & Oceania

137. What is the ratio of the number of Laptop of MMC brands sold in the Asia to those of Indian brands in the Africa for current year ?

(A) 1 : 1·04 (B) 1 : 33

(C) 1 : 4·07 (D) 1 : 7·06

(E) 1 : 1

138. What is the overall rate of growth in the industry (Current year over last year) ?

(A) – 2·2% (b) – 0·2%

(C) + 0·2% (D) + 2·2%

(E) Cannot be determined

139. Which region contributes to more than a fourth of all assembled Laptop sales ?

(A) Africa

(B) Latin America & Caribbean Island

(C) Central

(D) Asia

(E) Europe

140. In the current year, two regions alone account for more than 55% of Laptop sales. What was the contribution of these regions to sales last year ?

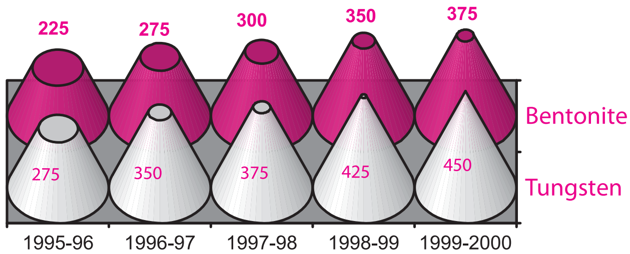
(A) 56 : 1%

(B) 56·8%

(C) 55·9%

(D) 56·0%

Directions (Q. 141 to 145) : The following graph shows price of Bentonite and Tungsten (In Rs./ Quintal)



141. In how many years the increase in price of Tungsten or Bentonite is more than ten percent of it support price in previous year ?

(A) 10 (B) 8

(C) 4 (D) 6

(E) 12

142. A Company produces 50 quintals of Tungsten and 75 quintals is of Bentonite per year for 1998-99 and 1999-2000. What is company's income during this period ?

(A) Rs. 98,125 (B) Rs.99,750

(C) Rs. 96, 275 (D) Rs 1,01, 875

(E) Rs. 95,455

143. During which year is the ratio of price to Bentonite to that of Tungsten highest ?

(A) 1996-97 (B) 1997-98

(C) 1995-96 (D) 1999-2000

(E) 1998-99

144. By what percent is the rate of increase of price of Bentonite from 1995-2000 less than the rate of increase of price of Tungsten for the same period approximately ?

(A) 14·28% (B) 16·66%

(C) 7·14% (D) 4·5%

(E) 10%

145. What will be the total price of one quintal each of Bentonite and Tungsten in 2000-2001, if they increase in same rate as in 1998-99 to 1999-2000 ?

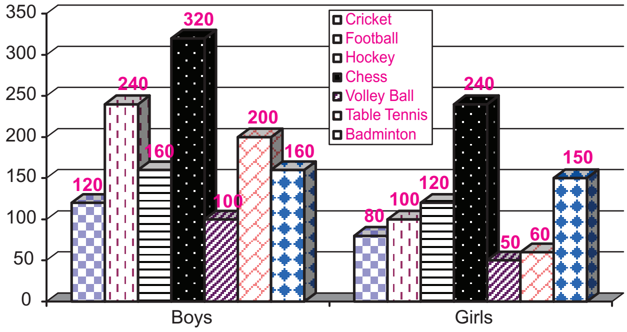
(A) Rs. 875 (B) Rs. 878

(C) Rs. 860 (D) Rs. 825

(E) Rs. 890

Directions (Q. 149 to 153) : In Institute of Perfection, Haridwar every student, except physically handicapped students have to participate in at least one sport.

There are 1200 students in IOP, of which 40% are girls. The table below gives the number of boys and girls participating in a sport.



Cricket, football, hockey and volley ball are outdoor games and rest are indoor games. One student can participate in at the most one outdoor and one indoor game. 30 boys and 10 girls are not participating in any of the games.

149. How many boys are participating in one game ?

(A) 40 (B) 80

(C) 120 (D) 90

(E) 110

Directions (Q. 146 to 148) : Following table shows the number of employees working in various departments of a automobile company from 2005 to 2010 :

Year	Departments (Number of Employees)				
	Press Shop	**Weld Shop**	**Purchase**	**Tool Room**	**Store**
2005	150	25	50	45	75
2006	225	40	45	62	70
2007	450	65	30	90	73
2008	470	73	32	105	70
2009	500	80	35	132	74
2010	505	75	36	130	75

146. In which year, the total number of employees reached approximately twice the total number of employees in the company in 2005?

(A) 2010 (B) 2009

(C) 2008 (D) 2007

(E) 2006

147. In which of the following years, each department had more number of employees than it had in the immediately preceding year?

(A) 2009 (B) 2008

(C) 2007 (D) 2006

(E) 2005

148. Which department had less than 10% of the total employees all through 2005 to 2010 ?

(A) Purchase (B) Weld Shop

(C) Tool Room (D) Store

(E) Press Shop

150. If every girl who participates in outdoor games participates in indoor games as well, then by how much should the participation of girls be increased or decreased in indoor game to satisfy the initial condition ?

(A) 20 (B) 10

(C) 0 (D) 5

(E) 15

151. 6% of the girls participating in indoor games are physically handicapped. Also the ratio of physically handicapped boys participating in indoor games is twice that of the physically handicapped girls participating in indoor games. If no physically handicapped person participates in outdoor games, then how many students are physically handicapped ?

(A) 121 (B) 840

(C) 850 (D) 650

(E) 700

152. A chess tournament was organized and all the students who participated in the IOP competition, took part in it. It was arranged in the Swiss league pattern in which the players having closest points play with each other and every player plays in every round with a player having closest points with respect to him. There is no draw possible and the player winning a game gets one point. Then after how many rounds we can surely find out a clear winner ?

(A) 8 Rounds (B) 9 Rounds

(C) 10 Rounds (D) 11 Rounds

(E) 12 Rounds

153. If mixed doubles and doubles games are possible for badminton and every student who participated in the badminton section participated at least in one double game and one mixed doubles game, then what is the minimum number of students who participated in more than two teams?

(A) 1 (B) 10

(C) 20 (D) 15

(E) Cannot be determined

Solutions

1. (B) 2. (A) 3. (E)

4. (B) Total sales = Rs. 10 crore

∴ Sales/Dealer/day = Rs. 5500

Average amount spend by a customer per visit = Rs. 91

5. (B) Cities within 10 E to 40 E which lie in southern hemisphere are Veinna , Sofia, Tripoli, Warsaw and Lusaka, Seven such cities are there, Out of these only one, Lusaka lies in the Southern hemisphere. Hence required percentage = $\frac{1}{5} \times 100 = 20\%$.

6. (D) (A) Number of cities with name starting with consonants in the northern hemisphere = 11

(B) Number of cities with names starting with consonants in east of the meridian = 13

Now (A) is 2 less than (B) Hence option (D) gives the right answer.

7. (A) Number of countries with name starting from a vowel situated in southern hemisphere = 3

Number of cities with names starting with a vowel = 2

Hence required ratio = 3 : 2.

8. (C) A number is 200% of other number then it should be 3 times of the other number. Hence if we compare expenses of year 2000, we find that expenses of five countries–Spain, Africa, Far-East, Australia and other world in the year 2000 is more then 3 times from that of in the year 1999.

9. (A) Out of the four options, only two option (a) and (b) show the net profit exceeding tax and charges Ratio of net profit to tax and charge of these countries are Far-East = $\frac{23}{21} = 1{\cdot}09$.

North Sea = $\frac{290}{280} = 1{\cdot}03$.

Hence option (A) is the correct answer.

10. (D) Total revenue in 1998 = 3,790, 5% of 3,790 = 189·5. Now it is clear that only two countries have revenue contribution less than 5% to the total revenue.

11. (D) If we compare income of 99, the only country which has expenses exceeding income, is other world, has least efficiency.

12. (D) Efficiency of Spain in

$$99 = \frac{2832}{1372} = 2{\cdot}06.$$

$$98 = \frac{3790}{2996} = 1{\cdot}26,$$

1999 > 1998. Hence (A) is true.

Profitability of North America in

$$99 = \frac{93}{137} = 0{\cdot}68$$

$$98 = \frac{52}{63} = 0{\cdot}82;$$

1999 < 1998. Hence (B) is true.

Efficiency of far east in

$$2000 = \frac{1354}{1024} = 1{\cdot}32$$

$$1999 = \frac{340}{296} = 1{\cdot}15;$$

2000 > 1999. Hence (C) is true.

Since all the options are correct.

Hence option (D) is the correct answer.

13. (B) It is clear from the table that expenses of Africa in 98 is more than the income. But in the year 99, expenses of Africa is less than the income. Hence it shows the maximum % increase in the profitability.

14. (B) There are four countries which show increase in profit before tax every year—Spain, Africa, North America and other world.

15. (B) Population of males in 1960

$$= \frac{1050}{1050 + 1000} \times 16{\cdot}4$$

$$= 8{\cdot}4 \text{ millions.}$$

And population of female is 1960

$$= 8 \text{ millions}$$

$\therefore$ Population of males in 1990

$= 8{\cdot}4 \times 1{\cdot}3 \times 1{\cdot}27 \times 1{\cdot}22$

$= 16{\cdot}9$ millions

And Population of females in 1990

$= 8 \times 1{\cdot}28 \times 1{\cdot}25 \times 1{\cdot}3$

$= 16{\cdot}64$ millions

$\therefore$ Total Population of the country in year 1990

$= 16{\cdot}9 + 16{\cdot}64 = 33{\cdot}54$

$\cong 34$ millions

16. (D)

17. (C) Population of male in 1980

$= 8{\cdot}4 \times 1{\cdot}3 \times 1{\cdot}27 = 13.86$ millions

Population of females in 1980

$= 8 \times 1{\cdot}28 \times 1{\cdot}25 = 12{\cdot}8$ millions

$\therefore$ Percentage of total literate

$$= \frac{45 \times 14 + 17 \times 13}{27} = 31{\cdot}5\%.$$

18. (E) Population of males in 1980

$= 13{\cdot}86$ million

Population of females in 1980

$= 12{\cdot}8$ million

The ratio is $13{\cdot}86 : 12{\cdot}8 \neq 15 : 10$

Hence statement (A) is not true

Number of males in 1970

$= 8{\cdot}4 \times 1{\cdot}3 = 10{\cdot}92$ million

Number of females in 1970

$= 8 \times 1{\cdot}28 = 10{\cdot}24$ million

Literate males $= 11 \times {\cdot}37 = 54$ million

Literate females $= 10{\cdot}24 \times {\cdot}12 = 1{\cdot}22$ million

Hence (B) is not true.

Statement (C) is also not true because the growth rate is for a decade and not for every year.

19. (B) 20. (A) 21. (D) 22. (A) 23. (B) 24. (B)

25. (C)

26. (E) Ratio of highest value to the lowest value in 2008 = 120 : 10 = 12 : 1 [August : April]

27. (A) Cost per room for Lokhandwala $= \frac{225}{535} = 0{\cdot}42$, for Raheja $= \frac{250}{500} = 0{\cdot}50$, for ITC $= \frac{300}{300} = 1$, it is, hence, clear that cost per room is least for Lokhandwala.

28. (C) Explanation same as above.

29. (C) In 1998, two projects namely Mumbai Heigh and Royal Holidays are completed.

The cost of project = (250 + 225) = 475 crore

Cost incurred $= 1060 \times (1{\cdot}1) = 1282{\cdot}6$ crore

30. (A) Cost of project – Majestic, Supremo, Windsor and Leela completed in 1999 = 250 + 300 + 275 + 235 = 1,060. Hence the cost incurred $= 1{,}060 \times (1.1)^2$ = 1,282·6 crore.

31. (B) Approximate cost of projects completed by 2,000 is $1282{\cdot}6 + 522{\cdot}5 + [250 \times (1{\cdot}1)^3] = 2{,}140$.

32. (A) Production capacity for Lipton is 64.80% for 1·64 thousand tonnes Since maximum capacity is 100% Hence for 100% it would be $\left(\frac{100}{64{\cdot}8}\right) \times 1{\cdot}64 = 2{\cdot}53$ thousand tonnes.

33. (D) Unutilised capacity for Brooke Bond = $\left[\frac{2{\cdot}97}{76{\cdot}5} \times 100 - 2{\cdot}97\right] = 0{\cdot}912$, for Nestle = 1·003, for Lipton = 0·89.

For MAC, it is 1·05 Hence, it is maximum for MAC.

34. (C) Capacity utilization for coffee is 61·30% for 11·60 ('000 tonnes)

Hence for 100% it is $\frac{100}{61{\cdot}3} \times 11{\cdot}6 = 187$ ('000 tonnes)

35. (D) Data insufficient to answer the question.

36. (B) Sales of coffee (other)

$= 132{\cdot}80 - (31{\cdot}15 + 26{\cdot}75 + 15{\cdot}25 + 17{\cdot}45)$

$= 42{\cdot}2$

Hence, required %

$$= \frac{42{\cdot}2}{132{\cdot}8} \times 100 = 32\% \text{ approxi.}$$

37. (B) 38. (C) 39. (E)

40. (A) 2% of x = 9 crore

x = Rs. 450 crore

41. (B) Since business volumes are in the ratio 1 : 2, in Egypt,

The business volume = Rs. 2·5 crore

Let, Market size of Egypt = X

$\therefore$ 30% of X = 2·5

$\therefore$ x = Rs. 8·33 crore

42. (A) The domestic consumption in 1991-92 $= \frac{1575}{0{\cdot}9}$ = 1750 megawatt. This constitutes 20% of total power consumed in 1991-92, and the rural consumption $= \left[1750 \times \frac{15}{20}\right] = 1312$ megawatts .

43. (D) It cannot be determined because the rates for the Urban sector is not known.

44. (B) The average Traiff in region 4 is

$$= \frac{415 + 423 + 441 + 451}{4}$$

$= 432{\cdot}5$ p/kWh.

$$\text{In region 2} = \frac{(472 + 468 + 478 + 470)}{4}$$

$$= 472 \text{ p/kWh}$$

$$\text{and region } 5 = \frac{(440 + 427 + 439 + 446)}{4}$$

$$= 438 \text{ p/kWh.}$$

Hence, the average tariff in region 2 is greater than region 5 this statement is not correct hence third statement cannot be evaluated.

45. (a) Total time taken by FRG team = (10·95 + 10·85 +10·58 +10·63) = 43·01. Total time taken by USA team = (10·78 + 10·75 + 10·94 + 10·36) = 42·83

Difference = 0·18

46. (b) First ranked person has total score of 8905 and second ranked person has total score of 8897. Therefore he must get a score of greater than 8880 but less then 8897. Presently he is scoring 582 +3003 = 3585. Therefore, if he gets a score of 5296, his total score would become 3585 +5296 = 8881, it ensures him a bronze medal.

47. (d) There would be 4 competitor namely Torsten Voss, Jorgen Hingsen, Grigory Degtyarov and Steve Fritz in which Michael Smith has performed better in long jump than his competitors.

48. (C) Inventory In = 125 + 160 + 120 + 90 + 130 + 180 = 805

Inventory Out = 150 + 140 + 200 + 110 + 190 + 120 = 910

Thus, in a week, total inventory goes down by 105

∴ Total stock after one week = 1200 – 105 = 1095

49. (B) Since every week inventory will get reduced by 105, hence after 11 weeks

Inventory remaining = 1200 – 1155 = 45

Now, on Monday, inventory will get reduced by 25 and on Tuesday it will increase by 20 but on Wednesday it will reduce further by 80 which is not in the store room.

∴ Total days for which inventory will last = 11 × 7 + 2 = 79 days

50. (C)

51. (D) Total inventory In during a week = 125 + 120 + 130 = 375

Total inventory out during a week = 140 + 110 + 120 = 370

∴ Total inventory up during a week = 5 units

Hence, in two weeks, *i.e.,* up to 14 June and on 15 June, inventory = 1200 + 10 + 125 = 1335.

52. (B) Every week inventory gets reduced by 105 units. Hence at the end of 5th week, the inventory will go down by 525 units and on Wednesday inventory will go down to 600 units. Hence, company will order special inventory after 5 × 7 + 3 = 38 days, *i.e.*, 8th of July

53. (C) 54. (B) 55. (C)

56. (B) Serial number : 1, 2, 3, 5 and 9 show those A type airport which account for than 40 million passengers.

57. (A) From the table, it is very much clear that airport have been classified and presented in descending order of passengers attendance .

Hence out of first ten (top ten), six airports are present in USA. Hence required percentage

$$= \frac{6}{10} \times 100 = 60\%$$

58. (C) As we have to answer the question in nearest percentage; we can take number of passengers in million to avoid calculation complexity.

Hence, required percentage

$$= \frac{62}{77 + 72 + 63 + 62 + 60}$$

$$= 18{\cdot}56 \text{ or } 20\%$$

59. (B) It is clear that all international airports handle more than 30 million passengers. Hence we have to count only those location from table which does not have USA as one of the countries. And here are six such locations.

60. (E) Since saving of Rajiv Narayan Gaur family in 2009 = 24% of 5 lakh = 1·2 lakh

In 2010, savings decrease by 20%, hence saving in 2010 = 0.96 lakh

$$\therefore \text{ Gross Annual Income in 2010} = \frac{0{\cdot}96}{24} = 4 \text{ Lakh}$$

61. (A) 39% = 32% + 7% (Income Tax + Food)

∴ In 2010, Income Taxes + Food

$$= 27\% + 9\% = 36\%$$

$$\text{Expenditure} = 36\% \text{ of 4 lakh} = 1{\cdot}44 \text{ lakh}$$

62. (C) Since in 2009, ratio is 3 : 1

Mr. Rajiv Narayan Gaur family's income

$$= \frac{3}{4} \times 5 = 3{\cdot}75 \text{ Lakh.}$$

Mrs. Rajiv Narayan Gaur family's income

$$= 1{\cdot}25 \text{ lakh}$$

Mr. Rajiv Narayan Gaur family's income 2010 decreases by 33·33% or $\frac{1}{3}$

$$\text{Decrease} = \frac{1}{3}(3{\cdot}75\%) = 1{\cdot}25$$

Since in 2009, ratio is 3 : 1

∴ Mr. Rajiv Narayan Gaur family's income

$$= 2{\cdot}5 \text{ lakh}$$

$\therefore$ Mrs. Rajiv Narayan Gaur family's income

$$= 1{\cdot}5 \text{ lakh}$$

Increase in Mrs. Rajiv Narayan Gaur family's income

$$= (1{\cdot}5 - 1{\cdot}25) \text{ lakh} = 0{\cdot}25 \text{ lakh}$$

Since Total Income = 4 lakh in 2010

$\therefore$ Percentage increase in Mrs. Rajiv Narayan Gaur family's $= \dfrac{0{\cdot}25}{1{\cdot}25} = \dfrac{1}{5} = 20\%$.

63. (C)

64. (C) Expenditure of the three in 2009 adds up to Rs. 80,000 (16% of Rs. 5 lakh) and in 2000 Rs. 72000 (18 % of Rs. 4 Lakh).

$$\therefore \quad \frac{80000}{72000} = \frac{10}{9} = 111{\cdot}11\%$$

65. (B) If we multiply the complex column of the head working day in the table with 50, we get the salary paid for the same work of the same employees. The figure so obtained should be compared with the respective head of salary. As such we get seven employees whose salary for complex work exceed Rs. 50 per day.

$600 > 10 \times 50, 450 > 8 \times 5, 550 > 9 \times 5, 250 > 4 \times 50, 360 > 6 \times 50, 490 > 8 \times 50$ and $1,234 > 19 \times 50$.

66. (A) Salary per day of 200180 is

$$\frac{1234}{19} = 64{\cdot}94,$$

and 2,00,080 is $\dfrac{600}{10} = 60$,

and 2,00,170 is $\dfrac{490}{8} = 61{\cdot}25$,

and 2,00,040 is $\dfrac{149}{10} = 14{\cdot}9$.

Hence employee 2,00,180 gets the maximum salary per day in complex work.

67. (A) 80% attendance in the month of June $= 25 \times \dfrac{80}{100}$ = 20 hours. Hence those employees who work more than 20 hours and earning more than Rs. 600 are required to the answer of the question and these five employees complying this requirement.

68. (A) There are 9 employees who worked for complex and medium both. Out of these 9, only 6 earn more in complex work than in medium work.

69. (A) Loans from rural banks in 1974 = $(260 \times 98 \times 243)$ = Rs. 6·19 million.

Hence, total amount of loan = (34·54 + 6·19) = Rs. 40·73 million. Hence percentage of agricultural loans $= \dfrac{34{\cdot}54}{40{\cdot}73} = 84{\cdot}79\% = 85\%$ (app.).

70. (B) Total number of loans upto 1980 = (2,520 + 4,485 + 6,760 + 25,480 + 38,478 + 1,47,240) = 2,51,963. And the total number of rural loans in 1998 = 3,19,200.

71. (D) Maximum increase in the number of loans for rural bank in 1980-81.

72. (B) Value of agricultural loan in 1983 at 1970 prices $= \dfrac{43 \times 915{\cdot}7}{149} = 264$.

73. (C) The number of rural bank loans per rural bank is least in 1970.

74. (B) Required % increase

$$= \frac{211600 - 18300}{18300} \times 100$$

$$= 1057\%$$

Since this growth is spread across 13 years, therefore simple annual rate of increase $= \dfrac{1057}{13}\% = 81{\cdot}3$ or 81% approximately.

75. Consumer price index 1970 is 43 which is to be taken as 105 as per instruction. According to price index for the year 1983 and 1975 should be taken as $\left[149 \times \dfrac{105}{43}\right] = 363{\cdot}83$ and $\left(78 \times \dfrac{105}{43}\right) = 190{\cdot}46$, respectively. Hence their difference = (363·83 – 190·46) = 173·37 = 174 approximately.

76. (B) Total value of loans = Rural bank loans + Agriculture loan. Rural bank loan in 1980 = $(605 \times 288 \times 567)$ = Rs. 98·79 million. Total value of agricultural loan in 1980 = Rs. 498·4 million. Hence total loan in 1980 = (98·79 + 498) = 597·19 .

77. (B) From the bar-chart, production for Orissa in 2007-08 = 9

And production for Orissa in, 2008-09 = 12

$$\text{Required ratio} = \frac{9}{12} = 0{\cdot}75$$

78. (B) From the bar-chart, for Rajasthan and West Bengal, there is a steady increase in the production of cotton.

79. (B) From the bar chart, total cotton production by Assam = 8 + 14 + 7 = 29000 tons.

Now, ten quintal = one ton

$\Rightarrow$ 29000 × 10 quintal = 2,90,000 quintal

80. (C) Average production of various states is as shown below :

$$\text{Rajasthan} = \frac{6 + 14 + 21}{3} = \frac{41}{3} = 13\frac{2}{3}$$

In 2006-07, Production = 6 < average, *i.e.*, below average

In 2007-08, Production = 14 > average, *i.e.*, above average

$$\text{Karnataka} = \frac{12+18+18}{3} = \frac{48}{3} = 16$$

In 2006-07, Production = 12 < average

In 2007-08, Production = 18 > average

$$\text{West Bengal} = \frac{5+9+15}{3} = \frac{29}{3} = 9\frac{2}{3}$$

In 2006-07, Production = 5 < average

In 2007-08, Production = 9 < average

Hence West Bengal is not counted.

$$\text{Orissa} = \frac{16+9+12}{3} = \frac{37}{3} = 12\frac{1}{3}$$

In 2006-07, Production = 16 > average

In 2007-08, Production = 9 < average

Hence Orissa is not counted.

$$\text{Assam} = \frac{8+14+7}{3} = \frac{29}{3} = 9\frac{2}{3}$$

In 2006-07, Production = 8 < average

In 2007-08, Production = 14 > average

Hence, the states showing below average production in 2006-07, but showed above average production in 2007-08 are only three—Rjashthan, Karnatka and Assam.

81. (C) Total Production of Rajasthan = 6 + 4 + 21 = 41 and total production of Karnatka = 12 +18 + 18 = 48.

Hence, statement (C) is false, all the rest are true, as can be observed from the bar-chart.

82. (A) It is very clear from the data table that quantity of apples supplied by J and K in the month of February *i.e.*, 11,183 tonnes is very close to the total figure 11,285,

which is $\frac{11183}{11285} \times 100 = 99\%$.

Hence it shows the maximum percentage.

83. (C) HP supplied a total of 2,31,028 tonnes, UP supplied a total of 258 tonnes and J and K supplied a total of 2,62,735 tonnes. Hence state J & K supplied maximum number of apples.

84. (C) It is clear that J and K supplied the highest percentage of apples.

85. (C) The stock taken from the month May to September is Zero hence during this period supply was greater than the demand.

86. (B) Total quantity of apples supplied to Delhi during the year was (2,31,028 + 258 + 262735) = 4,94,021 tonnes or 49,40,21,000 kg. If 1 tree yields 40 kg of apples, then the number of trees required to yield 49,40,21,000 kg. $\frac{49,40,21,000}{40} = 1,23,50,525$ trees

= 12·5 million trees (approximately).

87. (D) Required area $= \frac{1,23,50,525}{250}$

= 49,402 (approximately)

88. (D) Required number of lays = (Total number of lays) – (Number of lays which do not produce any size of yellow fabric) = (27 – 13) = 14.

89. (B) Required number of lays = (Total number of lays) – (Number of lays which do not produce XXL of any coloured fabric) = (27 – 11) =16.

90. (D) Required number of lays = lays which produce either XXL of yellow fabric or XXL of white fabric or XXL of yellow and white fabric = 15.

91. (B) There are four entries in the surplus row. Hence there are three varieties of fabric which exceed the order.

92. (C)

93. (D) The question asks us to find the operating profit as a percentage of selling price. The selling price of steel is not given.

94. (A) It can be clearly seen from the table, 'Power' experience continuous price rise during the ten year period.

95. (D) It can be clearly seen from the table, 'Time' and 'Wages' experience only one decline in price during the ten year period.

96. (B) 25% of total investment in 2008 = 25% of 179 = Rs. 44·75 lakh.

97. (A) 98. (C) 99. (B)

100. (B) By observation, we find that the increase is more in the year 2007 and 2009 over their corresponding previous years.

$$\text{Increase in 2007} = \frac{165-112}{165} = \frac{53}{165} = \frac{1}{3}$$

$$\text{Increase in 2009} = \frac{228-179}{179} = \frac{49}{179} = \frac{1}{3{\cdot}5}$$

101. (A) Deficit intensity of the year 1997-98

= (14·2- 9·2) = 5,

1996-97 = (16·2 – 8·2) = 8,

1995-96 = (15·5 – 7·9) = 7·6,

1994-95 = (13·8 – 7·5) = 6·3,

1993-94 = (12·4 – 7·3) = 5·1

Now growth rate in deficit intensity in the year

$$1994\text{-}95 = \frac{6{\cdot}3-5{\cdot}1}{5{\cdot}3} = 0{\cdot}23,$$

$$1995\text{-}96 = \frac{7{\cdot}6-6{\cdot}3}{6{\cdot}6} = 0{\cdot}20,$$

$$1996\text{-}97 = \frac{8 - 7{\cdot}6}{7} = 0{\cdot}058,$$

$$1997\text{-}98 = \frac{5 - 8}{4} = -0{\cdot}75.$$

It is therefore clear that growth rate in deficit intensity is higher in the year (1994-95).

102. (D) It is clear in the previous year that growth rate in deficit intensity is higher in the year 1994-95 and it is calculated as $\frac{6{\cdot}3 - 5{\cdot}1}{5{\cdot}3} \times 100 = 23\%$ (approximately).

103. (B) For the year 1997-98

$$\text{Import intensity} = \frac{\text{Import}}{\text{Sales}}$$

$$= 14{\cdot}2 = \text{Import}$$

$$= 14{\cdot}2 \text{ Sales.}$$

$$\frac{\text{Imported raw material}}{0{\cdot}5 \times \text{Sales}} = 20{\cdot}2$$

$$\Rightarrow \text{Raw material} = 10{\cdot}1 \times \text{sales}$$

(given raw material = 50% sales)

$$\text{Now Import} = \text{Raw material} + \text{Capital goods}$$

$$\Rightarrow \quad 14{\cdot}2 \text{ sales} = 10{\cdot}1 \text{ Sales} + \text{Capital goods}$$

$$\text{Capital goods} = 4{\cdot}1 \text{ Sales}$$

$$\frac{\text{Imported capital goods}}{\text{Gross Fixed Assets}} = \frac{4{\cdot}1 \text{ Sales}}{\text{GFA}} = 17{\cdot}6$$

$$\therefore \quad \frac{\text{Sales}}{\text{GFA}} = \frac{17{\cdot}6}{4{\cdot}1} = 4{\cdot}29 \text{ or } 4{\cdot}3$$

104. (D) Option (A) cannot be inferred because sales component is not given . Option (B) and option (C) cannot be inferred because of the same reason.

However, it is very clear that deficit intensity has increased from 1993-94 to 1996-97.

Hence option (D) is the correct choice.

105. (B) Percent change in the cost incurred in the fifth month $= \frac{1{,}00{,}000 - 40{,}000}{40{,}000} \times 100 = 150\%$.

106. (D) Cost incurred in the new coding stage

$$= \text{Rs. } 1{,}90{,}000$$

107. (D) Difference between old and new technique

$$= (1{,}90{,}000 - 1{,}30{,}000)$$

$$= \text{Rs. } 60{,}000.$$

108. (B) Cost incurred in specification stage

$$= (80{,}000 + 1{,}20{,}000) = \text{Rs. } 2{,}00{,}000$$

Which is the maximum cost.

109. (C) Average cost for consecutive month is lowest for month 11 to 15.

110. (B) Total annual exports for the year 1994-95

$$= (484 + 177 + 6) = 668$$

$\therefore$ Percentage to total annul business of that year

$$= \frac{668}{2041} \times 100 = 32{\cdot}73\%.$$

$$\text{Year 1997-98} = \frac{1970}{5031} \times 100 = 39{\cdot}16\%$$

It is clear that required exports percentage for the year 1996-97 and 1997-98 lie between 35% and 40%. Hence option (B) is the answer.

111. (A) Growth % for

$$1995\text{-}96 = \frac{845}{2041} \times 100 = 41{\cdot}40,$$

$$1996\text{-}97 = \frac{921}{2886} \times 100 = 31{\cdot}91,$$

$$1997\text{-}98 = \frac{1224}{3807} \times 100 = 32.15,$$

$$1998\text{-}99 = \frac{1021}{5031} \times 100 = 20{\cdot}29.$$

It is, therefore, clear that growth percentage for the year 1995-96 was the highest.

112. (C) it is clear option (A) and (B) are not correct . Now let us check option (C).

Total IT business in training during 1994-99

$$= (302 - 107) = 195.$$

Total IT business in maintenance during 1994-99

$$= (236 - 142) = 94.$$

Hence option (C) is the correct answer.

113. (D) As per instruction, any particular dominates other year if hardware activity in that year is greatest than other year. All options other then (D) are correct because in the year 1998-99 hardware activity = 1030 and in the year 1996-97 hardware activity = 1336.

114. (D) Option (A) is not true because peripherals activity in 1996-97 is which is less than that of in the year 1995-96, *i.e.*, 202.

Option (B) is not true because hardware activity I 1998-99 is less than that of in year 1995-96.

Option (C) is not true because peripheral activity in 1997-98 is less than that of in the year 1998-99.

Since none of the options is correct, our answer would be (D).

115. (B) Bangladesh > Philippines (97 > 86) for drinking water. And Philippines > any other country for sanitation. Hence both Bangladesh and Philippines are on the coverage frontier.

116. (C) Since, a country A is said to dominate B or A > B, if A has higher percentage in total coverage for both drinking water and sanitation facilities. Therefore option (B) and (D) are correct. India > China (81 > 67) for drinking water and (29 > 24) for sanitation likewise India > Nepal (81 > 63 and 29 > 18). Also China > Nepal (67 > 63 and 24 > 18)

117. (C) Let the urban and rural population be x and y respectively then $0{\cdot}7x + 0{\cdot}14y = 0{\cdot}29\ (x + y)$

$$= 0{\cdot}41\ x = 0.15\ y$$

$$\therefore \quad x = \frac{15}{41}y.$$

$\because$ Percentage for rural population

$$= \frac{y}{x+y} \times 100$$

$$= \frac{y}{\frac{15}{41}y + y} \times 11 = 73{\cdot}2\%.$$

118. (A) Percentage of rural population for Philippines, Indonesia, and China are 50%, 66·66% and 79·8% respectively. Hence P < I < C.

119. (D) India is not on coverage frontier because it is below Bangladesh and Philippines for drinking water and for sanitation it is below Philippines, Sri Lanka Indonesia, and Pakistan .

120. (A) Average employment level is 60 per factory. Since here are 100 factories. Hence total employment in 100 factories = 100 × 60 = 6,000. Employment in wholly private factories = 6,000 × 64·6% = 3,876.

There are 90·3 wholly private factories.

$\therefore$ Average employment $= \dfrac{3{,}876}{90{\cdot}3} = 42{\cdot}92$ or 43.

121. (A) Value added per employee in central and state/ local. Government $= \dfrac{1{\cdot}8}{1} = 1{\cdot}8$. Which is maximum among other options.

122. (B) Capital productivity of joint sector

$$= \frac{8{\cdot}4}{6{\cdot}8} = 1{\cdot}23.$$

Capital productivity of central and state/local

$$= \frac{1{\cdot}5}{1{\cdot}4} = 1{\cdot}07.$$

Capital productivity of wholly private

$$= \frac{63{\cdot}8}{46{\cdot}8} = 1{\cdot}36.$$

And all these may be arranged in descending order as wholly private, joint, central and state/local.

123. (C) Value added per employment and value added per fixed capital respectively for sector given in options are as wholly private = 0·9 and 1·25 and joint sector 1·59 and 1·19;

Central/state/local = 1·8, 1·28, other 0·92 and 0·75.

It is now clear that central and state/local has the highest ratio among all others.

124. (D) Percentage of joint sector firm = 1·8, now 1·8% of total = 2,700

$$\therefore \quad \text{Total firms} = \frac{2{,}70{,}000}{1{\cdot}8} = 150000$$

Now, number of central Government factories

$$= 1{,}50{,}000 \times 1\% = 15$$

Value added for central Government firms

$$= 140000 \times 14{\cdot}1\% = 19740$$

Hence, average value added per factory

$$= \frac{19740}{1500} = 13{\cdot}1.$$

125. (C) For ONGC from 2004 to 2005, it went down from 23 to 2.

126. (C) For RELIANCE ENERGY, the percentage increase from 2004 to 2005 was 800%. For ONGC, the percentage increase from 2005 to 2006 was also 800%.

127. (A) BHEL has the maximum number of accidents in five of the six years (from 2005 to 2009).

128. (A) TATA POWER has the least total number of accidents during 2004-2009. NTPC has a fewer number of accidents in 2006 than that in NTPC that year.

129. (C) Population of Indonasia

$$= \frac{1000}{11{\cdot}2} \times 2{\cdot}8 \times 10^6 = 2{\cdot}5 \times 10^8$$

Spending on infotech

$$= \frac{1}{100} \times 881 \times 2{\cdot}5 \times 10^8$$

$$= 2{\cdot}2025 \times 10^9$$

Similarly Population of Malaysia

$$= \frac{1000}{69{\cdot}4} \times 1{\cdot}9 \times 10^6 \times \frac{1{\cdot}3}{100} \times 3288$$

$$= 1{\cdot}17 \times 10^9$$

Spending of Philippines

$$= \frac{1000}{19{\cdot}1} \times 1{\cdot}7 \times 10^6 \times \frac{0{\cdot}6}{100} \times 981$$

$$= 5{\cdot}24 \times 10^8$$

Spending of India

$$= \frac{1000}{6{\cdot}2} \times 6{\cdot}2 \times 10^6 \times \frac{0{\cdot}8}{100} \times 461$$

$$= 3{\cdot}688 \times 10^9$$

130. (C) Population of Singapore

$$= \frac{1000}{700} \times 2{\cdot}4 \times 10^6 = 3{\cdot}43 \times 10^6$$

Population of Thailand

$$= \frac{1000}{22} \times 1{\cdot}9 \times 10^6 = 86{\cdot}36 \times 10^6$$

$\therefore$ Total population of Singapore and Thailand

$$= 89{\cdot}79 \times 10^6$$

Now, population of India

$$= \frac{1000}{6 \cdot 2} \times 6 \cdot 2 \times 10^6 = 10^9$$

Population of Malaysia

$$= \frac{1000}{69 \cdot 4} \times 1 \cdot 9 \times 10^6 = 27 \cdot 4 \times 10^6$$

Population of Philippines

$$= \frac{1000}{19 \cdot 1} \times 1 \cdot 7 \times 10^6 = 89 \times 10^6$$

131. (E) 132. (D)

133. (C) Average number of Dolomite produced

$$= \frac{\text{Total number of Dolomite}}{\text{Number of employees}}$$

$$= \frac{45000}{200} = 225$$

134. (E) Percentage of employees producing more than 500 Dolomite $= \frac{10}{200} \times 100 = 5\%$

135. (D) Percentage of employees producing less than 500 Dolomite = 20

Those producing 200 or more = 30 + 10 = 40

Percentage decrease $= \frac{40 - 20}{40} = 50\%$

136. (D)

Region	Percentage Growth
Latin America and Caribbean Island	$\frac{11617 - 11743}{11743} \times 100 = (-)\text{ve}$
Asia	$\frac{19117 - 19319}{19319} \times 100 = (-)\text{ve}$
Europe	$\frac{7984 - 7700}{7700} \times 100 = \frac{284}{7700} \times 100$
Africa	$\frac{7261 - 7002}{7002} \times 100 = \frac{259}{7002} \times 100$

Compare only the Europe and the Africa regions as for the other two regions there was a decline.

$$\frac{284}{77} = 3 \cdot 688 \text{ and } \frac{259}{70 \cdot 02} = 3 \cdot 699$$

137. (B) Market share of MNC brands in Asia = 10.3%

Current year volume in Asia = 19117000

$\therefore$ Number of Laptop of MNC brands = 0·103 × 19,117,000 = 19,69,051

Market share of Indian brands in Africa = 36·2%

Current year volume in Africa = 72,61,000

$\therefore$ Number of Laptop of Indian brands in Africa = 0·362 × 7261000 = 2628482

Required ratio = 1 : 1·33

138. (C) Total sales last year = 54,691

Total sales this year = 54,813

Rate of growth percentage

$$= \frac{54813 - 54691}{54691} \times 100$$

$$= \frac{122}{54691} \times 100 \approx 0 \cdot 2\%$$

139. (D) Total number of assembled Laptop's from

$$\text{Africa} = \frac{50 \cdot 6}{100} \times 7261000 = 3674066$$

$$\text{Europe} = \frac{60 \cdot 3}{100} \times 7984000 = 4814352$$

Latin America and Caribbean Island

$$= \frac{52 \cdot 2}{100} \times 11617000 = 6412584$$

$$\text{Asia} = \frac{42 \cdot 5}{100} \times 19117000 = 8124725$$

North America and Oceania

$$= \frac{73 \cdot 6}{100} \times 8833000 = 6501088$$

Total = 2,95,26,815

$\therefore$ One fourth of the sales

$$= \frac{1}{4} \times 29535648 = 7381704$$

140. (B) The two regions, are Asia and Latin America and Caribbean Island.

$\therefore$ Contribution of these two regions last year

$$= \frac{19319 + 11743}{54691} \times 100 = 56 \cdot 8\%$$

141. (C) $10\% = \frac{1}{10}$

Increase of Bentonite in 1996-97 over its previous year

$$= \frac{75}{275} = \frac{3}{11} > \frac{1}{10}$$

Increase of Bentonite in 1997-98 over its previous year

$$= \frac{25}{350} = \frac{1}{14} < \frac{1}{10}$$

Increase of Bentonite in 1998-99 over its previous year

$$= \frac{50}{375} = \frac{2}{15} > \frac{1}{10}$$

Increase of Bentonite in 1999-2000 over its previous year $= \frac{25}{425} = \frac{1}{17} < \frac{1}{10}$

Increase of Tungsten in 1997-98 over 1995-96

$$= \frac{50}{225} = \frac{2}{9} > \frac{1}{10}$$

Increase of Tungsten in 1997-98 over 1996-97

$$= \frac{25}{275} = \frac{1}{12} < \frac{1}{10}$$

Increase of Tungsten in 1998-99 over 1997-98

$$= \frac{50}{300} = \frac{1}{6} > \frac{1}{10}$$

Increase of Tungsten in 99-2000 over 1998-99

$$= \frac{25}{350} = \frac{1}{14} < \frac{1}{10}$$

In four years the increase in support price of Bentonite or Tungsten is more than the per cent of its support price in previous year.

142. (D) Total cost of Tungsten

$$= 50\,(350 + 375) = \text{Rs. } 36{,}250$$

Total cost of Bentonite

$$= 75\,(425 + 450) = \text{Rs. } 65{,}626$$

Total cost of Tungsten and Bentonite

$$= \text{Rs. } 36250 + \text{Rs. } 65625$$

$$= \text{Rs. } 101875$$

143. (A) The ratio of support price of Bentonite to that of Tungsten during :

$$1995\text{-}96 = \frac{275}{225} = \frac{11}{9} = 1{\cdot}22$$

$$1996\text{-}97 = \frac{350}{275} = \frac{14}{11} = 1{\cdot}27$$

$$1997\text{-}98 = \frac{375}{300} = \frac{15}{12} = 1{\cdot}25$$

$$1998\text{-}99 = \frac{425}{350} = \frac{17}{14} = 1{\cdot}21$$

$$1999\text{-}2000 = \frac{450}{375} = \frac{6}{5} = 1{\cdot}21$$

The ratio in 1996-97 is the highest.

144. (D) Rate of increase of support price of Bentonite from 1995-96 to 1999-2000 $= \frac{175}{275} = \frac{7}{11}$

Rate of increase of support price of Tungsten from 1995-96 to 1999-2000 $= \frac{150}{225} = \frac{2}{3}$

Support price of Bentonite is less than that of Tungsten by $\frac{2}{3} - \frac{7}{11} = \frac{1}{33}$

$$\therefore \quad \text{Percentage} = \frac{\frac{1}{33}}{\frac{2}{3}} \times 100 = 4{\cdot}5\%$$

145. (B) Support price of one quintal of Bentonite in 2001

$$= 450 + \frac{25}{425} \times 450 = \text{Rs. } 476$$

Support price of one quintal of Tungsten in 2001

$$= 375 + \frac{25}{350} \times 375 = \text{Rs. } 402$$

Total Price = Rs. 476 + Rs. 402 = Rs. 878

146. (D) Total number of employees in various years are as follows :

2005–345; 2006–442; 2007–708; 2008–750;

2009–821; 2010–821

Twice the total number of employees in 2005, *i.e.* twice of 345 is 690.

As this number is closest to 708, the total number of employees reached approximately twice the total number of employees in 2005, *i.e.,* in 2007.

147. (A) In 2009, the factory had more number of employees in each department than it had in 2008.

For all the other years, the number of employees has decreased for at least one department.

148. (A) Total number of employees all through the years 2005 to 2010 is 3,737.

All through the year 2005 to 2010 :

Total number of employees in purchase department

= 228

Total number of employees in sales department

= 358

Total number of employees in account department

= 564

Total number of employees in research department

= 437

Only purchase department has less than 9% of the total number of employee all through 2005 to 2010.

149. (B) There are total $1200 \times 0{\cdot}6 = 720$ boys out of which 30 are not playing any game .

∴ Total number of boys playing at least one game

= 690

By the given condition, one participate at most in two games, one outdoor and one indoor

Now, total number of boys playing

$$= 120 + 240 + 160 + 320 + 100 + 200 + 160$$

$$= 1300$$

As 690 boys are playing and they can play at most two games, then number of boys playing more than one game = 1300 – 690 = 610

∴ Number of students playing only one game

= 690 – 610 = 80

150. (A) Total number of girls playing outdoor = 350

Total number of girls playing indoor = 450

Now all girls playing outdoor as well.

$\therefore$ Total number of girls playing = girls playing outdoor games + girls playing indoors games only

$$= 350 + 100 = 450$$

But, from the given data,

Maximum number of girls playing at least one game

$$= 1200 \times 0{\cdot}4 - 10 = 470$$

Hence, to satisfy this condition, we have to increase the girl's participation by 20 in indoor games only.

151. (A) Number of girls participating in indoor games

$$= 450$$

6% of 450 = 27 are physically handicapped girls

And hence, 54 are physically handicapped boys.

Number of physically handicapped students

$$= 27 + 54 + 40 = 121.$$

152. (C) After the first round, 280 players get maximum points.

After the second round, 140 players get maximum points.

After the third round, 70 players get maximum points. After the fourth round, 35 players get maximum points.

In the fifth round, 34 players will play with each other and 35th player will play with a player having 3 points.

Hence, 17 or (17 + 1) players get maximum points.

Similarly 8 players get maximum points after the sixth round.

4 or 5 players get maximum points after the seventh round.

2 or 3 players get maximum points after the eighth round.

1 or 2 players get maximum points after the ninth round.

Hence, definitely after the tenth round, a winner can be determined.

153. (A) Now there are 160 boys and girls who play badminton. Hence, 150 mixed doubles teams can be easily formed. But then 10 boys were left with no girl partner. Hence, at least 1 girl has to form more than two-mixed doubles teams *i.e.*, she will participate in 11 mixed doubles apart from one doubles team.

Hence minimum number of students, participating in more than two teams, is one.

PART–IV : PRACTICE SETS

Mock CAT–1

Directions (Q. 1 to 5) : Study the following information and answer the question given below :

Following are the conditions laid down for declaring the results of examinees in annual examinations of an institute :

There are five groups P, Q, R, S, and T. Among them P, Q and R consist of two question papers each. Following are the criteria to declare a candidate passed. It is necessary for the success of candidate that :

(i) he/she must get 50 marks out of 100 in each paper of group P (paper I and II) and S.

(ii) he/she must get 40 marks in each paper of group Q and 30 marks in each paper of group R.

(iii) he/she must get 25 marks in group T.

(iv) he/she must get the minimum pass marks as determined above.

However, if a candidate has passed in all the paper/group except the following :

P. Those who have secured minimum 40% marks in two papers of group P together but passed in only one of them, then his/her case is to be referred to Asst. Headmaster provided he/she has secured minimum 70% marks in group T.

Q. Those who haven't passed in group P or group R but have obtained minimum 35% marks in each paper and minimum 60% marks as a whole in the group, then his/her case is to be referred to Headmaster.

R. Those who haven't passed in group P or group Q but obtained minimum 50% or more marks in that group, are entitled to ATKT.

Based on the above criteria and the information given in each of the following questions, what course of action will be taken in case of the following candidates ?

Give answer:

(a) If the candidate is to be declared passed.

(b) If the candidate is to be declared failed.

(c) If the candidate is to be referred to Asst. Headmaster.

(d) If the candidate is to be referred to Headmaster.

(e) If the candidate is entitled ATKT.

Questions Paper

		Gr. P		Gr. Q		Gr. R		Gr. S	Gr. T
		I	II	I	II	I	II		
	Marks	(100)	(100)	(100)	(100)	(75)	(75)	(100)	(50)
1.	Candidate F	50	45	64	72	40	21	56	27
2.	Candidate G	58	28	60	74	32	36	76	36
3.	Candidate H	63	47	46	54	50	60	69	43
4.	Candidate I	52	74	54	62	67	28	64	35
5.	Candidate J	46	76	72	59	34	43	55	32

Directions (Q. 6 to 10) : Ravi, Suresh and Asif are in control of the following number-letter-symbol series respectively.

Ravi : 2 & S * 9 P T B π 8 Q Δ 6

Suresh : ⊗ 1 ψ F @ V 4 ∃ ℘ M T D ⇔

Asif : G 3 H # K N ● 5 R = 7 W Y

6. Starting from the left end and following the given order in each series, if a group of three elements is formed by taking symbol from Suresh's series, number from Ravi's series and letter from Asif's series, each symbol-number-letter only once, which of the following will be the elements of the 4th such group formed ?

(A) ∃ 8 N (B) ℘ 6 R

(C) ∃ 6 N (D) * 1 V

(E) None of these

7. If from each series, amongst letters/numbers/symbols the one having highest members is sorted out and then arranged in the descending order on the basis of the number of elements they have, which of the following will indicate the correct descending order?

(A) Asif-letters, Suresh-symbols, Ravi-symbols

(B) Asif-letters, Suresh-symbols, Ravi-letters

(C) Ravi-numbers, Suresh-letters, Asif-symbols

(D) Ravi-letters, Suresh-symbols, Asif-letters

(E) None of these

8. If each symbol which immediately precedes a number in Ravi's series, each number which immediately follows a letter in Suresh's series, and each letter which immediately precedes a symbol in Asif's series are selected what will be total number of these elements ?

(A) 7 (B) 8

(C) 6 (D) 5

(E) None of these

9. If all the numbers from Asif's series, all the letters from Suresh's series and all the symbols from Ravi's series are respectively arranged in the same given order one after the other from the left end, which of the following will be the seventh to the right of the eleventh element from your right ?

(A) D (B) M

(C) ℘ (D) Δ

(E) None of these

10. Which of the following is true?

(A) The total number of symbols immediately preceded by numbers in Ravi's series is equal to the total number of letters immediately preceded by numbers in Suresh's series.

(B) The total number of symbols immediately followed by numbers in Asif's series is less than the total number of letters immediately preceded by numbers in Ravi's series.

(C) The total number of letters immediately following symbol in Ravi's series is more than the total number of symbols immediately preceded by numbers in Suresh's series.

(D) The total number of symbols immediately preceding letter in Ravi's series is less than the total number of symbols immediately followed by numbers in Asif's series.

(E) The total number of symbols in Ravi's series is more that the total number of symbols in Suresh's series.

Direction (Q. 11 to 14) : The following table gives the sales details for Nuts and Bolts of Car, Bike, Scooter and Bus.

Year	Car	Bike	Scooter	Bus
2005	42137	8820	65303	25343
2006	53568	10285	71602	27930
2007	58770	16437	73667	28687
2008	56872	15475	71668	30057
2009	66213	17500	78697	33682
2010	68718	20177	82175	36697

11. What is the growth rate of sales of Nuts and Bolts for Car 2005 to 2010 ?

(A) 29% (B) 51%

(C) 63% (D) 163%

(E) 150%

12. Which of the categories shows the lowest growth rate from 2005 to 2010 ?

(A) Car (B) Bike

(C) Scooter (D) Bus

(E) Cannot be determined

13. Which category had the highest growth rate in period shown ?

(A) Car (B) Bike

(C) Scooter (D) Bus

(E) Cannot be determine

14. Which of the categories had either a consistent growth or a consistent decline in the period shown ?

(A) Car (B) Bike

(C) Scooter (D) Bus

(E) Cannot be determine

15. Two liquids A and B are in the ratio 5 : 1 in the container 1 and 1 : 3 in container 2. In what ratio should the contents of the two containers be mixed so as to obtain a mixture of A and B in the ratio 1 : 1 ?

(A) 2 : 3 (B) 3 : 4

(C) 4 : 3 (D) 3 : 2

(E) None of these

Note Questions 16 to 20 carry two mark each

16. The Director of the Institute of Perfection, Dr. Dim, has announced that six guest lectures on different area-Leadership, Decision Making, Quality Circles, Motivation, Assessment Centre and Group Discussion. Only one lecture can be organised each day from Monday to Sunday.

I. Lecture on Motivation should be organized immediately after the lecture on Assessment Centre.

II. Lecture on Quality Circles should be organised on Wednesday and should not be followed by the lecture on Group Discussion.

III. Lecture on Decision Making should be organised on Friday and there should be a gap of two days between the lecture on Leadership and Group Discussion.

IV. One day there will be no lectures (Saturday is not that day) and the lecture on Group Discussion will be organised on the preceding day.

So find out How many lectures will be organised between the lectures on Motivation and Quality Circles and which day will the lecture on Leadership be organized ?

(A) 1, Tuesday (B) 1, Thursday

(C) 2, Thursday (D) 3, Tuesday

(E) 1, Wednesday

17. At Feel Good BAR & Restaurant, above the kitchen door there are four small lights, arranged side by side and numberd consecutively, left to right, from one to four. The lights are used to signal waiters when orders are ready. On a certain shift there are exactly five waiters–Raman, Pawan, Ritesh, Hetesh, Mitlesh.

1. To signal Raman, all four lights are illuminated.
2. To signal Pawan, only light one and two are illuminated.
3. To signal Ritesh, only light one is illuminated.
4. To signal Hetesh, only light two, three and four are illuminated.
5. To signal Mitlesh, only light three and four are illuminated.

So, if light two and three are both off, then the waiter signalled is .... ?

(A) Hitesh (B) Mitlesh
(C) Raman (D) Pawan
(E) Ritesh

18. (a) There is a group of six persons in a family A, B, C, D, E and F.
(b) There are two married couples in the family.
(c) A is the most talkative in the family while D talks less than E or C.
(d) F is more talkative than D or B.
(e) The least talkative in the family is married to the second most talkative in the family.
(f) There are three females and three males.
(g) Nobody is a widow or a widower.
(h) D is an unmarried male, B is a female.
(i) A is of the same sex as the unmarried person other than D.
(j) E is married and is not of the same sex as A or married to A.
(k) The marital status of the most talkative and the least talkative of the family are the same.

So, find out (i) Who is the least talkative member of the family ?

(ii) The marital status of E is ?

(A) A, Married (B) B, Unmarried
(C) C, Unmarried (D) B, Married
(E) A, Unmarried

19. A lady, Praveen Boby, has some flowers with her when Pravin Boby leaves her home. She has to worship 3 deities to whom she presents flowers. She starts from her home (As Director Raj Kapoor decided) with 'n' number of flowers and goes to the bank of the river (As Raj Kapoor decided). After taking a bath she dips the number of flowers in the river and the number of flowers increased by 50%. She then goes to the first deity and presents 'm' number of flowers to him. Then she again goes to the river (As Raj Kapoor decided) and dips the remaining flowers in the river. The number of flowers again increased by 50%. She then goes to the second deity and presents it with 'm' number of flowers. Then she again goes to the river (As Raj Kapoor decided) and dips the remaining flowers. The number of flowers again increased by 50%. She then goes to the third deity, presents it with 'm' flowers. Now she finds that she is not left with any flowers. (As Raj Kapoor had not decided).

So Raj Kapoor put two Questions in front of his team, "Tell me, the minimum number of flowers Pravin Boby could have got from her home and the minimum number of flowers the Pravin Boby could have presented to each deity are .... ?"

(A) 38, 27 (B) 39, 28
(C) 40, 30 (D) 35, 25
(E) 50, 50

20. Once upon a time, during the Indus civilization A peasant with a goat, a bundle of grass and his bravo dog were crossing the Sindhu River in a boat. The boat could only carry the peasant with only one more item in a trip. The goat if left with the grass would eat it away. If the dog and goat were left behind, the dog is prone to bite the goat hence two cannot be left together. What is the minimum number of crossings to transfer all four to the other side intact?

(A) 8 (B) 7
(C) 6 (D) 9
(E) 5

Directions (Q. 21 to 25) : Following are the conditions for admission to Paramedical Course in the ABC Medical College :

A candidate must—

(1) have passed B.Sc. with at least 50% marks (Second Division).

There is relaxation of 5% marks for candidates belonging to Scheduled Castes and Scheduled Tribes.

(2) have completed 19 years of age in the case of girl candidates and 20 years of age for the boys as on January 1, 2002.

(3) deposit Rs. 25000 with the application, if the candidate has secured second division in graduation. However, if the applicant has secured more than 75% marks at graduation, the deposit amount would be Rs. 15000 and if the marks are above 60% but below 70% the deposit amount will be Rs. 20000. If the candidate is a son or daughter of the staff member of the college offering the course, the amount of deposit will be 50% of that applicable to others, keeping the merit criteria the same.

(4) For SC/ST candidates who produce valid certificate of their category, the deposit amount is 20% of that applicable to non-staff relation candidates and the condition of marks for each slab described above in (3) is **relaxable** by 5% for SC/ST candidates.

(5) If a candidate fulfils the criteria at (1) and (2) and can pay at least three-fourth amount of applicable deposit, the candidate may be **provisionally admitted.**

(6) If a candidate who is eligible under (1) and (2) criteria, and can pay at least half of the applicable deposit, the candidate can be referred to **the Chairman of the Institute.**

(7) The candidates who are eligible under criteria (1) and (2), but who cannot pay even half of the applicable deposit, **cannot be admitted.**

Based on the above criteria, decide which of the following courses of action should be taken in the cases of the candidate whose description is provided in the following question. Please note that you are NOT to assume any data other than those described. However, you may treat the candidate as "General Category" candidate if his category is not explicitly mentioned as SC/ST.

Mark (a) as your answer if the candidate can be admitted.

Mark (b) as your answer if the candidate can be provisionally admitted.

Mark (c) as your answer if the candidate should be referred to the Chairman.

Mark (d) as your answer if the candidate cannot be admitted.

Mark (e) if the data given are not sufficient to decide the course of action.

21. Shekhar, the son of an ST member of the ABC Medical College, was born on 27th February, 1979 and has passed his B.Sc. with 64% marks. He can pay a maximum of Rs. 7500.
22. Mangesh, an M.Sc. with 66·5% marks, is the son of a businessman. His age is 22 years as on the stipulated date. He can pay Rs. 25000 as the deposit.
23. Ranjit, the son of a staff member of the ABC Medical College, was born on 7th January, 1982 and has passed his B.Sc. examination with 76% marks. He can pay Rs. 20,000 as deposit immediately.
24. Brinda, the daughter of a teacher, has passed her B.Sc. with 78% marks. She was born on 23rd September, 1980. She can pay a maximum of Rs. 8,000 as deposit.
25. Mohini, the daughter of an ex-employee of ABC Medical College, was born on 1st April 1981. She passed her B.Sc. examination with 68% marks. She can remit Rs. 11,000 as deposit.

Passage–1

I think that it would be wrong to ask whether 50 years of India's Independence are an achievement or a failure. It would be better to see things as evolving. It's not an either-or question. My idea of the history of India is slightly contrary to the Indian idea. India is a country that, in the north, outside Rajasthan, was ravaged and intellectually destroyed to a large extent by the invasions that began in about AD 1000 by forces and religions that India had no means of understanding.

The invasions are in all the school books. But I don't think that people understand that every invasion, every war, every campaign, was accompanied by slaughter, a slaughter always of the most talented people in the country. So these wars, apart from everything else led to a tremendous intellectual depletion of the country. I think that in the British period, and in the 50 years after the British period, there has been a kind of regrouping or recovery, a very slow revival of energy and intellect. This isn't an idea that goes with the vision of the grandeur of old India and all that sort of rubbish. That idea is a great simplification and it occurs because it is intellectually, philosophically easier for Indians to manage. What they cannot manage, and what they have not yet come to terms with, is that ravaging of all the north of India by various conquerors. That was ruin not by the act of nature, but by the hand of man. It is so painful that few Indians have begun to deal with it. It is much easier to deal with British imperialism. That is a familiar topic, in India and Britain. What is much less familiar is the ravaging of India before the British.

What happened from AD 1000 onwards, really, is such a wound that it is almost impossible to face. Certain wounds are so bad that they can't be written about. You deal with that kind of pain by hiding from it. You retreat from reality. I do not think, for example, that the Incas of Peru or the native people of Mexico have ever got over their defeat by the Spaniards. In both places the head was cut off. I think the pre-British ravaging of India was as bad as that. In the place of knowledge of history, you have various fantasies about the village republic and the Old Glory. There is one big fantasy that Indians have always found solace in about India having the capacity for absorbing its conquerors. This is not so India was laid low by its conquerors. I feel the past 150 years have been years of every kind of growth. I see the British period and what has continued after that as one period. In that time, there has been a very slow intellectual recruitment. I think every Indian should make the pilgrimage to the site of the capital of the Vijayanagar Empire, just to see what the invasion of India led to. They will see a totally destroyed town. Religious wars are like that people who see that might understand what the centuries of slaughter and plunder meant. War isn't a game. When you lost that kind of war, your town was destroyed; the people who built the towns were destroyed. You are left with a

headless population. That's where modern India starts from The Vijayanagar capital were destroyed in 1565. It is only now that the surrounding region has begun to revive. A great chance has been given to India to start up again, and I feel it has started up again. The questions about whether 50 years of India since Independence have been a failure or an achievement are not the questions to ask.

In fact, I think India is developing quite marvelously, people thought even Mr. Nehru thought-that development and new institutions in a place like Bihar, for instance, would immediately lead to beauty. But it doesn't happen like that. When a country as ravaged as India, with all its layers of cruelty, begins to extend justice to people lower down, it's a very messy business. It's not beautiful, it's extremely messy. And that's what you have now, all these small politicians with small reputations and small parties. But this is part of growth this is part of development. You must remember that these people, and the people they represent, have never had rights before. When the oppressed have the power to assert themselves, they will behave badly. It will need a couple of generations of security, arid knowledge of institutions, and the knowledge that you can trust institutions-it will take at least a couple of generations before people in that situation begin to behave well.

People in India have known only tyranny. The very idea of liberty is a new idea. The rulers were tyrants, the tyrants were foreigners, and they were proud of being foreign. There's story that anybody could run and pull a bell and the emperor would appear at his window and give justice. This is a child's idea of history- the slave's idea of the ruler's mercy. When the people at the bottom discover that they hold justice in their own hands, the earth moves a little. You have to expect these earth movements in India. It will be like this for a hundred years. But it is the only way. It's painful and messy and primitive and petty, but it's better that it should begin. It has to begin. If we were to rule people according to 'what we think fit, that takes us back to the past when people, had no voices. With self-awareness all else follows. People begin to make new demands on their leaders, their fellows, on themselves. They ask for more in everything. They have a higher idea of human possibilities. They are to content with what they did before or what their fathers did before. They want to move. That is marvelous. That is as it should be. I think that within every kind of disorder now in India, there is a larger positive movement. But the future will be fairly chaotic. Politics will have to be at the level of the people now, People like Nehru were colonial-style politicians. They were to a large extent created and protected by the colonial order. They did not begin with the people. Politician now have to begin with the people. They cannot be too far above the level of the people. They are very much part of the people. It is important that self-criticism does not stop. The mind has to work, the mind has to be active, there has to be an exercise of the mind. I think it's almost a definition of a living country that it looks at itself, analyses itself all times. Only countries that have ceased to live, can say it's all wonderful.

26. The writer's attitude is :
(A) Excessively critical of India
(B) Insightful.
(C) Cynica1
(D) Cold
(E) Cannot be determined

27. The writer has given the example of the Vijayanagar kingdom in order to drive home the point that :
(A) Indians should know their historical sites
(B) Indians should be aware of the existence of such a historical past
(C) It is time that India came to terms with the past
(D) All of the above
(E) None of these

28. According to the writer, India's regeneration and revival took place :
(A) In the British period
(B) After the British period
(C) During and after the British period
(D) A long time after the British left
(E) Data inadequate

29. According to the passage, self-awareness is followed by :
(A) Self-righteousness
(B) A higher idea of human possibilities
(C) A desire for more in everything
(D) Both (B) and (C)
(E) All (A) (B) and (C)

30. According to the passage, India's current situation is :
(A) Bleak
(B) Horrific
(C) Primitive and Messy
(D) (A) and (C) are wrong
(E) All are wrong

Passage–2

When Deng Xiaoping died a few years ago, the Chinese leadership barely paused for a moment before getting on with the business of governing the country. Contrast that with the chaotic contortions on India's political stage during the past month, and it is easy to conclude that democracy and democratic freedoms are serious obstacles to economic progress. When the Chinese leadership wants a power plant to be set up, it just goes ahead. No fears of protracted litigation, of environmental

protests, or of lobbying by interested parties. It-or the economy-is not held to ransom by striking truckers or air traffic controllers. Certainly there is much that is alluring about an enlightened dictatorship. But there the trouble begins. First, there is no guarantee that a dictatorship will be an enlightened one. Myanmar has been ruled by a dictator for decades, and no one would claim that it is better off than even Bangladesh which has itself suffered long stretches of dictatorship. Nor can Mobuto Sese Seko, much in the news these days, be described as enlightened by any reckoning. The people of Israel, almost the only democracy in a region where dictatorships (unenlightened ones) are the norm, are much better off than their neighbours.

Second, dictatorships can easily reverse policies. China was socialist as long as Mao Ze-dong was around. When Deng Xiaoping took over in what was essentially a palace coup, he took the country in the opposite direction. There is little to ensure that the process will not be repeated. In India such drastic reversals are unlikely.

Six years ago Indian politicians agreed that industries should be de-licensed, that imports should be freed or the investment decisions should be based on economic considerations, Now few think otherwise. Almost all politicians are convinced of the merits of liberalisation though they may occasionally lose sight of the big picture in pandering to their constituencies. India has moved slower than China on liberalisation, but whatever moves it has made, are more permanent.

Democracies are also less likely to get embroiled in destructive wars. Had Saddam Hussain been under the obligation of facing free elections every five years, he would have thought ten times before entangling his people in a long confrontation with. The West Germany, Italy and Japan were all dictatorships when they launched the Second World War. The price was paid by the economies.

Democracies make many small mistakes. But dictatorships are more susceptible to making huge ones and risking everything on one decision-like going to war. Democracies are the political equivalent of free markets. Companies know they can't fool the consumer too often; he will, simply switch to the competition. The same goes for political parties. When they fail to live up to their promises in government, the political consumer opts for the competition. Democratic freedoms too are important for the economy, especially now that information is supreme. Few doubt that the internet will play an important part in the global economy in the decades to come. But China, by preventing free access to it, is already probably destroying its capabilities in this area. As service industries grow in importance, China may well be at a disadvantage though that may not be apparent today when its manufacturing juggernaut is rolling ahead. India has stifled its entrepreneurs through its licensing policies. That was an example of how the absence of economic freedom can harm a country. But right-wing dictatorships like South Korea erred in the opposite direction. They forced their businesses to invest in industries, which they (the dictators) felt had a golden future. Now many of those firms are trying to retreat from those investments. Statism is bad, no matter what the direction in which it applies pressure. At this moment, China and other dictatorships may be making foolish investment decisions. But as industries are subsidized and contrary voices not heard, the errors will not be realised until the investments assume gargantuan proportions.

India's hesitant ways may seem inferior to China's confident moves. But at least we know what the costs are that is not the case with China. It was only years after the Great Leap Forward and only such experiments that the cost in human lives (millions of them) became evident to the world. What the cost of China's present experiments is, we may not know for several years more. A nine per cent rate of growth repeated year after year may seem compelling. But a seven per cent rate of growth that will not falter is more desirable. India seems to be on such a growth curve, whatever the shenanigans of our politicians.

31. The passage says that :
    (A) Benevolent dictators are not easy to find.
    (B) Not all dictators will be enlightened.
    (C) Dictators can make or break a country.
    (D) An enlightened dictatorship is better than a corrupt democracy.
    (E) Idea in passage is not clear

32. It can be implied from the passage that :
    (A) A lower rate of growth is preferred to a higher rate of growth.
    (B) A higher rate of growth is preferred to a lower rate of growth.
    (C) A low but stable rate of growth is preferred to a high rate of growth.
    (D) A low but faltering rate of growth is a sign of stability amidst growth.
    (E) None of these

33. Vis-a-vis democracies, dictatorships run the risk of :
    (A) Losing all for a single mistake
    (B) Making bigger mistakes
    (C) Making huge mistakes and risking everything
    (D) None of the above.
    (E) All (A) , (B) and (C)

34. The writer's conclusion in the passage is that :
    (A) Under no circumstances should a country encourage a corrupt democrat
    (B) Under no circumstances should statism be a welcome move
    (C) A statist will not give due importance to the voice of the people
    (D) A statist will always look to his own welfare
    (E) Cannot be determined

35. Democracy has been compared to the free market, as :
    (A) Both have a high degree of competition
    (B) Both offer a multitude of options to choose from
    (C) Consumer satisfaction plays an important role in both
    (D) All of the above
    (E) None of these

Passage–3

Of each of the great leaders, it is said by his follower, long after he is gone, he made us do it. If leadership is the art of persuading your people to follow your bidding, without their realising your involvement, the archetype of its practice is N.R. Narayanan Murthy, the chairman and managing director of the Rs. 143·81 crore Infosys Technologies (Infosys). For, the 52-year-old CEO of the globalised software corporation—which he founded with six friends, and a combined capital of Rs. 10,000, in 1981 and which now occupies the front rank of the country's most admired corporations, leads with the sublets of weapons personal example. Infosys rank only 578th among the country's listed companies, and sixth in the software sector, in terms of its turnover. But it is setting new standards for India Inc. through its practices of inter alai awarding stock options to its employees, putting the value of its intellectual assets and its brands on its balance sheet, and conforming to the disclosure standards of the Securities and Exchange Commission (SEC) of the US. Behind all this is the stubborn personal subscription of its CEO to the underlying cause of wealth-creation, people-power and transparency. "What were choice earlier and compulsions now," asserts Murthy. In fact, the mirror image of Murthy, the Man, can be found all over Infosys, his company. His egalitarianism-which finds expression in such habits as using the same table and chair as anyone else in the organisation-is practiced firmly. When it comes to charting a course for the company's future, everyone has voice. "We have no hierarchy just for the sake of control."

Brimming with the conviction that customer satisfaction is the key to success, Murthy has built a fleet-footed human resource management system that treats employees as customers, using the resources of the organisation to meet their professional and personal needs. His instruments are not just top-of-the market salaries, but also operational empowerment as well as every facility that an employee needs to focus on the job. Just what method does Murthy use to ensure that his DNA is replicated in his company ? Not for his are the classical leadership genre-transactional or transformational, situational or visionary. His chosen style, instead, is to lead by example, ensuring that the CEO's actions set the template for all infoscions. Murthy believe that the betterment of man can be brought about through the "creation of wealth, legally and ethically". The personal example that he has set, enabled his company to mirror those beliefs, trying his own rewards, and measuring his value to the company, to his ability to create wealth, and erecting systems for the company's wealth to be shared by is people. Sums up Nandan Nilekani, 41, Deputy managing director , Infosys "This is the future model of the corporation. Run an excellent company, and let the market increase its value to create wealth".

Although Murthy is one of the prime beneficiaries of the philosophy–his 10 percent stake in Infosys is worth Rs.130 crore today in his book, the leader leads not by grabbing the body but by teaching others to take what they deserve. That's why, on the Infosys' balance sheet, the value of Murthy's intellectual capital is nowhere near the top, on the rational, that the CEO, at 52, is worth far less his company than, say, a bright young programmer of 26. To spread the company's wealth, Murthy has instituted stock options-the first to do so in the country for employees, creating 300 millionaires already. By 2000, he wants the number to climb to 1000.

To act as a beacon for his version of the learning organisation, Murthy not only spends an hour a day trawling the Internet to learn about new technological developments in his field, he also makes as many luncheon appointments as he can with technical people and academicians-done from the Indian Institute of Technology for instance-systematically plumbing their depth for an understanding of new developments in infotech. Murthy's objective is not just to stay abreast of the state of the art, but also to find a way to use that knowledge for the company. Following Murthy's example, Infosys has set up a technology advancement until, whose mandate is to track, evaluate, and assimilate new techniques and methodologies. In fact, Murthy views learning not just as amassing data, but as a process that enables him to use the lessens from failure to achieve success. This self-corrective loop is what demonstrates through his leadership during a crisis.

In 1995, for example, Infosys lot a Rs.15 crore account then 20 per cent of its revenue, when the $69 billion GE yanked its business from it. Instead of recriminations, Murthy activated Infosys' machinery to understand why the business was taken away and leverage the learning for getting new clients instead. Feeling determined instead of guilty, his employees went on to sign up high profile customer like the $ 20 billion Xerox, the $ 7 billion Levis Strauss, and the $ 14 billion Nynx.

"You must have a multi-dimensional view of paradigms", says the multi-tasking leader. The objective is obvious–ensure that Infosys' perspective on its business and the world come from as many vantage points as possible so that corporate strategy can be synthesized not from a narrow vision, but from a wide angle lens. In fact Murthy still regrets that,in its initial years, Infosys didn't dstill a multi-pronged understanding of the environment

into its strategies, which forced it onto an incremental path that led revenue to snake up from Rs.0.02 crore to just 5 crore in the first 10 years. It was after looking around itself, instead of focusing on its initial business of banking software that Infosys managed to accelerate. Today the company operates with stretch targets setting distant goals and working backward to get to them. The crucial pillar on which Murthy bases his ethical leadership, is openness. Transparency, he reckons, is the clearest signal that one has nothing to hide. The personal manifestations of that are inter alai the practice of always giving complete information whenever any employee, customer, or investor asks for it; the loudly proclaimed insistence that every Infoscion pay taxes and file returns and a perpetually open office into which anyone can walk.

But even as he tries to leas Infosys into cloning his own approach to enterprise, is Murthy choosing the best future for it ? If infosys grown with the same lack of ambition, the same softness of style, and the same absence of aggression, is it not cutting off avenues of growth that others may seize ? As Infosys approaches the 21st century, it is obvious that Murthy's leadership will have to set ever-improving role models for his ever-learning company. After all, men grow old; companies shouldn't.

36. One of the way in which Infosys spreads the company's wealth among its employees :
    (A) By awarding stock options
    (B) By giving extravagant bonus at the end of each year
    (C) Both (A) and (B)
    (D) None of the above
    (E) Cannot be determined

37. According to the passage :
    (A) At Infosys, control is exerted through a system of hierarchy
    (B) Control is not exerted through a system of hierarchy
    (C) Hierarchy does not have pride of place in Infosys
    (D) Popular opinion is the most respected voice in Infosys
    (E) Cannot be determined

38. Murthy believes in :
    (A) Betterment of man through learning
    (B) Betterment of man through ethical creation of wealth
    (C) Betterment of man through experimentation.
    (D) All of these
    (E) None of these

39. The example of the Rs. 15 crore account highlights :
    (A) Murth's ability to see his company through a crisis.
    (B) Murth's ability to turn failure into success.
    (C) Murth's potential to handle a crisis.
    (D) All the above.
    (E) None of these

40. According to Murthy, learning is :
    (A) The essence of an employee.
    (B) The art of amassing data.
    (C) A process that helps him to learn from failure
    (D) All of these
    (E) None of these

Direction (Q. 41 to 45) : Each question is a logical sequence of statements with a missing link, the location of which is shown parenthetically. From the five choices available, you are required to choose the one which best fits the sequence logically.

41. The modern minded man, although he believes profoundly in the wisdom of his period, must be presumed to be very modest about his personal powers. His highest hope is to think first what is about to be thought, to say what is about to be said, and to feel what is about to be felt, he has no wish to think better thoughts than his neighbours, to say things showing more insight, or to have emotions which are not those of some fashionable group, but only to be slightly ahead of others in point of time (.........). A mentally solitary life, such as that of Copernicus, or Spinoza, or Milton, seems pointless according to modern standards.
    (A) Quite deliberately he suppresses what is individual in himself for the sake of the admiration of the herd.
    (B) He is largely indifferent to his environment.
    (C) Often he surrounds himself with worldly luxuries and leads the life of an unthinking hedonist.
    (D) His energy levels are abysmally low and his attitude that of an escapist.
    (E) None of these

42. The problems of outdoor crowding results from a megalopolis civilisation that excludes nature. The long-term solution is to rebuild our habitat to include nature so that the sense of the wild is always close by. City governments need to promote more neighbourhood parks, wildlife corridors and biodiversity in home landscaping (.........). We will know we've succeeded when we don't need to go elsewhere to find happiness. In the meantime, why not avoid crowds by taking up a hobby like bird watching? Not only will you find peace and solitude close to your home, but you will get to take a nature break every day.
    (A) Enough attention should also be paid to institute a foolproof security system
    (B) Where we live should be beautiful, diverse and natural
    (C) We should of course, have all the amenities of modern living near at hand

(D) They should also promote increased awareness of civic problems among the citizenry

(E) None of these

43. These "fight or flight" reactions, as they are called, turn up in moderate degrees upon receipt of all kinds of messages from the consciousness. Most people cannot even tell a lie without stepping up the autonomic activity of the sweat glands, the respiratory system and the heart. (.........) rates of perspiration, breathing and heartbeat is what a lie detector does. On many other occasions, appropriate or inappropriate, we have "butterflies" in the stomach, clammy hands, pounding heart, a flush or loss of colour.

(A) Manipulating

(B) Measuring changes in

(C) Producing variations in

(D) Enhancing the

(E) None of these

44. What Hussein and others have discovered is a society of women that hovers precariously on the verge of greater liberation, but also on the brink of intensified repression. Access to birth control for instance is critical to a woman's control over her own life. For years Hussein and other family planning activists have encouraged religious leaders to issue fatwas, or religious decrees, approving contraception and they have. Even in the time of the Prophet it was determined, coitus interrupts was practised to prevent pregnancy. (.........). Partly as a result of such rulings, family planning is now widespread in much of the Muslim world. According to statistics released by the Population Council in Cairo, more than 47 percent of Egyptian women use some form of family planning, while in Jordan, Syria, Tunisia, Morocco, Algeria and Lebanon the figure ranges form 35 to 55 percent.

(A) Islam has ways stood for moderation in the matter of sensual pleasures

(B) If there had been other means, it's assumed, they would have been used

(C) This goes to prove that Islam had nothing against contraception

(D) Though abortion was not tolerated; other means of birth control were by no means discouraged in the countries where Islam held sway

(E) None of these

45. For an infertile woman trying to get pregnant, the desire for a baby often overwhelms everything else. At the same time, the top fear of many Americans is cancer. So when a recent study in the New England Journal of Medicine reported that taking clomiphene one of the most commonly prescribed fertility drugs in the United States--may dramatically increase the risk of ovarian cancer, thousands of women were thrown into conflict (.........)

(A) It is a pity, in such matters, that the medical profession is woefully short of any practical assistance.

(B) Surely, taking clomiphene cannot be as bad as smoking which for a long time has been suspected to cause cancer?

(C) Is creating a new life worth jeopardising your own ?

(D) Since so many other factors are suspected to cause cancer, could the findings of this new study be taken seriously ?

(E) None of these

Directions (Q. 46 to 50) : Below is given a passage followed by several possible inferences which can be drawn from the facts stated in the passage. You have to examine each inference separately in the context of passage and decide upon its degree of truth or falsity.

Mark Answer (a) : If the inference is 'definitely true', *i.e.* it properly follows from the statement of facts given.

Mark Answer (b) : If the inference is 'probably true', though not 'definitely true' in the light of the facts given.

Mark Answer (c) : If the 'data are inadequate' *i.e.* from the facts given you cannot say whether the inference is likely to be true or false.

Mark Answer (d) : If the inference is 'probably false', though not 'definitely false', in the light of given facts given.

Mark Answer (e) : If the inference is 'definitely false', *i.e.* it cannot possibly be drawn from the facts given or it contradicts the given facts.

Hopes of a global recovery for the information technology industry are growing by the day. Overall economic data, especially from the United States and Japan, give justifiable cause for optimism. This is good news specifically for the IT industry, whose progress is demonstrably linked to the general economic performance of the national economics. The past three years period of recessional or flat economic performance has not been without its benefits for India, as companies have sought a low-wage environment for their less-skilled tasks. As a result, culture and thinking of multinational companies has changed and become more comfortable with a global search for the best solutions and service. This plays directly to the strengths of India's most famous international business offerings-offshoot software development and business process outsourcing.

46. India did not have adequate skilled manpower to undertake software development requirement of multinational companies till a couple of year ago.

47. India does not provide its offshore services and Business Process Outsourcing centres to the countries other than USA and Japan.

48. India is likely to gain by providing skilled manpower at cheaper rate.

49. India's share in the software development sector has increased considerably during the last three years.

50. Major part of the software development work was carried out within the developed countries before the recent lull in economic development.

51. Which of the following can be the value of n such that $(n + 7)$ is a factor of $(n + 1)^2$?

(A) 12 (B) 47

(C) 25 (D) 30

(E) None of these

52. The sequence N of natural numbers is divided into classes as follows

		1	2		
	3	4	5	6	
7	8	9	10	11	12

....

....

Find the sum of the number in the n^{th} row

(A) $n(n^2 + 1)$ (B) $n(2n^2 + 1)$

(C) $n^2(n + 1)$ (D) $2n^2(n + 1)$

(E) None of these

Direction (Q. 53 to 55) : Ramesh, Ram, Kareem and Mohan collected coins of different countries.

A. They collected 100 altogether.

B. None collected less than 10.

C. Each collected an even number.

D. Each collected a different number.

53. Based on the above, we can say that the number of coins collected by the boy who collected the most could not have exceeded

(A) 64 (B) 54

(C) 60 (D) 58

(E) None of these

54. If Ramesh collected 54 coins, we can say (on the basis of information obtained so far) that the difference in numbers collected by the boy who collected the most and the boy who collected the second most should be at least.

(A) 30 (B) 18

(C) 26 (D) 12

(E) None of these

55. Ramesh collected 54 coins. If Kareem collected two more than double the number collected by Mohan, the number collected by Kareem was

(A) 10 (B) 30

(C) 22 (D) 26

(E) None of these

56. A number when divided successively by 7 and 3 leaves remainders 2 and 1, what is the remainder when the number is divided by 8.

(A) 2 (B) 4

(C) 1 (D) 3

(E) None of these

57. A number consists of 3 digits whose sum is 10. The middle digit is equal to the sum of the other 2 and the number will be increased by 99 if its digits are reverse. What is the number ?

(A) 253 (B) 374

(C) 1743 (D) 495

(E) None of these

58. An odd number of stones lies along a straight path, the distance between consecutive stones being 10m. The stones are to be collected at place where the middle stone lies. A man can carry only one stone at a time. He starts carrying the stones beginning from the extreme stone. If he covers a path of 3 km, how many stones are there ?

(A) 24 (B) 25

(C) 30 (D) 35

(E) None of these

59. The coefficient of x in the equation $x^2 + px + q = 0$ was x taken as 17 in places of 13 and its roots were found to be – 2 and – 15. The roots of the original equation are—

(A) – 2, 15 (B) 2, 15

(C) – 10, – 3 (D) 10, 3

(E) None of these

60. If the equation $x^2 + bx + c = 0$ and $x^2 + cx + b = 0$ $(b \neq c)$ have a common root, then

(A) $b + c + 1 = 0$ (B) $b + c - 1 = 0$

(C) $b + c = 0$ (D) $b = C$

(E) None of these

61. Let $f(x) = \dfrac{x + 1}{x - 3}, x \in$ R and $g(x) = \log_e (x - 2), x \in$ R be two real valued functions. Then dom $(f + g)$ is given by—

(A) $]2, \infty[$ (B) R – {3}

(C) $]2, 3[\cup]3, \infty[$ (D) $]2, 3[\cup]3, \infty[$

(E) None of these

62. Two numbers x and y are such that $x = y^2$. If Y is increased by 10%, then x is—

(A) Increased by 100% (B) Increased by 10%

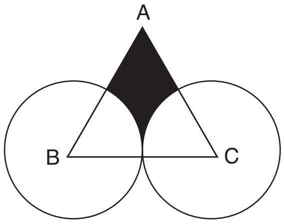
(C) Increased by 11% (D) Increased by 21%

(E) None of these

63. Ten different letters of alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is :

(A) 69760 (B) 30240
(C) 99748 (D) 62255
(E) None of these

64. Find the area of the shaded portion in the figure given below where ABC is an equilateral triangle of side 28 cm. and the radius of each circle is 14 cm.



(A) $339 \cdot 48$ cm^2 (B) $21 \cdot 48$ cm^2
(C) $229 \cdot 40$ cm^2 (D) $314 \cdot 80$ cm^2
(E) None of these

Directions : IOP (Institute of Perfection) Cricket club intends to give its membership to a select few players based on the following criteria :

The player must not be below 16 years and not more than 24 years of age as on 1.2.99., he must pay Rs. 15,000 as entrance fee and Rs 1,000 as monthly fee throughout his membership period. In case he pays Rs 25,000 as additional entrance to this, he should satisfy at least one of the following conditions:

(i) He has won any one inter-college cricket tournament by leading his college team and has scored at least one century in college level tournaments.

(ii) He has scored at least one century and two fifties in inner-university or inter-state tournaments.

(iii) He has led his cricket team at college level at least thrice and has taken 10 or more wickets either by bowling or while wicket-keeping or has made aggregate 1000 runs in college level matches.

(iv) He has represented his state in national level matches at least thrice with a remarkable bowling or batting or wicket keeping record.

(v) He has six centuries at his credit at college level matches and is a spin or medium fast bowler having taken at least one wicket per match in college level matches

Based on the above conditions and the data given in each of the following cases you have to take decision. You are not supposed to assume anything. All the facts are given as on 1-2-1999

65. Ishoo has represented her college as captain for 3 years and represented her university for two years. She has taken 15 wickets in seven matches as spin bowler. She has two centuries at her credit while playing for her college. Her team has won twice and thrice under her leadership in college-level and university-level matches respectively. Her date of birth is 2.1.1976. She is ready to pay Rs.40,000 as entrance fee.

(A) Membership be given—Satisfies only (i)
(B) Membership be given—Satisfies only (iii)
(C) Membership be given—Satisfies both (i) & (iii)
(D) Membership not to be given
(E) None of these

66. Anil has been playing for his college, university and state during his 7-year cricket tenure. He started playing for his college in January 1992 when he was 17 years old. He has 7 centuries and 5 fifties to his credit aggregating 1600 runs. He led his university and sate for two years and 3 years respectively. He has taken 11 wickets as medium fast bowler while playing for his state in national level matches. He is willing to pay requisite entrance fee and monthly fee.

(A) Membership to be given—satisfies only (ii) & (iv)
(B) Membership to be given—satisfies only (iv) & (v)
(C) Membership to be given—satisfies only (ii), (iv) & (v)
(D) Membership not to be given
(E) None of these

67. The ratio of the number of ladies to gents at a party was 1:2. However when 9 more ladies joined the party, the ratio became reversed. How many gents were there at the party ?

(A) 3 (B) 6
(C) 9 (D) 12
(E) None of these

68. If $a = \log_{12} 6$, $b = \log_{18} 12$, $c = \log_{24} 18$

Which of the following is true—

(A) $1 + a^2b^2 - 4c^2 - 2ab = 0$
(B) $a^2b^2c^2 = 1 + abc$
(C) $4b^2c^2 + 1 - a^2b^2c^2 - 4bc = 0$
(D) $a^2 + b^2 + c^2 = abc$
(E) None of these

69. A family consists of a father, mother, two sons and a daughter who is the youngest of all the siblings. The age of the father is four times the age of the second son. The ratio of the ages of the first son to that of his sister is 3 : 1. The mother is 3·5 times older as compared to the second son. The age of the second son is 2/3rd that of the first son. If the age of the daughter is 5 years, find the sum of the ages of all family members—

(A) 115 years (B) 105 years
(C) 205 years (D) 210 years
(E) None of these

70. The ratio of the number of boys and girls in a school is 2 : 1. Of the girls, 3/4$^{\text{th}}$ are day scholars and 2/3$^{\text{rd}}$ of them travel to school by bus. If 2/3$^{\text{rd}}$ of the boys

are day scholars and 3/4th of them travel to school by bus, what part of the student body travel to school by bus ?

(A) 2/3 (B) 1/2

(C) 1/3 (D) 1/4

(E) None of these

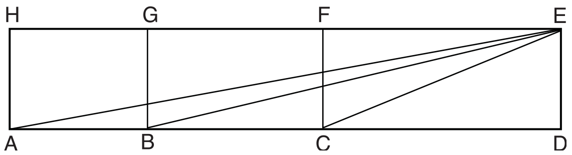
71. The probability that A speaks truth is $\frac{4}{5}$ while this probability for B is $\frac{3}{4}$· The probability that they contradict each other when asked to speak on a fact is—

(A) $\frac{3}{20}$ (B) $\frac{1}{5}$

(C) $\frac{7}{20}$ (D) $\frac{4}{5}$

(E) None of these

72. Three identical squares are placed, in such a manner so as to form a rectangle. Which of the following can be said about the CE, BE and AE ?



(A) They form an A.P.

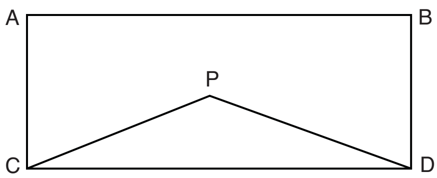
(B) They form a G.P.

(C) They form an increasing sequence

(D) There is no definite relationship

(E) None of these

73. Two pipes A and B fill the cistern in 4 and 8 min. respectively while an empty pipe C can empty the cistern in 5 min. All the three pipes are opened together and after 3 min. pipe C is closed. Find when the cistern is full ?



(A) 3·94 min. (B) 6·9 min.

(C) 7·0 min. (D) 4·3 min.

(E) None of these

74. If $f(x) = \frac{1}{x}$ $g(x) = \frac{1}{1-x}$ and $h(x) = x^2$, then find : fogoh (2)—

(A) – 1 (B) 1

(C) $\frac{1}{2}$ (D) – 3

(E) None of these

75. A sequence of odd numbers is formed as follows 1, 3, 3, 5, 5, 5, 5, 7, 7, 7, 7, 7, 7 ……………… what is the number in the 200th place ?

(A) 27 (B) 28

(C) 29 (D) 31

(E) None of these

Discussion–1

1. (B) Candidate F is not fulfilling criteria (i). Neither is he fulfilling criteria (P). He hasn't secured 70% marks in Group T.
2. (C) Candidate G hasn't secured 50 marks in paper II of group P. But he has secured more than 40% marks in group P and more than 70% marks in group T.
3. (C)
4. (D) Candidate I has failed in paper II of group R but he has secured more than 35% in each paper and overall 60% marks in his group.
5. (E)
6. (C) 4th symbol in Suresh's series: ∃; 4th Number in Ravi's series : 6
   4th letter in Asif's series : N; Hence the series is ∃ 6 N
7. (B) Ravi : Number → 4, Symbols → 4, letters → 5.
   Suresh : Number → 2, Symbols → 6, letters → 5.
   Asif : Number → 3, Symbols → 3, letters → 7.
8. (A) Required element are underline in every series.
   Ravi : 2 # S * 9 P T B π 8 Q Δ 6
   Suresh : ⊗ 0 1 ψ F @ V 4 ∃ & M T D ⇔
   Asif : G 3 H $ K N 1 5 R = 7 W Y
   Hence, number of these elements = (3 + 1 + 3) 7
9. (E) The required series will be 3 5 7 F V M T D & * ∃ Δ. Now , seventh to the right of the eleventh element from right = (11 – 7) 4th from right = &
10. (D)
11. (C) Required percentage growth = (68,718 – 42,137) $\times \frac{100}{42137}$ = 63% (approximate)

12. (C)

Nuts & Bolts	2005	2010	Percentage Growth
Car	42137	68718	66%
Bike	8820	20177	125%
Scooter	65303	82175	26%
Bus	25343	36697	36%

13. (B) Again referring to the above table, we can see that the % growth rate is maximum for Bikes Nuts and Bolts, *viz* 25%.
14. (D) It can be seen from the given table that though car Nuts and Bolts have shown a consistent growth, it has declined in 2008. On the other hand, Bike and Scooters Nuts and Bolts have shown a consistent increase except for 2007 when it had declined. But the Bus's Nuts and Bolts have shown a consistent growth over the period.
15. (B) Let the ratio of contents of two containers be x and y.

Then quantity of a liquid A in the mixture = $\frac{5}{6}x + \frac{1}{4}y$

And quantity of liquid B in the mixture = $\frac{1}{6}x + \frac{3}{4}y$

Given $$\frac{\frac{5}{6}x + \frac{1}{4}y}{\frac{1}{6}x + \frac{3}{4}y} = 1$$

$$\Rightarrow \quad \frac{5}{6}x + \frac{1}{4}y = \frac{1}{6}x + \frac{3}{4}y$$

$$\Rightarrow \quad \frac{4}{6}x = \frac{2}{4}y$$

$$\Rightarrow \quad \frac{x}{y} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$$

16.

Monday	—	Group Discussion
Tuesday	—	x
Wednesday	—	Quality circles
Thursday	—	Leadership
Friday	—	Decision Making
Saturday	—	Assessment Centre
Sunday	—	Motivation

Ans. (B)

17. (C)

Lights	1	2	3	4
Raman	ON	ON	ON	ON
Pawan	ON	ON	X	X
Ritesh	ON	X	X	X
Hetesh	X	ON	ON	ON

18. (D) There are two married couples, three females and three males. It means, there are two married males and females each and one unmarried male and female each. Now, S-III & IV make it obvious that A or E or C or F are not the least talkative. This leaves D or B. But S-V says that the least talkative person is married while S-VIII says that D is unmarried. It is therefore obvious that B is least talkative person. She is a married female. Now, D is a male and hence the other unmarried person must be a female and thus by virtue of IX, A is a female. A is the most talkative person and by XI, A is a married female. Now, E is a married male, So he must be married to either A or B (because there are only two married females). But X says that E is not married to A. Hence E must be married to B.

19. (A) 38, 27

20. (B)

Left Bank	One the river		Right Bank	
Dog; Grass	Man; goat ?	→	(i)	—
Dog; Grass	Man	←	(ii)	Goat
Grass	Man; Dog	→	(iii)	Goat
Grass	Man; Goat	←	(iv)	Dog
Goat	Man; Grass	→	(v)	Dog
Goat	Man;	←	(vi)	Dog; Grass
	Man; Goat	→	(vii)	All

So minimum no. of crossing are 7. (B)

21. (A) 22. (E) 23.(D) 24. (C) 25. (C)
26. (B) 27. (C) 28. (C) 29. (D) 30. (C)
31. (B) 32. (C) 33. (C) 34. (B) 35. (D)
36. (A) 37. (B) 38. (B) 39. (D) 40. (C)

41. (C) The third statements links with the subsequent line

42. (B) We should live in natural surroundings, according to the argument.

43. (B) The lie detector measures changes in these variables.

44. (D) Other means of family planning methods link with the opening sentence

45. (C) It may increase the risk of cancer.

46. (C) 47. (D) 48. (A) 49. (E) 50. (B)

51. (C) Since $(n + 7)$ is a factor $(n + 1)^2$.
From the options substituting $n = 29$,
We get $\frac{30^2}{36} = 25$.

52. (B) Substituting in the given choice b.
If $n = 1$ is the first row, $n\ (2n^2 + 1) = 1\ (2 + 1) = 3$
If $n = 2$ is the second row, $n\ (2n^2 + 1)\ 2\ (8 + 1) = 18$
If $n = 3$ is the third row, $n\ (2n^2 + 1) = 3\ (2\times 9 + 1)\ 57$
Hence by mathematical induction choice is (B).

53. (A) Minimum number of coins selected by three persons, after satisfying all the conditions, is 10 + 12 + 14 = 36. The total number of coins is 100. Hence the maximum number of coins collected by anyone of them cannot exceed 100 – 36 = 64……

54. (A) If A collected 54 coins, which is more than half of the total number of coins collected by all of them. Thus A collected maximum number of coins. The minimum number of coins that will be collected by two other persons will be 10 + 12 = 22. Thus the second highest number of coins that one can collect is 100 – 54 – 22 = 24. Then the difference between the one who collected maximum number of coins and the one who collected the second highest number of coins must be at least 54 – 24 = 30.

55. (C) 56. (C)

57. (A) Let the unit's digit, ten's digit and hundred's digit be x, y and z respectively.

$$\Rightarrow \quad x + y + z = 10$$

Also $$y = x + z$$

$$\Rightarrow \quad 2y = 10 \Rightarrow y = 5$$

and $100z + 10y + x = 100x + 10y + z - 99$

$$\Rightarrow \quad x - z = 1$$

$$\Rightarrow \quad x + z = 5$$

and $$x - z = 1$$

$$\Rightarrow \quad x = 3, z = 2$$

Hence, the number is 253.

58. (B) 59. (C)

60. (A) If α is the common root, then

$$\alpha^2 + b\alpha + c = 0$$

and $\alpha^2 + c\alpha + b = 0$

Subtracting $(b - c)\,\alpha + c - b = 0$

$\Rightarrow \quad \alpha = 1$

$\therefore \quad 1 + b.1 + c = 0$

$\Rightarrow \quad b + c + 1 = 0$

61. dom $(f+g)$ = dom. f $\cap$ dom g dom.

f = R- {3}

and dom. g = {x x - 2 > 0}

=]2, ∞[

dom. f $\cap$ dom. g =]2, 3[$\cap$]3, ∞ [.

62. (D) Let the value of y be = 1

After an increase of 10%, $y = 1.1$

Given that $x = y^2 = x = 1$

If $y = 1{\cdot}1$; the $x = (1{\cdot}1)^2 = 1{\cdot}21 => y$ is increased by 21%.

63. Required number of words

= Total number of words – Total number of words in which no letter is repeated

$= 10^5 - {}^{10}P_5 = 100000 - 30240 = 69760$

(As each place of word can be filled in 10 ways by any letter)

64. (B) Area of triangle ABC

$$= \frac{\sqrt{3}}{4} \times 28^2 = 339{\cdot}481 \text{ cm}^2$$

Area of half circle $= \frac{1}{2}\pi \times 14^2 = 308$

Area of shaded portion $= (339{\cdot}481 - 308)$

$= 31{\cdot}48 \text{ cm}^2$.

65. (D) 66. (D)

67. (B) Suppose originally x ladies were present then number of gents $= 2x$.

When 9 more ladies join, the ratio of the no. of ladies to gents is $(x + 9)/2x$

(*i.e.*) $\quad \frac{(x+9)}{2x} = \frac{2}{1}$ so $x = 3$.

(*i.e.*) Number of gents $= 2x = 2 \times 3 = 6$.

68. (C)

69. (B) Let Father's age $= x$

Mother's age $= 3{\cdot}5\,(x/4)$

Elder son $= 3x/8$

Younger son $= x/8$

Youngest daughter $= x/8$

Given that $x/8 = 5 \Rightarrow x = 40$

Father's age = 40;

Mother's age $= 3{\cdot}5\,(40/4) = 35$

Elder son's age $= (3 \times 40)\,/\,8 = 15$

Younger son's age = 10

Younger daughter's age = 5

Sum of the ages = 40 + 35 + 15 + 10 + 5

= 105 years.

70. (B)

Number	**Girls**	**Boys**
Day Scholars	3/4 of 9/3 of total	2/3 of 2/3 of total
No. of travelling by bus	2/3 of 3/4 of total	3/4 of 2/3 of 2/3 of total
	= 1/6 of total	= 1/3 of total

(*i.e.*) Total boys and girls who travel to school by bus = 1/6 + 1/3 or 1/2 the total.

71. (C) The probability of speaking truth of A, P (A) $= \frac{4}{5}$

The probability of not speaking truth of A,

$$P(\overline{A}) = 1 - \frac{4}{5} = \frac{1}{5}$$

The probability of speaking truth of B, P(B) $= \frac{3}{4}$

The probability of not speaking truth of B, P $(\overline{B}) = \frac{1}{4}$.

The probability of that they contradict each other.

$$= P(A).\,P(\overline{B}) + P(\overline{A}).\,P(B)$$

$$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{1}{5} + \frac{3}{20} = \frac{7}{20}$$

72. (C) CE $= \sqrt{a^2}$

BE $= \sqrt{4a^2 + a^2} = \sqrt{5a}$

AE $= \sqrt{9a^2 + a^2} = \sqrt{10a}$

$a\sqrt{2}, a\sqrt{5}, a\sqrt{10}$

$\Rightarrow$ There is no definite relationship among them.

73. Let the cistern be filed in time t. Pipe A and B are opened for t min. and pipe C is opened for 2 min.

$\Rightarrow \quad \frac{t}{4} + \frac{t}{8} - \frac{3}{5} = 1$

$\Rightarrow \quad 15t = 64$

$\Rightarrow \quad t = 4{\cdot}26$

74. (D) fogoh(2) is the same as $f[\{g\,(h(2)\}]$.

To solve this, open the innermost bracket first.

This means that we first resolve the function h (2).

Since h (2) = 4, we will get :

$f[g\,\{h(2)\}] = f\{g(4)\} = f(-1/3) = -3$.

75. (C)

We get the series 1 + 3 + 5 + 7 + ……… the series upto in terms to get $n^2 < 200$ hence x must lie between 14 and 15. Now last 27 must appear in 183th place, hence the next number is 29.

Number—1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29 ……

Place—1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28 ……

Cumulative—1, 3, 7, 13, 21, 31, 43, 57, 73, 91, 111, 133, 157, **183, 211** ……

Mock CAT–2

Direction (Q. 1 to 5) : The amount of money invested (rupees in crores) in the core infrastructure areas of two districts, Dehradoon and Haridwar of Uttranchal, is as follows :

Dehradoon			Haridwar		
Core Field	**NDA GOVT 2003**	**UPA GOVT 2004**	**Core Field**	**NDA GOVT 2003**	**UPA GOVT 2004**
Power	815·2	1054·2	Power	2065·8	2365·1
Hydro	389·5	476·7	Hydro	745·3	986·4
Road	632·4	565·9	Road	1232·7	1026·3
Solar	468·1	589·6	Solar	1363·5	1792·1
Technology	617·9	803·1	Technology	1674·3	2182·1
Total	**2923·1**	**3489·5**	**Total**	**7081·6**	**8352·0**

1. By what per cent was the total investment in the two districts more in UPA GOVT.2004 as compared NDA GOVT.2003 ?
   (A) 14% (B) 21%
   (C) 24% (D) 18%
   (E) None of these

2. The investment in Power and Road in NDA GOVT. 2003 in these two districts formed what per cent of the total investment made in that year ?
   (A) 41% (B) 47%
   (C) 52% (D) 55%
   (E) None of these

3. In Haridwar District, the investment in which field in UPA GOVT. 2004 showed the highest percentage increase over the investment in that field NDA GOVT. 2003 ?
   (A) Power (B) Hydro
   (C) Solar (D) Technical
   (E) None of these

4. Approximately how many times was the total investment in Dehradoon to the total investment in Haridwar ?
   (A) 2·8 (B) 2
   (C) 2·4 (D) 1·7
   (E) None of these

5. If the total investment in Haridwar shows the same rate of increase next GOVT time 2005, as it shown from NDA GOVT. 2003 to UPA GOVT. 2004, what approximately would be total investment in Haridwar in Next GOVT 2005 ?
   (A) Rs. 9,850 crore (B) Rs. 10,020 crore
   (C) Rs. 9,170 crore (D) Rs. 8,540 crore
   (E) None of these

Direction (Q. 6 to 10) : Hundai and Maruti can produce either Body Case or Carburetor. The time taken by Hundai and Maruti (in minutes) to produce one unit of Body Case and Carburetor are given in the table below :

(Each machine works 8 hour per day)

Product	Hundai	Maruti
Body Case	10	8
Carburetor	6	6

6. What is the maximum number of units that can be manufactured in one day ?
   (A) 140 (B) 160
   (C) 120 (D) 180
   (E) None of these

7. If Hundai works at half its normal efficiency, what is the maximum number of units produced, if at least one unit of each must be produced ?
   (A) 96 (B) 89
   (C) 100 (D) 119
   (E) None of these

8. What is the least number of machine-hours required to produce 30 pieces of Body Case and 25 Pieces of Carburetor respectively ?
   (A) 6 hrs 30 min (B) 7 hrs 24 min
   (C) 6 hrs 48 min (D) 4 hrs 6 min
   (E) None of these

9. If the number of units of Body Case to be three times that of Carburetor, what is the maximum idle time to maximize total units manufactured ?
   (A) 0 min (B) 24 min
   (C) 1 Hr. (D) 2 Hr.
   (E) None of these

10. If equal quantities of both are to be produced, then out of the four choices given below, the least efficient way would be :

(A) 48 of each with 3 min idle

(B) 64 of each with 12 min idle

(C) 53 of each with 10 min idle

(D) 71 of each with 9 min idle

(E) None of these

Direction : Study the following table carefully and answer questions given below it :

Number of Candidates Appeared and Qualified Under Various Disciplines in an Examination Over the Years

Disciplines →	Arts		Science		Commerce		Agriculture		Engineering		Total	
↓ Years	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App	Qual.
2004	850	200	1614	402	750	212	614	170	801	270	4629	1254
2005	1214	315	1826	420	800	220	580	150	934	350	5354	1455
2006	975	250	1970	500	860	260	624	160	742	300	5171	1470
2007	820	196	1560	450	842	300	490	160	850	312	4562	1418
2008	1412	378	2120	625	1105	320	760	200	642	301	6039	1824
2009	738	359	3506	880	1240	308	640	210	962	400	7086	2157

11. The number of candidates appeared under Agriculture in 2007 was approximately what per cent of the number of candidates qualified under Arts in 2006 ?

(A) 20 (B) 100

(C) 400 (D) 200

(E) 125

12. What was the per cent drop in the number of candidates qualified in science discipline from 2006 to 2007 ?

(A) 10 (B) 20

(C) 50 (D) 25

(E) 75

13. In which of the following disciplines, there was a continuous increase in the number of candidates appearing over the given years ?

(A) Arts (B) Commerce

(C) Agriculture (D) Engineering

(E) None of these

14. In which of the following years, the percentage of the qualified to the appeared ones in engineering discipline was the maximum ?

(A) 2004 (B) 2005

(C) 2007 (D) 2008

(E) 2006

15. In which of the following years, the percentage of total number of candidates of all the disciplines together, qualified to the appeared, was the maximum ?

(A) 2005 (B) 2006

(C) 2007 (D) 2008

(E) 2009

Directions : Study the following table carefully and answer the questions given below it :

Number of People Travelled to Five Destinations Over The Years (In Thousands)

Destination → ↓ Years	A	B	C	D	E
2004	20	24	20	18	21
2005	36	22	16	24	23
2006	18	16	12	22	16
2007	24	30	18	20	30
2008	28	32	26	19	34
2009	22	26	28	25	38

16. In which of the following years, the number of travellers for destination C was equal to the number of travellers for destination A in 2006 ?

(A) 2004 (B) 2005

(C) 2007 (D) 2008

(E) None of these

17. In case of which of the following destinations, there was a continuous increase in the number of travellers over the years ?

(A) A (B) B

(C) C (D) E

(E) None of these

18. In which of the following years, the number of travellers for E was equal to the total number of travellers for C in 2004 and 2007 together ?

(A) 2004 (B) 2009

(C) 2007 (D) 2008

(E) None of these

19. What was the percent increase in the number of travellers for destination D from 2004 to 2005 ?

(A) $66\frac{2}{3}$ (b) $6\frac{1}{3}$

(C) 33 (D) 50

(E) None of these

20. What was the difference in the number of travellers for destination D from 2005 to 2007 ?

(A) 4,000 (B) 400

(C) 6,000 (D) 8,000

(E) None of these

Direction (Q. 21 to 24) :

- To obtain a government post in the Republic of Malabar, you must either be a member of the ruling Independence Party or a Personal Associate of President Irfan.
- Party members seeking a government post must either give a substantial donation in gold bullion to the party's campaign fund or make a televised speech denouncing President Irfan's political enemies.
- Gold bullion may be purchased only at the National Bank, which does business only with those who have been certified as politically sound by the Minister of Justice.
- Only those who either have been certified as politically sound by the Minister of Justice or have donated 300 hours of service to the Independence Party, are allowed to make political speeches on television.
- To become a Personal Associate of President Irfan, you must either give a substantial donation in gold bullion to the President's Personal expense account or perform personal services for a member of his immediate family.
- Before appointing a personal associate to a government post, President Irfan always checks to make sure that he or she has been certified as politically sound by the Minister of Justice.

21. Mr. Jamil is a member of the Independence Party. To obtain a government post, his next step must be to either :

(A) Be certified as politically sound by the Minister of Justice, or give a substantial donation in gold bullion to the party's campaign fund.

(B) Donate 300 hours of service to the Independence Party, or give a substantial donation in gold bullion to the President's Personal expense account.

(C) Be certified as politically sound by the Minister of justice, or donate 300 hours of the service to the party.

(D) Perform personal services for a member of President Irfan's immediate family, or make a televised speech denouncing president's political enemies.

(E) Cannot be determined

22. All those who wish to obtain government posts must—

I. become personal associates of President Irfan

II. be certified as politically sound by the Minister of Justice

III. purchase gold bullion at the National bank

(A) I only (B) II only

(C) III only (D) I and II only

(E) Neither I, II, nor III

23. Mr. Razim has been certified as politically sound by the Minister of justice. He may obtain a government post immediately, only if he :

(A) has donated 300 hours of service to the Independence Party.

(B) is allowed to make political speeches on television.

(C) is a member of the Independence party.

(D) is a personal associate of the President Irfan.

(E) Cannot be determined

24. Because of a financial crisis, the National Bank is closed indefinitely. Those who wish to obtain government posts during this period must :

(A) Either perform some kind of services or make televised speeches denouncing President Irfan's, political enemies.

(B) Become members of the Independence Party.

(C) Donate 300 hours of service to the Independence Party.

(D) Become personal associates of President Irfan.

(E) Not defined

25. You are given 50 white marbles, 50 black marbles and two jars. You need to put 100 marbles in any of these two jars. The jars will then be shaken & you will be asked to pick one marble from either jar. How would you distribute the marbles in two jars to maximize the possibility of picking a white marble blind folded ?

(A) 25 white and 25 black in each.

(B) White in one and till 99 in the other.

(C) 50 white in one & 50 black in the other.

(D) All hundred in one.

(E) Cannot be determined

Passage–1

Management education gained new academic stature with US Universities and greater respect from outside during the 1960's. Some observer attribute the competitive superiority of US corporations, to the quality of business education. In 1978, a management professor, Herbert. A Simon of Carnegie Mellon University, won the Nobel Prize in economics for his work in decision theory. And the popularity of business education continued to grow. Since 1960, the number of master's degree awarded annually has grown from under 5000 to over 50,000 in the mid 1980's and the MBA has become known as 'the passport to the good life'.

By the 1980's however, US business school faced critics who charged that learning has little relevance to real business problems. Some went so far as to blame business schools for the decline in US competitive.

Amidst the criticism, four distinct arguments may be discerned. The first is that business schools must be either unnecessary or deleterious because Japan does do well without them. Underlying this argument is the idea that management ability cannot be taught, one is either born with it or must acquire it over year of practical experience. A second argument is that business schools are overly academic and theoretical. They teach quantitative models that have little application to real world problem. Third, they encourage undesirable attitude in students, such as placing value on the short term and 'bottom line' targets, while neglecting longer-term development criteria. In summary, some business executives complains that MBA's are incapable of handling day to day operational decisions, unable to communicate and to motivate people, and unwilling to accept responsibility for following through on implementation plans. We shall analyse this criticism after having reviewed experience in other countries.

In contrast to the expansion and development of business education in the United State and more recently in Europe, Japanese business schools graduate no more than hundred MBA's each year. The Keio business School (KBS) was the only graduate school of management in the entire country until the mid 1970's and it still boasts the only two year masters programme. The absence of business schools in Japan would appear in contradiction with the higher priority placed upon learning by its Confucian culture. Confucian colleges taught administrative skills as early as 1630 and Japan wholeheartedly accepted Western learning following the Meiji restoration of 1868 when hundreds of students were dispatched to universities in US, Germany, England and France to learn the secrets of Western technology and modernization. Moreover, the Japanese education system is highly developed and intensely competitive and can be credited for raising the literary and mathematical abilities of the Japanese to the highest level in the world.

Until recently, Japanese corporations have not been interested in using either local or foreign business schools for the development of their future executives. There in–company-training programmes have sought the socialization of newcomers—the younger the better. The training is highly specific and those who receive it have neither the capacity not the incentive to quit. The prevailing belief says Imai is the management should be born out the experience and many years of efforts and not learnt from educational institutions. A 1960 survey of Japanese senior executives confirmed that a majority (54%) believed that managerial capabilities could be attained only on the job and not in universities.

However, this view seems to be changing that same survey revealed that even as early as 1960, 37% of senior executives felt that the universities should teach integrated professional management. In the 1980's a combination of increased competitive pressure and great multi-nationalisation of Japanese business are making it difficult for many companies to rely solely upon internally trained manager. This has led to a rapid growth of local business programmes and grater use of American MBA progrmmes. In 1982-83, the Japanese comprised the largest single group of foreign student at Wharton, where they not only learnt the latest technique of financial analysis, but also developed world-wide contacts through their classmates and became Americanised, something highly useful in future negotiation. The Japanese, then do not 'so without' business school as it sometimes contended. But the process of selecting and orienting new graduates, even MBA's into corporations is radically different than in US rather than being placed in highly paying staff positions, new Japanese recruits are assigned responsibility for operational and even menial tasks. Success is based upon Japan's system of highly competitive recruitment and intensive in company management development, which in turn are grounded in its tradition of universal and rigorous academic education, life-long employment and strong group identification.

The harmony among these tradition elements has made Japanese industry highly productive and given corporate leadership a long-term view. It is true that this has been achieved without much attention to university business education, but extraordinary attention has been devoted to the development of managerial skills, both within the company and through participation in programmes sponsored by the Productivity Center and other similar organisations.

26. The 1960's and 1970's can best be described as a period :
    (A) When quality business education contribute to the superiority of US corporate.
    (B) When the number of MBA's rose from under 5,000 to over to 50,000
    (C) When management education gained new academic stature and greater respect.

(D) When the MBA became more disreputable

(E) None of the above

27. According to the passage :
    (A) Learning, which was useful in the 1960's and 1970's became irrelevant in the 1980's
    (B) Management education faced criticisms in the 1980's
    (C) Business schools are incentives to the needs of industry.
    (D) By the 1980's business schools contributed to the decline in US competitiveness.
    (E) Cannot be determined

28. The growth in popularity of business schools among students was most probably due to :
    (A) Herbet A. Simon, a management professor, winning the Nobel Prize in economics
    (B) The gain in academic stature
    (C) The large number of MBA degrees awarded
    (D) A perception that it was a passport to good life.
    (E) Not known

29. A criticism that management education did not face was that :
    (A) It imparted poor qualitative skills to MBA's
    (B) It was unnecessary and deleterious.
    (C) It was irrevocably irrelevant.
    (D) It inculcated undesirable attitude in students
    (E) None of these

30. What is the suitable title of this passage ?
    (A) Global Management Education
    (B) MBA–with or without Japan
    (C) USA and Japan–Comparative study of Management Education
    (D) Role of US universities in MBA
    (E) History of MBA Education

Passage–2

Government looking for easy popularity have frequently been tempted into announcing give-away of all sorts free electricity, virtually free water, subsidized food, cloth at half price and so on. The subsidy culture has gone to extremes. The richest farmers in the country get subsidised fertilizer. University education, typically accessed by the wealthier sections, is charged at a fraction of cost, Postal services are subsidised, and so are railway services. But fares cannot be raised to economical levels because there will be violent protest, so bus travel is subsidised too. In the past, price control on a variety of items, from steel to cement, meant that industrial consumer of these item got them at less than actual cost, while the losses of the public sector companies that produced them were borne by the taxpayer! A study done a few year ago came to the conclusion that subsidies in the Indian economy total as much as 14·5 per cent of gross domestic product. At today's level, that would work out to about Rs.150,000 crore. And who pays the bill? The theory and the political fiction on the basis of which it is sold to unsuspecting voters, is that subsidies go to the poor and are paid for by the rich. The fact is that most subsidies go to the 'rich'(defined in the Indian context as those who are above the poverty line), and much of the tab goes indirectly to the poor. Because the hefty subsidy bill result in fiscal deficits, which in turn push up rates of inflation which, as everyone knows, hits the poor the hardest of all. Indeed, that is why taxmen call inflation the most regressive form of taxation.

The entire subsidy system is built on the thesis that people cannot help themselves. Therefore, governments must do so. That people cannot afford to pay for variety of goods and services, and therefore the government must step in. This thesis has been applied not just in the poor countries but in the rich ones as well; hence the birth of the welfare state in the west, and an almost Utopian social security system, free medical care, food aid, old age security, etc . But with the passage of time, most of the wealthy nations have discovered that their economies cannot sustain this social safety net, which in fact reduces the desire among people to pay their own way, and takes away some of the incentive to work, in short, the bill was unaffordable, and their societies were simply not willing to pay. To the regret of many, but because of the laws of economics and harsh, most Western societies has been busy pruning the welfare bill. In India, the lessons of this experience over several decades, and in many countries do not seem to have been learnt or they are simply ignored in the pursuit of immediate votes. People who are promised cheap food or clothing do not in most cases look beyond the gift horses to the question of who picks up the tab. The uproar over higher petrol, diesel and cooking gas prices ignored this basic question; if the user of cooking gas does not want to pay for its cost, who should pay ? Diesel in the country is subsidised, and if the trucker or owner of diesel generator does not want to pay for its full cost, who does he or she think should pay the balance of the cost ? It is a simple question, nevertheless it remain unasked.

The DeveGowda government has shown some courage in bitting the bullet when it come to the price of petroleum products. But it has been bitten by much bigger subsidy bug. It want to offer at half its cost to every one below the poverty line, supposedly estimated at some 380 million people. What will be the cost ? And of course, who will pickup the tab ? The Andhra Pradesh Government has been bankrupted by selling rice as Rs. 2 per kg. should the Central Government be bankrupted too, before facing up to the question of what is affordable and what is not ? Already, India is perennially short of power because the subsidy on electricity has bankrupted most electricity boards, and made private investment

wary unless it gets all manner of state guarantees. Delhi's subsidised bus fares have bankrupted the Delhi Transport Corporation, whose buses have slowly dispersed from the capital's streets. It is easy to be soft and sentimental, by looking at programmes that will be popular. After all, who doesn't like a free lunch/ but the evidence is surely mounting that the lunch isn't free at all. Somebody is paying the bill. And if you want to know who, take a look at the country's poor economic performance over the years.

31. Which of the following should not be subsidised now, according to the passage ?
    (A) University education
    (B) Postal services
    (C) Steel
    (D) All of the above
    (E) None of these

32. The statement that subsidies are paid for by the rich and go the poor is :
    (A) Fiction
    (B) Fact
    (C) Fact, according to the author
    (D) Fiction, according to the author
    (E) Cannot be determined

33. Why do you think that the author call the Western social security system, Utopian ?
    (A) The countries' belief in the efficiency of the system was bound to turn out to be false
    (B) The system followed by these countries is the best available in the present context
    (C) Every thing under this system was supposed to be free, people were charging money for them
    (D) The theory of system followed by these countries was devised by Dr.Utopia
    (E) It is short form of 'you to have pain'

34. It can be inferred from the passage that the author :
    (A) Believes that people can help themselves and do not need the government
    (B) Believe that the theory of helping with subsidy is destructive
    (C) Believe in democracy and free speech
    (D) Is not successful politician
    (E) It is not successful man

35. Which of the following is not a victim of extreme subsidies ?
    (A) The poor
    (B) The Delhi Transport Corporation.
    (C) The Andhra Pradesh Government
    (D) All of these
    (E) None of these

Passage–3

The membrane-bound nucleus is the most prominent feature of the eukaryotic cell. Schleiden and Schwann, when setting forth the cell doctrine in the 1830s, considered that it had a central role in growth and development. Their belief has been fully supported even though they had only vague notions as to what that role might be, and how the role was to be expressed in some cellular action. The membraneless nuclear area of the prokaryotic cell, with its tangle of fine threads, is now known to play a similar role.

Some cells, like tire sieve tubes of vascular plants and the red blood cells of mammals, do not possess nuclei during the greater part of their existence, although they had nuclei when in a less differentiated state. Such cell can no longer divide and their life span is limited. Other cells are regularly multinucleate. Some, like the cells of striated muscles of the latex vessels of higher plants, become so through cell fusion. Some like the unicellular protozoan paramecium, are normally binucleate, one of the nuclei serving as a source of hereditary information for the next generation, the other governing the day-to-day metabolic activities of the cell. Still other organisms, such as some fungi, are multinucleate because cross walls, dividing the mycelium into specific cells, are absent or irregularly present. The uninucleate situation, however, is typical for the vast minority of cells, and it 'would appear that this is the most efficient and most economical manner of partitioning living substance into manageable units. This point of view is given credence not only by the prevalence of uninucleate cells but because for each kind of cell there is a ratio maintained between the volume of the nucleus and that for the cytoplasm. If we think of the nucleus as the control centre of the cell, this would suggest that for a given kind of cell performing a given kind of work, one nucleus can take care of a specific volume of cytoplasm and keep it in functioning order. In items of material and energy, this must mean providing the kind of information needed to keep flow of materials and energy moving at the correct rate and in the proper channels. With the multitude of enzymes in the cell, material and energy can of course be channeled in a multitude of ways; it is the function of some information molecules to make channels of use more preferred than other at any given time. How this regulatory control is exercised, is not entirely clear.

The nucleus is generally a rounded body. In plant cells, however where the centre of the cell is often occupied by a large vacuole, the nucleus may be pushed against the cell wall, causing it to assume a lens shape. In some white blood cells, such as polymorphonucleated leukocytes, and in cells of the spinning gland of some insects and spider, the nucleus is very much lobed. The reason for this is not clear, but it may relate to the fact that for given volume of nucleus, a lobate form provides much greater surface area for nucleur-cytoplasmic

exchange, possibly affecting both the rate and the amount of metabolic reactions. The nuclear, whatever its shape, is segregated from the cytoplasm by a double membrane, the nuclear envelope, with the two membranes separated from each other by a perinuclear space of varying width. The envelope is absent only during the time of cell division, and then just for a brief peroid. The outer membrane is often continuous with the membranes of the endoplasmic reticulum, a possible retention of an earlier relationship, since the envelope, at least in part, is formed at the end cell division by coalescing fragments of the endoplasmic reticulum.The cytoplasmic side of the nucleus is frequently coated with ribosomes, another fact that stresses the similarity and relation of the nuclear envelope to the endoplasmic reticulum. The inner membrane seems to possess a crystalline layer where it abuts the necleoplasm, but its function remains to be determined.

Everything that passes between the cytoplasm and the nucleus in the eukaryotic cell must transverse the nuclear envelope. This includes some fairly large molecules as well as bodies such as ribosome, which measure about 25 mm in diameter. Some passageway is, therefore, obviously necessary since there is no indication of dissolution of the nuclear envelope in order to make such movement possible. The nuclear pores appear to be reasonable candidates for such passage ways. In plant cells these are irregularly, rather sparsely distributed over the surface of the nucleus, but in the amphibian oocyte, for example, the pores are numerous, regularly arranged, and octagonal and are formed by the fusion of the outer and inner membrane.

36. Which of the following kinds of cells never have a nuclei ?
    (A) Sieve Tubes.
    (B) Red bloods cells of mammals.
    (C) Prokaryotic Cells.
    (D) Both (A) and (B).
    (E) None of these.

37. According to the first paragraph, the contention of Schleiden and Schwann that the nucleus is the most important part of the cell has :
    (A) Been proved to be true.
    (B) Has been true so far but false in the case of the prokaryotic cell.
    (C) Is only partially true.
    (D) Has been proved to be completely false.
    (E) None of these.

38. What is definitely a function of the nuclei of the normally binucleate cell ?
    (A) To arrange for the growth and nourishment of the cell.
    (B) To hold hereditary information for the next generation.
    (C) To make up the basic physical structure of the organism
    (D) To fight the various foreign diseases attacking the body.
    (E) None of these.

39. The function of the crystalline layer of the inner membrane of the nucleus is :
    (A) Generation of nourishment of the cell.
    (B) Holding together the disparate structures of the endoplasmic reticulum.
    (C) Helping in transversal of the nuclear envelope.
    (D) Cannot be determined from the passage.
    (E) None of these

40. Why according to the passage, is the polymorphonucleated leukocyte probably lobed ?
    (A) Because it is quite convoluted in its functions.
    (B) Because it is the red blood cell which is the most important cell in the body.
    (C) Because it provides greater area for metabolism reactions.
    (D) Because it provides greater strength to the spider web due to greater area.
    (E) None of these.

Direction (Q. 41 to 44) : Each question is followed by two statements—A and B. Answer each question using the following instructions :

Choose (a), if the question can be answered by using statement A only.

Choose (b), if the question can be answered by using statement B only.

Choose (c), if the question can be answered by using either of the statements alone.

Choose (d), if the question can be answered only by using both the statements together.

Choose (e), if the question cannot be answered.

41. A biased coin is tossed and it shows head on 1st turn. The probability of getting a head on the 3rd turn, when it is head on 2nd turn and when it is tail on 2nd are 0·4 and 0·75 respectively. The probability of getting a head on the 4th turn, when it was head and head on the 2nd and 3rd turn and head and tail on the 2nd and 3rd turn is 0·6 and 0·2 respectively. Whereas, the probability of getting a head on 4th turn when it was tail and head on the 2nd and 3rd turn and tail and tail on the 2nd and 3rd turn is 0·7 and 0·55 respectively. Is the probability of getting a head is greater than 0·50 on the 4th turn ?
    (A) It was head on turn 2
    (B) It was tail on turn 2

42. In a college of 500 students, 150 drink milk, 200 drink tea, 150 drink coffee. What is the ratio of number of students drinking milk and tea both to those drinking only coffee ?

(A) No person drinks both milk and coffee

(B) 10% drink both milk and tea

43. What is Sonali's share in profit at the end of one financial year, from a business which she has done along with her friends Annu and Aisha giving an annual return of 10%. Profits are shared in proportion to the investment amount and duration for which the amount was invested by each of them

(A) Aisha invested \$ 8000 in the beginning of the year and she exited from the business in the first week of October. Aisha's profit was 3/2 times that of Sonali's profit and her investment, was four times that of Annu

(B) Annu and Sonali each invested for one year in the ratio of 1 : 2

44. Justice department had rejected a company N'ron's application for protection from its creditors under chapter II and ordered the lenders to recover their dues by liquidating assets of the company. What is the value' of assets at the present value ?

(A) The company had a liability of \$ 56 billion

(B) The creditors received 8 per cent per dollar

45. Read, Study, Analyize the set of informations given below very carefully—

(1) There is a group of seven persons in a Royal Family, Ayol, Byol, Cyol, Dyol, Eyol, Fyol and Gyol. They all appeared in an IOP I.Q. test to test their intelligence.

(2) There are two married couples in the family and three females in total.

(3) Gyol, a female, is the most intelligent.

(4) Byol, the father of Eyol, is more intelligent than his son.

(5) Cyol has one son and one daughter. She is more intelligent than her husband.

(6) The father of Byol is more intelligent than Byol himself.

(7) Eyol, the grandson of Fyol, is the least intelligent. Fyol, the grandfather is the second most intelligent in the family.

(8) The mother of Byol is less intelligent than B.

(9) None among the married topped the I.Q. test.

(10) The grandmother of Gyol has two sons, one of whom is Dyol, who is more intelligent than his brother but less intelligent than his wife.

(11) Nobody is a widow or a windower in the family.

Now reply these simple questions to test your intelligence.

How is Cyol related with Ayol ?

(A) Daughter-in-law (B) Mother-in-law

(C) Sister-in-law (D) Cannot be determined

(E) None of these

Directions (Q. 46 to 50) : Below is given a passage followed by several possible inferences which can be drawn from the facts stated in the passage. You have to examine each inference separately in the context of passage and decide upon its degree of truth or falsity.

Mark Answer (a): If the inference is 'definitely true', *i.e.* it properly follows from the statement of facts given.

Mark Answer (b): If the inference is 'probably true', though not 'definitely true' in the light of the facts given.

Mark Answer (c) : If the 'data are inadequate' *i.e.* from the facts given you cannot say whether the inference is likely to be true or false.

Mark Answer (d) : If the inference is 'probably false', though not 'definitely false', in the light of given facts.

Mark Answer (e) : If the inference is 'definitely false', *i.e.* it cannot possibly be drawn from the facts given or it contradicts the given facts.

India needs higher investment in the port sector and still lags far behind the international ports in container traffic, though there has been considerable growth in this segment over the past few years. There is a need to continually benchmark Indian ports against the best ports worldwide and continue to engage in policy efforts so as to attain prices per container of port services which are the lowest in the world. The average turn-around time for ship docking at most of the ports in India has been falling for the past three years. However, pre-berthing time has been marginally increasing over the years

46. Indian ports have not been graded so far on the international standard.

47. Indian ports offer the lowest price per container as service charges.

48. Indian ports need considerable development to match with the best ports in the world.

49. The best port in the world is located in United States of America.

50. Pre-berthing time is gradually decreasing over the years.

51. If a, b and c are distinct positive numbers such that $b + c - a, c + a - b$ and $a + b - c$ are positive, the expression $(b + c - a)(c + a - b)(a + b - c) - abc$ is—

(A) Positive (B) Negative

(C) Non-positive (D) None of these

(E) Cannot be determine

52. Find the sum to infinity of the serie :

$$1 + 2 \times \frac{1}{3} + 3 \times \frac{1}{3^2} + \ldots\ldots + n \times \frac{1}{3^{n-1}} —$$

(A) 2/3 (B) 3/2
(C) 4/9 (D) 9/4
(E) 2/3

53. An 8" Pizza sells for Rs. 20 at my favorite pizza store. The store claims they have a great deal on the large 12" Pizza, which is specially priced at Rs. 38·25. Given that the Pizzas are priced according to their areas, find the discount percent the store is offering on the large 12" Pizza—

(A) 15% (B) 20%
(C) 25% (D) 28%
(E) 35%

54. If a, b, c are distinct +ve numbers different from 1 such that :
$(\log_b a . \log_c a - 1) + (\log_a b . \log_c b - 1) + (\log_a c . \log_b c - 1) = 0$ then ab = ?

(A) C-1 (B) C2
(C) C (D) $\sqrt{c}$
(E) 1

55. Let a, b, c be real numbers, $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies—

(A) $\alpha < \gamma < \beta$ (B) $\gamma = \alpha$
(C) $\gamma = \alpha + \frac{1}{2}\beta$ (D) $\gamma = \frac{1}{2}(\alpha + \beta)$
(E) None of these

56. The number of factors of 8100 expressed as a product of two different factors is—

(A) 44 (B) 11
(C) 40 (D) 22
(E) 33

57. The value of $\left(\frac{101}{100}\right)^{100}$ is nearest to—

(A) 3 (B) 30
(C) 300 (D) 3000
(E) None of these

58. Two roads connect the towns A and B. The first road is 10 km. longer than the second. A car travels along the first road and covers the distance between the towns in $3\frac{1}{2}$ hrs. Another car travels along the second road, and covers the distance in $2\frac{1}{2}$ hrs. What is the speed of each car, if it is known that the speed of the first car is 20 kmph. less than that of the second ?

(A) 30 kmph., 50 kmph.
(B) 40 kmph., 60 kmph.
(C) 60 kmph., 80 kmph.
(D) 80 kmph., 100 kmph.
(E) None of these

59. If $x = 7 + 4\sqrt{3}$ and $xy = 1$, then value of $\frac{1}{x^2} + \frac{1}{y^2}$ is—

(A) 64 (B) 124
(C) 194 (D) 214
(E) None of these

60. Let $x, y \in$ N and $7x + 12y = 220$. The number of solution is—

(A) 2 (B) 1
(C) 3 (D) Infinitely many
(E) None of these

61. The solution of the equation :

$$|x| - 2\,|x + 1| + 3\,|x + 2| = 0 \text{ has—}$$

(A) Only one solution
(B) Two solutions
(C) Infinite number of solutions
(D) No solution
(E) Cannot be determine

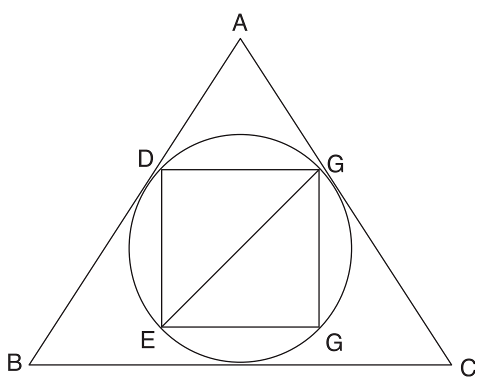
62. The number of position integer value pairs $(x. y)$ satisfying $4x - 17y = 1$ and $x < 100$ is—

(A) 59 (B) 57
(C) 55 (D) 58
(E) None of these

63. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then—

(A) $f(x + 2) = f(x - 2)$ (B) $f(2 + x) = f(2 - x)$
(C) $f(x) = f(-x)$ (D) $f(x) = -f(-x)$
(E) None of these

64. In the above figure, DEFG is a square incribed in a circle, which is inscribed in an equilateral triangle ABC of perimeter 12 m. Find the area of shaded region ?



(A) $\frac{8}{\sqrt{6}} m^2$ (B) $\frac{4}{\sqrt{6}} m^2$

(C) $\frac{8}{\sqrt{3}} m^2$ (D) $\frac{4}{3} m^2$

(E) None of these

Directions (Q. 65 to 66) : IOP (Institute of Perfection) Cricket club intends to give its membership to a select few players based on the following criteria :

The player must be before 16 years and not more than 24 years of age as on 1-2-1999. He must pay Rs. 15,000 as entrance fee and Rs. 1,000 as monthly fee throughout his membership period. In case he pay Rs. 25,000 as additional entrance to this, he should satisfy at least one of the following conditions :

(i) He has won any one inter-college cricket tournament by leading his college team and has scored at least one century in college level tournaments.

(ii) He has scored atleast one century and two fifties in inner-university or inter-state tournaments.

(iii) He has led his cricket team at college level at least thrice and has taken 10 or more wickets either by bowling or while wicket-keeping or has made aggregate 1000 runs in college level matches.

(iv) He has represented his state in national level matches atleast thrice with a remarkable bowling or batting or wicket keeping record.

(v) He has six centuries at his credit in college level matches and is spin or medium fast bowler having taken atleast one wicket per match in college level matches

Based on the above conditions and the data given in each of the following cases, you have to take decision. You are not supposed to assume anything. All the facts are given as on 1-2-1999

65. Divyen is an exceptionally brilliant wicket keeper. He has to his credit 22 stumpings and 20 catches while representing his state in national level matches for consecutive 4 years. His first century was leading his college team to win in February 1991 at the age of 16 years. He will be able to pay Rs.15000 and monthly dues. Before playing for the state, he played many matches for his college.

(A) Membership not to be given

(B) Membership to be given—satisfies only (iv)

(C) Membership to be given—satisfies only (ii) & (iv)

(D) Membership to be given—satisfies only (i) & (iv)

(E) None of these

66. Sarvesh has a record of having taken two or more wickets at every match played by him for his college. He is a medium fast bowler. He is a good batsman also and has scored three centuries and four fifties while playing inter-university matches. His college has always won the match under his captaincy during the last 4 years. He has amassed 1200 runs in these matches. He is ready to pay Rs. 40000 at entry level. His date of birth is 30-01-1976—

(A) Membership to be given—satisfies only (i) only

(B) Membership to be given—satisfies only (i) & (ii) only

(C) Membership to be given—satisfies only (i) & (v) only

(D) Membership to be given—satisfies only (i), (ii) & (iii) only

(E) None of these

67. If the ratio of sum of the first three terms of a G.P. to that of the first six terms of a G.P. is 125 : 152, then find the common ratio of the progression—

(A) 3/2 (B) 4/5

(C) 2/5 (D) 3/5

(E) 1/5

68. $\log_3 (1 + 1/4) + \log_3 (1 + 1/5) + \log_3 (1 + 1/6) + \dots\dots\dots + \log_3 (1 + 1/2915) = ?$

(A) 0 (B) 4

(C) 8 (D) 6

(E) 1

Direction (Q. 69 to 70) : A company imports component A from Germany and component B from USA. It then assembles them along with other components to produce a machine used in a chemical process. Component A contributes 30% to the production cost and component B contributes 50% to the production cost. The current practice is to sell the machine at a price that is 20% over the production cost. Due to foreign exchange fluctuations, the German Mark has become costlier by 30% and the US Dollar by 22%. But the company is unable to increase the selling price by more than 10%.

69. The current margin of profit is—

(A) 10% (B) 15%

(C) 12% (D) 8%

(E) 5%

70. Suppose the US Dollar becomes cheaper by 12% of its original value, and the German Mark becomes costlier by 20% of its original value. To achieve a profit margin of 10%, the selling price must exceed the production cost by—

(A) 10% (B) 20%

(C) 12% (D) 8%

(E) 5%

71. There are 10 lamps in an auditorium. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is—

(A) 102 (B) 210

(C) 10! (D) 1023

(E) None of these

72. Each of the angles of a triangle when expressed in degrees is a perfect square. Find the smallest of the three angles—

(A) 25^0 (B) 16^0

(C) 36^0 (D) 45^0

(E) None of these

73. An author gets a royalty of 13% on the printed price of books written by him. If the printed price of a copy of his particular book is Rs. 4·50 and if 600 copies of the books are sold, how much royalty did he earned ?

(A) Rs. 2700 (B) Rs. 2349

(C) Rs. 351 (D) Rs. 212

(E) None of these

74. If $|r - 6| = 11$ and $|2q - 12| = 8$, what is the minimum possible value of q/r ?

(A) – 2/5 (B) 2/17

(C) 10/17 (D) 7/8

(E) None of these

75. The letters of the word LUCKNOW are arranged among themselves. Find the probability of always having NOW in the word.

(A) $\frac{5}{42}$ (B) $\frac{1}{6}$

(C) $\frac{1}{42}$ (D) $\frac{11}{42}$

(E) None of these

Discussion–1

1. (D) Total investment in the two districts in 2003 = 2932·1 + 7081·6 = 10,000

Total investment in the two districts in 2004 = 2932·1 + 7081·6 = 10,000

Total investment in the two districts in 2004 = 3489·5 + 8352 = 11840

$$\text{Required \%} = \frac{(11840 - 10000)}{10000} = 18\%$$

2. (B) Total investment in Power and Road in both the districts in 2003 = (815·2 + 632·4 + 2065·8 + 1232·7) = 4746·1. Total investment made in that year = 2923·1 + 7081·6 = 1004·7 = 10000. Hence, required percentage is $\frac{4746 \cdot 1}{10000} = 47\%$.

3. (B) Percentage increase in investment in Power = $\frac{300}{2070} = 14\%$. Percentage increase in investment in Hydro = $\frac{(986 \cdot 4 - 745 - 3)}{745 \cdot 31} \times 100 \approx \frac{240}{745} \approx 32\%$.

4. (C) Total investment in Dehradoon = 2923·1 + 3489·5 = 6412·6 = 6410.

Total investment in Hardwar = 7081·6 + 8352 = 15430.

$$\text{Required ratio} = \left[\frac{15{,}430}{6{,}410}\right] = 2 \cdot 4 \text{ times}$$

5. (A) Percentage increase in the total investment in Hardwar in

$$\text{UPA GOVT. 2004} = \left[\frac{(8{,}352 - 7{,}081 \cdot 6)}{7{,}081 \cdot 6}\right] \times 100$$

$$= \approx \frac{1270}{7080} \approx 18\%$$

Total investment in Hardwar in 2005 will be 1·18 × 8352 = 9855·36 = 9850

6. (B) Since time taken to manufacture Carburetor on both the factory is the least, we have to manufacture only carburetor in order to maximize the output for the day. In such a case total number of units of carburetor produced on Hundai = $\frac{(8 \times 60)}{6}$ = 80 units and that produced on Maruti = $\frac{(8 \times 60)}{6}$ = 80 Units.

So, the maximum number of units that can be produced = (80 + 80) = 160 units.

7. (D) If Hundai works at half of its normal efficiency, time taken by Hundai to manufacture 1 unit of Body Case = 20 min and Carburetor = 12 min. And now for maximum number of units, we have to produce Carburetor on Maruti first as it takes only 6 min per piece. Also since atleast one unit of Body case has to be manufactured and it is more efficient to do so on Maruti, we would do that. So time taken to manufacture 1 unit of Body case on Maruti = 8 min. Hence, the remaning on Maruti = (480 – 8) = 472. In this remaining time number of units of Carburetor that can be manufactured = [472/6] = 78. Now since it takes less time to manufacture Carburetor on Hundai as well, we will maximize Carburetor on Hundai. Since number of units that can be produced = (8 × 60)/12 = 40. Hence, total number of units manufactured = (1 + 78 + 40) = 119 units.

8. (A). In order to take minimum time manufacture Body case on Maruti and Carburetor on Hundai. Number of machine hours required to manufacture 30 units of Bodycase on Maruti = (30 × 8) = 240 min = 4 Hrs. Number of Factory hours required to manufacture 25 units of Carburetor on Hundai. = (25 × 6) = 150 min = 2·5 Hrs. So total time taken = (4 + 2·5) = 6·5 hrs.

9. (A) 10. (C)

11. (D) Suppose 490 = X% of 250

$\Rightarrow \quad X\% = \frac{490}{250} \times 100 = 200\%$ appox.

12. (A) $\frac{50}{500} x\ 100$

13. (E)

14. (D) Percentage of the qualified to the appeared in Engineering discipline in 2004 = $\frac{270}{801} \times 10 = 33{\cdot}71$.

In 2005 = 37·43, In 2006 = 40·43, In 2007 = 36·71, In 2008 = 46·88 and In 2009 = 41·58.

15. (C) Percentage of the qualified to the appeared in all the disciplines together in 2004

$$= \frac{1254}{4629} \times 100 = 27{\cdot}09.$$

In 2005 = 27·18, In 2006 = 28·43, In 2007 = 31·08, In 2008 = 30·20 and In 2009 = 30·44

16. (C) 18

17. (D)

18. (B) 38

19. (C) $\frac{6}{18} \times 100 = 33\frac{1}{3}$

20. (A) 24000 – 20000 = 4000

21. (C) 22. (E) 23. (D) 24. (A)

25. (B) Probability of getting white is 1/2 in each of the given choices except (b), where the probability is 1/2 × 1 + 1/2 × 49199, which is greater than 1/2.

26. (C) Refer paragraph 1

27. (B) Refer paragraphs 2 & 3

28. (B) Refer paragraphs 6 evidences.

29. (C) Refer paragraph 6 & 7

30. (C) 31. (D) 32. (D) 33. (A) 34. (B) 35. (E)

36. (E) 37. (A) 38. (B) 39. (D) 40. (C)

41. (C) **Head**

Head 0·75 0·25 Tail

Head 0·4 0·6 Tail Head 0·75 0·25 Tail

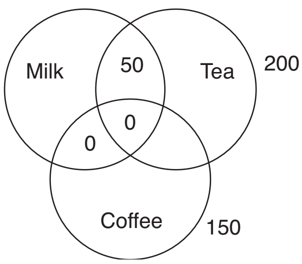
Head Tail Head Tail Head Tail Head Tail

0·6 0·4 0·20 0·8 0·7 0·3 0·55 0·45

(A) If it is head on turn 2 the probability of getting head on turn 4 is $(0{\cdot}4 \times 0{\cdot}6 + 0{\cdot}6 \times 0{\cdot}2) = 0{\cdot}36$

(B) If it is tail then the probability of getting head on turn 4 is $(0{\cdot}75 \times 0{\cdot}7 + 0{\cdot}25 \times 0{\cdot}55) = 0{\cdot}6625$

42. (E)



Using statements (A) and (B) alone nothing can be, said. Even after combining the two statements, the ratio cannot be determined.

43. (D) Statements (A) provides investment amount put in by three business partners while statement (B) provides duration of each investment period. So, the question can be answered by using both the statements together.

44. (D) From A Liability = $ 56 billion

From B Creditors received 8% of total liability. Combining A and B present value = 8% of $ 56 billion.

45. (I) We note that there are three females. Now, Cyol is a female, Gyol is a female while Byol, Fyol, Eyol and Dyol all are males. It implies that the third female is Ayol.

(II) Now, Since Gyol is the most intelligent and since none of the married topped the I.Q. test, it implies that Gyol is unmarried. It means that both Ayol and Cyol are married because there are two married couples.

(III) The grandmother of Gyol has two sons. So the grandmother can not be Cyol because Cyol has one son and one daughter too. It implies that Ayol is the grandmother.

(IV) Since Fyol is a grandfather and since Ayol is a grandmother and since none in the family is a widow or a windower. Ayol and Fyol must be a couple. On the same logic, Byol and Cyol must be a couple.

(V) Fyol (M) ⇔ Ayol (F)

Dyol (M) Byol (M) ⇔ Cyol (F)

Eyol (M) Gyol (F)

Their order of Intelligence is

Gyol > Fyol > Cyol > Dyol > Byol > Ayol > Eyol

(A) daughter in law (B) Cyol

46. (E) The passage clearly implies that the Indian ports lag far behind the international ports. This comparison is not possible without grading Indian port on international standards.

47. (E) It is clear from the sentence, "There is a …… lowest is the world" that Indian ports don't offer the lowest price per container as service charges.

48. (A) It is clear from the first sentence of the passage, "India needs …… past few years."

49. (C) No information regarding any specific country having best port in the world has been given in the passage.

50. (E) It is clear from the last sentence of the passage, "However, ……… over the years".

51. (B) Substitute suitable values and get the result.

52. (D)

53. (A) Price of pizza $= r^2$

$$20 = k(4)^2$$
$$x = k_1(6)^2$$
$$x = (36 \times 20)/16 = \text{Rs. } 45$$

Discount given = Rs. (45 - 38·25) = Rs. 6·75

Discount percentage $= \left(\dfrac{6{\cdot}75}{45}\right) \times 100 = 15\%$

54. (A)

55. (A) Here $a^2\alpha^2 + b\alpha + c = 0$ ...(i)

and $a^2\beta^2 - b\beta - c = 0$ ...(ii)

Now, Let $f(x) = a^2x^2 + 2bx + 2c$

$$f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c$$
$$= -a^2\alpha^2 < 0$$

and $f(\beta) = a^2\beta^2 + 2b\beta + 2c$

$$= 3a^2\beta^2 > 0,$$

$f(\alpha)$ and $f(\beta)$ are of opposite sign and $\alpha < \beta$.

There exists γ between a and β such that

$$f(\gamma) = 0$$

Here $\alpha < \gamma < \beta$

Where γ is a root of $a^2x^2 + 2bx + 2c = 0$.

56. (D) $8100 = 3^4 \times 5^2 \times 2^2$

Required number of ways

$$= 1/2\{(4 + 1)(2 + 1)(2 + 1) - 1\} = 22.$$

57. (A)

58. (C) Distance from road 1 $= x + 10$

Distance from road 2 $= x$

Time taken to travel by first road $= 3\frac{1}{2}$ hrs.

Time taken to travel by second road $= 2\frac{1}{2}$ hrs.

Speed of the first car traveling by road 1

$= (y - 20)$ kmph.

Speed of the second car traveling by road 2

$= y$ kmph.

$$s = d/t$$
$$y = 2{\cdot}5x$$
$$y - 20 = \frac{x + 10}{3{\cdot}5} \quad \ldots(1)$$

or $y = \dfrac{x}{2{\cdot}5}$ ...(2)

$\Rightarrow$ $x = 2{\cdot}5y$

From (1) and (2)

$$3{\cdot}5y - 70 = 2{\cdot}5y + 10;$$
$$y = 80, 60$$

59. (C)

60. Here x, y are positive integers and $7x + 12y = 220$

$$x + y + \frac{5y}{7} = 31 + \frac{3}{7}$$

i.e. $(x + y - 31) = \dfrac{3 - 5y}{7}$ must be an integer.

Thus $\dfrac{15y - 9}{7}$ is an integer,

i.e. $(2y - 1) + \dfrac{y - 2}{7}$ is an integer an so $\dfrac{y - 2}{7}$ is an integer,

Let $\dfrac{y - 2}{7} = p$ so $y = 7p + 2$, then $x = 28 - 12p$, were p is any integer so that x and y turns to be positive integers. Thus :

p	0	1	2
$x = 28 - 12p$	28	16	4
$y = yp + 2$	2	9	16

Thus $(x, y) = (28, 2), (16, 9)$ and $(4, 16)$, *i.e.* 3 solutions in all.

61. (A) Here change points are – 2, – 1 and 0.

(i) $x < -2$, equation becomes

$-x + 2x + 2 - 3x - 6 = 0$

$\Rightarrow x = -2$ but it is out of domain, so not valid.

(ii) $-2 \le x < -1$, then $3x + 6 + 2x + 2 - x = 0$

$\Rightarrow x = -2$ which is in domain, so, – 2 is a solution.

(iii) $-1 \le x < 0$, we have

$3x + 6 - 2x - 2 - x = 0 \Rightarrow 4 = 0$ which is absurd.

(iv) $x \ge 0$, then $x - 2x - 2 + 3x + 6 = 0$

$\Rightarrow x = -2$ which is out of domain, so not valid. Thus $x = -2$ is the only solution.

62. (A) $4x - 17y = 1$ or $4x = 17y + 1$. Now get some value of (x, y) satisfying the equation. We get (13, 3), (30, 7) (47, 1a) and so on. The values of x are in AP with $a = 13$ and $d = 17$. Use formula of n terms to get number of terms upto 100.

63. Since graph is symmetrical about the line $x = 2$

$\Rightarrow f(2 + x) f(2 - x)$

64. (D) Radius of inscribed circle

$$= \frac{a}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$\Rightarrow$ Diameter $= \dfrac{4}{\sqrt{3}}$

Side of a square $= a\sqrt{2} = \dfrac{4}{\sqrt{3}}$

$\Rightarrow$ $a = \dfrac{2\sqrt{2}}{\sqrt{3}}$

Area of shaded portion $= 1/2 \times b \times h$

$$= \frac{1}{2} \times \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{2\sqrt{2}}{\sqrt{3}} = \frac{4}{3} \text{ m}^2$$

65. (B) 22 stumping and 20 catches, *i.e.,* 42 wickets clearly indicates that he has played a least 3 matches at national level. Hence he fulfils criteria (iv). Winning a match doesn't mean winning a tournament. Hence (i) is not satisfying by him.

66. (D) 'His college has always won the match under his captaincy during the last 4 years, clearly means that his college team has won a tournament under his captaincy.

67. (D) It is given that

$$\frac{S_6}{S_3} = \frac{152}{125}$$

$$\Rightarrow \quad [a(r^6 - 1)/(r - 1)]/[a(r^3 - 1)/(r - 1) = \frac{152}{125}$$

$$\Rightarrow \quad r = 3/5$$

68. (D) $\log_3 \left(\frac{5}{4} \times \frac{6}{5} \times \frac{7}{6} \times \frac{8}{7} \times \ldots\ldots \times \frac{2916}{2915}\right)$

$$\log_3 \frac{2916}{4} = \log_3 729 = 6.$$

69. (A) The component A, from Germany, forms 30% of the total production cost. The price of German Mark is risen by 30%, which in turn results into the rise of (30) × 30/ 100 = 9% increase in the total production cost. Also component B forms 50% of the total production cost : The price of USA dollar is risen by 22%, which in turn results into the rise of (50)22/ 100 = 11 % increase in the total production cost. Thus the total rise in the production cost is 9 + 11 = 20%. But the selling price is already kept 20% higher than the production cost. Thus after the rise in the prices of international currencies by the 10%, the maxi-mum current gain possible is 10%.

70. (A) If the Dollar becomes cheap by 12% over its original cost, the total production cost reduces by (50) 12/100 = 6%. While due to the rise in the cost of German Mark by 20%, the production cost increases by (30)20/100 = 6%. Thus effectively the cost price is not altered. Also the selling price, which is 20% higher than the cost price, is not altered. Thus the gain will be 20%.

71. Each bulb has two choices, either switched on or off

Required number = 210 – 1 = 1023

(Since, in one way when all switched are off, the hall will not be illuminated)

72. (B) The angles are $(16^0, 64^0, 100^0)$

73. (C) Royalty earned by the author

$$= 13\% \text{ of Rs. } 4{\cdot}50 \times 600$$

$$= \frac{13}{100} \times 4{\cdot}50 \times 600$$

$$= \text{Rs. } 351.$$

74. (B) The values of q are 10 and 2, and of r = 17 or – 5. Try different substitutions of different values. The minimum value of q/r will be 10/(– 5) = – 2

75. (C) The required probability will be given by the equation

= number of words having NOW/Total number of words

= 5!/7! = 1/(7 × 5) = 1/42

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